Preparation for Examination

The examination is done in writing, where your ability to calculate some problems, is examined.

Some theoretical questions are given, where your understanding of theory is checked.

We recommend the preparation for examination in the following way: Calculate the examples, given in the Collection of Exercises, and test yourself by comparing your solutions with those given in the Collection. The three problems at the test in writing are very similar to those, but a little bit shorter. For theory examination only those questions, which you find hereafter included, will be asked. Thus the content for learning is exactly defined.

Please note, that the text book contains much more information about motors, than you will need for the examination. We advice you to prepare the collection of questions, when you are learning for the exam, and try to find the answers from the text book. If you have difficulties in answering the questions, do not hesitate to consult us.

If you have any further questions concerning the examination, theory or methods of calculation, do not hesitate to contact us.

Yours sincerely

A. Binder Darmstadt, 17.02.2012

Theory Questions for the Examination

Chapter 2. Rotating Fields in Electrical Machines

2.1 What is a rotating field and how is it generated? Explain the movement of the air gap field of a three-phase winding graphically!

2.2 How are three-phase windings normally designed? Explain similarities and differences between single and two-layer windings.

2.3 Describe the squirrel cage as a special case of a poly-phase winding.

2.4 Explain the difference between concentric coils and coils with identical coil span. What is the effect of these coil forms on the design of the winding overhang (sketch)? Which of these coil forms are preferred for wire-wound coil windings/shaped-wire windings, low voltage/high voltage windings?

2.5 What is the magnetic voltage (m.m.f.)? How is the distribution of the magnetic voltage inside the air gap of a machine determined? Which simplifications does this derivation base on?

2.6 Explain the mathematical principle of the generation of a rotating field (fundamental !) as the superposition of standing pulsating fields (corresponding to the excitation of single phases) with the cancellation of certain positive and negative phase-sequence fields!

2.7 What is the synchronous angular velocity and corresponding circumference speed?
Give numerical values for

a) a small standard induction machine, shaft height 132 mm, four poles, 7.5 kW, 50 Hz, stator bore diameter 130 mm,

b) a large synchronous hydro-generator, 38 MVA, 64 poles, 50 Hz, 6.8 m stator bore diameter (bulb-type generator, *Danube* river power station, *Greifenstein*, *Austria*)

2.8 Give the synchronous speed and the circumference velocity of a hydrogen-cooled turbo generator: 50 Hz, 880 MVA, stator bore diameter 1.35 m, two poles. Why are turbo generators the mechanically highest stressed type of electrical machines! How is the rotor therefore constructed?

Chapter 3. Mathematical Analysis of Air Gap Fields

3.1 Explain the distribution, pitching and winding factor using the *FOURIER* analysis. Explain the term “slot harmonics”.

3.2 Why does a 6-phase winding approach the desired sinusoidal field distribution better than a three-phase winding (note: regard the general expression to calculate the ordinal numbers of harmonics)? How do the time phasors of the corresponding phase currents look like?

3.3 Symmetrical rotating field windings excite both a fundamental air gap field and harmonic fields. Which ordinal numbers occur when using a three-phase winding? Discuss the terms positive and negative phase-sequence system. How do the amplitudes of the air gap fields decrease with increasing ordinal numbers?

3.4 Explain the influence of short-pitching and the number of slots per pole on the amplitude of the harmonic fields!

3.5 Why do normally only odd ordinal numbers of harmonics occur that are not divisible by three?

Chapter 4. Voltage Induction in Rotary Machines

4.1 The air gap field of a synchronous generator is not ideally sinusoidal, even though the shape of the pole shoes has been optimised. Nevertheless, the induced line-to-line voltage is almost ideally sinusoidal. Why?
Why does the corresponding phase voltage in general contain a higher number of harmonics than the line-to-line voltage?

4.2 The voltage induction can be experienced as two different effects, depending on the location of the observer, namely stationary induction and induction by movement. Explain these terms using the example of a simple conductor loop in a magnetic field. What is the meaning of the terms “transformer” and “rotationally” voltage induction, which are used with rotating electrical machines?

4.3 A sinusoidal field wave rotating with constant speed induces a sinusoidal voltage in a coil located in the stator slots of a machine. Which kind of induction (stationary or motion induction) does an observer see, who travels with the rotating field? Give the formula!

4.4 Why is the total field inductance of a three-phase winding higher than the “fundamental wave” inductance? What is the meaning of “harmonic leakage”?

4.5 How is the fundamental magnetising inductance calculated? Explain the calculation in the following steps: A three-phase winding is supplied with a symmetrical three-phase current system I and therefore excites a rotating field with the air gap field fundamental B_δ (saturation of iron neglected). This induces a voltage U in each phase (three-phase voltage system), thus yielding a reactance $X = U/I$ per phase, which leads to a rotating field inductance per phase.

4.6 Explain the effect of the short-pitching factor of a chorded (short-pitched) winding on the voltage induction of harmonics in comparison to a full-pitch coil.

4.7 Explain the effect of the distribution factor of a coil group made of two unchorded coils from the phasor diagram of the induced voltages of the two coils.

4.8 Explain the mutual inductance between two phases of two three-phase windings, if one phase is located in the stator whereas the other one is in the rotor.

4.9 Explain the operation principle of a rotary transformer (variable transformer)? What are the main advantages and disadvantages in comparison to standard transformers in three-limb core design?

Chapter 5. The Slipring Induction Machine

5.1 Explain the operation principle of a slipring induction machine in general. How can the $M(n)$-characteristic be explained physically ?

5.2 Some major assumptions have been made when deriving the T-equivalent circuit of the induction machine. Which?

5.3 Give the stator and rotor voltage equations per phase of the induction machine. Draw the T-equivalent circuit. Which parameters occur in the equivalent circuit of the induction machine?

5.4 What is the advantage of transforming the rotor quantities current, voltage, flux linkage, resistance and inductance to the stator side using a transformation ratio *ü*? In general, can *ü* be chosen arbitrarily? If so, show why! Which transformation ratio *ü* is commonly used with induction machines?

5.5 Why do we have to distinguish two different transformation ratios in case of squirrel cage machines? Which transformation ratios do we need?

5.6 What is the slip? Equation? What is its physical meaning? Which operating points correspond to slip –1,0,1,2 ?

5.7 How big is the slip of an induction machine running at no-load, at rated load (typical values) and at stand-still? How big are the rotor frequencies in these cases assuming stator frequencies of a) 50 Hz and b) 60 Hz?

5.8 When deriving the equivalent circuit of the induction machine from the rotating field theory (air gap field!), the leakage inductances of the winding leakage fluxes had to be added afterwards. Which leakage fluxes are these? Sketch the distribution of these leakage fluxes in the machine.

5.9 What is the magnetising current of an induction machine? In which case does this current actually flow in the phase windings of the machine?

5.10 What is the internal (or “electric”) torque of an induction machine? How does it depend on the “air gap power”? Is it bigger or smaller than the shaft torque in motor and generator operation?

5.11 Which losses occur in induction machines? Determine an equation for the efficiency for motor and generator operation. Which of the different loss groups are already included in the equivalent circuit of the induction machine?

5.12 Is it true, that the breakdown slip of induction machines is the same for motor and generator operation when the stator resistance is considered ($R_s \neq 0$)? If so, is this also true for the breakdown torques? If not, why? What happens when the stator resistance is neglected ($R_s = 0$)?

5.13 The equivalent circuit of induction machines has been derived for the fundamentals of the stator and rotor air gap fields. Which part of the leakage flux is due to the effect of harmonic air gap fields to the equivalent circuit? Discuss the *BLONDEL* leakage coefficient σ and explain how to determine this coefficient by measurement!

5.14 Draw a qualitative voltage and current phasor diagram of an induction machine operating at a constant stator voltage at rated load! Use the consumer reference-arrow system!

5.15 Draw qualitatively a current locus diagram of an induction machine with consideration of the stator resistance (*OSSANNA* circle) and label special points ($s = 0$, $s = s_b$) as well as $s = 1$ and $s = \infty$! Add the torque and the power line. How can these lines be used to determine the internal torque, the mechanical output power and the resistive losses in both stator and rotor winding?

5.16 How can we label the locus diagram with the slip using a slip-line? Which operating points do we need to know? Give a qualitative sketch!

5.17 How big is typically the no-load current and the starting current of induction machines operated at the grid, with respect to the rated current? How big is typically the breakdown and the starting torque of a deep-bar rotor induction machine, with respect to rated torque?

5.18 Give the stator and rotor voltage equations of an induction machine according to the equivalent circuit! Determine the dependence of the rotor current on the stator current. Discuss this relation for $s = 0$, $s = 1$ and $s = \infty$, concerning current amplitude and phase shift with respect to the stator current!

5.19 Sketch the locus diagram neglecting the stator resistance (*HEYLAND*-circle). How can torque and power line be used to determine the mechanical output power, the inner torque, the rotor losses and the stator current ?

5.20 Use a sketch of the locus diagram to determine the stator current of an induction machine (motor/generator), the corresponding $\cos\varphi$ and the mechanical power in a slip range -1 ... 1 ! What happens to the current, if the voltage drops to 50 % of the prior value? How does the torque depend on the voltage?

5.21 What is *KLOSS´* relation (formula)? On which simplifying assumption is it based? How does the corresponding torque-speed diagram look like? Sketch!

5.22 The following example shows, that many important values for correct operation can be found on the nameplate of an induction machine: Induction machine, 400V/690V D/Y, 950A D, 560 kW, 1492kW, 50Hz, $\cos\varphi = 0.88$. Use these values to determine the efficiency, the shaft torque, the total losses, the number of poles, and the rated slip of the machine!

5.23 Sketch the energy flow of an induction machine for motor and generator operation! Discuss the losses!

5.24 Explain the possibilities to change the operational behaviour of a slipring induction machine using external rotor resistances. Why does it still make sense to use a slipring induction motor as a variable speed pump drive, when considering the additional losses in the rotor resistors?

Chapter 6. The Squirrel Cage Induction Machine

6.1 What is current displacement? Where and why does it occur in squirrel cage induction machines? How is current displacement used to improve the operational behaviour of squirrel cage induction machines?

6.2 Discuss the dependence of the bar resistance and the slot leakage inductance on the rotor frequency! Sketch the corresponding curves! By how much does the rotor bar resistance of a deep copper bar rotor increase at 50 Hz and 75 °C (bar width = slot width) at a bar height of 5 cm?

6.3 What is the influence of current displacement on the *M(n)*-characteristic of an induction machine? (Sketch). Draw rotor bar shapes that lead to a a) weak and b) strong influence of current displacement on the resistance and the slot leakage inductance and explain the results!

6.4 Is it true, that under identical motor geometry a copper bar in a rectangular slot has a stronger current displacement than an aluminium bar? Remark: electric conductivity: copper (20°C): $57 \cdot 10^6$ S/m , aluminium $34 \cdot 10^6$ S/m.

6.5 Which problems occur during line-starting of induction machines? How can this problem be handled with a slipring induction machine?

6.6 Why is the breakdown torque of a slipring or round bar cage rotor induction machine higher than the breakdown torque of deep-bar of double-bar cage rotors with similar stator parameters?

Chapter 7. Induction Machine Based Drive Systems

7.1 Explain the "quasi-static" stability of operating points of induction machines using the steady-state $M_e(n)$ motor characteristic together with typical load characteristics $M_s(n)$.

7.2 How big is the energy that is dissipated in the rotor cage of an induction machine that drives a centrifuge, if the centrifuge is accelerated from 0 to 3000 /min at no load? The polar momentum of inertia of the centrifuge and the rotor is 15 kgm^2.

7.3 Sketch the *M(n)*-characteristics of typical load machines ! Give and discuss applications!

7.4 Describe the operating principle of a pole-changing winding according to *DAHLANDER* ! Which are the advantages and disadvantages when compared to motors with two independent windings for different speeds? Which are the main applications of drives with two distinctive speeds? Which alternative is provided by inverter-fed variable speed drives?

7.5 Rotor-fed induction machines are sometimes used in wind generators, as variable frequency rotor supply allows for limited variable speed generator operation. Explain the operating principle of rotor supply with a variable frequency voltage! How can we obtain sub- or supersynchronous operation at constant current? Why is this option (bi-directional converter supplies the rotor circuit) cheaper than supplying the stator with variable frequency (which are technical limits)?

7.6 Describe the operation of induction machines with stator voltages with variable frequency f and variable amplitude U (inverter supply)! How are U and f changed to obtain a constant breakdown torque? Neglect the stator resistance! Sketch the *M(n)*-characteristics for different values for U and f !

7.7 Explain "field weakening" for inverter-fed induction machines. How do U and f need to be changed in order to achieve field weakening? Why do we often make use of field weakening to obtain a wide speed range with constant power?

7.8 How has the voltage of inverter-fed induction machines to be changed in order to obtain a constant flux at variable frequency and thus variable speed: a) with $R_s = 0$, b) with $R_s \neq 0$?
In which range of speed and frequency is the stator resistor R_s of dominating influence? Why?

7.9 What is the advantage of inverter supply of induction machines (voltage source inverter) with pulse width modulated voltage when compared with block voltage supply? What is the effect of this method with a sufficiently high pulse frequency on the shape of the stator current of the induction machines?

7.10 Explain the principle of Y/Δ-starting! Which advantages do we get and what are the disadvantages?

7.11 What is "four-quadrant operation"? Give the sign of speed and torque in all four quadrants. How do we achieve this operation with a voltage source inverter and an induction machine?
Which are the prerequisites concerning the inverter, if we want to feed power back into the grid (regenerative braking)?

7.12 Describe the terms
a) counter-current braking (braking by reversal) and
b) supersynchronous braking
by the torque-speed characteristic of induction machines.

7.13 Is it possible to operate an induction machine on a grid with $\cos\varphi = 1$? Explain your answer! With which additional devices would this operation be possible?

Chapter 8. The Synchronous Machine

8.1 Explain the operating principle of a synchronous machine! Give the basic equation of the torque-load angle characteristic at fixed voltage and frequency.

8.2 Explain the difference between a round-rotor and a salient-pole synchronous machine
a) in terms of construction (sketch !) and
b) in terms of torque generation ($M(\vartheta)$-characteristic).

8.3 Draw the voltage and current phasor diagram of an overexcited round-rotor synchronous machine in generator operation in the consumer reference-arrow system!

8.4 What is the effect of salient rotor poles on the magnetising inductance of the stator winding? Explain the terms d- and q-axis as well as L_d and L_q !

8.5 Draw the voltage and current phasor diagram of an overexcited salient-pole synchronous machine in generator operation in the consumer reference-arrow system!

8.6 What is the definition of the load angle ϑ in synchronous machines? Why does it not make sense to define a load angle with induction machines?

8.7 Draw qualitatively the voltage phasor diagram of the overexcited round-rotor synchronous machine in motor and generator operation (consumer reference-arrow system) and label the load angle. Determine from the sign of the load angle and from the round-rotor machine power equation ($R_s = 0$) the direction of energy flow.

8.8 Explain graphically for the example of a round-rotor machine voltage phasor diagram (consumer reference-arrow system, $R_s = 0$) the terms "overexcited", "underexcited", "motor" and "generator" operation (4 different cases) !

8.9 Why is a capacitor bank connected in parallel to an induction machine generator operating in an isolated grid, whereas no capacitor bank is required with isolated synchronous generators?

8.10 Explain phase shifter operation of a synchronous machine using the phasor diagram (consumer reference-arrow system, $R_s = 0$)! What are phase shifters used for? How big is the shaft power?

8.11 An unexcited synchronous machine is running at a grid without any mechanical load (no power consumption or generation, machine is assumed to have no losses). Draw the phasor diagram. Who is providing the necessary magnetisation of the machine?

8.12 Draw the phasor diagram of a permanently short-circuited synchronous machine running at rated speed, if the rotor is excited with rated current I_{fN}

a) for $R_s = 0$,
b) for $R_s \neq 0$.

When is it allowed to neglect th e stator resistance R_s and still get an accurate phasor diagram, at low or at high speed? Why?

8.13 How does the torque of a grid-connected round-rotor synchronous machine change with increasing mechanical load in motor and generator operation (formula)? Why does it show a pull-out torque?

8.14 Discuss the quasi-static stability of the grid-connected round-rotor synchronous machine using the steady-state $M(\vartheta)$ characteristic!

8.15 Why is the synchronous machine in stable operation conditions often compared with a non-linear torsion spring? How do we determine the equivalent torsion spring constant c_ϑ ? Is this "spring" stiffer at small or at big load and at small or at high field current I_f? Does it make sense to compare a synchronous generator in isolated operation with a torsional spring?

8.16 Why has the load angle of a synchronous machine, operated at a fixed voltage, a tendency to oscillate? How does the load torque effect the eigenfrequency of the synchronous machine? How big is this eigenfrequency roughly at no load and at rated torque?

8.17 Why are big industrial sites with many inductive consumers (e.g. many induction machines) often equipped with synchronous machines to compensate the reactive load? What would be alternative countermeasures?

8.18 Explain the operation of a reluctance machine as a special case of a unexcited grid-connected salient-pole synchronous machine (motor/generator operation, pull-out torque)!

8.19 Sketch the $M(\vartheta)$-characteristics of the synchronous reluctance machine for generator and motor operation!

8.20 Due to technical improvements, many older power stations generators are nowadays refurbished to increase the efficiency. Discuss the effect of an efficiency increase of 0.3% of a 750 MW turbo generator! By which value is the on-site heat dissipation per year (in kWh) reduced?

8.21 Sketch the $M(\vartheta)$-characteristics of a salient-pole synchronous machine for motor/generator operation! Which load angle range is stable (pull-out load angle)? What does this characteristic look like, if the excitation is turned off? Which range is now stable (pull-out load angle)?

8.22 Draw the characteristic $U_s(I_s)$ of a synchronous generator in isolated operation at a constant excitation (hence, without voltage control) and resistive load, as well at pure capacitive and inductive load!

8.23 The synchronous generated voltage U_p can be included in the equivalent circuit of the synchronous machine as an internal voltage source. In which state of operation can this voltage directly be measured directly at the machine terminals?

8.24 Use the phasor diagram of a round-rotor synchronous machine ($R_s = 0$) in isolated generator operation with constant excitation to explain why pure capacitive load will lead to terminal voltages higher than at no-load!
Discuss the related phenomenon of self-excitation of idle, non-excited synchronous generators feeding long, open ended lines!

9. Electrically and Permanent Magnet Excited Synchronous Machines

9.1 Sketch the no-load and short-circuit characteristics of a synchronous machine? How are they determined by measurement?

9.2 How is the saturated synchronous reactance determined considering saturation due to no-load excitation I_{f0}?
Which voltage can be measured at the open terminals of a rotating synchronous machine with no-load excitation and rated speed?

9.3 What is the "no-load – short-circuit" ratio? What is the definition of the synchronous reactance? Explain its value for the saturated and the unsaturated synchronous machine (sketch) !

9.4 Why is the open-circuit characteristic of synchronous machines saturated, whereas the short-circuit characteristic is not saturated?

9.5 Is a large synchronous reactance at grid operation an indication for high overload capability (large stability margin), if the excitation and therefore the synchronous generated voltage is kept constant? Explain your answer using the $M(\vartheta)$-curve of a round-rotor synchronous machine ($R_s = 0$) !

9.6 Which excitation techniques are common with synchronous machines? Draw simple circuit sketches! Discuss advantages and disadvantages of the various possibilities!

9.7 What is high-speed excitation and what is it needed for? What is the ceiling voltage? What is the aim of high-speed de-excitation? How is it realised?

9.8 What is the purpose of the damper cage of a synchronous machine? Does a synchronous machine always need a damper cage, both at grid and isolated operation?

9.9 How does the eigenfrequency of a grid-connected oscillating synchronous machine change with/without a damper cage? How big is the eigenfrequency roughly?

9.10 Name permanent magnet materials and discuss their properties !

9.11 How is the magnetic operating point of a permanent magnet machine determined?

9.12 Describe the operating principle of a synchronous machine with rotor position control!

9.13 Why do you get maximum torque from synchronous machines when using I_q-current ? Give a phasor diagram!

Chapter 10. Dynamic Performance of Synchronous Machines

10.1 For rapidly changing currents (loads) at constant speed the synchronous machine is described as 3 winding transformer in the *d*-axis and as a 2 winding transformer in the *q*-axis, additionally to the rotary voltage induction.

a) Why can this transformer effect be omitted during steady-state synchronous operation, where currents are changing sinusoidally with synchronous frequency?
b) Which windings are “involved” in the 3-, which are involved in the 2-winding transformer?
c) Draw equivalent circuits for these transformer configurations!

10.2 Explain the terms “transient” and “subtransient” reactance using the *d*-axis transformer equivalent representation of the synchronous machine? Give typical per unit values!

10.3 Why does the *q*-axis only have a subtransient, but no transient reactance? Determine the subtransient reactance of the *q*-axis from the 2-winding equivalent transformer!

10.4 Name typical orders of magnitude of the different reactances of synchronous machines in p.u.-values! Which reactances are the smallest and why?

10.5 Give an approximate formula for the calculation of the magnitude of the sudden short-circuit current of synchronous machines with damper cage! Is the sudden short-circuit current larger than the steady-state short circuit current? If so, why and by typically which value?

10.6 When does the sudden short-circuit need to happen - with respect to the terminal voltage - to cause the maximum current amplitude? When does it need to happen to cause the minimum current amplitude? Why is it inevitable in case of three-phase machines that at least one phase will suffer from (nearly) maximum short circuit current amplitude?

Chapter 11. DC Machines

11.1 Explain the operating principle of a dc machine! Which machine components are needed?

11.2 How does the mechanical rectification of the induced armature ac voltage work in dc machines? How do we achieve a rectified armature voltage with almost no residual ripple? How big is the frequency of this ripple voltage? What are physical reasons for this (small) ripple in the armature voltage?

11.3 How big is the frequency of the ac magnetisation of the rotating iron core? What is the effect of this ac magnetisation? Why has the rotor iron core to be laminated?

11.4 Name the basic formulas of the dc machine: induced voltage, electromagnetic torque, terminal voltage for motor and generator operation!

11.5 Sketch the distribution of the air gap field of an uncompensated dc machine without inter-pole windings for $I_f > 0$ at a) no load ($I_a = 0$) and b) at load ($I_a > 0$) over two pole pitches. Label the flux loss due to saturation in case b) caused by the armature reaction.

11.6 Sketch the (saturated) no-load characteristic $U_0(I_f)$ of a dc generator at constant speed $n = \text{const.}$ How can it be measured? Where is the magnetising field saturation mainly located in the iron core of dc machines?

11.7 What is “commutation” of dc machines? What are problems caused by commutation for the operation of dc machines? Explain the term “reactance voltage”.

11.8 Describe the operation principle of inter-pole windings. Can we achieve almost sparkless commutation for any arbitrary combination of n and I_a, hence for any speed and load?

11.9 Which are typical power limits for dc machines? Discuss these limits compared with squirrel-cage induction machines!

11.10 Describe the operating principle of the simplex armature lap and wave winding using coil sketches including the connection to the commutator! What are the electric differences between lap and wave windings? What are the consequences for technical applications?

11.11 How does the compensation winding of dc machines work (Sketch)? What are “first kind compensators”? What are they needed for? (Sketch)

11.12 Sketch the four basic connection options of dc machines! What are the field regulating resistor (field rheostat) and the starting resistor needed for?

11.13 Sketch the $U(I_a)$ characteristic of a separately excited dc generator with and without compensation winding! How can it be measured?

11.14 How does self-excitation with shunt-wound dc generators work?

11.15 Sketch the $n(M)$-characteristic

a) of a separately excited, compensated motor and

b) of a series-wound motor.

Discuss at these two characteristics: motor reaction to unloading and the speed drop with increasing load!

11.16 Describe the quasi-static stability of a shunt-wound dc machine with the influence of armature reaction. What is the mathematical requirement for stability?

11.17 What is a *WARD-LEONARD*-converter ? What was/is it used for?

11.18 How can we achieve variable speed with dc machines by power electronics? Give examples by sketching the circuits!

11.19 Sketch the one-quadrant family of characteristics of a converter-fed, separately excited, compensated dc machine including field weakening operation.

11.20 At high speed, the reactance voltage needs to be limited to approximately which value, in order to limit spark generation? How do the corresponding characteristics look like: $M(n)$, $I_a(n)$, $U(n)$, $\Phi(n)$, considering this effect?

11.21 How big is typically the voltage drop across the brushes of dc machines? How does voltage vs. current density in the brushes look like?

11.22 Explain “over-“ and “under-commutation” of dc machines? Why can over-commutation cause instability of a separately excited motor?

11.23 Sketch the field lines of the inter-pole field of a dc machine schematically (two pole equivalent machine)! How does the ratio of number of turns per pole of the inter-pole winding $N_{W,pole}$ and the armature winding $N_{a,pole} = z/(8ap)$ of an uncompensated dc machine need to be chosen to obtain a positive inter-pole field in the inter-pole air gap?

11.24 Name typical limit values

a) for the reactance voltage for steady-state operation and intermittent operation (e.g. rolling mill applications),

b) the medium and the maximum permissible voltage between two commutator segments and

c) the steady-state current densities of the bush contact faces.

Exercise 1.1: Three-phase winding of an induction motor

Sketch the m.m.f.-curve for the given winding!

Winding data:

Winding type: double layer, chorded

$2 \cdot p = 3$ (number of pole pairs)

$Q_s = 36$ (number of stator slots)

$W/\tau_p = 5/6$ (short pitching)

1 turn per coil

The instantaneous values of the currents are (in arbitrary units):

$I_U = +1$ unit

$I_V = -1$ unit

$I_W = 0$.

Solution:

$I_U = -I_V = I$, $I_W = 0$ (I : 1 unit measured in Ampere)

$q = Q/(2 \cdot p \cdot m) = 36/(6 \cdot 3) = 2$

$W/\tau_p = 5/6$ (short pitching)

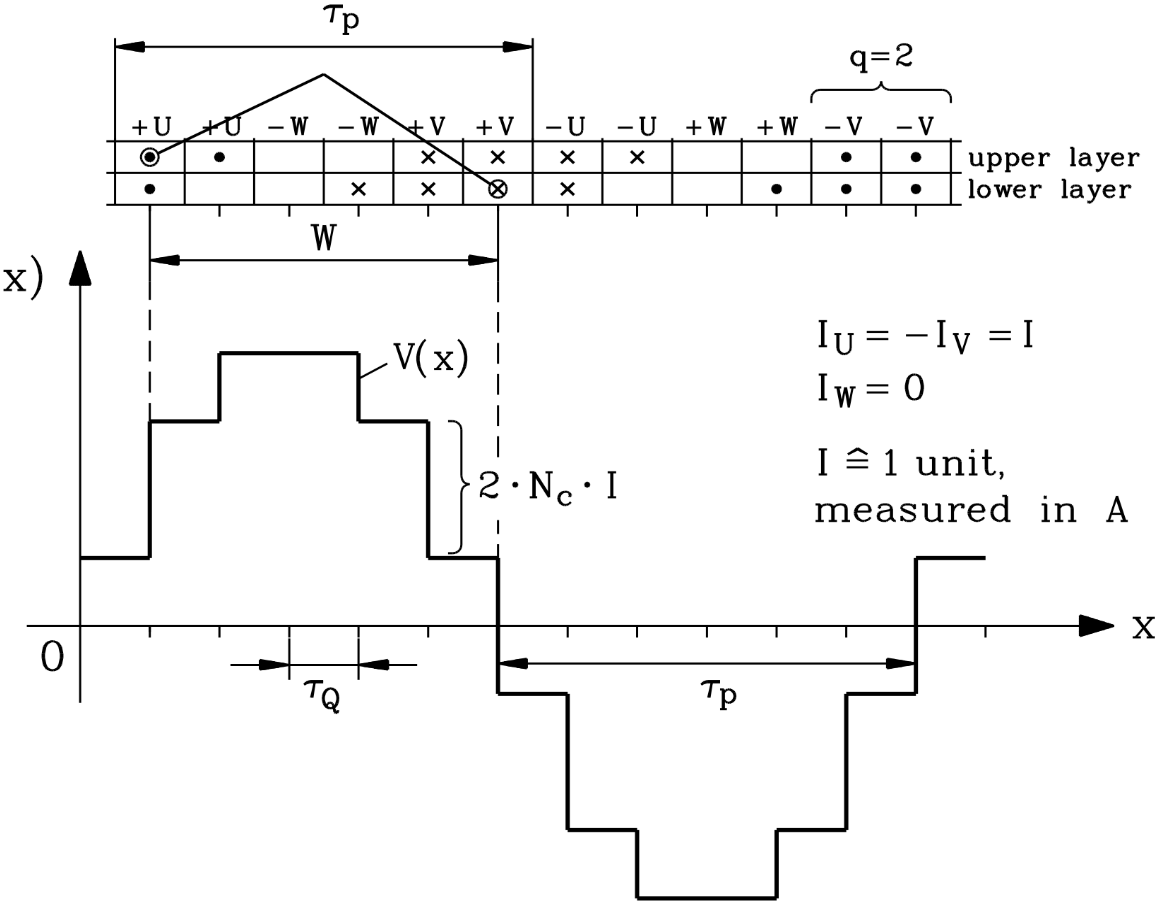


Fig. 1.1-1: m.m.f.-curve in arbitrary units

Exercise 1.2: Three-phase winding of a standard induction motor

Motor data:

U_N = 380 V Y (nominal voltage, star connection)

P_N = 15 kW (nominal output power)

$2\cdot p = 6$ (number of poles)

f_N = 50 Hz (nominal frequency)

data of stator core:

d_{si} = 200 (bore diameter), l_{fe} = 150 mm (length of core)

Q_s = 36 (number of stator slots)

Design a chorded double-layer winding with 5/6 short pitching! What is the number of turns per coil N_c , if all coils of one phase are connected into series? Select air gap flux density amplitude between 0.9 and 1 T !

Solution:

Q = 36, W/τ_p = 5/6. By neglecting the primary voltage drop $(R_s + X_{s\sigma})\cdot I_s$ the induced voltage U_h is equal to the supply phase voltage:

$$U_h = \frac{U_N}{\sqrt{3}} = \frac{380\text{V}}{\sqrt{3}} = 220\text{V}$$

$$\tau_p = \frac{d_{si}\cdot\pi}{2\cdot p} = 104{,}7\text{ mm},\quad d_{si} = 200\text{mm}, 2\cdot p = 6,\qquad l_{fe} = 150\text{ mm}$$

$$\nu = 1: k_{w1} = k_{p1}\cdot k_{d1} = \sin\left(\frac{\pi}{2}\cdot\frac{W}{\tau_p}\right)\cdot\left(\frac{\sin(\frac{\pi}{6})}{q\cdot\sin(\frac{\pi}{6\cdot q})}\right) = 0.966\cdot 0.966 = 0.933,\quad q = 2,\quad \frac{W}{\tau_p} = \frac{5}{6}$$

$$U_h = \sqrt{2}\cdot\pi\cdot f\cdot N_s\cdot k_w\cdot\frac{2}{\pi}\cdot\tau_p\cdot l_{fe}\cdot\hat{B}_{\delta,1} \Rightarrow N_s\cdot\hat{B}_{\delta,1} = 106{,}1\frac{\text{Vs}}{\text{m}^2},\qquad f = 50\text{Hz}$$

For good utilisation of iron, the air-gap induction has to be between 0.9 ... 1 T, so e.g. $\hat{B}_{\delta,1} \approx 0{,}95\,\text{T}$. An air-gap induction of $\hat{B}_{\delta,1} \approx 0{,}95\,\text{T}$ leads to induction in the teeth of $B_d = 2\cdot\hat{B}_{\delta,1} \approx 1{,}9\,\text{T}$, the value when saturation of iron occurs.

Selection of number of turns per coil:

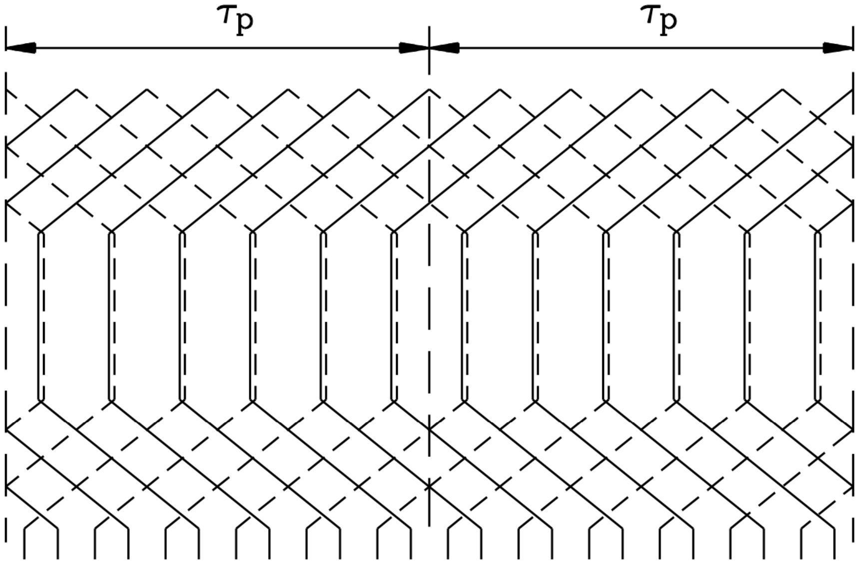
$\Rightarrow N_c = 106.1/0.95 = 111.75,\quad N_s = 2p\cdot q\cdot N_c/a$. With series connection a = 1 we get:

In case of $N_c = 9$: $N_s = 108$ and $\hat{B}_{\delta,1} = 0.98\,\text{T}$

In case of $N_c = 10$: $N_s = 120$ and $\hat{B}_{\delta,1} = 0.88\,\text{T}$. To get the higher field amplitude, $\underline{N_c = 10}$ is chosen.

Exercise 1.3: Three-phase winding for a synchronous emergency generator

Fig.1.3-1 shows the plan for a three-phase winding. It is the cut-out over two pole pitches.



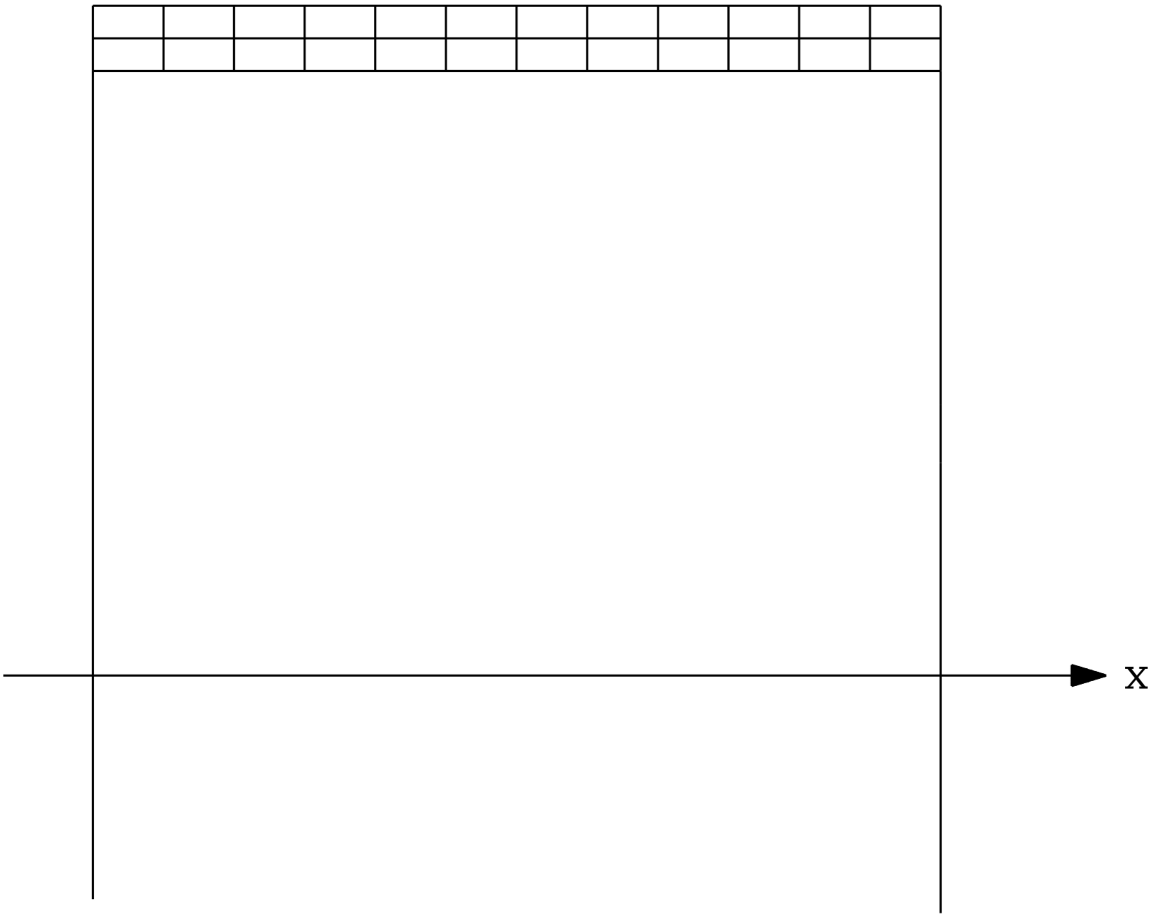


Fig. 1.3-1: Plan for a three-phase two-layer winding

1) What is the number of slots per pole and phase q?
2) What is the type of winding:
   - single-layer or double-layer winding?
   - short-pitch or full-pitch winding?
3) To obtain a three-phase winding, the coils has to be connected. Draw the wiring of the coils into the plan. Make sure, that a three-phase winding with the terminals U-X, V-Y, W-Z is obtained!
4) What is the instantaneous value for the phase currents if the current in phase U is zero? Draw the current distribution for this moment into the band below the plan of the three-phase winding!
5) Draw the m.m.f. curve corresponding to the current distribution into the diagram Fig. 1.3-1 below the phase band.
6) The number of poles of the emergency generator is 12. What is the number of stator-slots ?

Solution:

1) The number of slots per pole and phase is $q = 2$.
2) Winding type: double layer short-pitch: $W/\tau_p = 5/6$
3) In case of star connection the terminals X, Y and Z have to be connected as a star point. The connection of the coils has to be done that way that north and south pole are achieved.
4) The instantaneous value of the current in phase U is zero $\Rightarrow I_V = -I_W = \frac{\sqrt{3}}{2}\hat{I}$.

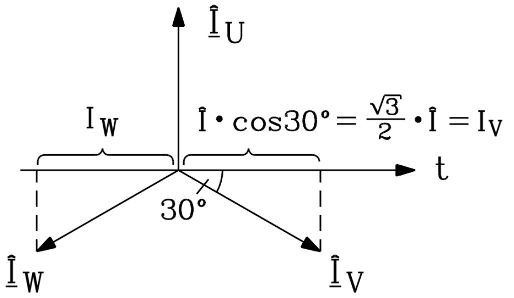


Fig. 1.3-2: Instantaneous current values

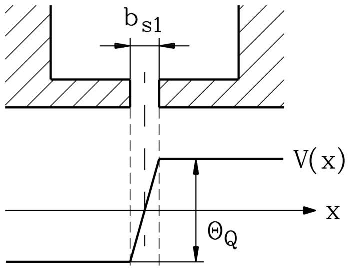
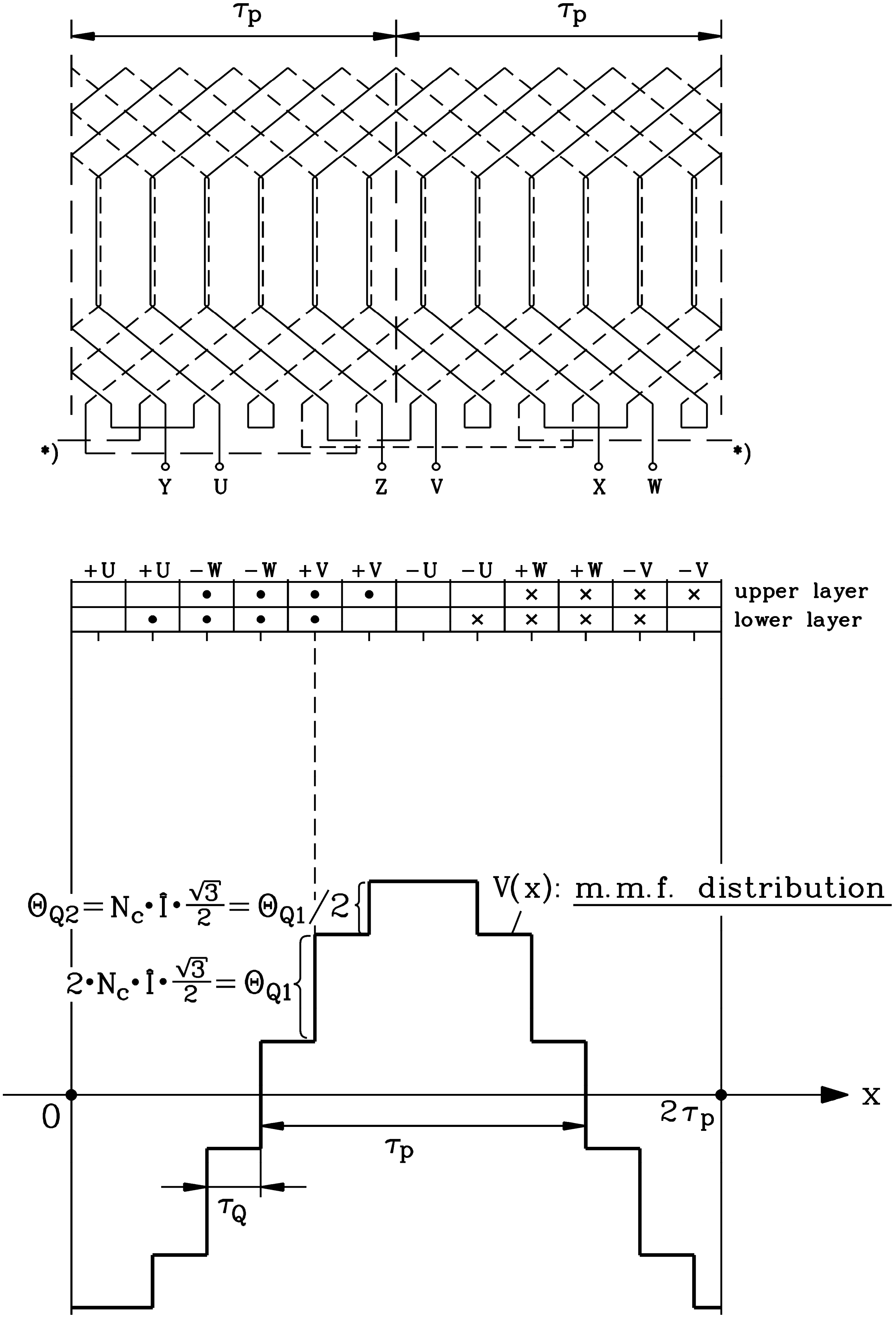


Fig. 1.3-3: Influence of slot opening on m.m.f.

5) The magneto-motive force curve (m.m.f. curve) is: $V(x) = \int A(x) \cdot dx$. If the influence of slot opening is neglected (Fig. 1.3-3), the m.m.f. “jumps” in the centre of the each slot with the corresponding slot *Ampere* turns Θ_Q (solution see Fig. 1.3-4).
6) $Q = 2 \cdot p \cdot q \cdot m = 12 \cdot 2 \cdot 3 = \underline{72}$



+U	+U	−W	−W	+V	+V	−U	−U	+W	+W	−V	−V	
		●	●	●	●			×	×	×	×	upper layer
	●	●	●	●			×	×	×	×		lower layer

Fig. 1.3-4: Completed plan for a three-phase two-layer winding and m.m.f. distribution

Exercise 1.4: Three-phase winding for a low-speed bulb-type synchronous hydro-generator

The winding data for the synchronous hydro-generator are:

$q = 2$ (number of slots per pole and phase)

$W/\tau_p = 5/6$ (chorded winding)

1) The instantaneous value for the current in phase U is $I_U = I_{\max} = \sqrt{2} \cdot I_{\mathrm{rms}}$. What is the value for the corresponding currents in phases V and W? Draw the full-scale m.m.f. curve for the given winding and the instantaneous value of the currents over two pole pitches!
   a) Draw the instantaneous current distribution into a planar slot band per one pole pitch !
   b) Draw the m.m.f. $V(x)$ obtained from the current distribution from a)! Neglect the influence of slot opening.

Recommended scale:

- Slot pitch $\tau_Q \Leftrightarrow$ 1cm
- Scale for the m.m.f. curve $\Theta_{Q,\max} = 2 \cdot \sqrt{2} I_{rms} \cdot N_c \Leftrightarrow$ 1 cm.

2) What is the value of the amplitude of the m.m.f. curve (in per unit)? Compare this value with the amplitude of *FOURIER* fundamental !
3) Describe with your own words
   a) the effect of the short-pitching on the m.m.f. curve which can be seen in m.m.f. $V(x)$ of 1b),
   b) the effect of short-pitching on the voltage induced by a rotating magnetic field !
4) Draw the phasor diagram of the voltages in the coil sides of one coil group of phase U, induced by the rotating magnetic field, for the given winding. Deduce the equation of the distribution factor k_{d1} from this phasor diagram.

Solution:

1) Three-phase winding of double layer type:

$$I_U = I_{\max} = \sqrt{2} \cdot I_{\mathrm{rms}} = \hat{I} \quad \rightarrow \quad I_V = I_W = -0.5 \cdot \sqrt{2} \cdot I_{\mathrm{rms}} \qquad \text{short-pitching}: W/\tau_p = 5/6$$

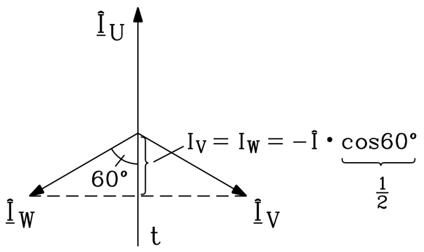


Fig. 1.4-1: Instantaneous current values

Solution for m.m.f. distribution and slot current distribution see Fig. 1.4-2.

2) $\hat{V} = (\frac{1}{2} + \frac{1,5}{2} + \frac{1}{2}) \cdot \Theta_{Q,\max} = \underline{1.75 \cdot \Theta_{Q,\max}}$

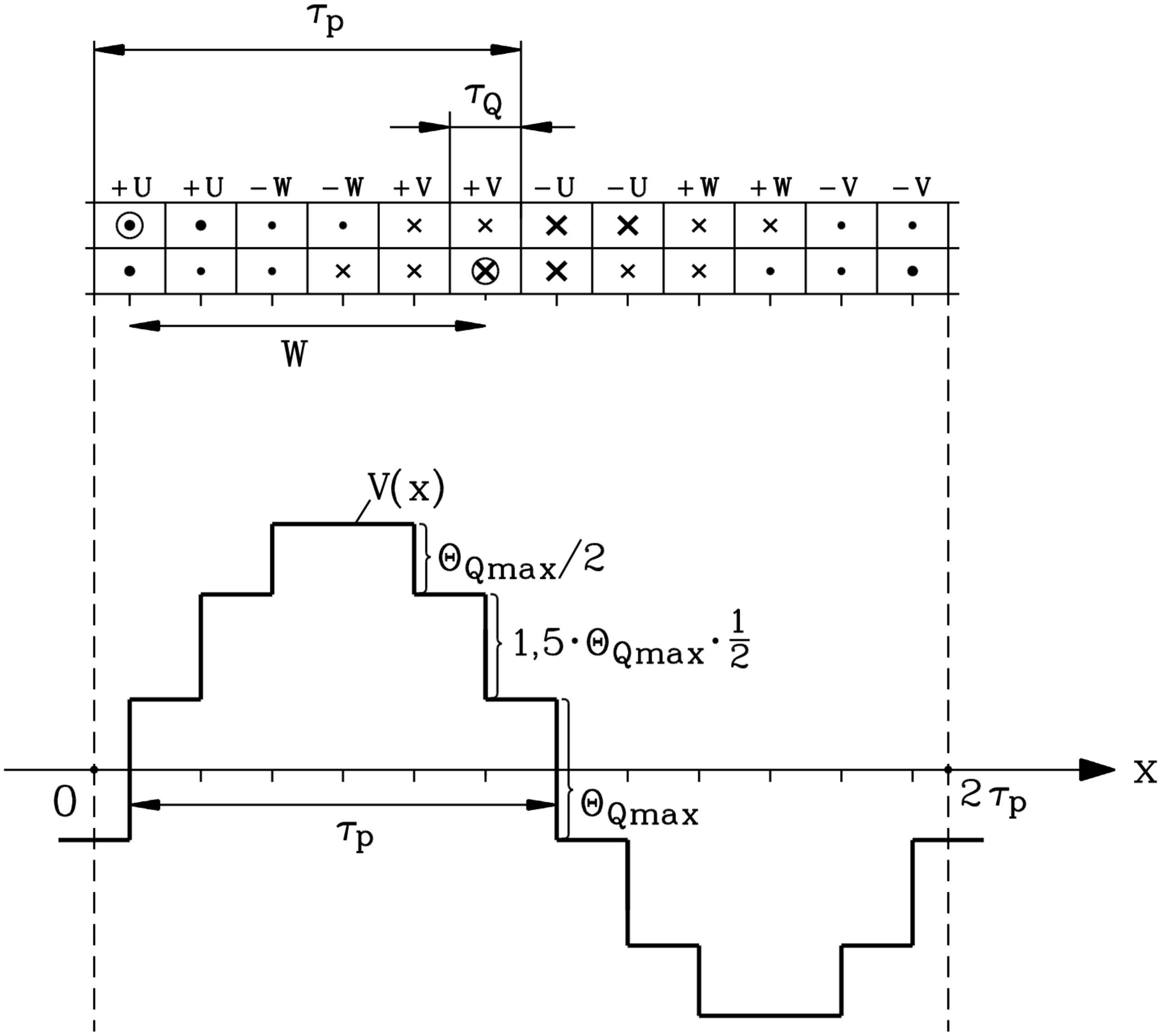


Fig. 1.4-2: Distribution of slot ampere turns (above) and m.m.f. distribution (below) for instantaneous current values of Fig. 1.4-1

$$\Theta_{Q,\max} = 2 \cdot I_{\mathrm{rms}} \cdot \sqrt{2} \cdot N_{\mathrm{c}} \Rightarrow \hat{V} = 3.5 \cdot \sqrt{2} \cdot I_{\mathrm{rms}} \cdot N_{\mathrm{c}}$$

In comparison to amplitude of fundamental wave: $\hat{V}_{\mathrm{s}} = \frac{\sqrt{2}}{\pi} \cdot \frac{m_{\mathrm{s}} \cdot N_{\mathrm{s}} \cdot k_{\mathrm{w}}}{p} \cdot I_{\mathrm{rms}}$ with:

$$k_{\mathrm{w}} = k_{\mathrm{p}} \cdot k_{\mathrm{d}} = 0.966 \cdot 0.966 = \underline{0.933}$$

$$N_{\mathrm{s}} = 2 \cdot p \cdot q \cdot \frac{N_{\mathrm{c}}}{a}$$

$a = 1$ (series connection)

we get $\Rightarrow \hat{V}_{\mathrm{s}} = \frac{\sqrt{2}}{\pi} \cdot \frac{3 \cdot N_{\mathrm{c}} \cdot k_{\mathrm{w}} \cdot 2 \cdot q}{1} \cdot I_{rms} = 3.56 \cdot \sqrt{2} \cdot N_{\mathrm{c}} \cdot I_{rms}$

Result: The amplitude of the fundamental wave $\hat{V}_{\mathrm{s}}$ is $\frac{3.56}{3.5} = 1.018$ bigger than the value of $\hat{V}$!

3 a) By short-pitching the shape of the m.m.f. curve becomes more sinusoidal.

b) By short-pitching the voltage amplitudes, induced by higher harmonics, will be reduced.

4)

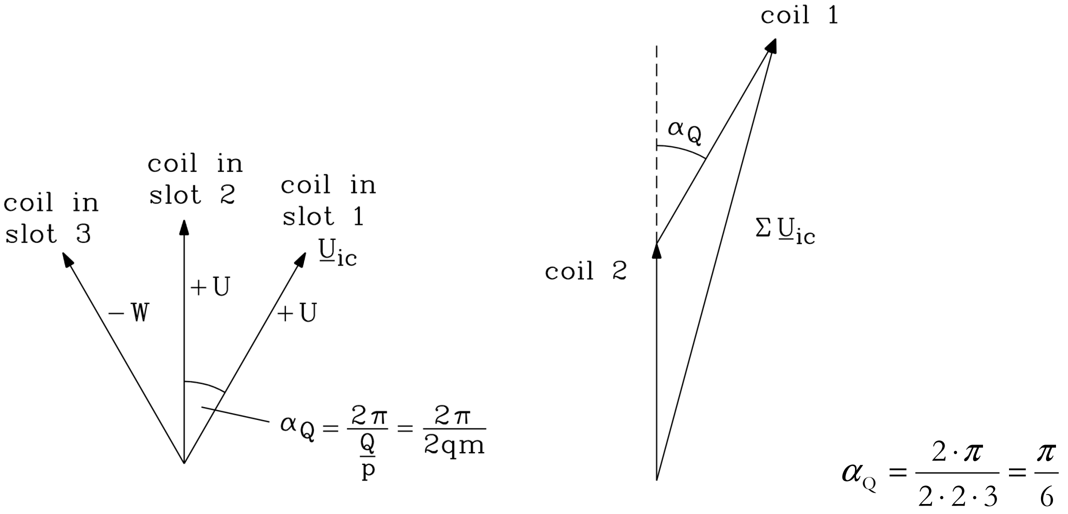


Fig. 1.4-3: Voltage phasors in the two series-connected adjacent coils of coil group U and their sum

$$\sum \underline{U}_{\mathrm{i,c}} = \underline{U}_{i,c,slot1} + \underline{U}_{i,c,slot2}$$

$$k_{\mathrm{d},1} = \frac{\sum \underline{U}_{\mathrm{ic}}}{\sum |\underline{U}_{\mathrm{ic}}|} = \frac{\sqrt{(1+\cos\alpha_{\mathrm{Q}})^2 + (\sin\alpha_{\mathrm{Q}})^2}}{2} = \frac{\sqrt{(1+\cos(\pi/6))^2 + (\sin(\pi/6))^2}}{2} =$$

$$= \frac{\sqrt{(1+\sqrt{3}/2)^2 + (1/2)^2}}{2} = 0{,}5\cdot\sqrt{(1{,}866)^2 + (1/2)^2} = \underline{0.966}$$

Compared with formula from text book (for fundamental $\nu = 1$, we get in accordance:

$$k_{\mathrm{d},\nu} = \frac{\sin\left(\frac{\nu\cdot\pi}{2\cdot m}\right)}{q\cdot\sin\left(\frac{\nu\cdot\pi}{2\cdot m\cdot q}\right)} = \underline{0.966} \quad (m = 3,\ q = 2)$$

Exercise 1.5: Deflection magnet for a particle accelerator

The *German Society for Research on Heavy Ions (Gesellschaft für Schwerionenforschung (GSI))* in *Darmstadt* has a big dipole-magnet for the targeted deflection of the ion beam of charged heavy ions.

The data for the magnet are:

Nominal current I_{N} = 2500 A (DC)

Maximum Voltage U_{max} = 600 V.

Due to the high current density of 18 A/mm^2 the conductors of the magnet are directly-cooled with water. The flow rate of water is 520 l/min at 21 bar. The axial cross section of the magnet with geometrical magnet data is shown in Fig. 1.5-1.

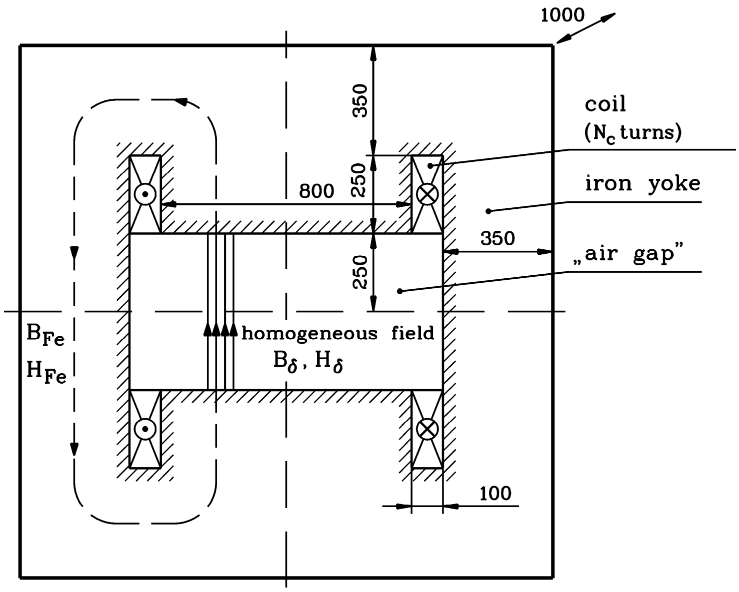


Fig. 1.5-1: Axial cross section of magnet in idealised shape. Lengths in mm.

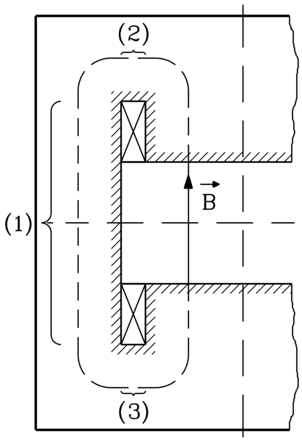


Fig. 1.5-2: Flux line path in iron

1) What is the value of the m.m.f. V_δ in the air gap, if a homogenous magnetic air gap flux density of $B_\delta = 1.66$ T has to be achieved? Estimate the required number of turns N_c for the windings? Neglect the magnetisation required for the back-iron.
2) How big is the air gap flux Φ_δ ?
3) By neglecting the leakage flux of the coils, the flux density in back-iron is $B_{Fe} = 1.9$ T.

a) The average length of flux lines in iron is $\Delta_{Fe} = 1,2$ m. How is this value obtained?

b) What is the value for the magnetisation of back-iron, if in accordance to $B(H)$-characteristic of the applied iron sheet $H_{Fe}(B_{Fe}) = 12$ kA/m? Is negligence of magnetisation for the back-iron according to 1) acceptable?

4) What is the value of the total inductance of winding, if the coils of both poles are connected into series?
5) What is the value of the electrical resistance of excitation winding at 50 °C ($\kappa_{Cu}(50°C) = 50\cdot 10^6$ S/m)? The average length of turns is $l_w = 4$ m.
6) What is the value for the required winding voltage and the resistive winding losses?

Solution:

1) $B_\delta = 1.66\mathrm{T}$

$$\delta = 2\cdot 250\,\mathrm{mm} = 500\,\mathrm{mm}$$

$$V_\delta = H_\delta \cdot \delta = 1321656\cdot 0.5 = 660828\mathrm{A}$$

$$H_\delta = \frac{B_\delta}{\mu_0} = \frac{1.66}{4\cdot\pi\cdot 10^{-7}} = 1321.7\frac{\mathrm{kA}}{\mathrm{m}}$$

$$\oint_C \vec{H}\cdot d\vec{s} = H_\delta\cdot\delta + H_{Fe}\cdot\Delta_{Fe} = 2\cdot N_c\cdot I \text{ with } H_{Fe}\cdot\Delta_{Fe}\approx 0 \rightarrow N_c = \frac{V_\delta}{2\cdot I} = \frac{660828}{2\cdot 2500} = 132.17$$

$$\underline{\rightarrow N_c = 132\text{ turns}}$$

2) $\Phi_\delta = \int_A \vec{B}_\delta \cdot d\vec{A} \cong B_\delta \cdot A_{\mathrm{Pole}} = B_\delta \cdot b_p \cdot l_{\mathrm{Fe}} = 1.66 \cdot 0.8 \cdot 1.0\,\mathrm{Wb} = \underline{\underline{1.328\,\mathrm{Wb}}}$

$b_p = 800\,\mathrm{mm}, \quad l_{\mathrm{Fe}} = 1000\,\mathrm{mm}$

3) a) Flux line in iron: According to Fig. 1.5-2 we get:

$\Delta_{\mathrm{fe}} \cong (1) + (2) + (3) = (1000 + 100 + 100)\,\mathrm{mm} = 1200\,\mathrm{mm}$

b) $B_{\mathrm{Fe}} = 1.9T$

Constancy of magnetic flux between adjacent flux lines: $\rightarrow \Phi_y = 0.5 \cdot \Phi_\delta = B_{\mathrm{Fe}} \cdot A_{\mathrm{Fe}}$

$A_{\mathrm{Fe}} = b_{\mathrm{Fe}} \cdot l_{\mathrm{Fe}} = 0.35 \cdot 1.0\,\mathrm{m}^2 = 0.35\,\mathrm{m}^2, (b_{\mathrm{Fe}} = 350\,\mathrm{mm})$

$$B_{\mathrm{Fe}} = 0.5 \cdot \Phi_\delta \cdot \frac{1}{A_{\mathrm{Fe}}} = 0.5 \cdot 1.328 \cdot \frac{1}{0.35}\,\mathrm{T} = 1.9\,\mathrm{T}$$

$$H_{\mathrm{Fe}} \cdot \Delta_{\mathrm{Fe}} = 12000 \cdot 1.2 = 14400 \frac{\mathrm{A}}{\mathrm{m}} \cdot \mathrm{m} = 14400\,\mathrm{A}$$

$$\frac{V_{\mathrm{Fe}}}{V_\delta} = \frac{14400}{660828} = \underline{2.18\%} \rightarrow \text{negligible!}$$

4)

$$L = 2 \cdot p \cdot L_c, \quad L_c = \frac{\Psi_c}{I}$$

$\Psi_c = N_c \cdot \Phi_\delta$ (if leakage flux is neglected)

$\Psi_c = 132 \cdot 1.328 = 175.3\,\mathrm{Vs}$

$$L_c = \frac{175.3}{2500} = 70.12\,\mathrm{mH}, \quad L = 2 \cdot 70.12 = \underline{140.24\,\mathrm{mH} = 0.14024\,\mathrm{H}}$$

5)

$$R = 2 \cdot p \cdot R_c$$

$$R_c = \frac{1}{\kappa_{\mathrm{Cu}}} \cdot \frac{N_c \cdot l_w}{q_{\mathrm{Cu}}} = \frac{10^{-6}}{50} \cdot \frac{132 \cdot 4}{138.89 \cdot 10^{-6}} = 0.076\,\Omega$$

$$q_{\mathrm{Cu}} = \frac{I}{J_{\mathrm{Cu}}} = \frac{2500}{18} = 138.89\,\mathrm{mm}^2, \quad \left(J_{\mathrm{Cu}} = 18 \frac{A}{\mathrm{mm}^2} \right)$$

$$R_{(50°\mathrm{C})} = 2 \cdot 0.076 = \underline{\underline{0.152\,\Omega}}$$

6)

$U = R \cdot I = 0{,}152 \cdot 2500 = 380.16 \approx \underline{\underline{380\,\mathrm{V} < 600\,\mathrm{V}}}$ OK!

$$P = U \cdot I = R \cdot I^2 = 0.152 \cdot 2500^2 = \underline{\underline{950.4\,\mathrm{kW}}}$$

Note:

Dissipation of heat due to these high resistive losses is only possible by direct water-cooling of conductors. With flow rate of 520 liters/m the temperature rise of water is 23 K.

Exercise 1.6: Single-phase generators for railway power supply

The synchronous hydro-generators of the *Spullersee/Vorarlberg (Austria)* pumping power station for the *Austrian Federal Railways (Österreichische Bundesbahnen (ÖBB))* are single-phase generators. The data of the generators are:

$S_N = 16$ MVA

$U_N = 6300$ V

$I_N = 2450$ A

$f_N = 16\ 2/3$ Hz

$n_N = 500$/min

The bore diameter of generators is 2.2 m, the axial length of core is 1540 mm. To obtain the winding of a single-phase generator, only the phases U and V are used, whereas phase W is missing. To gain a single phase winding, phase U and V are connected into series. The winding data is:

type of winding: double layer

$q = 8$ (slots per phase and pole)

$N_c = 1$

coil span: slot 1 to slot 21

$a = 1$, series connection per phase

1) What is the number of pole pairs of generators?
2) Calculate the air gap pole flux of the fundamental wave.

   The amplitude of rotor field is $B_{p,\mu=1} = 1$T!
3) Evaluate the total number of turns N of single-phase stator winding !
4) Calculate the short-pitching W/τ_p and the induced line-to-line voltage by usage of flux of 2) !
5) Due to the fact that the contour of rotor poles is not perfect the magnetic field in air gap has field wave harmonics with ordinal number 3, 5 and 7. The values of their amplitudes are listed below.

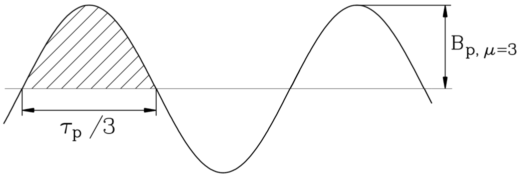


Fig. 1.6-1: Third flux density harmonic in air gap

$$\frac{B_{p,\mu=3}}{B_{p,\mu=1}} = 0{,}15 \qquad \frac{B_{p,\mu=5}}{B_{p,\mu=1}} = 0{,}08 \qquad \frac{B_{p,\mu=7}}{B_{p,\mu=1}} = 0{,}05$$

Calculate the values of the associated line-to-line induced voltages!

$$\frac{U_{i,\mu=3}}{U_{i,\mu=1}} = ? \qquad \frac{U_{i,\mu=5}}{U_{i,\mu=1}} = ? \qquad \frac{U_{i,\mu=7}}{U_{i,\mu=1}} = ?$$

6) Calculate the r.m.s. value of voltage harmonics with respect to total r.m.s. voltage value, using data of 5) !

Solution:

1) $f = n \cdot p \;\Rightarrow\; 2 \cdot p = \frac{2 \cdot f}{n} = \frac{2 \cdot 16{,}67}{500/60} = 4 \;\Rightarrow\; \underline{2p = 4}$

2) $\Phi_{\mu=1} = \frac{2}{\pi} \cdot \tau_{\mathrm{p}} \cdot l \cdot B_{\mathrm{p},\mu=1}$

$$\tau_{\mathrm{p}} = \frac{d\mathrm{si} \cdot \pi}{2 \cdot \mathrm{p}} = \frac{2{,}2 \cdot \pi}{2 \cdot 2} = 1{,}728\mathrm{m}, \quad l = 1{,}54\mathrm{m}$$

$$\Phi_{\mu=1} = \frac{2}{\pi} \cdot 1{,}728 \cdot 1{,}54 \cdot 1{,}0 = \underline{1{,}694\mathrm{Wb}}$$

3) According to Fig. 1.6-2, we get for the single phase winding the total number of turns *N*:

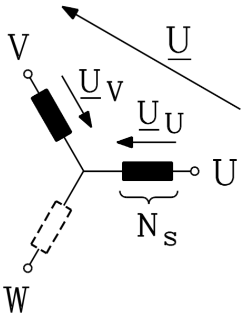


Fig. 1.6-2: Generation of single phase winding out of three-phase winding scheme

$$N_s = 2p \cdot q \cdot N_c / a = 4 \cdot 8 \cdot 1/1 = 32 \;\Rightarrow\; N = 2N_s = 2 \cdot 32 = \underline{\underline{64}}$$

4)

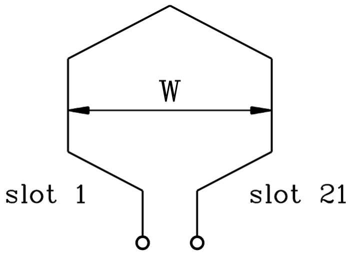


Fig. 1.6-3: Coil span

Slots per pole: $m \cdot q = 3 \cdot 8 = 24$ slots:

Coil span, counted in numbers of slots: $W = 21 - 1 = 20$.

$$\frac{W}{\tau_{\mathrm{p}}} = \frac{20}{24} = \frac{5}{6} = \underline{\underline{0.833}}$$

The induced line-to-line voltage has to be determined acc. to Fig. 1.6-2, considering the phase shift between phases U and V:

$$U_{\mathrm{i}} = \left|\underline{U}_{\mathrm{iU}} - \underline{U}_{\mathrm{iV}}\right| = \sqrt{3} \cdot U_{\mathrm{iU}} = \sqrt{3} \cdot \sqrt{2} \cdot \pi \cdot f \cdot N_{\mathrm{s}} \cdot k_{\mathrm{w},1} \cdot \Phi_{\mu=1}$$

$$k_{\mathrm{p},\mu=1} = \sin\left(\frac{W}{\tau_{\mathrm{p}}} \cdot \frac{\pi}{2}\right) = \sin\left(\frac{5}{6} \cdot \frac{\pi}{2}\right) = 0.9659, \quad k_{\mathrm{d},\mu=1} = \frac{\sin\left(\frac{\pi}{6}\right)}{q \cdot \sin\left(\frac{\pi}{6 \cdot q}\right)} = \frac{0.5}{8 \cdot \sin\left(\frac{\pi}{6 \cdot 8}\right)} = 0.9556$$

$$\Rightarrow k_{\mathrm{w},\mu=1} = k_{\mathrm{p},\mu=1} \cdot k_{\mathrm{d},\mu=1} = 0.923$$

$$U_{\mathrm{i},\mu=1} = \sqrt{3} \cdot \sqrt{2} \cdot \pi \cdot 16.66 \cdot 32 \cdot 0.923 \cdot 1.694 = \underline{6414\mathrm{V}}$$

5)

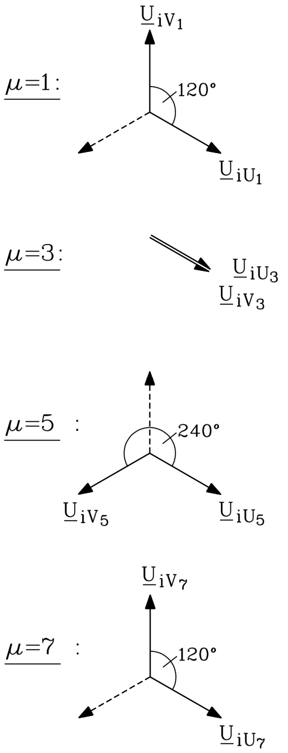


Fig. 1.6-4: Voltage phasors for the harmonics of 1st, 3rd, 5th, 7th order

Phase shift between phase $\underline{U}_{\mathrm{iU},\mu}$ and $\underline{U}_{\mathrm{iV},\mu}$:

$$\mu = 1: \quad 120° = 120°$$

$$\mu = 3 : 3 \cdot 120° = 360° = 0°$$

$$\mu = 5 : 5 \cdot 120° = 600° = 240°$$

$$\mu = 7 : 7 \cdot 120° = 840° = 120°$$

Resulting line - to - line voltage, determined from phase voltages $\underline{U}_{\mathrm{iU},\mu}$ and $\underline{U}_{\mathrm{iV},\mu}$:

$$U_{\mathrm{i}\mu} = \left|\underline{U}_{\mathrm{i}\mu}\right| = \left|\underline{U}_{\mathrm{iU},\mu} - \underline{U}_{\mathrm{iV},\mu}\right|$$

$$\mu = 1: \quad U_{\mathrm{i}1} = \sqrt{3} \cdot U_{\mathrm{iU}1}$$

$$\mu = 3: \quad U_{\mathrm{i}3} = 0$$

$$\mu = 5: \quad U_{\mathrm{i}5} = \sqrt{3} \cdot U_{\mathrm{iU}5}$$

$$\mu = 7: \quad U_{\mathrm{i}7} = \sqrt{3} \cdot U_{\mathrm{iU}7}$$

R.m.s. values of induced phase voltages:

$$U_{\mathrm{iU}\mu} = \sqrt{2} \cdot (\mu \cdot f) \cdot N_{\mathrm{s}} \cdot k_{\mathrm{w},\mu} \cdot \frac{2}{\pi} \cdot \frac{\tau_{\mathrm{p}}}{\mu} \cdot l \cdot B_{\mathrm{p},\mu}, \qquad \mu \cdot f = f_{\mu}$$

$$k_{\mathrm{d},\mu} = \sin\left(\mu \cdot \frac{W}{\tau_\mathrm{p}} \cdot \frac{\pi}{2}\right), \quad k_{\mathrm{p},\mu} = \frac{\sin\left(\dfrac{\mu \cdot \pi}{6}\right)}{q \cdot \sin\left(\dfrac{\mu \cdot \pi}{6 \cdot q}\right)}$$

$$\Rightarrow k_{\mathrm{w},\mu} = k_{\mathrm{p},\mu} \cdot k_{\mathrm{d},\mu} \qquad \Rightarrow \underline{\underline{\frac{U_{\mathrm{iU}\mu}}{U_{\mathrm{iU1}}} = \frac{k_{\mathrm{w}\mu}}{k_{\mathrm{w1}}} \cdot \frac{B_{\mathrm{p},\mu}}{B_{\mathrm{p},1}}}}$$

μ	$k_{\mathrm{p},\mu}$	$k_{\mathrm{d},\mu}$	$\dfrac{U_{\mathrm{iU},\mu}}{U_{\mathrm{iU},1}}$	$\dfrac{U_{\mathrm{i},\mu}}{U_{\mathrm{i},1}}$	$\dfrac{B_{\mathrm{p},\mu}}{B_{\mathrm{p},1}}$
1	0.9659	0.9556	1	1	1
3	-0.707	0.64	-0.0735	0	0.15
5	0.259	0.194	0.0044	0.0044	0.08
7	0.259	-0.141	-0.0020	-0.0020	0.05

Due to the influence of short-pitching the voltage harmonics are much smaller than the field harmonics.

6) Total harmonic distortion *k*:

$$k = \frac{\sqrt{U_{i3}^2 + U_{i5}^2 + U_{i7}^2}}{\sqrt{U_{i1}^2 + U_{i3}^2 + U_{i5}^2 + U_{i7}^2}} = \frac{\sqrt{\left(\dfrac{U_{\mathrm{i3}}}{U_{\mathrm{i1}}}\right)^2 + \left(\dfrac{U_{\mathrm{i5}}}{U_{\mathrm{i1}}}\right)^2 + \left(\dfrac{U_{\mathrm{i7}}}{U_{\mathrm{i1}}}\right)^2}}{\sqrt{1 + \left(\dfrac{U_{\mathrm{i3}}}{U_{\mathrm{i1}}}\right)^2 + \left(\dfrac{U_{\mathrm{i5}}}{U_{\mathrm{i1}}}\right)^2 + \left(\dfrac{U_{\mathrm{i7}}}{U_{\mathrm{i1}}}\right)^2}} =$$

$$= \frac{\sqrt{0{,}0044^2 + 0{,}002^2}}{\sqrt{1 + 0{,}0044^2 + 0{,}002^2}} = \underline{0{,}00483}$$

Exercise 1.7: FARADAY´ disc

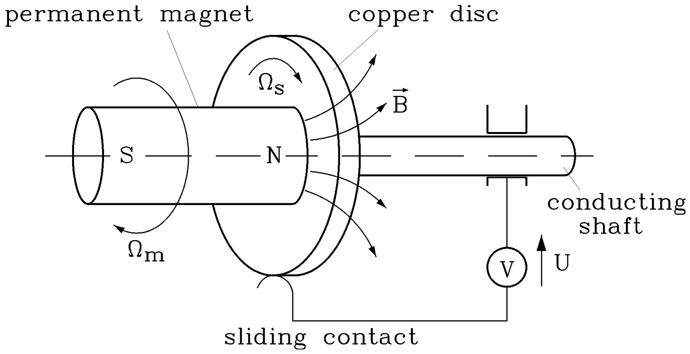


Fig. 1.7-1: *Faraday´*s disc

A copper disc (diameter $d = 2R = 60$ cm) rotates in a homogeneous magnetic field $B = 1.8$ T with a peripheral speed of $v_\mathrm{u} = 100$ m/s.

1) How big is the rotational speed $n = \Omega_s/(2\pi)$ of the disc in 1/s and 1/min?

2) How big is the induced voltage between the disc centre and disc outer radius R?

3) The disk will be connected via 2 carbon brushes as sliding contacts, one at the disc centre and one at the outer radius, with an external OHM' resistance $R = 1\ \Omega$. The internal resistance of the disc is neglected. The voltage drop of both brushes amounts to 2 V at current flow. How big is the load current I ?

4) The friction losses of the disk (brush-, air-, bearing friction) amount to $P_{\mathrm{fr}} = 100$ W. How big is the for 3) necessary mechanical power P_{m}, which should be applied to the disk and the corresponding mechanical torque M ?

<u>Solution:</u>

1) $v_u = d\cdot\pi\cdot n \quad\Rightarrow\quad n = \dfrac{v_u}{d\cdot\pi} = \dfrac{100}{0.6\cdot\pi} = \underline{\underline{53\ /s = 3183\ /\min}}$

$v_u = 100$ m/s, $d = 0.6$ m

2) $\vec{E} = \vec{v}\times\vec{B},\quad v = 2\pi\cdot n\cdot r$

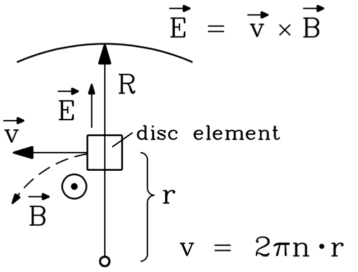


Fig. 1.7-2: Induction of motion in one disc element

$$U_i = \int_0^R (\vec{v}\times\vec{B})\cdot d\vec{s} = \int_0^R \vec{E}\cdot d\vec{s} = \int_0^R E\cdot ds = \int_0^R E\cdot dr = \int_0^R 2\pi\cdot n\cdot r\cdot B\cdot dr = 2\pi\cdot n\cdot B\left.\frac{r^2}{2}\right|_0^R = \pi\cdot n\cdot B\cdot R^2 =$$

$$= 2R\cdot\pi\cdot n\cdot B\frac{R}{2} = v_u\cdot B\cdot\frac{R}{2} = 100\cdot 1{,}8\cdot\frac{0{,}3}{2} = \underline{\underline{27\ \mathrm{V}}}$$

3) $R_i \approx 0 \quad\Rightarrow\quad R\cdot I + U_b = U_i \quad\Rightarrow\quad I = \dfrac{U_i - U_b}{R} = \dfrac{27-2}{1} = \underline{\underline{25\ \mathrm{A}}}$

4)

$$\begin{aligned}
P_{out} = P_e = R\cdot I^2 = 1\cdot 25^2 = {}& 625\ \mathrm{W}\\
P_b = U_b\cdot I = 2\cdot 25 = {}& 50\ \mathrm{W}\\
\underline{P_{fr} = \qquad\qquad\qquad} & \underline{100\ \mathrm{W}}\\
P_{in} = P_m = {}& \underline{\underline{775\ \mathrm{W}}}
\end{aligned}$$

$$P_m = 2\pi\cdot n\cdot M \quad\Rightarrow\quad M = \frac{775}{2\pi\dfrac{3183}{60}} = \underline{\underline{2.33\,\mathrm{Nm}}}$$

Exercise 1.8: Gearless synchronous wind generator

A synchronous generator with electrically excited rotor is directly coupled, without any gear, to a wind turbine. It must therefore be designed for ultra low speed of only 15/min.

Generator data:

$P_N = 1{,}5\text{MW}$ $\quad n_N = 15\frac{1}{\text{min}}$

$2p = 90$ $\quad d_{si} = 5\text{m}$

$l_{Fe} = 145\text{mm}$ $\quad q = 2$

$\frac{W}{\tau_P} = \frac{5}{6}$ $\quad N_C = 3$ (number of turns per coil)

The generator is equipped with a three-phase, two-layer winding in star connection with short pitched coils $W/\tau_P = 5/6$. All coils per phase are connected in series (a = 1).

1. Calculate number of turns per phase N_s !
2. Determine winding factor $k_{w,\nu}$ for fundamental field wave $\nu = 1$!
3. How big is flux per pole Φ at an fundamental air gap field amplitude $B_{\delta,\nu=1}$=1,0 T ? Determine induced no-load voltage per phase $U_{i,ph}$ at rated speed !
4. Sketch the position of coils and their proper connection for phase U for one pole pair. Use the slot scheme of Fig. 1.8.1 !

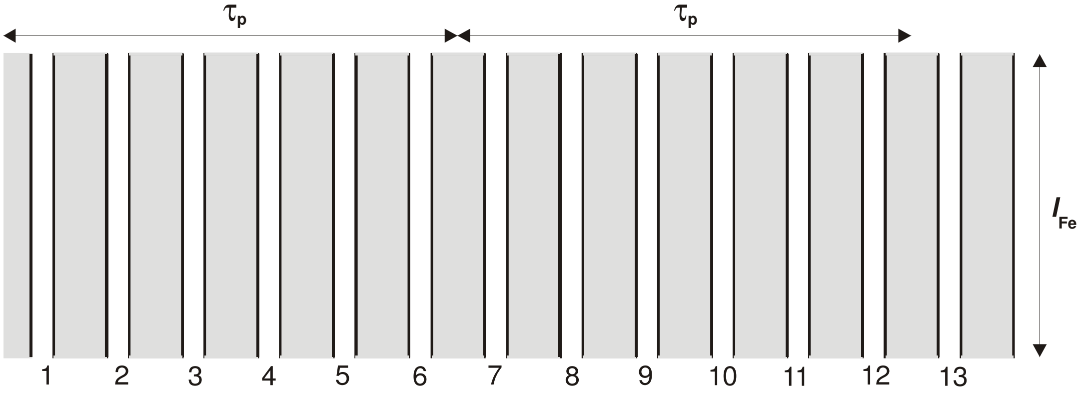


Fig. 1.8-1: Slot plan for one pole pair

Solution:

1) $N = 2p \cdot q \cdot N_c / a = 90 \cdot 2 \cdot 3/1 = \underline{\underline{540}}$

2) $k_{p,1} = \sin(W/\tau_p \cdot (\pi/2)) = \sin(5/6 \cdot (\pi/2)) = 0.966$

$$k_{d,1} = \frac{\sin\left(\frac{\pi}{2m}\right)}{q \cdot \sin\left(\frac{\pi}{2mq}\right)} = \frac{\sin\left(\frac{\pi}{6}\right)}{2 \cdot \sin\left(\frac{\pi}{2 \cdot 3 \cdot 2}\right)} = 0.966 \qquad k_{w1} = k_{p1} \cdot k_{d1} = 0.966 \cdot 0.966 = \underline{\underline{0.933}}$$

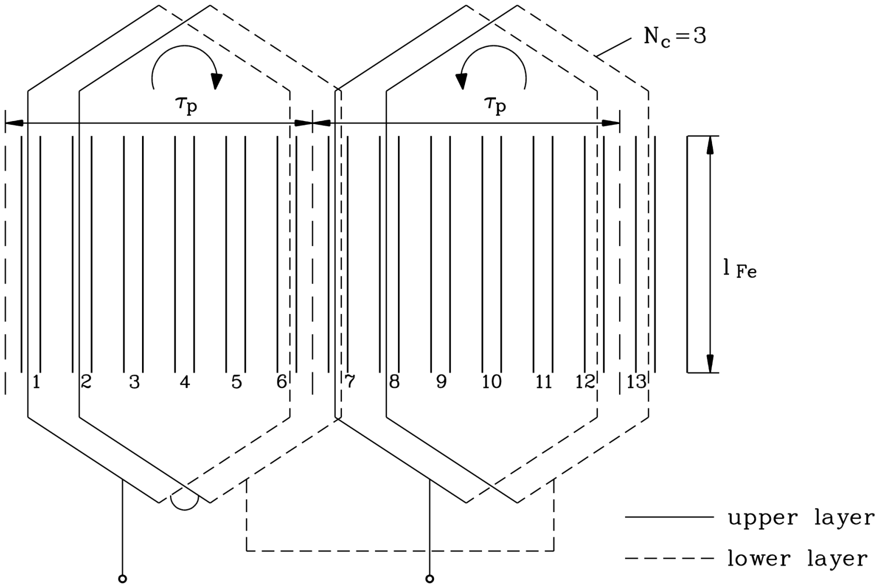
3) $\tau_p = d_{si} \cdot \pi/(2p) = 5 \cdot \pi/90 = 0.1745\,\text{m}$

$$\Phi = \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1} = \frac{2}{\pi} \cdot 0.1745 \cdot 0.145 \cdot 1.0 = \underline{\underline{16.1}}\,\text{mWb}$$

$$f_N = n_N \cdot p = (15/60) \cdot 45 = 11.25\,\mathrm{Hz}$$

$$U_i = \sqrt{2}\pi \cdot f_N \cdot N_s k_{w,1} \cdot \Phi = \sqrt{2}\pi \cdot 50 \cdot 11.25 \cdot 540 \cdot 0.933 \cdot 0.0161 = \underline{\underline{405.4}}\,\mathrm{V}$$

4)



<u>Fig. 1.8-2:</u> Winding scheme per phase and pole pair