Lecture Series

Finite-Element Electrical Machine Simulation

in the framework of the DFG Research Group 575
„High Frequency Parasitic Effects
in Inverter-Fed Electrical Drives"
http://www.ew.e-technik.tu-darmstadt.de/FOR575

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V07:
Coupling to External Circuits

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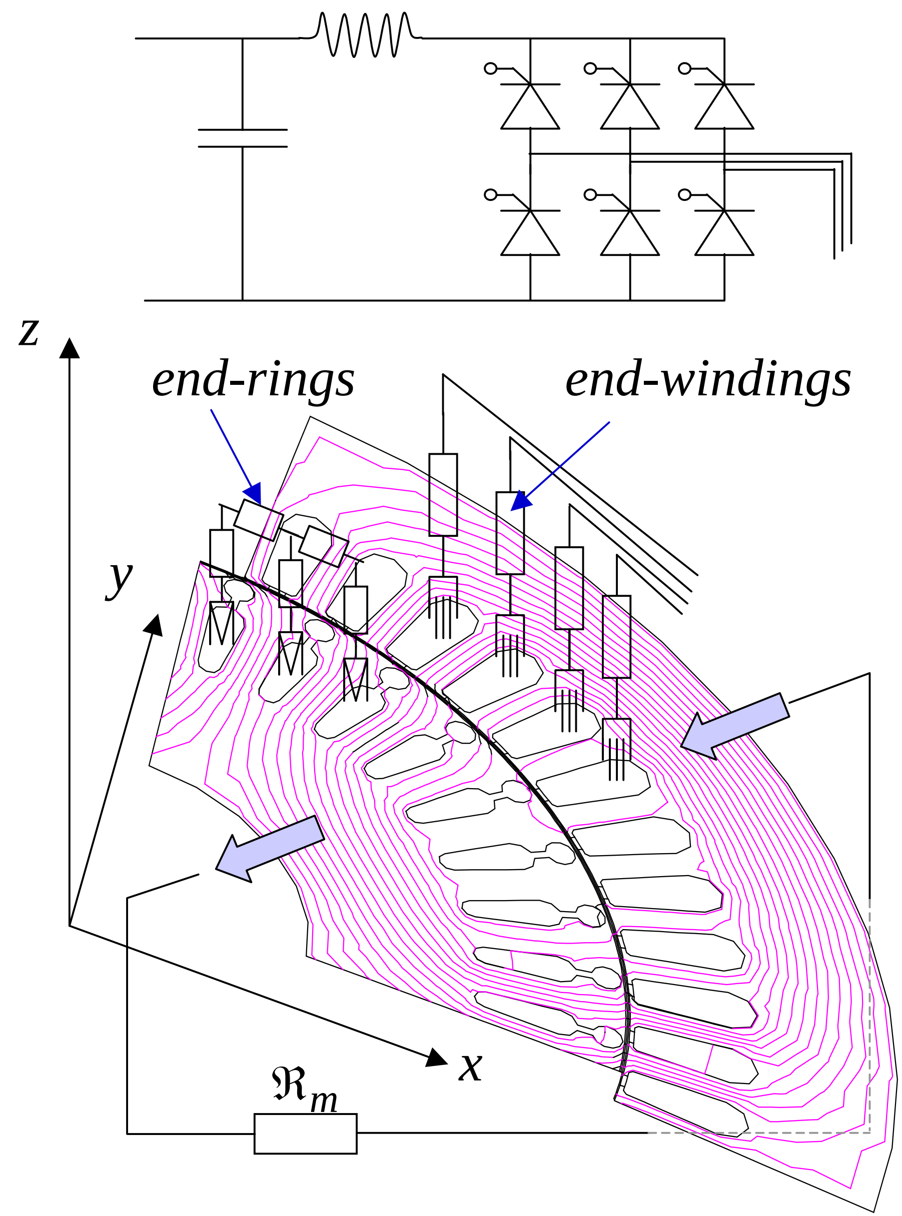
field-circuit coupling

FE/FIT model

- geometrical details
- ferromagnetic saturation (non-linear!!)
- (motional) eddy currents

circuit

- external sources/loads, (e.g. power electronic equipment)
- parts outside the FE model (e.g. end windings/rings)
- representing (linear) parts for which an equivalent circuit suffices (e.g. homopolar shaft flux)



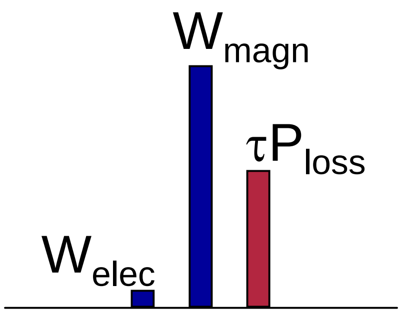
Overview

- discrete magnetoquasistatic formulation (recapitulation)
- solid conductors
- stranded conductor model
- circuit description
- example

Magnetoquasistatics (1)

- neglect displacement currents with respect to conducting currents
  - Ampère-Maxwell

$$\nabla \times \vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}$$



- magnetic vector potential $\vec{A}$
  - conservation of magnetic flux

$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = 0 + \nabla \times \vec{A}$$

- electric scalar potential φ (voltage)
  - Faraday-Lenz

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\nabla \times \frac{\partial \vec{A}}{\partial t} \quad \Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi$$

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Magnetoquasistatics (2)

Ampère

$$\nabla \times \vec{H} = \vec{J}$$

$$\Downarrow$$

$$\nabla \times \left(\nu \vec{B} \right) = \sigma \vec{E}$$

$$\Downarrow$$

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \underbrace{-\sigma \nabla \varphi}_{\vec{J}_s}$$

permeability

$$\vec{B} = \mu \vec{H} = \frac{1}{\nu} \vec{H}$$

reluctivity

$$\vec{J} = \sigma \vec{E}$$

conductivity

source current density

Physical Meaning

flux

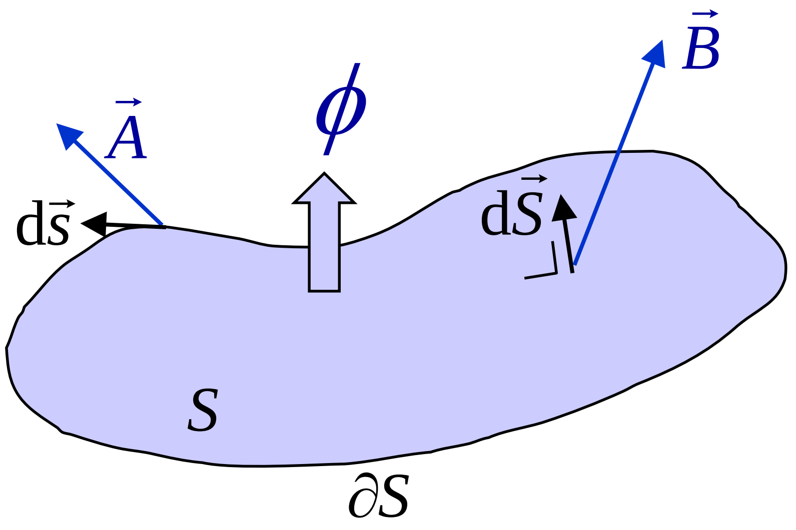
$$\phi = \iint_S \vec{B} \cdot \mathrm{d}\vec{S}$$

definition magnetic vector potential

$$\phi = \iint_S \nabla \times \vec{A} \cdot \mathrm{d}\vec{S}$$

Stokes

$$\phi = \oint_{\partial S} \vec{A} \cdot \mathrm{d}\vec{s}$$



induced voltage

$$u_{\mathrm{ind}} = -\frac{\mathrm{d}}{\mathrm{d}t} \oint_{\partial S} \vec{A} \cdot \mathrm{d}\vec{s}$$

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A-φ Formulation

magnetic vector potential, electric scalar potential

$$\nabla . \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = 0 + \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{J} = 0$$

$$\vec{H} = \nu \vec{B}$$

$$\vec{J} = \sigma \vec{E} + \vec{J}_s$$

$$\boxed{\begin{aligned} &\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} + \sigma \nabla \varphi = \vec{J}_s \\ &-\nabla \cdot \left(\sigma \frac{\partial \vec{A}}{\partial t} \right) - \nabla \cdot \left(\sigma \nabla \varphi \right) = 0 \end{aligned}}$$

$\nabla \times \vec{A}$ and $-\frac{\partial \vec{A}}{\partial t} - \nabla \varphi$ unique

$\vec{A}, \varphi$ and $\nabla \varphi$ not unique

$\left(\vec{A}, \varphi \right)$ is a solution

$\Rightarrow \left(\vec{A} + \nabla \psi, \varphi - \frac{\partial \psi}{\partial t} + ct \right)$ is a solution as well

A* Formulation

modified magnetic vector potential

$$\nabla . \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = 0 + \nabla \times \vec{A}^*$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}^*}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{J} = 0$$

$$\vec{H} = \nu \vec{B}$$

$$\vec{J} = \sigma \vec{E} + \vec{J}_s$$

$$\boxed{\begin{aligned} &\nabla \times \left(\nu \nabla \times \vec{A}^* \right) + \sigma \frac{\partial \vec{A}^*}{\partial t} = \vec{J}_s \\ &-\nabla \cdot \left(\sigma \frac{\partial \vec{A}^*}{\partial t} \right) = 0 \end{aligned}}$$

$\nabla \times \vec{A}^*$ unique $\qquad$ $\vec{A}^*$ not unique

$\vec{A}^*$ is a solution

$\Rightarrow$ $\vec{A}^* + \nabla \psi$ is a solution as well

Discretisation (1)

$$\nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \frac{\partial \vec{A}}{\partial t} = \underbrace{-\sigma \nabla \varphi}_{\vec{J}_\mathrm{s}}$$

spatial discretisation

$$\sum_j \left(u_j \underbrace{\int_\Omega \nu \nabla \times \vec{v}_j \cdot \nabla \times \vec{v}_i \, \mathrm{d}\Omega}_{= k_{ij}} + \frac{du_j}{dt} \underbrace{\int_\Omega \sigma \vec{v}_j \cdot \vec{v}_i \, \mathrm{d}\Omega}_{= m_{ij}} \right) = \underbrace{\int_\Omega \vec{J}_\mathrm{s} \cdot \vec{v}_i \, \mathrm{d}\Omega}_{= f_i}$$

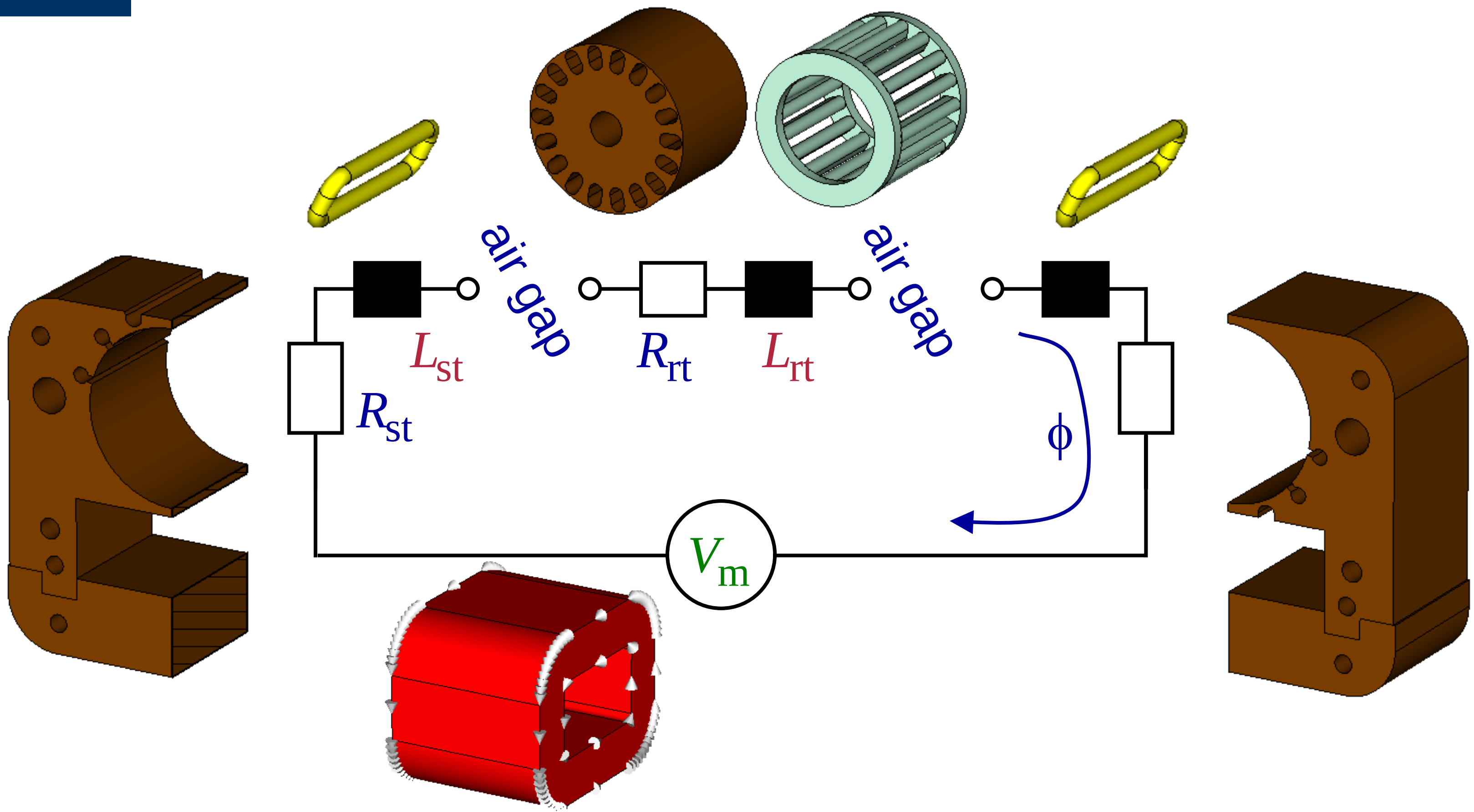
$$\left[k_{ij} \right] \left[u_j \right] + \left[m_{ij} \right] \left[\frac{du_j}{dt} \right] = \left[f_i \right]$$

Discretisation (2)

$$\sum_j \left(\underbrace{u_j}_{\boxed{\widehat{\mathbf{a}}_j}} \sum_p \sum_q c_{ip} c_{jq} \underbrace{\int_\Omega \nu \vec{z}_q \cdot \vec{z}_p \, \mathrm{d}\Omega}_{\boxed{\mathbf{M}^{\mathrm{FE}}_{\nu,p,q}}} + \frac{du_j}{dt} \underbrace{\int_\Omega \sigma \vec{v}_j \cdot \vec{v}_i \, \mathrm{d}\Omega}_{\boxed{\mathbf{M}^{\mathrm{FE}}_{\kappa,i,j}}} \right) = \underbrace{\int_\Omega \vec{J}_{\mathrm{s}} \cdot \vec{v}_i \, \mathrm{d}\Omega}_{\boxed{\widehat{\widehat{\mathbf{j}}}_{\mathrm{s},i}}}$$

$$\tilde{\mathbf{C}} \mathbf{M}^{\mathrm{FE}}_{\nu} \mathbf{C} \widehat{\mathbf{a}} + \mathbf{M}^{\mathrm{FE}}_{\sigma} \frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_s$$

Discretisation (3)



$$\left(R_{st}+R_{rt}+R_{ag}\right)\phi\;+\left(L_{st}+L_{rt}\right)\frac{d\phi}{dt}\;=V_m$$

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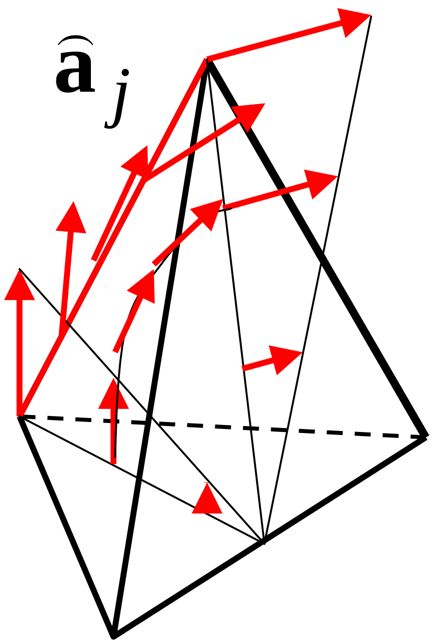
Discretisation (4)

$$\tilde{\mathbf{C}}\mathbf{M}_{\nu}^{\mathrm{FE}}\mathbf{C}\widehat{\mathbf{a}} + \mathbf{M}_{\sigma}^{\mathrm{FE}}\frac{d\widehat{\mathbf{a}}}{dt} = \widehat{\widehat{\mathbf{j}}}_s$$

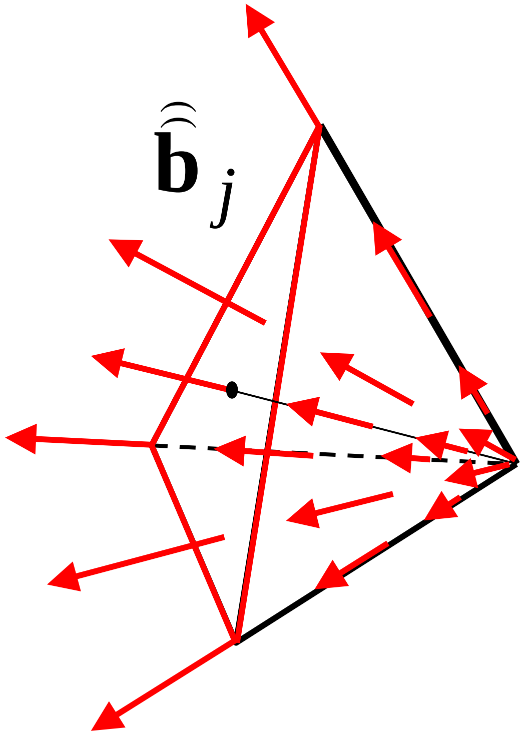
$\widehat{\mathbf{a}}$

along primary edges



$\widehat{\widehat{\mathbf{b}}} = \mathbf{C}\widehat{\mathbf{a}}$

through primary faces



Discretisation (5)

$$\tilde{\mathbf{C}}\underbrace{\mathbf{M}_{\nu}^{\mathrm{FE}}\mathbf{C}\widehat{\mathbf{a}}}_{\widehat{\mathbf{h}}}+\mathbf{M}_{\sigma}^{\mathrm{FE}}\frac{d\widehat{\mathbf{a}}}{dt}=\widehat{\widehat{\mathbf{j}}}_{s}$$

$\widehat{\widehat{\mathbf{j}}}$

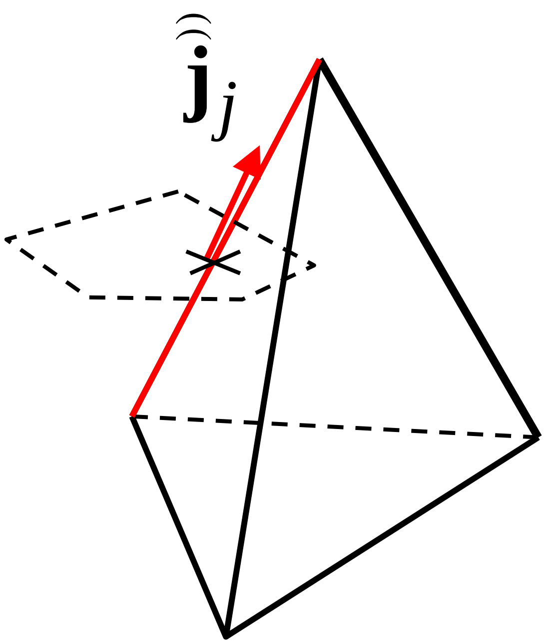
through dual faces

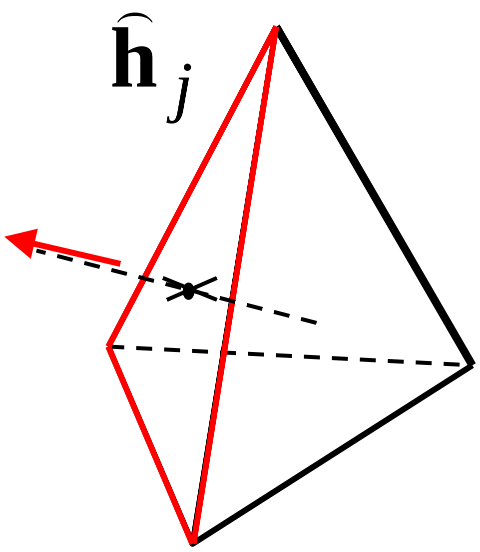
= along primary edges

$\widehat{\mathbf{h}}$

along dual edges

= through primary faces





Discrete A-φ Formulation

magnetic vector potential, electric scalar potential

$$\mathbf{S}\hat{\hat{\mathbf{b}}} = 0 \qquad \hat{\hat{\mathbf{b}}} = 0 + \mathbf{C}\hat{\mathbf{a}}$$

$$\mathbf{C}\hat{\mathbf{e}} = -\frac{d\hat{\hat{\mathbf{b}}}}{dt} \qquad \hat{\mathbf{e}} = -\frac{d\hat{\mathbf{a}}}{dt} - \mathbf{G}\phi$$

$$\tilde{\mathbf{C}}\hat{\mathbf{h}} = \hat{\hat{\mathbf{j}}}$$

$$\tilde{\mathbf{S}}\hat{\hat{\mathbf{j}}} = 0$$

$$\hat{\mathbf{h}} = \mathbf{M}_\nu \hat{\hat{\mathbf{b}}}$$

$$\hat{\hat{\mathbf{j}}} = \mathbf{M}_\sigma \hat{\mathbf{e}} + \hat{\hat{\mathbf{j}}}_S$$

$$\boxed{\begin{aligned} &\tilde{\mathbf{C}}\mathbf{M}_\nu\mathbf{C}\hat{\mathbf{a}} + \mathbf{M}_\sigma\frac{d\hat{\mathbf{a}}}{dt} + \mathbf{M}_\sigma\mathbf{G}\phi = \hat{\hat{\mathbf{j}}}_S \\ &-\tilde{\mathbf{S}}\mathbf{M}_\sigma\frac{d\hat{\mathbf{a}}}{dt} - \tilde{\mathbf{S}}\mathbf{M}_\sigma\mathbf{G}\phi = 0 \end{aligned}}$$

$\mathbf{C}\hat{\mathbf{a}}$ and $-\frac{d\hat{\mathbf{a}}}{dt} - \mathbf{G}\phi$ unique

$\hat{\mathbf{a}}, \phi$ and $\mathbf{G}\phi$ not unique

$(\hat{\mathbf{a}}, \phi)$ is a solution

$\left(\hat{\mathbf{a}} + \mathbf{G}\psi, \phi - \frac{d\psi}{dt} + ct\right)$ is a solution as well

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Overview

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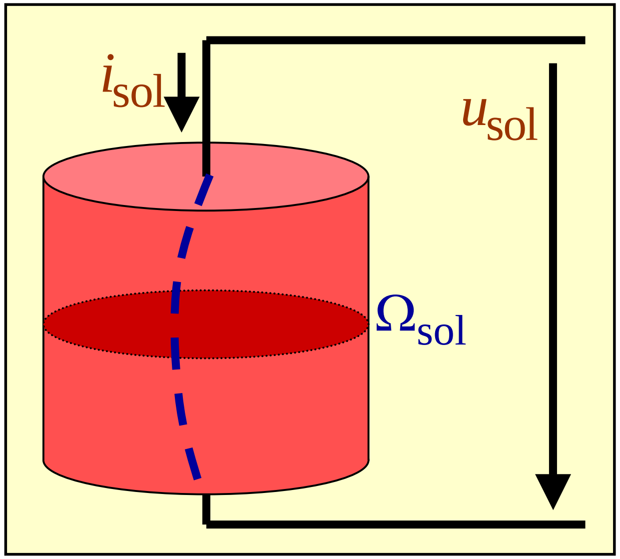
- solid conductors
- stranded conductor model
- circuit description
- example

Solid Conductor Coupling



voltage drop u_{sol}

$$\int_0^{\ell_{sol}} \vec{E}\cdot d\ell - u_{sol} = -\frac{d\phi}{dt}$$

$$\int_0^{\ell_{sol}} \vec{E}\cdot d\ell - u_{sol} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

$$-\int_0^{\ell_{sol}} \nabla\varphi\cdot d\ell - \int_0^{\ell_{sol}} \frac{d\vec{A}}{dt}\cdot d\ell - u_{sol} = -\frac{d}{dt}\int_S \nabla\times\vec{A}\cdot d\vec{S}$$

$$-\int_0^{\ell_{sol}} \nabla\varphi\cdot d\ell - \cancel{\int_0^{\ell_{sol}} \frac{d\vec{A}}{dt}}\cdot d\ell - u_{sol} = -\cancel{\frac{d}{dt}\int_{\partial S}} \vec{A}\cdot d\ell$$

$$u_{sol} = -\int_0^{\ell_{sol}} \nabla\varphi\cdot d\ell$$

choice for $\nabla\varphi$?

current i_{sol}

$$i_{sol} = \int_{\Omega_{sol}} \vec{J}\cdot d\vec{S}$$

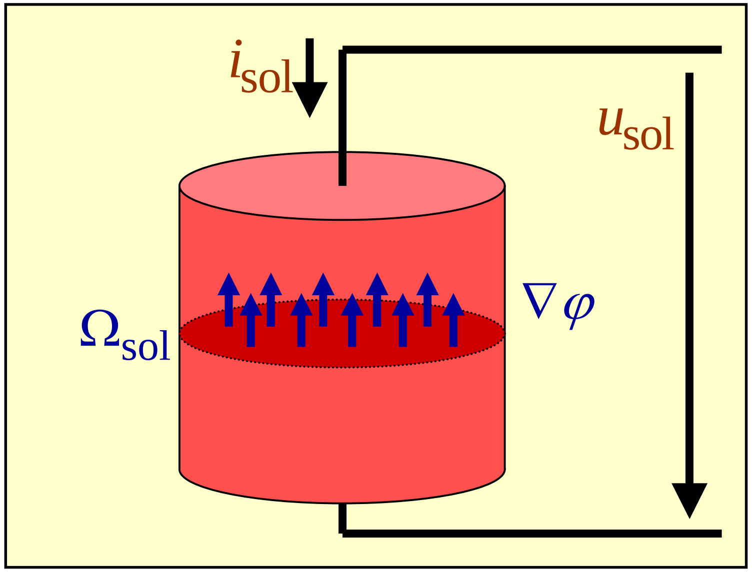
Coupling at Reference Plane

1. $\vec{A}$ and $\nabla\varphi$ do not necessary have to be continuous !

2. only $\vec{B} = \nabla \times \vec{A}$

   and $\vec{E} = -\nabla\varphi - \dfrac{\partial \vec{A}}{\partial t}$

   have to fulfill certain conditions



3. choose φ piecewise constant such that $\nabla\varphi$ represents a jump at solid-conductor cross-section Ω_{sol} and such that

$$u_{\mathrm{sol}} = -\int_0^{\ell_{\mathrm{sol}}} \nabla\varphi \cdot d\ell$$

4. total current

$$i_{\mathrm{sol}} = \int_{\Omega_{\mathrm{sol}}} \vec{J} \cdot d\vec{s} = \int_{\Omega_{\mathrm{sol}}} (-\sigma\nabla\varphi) \cdot d\vec{S} - \int_{\Omega_{\mathrm{sol}}} \sigma \frac{\partial \vec{A}}{\partial t} \cdot d\vec{S}$$

Coupling at Reference Plane

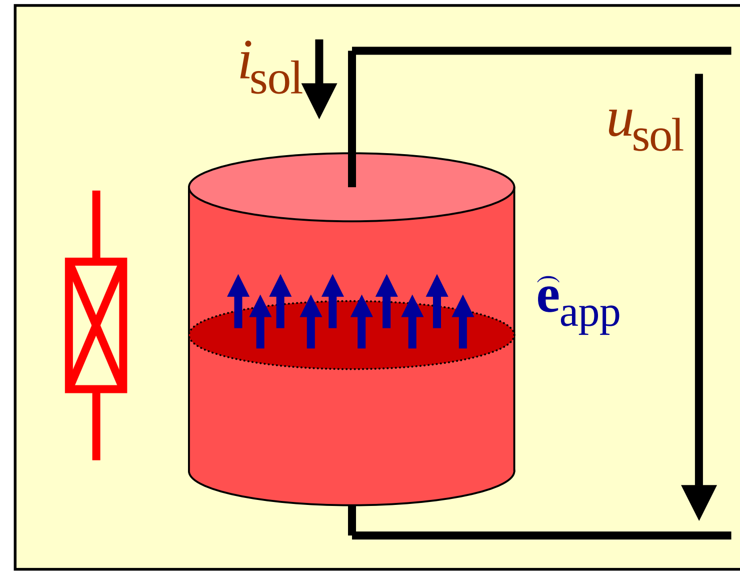
at primary edges

$$\widehat{\mathbf{e}}_{\mathrm{app}} = \tilde{\mathbf{Q}}_{\mathrm{sol}}\, u_{\mathrm{sol}}$$

at dual facets

$$i_{\mathrm{sol}} = \tilde{\mathbf{Q}}_{\mathrm{sol}}^{T}\, \widehat{\widehat{\mathbf{j}}}_{\mathrm{sol}}$$

$\tilde{\mathbf{Q}}_{\mathrm{sol}}$ = 2D incidence matrix



$$\widehat{\widehat{\mathbf{j}}}_{\mathrm{s}} = \mathbf{M}_{\sigma}\tilde{\mathbf{Q}}_{\mathrm{sol}}\, u_{\mathrm{sol}}$$ (not divergence-free)

$$\widehat{\widehat{\mathbf{j}}}_{\mathrm{sol}} = \mathbf{M}_{\sigma}\left(\tilde{\mathbf{Q}}_{\mathrm{sol}}\, u_{\mathrm{sol}} - \frac{d\widehat{\mathbf{a}}}{dt}\right)$$ (divergence-free)

$$\tilde{G}_{\mathrm{sol}} = \tilde{\mathbf{Q}}_{\mathrm{sol}}^{T}\mathbf{M}_{\sigma}\tilde{\mathbf{Q}}_{\mathrm{sol}}$$

= conductance of the reference layer

$$\begin{bmatrix} \tilde{\mathbf{C}}\mathbf{M}_{\nu}\mathbf{C} + j\omega\mathbf{M}_{\sigma} & -\mathbf{M}_{\sigma}\tilde{\mathbf{Q}}_{\mathrm{sol}} \\ -j\omega\tilde{\mathbf{Q}}_{\mathrm{sol}}^{T}\mathbf{M}_{\sigma} & \tilde{G}_{\mathrm{sol}} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}} \\ u_{\mathrm{sol}} \end{bmatrix} = \begin{bmatrix} 0 \\ i_{\mathrm{sol}} \end{bmatrix}$$

Coupling at conductor volume

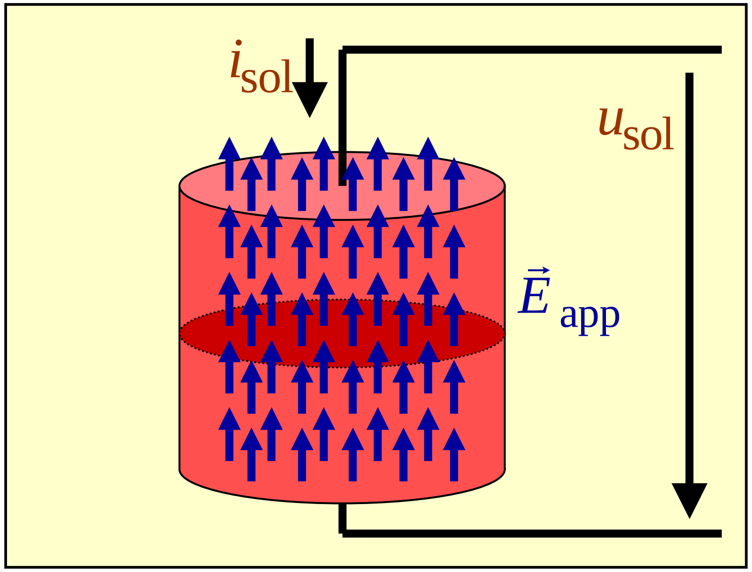
1. it is possible to define the electric scalar potential φ such that the source current is divergence-free

2. solve $-\nabla\cdot(\sigma\nabla\varphi)=0$ with boundary conditions

3. dense coupling !

4. total current

$$\vec{J}=\underbrace{-\sigma\nabla\varphi}_{\vec{J}_{\mathrm{s}}}\underbrace{-\sigma\frac{\partial\vec{A}}{\partial t}}_{\vec{J}_{\mathrm{e}}}$$



$$i_{\mathrm{sol}}=\int_{\Omega_{\mathrm{sol}}}\vec{J}\cdot d\vec{s}=\int_{\Omega_{\mathrm{sol}}}(-\sigma\nabla\varphi)\cdot d\vec{S}-\int_{\Omega_{\mathrm{sol}}}\sigma\frac{\partial\vec{A}}{\partial t}\cdot d\vec{S}$$

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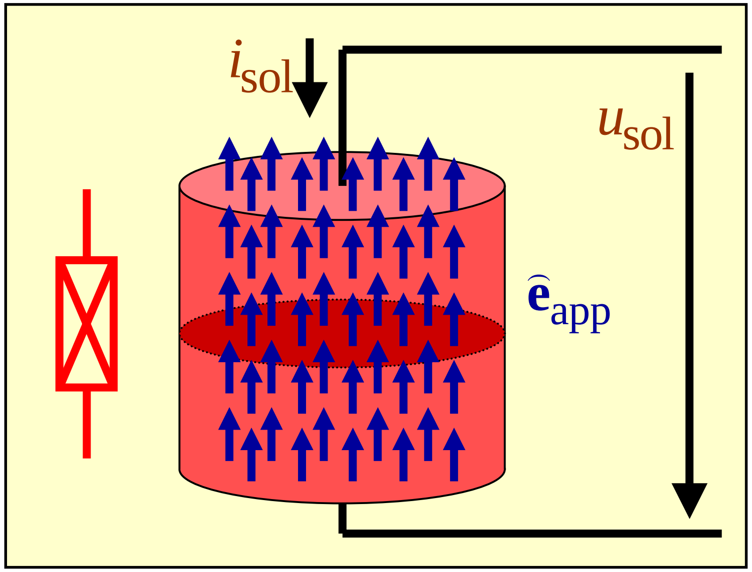
Coupling at conductor volume

electrokinetic solution:

$$\tilde{\mathbf{S}}\mathbf{M}_\sigma\tilde{\mathbf{S}}^T\phi = -\tilde{\mathbf{S}}\mathbf{M}_\sigma\widehat{\mathbf{e}}_{\mathrm{plane}}$$



$$\widehat{\mathbf{e}}_{\mathrm{app}} = \widehat{\mathbf{e}}_{\mathrm{plane}} + \tilde{\mathbf{S}}^T\phi = \mathbf{Q}_{\mathrm{sol}}\, u_{\mathrm{sol}}$$

$$\widehat{\widehat{\mathbf{j}}}_{\mathrm{s}} = \mathbf{M}_\sigma\widehat{\mathbf{e}}_{\mathrm{app}} = \mathbf{M}_\sigma\mathbf{Q}_{\mathrm{sol}}\, u_{\mathrm{sol}}$$

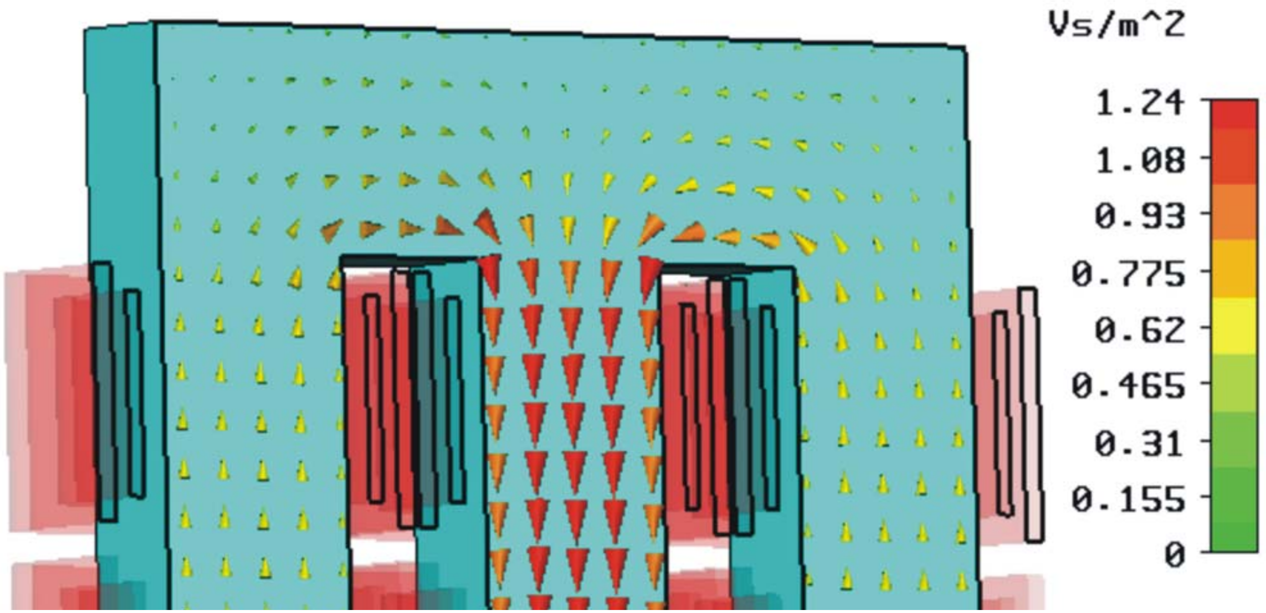
$$\widehat{\widehat{\mathbf{j}}}_{\mathrm{sol}} = \mathbf{M}_\sigma\left(\mathbf{Q}_{\mathrm{sol}}\, u_{\mathrm{sol}} - \frac{d\widehat{\mathbf{a}}}{dt}\right)$$

$$\begin{bmatrix} \tilde{\mathbf{C}}\mathbf{M}_\nu\mathbf{C} + j\omega\mathbf{M}_\sigma & -\mathbf{M}_\sigma\mathbf{Q}_{\mathrm{sol}} \\ \boxed{-j\omega\mathbf{Q}_{\mathrm{sol}}^T\mathbf{M}_\sigma} & G_{\mathrm{sol}} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}} \\ u_{\mathrm{sol}} \end{bmatrix} = \begin{bmatrix} 0 \\ i_{\mathrm{sol}} \end{bmatrix}$$

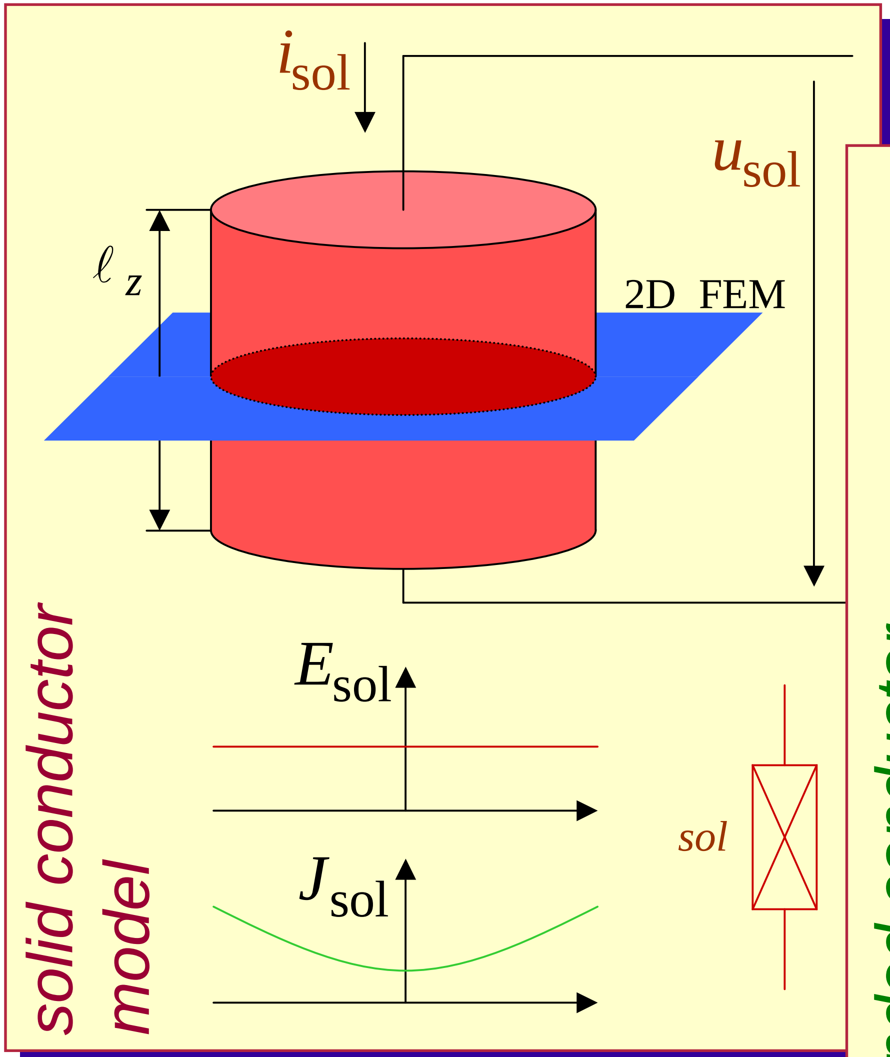
relatively dense

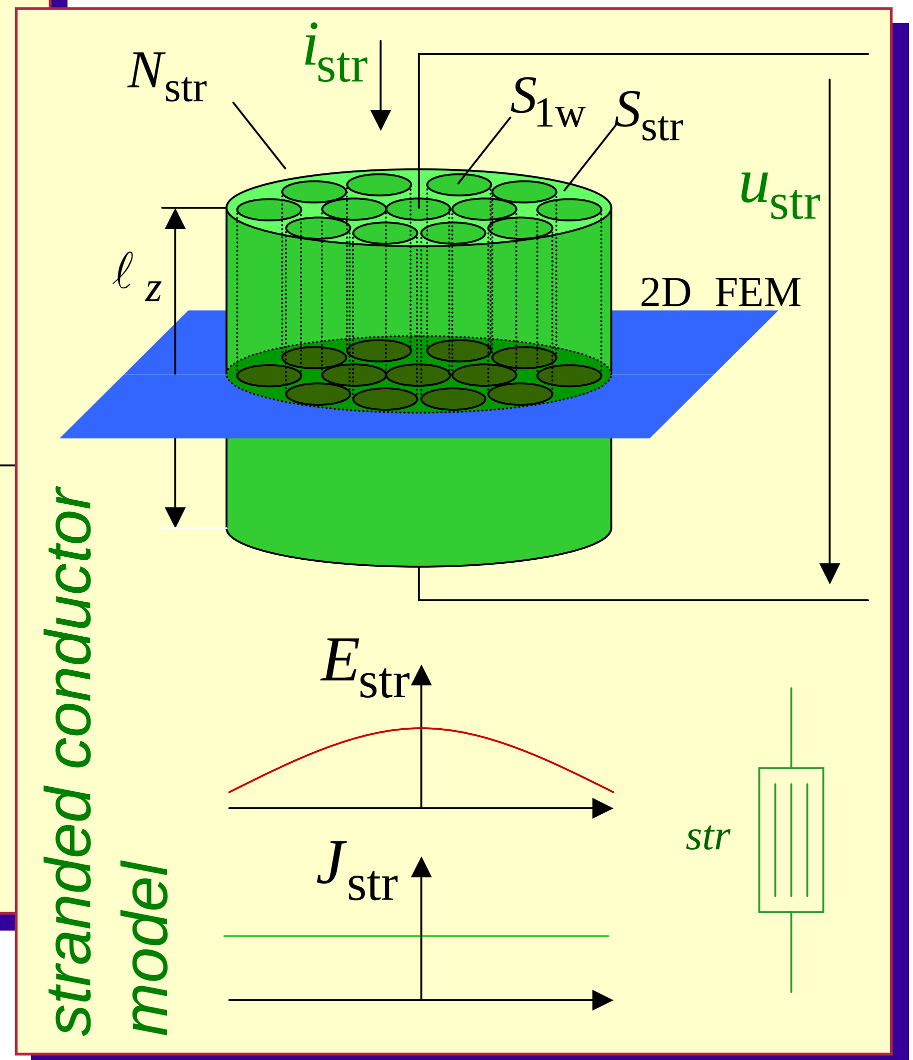
Numerical Example

SSOR-COCG	coupling matrices		number of iterations	solution time (s)
single-phase transformer	$\tilde{\mathbf{Q}}_{\mathrm{sol}}, \tilde{\mathbf{Q}}_{\mathrm{str}}$	(2D)	198	15
	$\mathbf{Q}_{\mathrm{sol}}, \mathbf{Q}_{\mathrm{str}}$	(3D)	127	12
three-phase transformer	$\tilde{\mathbf{Q}}_{\mathrm{sol}}, \tilde{\mathbf{Q}}_{\mathrm{str}}$	(2D)	756	145
	$\mathbf{Q}_{\mathrm{sol}}, \mathbf{Q}_{\mathrm{str}}$	(3D)	465	176



Conductor Models





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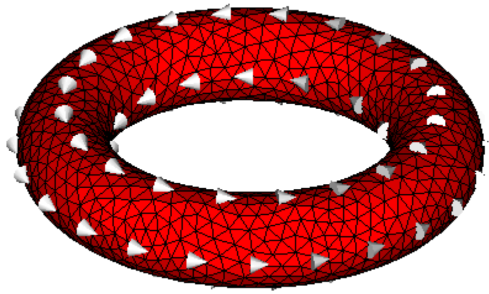
Stranded Conductor Model (1)



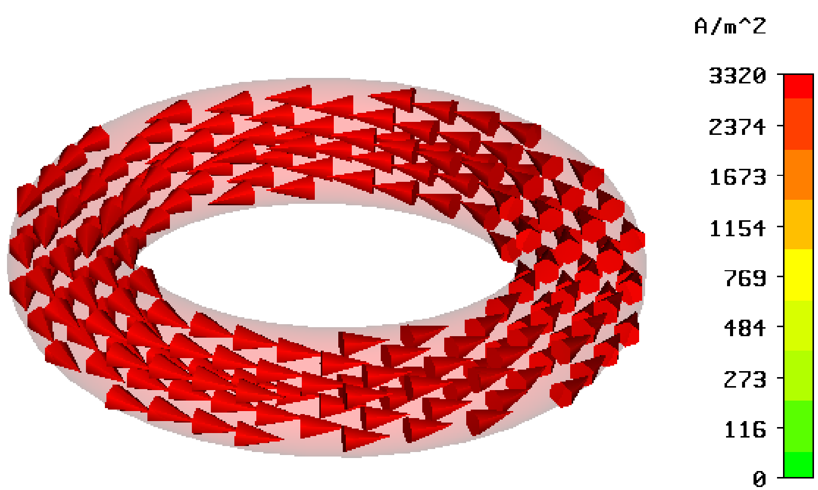
assumptions
- homogeneous current distribution
- no eddy currents

notice (*model*)
- there will be an induced voltage !!
- current not constant when cross-section not constant

winding function $\vec{t}_{\mathrm{str},q}$ [1/m²]
- computed geometrically
- by field solution (lecture V10)

$$\vec{J}_{\mathrm{str},q} = \vec{t}_{\mathrm{str},q}\; i_q(t)$$

$$\vec{J}_{\mathrm{s}} = \sum_{q=1}^{n_{\mathrm{str}}} \vec{t}_{\mathrm{str},q}\; i_q(t)$$



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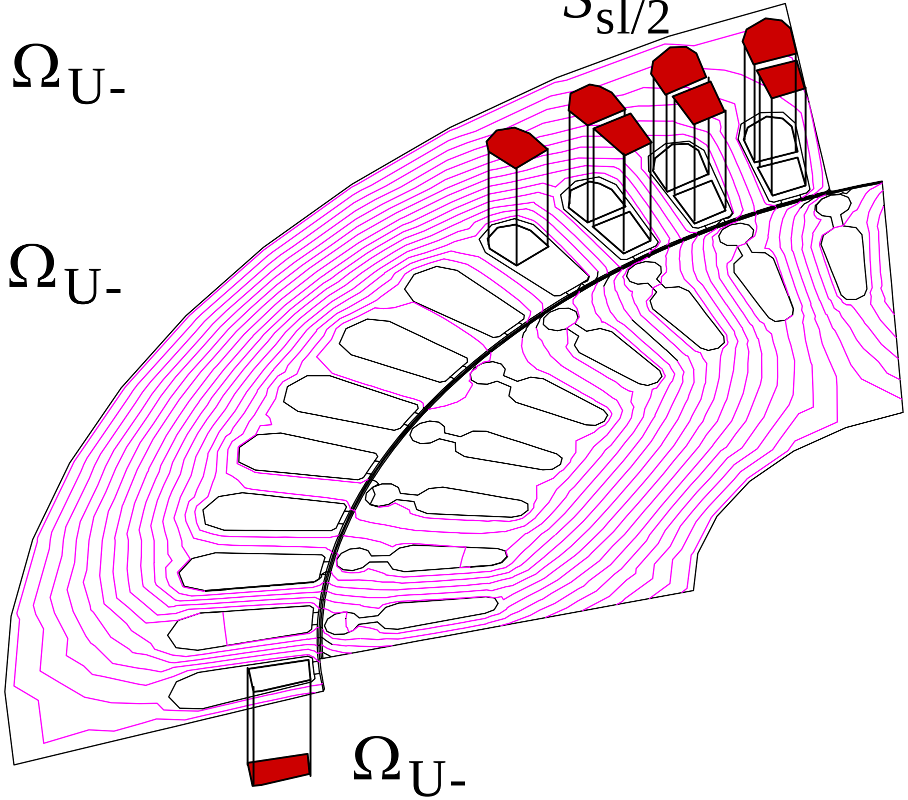
Stranded Conductor Model (2)

in 2D: $\vec{J} = \left(0, 0, J_z(x, y)\right)$

$$\begin{cases} \vec{t}_{\mathrm{str},U} = +\dfrac{N_\mathrm{t}}{S_{\mathrm{sl}/2}}\vec{e}_z & \text{in } \Omega_{\mathrm{U+}} \\ \vec{t}_{\mathrm{str},U} = -\dfrac{N_\mathrm{t}}{S_{\mathrm{sl}/2}}\vec{e}_z & \text{in } \Omega_{\mathrm{U-}} \\ \vec{t}_{\mathrm{str},U} = 0 & \text{in } \Omega \setminus \Omega_{\mathrm{U+}} \setminus \Omega_{\mathrm{U-}} \end{cases}$$



Stranded Conductor Model (3)

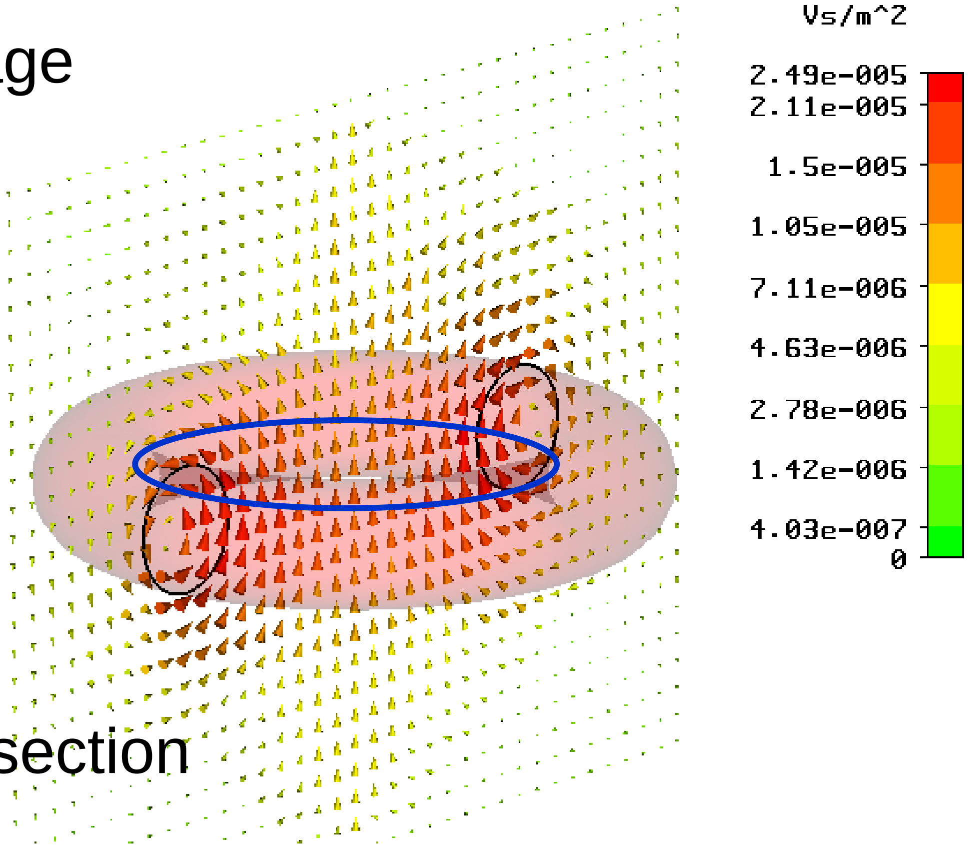
induced voltage ~ flux linkage
- which flux is linked?

for a single path

$$\phi = \oint_{\partial S} \vec{A} \cdot \mathrm{d}\vec{s}$$

for a coil
- integrating along the coil
- average at the coil cross-section

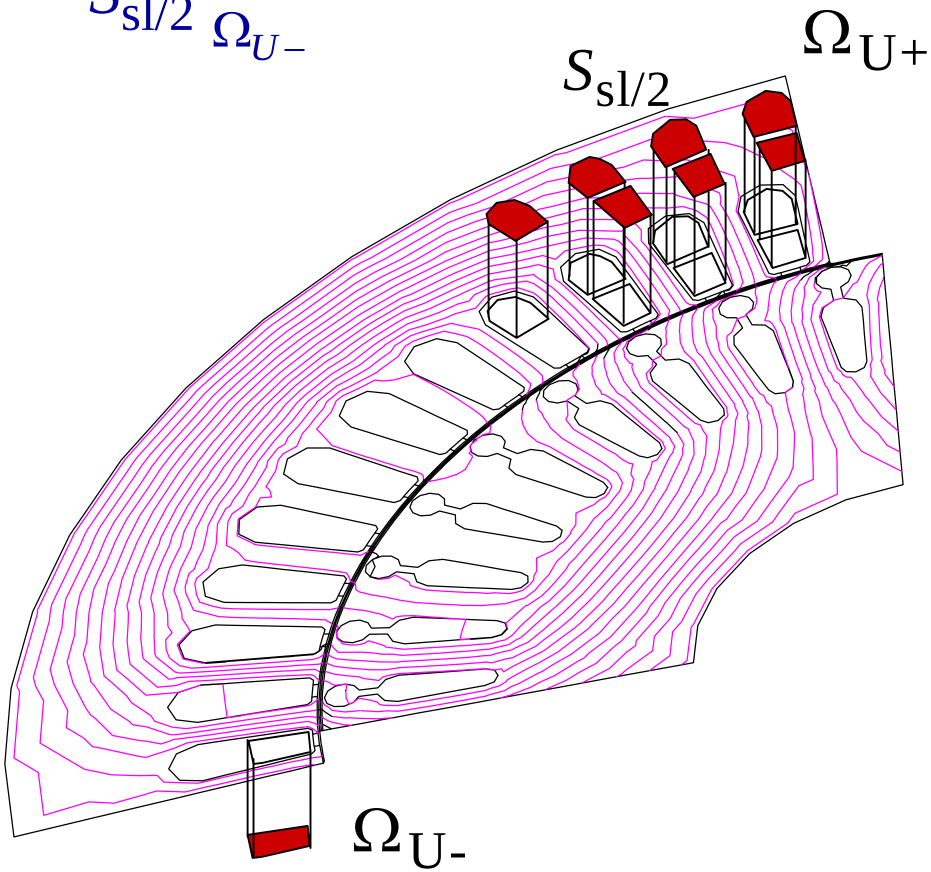
$$\psi_{\mathrm{str},q} = \int_{\Omega} \vec{A} \cdot \vec{t}_{\mathrm{str},q} \, \mathrm{d}V$$



Stranded Conductor Model (4)

in 2D: $$\psi_{\mathrm{str},q} = \int_{\Omega} \vec{A} \cdot \vec{t}_{\mathrm{str},q} \, \mathrm{d}V$$

$$\psi_{\mathrm{str},U} = N_{\mathrm{t}} \frac{1}{S_{\mathrm{sl/2}}} \int_{\Omega_{U+}} A_z \, \mathrm{d}\Omega - N_{\mathrm{t}} \frac{1}{S_{\mathrm{sl/2}}} \int_{\Omega_{U-}} A_z \, \mathrm{d}\Omega$$



Stranded Conductor Model (5)

$$\overset{\frown}{\mathbf{j}}_i = \int_{\Omega} \vec{J}_{\mathrm{s}} \cdot \color{red}{\vec{v}_i} \color{black}{\, \mathrm{d}\Omega}$$

$$\vec{J}_{\mathrm{s}} = \sum_{q=1}^{n_{\mathrm{str}}} \vec{t}_{\mathrm{str},q} \; i_q(t)$$

$$\overset{\frown}{\mathbf{j}}_i = \sum_{q=1}^{n_{\mathrm{str}}} \underbrace{\int_{\Omega_q} \vec{t}_{\mathrm{str},q} \cdot \color{red}{\vec{v}_i} \color{black}{\, \mathrm{d}\Omega}}_{\mathbf{P}_{\mathrm{str},i,q}} \; i_q(t)$$

$$\boxed{\overset{\frown}{\mathbf{j}} = \mathbf{P}_{\mathrm{str}} \mathbf{i}_{\mathrm{str}}}$$

Stranded Conductor Model (6)

$$u_{\mathrm{str},q} = R_{\mathrm{str},q}\, i_q + \frac{d\psi_{\mathrm{str},q}}{dt}$$

$$\Downarrow \quad \psi_{\mathrm{str},q} = \int_{\Omega_q} \vec{A} \cdot \vec{t}_{\mathrm{str},q}\, \mathrm{d}V$$

$$u_{\mathrm{str},q} = R_{\mathrm{str},q}\, i_q + \sum_j \frac{d\widehat{\mathbf{a}}_j}{dt} \underbrace{\int_{\Omega_q} \vec{v}_j \cdot \vec{t}_{\mathrm{str},q}\, \mathrm{d}\Omega}_{\mathbf{P}_{\mathrm{str},j,q}}$$

$$\boxed{\mathbf{u}_{\mathrm{str}} = \mathbf{R}_{\mathrm{str}}\, \mathbf{i}_{\mathrm{str}} + \mathbf{P}_{\mathrm{str}}^T \frac{d\widehat{\mathbf{a}}}{dt}}$$

Stranded Conductor Model (7)

field model
+ stranded conductors
+ voltage sources

$$\begin{bmatrix} \tilde{\mathbf{C}}\mathbf{M}_\nu\mathbf{C} & -\mathbf{P}_{\mathrm{str}} \\ j\omega\mathbf{P}_{\mathrm{str}}^T & \mathbf{R}_{\mathrm{str}} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}} \\ \mathbf{i}_{\mathrm{str}} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{u}_{\mathrm{str}} \end{bmatrix}$$

symmetrisation: multiply the circuit equations by $-\frac{1}{j\omega}$

no eddy-current term !

Overview

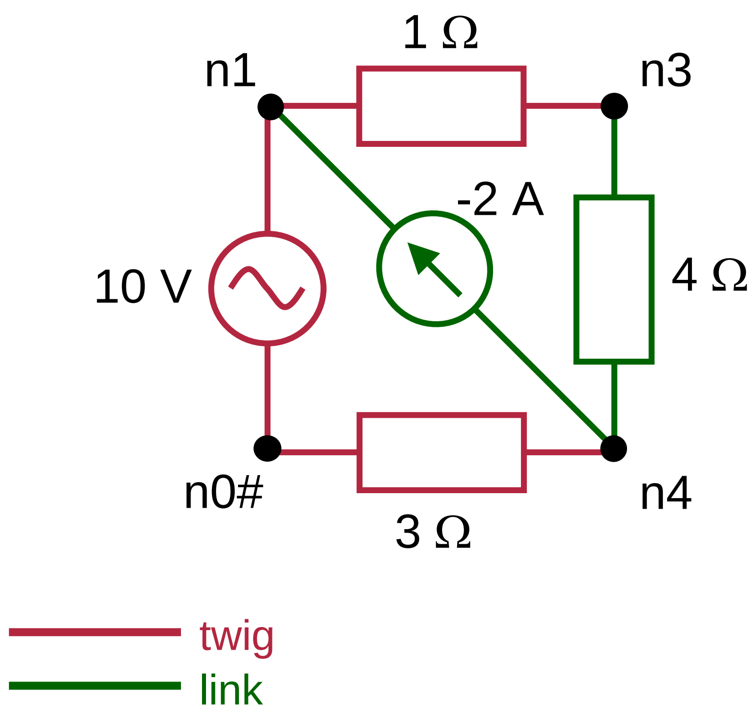
- discrete magnetoquasistatic formulation (recapitulation)
- solid conductors
- stranded conductor model
- circuit description
- example

Coupling Requirements

$\begin{bmatrix} K & B^T \\ B & C \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$	*(compacted) modified nodal analysis*	*(compacted) loop analysis*	*hybrid analysis*
unknowns	nodal voltages (+a few currents)	loop currents (+a few voltages)	twig voltages link currents
keeps the FE matrix part unchanged ▪ sparsity ▪ preconditioners (multigrid) ▪ possible benefits thanks to structured grids (FIT)	no (yes)	no (yes)	yes
preserves symmetry ▪ Krylov subspace solvers for symmetric systems (CG, MINRES, QMR) ▪ storage	yes	yes	yes
preserves positive definiteness ▪ solvers (CG) ▪ preconditioners (IC)	yes (no)	yes (no)	no [yes]

Circuit Description (1)

1. Trace a tree through the circuit



starting from the circuit node n0#,
the twigs are selected in the order
1. voltage source 10 V
2. resistor 1 Ω
3. resistor 3 Ω

Priority list

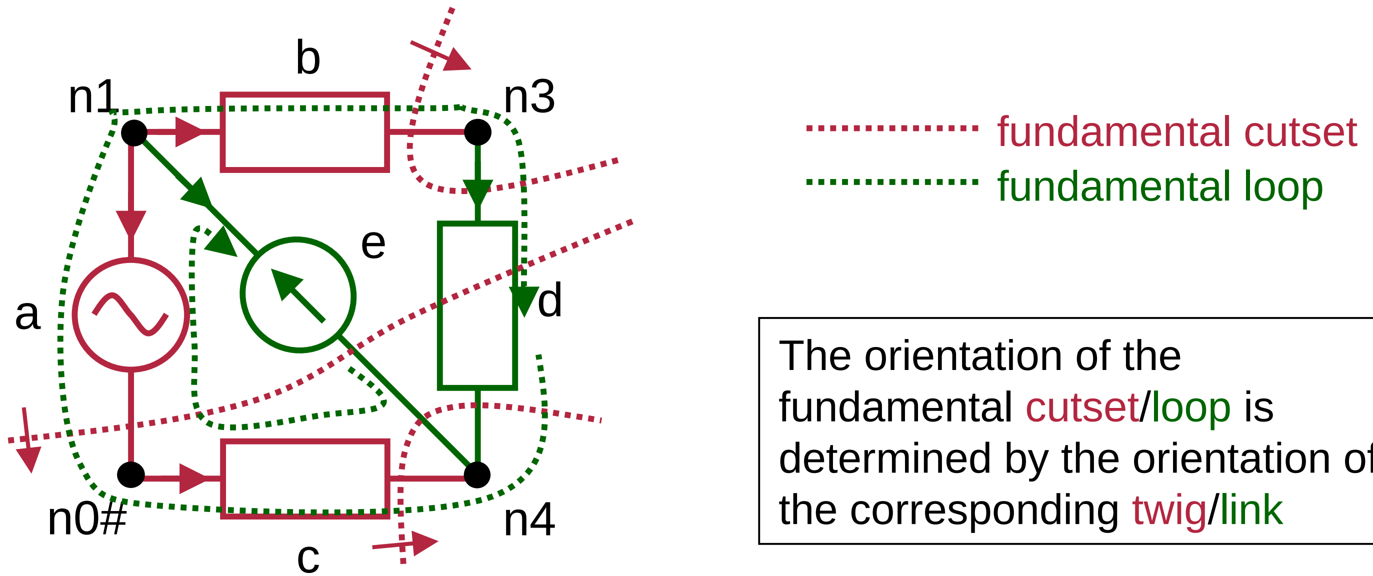
highest priority, preferably twig

- voltage sources
- solid conductors (coupled)
- capacitors (largest capacitance first)
- resistors (largest conductance first)
- inductors (smallest inductance first)
- stranded conductors (coupled)
- current sources

smallest priority, preferably link

Circuit Description (2)

2. Determine fundamental cutsets and fundamental loops



The orientation of the fundamental cutset/loop is determined by the orientation of the corresponding twig/link

A fundamental cutset is formed by 1 twig and the unique set of links completing the set of branches which would upon removal result in two disconnected circuit parts.

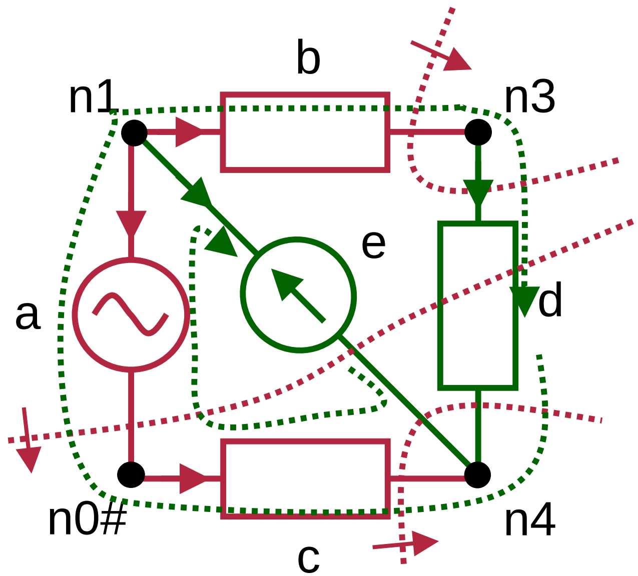
A fundamental loop consists of 1 link and the unique path through the tree closing the loop.

Property: priority(twig) $\geq$ priority(branch), $\forall$branch $\in$ fundamental cutset

Property: priority(link) $\leq$ priority(branch), $\forall$branch $\in$ fundamental loop

Circuit Description (3)

3. Construct the fundamental cutset and fundamental loop matrices



fundamental cutset matrix

$$D = \left[\begin{array}{ccc|cc} 1 & & & 1 & 1 \\ & 1 & & -1 & \\ & & 1 & 1 & 1 \end{array}\right]$$

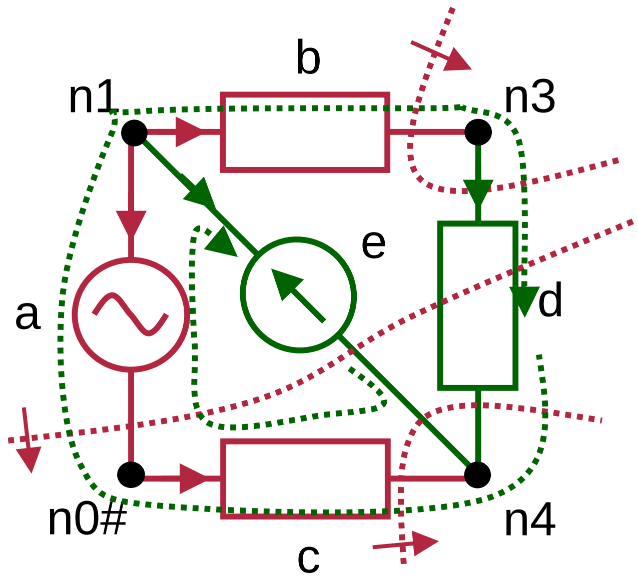
fundamental loop matrix

$$B = \left[\begin{array}{ccc|cc} -1 & 1 & -1 & 1 & \\ -1 & & -1 & & 1 \end{array}\right]$$

remark: $B_{\mathrm{ln,tw}} = -D^{\mathrm{T}}_{\mathrm{tw,ln}}$

Circuit Description (4)

4. Partition the fundamental incidence matrices



$$D = \left[\begin{array}{ccc:cc} 1 & & & \boxed{1} & \boxed{1} \\ & 1 & & \boxed{-1} & \\ & & 1 & \boxed{1} & \boxed{1} \end{array}\right]$$

$D_{\mathrm{twv,lno}}$

$D_{\mathrm{twv,lni}}$

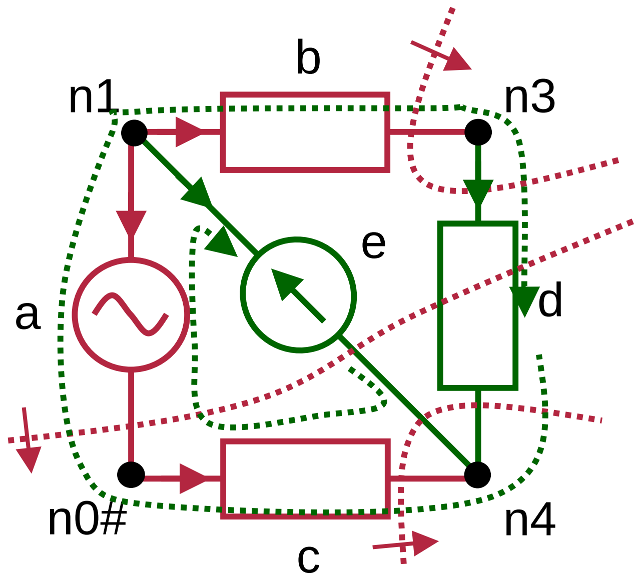
$D_{\mathrm{two,lni}}$

$D_{\mathrm{two,lno}}$

twv	twigs at which the voltage is known (voltage sources)
two	twigs at which an unknown voltage is assigned (“*free twigs*”)
twu	eliminated twigs (“*eliminated twigs*”) (not in this example)
lnu	eliminated links (“*eliminated links*”) (not in this example)
lno	links at which an unknown current is assigned (“*free links*”)
lni	links at which the current is known (current sources)

Circuit Description (5)

5. Write impedance/admittance matrices and voltage/current vectors



1. admittance matrix
   for the free twigs

$$Y_{\mathrm{two}} = \begin{bmatrix} 1/1 & \\ & 1/3 \end{bmatrix}$$

2. impedance matrix
   for the free links

$$Z_{\mathrm{lno}} = [4]$$

3. voltage vector
   for the voltage sources

$$u_{\mathrm{twv}} = [10]$$

4. current vector
   for the current sources

$$i_{\mathrm{lni}} = [2]$$

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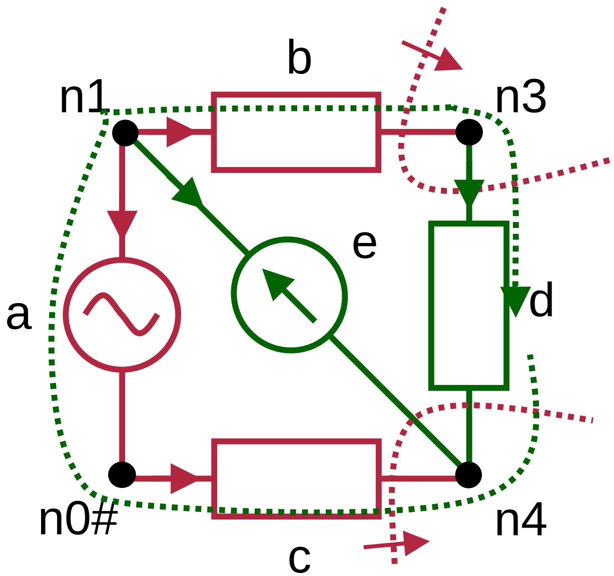
Circuit Description (6)

6. Write system of equations



intuitive approach:

1. write the Kirchhoff current law for each fundamental cutset associated with a free twig

2. write the Kirchhoff voltage law for each fundamental loop associated with a free link

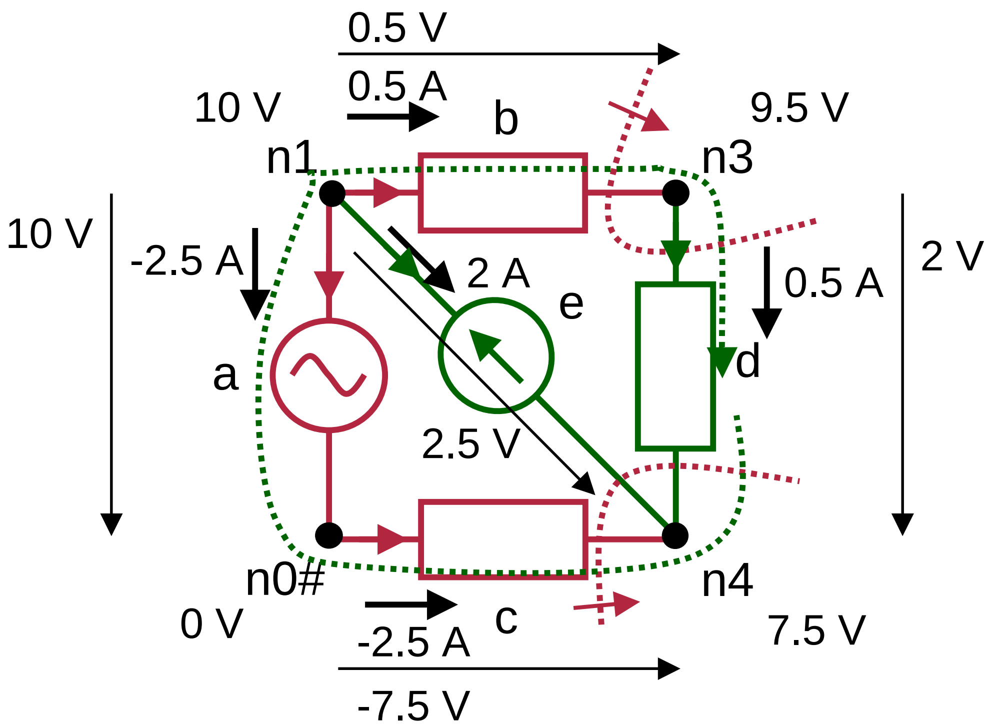
$$\left[\begin{array}{cc:c} 1 & 0 & -1 \\ 0 & 0.333 & 1 \\ \hdashline -1 & 1 & -4 \end{array}\right]\left[\begin{array}{c} u_b \\ u_c \\ \hdashline i_d \end{array}\right] = \left[\begin{array}{c} 0 \\ -2 \\ \hdashline -10 \end{array}\right]$$

$$\begin{bmatrix} Y_{\text{two}} & D_{\text{two,lno}} \\ -B_{\text{lno,two}} & -Z_{\text{lno}} \end{bmatrix}\begin{bmatrix} u_{\text{two}} \\ i_{\text{lno}} \end{bmatrix} = \begin{bmatrix} -D_{\text{two,lni}}\, i_{\text{lni}} \\ B_{\text{lno,twv}}\, v_{\text{twv}} \end{bmatrix}$$

remark: symmetric because $B_{\text{lno,two}} = -D_{\text{two,lno}}^{\mathrm{T}}$

Circuit Description (7)

7. Solve system of equations & propagate the circuit solution



solution

$$\begin{bmatrix} u_b \\ u_c \\ \hdashline i_d \end{bmatrix} = \begin{bmatrix} 0.5 \\ -7.5 \\ \hdashline 0.5 \end{bmatrix}$$

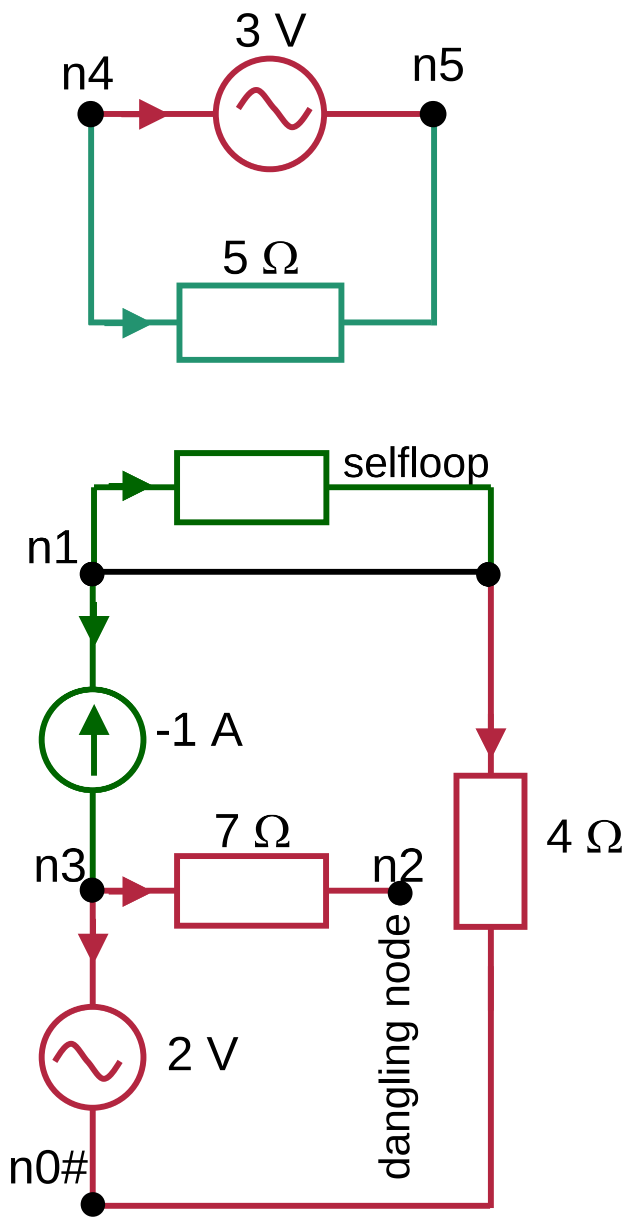
twig currents:

$$i_{\text{two}} = -D_{\text{two,lno}} i_{\text{lno}} - D_{\text{two,lni}} i_{\text{lni}}$$
$$i_{\text{twv}} = -D_{\text{twv,lno}} i_{\text{lno}} - D_{\text{twv,lni}} i_{\text{lni}}$$

link voltages

$$u_{\text{lno}} = -B_{\text{lno,two}} u_{\text{two}} - B_{\text{lno,twv}} u_{\text{twv}}$$
$$u_{\text{lni}} = -B_{\text{lni,two}} u_{\text{two}} - B_{\text{lni,twv}} u_{\text{twv}}$$

Particularities



1. Distinct circuit parts

2. Dangling nodes

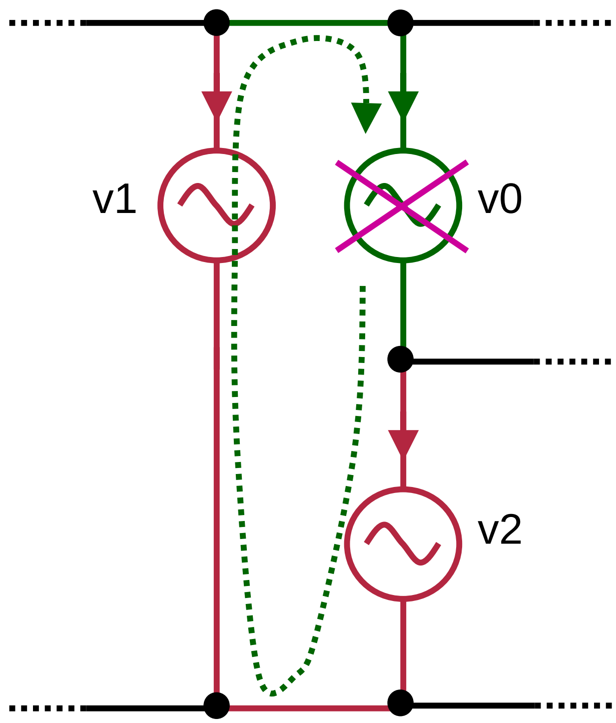
a branch to a dangling node
always a twig
associated fundamental cutset
only contains the twig

3. Self-loops

a self-loop
is always a link
the associated fundamental loop
only contains the link

Consistency Check (1)

1. Fundamental loop consisting of voltage sources

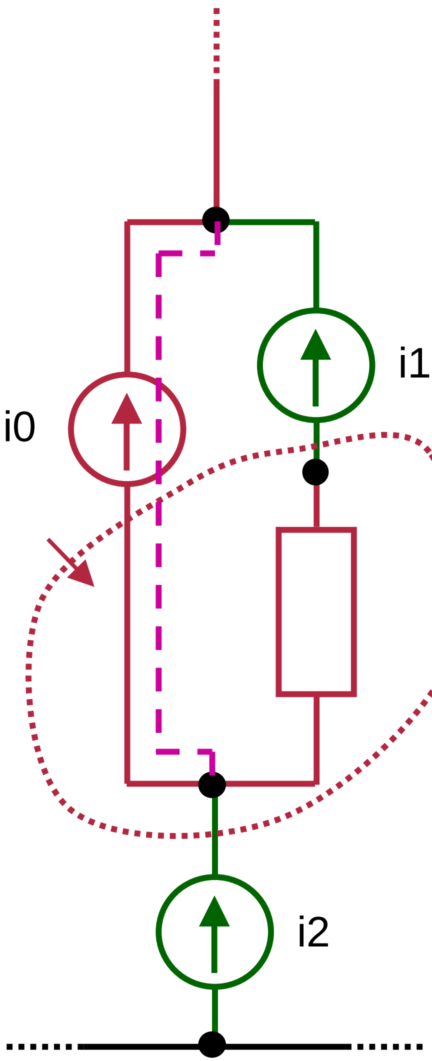


Problem: a voltage source is necessarily selected as link

Treatment: check the Kirchhoff voltage law in the associated fundamental loop
` e.g. v0-v1+v2 = 0 ??
if valid
omit the voltage source link
if not valid
the circuit has no solution

Consistency Check (2)

2. Fundamental cutset consisting of current sources



Problem: a current source is necessarily selected as twig

Treatment: check the Kirchhoff current law in the associated fundamental cutset

e.g. i0 + i1 - i2 = 0 ??

if valid

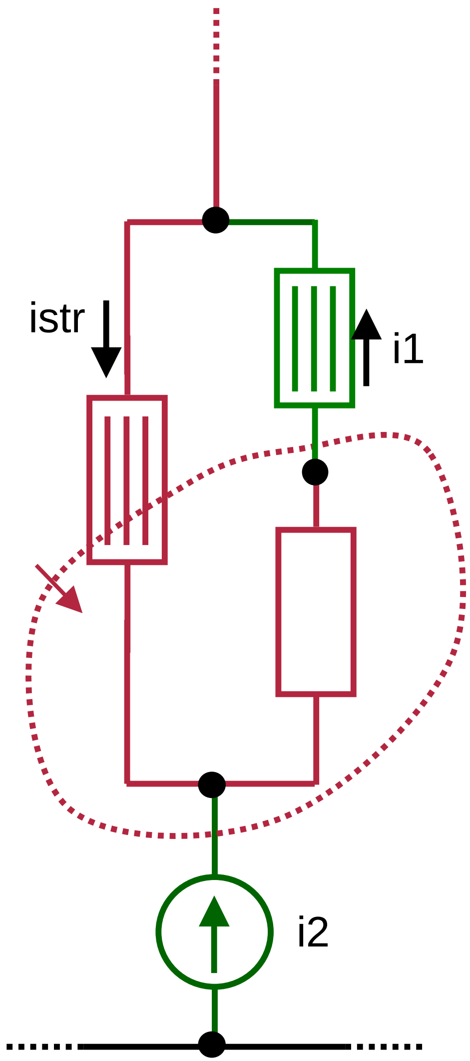
replace the current source twig by a short-circuit connection

if not valid

the circuit has no solution

Partial Transformation (1)

1. Stranded conductor being selected as twig



Problem: a stranded conductor (current-driven branch) is necessarily selected as twig

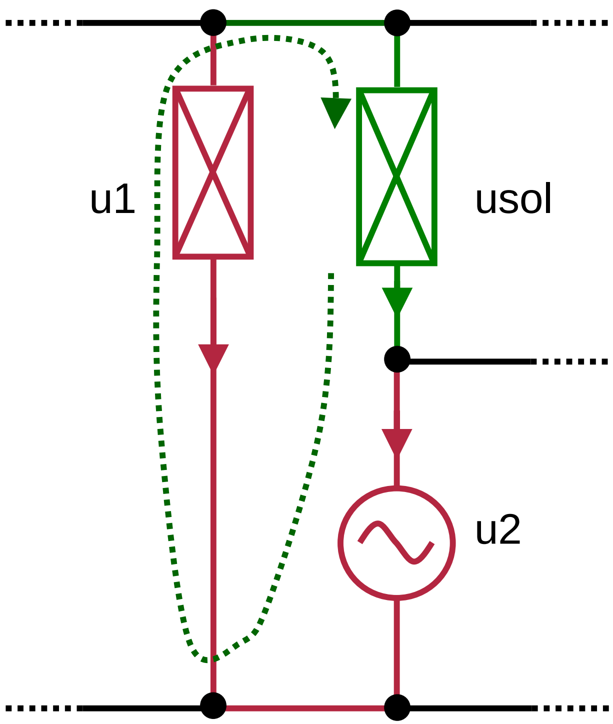
Property: priority(twig) ≥ priority(branch), ∀branch ∈ associated fundamental cutset

Treatment: apply the Kirchhoff current law in the associated fundamental cutset to express the stranded-conductor current in terms of link currents
e.g. istr = i1 + i2

~ small and independent Schur complements

Partial Transformation (2)

2. Solid conductor being selected as link



Problem: a solid conductor (voltage-driven branch) is necessarily selected as link

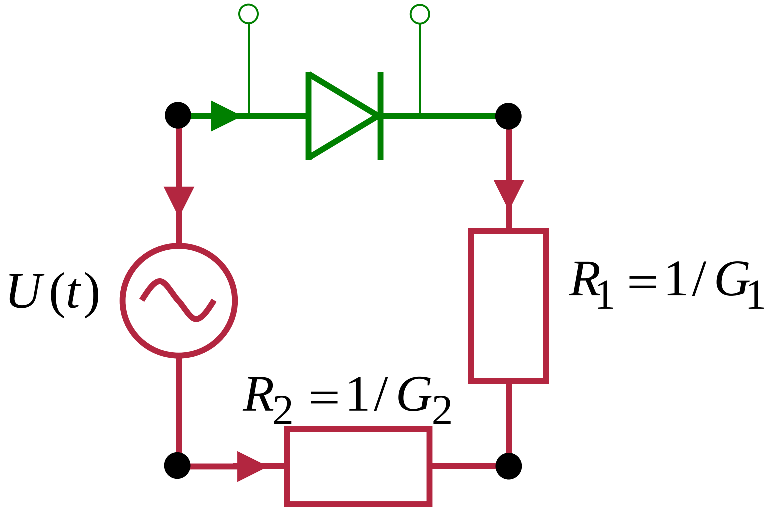
Property: priority(link) ≤ priority(branch), ∀branch ∈ associated fundamental loop

Treatment: apply the Kirchhoff voltage law in the associated fundamental loop to express the solid-conductor voltage in terms of twig voltage
e.g. usol = u1 + u2

~ small and independent Schur complements

Topological Changes (1)

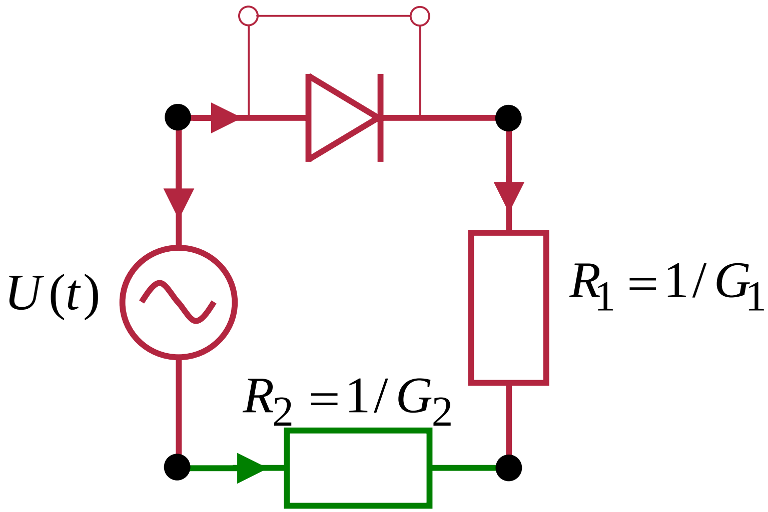
1. Switching elements closes (switch, diode, thyristor,…)



Problem: the priority of a branch increases during (transient) simulation

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Treatment: consider associated fundamental loop and possibly change link/twig-mode with the branch with the lowest priority



$$\begin{bmatrix} G_1 & 1 \\ 1 & -R_2 \end{bmatrix} \begin{bmatrix} u_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ U \end{bmatrix}$$

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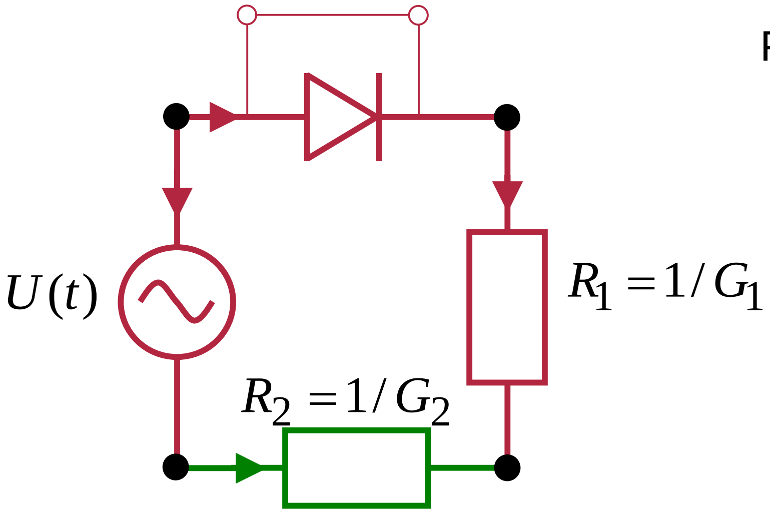
Topological Changes (2)

2. Switching element opens (switch, diode, thyristor,…)

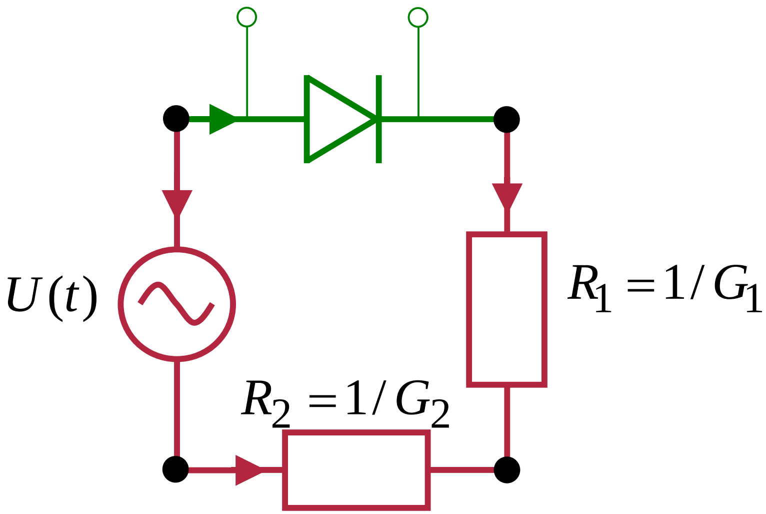


Problem: the priority of a branch decreases during (transient) simulation

$$\begin{bmatrix} G_1 & 1 \\ 1 & -R_2 \end{bmatrix} \begin{bmatrix} u_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ U \end{bmatrix}$$

Treatment: consider associated fundamental cutset and possibly change link/twig-mode with the branch with the highest priority

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Field-Circuit Coupling

1. magnetoquasistatic PDE in terms of the magnetic vector potential
2. Kirchhoff current law (applied for fundamental cutsets)
3. Kirchhoff voltage law (applied for fundamental loops)

\+ branch relations for solid and stranded conductors

\+ branch relations for resistors, capacitors, inductors, ...

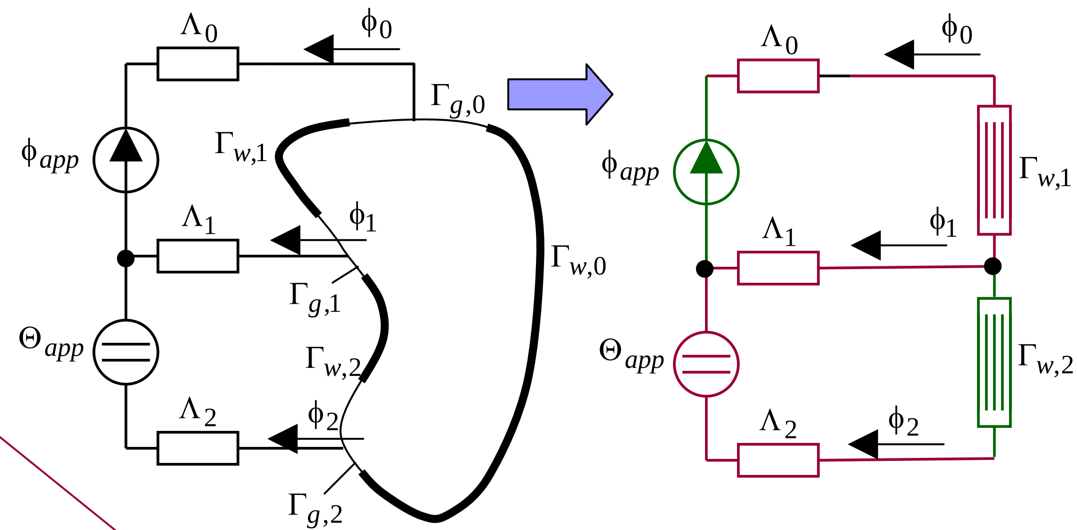
$$\left[\begin{array}{c|cc} \tilde{\mathbf{C}}\mathbf{M}_\nu\mathbf{C} + \alpha\mathbf{M}_\sigma & -\mathbf{M}_\sigma\mathbf{Q}_{\mathrm{two}} & -\mathbf{Q}_{\mathrm{lno}} \\ \hline -\mathbf{Q}_{\mathrm{two}}^T\mathbf{M}_\sigma & \chi\mathbf{G}_{\mathrm{two}} + \chi\alpha\mathbf{C}_{\mathrm{two}} & \chi\mathbf{D}_{\mathrm{two,lno}} \\ -\mathbf{Q}_{\mathrm{lno}}^T & -\chi\mathbf{B}_{\mathrm{lno,two}} & -\chi\mathbf{R}_{\mathrm{lno}} - \chi\alpha\mathbf{L}_{\mathrm{lno}} \end{array}\right] \left[\begin{array}{c} \hat{\mathbf{a}} \\ \hline \mathbf{u}_{\mathrm{two}} \\ \mathbf{i}_{\mathrm{lno}} \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline -\chi\mathbf{D}_{\mathrm{two,lni}}\mathbf{i}_{\mathrm{lni}} \\ \chi\mathbf{B}_{\mathrm{lno,twu}}\mathbf{u}_{\mathrm{twu}} \end{array}\right]$$

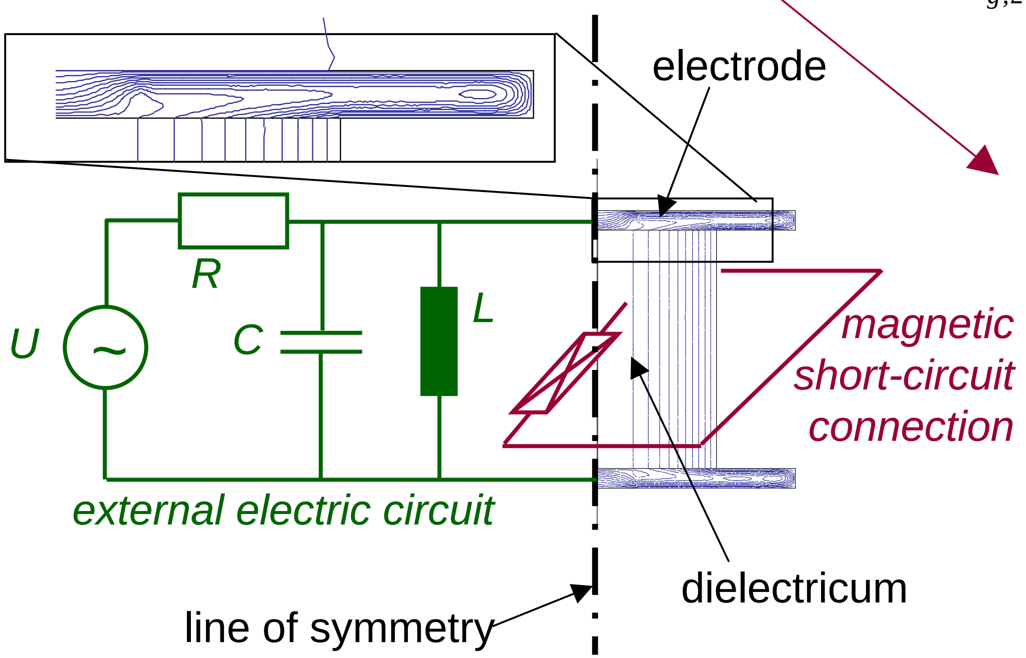
symmetrisation factor

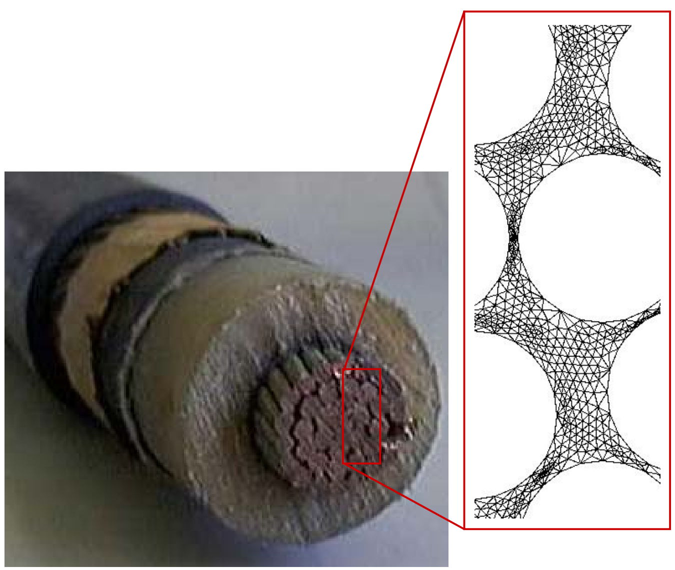
factor determined by the time integrator

Other Applications

1. magnetic field + magnetic circuit
2. magnetic field + analytical model
   + electric circuit
3. electrokinetic field
   + magnetic circuit
   + electric circuit







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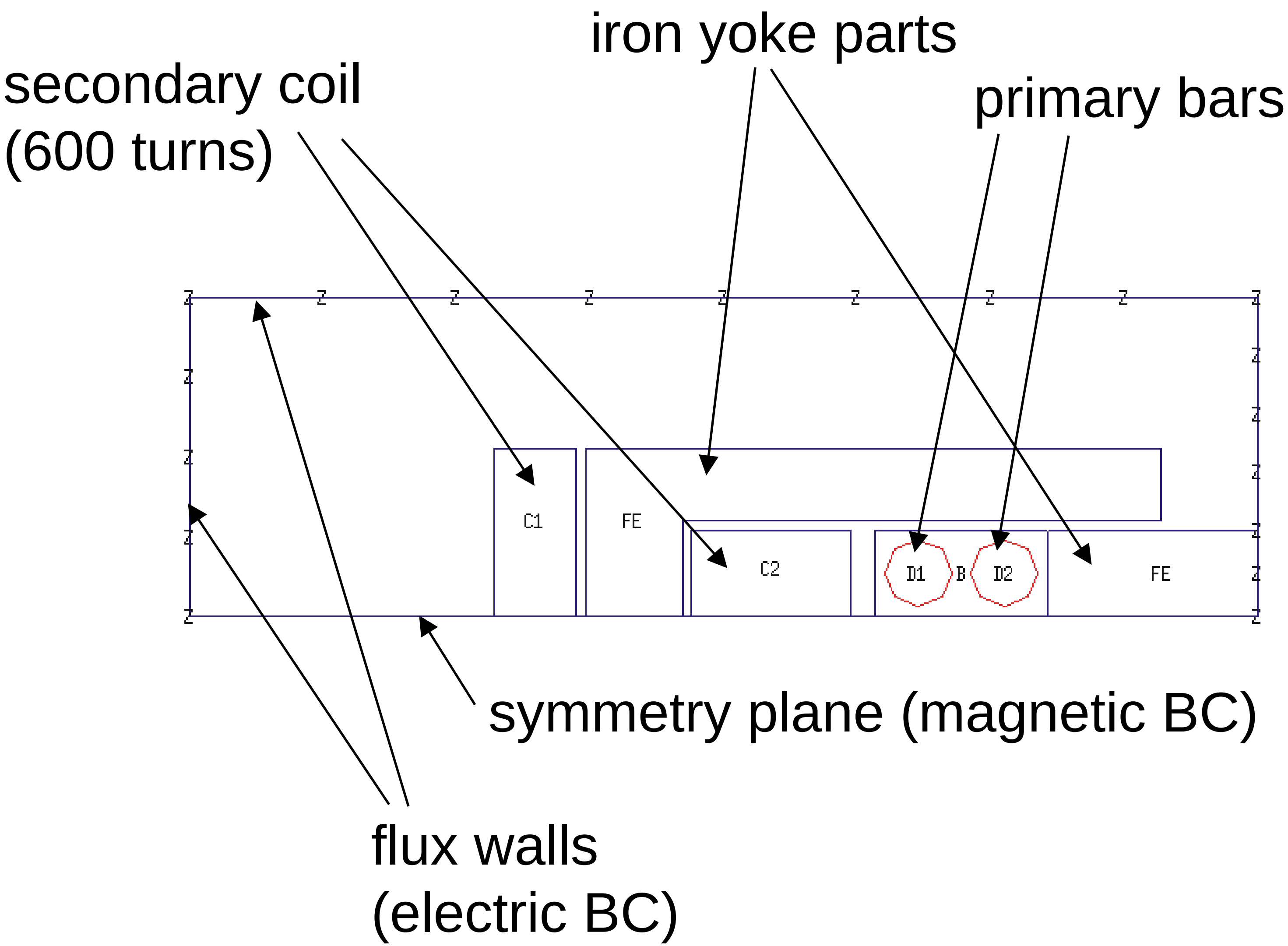
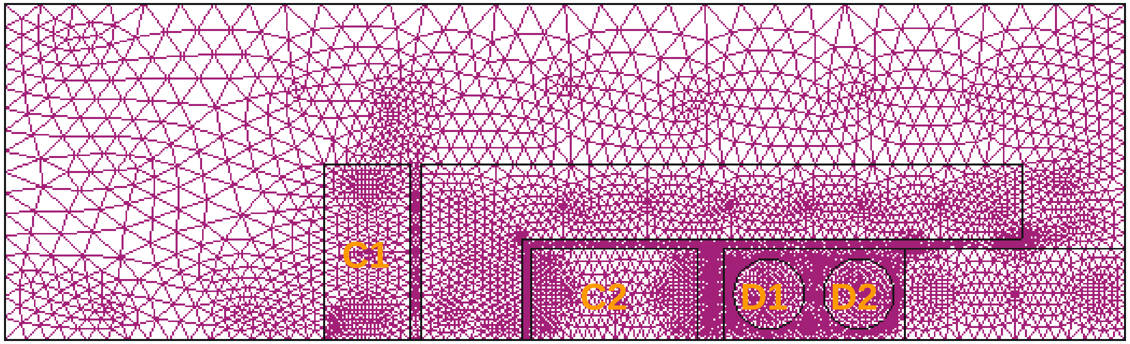
Overview

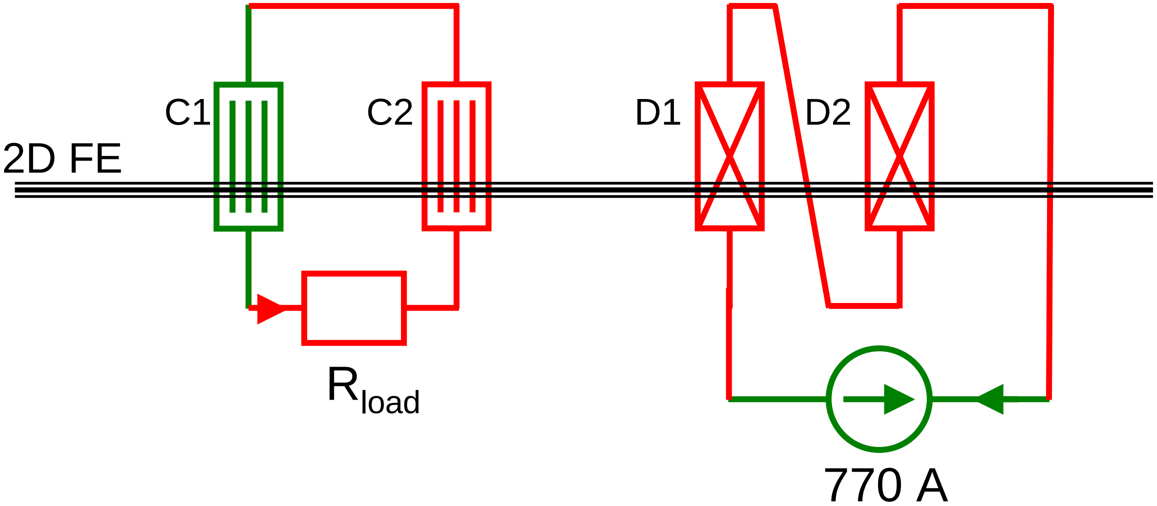
- discrete magnetoquasistatic formulation (recapitulation)
- solid conductors
- stranded conductor model
- circuit description
- example

Transformer (1)



Transformer (2)





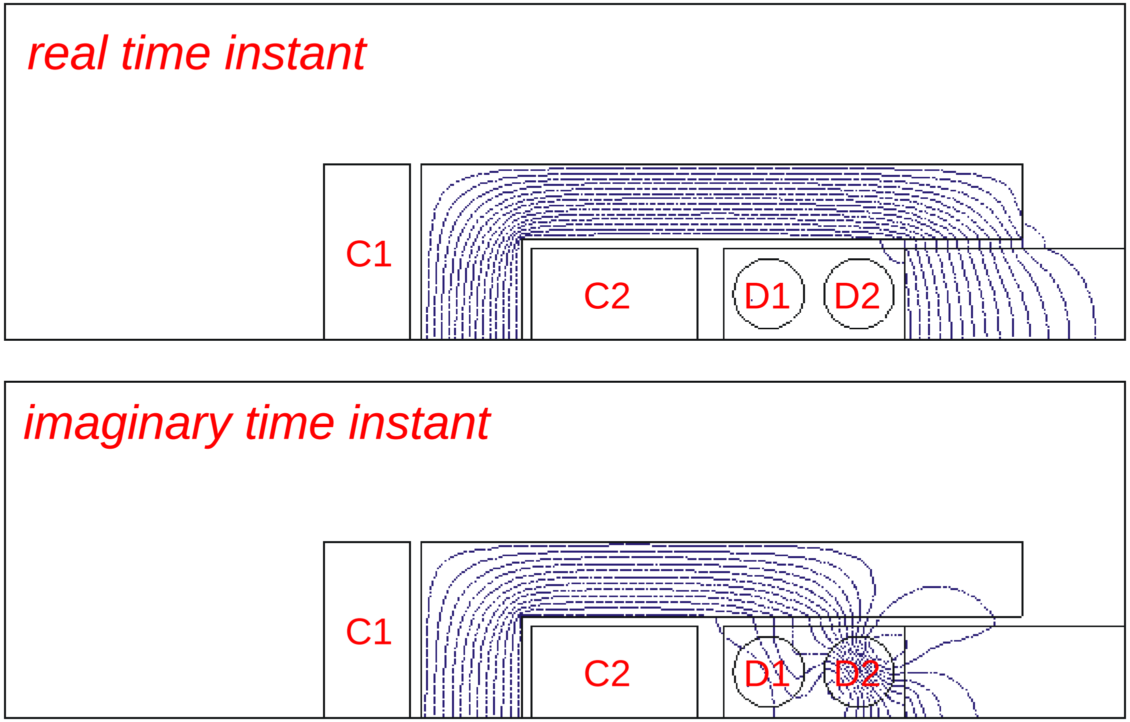
Transformer (3)

no-load operation

$$R_{\text{load}} = \infty$$

$$u_{D2} = 8.1\,\text{mV}\ \angle 89^\circ$$

$$u_{D1} = 8.4\,\text{mV}\ \angle 89^\circ$$



$$u_{C2} = 5.0\ \text{V}\ \angle 90^\circ$$

$$u_{C1} = 6.4\,\text{mV}\ \angle 90^\circ$$

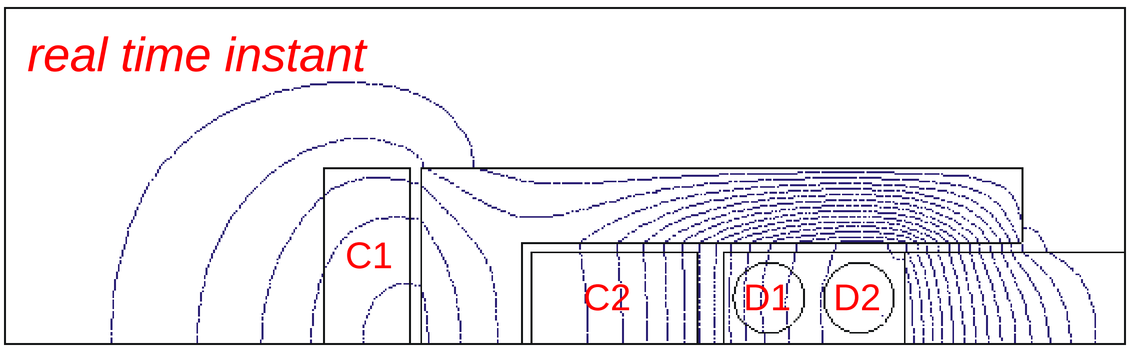
Transformer (4)

load operation

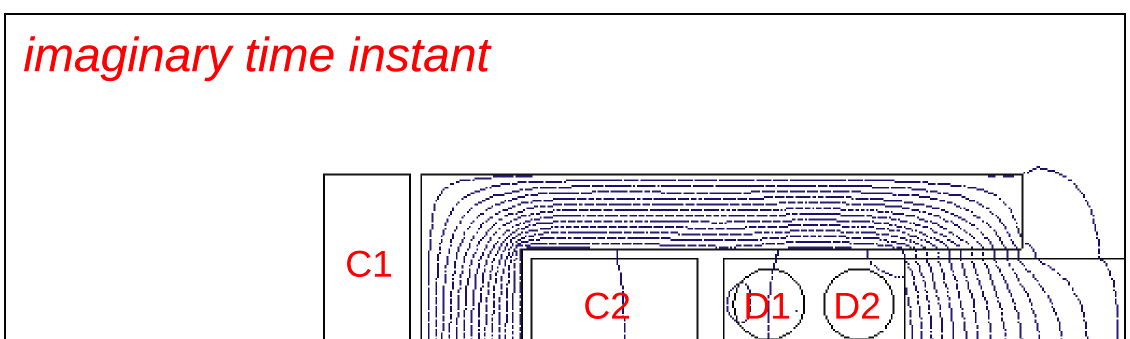
$$R_{\text{load}} = 0.1\,\Omega$$

$$u_{D2} = 1.2\text{ mV } \angle 74^\circ$$

$$u_{D1} = 1.0\text{ mV } \angle 72^\circ$$



real time instant



imaginary time instant

$$u_{C2} = 0.23\text{ V } \angle 60^\circ$$

$$u_{C1} = 0.20\text{ V } \angle 85^\circ$$

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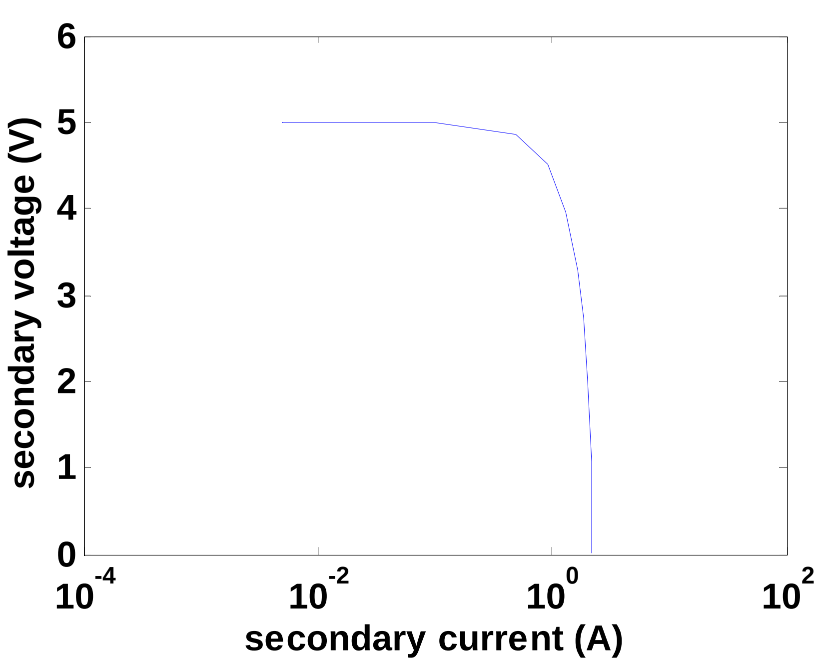
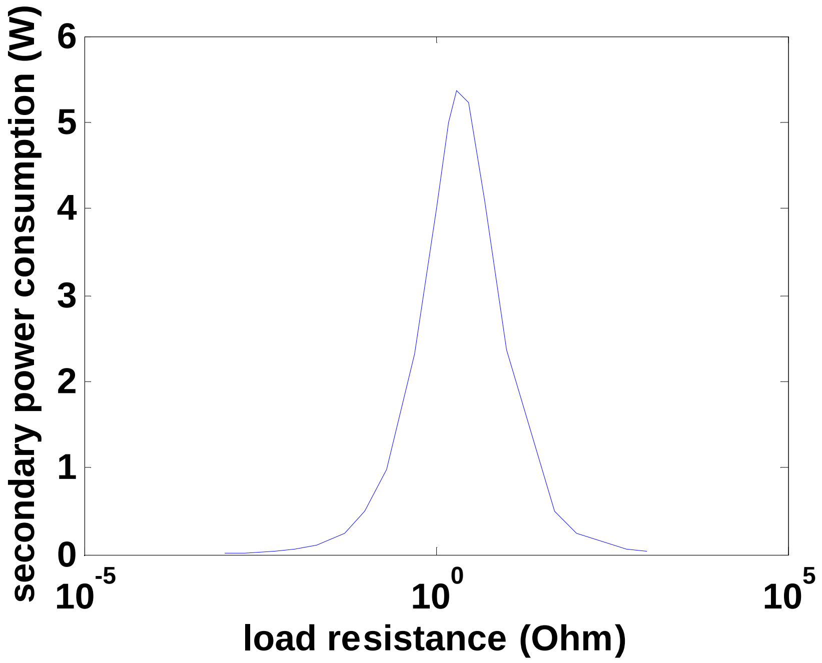
Transformer (5)





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V08: Modelling and Simulation of Induction Machines

Dr.-Ing. Herbert De Gersem
summer semester 2006
Institut für Theorie Elektromagnetischer Felder

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