TECHNISCHE UNIVERSITÄT DARMSTADT

Lecture Series

Finite-Element Electrical Machine Simulation

in the framework of the DFG Research Group 575
„High Frequency Parasitic Effects
in Inverter-Fed Electrical Drives"
http://www.ew.e-technik.tu-darmstadt.de/FOR575

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summer semester 2006

Institut für Theorie Elektromagnetischer Felder

Technische Universität Darmstadt, Fachbereich Elektrotechnik und Informationstechnik
Schloßgartenstr. 8, 64289 Darmstadt, Germany - URL: www.TEMF.de

V08:
Modelling and Simulation of Induction Machines

Overview

- literature overview
- induction machine models
  - equivalent scheme
  - coupled inductance model
  - d-q-model
- computation of stationary operation (equivalent scheme)
  - no-load operation
  - short-circuit operation
  - load operation
- computation of dynamic operation

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Literature

[1] S. Williamson, "Induction motor modelling using finite elements", ICEM 1994, Paris, 5-8 Sept, 1994, Vol. 1, pp. 1-8.

[2] E. Vassent, G. Meunier, J.C. Sabonnadiere, "Simulation of induction machine operation using complex magnetodynamic finite elements", IEEE Trans. Magn., Vol. 25, No. 4, 1989, pp. 3064-3066.

[3] A. Arkkio, "Finite element analysis of cage induction motors fed by static frequency convertors", IEEE Trans. Magn., Vol. 2, No. 2, 1990, pp. 551-554.

[4] D. Dolinar, R. De Weerdt, R. Belmans, E.M. Freeman, "Calculation of two-axis induction motor model parameters using finite elements", IEEE Trans. Energy Conversion, Vol. 12, No. 2, June 1997, pp. 133-140.

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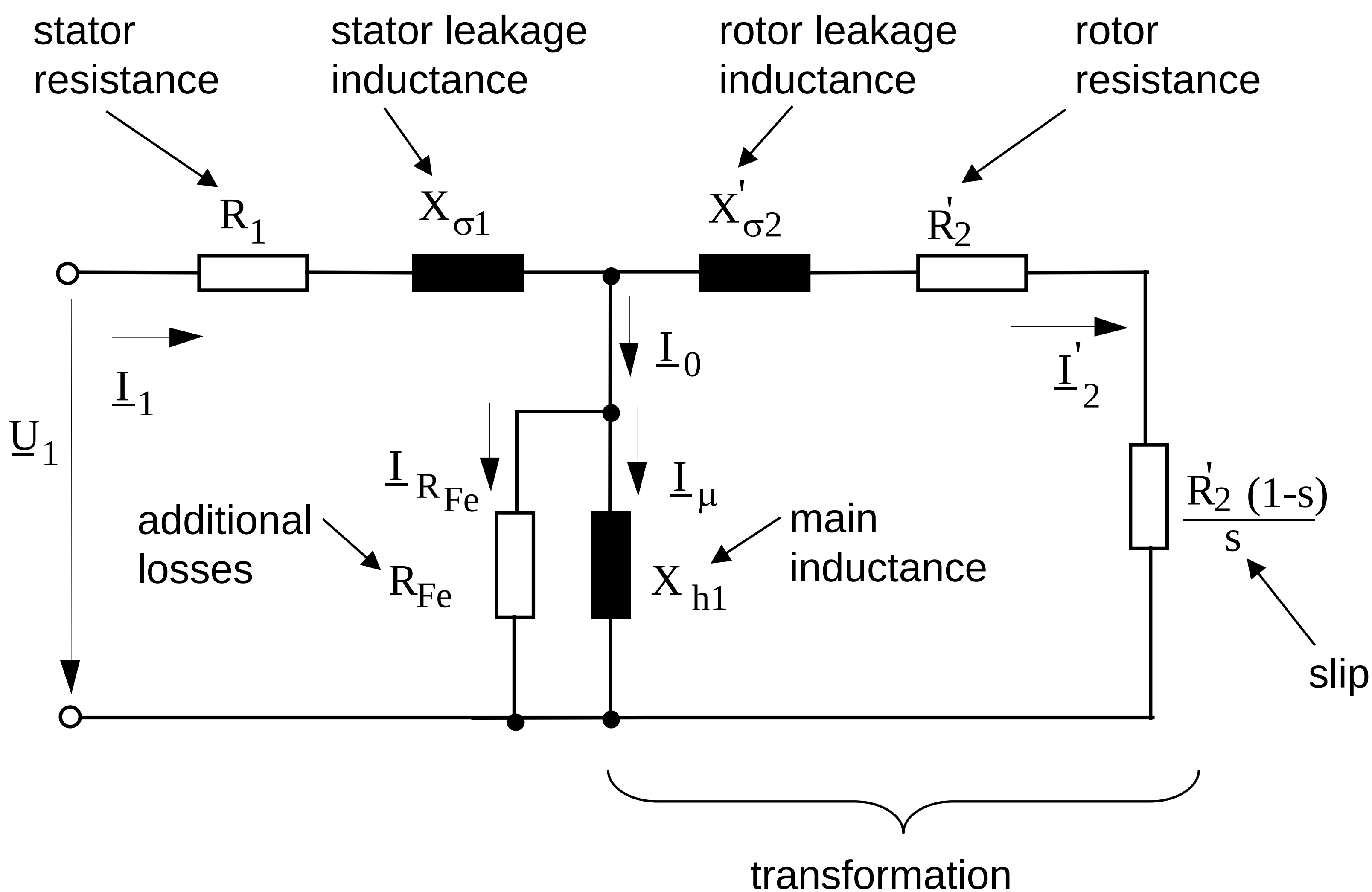
Equivalent Scheme



C.P. Steinmetz, "The alternating current induction motor", Trans. Am. Inst. Elect. Eng., 1897, pp. 185-217.

Coupled Inductance Model

$$\mathbf{u} = \mathbf{R}\mathbf{i} + \frac{d}{dt}(\mathbf{L}\mathbf{i})$$

$$\mathbf{u} = \begin{bmatrix} u_U \\ u_V \\ u_W \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{i} = \begin{bmatrix} i_U \\ i_V \\ i_W \\ i_{r1} \\ i_{r2} \\ i_{r3} \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} R_{st} & & & & & \\ & R_{st} & & & & \\ & & R_{st} & & & \\ & & & R_{rt} & & \\ & & & & R_{rt} & \\ & & & & & R_{rt} \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} \ell_{st}+\ell_{st,\sigma} & -\frac{\ell_{st}}{2} & -\frac{\ell_{st}}{2} & \ell_m\cos(p\theta) & \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) \\ -\frac{\ell_{st}}{2} & \ell_{st}+\ell_{st,\sigma} & -\frac{\ell_{st}}{2} & \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m\cos(p\theta) & \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) \\ -\frac{\ell_{st}}{2} & -\frac{\ell_{st}}{2} & \ell_{st}+\ell_{st,\sigma} & \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m\cos(p\theta) \\ \ell_m\cos(p\theta) & \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_{rt}+\ell_{rt,\sigma} & -\frac{\ell_{rt}}{2} & -\frac{\ell_{rt}}{2} \\ \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m\cos(p\theta) & \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) & -\frac{\ell_{rt}}{2} & \ell_{rt}+\ell_{rt,\sigma} & -\frac{\ell_{rt}}{2} \\ \ell_m\cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m\cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m\cos(p\theta) & -\frac{\ell_{rt}}{2} & -\frac{\ell_{rt}}{2} & \ell_{rt}+\ell_{rt,\sigma} \end{bmatrix}$$

coupled inductance model

Park transformation

$$\begin{bmatrix} i_\mathrm{a} \\ i_\mathrm{b} \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} i_\mathrm{U} \\ i_\mathrm{V} \\ i_\mathrm{W} \end{bmatrix}$$

$$\begin{bmatrix} u_\mathrm{d} \\ u_\mathrm{q} \end{bmatrix} = \begin{bmatrix} R_\mathrm{d} & 0 \\ 0 & R_\mathrm{q} \end{bmatrix} \begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} L_\mathrm{d} & M \\ M & L_\mathrm{q} \end{bmatrix} \begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} \right)$$

R.H. Park, "Two reaction theory of electrical machines, generalised method of analysis, part 1", AIEE Trans., Vol.48, July 1929, pp. 716-727.

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Required FE Software

		linear	nonlinear	external circuit	motion
2D	static	X	X		
	time-harmonic	X	X	X	
	transient	X	X	X	X
3D	static	X	X		
	time-harmonic	X	X	X	
	transient	X	X	X	X

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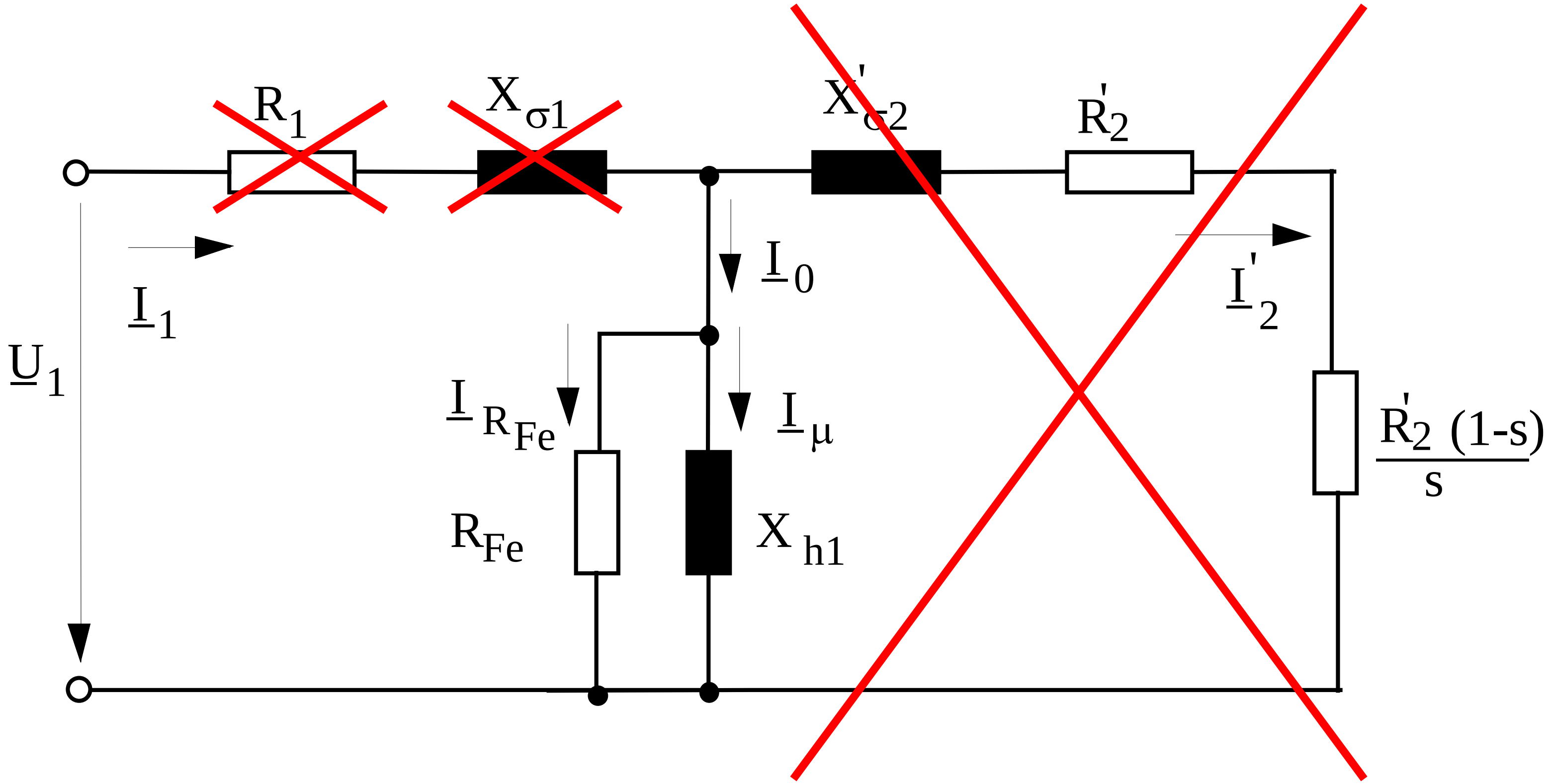
No-Load Operation (1)

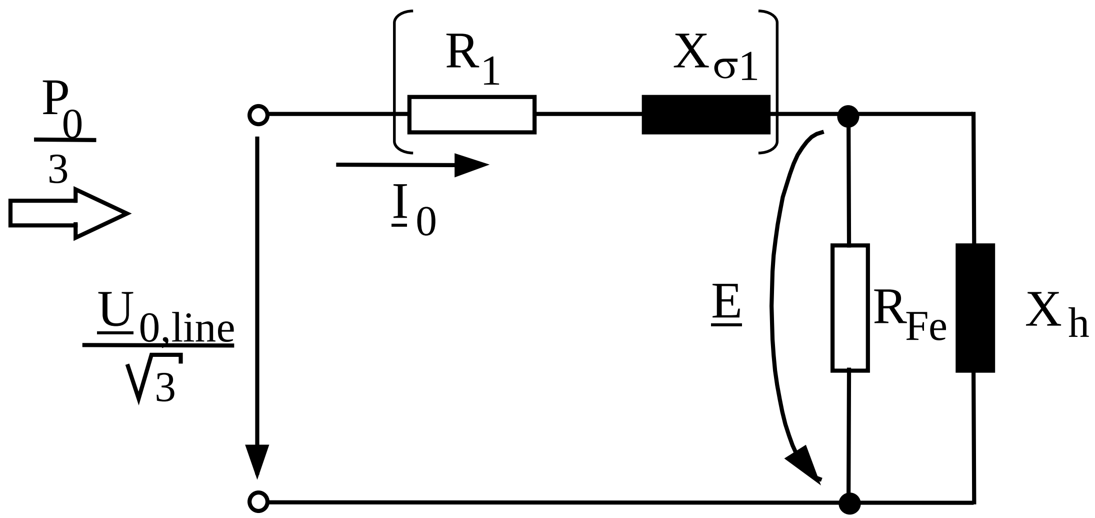
$s = 0 \Rightarrow R_2' \frac{1-s}{s} = \infty$

$R_1 << R_{Fe}$

$X_{\sigma 1} << X_{h1}$



No-Load Operation (2)



compute stator resistance R_1 analytically

neglect $X_{\sigma 1}$ with respect to X_{h1}

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No-Load Operation (3)

expected phenomena

- no induced currents in the rotor bars
- ferromagnetic saturation

simulation features

- static simulation should be sufficient
- nonlinear simulation

simulation approach

- 2D magnetostatic simulation: $\nabla \times \left(\nu \nabla \times \vec{A} \right) = \vec{J}$
- nonlinear BH-characteristic (+ adaptive mesh refinement for achieving a sufficient resolution)
- instantaneous current distribution in the stator windings

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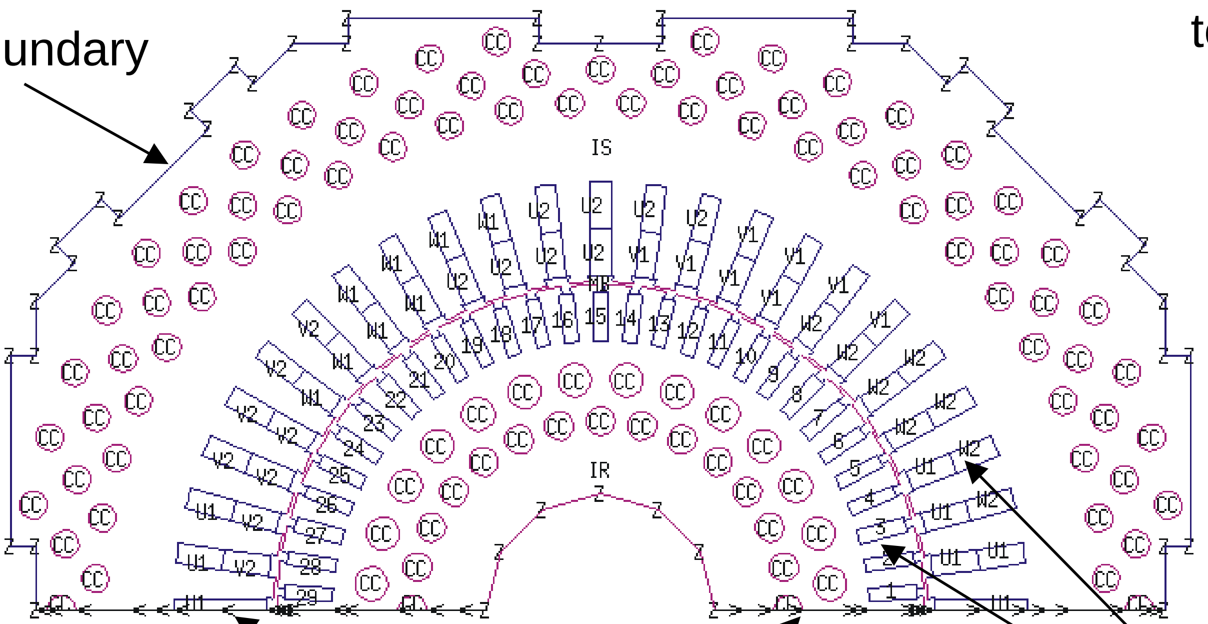
No-Load Model (1)

Geometry
Boundary conditions

48 stator slots
58 rotor slots

→ 2 of 4 poles to be modelled

electric boundary conditions



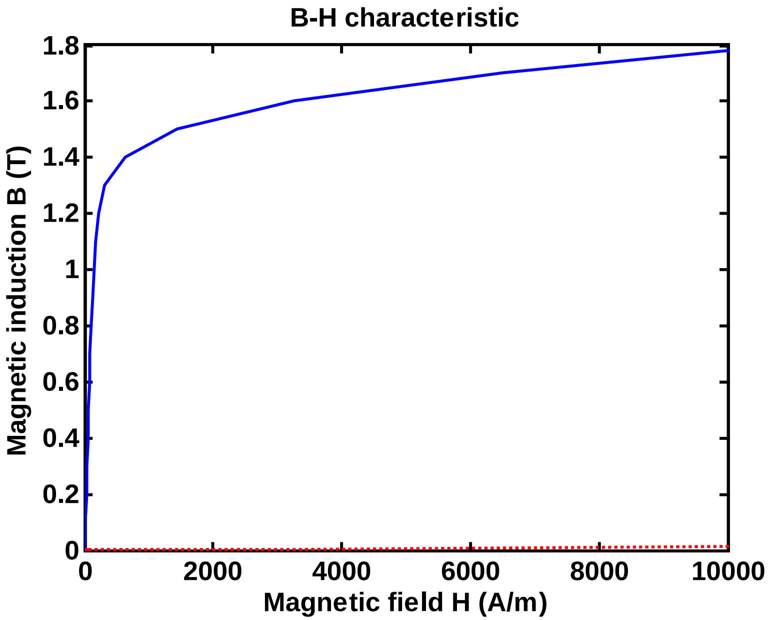
region labels

periodic boundary conditions

No-Load Model (2)

Materials

air : $\mu = \mu_0$
Cu : $\mu = \mu_0$
Fe : $\mu = \mu\left(\left|B\right|\right)$



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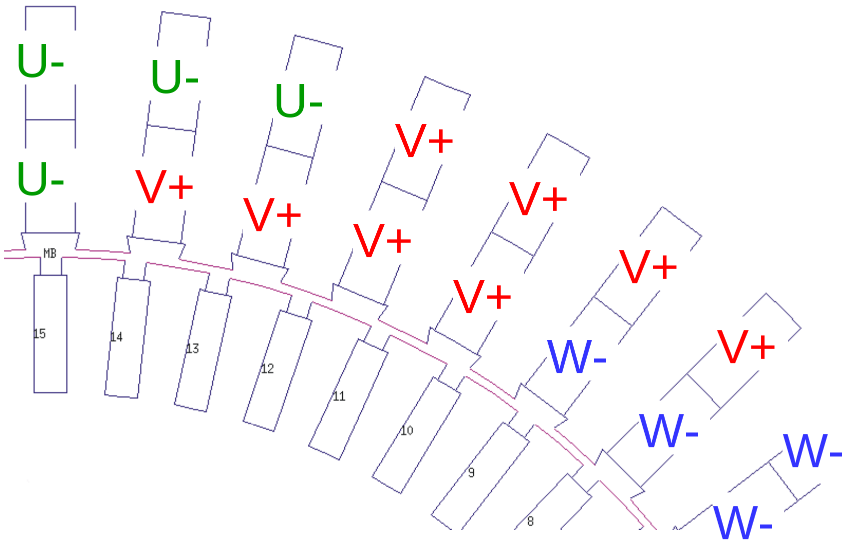
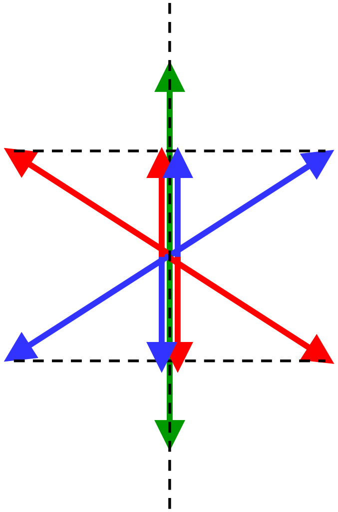
No-Load Model (3)

Excitations





$$i_{U+} = U_{eff}\sqrt{2}$$

$$i_{V+} = -\frac{1}{2}U_{eff}\sqrt{2}$$

$$i_{W+} = -\frac{1}{2}U_{eff}\sqrt{2}$$

$$i_{U-} = -U_{eff}\sqrt{2}$$

$$i_{V-} = \frac{1}{2}U_{eff}\sqrt{2}$$

$$i_{W-} = \frac{1}{2}U_{eff}\sqrt{2}$$

\+ winding functions $\vec{t}_{U+}$ $\vec{t}_{V+}$ $\vec{t}_{W+}$ $\vec{t}_{U-}$ $\vec{t}_{V-}$ $\vec{t}_{W-}$

⇨ excitation current $\vec{J} = \sum_{q=1}^{6} \vec{t}_q \, i_q$

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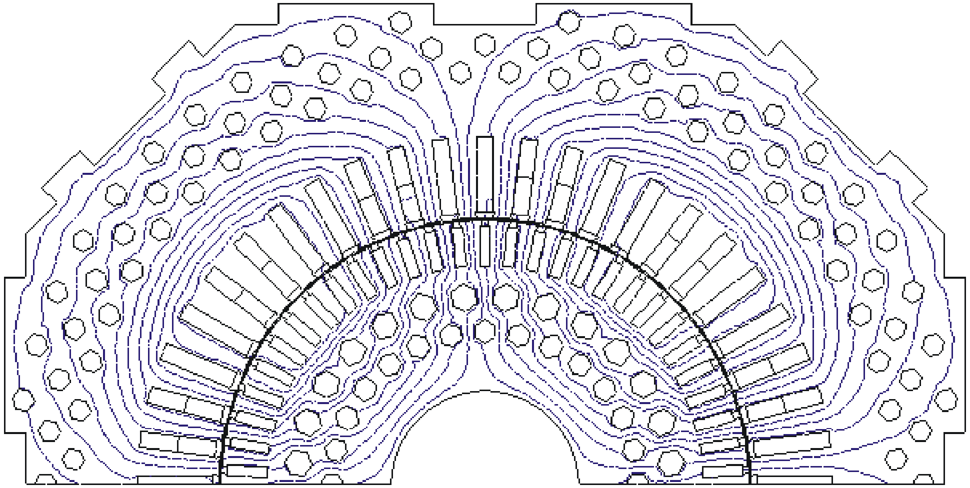
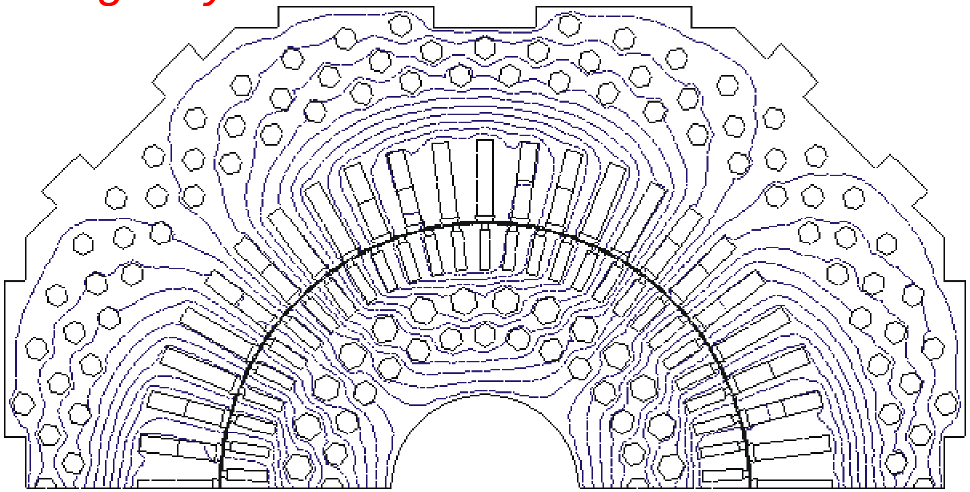
No-Load Results (1)

real time instant



imaginary time instant



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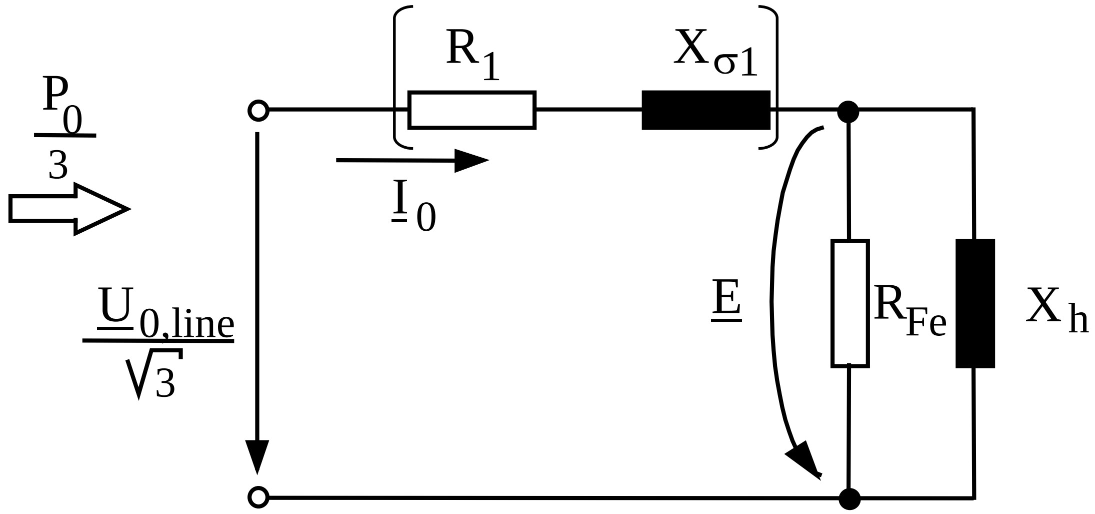
No-Load Results (2)



compute flux linked to e.g. phase U:

$$\psi_U = \int_{\Omega_{U+}} \vec{A}\cdot\vec{t}_{U+}\,d\Omega - \int_{\Omega_{U-}} \vec{A}\cdot\vec{t}_{U-}\,d\Omega$$

$$X_{h1} = j\omega L_U = j\omega\frac{\psi_U}{i_U}$$

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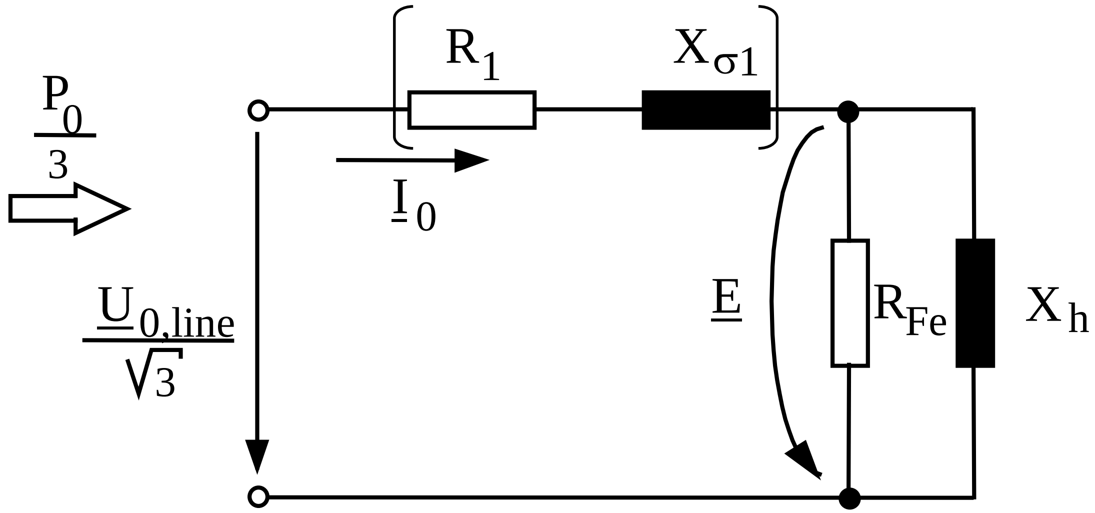
No-Load Results (3)



compute hysteresis losses by the Steinmetz formula

$$p_{\text{hyst}} = \sigma_{\text{hyst}} k_{\text{hyst}} \frac{f}{50\ \text{Hz}} \left(\frac{\left|\vec{B}\right|}{1\ \text{T}} \right)^2$$

integrate for the stator iron (not for the rotor)

$$P_{\text{hyst}} = \int_{\Omega_{\text{hyst}}} p_{\text{hyst}}\, \ell_z \, d\Omega$$

⇨ resistance $R_{\text{Fe}} = \frac{3\, U_{\text{eff}}}{P_{\text{hyst}}}$

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No-Load Results (4)

$$
\mathbf{L} = \begin{bmatrix}
\ell_{st}+\ell_{st,\sigma} & -\frac{\ell_{st}}{2} & -\frac{\ell_{st}}{2} & \ell_m \cos(p\theta) & \ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) \\
-\frac{\ell_{st}}{2} & \ell_{st}+\ell_{st,\sigma} & -\frac{\ell_{st}}{2} & \ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m \cos(p\theta) & \ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) \\
-\frac{\ell_{st}}{2} & -\frac{\ell_{st}}{2} & \ell_{st}+\ell_{st,\sigma} & \ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m \cos(p\theta) \\
\ell_m \cos(p\theta) & \ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_{rt}+\ell_{rt,\sigma} & -\frac{\ell_{rt}}{2} & -\frac{\ell_{rt}}{2} \\
\ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m \cos(p\theta) & \ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) & -\frac{\ell_{rt}}{2} & \ell_{rt}+\ell_{rt,\sigma} & -\frac{\ell_{rt}}{2} \\
\ell_m \cos\left(p\theta-\frac{2}{3}\pi\right) & \ell_m \cos\left(p\theta+\frac{2}{3}\pi\right) & \ell_m \cos(p\theta) & -\frac{\ell_{rt}}{2} & -\frac{\ell_{rt}}{2} & \ell_{rt}+\ell_{rt,\sigma}
\end{bmatrix}
$$

- define "three-phase system" at the rotor side
  by linear combination of rotor bar winding functions
- excite 1 phase of the system (either rotor or stator)
- compute fluxes linked to all phases $\psi_y = \int_{\Omega_y} \vec{A}_x \cdot \vec{t}_y \, d\Omega$
- mutual inductance $M_{yx} = \frac{\psi_y}{i_x}$
- permutations for other phases
- multiply by cos(pθ) to introduce motion

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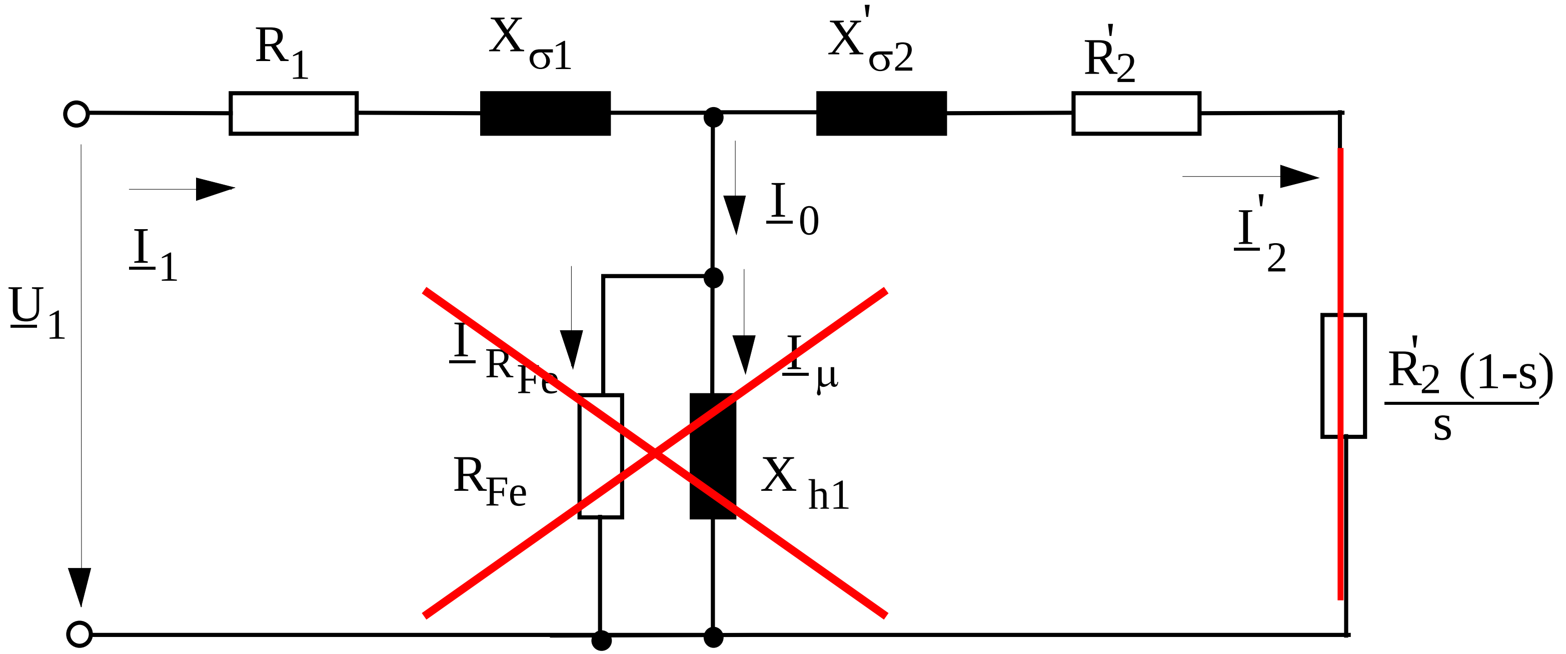
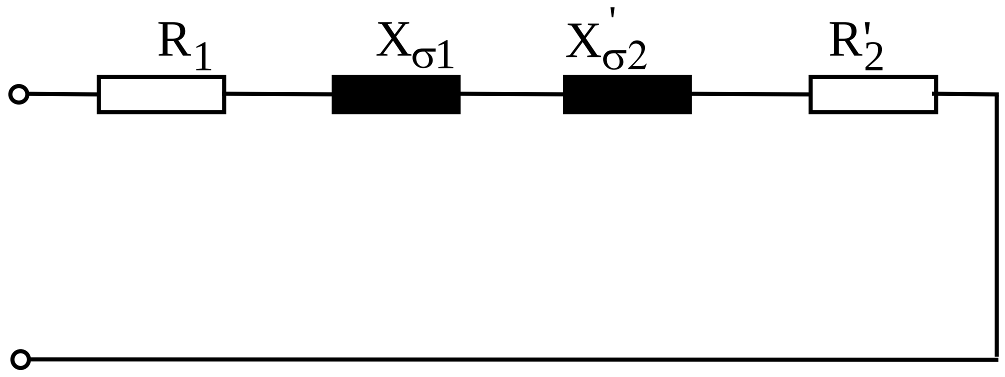
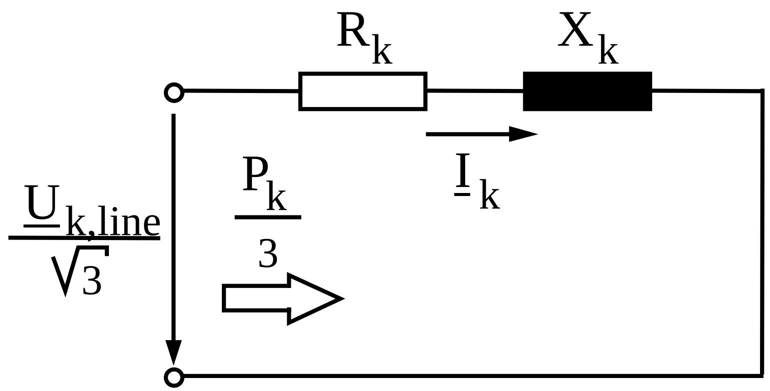
Short-Circuit Operation (2)

$s = 1$ ⇒ $R_2' \frac{1-s}{s} = 0$



Short-Circuit Operation (2)





Short-Circuit Operation (3)

expected phenomena

- induced currents in the rotor bars
- ferromagnetic saturation (especially for closed rotor slots)
- currents in e.g. rotor ring

simulation features

- time-harmonic simulation
- nonlinear simulation (effective saturation characteristic)
- external circuit coupling

simulation approach

- 2D time-harmonic simulation: $\nabla \times \left(\nu \nabla \times \vec{A} \right) + j\omega\sigma\vec{A} = \vec{J}$
- effective BH-characteristic (+ adaptive mesh refinement for achieving a sufficient resolution in the air gap and in wedges)
- current or voltage excitation of the stator through external circuit

- possible source of discrepancy with measurements:
  - measurements : under lower voltage (nominal current)
  - simulation : possibly under nominal voltage

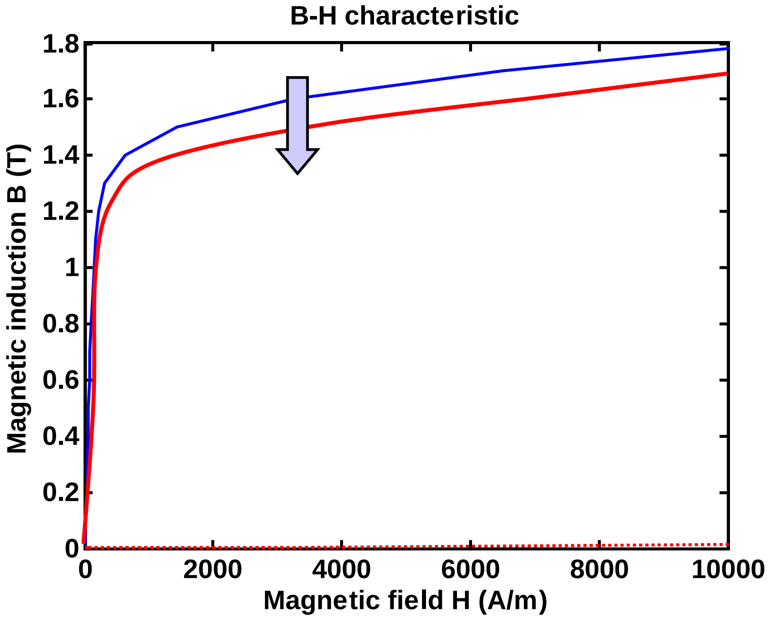
Short-Circuit Model (1)

Effective material charactistic

air : $\mu = \mu_0$

Cu : $\mu = \mu_0$

Fe : $\mu = \mu_{\text{eff}}\left(\left|B\right|\right)$



$$\frac{1}{2}\mu_{\text{eff}}\left(\left|\underline{B}_{\text{eff}}\right|\right)\underline{B}_{\text{eff}}\,\underline{B}_{\text{eff}}^{*} = \frac{1}{T}\int_0^T \frac{1}{2}\mu\left(\left|B\right|\right)\left(\text{Re}\left\{\underline{B}_{\text{eff}}\sqrt{2}\,e^{j\omega t}\right\}\right)^2 dt$$

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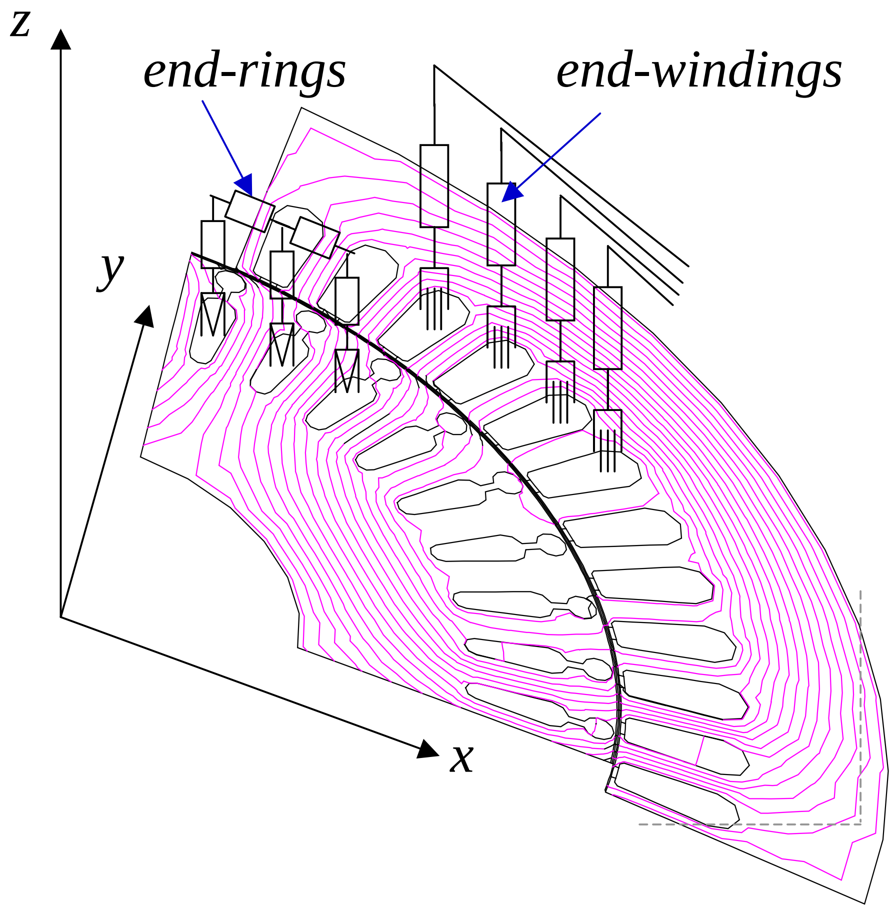
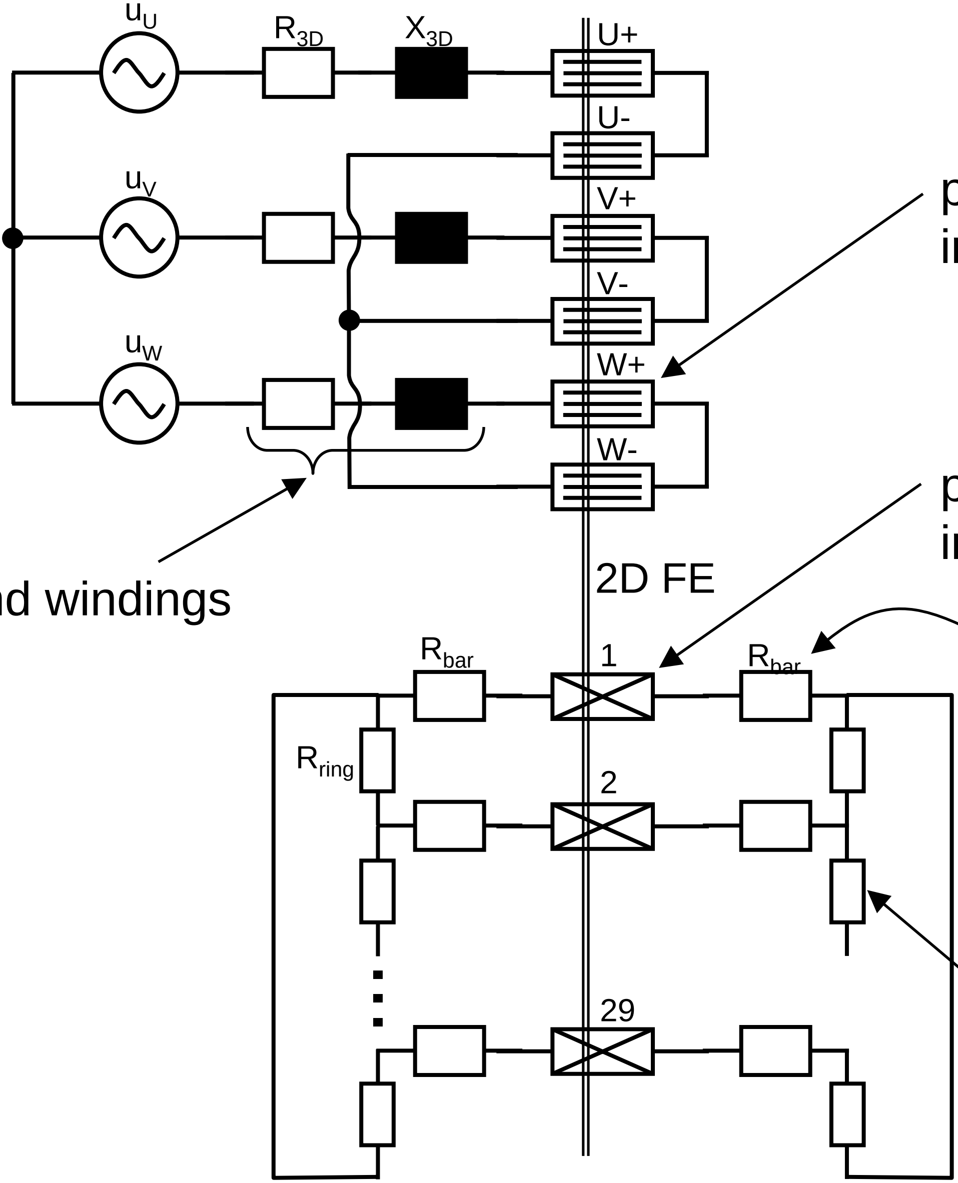
Short-Circuit Model (2)

field-circuit coupling



Short-Circuit Model (3)

when an even number of poles are modelled



part of the stator windings in the magnetic model

end windings

part of the rotor bars in the magnetic model

part of the rotor bars outside the magnetic model

rotor ring

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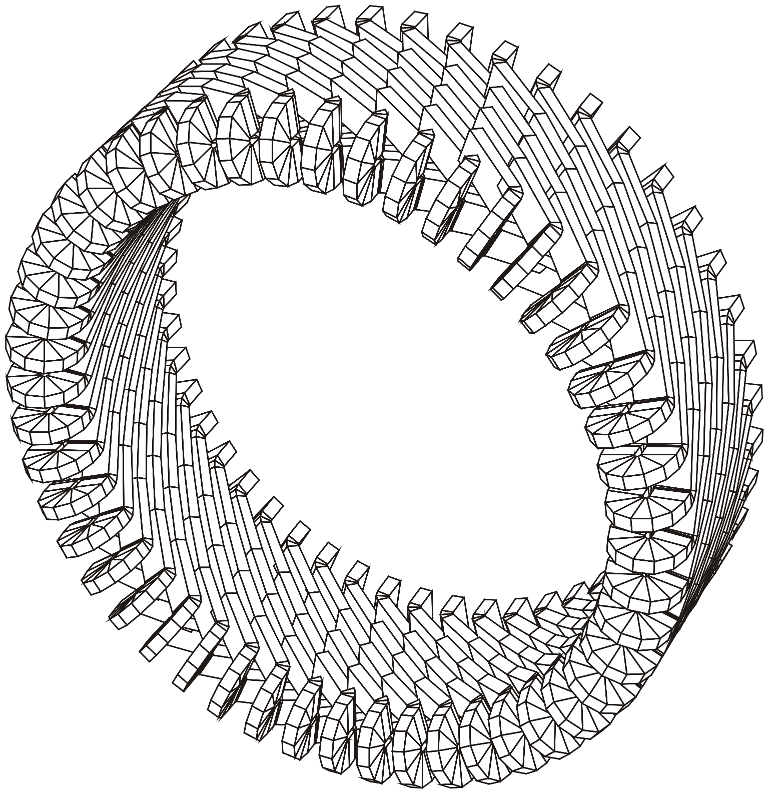
Short-Circuit Model (4)

external circuit parameters

R_{3D}	linear, analytical computation
X_{3D}	linear, analytical or 3D FE computation
R_{bar}	frequency and temperture dependent, analytical or 2D linear time-harmonic FE computation
R_{ring}	frequency and temperture dependent, analytical or 3D linear time-harmonic FE computation



picture: PhD Ronny Mertens

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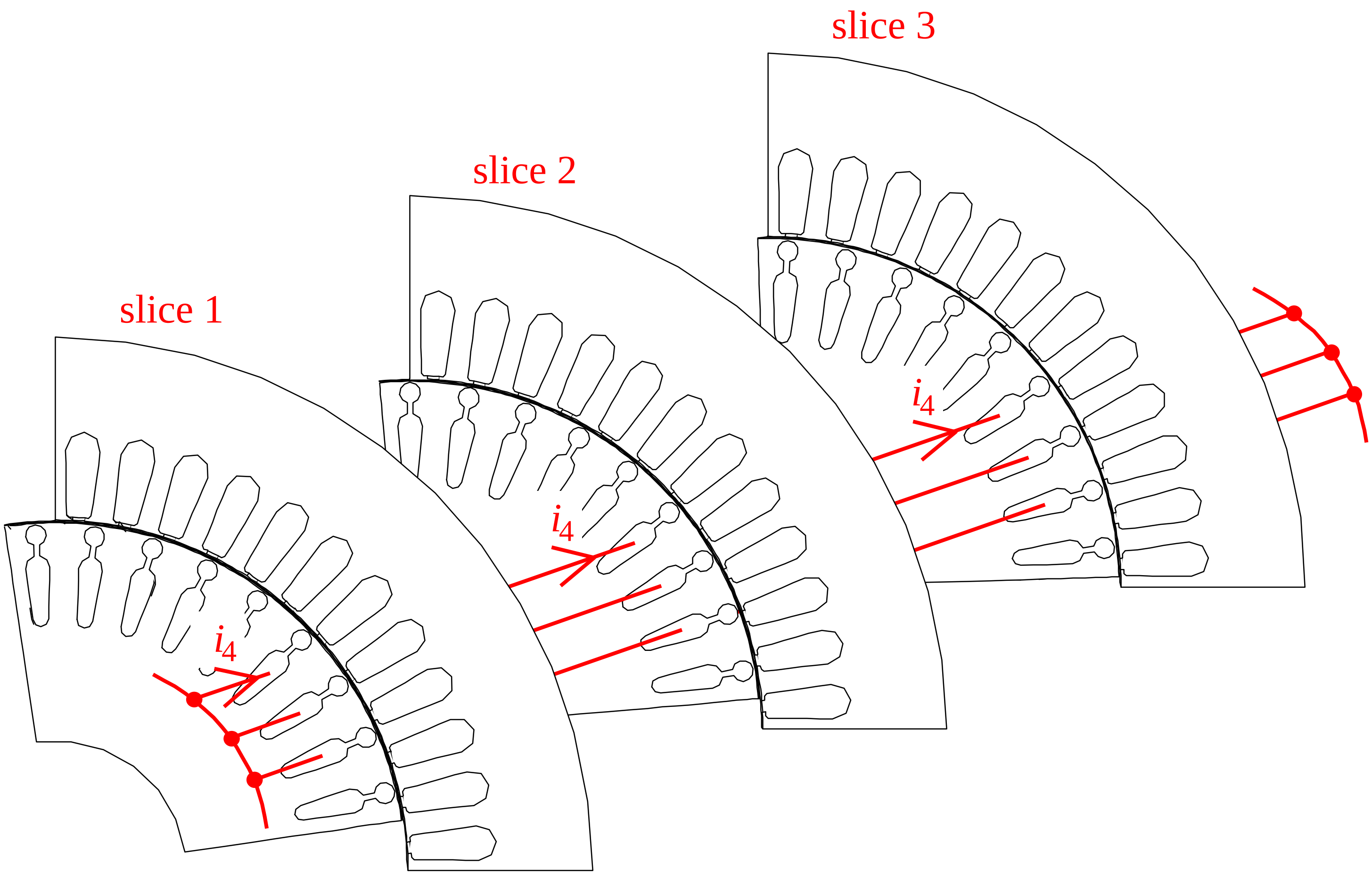
Short-Circuit Model (5)

multi-slice technique



picture: PhD Ronny Mertens

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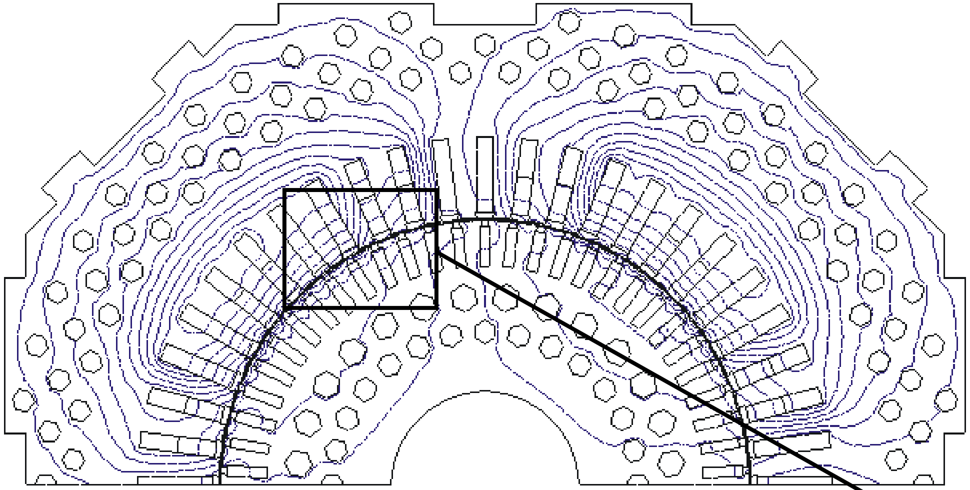
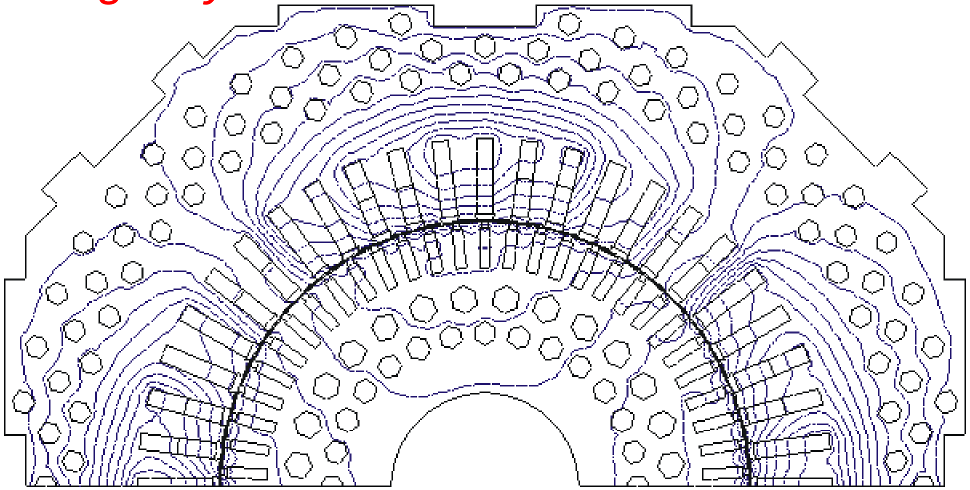
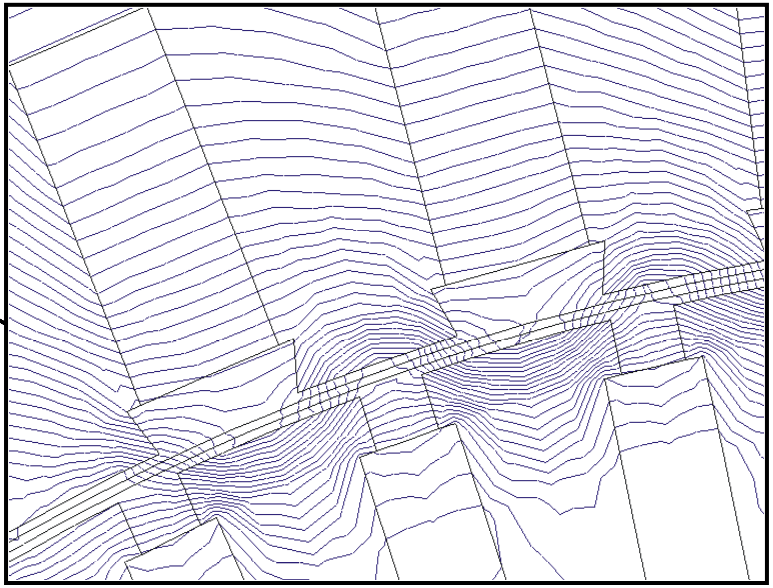
Short-Circuit Results (1)

real time instant

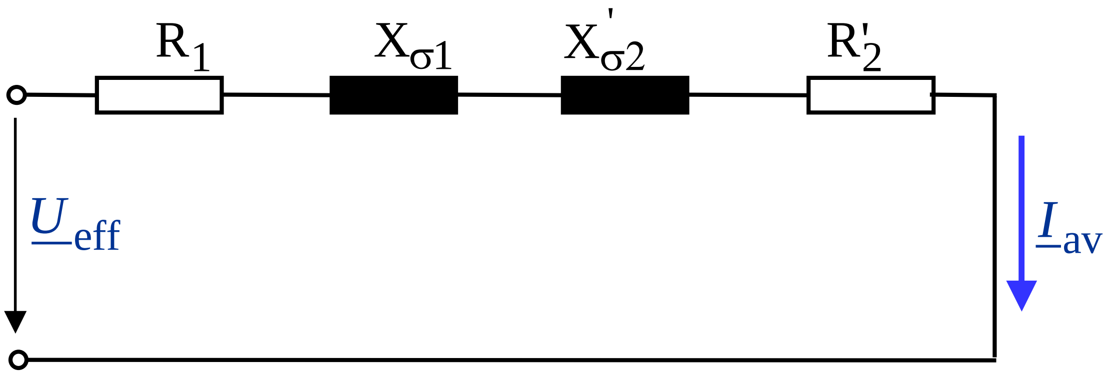


imaginary time instant





Short-Circuit Results (2)



simulation result: $$\underline{I}_{av} = \frac{\underline{I}_U + \underline{I}_V e^{j120°} + \underline{I}_W e^{-j120°}}{3}$$

$$\begin{cases} \underline{U}_{eff} = \left(R_1 + R_2' + jX_{\sigma 1} + jX_{\sigma 2}\right)\underline{I}_{av} \\ \dfrac{R_1}{R_2'} = \dfrac{X_{\sigma 1}}{X_{\sigma 2}} \end{cases}$$

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Load Operation (1)

expected phenomena

- induced currents in the rotor bars
- ferromagnetic saturation (especially for closed rotor slots)
- currents in e.g. rotor ring

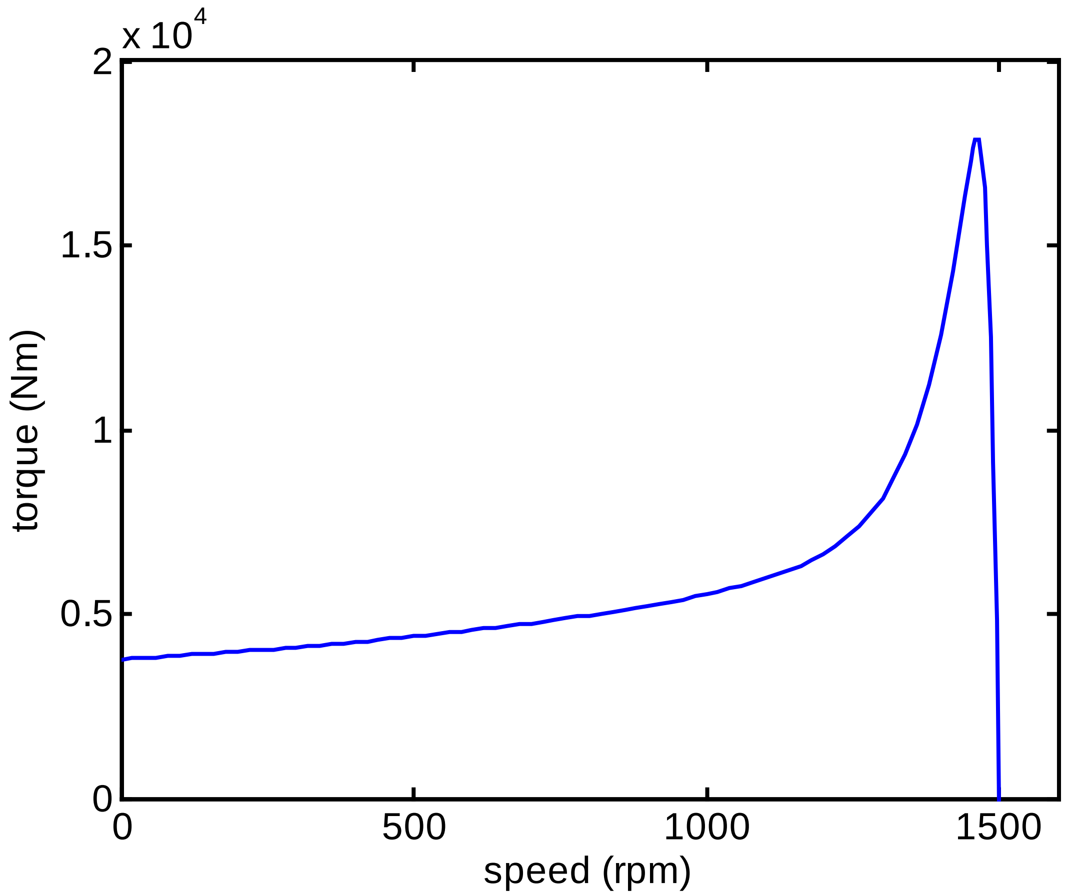
simulation features

- time-harmonic simulation
- nonlinear simulation (effective saturation characteristic)
- external circuit coupling
- slip frequency at the rotor
- torque computation

simulation approach

- 2D time-harmonic simulation: $\nabla \times \left(\nu \nabla \times \vec{A} \right) + j\omega s \sigma \vec{A} = \vec{J}$
- effective BH-characteristic
- current or voltage excitation of the stator through external circuit
- impedance of the rotor circuit scaled by *s !!*

Load Results (1)



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Transient Simulation (1)

mechanical equation of motion

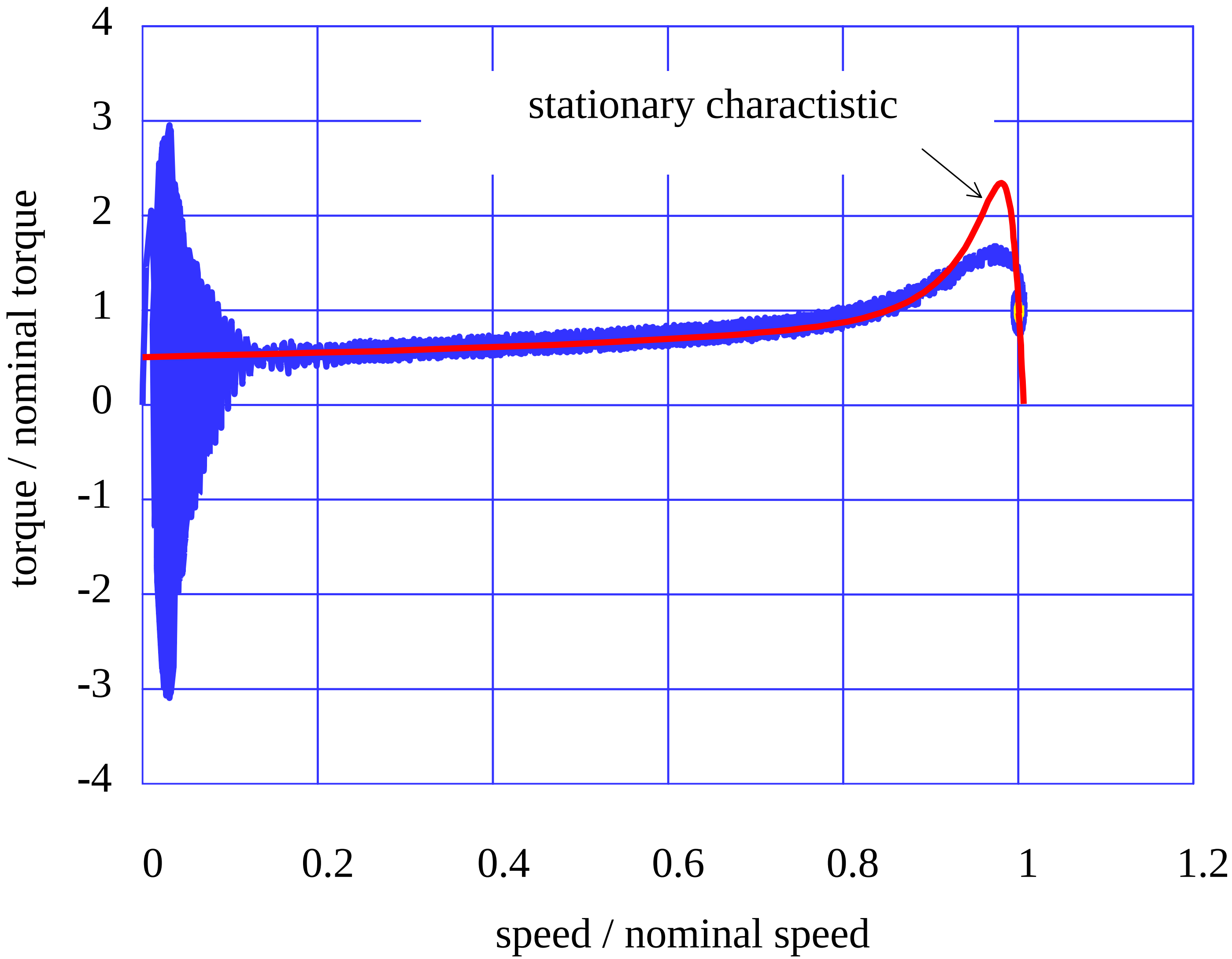
$$J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+C\frac{\mathrm{d}\theta}{\mathrm{d}t}=T_M-T_L$$

explicit time-stepping scheme

$$\theta_n=\theta_{n-1}+\alpha\,\Delta t\,\omega_n+(1-\alpha)\Delta t\,\omega_{n-1}$$

e.g. moving-band technique
for implementing rotor displacement

Transient Results (1)



picture: PhD Ronny Mertens

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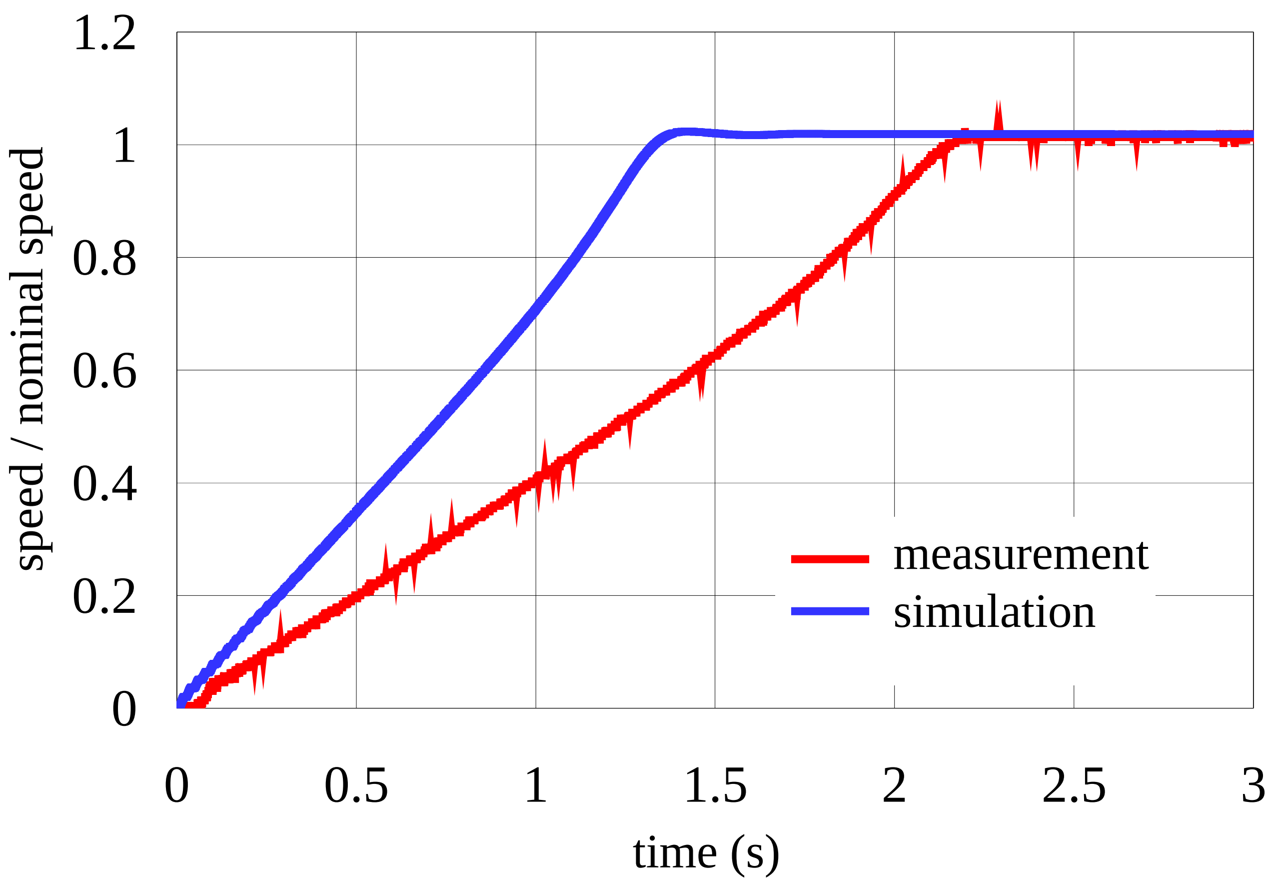
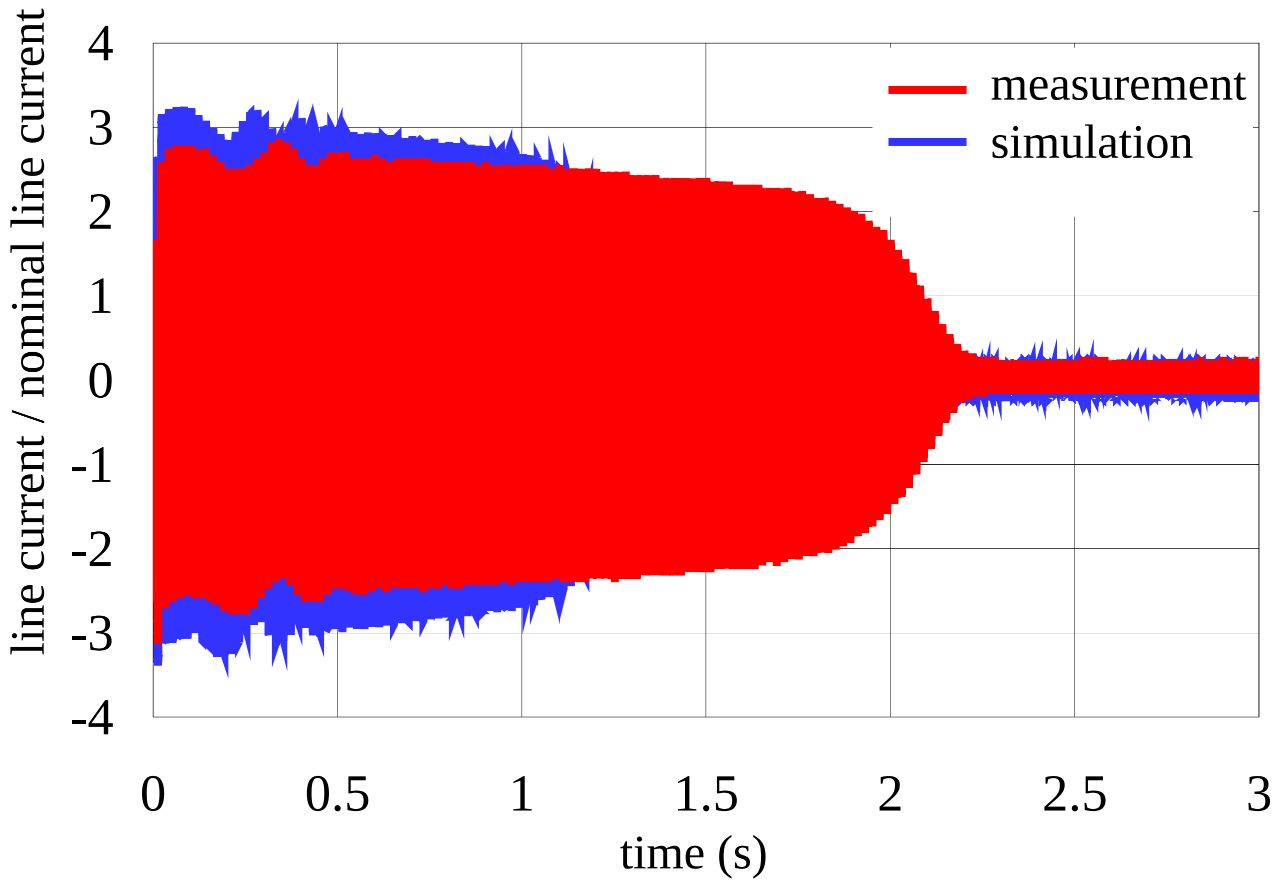
Transient Results (2)



picture: PhD Ronny Mertens

Transient Results (3)



picture: PhD Ronny Mertens

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Lecture Series

Finite-Element Electrical Machine Simulation

http://www.ew.e-technik.tu-darmstadt.de/FOR575
NEXT LECTURE : THURSDAY, July 13th 2006

V09: Modelling of hysteresis

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summer semester 2006
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