<u>Exercise 2.1</u>: Fan blower motor

In a steel work a three-phase asynchronous machine with slip-ring rotor is used driving the fan blower, which supplies the blast-furnace with fresh air. In Fig. 2.1-1, the torque-speed characteristic is given for the operation at rated voltage with short-circuited slip-rings.

1) Draw into this diagram with correct scale the *M(n)*-characteristics for the operation at rated voltage with external rotor resistors R_v in the rotor circuit, which correspond to
   a) 3 times
   b) 9 times

   the rotor resistance R_r. Mark particularly the points for $0.4 \cdot M_b$, $0.6 \cdot M_b$, $0.8 \cdot M_b$ and M_b.

2) How big is the starting torque M_1 of the asynchronous machine with $R_v = 0$, $3R_r$, $9R_r$?

<u>Solution:</u>

1) a) $R_v = 3 \cdot R_r$, $\dfrac{R_r}{s} = \dfrac{R_r + R_v}{s^*}$, $s^* = \left(1 + \dfrac{R_v}{R_r}\right) \cdot s$

s^* : slip at the same torque M as at slip s, $R_v = 0$

$s^* = 4 \cdot s$

Take torque from Fig. 2.1-1 at slip s, and put it at slip s^* in Fig. 2.1-2.

b) $R_v = 9 \cdot R_r$: Same procedure as in a), but with slip $s^* = 10 \cdot s$, see Fig. 2.1-2.

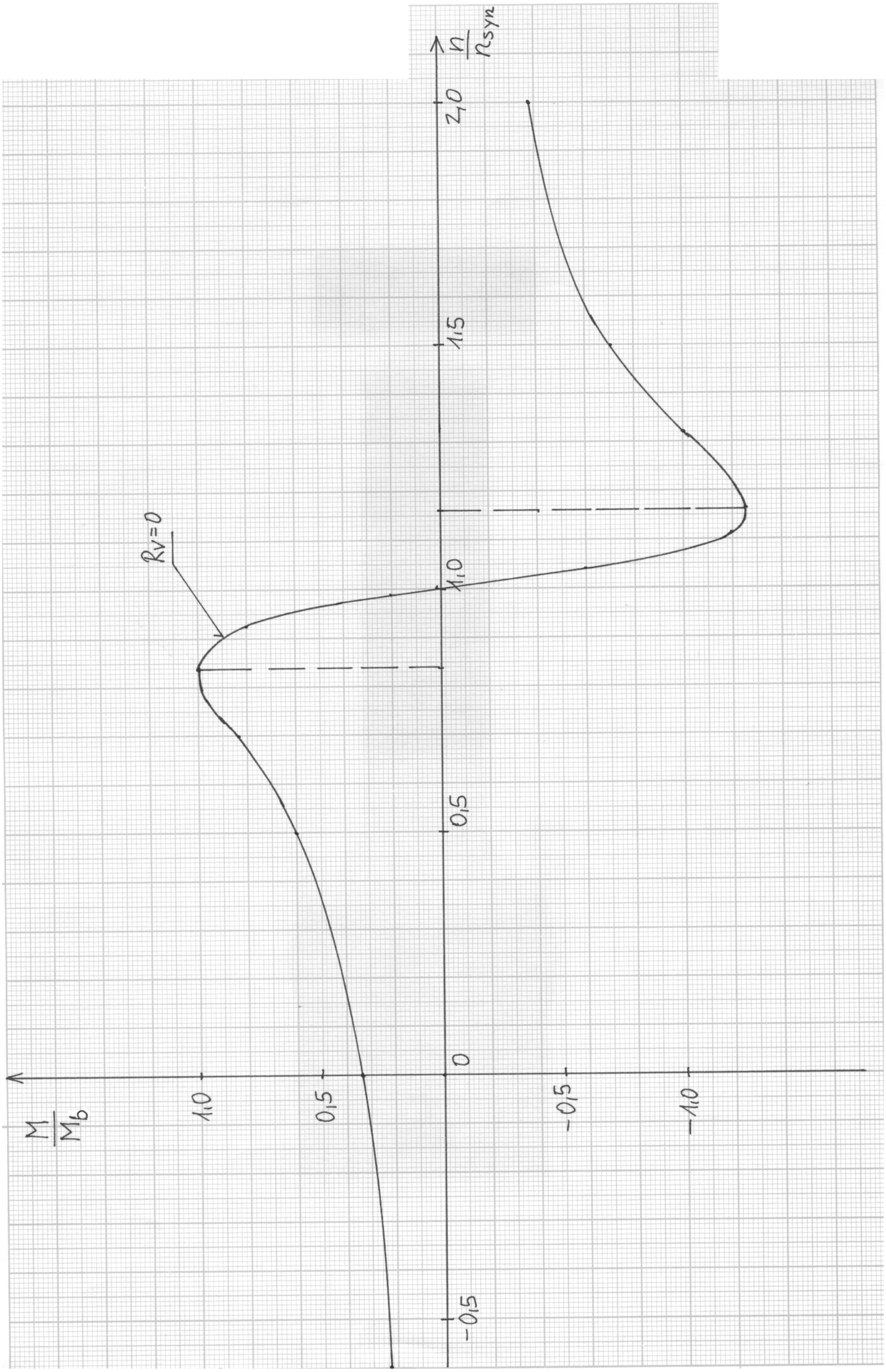


Fig. 2.1-1: Torque-speed-characteristic of fan blower motor in per-unit of break down torque and synchronous speed

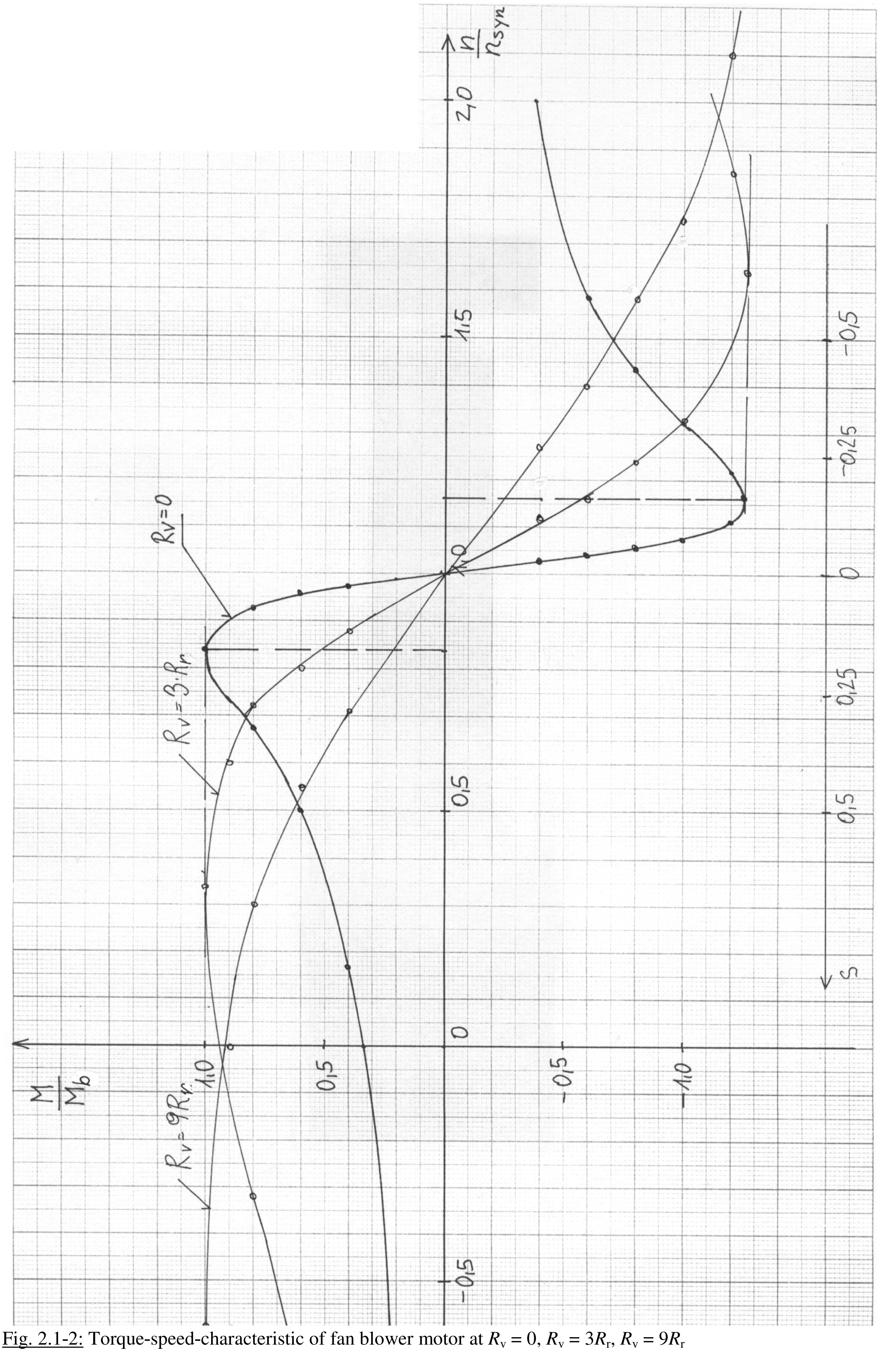


Fig. 2.1-2: Torque-speed-characteristic of fan blower motor at $R_v = 0$, $R_v = 3R_r$, $R_v = 9R_r$

<u>Exercise 2.2</u>: Motor for water feeder pump of thermal power plant

For drive of the water feeder pump of the stem generating boiler in a thermal power station with the data

P = (2.2 ...1.6) MW
at
n = (990...720) 1/min

a slip-ring <u>asynchronous machine</u> is used as an <u>sub-synchronous converter cascade</u>.

1) Sketch the electric circuit of the cascaded drive.

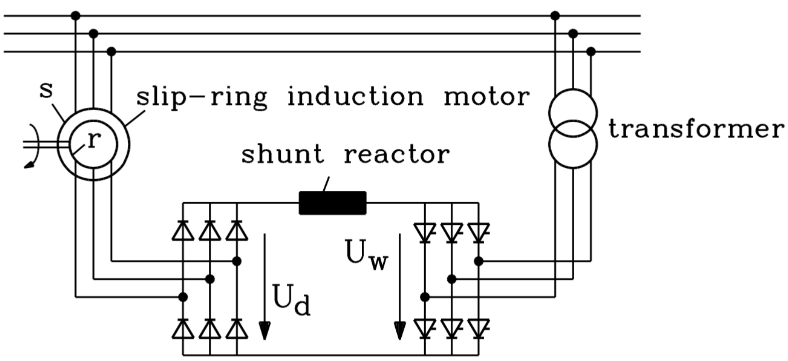
2) What is rated power and pole number (f = 50 Hz) of the asynchronous motor?

3) How big is the rated current for a grid voltage of U_N = 6300 VY (line-to-line value) and the motor nominal data $\cos\varphi_N = 0.90$; $\eta_N = 0.95$?

4) For which apparent power should the converter be designed ?

5) Sketch the characteristic M = f (n) (torque versus speed), when the converter is operated at
(1) zero d.c.-link voltage,
(2) the lowest speed according to the indicated operating range.

<u>Solution:</u>
1)



<u>Fig. 2.2-1</u>: Sub-synchronous cascade with slip-ring induction machine

2) $n_{\max} = 990 \ /\min \rightarrow n_{syn} = 1000 \ /\min \rightarrow$ at 50 Hz : $\underline{\underline{2p = 6}}$

$$n_{syn} = \frac{f}{p} \cdot 60 = \frac{50}{3} \cdot 60 = 1000 \ /\min$$

3) $P_N = 2.2 \ \text{MW} = \underset{0{,}95}{\eta_N} \cdot \underset{0{,}9}{\cos\varphi_N} \cdot \sqrt{3} \cdot \underset{6{,}3\ \text{kV}}{U_N} \cdot I_N \Rightarrow I_N = \underline{\underline{236 \ \text{A}}}$

4) The slip power (minus the rotor winding losses, which are neglected here) will be fed back to the grid via the rotor-side converter.

$$s_{max} = \frac{n_{syn} - n_{min}}{n_{syn}} = \left(1 - \frac{720}{1000}\right) = 0{,}28$$

$$P_N \cdot s_{max} = P_r\,, \quad S_r = S_N \cdot s_{max} = \sqrt{3} \cdot \underset{6{,}3\ \mathrm{kV}}{U_N} \cdot \underset{236\ \mathrm{A}}{I_N} \cdot \underset{0{,}28}{s_{max}} = \underline{\underline{721\ \mathrm{kVA}}}$$

5)

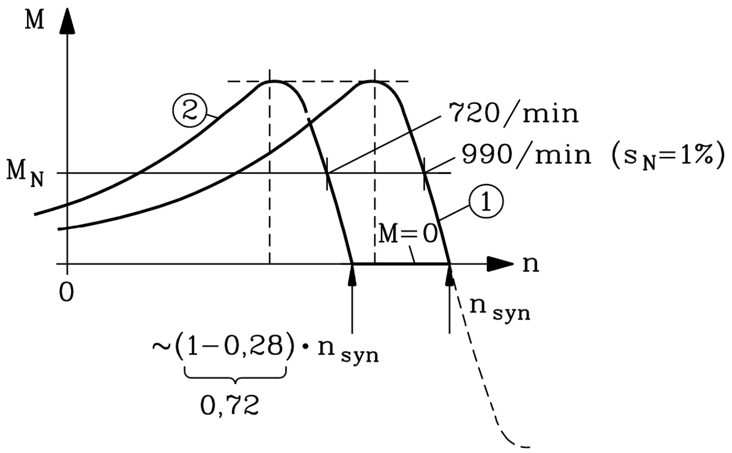


Fig. 2.2-2: Torque-speed characteristic of sub-synchronous cascade with slip-ring induction machine (1) dc link voltage is zero, (2) minimum speed operation

Note:
For case (2), the torque M is zero 0 in the speed range between n_{syn} and $0{,}72 \cdot n_{syn}$, as the rectified rotor voltage is smaller than the dc link voltage $U_d < U_W$.

Exercise 2.3: Groundwater pump station

The variable speed drive of a centrifugal pump in a groundwater pump station is done with a three-phase cage induction machine, which is fed via a voltage source dc link inverter.

Rating of induction machine:
P_N = 125 kW, U_N = 500 V Y (line-to-line value), I_N = 218 A
f_N = 75 Hz, $2p = 8$, $\cos\varphi_N = 0.72$

The stator winding resistance is neglected ($R_s = 0$). The no-load current is I_0 = 125 A ($\cos\varphi_0 \approx 0$).

1) Draw with $\underline{I}_0$ and $\underline{I}_N$ the circle diagram (*HEYLAND*-circle) for the operation with 500 V Y and 75 Hz. (recommended scale: 1 cm ≙ 50 A)

2) How big is the corresponding break-down torque?

3) The machine is fed now with 500 V Y and 120 Hz.. Voltage harmonics from the inverter and variable iron saturation in the machine are neglected.
a) Draw the circle diagram for this operation into the diagram from point 1). Use the same scale!).
b) How big is the corresponding brake-down torque?
c) Determine the stator current and the power factor for an output power of $P = P_N$ = 125 kW assuming that the efficiency is the same as at the rated point of operation.

Solution:

1) $P_0: \cos\varphi_0 = 0$, $\varphi_0 = 90°$, $I_0 = 125\ \text{A} \hat{=} 2{,}5\ \text{cm}$ $\quad 1\ \text{cm} \hat{=} 50\ \text{A}$

$P_N: \cos\varphi_N = 0{,}72$, $\varphi_N = 44°$, $I_N = 218\ \text{A} \hat{=} 4{,}4\ \text{cm}$

Determination of centre point *M* of circle diagram:

M lies on abscissa, as R_s is neglected ! Draw a straight line perpendicular to $\overline{P_0P_N}$ by dividing $\overline{P_0P_N}$ into two halves. The circle centre point *M* results as the intersection point with the -Im-axis (abscissa) (Fig. 2.3-1).

2) As $R_s \cong 0$, the air-gap power at break down slip is : $P_{\delta,b} = m_s \cdot U_s \cdot I_{s,b,active} = \omega_{syn} \cdot M_b$

$$I_{s,b,active} \hat{=} \overline{\text{KM}} = 9{,}7\ \text{cm} \hat{=} 485\ \text{A},\quad \omega_{syn} = 2\pi\frac{f}{p} = 2\pi\frac{75}{4} = 117{,}8\ \text{s}^{-1}$$

Graphically determined break-down torque:

$$M_b = \frac{\sqrt{3}\cdot 500\cdot 485}{117{,}8} = \underline{\underline{3565\ \text{Nm}}},$$ (This value contains of course the drawing inaccuracy!)

3)

a) $f = 120\ \text{Hz}$, reactances $X = \omega L$ increase with the ratio 120/75 = 1.6 !

no-load current $I_0 = \dfrac{U_s}{\omega(L_{\sigma s} + L_h)}$ decreases with 1/1.6 = 0.63

$I_0^* \hat{=} 0{,}63\cdot 2{,}5 = 1{,}6\ \text{cm}$: Draw point P_0^* in Fig. 2.3-1.

Current at $s \to \infty : I_\infty = \dfrac{U_s}{\omega\left(L_{\sigma s} + \dfrac{L_h\cdot L'_{r\sigma}}{L_h + L'_{r\sigma}}\right)}$ decreases also with 0.63

$I_\infty^* \hat{=} 0{,}63\cdot 22{,}1 = 13{,}9\ \text{cm}$: P_∞^* ; Draw this point in Fig. 2.3-1.

$\omega_{syn}^* = 1{,}6\omega_{syn} = 188{,}5\ \text{s}^{-1}$

Determination of centre point M* of new circle diagram at 120 Hz:

M^* divides the distance $\overline{P_0^*P_\infty^*}$ in two halves; $I_{s,b,active}^* \hat{=} \overline{M^*K^*} = 6{,}1\ \text{cm} \hat{=} 305\ \text{A}$

b) $$M_b^* = \frac{m_s\cdot U_s\cdot I_{s,b,active}^*}{\omega_{syn}^*} = \frac{\sqrt{3}\cdot 500\cdot 305}{188{,}5} = \underline{\underline{1401\ \text{Nm}}}$$

c)

$$\eta_N = \frac{P_{out}}{P_{in}} = \frac{P_N}{\sqrt{3}\cdot U_N\cdot I_N\cdot\cos\varphi_N} = \frac{125000}{\sqrt{3}\cdot 500\cdot 218\cdot 0{,}72} = 91{,}96\ \%$$

$$P_{in} = 135{,}9\ \text{kW} = \sqrt{3}\cdot U_N\cdot I_N^*\cdot\cos\varphi_N^* \Rightarrow I_N^*\cdot\cos\varphi_N^* = 157\ \text{A} = I_{N,active}^*$$

$157\ \text{A} \hat{=} 3{,}1\ \text{cm} \to$ circle diagram for $120\ \text{Hz}$: $I_N^* \hat{=} 4\ \text{cm}$

$$I_N^* = 4\cdot 50 = \underline{\underline{200\ \text{A}}},\qquad \cos\varphi_N^* = \frac{157}{200} = \underline{\underline{0{,}785}}$$

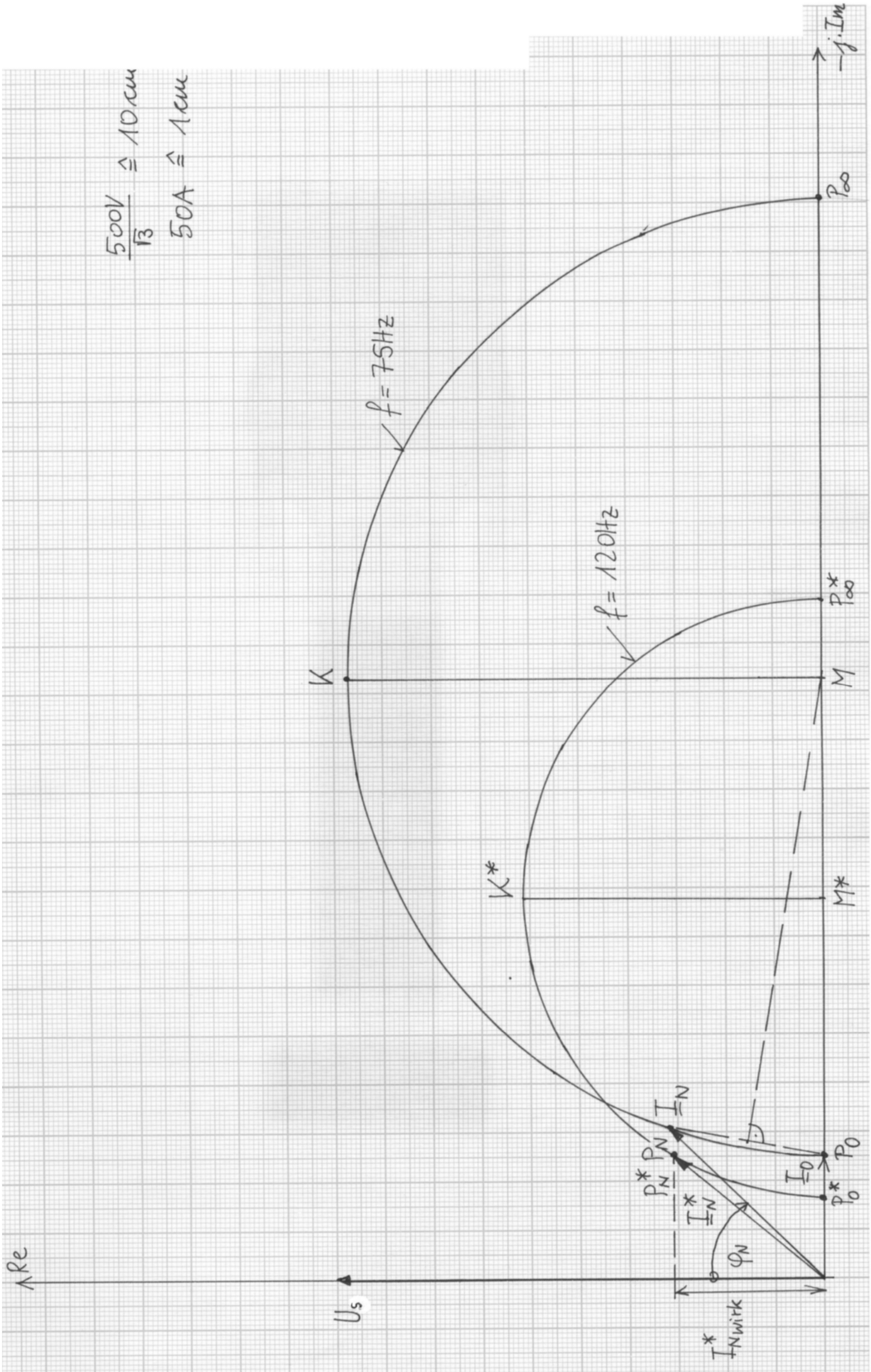


Fig. 2.3-1: Circle diagram of groundwater pump motor at 75 Hz and 120 Hz stator frequency

Exercise 2.4: Star-delta starting of induction machine

Describe the method of the star-delta start-up and its advantages and disadvantages (by mathematical argument)

Solution:

Star-delta start-up: At direct switching of the asynchronous machine to the grid, the starting current is too high (usually 5 ÷ 7 times the rated current). By switching on the motor in Y-connection instead of Δ-connection (which is required for rated operation) this current will be decreased down to 33%, however, the start-up torque decreases also to 33%.

Mathematical proof:

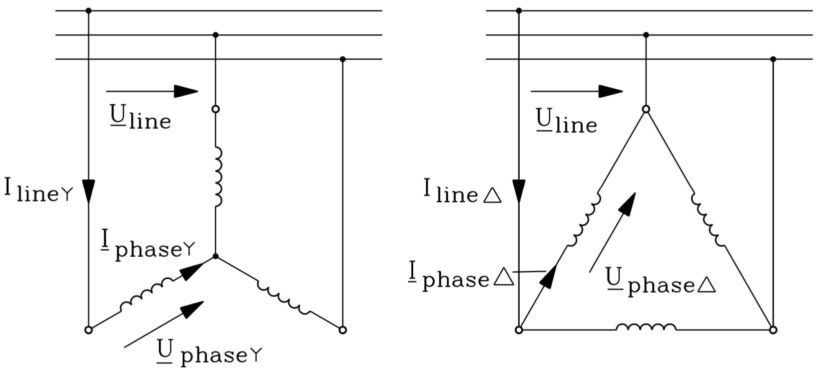


Fig. 2.4-1: Star-delta-start up of induction machine

$$U_{LL} = \sqrt{3} \cdot U_{ph,Y} \qquad U_{LL} = U_{ph,\Delta}$$

$$I_{LL,Y} = I_{ph,Y} \qquad I_{LL,\Delta} = \sqrt{3} \cdot I_{ph,\Delta}$$

$$Z_{ph} = \frac{U_{ph,Y}}{I_{ph,Y}} = \frac{U_{ph,\Delta}}{I_{ph,\Delta}} \Rightarrow$$

$$I_{LL,Y} = I_{ph,\Delta} \cdot \frac{U_{ph,Y}}{U_{ph,\Delta}} = I_{LL,\Delta} \cdot \frac{1}{\sqrt{3}} \cdot \frac{U_{LL}/\sqrt{3}}{U_{LL}} = I_{LL,\Delta} \cdot \frac{1}{3}$$

$$\underline{\underline{I_{LL,Y} = \frac{1}{3} \cdot I_{LL,\Delta}}}$$

$$M_1 \sim U_{ph}^2 \Rightarrow \frac{M_{1Y}}{M_{1\Delta}} = \left(\frac{U_{ph,Y}}{U_{ph,\Delta}}\right)^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \underline{\underline{\frac{1}{3}}}$$

Necessary condition of machine to be used for Y-Δ start-up: All 6 ends of the three phase winding U, X, V, Y, W, Z must be accessible, so that they can be connected to the mechanical Y-Δ-switch.

Exercise 2.5: Test field determination of circle diagram including iron and friction losses

From test data of a slip-ring rotor asynchronous machine the test field engineer obtained the circle diagram (Fig. 2.5-1). The values of the circle diagram are given in A (ampere):

Distance $\overline{AA'}$ = 2 A, distance $\overline{PA}$ = 194 A, distance $\overline{PD}$ = 192 A, distance $\overline{BD}$ = 7 A, distance $\overline{CD}$ = 18 A

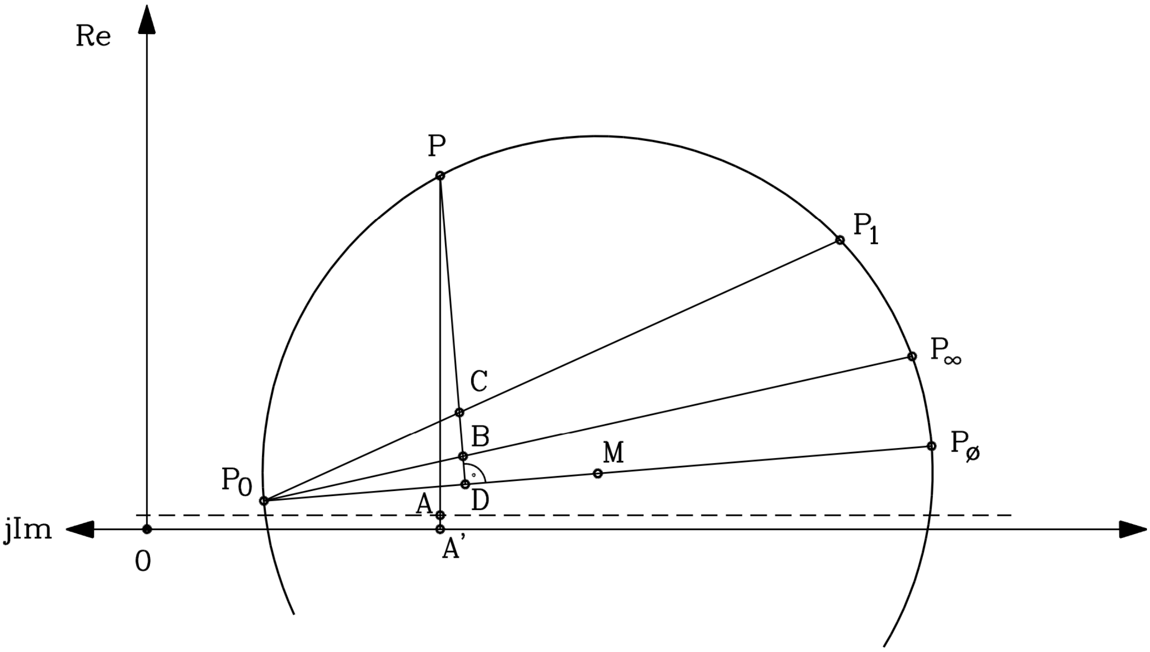


Fig. 2.5-1: Circle diagram, including stator resistive losses (*OSSANNA*-circle), and iron and friction losses

The iron and friction losses are considered by an additional loss current I_V according to the right equivalent circuit (Fig. 2.5-2 B): The distance $\overline{AA'}$ represents this additional loss current I_V, which is the active component of the stator current for the covering of iron and friction losses.

A: B:

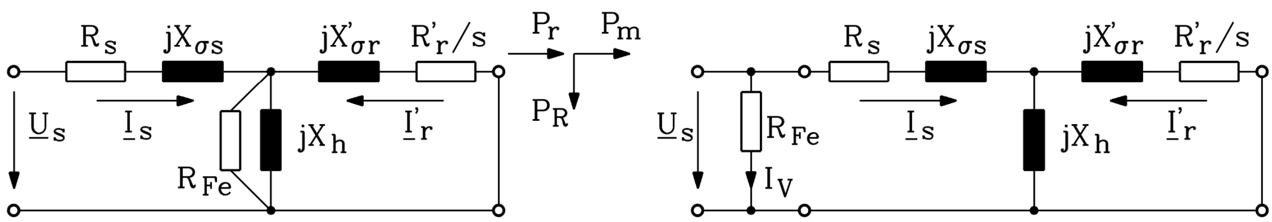


Fig. 2.5-2: Equivalent circuit per phase of induction machine:
A: Correct consideration of iron losses P_{Fe} with equivalent resistance R_{Fe} parallel to X_h, and subtracting friction losses P_R at the shaft
B: Approximated consideration of iron and friction losses with R_{Fe} parallel to grid voltage

$$P_{Fe} = 3 \cdot \frac{U_h^2}{R_{Fe}}$$

$$P_r = \left(\frac{R_s'}{s} - R_r'\right) \cdot I_r'^2 \cdot 3$$

$$P_m = P_r - P_R$$

exact consideration

$$P_{Fe+R} = 3 \cdot \frac{U_s^2}{R_{Fe}}$$

Approximated consideration

$$I_V = \frac{U_s}{R_{Fe}}, \qquad P_{Fe+R} = 3 \cdot U_s \cdot I_V$$

distance $\overline{AA'} \mathrel{\hat{=}} I_V$!

Determine for Y-connection with $m_s = 3$, $U_{LL} = 380\text{V}$ (line-to-line), $n_{syn} = 1500/\text{min}$ in operation point P the corresponding values

a1) of the stator losses <u>without</u> iron and friction losses,

a2) of the stator losses <u>inclusive</u> iron and friction losses,
b) of the rotor losses,
c) of the developed torque (in Nm),
d) of the efficiency.

<u>Solution:</u>
a1) without iron- and friction losses:

$$\overline{\mathrm{PA}} \mathrel{\hat{=}} I_{s,active} \Rightarrow P_s = 3 \cdot U_s \cdot I_{s,active} = 3 \cdot U_s \cdot I_s \cdot \cos\varphi : \text{ supplied stator active power}$$

$$\overline{\mathrm{PB}} \times 3 \cdot U_s = P_\delta : \text{ Air-gap power;} \quad U_s = \frac{380}{\sqrt{3}} = 220 \text{ V (phase voltage)}$$

$$\overline{\mathrm{CB}} \times 3 \cdot U_s = P_{Cu,r} : \text{ rotor losses}$$

$$\overline{\mathrm{PC}} \times 3 \cdot U_s = P_m : \text{ mechanical power}$$

$$\text{Stator losses: } P_{Cu,s} = P_s - P_\delta = 3 \cdot U_s \cdot \left(\overline{\mathrm{PA}} - \overline{\mathrm{PB}}\right) = 3 \cdot 220 \cdot (194 - 185) = \underline{\underline{5.94 \text{ kW}}}$$

$$\overline{\mathrm{PB}} = \overline{\mathrm{PD}} - \overline{\mathrm{BD}} = 192 - 7 = 185 \text{ A}$$

a2) Approximated consideration of the iron and friction losses according to equivalent circuit Fig. 2.5-2 B:

Stator copper losses + iron losses + friction losses:

$$P_{Cu,s} + P_{Fe+R} = 3 \cdot 220 \cdot \left(\underbrace{\overline{\mathrm{PA'}}} - \overline{\mathrm{PB}}\right) = 3 \cdot 220 \cdot (196 - 185) = \underline{\underline{7260 \text{ W}}}$$

$$\overline{\mathrm{PA}'} = 194 + 2 = 196 \text{ A}$$

$$P_{Fe+R} = 3 \cdot 220 \cdot 2 = \underline{\underline{1320 \text{ W}}}$$

$$\overline{\mathrm{AA'}} = 2 \text{ A}$$

b)

$$P_{Cu,r} = 3 \cdot 220 \cdot 11 = \underline{\underline{7260 \text{ W}}}$$

$$\overline{\mathrm{CB}} = \overline{\mathrm{CD}} - \overline{\mathrm{BD}} = 18 - 7 = 11 \text{ A}$$

c)

$$M_e = \frac{P_\delta}{\omega_{syn}} = \frac{122100}{157} = \underline{\underline{778 \text{ Nm}}}$$

$$\omega_{syn} = 2\pi \cdot \frac{1500}{60} = 157 \text{ s}^{-1}, \quad P_\delta = 3 \cdot 220 \cdot 185 = 122.1 \text{ kW}$$

$$\overline{\mathrm{PB}} = 185 \text{ A}$$

d)

$$\eta = \frac{P_m}{P_m + P_{Cu,s} + P_{Fe+R} + P_{Cu.r}} = \frac{114840}{114840 + 7260 + 7260} = \underline{\underline{88.78 \text{ \%}}}$$

$$P_m = P_\delta - P_{Cu,r} = 122100 - 7260 = 114840 \text{ W}$$

Exercise 2.6: *OSSANNA*-circle and slip line

A three-phase induction machine with slip-ring rotor (U_N = 380 V Y, $2p$ = 4, f = 50 Hz) was measured in the manufacturer´s test bay:

a) No-load measurement: I_{s0} = 14 A
P_0 = 0.66 kW

b) Locked rotor test: I_{s1} = 210 A
P_1 = 46.7 kW

c) In addition, a stator current of 189 A and a phase angle of 55° was measured.

d) Rotor phase resistance: R_r = 0.3 Ω, Y-connection of rotor winding

e) Stator and rotor winding losses at s = 1 were found to be equal.

1. Draw the *OSSANNA*-circle and neglect iron and friction losses. Recommended scale: 1mm ≙ 1 A.

2. Determine from *OSSANNA*-circle the speed-torque-curve M = f (n) with correct scale for slip range 1.5 > s > 0 and mark the points for slip: 0. 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.2, 1.5. Use the slip line for determining the slip values ! How big are break down torque and starting torque ?

3. Determine the value of external rotor resistance per phase R_v to get a starting torque, that is at rated voltage 80 % of break down torque !

4. How big is the corresponding starting current I_{s1} at that external rotor resistance R_v ?

5. Draw the torque-speed-curve M = f (n) for operation with external rotor resistance R_v.

Solution:

1) Construction of circle diagram:
From no-load current phasor (length 1.4 cm, phase angle 85.9°) draw point P_0 of circle diagram Fig. 2.6-3, from locked-rotor current phasor (length 21 cm, phase angle 70.3°) determine point P_1. A third point P_3 is determined by current 196 A (length 19.6 cm) at 55° phase angle. From three points the centre of circle M is determined by:

- drawing the perpendicular lines on the line sections $\overline{P_0P_1}$, $\overline{P_0P_3}$, so that these sections are cut into two equal halves.
- The intersection of these two perpendicular straight lines gives the centre of circle M. (Fig. 2.6-3).

With that point M determine the diameter point $P_\varnothing$.
At s = 1: Stator and rotor losses are equal: This is used to determine point P_∞.

$$P_1 = \sqrt{3}\cdot U_N \cdot I_{s1}\cdot\cos\varphi_1 = 46700\,\text{W},\ U_s = 220\text{ V},\ P_1 = 3\cdot U_s\cdot\overline{P_1A} \Rightarrow \overline{P_1A} = 7.1\,\text{cm}$$

$$P_{Cu,s} = 3\cdot U_s\cdot(\overline{P_1A}-\overline{P_1B}) = P_{Cu,r} = 3\cdot U_s\cdot\overline{P_1B} \Rightarrow \overline{P_1A}-\overline{P_1B} = \overline{P_1B}:\quad \overline{P_1B} \mathrel{\hat=} 7.1/2 = 3.55\,\text{cm}$$

- The distance $\overline{P_1B}$ is perpendicular on the circle diameter (a line between P_0 and $P_\varnothing$). The straight line from P_0 via B gives P_∞ (the torque line).

2) With the construction of slip line g_s (Fig. 2.6-1) the slip in circle diagram is determined (Fig. 2.6-4).

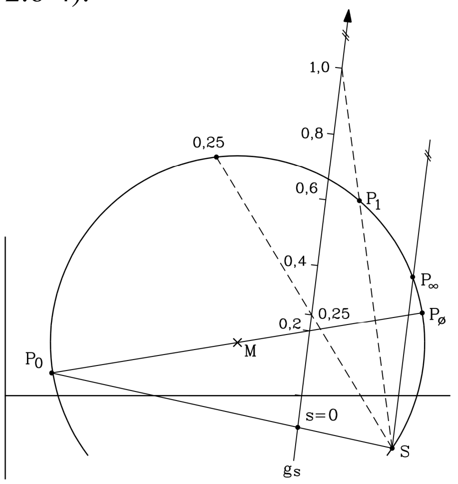


Fig. 2.6-1: Slip line construction to determine slip on circle diagram

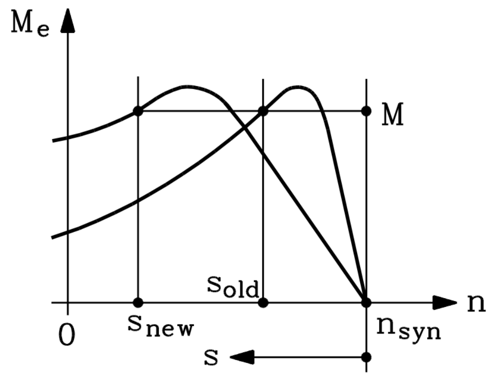


Fig. 2.6-2: Shearing of speed-torque-curve

Torque is determined in operation point P from circle diagram with

$$M_e = \frac{P_\delta}{2\pi \cdot f_s / p} = \frac{3 \cdot U_s \cdot \overline{PB}}{2\pi \cdot f_s / p}$$ and speed with $n = (1-s) \cdot f_s / p$.

s	-	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5
$\overline{PB}$	A	0	85	101	90	76	65	55	43	35	29	25
M_e	Nm	0	357	424	378	319	273	231	180	147	122	105
n	1/min	1500	1350	1200	1050	900	750	600	300	0	-300	-750

This curve $M_e(n)$ is given in Fig. 2.6-5. Breakdown torque occurs at slip 0.17 with 426 Nm, locked rotor torque at s = 1 with 147 Nm.

3) Starting torque shall be 80% of breakdown torque: $0.8 \cdot 426 = 341\,\mathrm{Nm}$ at s = 1. In Fig. 2.6-5 this torque value occurs at slip 0.36. So external rotor resistance is required:

$$\frac{R_r + R_v}{R_r} = \frac{s^*}{s} = \frac{1}{0.36} \quad \Rightarrow \quad R_v = \underline{\underline{0.53}} \text{ Ohm per phase}$$

4) The current at s = 1 with external rotor resistance 0.53 Ohm corresponds to the current at slip 0.36 without external rotor resistance. From circle diagram Fig. 2.6-4 we read: I_s = 198 A.

4) We draw the new $M(n)$-curve with external rotor resistance 0.53 Ohm in Fig. 2.6-5 by shearing the old $M(n)$-curve in accordance with $\frac{s_{new}}{s_{old}} = \frac{R_v + R_r}{R_r} = 2.77$ for the same torque value.

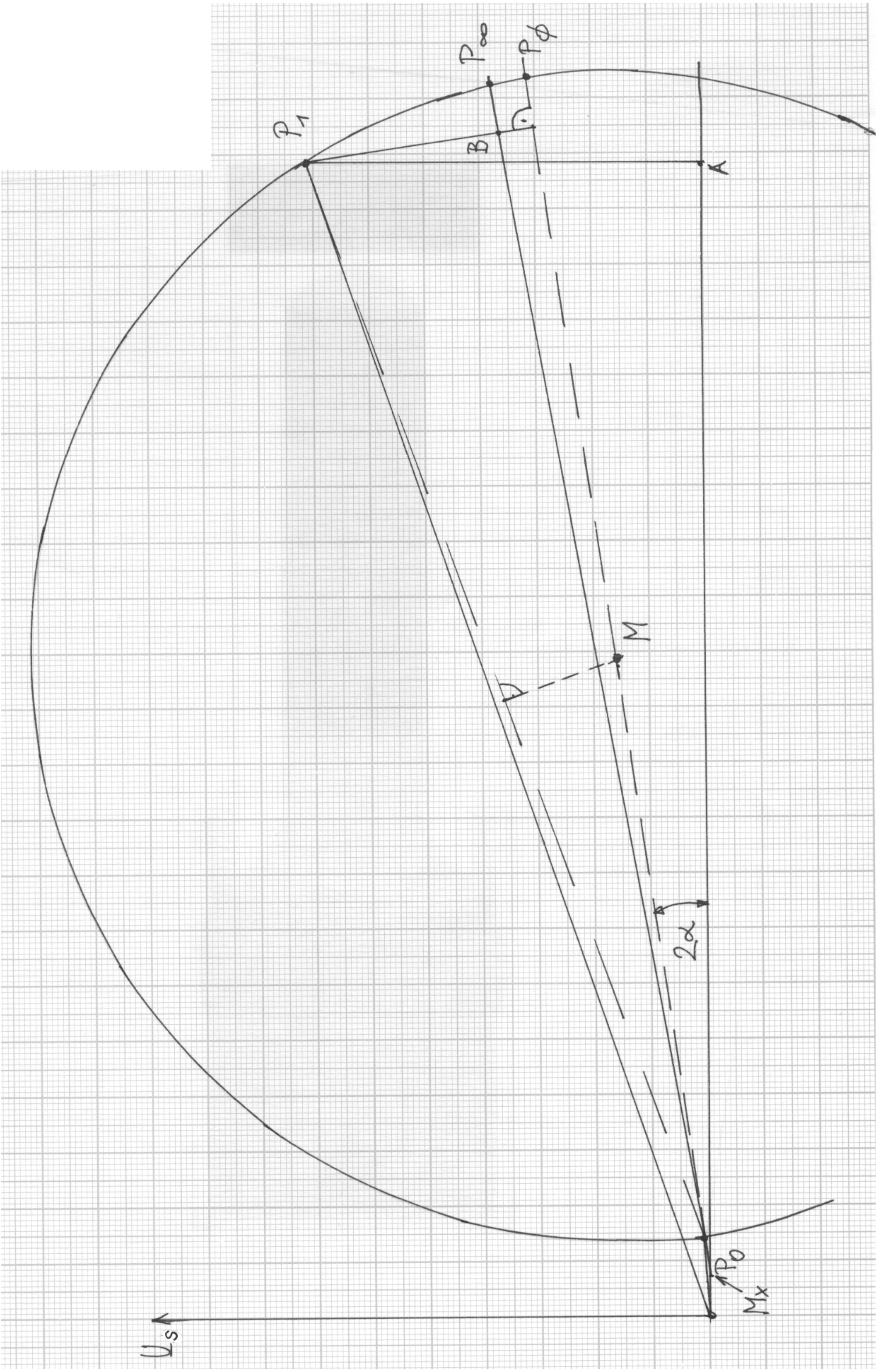


Fig. 2.6-3: Construction of *OSSANNA* circle

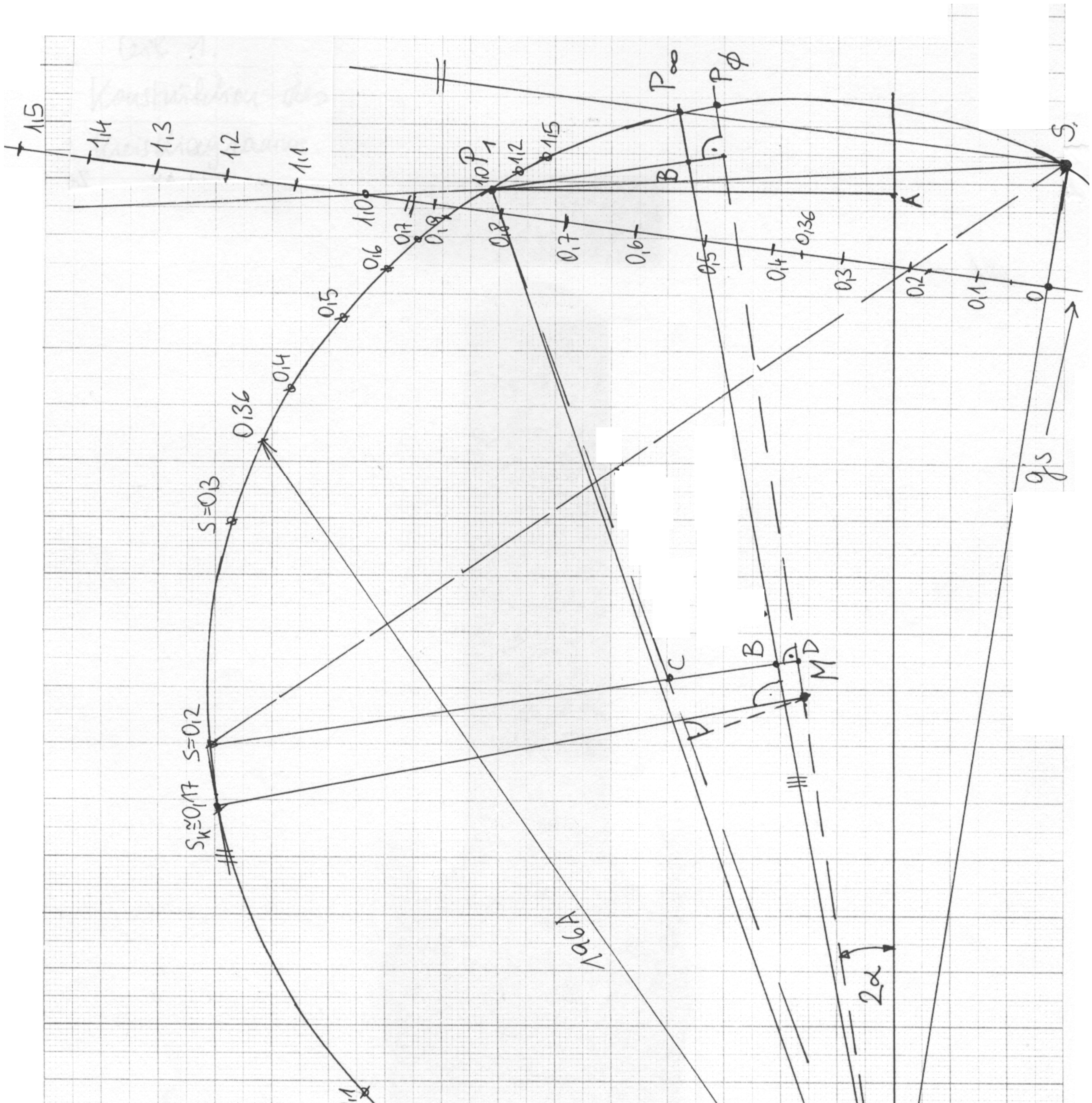


Fig. 2.6-4: Construction of slip line and slip determination at *OSSANNA* circle

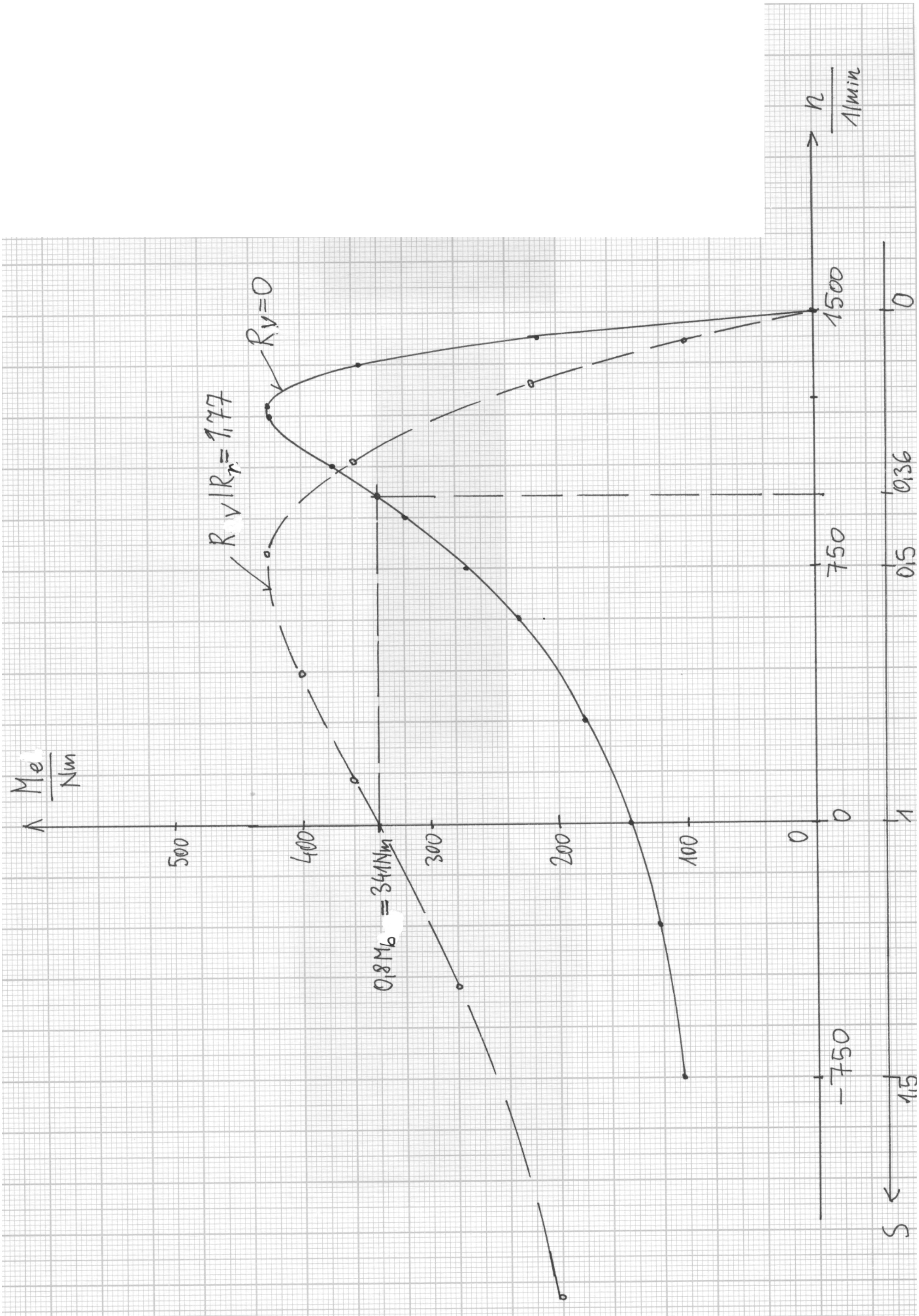


Fig. 2.6-3: Torque-speed-curve, taken from *OSSANNA* circle, without and with external rotor resistance

Exercise 2.7: Drive for a sugar centrifuge

A 4 pole squirrel cage induction machine 630 kW, 500 V, delta-connected, 50 Hz, has been selected for inverter operation up to 75 Hz stator frequency to drive a sugar centrifuge. For constant power operation the voltage is kept constant between 50 Hz and 75 Hz. Motor calculation data produced the T-equivalent circuit per phase at 50 Hz (Fig. 2.7-1).

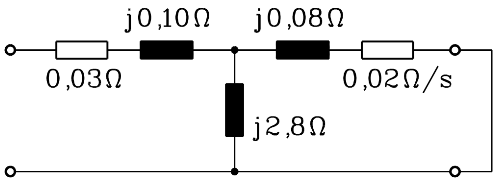


Fig. 2.7-1: T-equivalent circuit of selected squirrel cage motor

1) Calculate synchronous speed at 50 Hz and 75 Hz.
2) Determine simplified *OSSANNA*-circle diagram for a) 50 Hz and b) 75 Hz at 500 V rated voltage from the points $s = 0$ (P_0) and $s = \infty$ (P_∞). Scale: 1cm ≙ 200 A.
3) Determine break down torque graphically for 50 Hz and 75 Hz. Discuss the result !
4) In the manufacturer´s test bay the motor is operated directly from the 50 Hz grid at 500 V. At $s = 1$ the skin effect cause an increase of rotor resistance up to 300% of data of Fig. 2.7-1. Determine locked-rotor torque and compare it with the value with neglected skin effect.
5) The test bay measurement at locked rotor resulted in a power factor of 0.4. Is the measured locked rotor current bigger than the calculated one of 4), with consideration of skin effect ?

Solution:

1) $n_{syn,50} = f_s / p = 50/2 = 25/s = \underline{\underline{1500/\min}}$, $n_{syn,75} = f_s / p = 75/2 = 37.5/s = \underline{\underline{2250/\min}}$
2) Simplified *OSSANNA*-circle diagram: Centre point of circle M lies on abscissa. Current and voltage in circle diagram are phase values. Delta connection: Line voltage 500 V is also phase voltage. Voltage phasor is put into real axis: $\underline{U}_s = U_s = 500$ V.

a) 50 Hz:

No-load: $s = 0$

$$\underline{I}_{s0} = \frac{U_s}{R_s + j(X_{s\sigma} + X_h)} = \frac{500}{0.03 + j(0.1 + 2.8)} = (1.78 - j172.4)A$$

$s = \infty$:

$$\underline{I}_{s\infty} = \frac{U_s}{R_s + jX_{s\sigma} + j\left(\dfrac{X_h X'_{r\sigma}}{X_h + X'_{r\sigma}}\right)} = \frac{500}{0.03 + j0.1 + j\left(\dfrac{2.8 \cdot 0.08}{2.8 + 0.08}\right)} = (461.5 - j2735.4)\text{A}$$

b) 75 Hz: The reactances increase with the ratio 75/50 = 1.5!

No-load: $s = 0$

$$\underline{I}_{s0} = \frac{U_s}{R_s + j(X_{s\sigma} + X_h)} = \frac{500}{0.03 + j(0.15 + 4.2)} = (0.8 - j115)A$$

$s = \infty$:

$$\underline{I}_{s\infty} = \frac{U_s}{R_s + jX_{s\sigma} + j\left(\dfrac{X_h X'_{r\sigma}}{X_h + X'_{r\sigma}}\right)} = \frac{500}{0.03 + j0.15 + j\left(\dfrac{4.2 \cdot 0.12}{4.2 + 0.12}\right)} = (205.5 - j1839)A$$

With the two points $s = 0$ (P_0) and $s = \infty$ (P_∞) the perpendicular line on the distance $\overline{P_0 P_\infty}$ is drawn, that intersects this distance into two equal halves. The intersection of this perpendicular line with the abscissa (-Im-axis) gives the centre point M of simplified *OSSANNA*-circle diagram (Fig. 2.7-3, 2.7-4).

3) Break down torque: $M_b = \dfrac{P_{\delta,b}}{2\pi \cdot f_s / p} = \dfrac{3 \cdot U_s \cdot \overline{P_b B}}{2\pi \cdot f_s / p}$

Point P_b is determined graphically by parallel line to torque line $\overline{P_0 P_\infty}$ (see Fig. 2.7-3, 2.7-4).

At f_s = 50 Hz: $\overline{P_b B} = 1100A \mathrel{\hat{=}} 5.5cm$: $M_{b,50}$ = <u>10504 Nm</u>

At f_s = 75 Hz: $\overline{P_b B} = 780A \mathrel{\hat{=}} 3.9cm$: $M_{b,75}$ = <u>4965 Nm</u>

Discussion of result:

Ratio: $\dfrac{M_{b,75}}{M_{b,50}} = \dfrac{4965}{10504} = 0.47$. This is in accordance with theory, that at constant voltage break down torque decreases with square of stator frequency (Fig. 2.7-2) due to flux weakening.

$$\frac{M_{b,75}}{M_{b,50}} = \left(\frac{50}{75}\right)^2 = 0.44$$

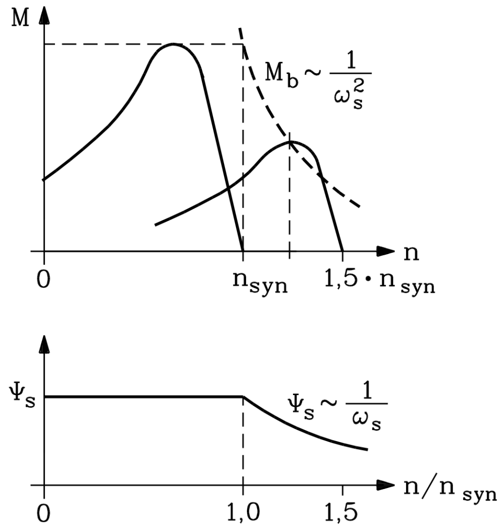


Fig. 2.7-2: Decrease of break down torque at constant voltage with increasing frequency

4) Skin effect: Rotor resistance: $R'_r = 3 \cdot 0.02 = 0.06\Omega$

Starting torque: $M_1 = \dfrac{P_{Cu,r}}{2\pi \cdot f_s / p}$, $P_{Cu,r} = P_{e,in} - P_{Cu,s} = 3 \cdot U_s \cdot I_{s,1} \cdot \cos\varphi_1 - 3R_s I_{s,1}^2$

$$\underline{I}_{s,1} = \frac{U_s}{R_s + jX_{s\sigma} + \dfrac{jX_h \cdot (R'_r + jX'_{r\sigma})}{R'_r + j(X_h + X'_{r\sigma})}} = \frac{500}{0.03 + j0.1 + \dfrac{j2.8 \cdot (0.06 + j0.08)}{0.06 + j(2.8 + 0.08)}} = (1096 - j2263)A$$

$$I_{s,1} = 2513A, \quad I_{s,1}\cos\varphi_1 = 1096A, \quad P_{Cu,r} = 3 \cdot 500 \cdot 1096 - 3 \cdot 0.03 \cdot 2513^2 = 1075.2\,\text{kW}$$

$$M_1 = \frac{1075200}{2\pi \cdot 50/2} = \underline{\underline{6844}}\,\text{Nm}$$

Without skin effect: Rotor resistance: $R'_r = 0.02\Omega$

$$\underline{I}_{s,1} = \frac{500}{0.03 + j0.1 + \dfrac{j2.8 \cdot (0.02 + j0.08)}{0.02 + j(2.8 + 0.08)}} = (718.2 - j2613)A$$

$$I_{s,1} = 2710A, \quad I_{s,1}\cos\varphi_1 = 718.2A, \quad P_{Cu,r} = 3\cdot 500\cdot 718.2 - 3\cdot 0.03\cdot 2710^2 = 416.33\,\text{kW}$$

$$M_1 = \frac{416330}{2\pi\cdot 50/2} = \underline{\underline{2650}}\,\text{Nm}$$

Comparison of starting torque: Due to skin effect the starting torque increases by 158% !

5) With measured power factor 0.4 a phase angle $\varphi_1 = 66.4°$ is derived, which yields with *OSSANNA*-circle Fig. 2.7-3 at 50 Hz a starting current 2520 A. This is slightly larger than the calculated locked-rotor current with skin effect consideration, 2513 A. Deviation is only 0.3%, so we conclude: Calculation and measurement fit well !

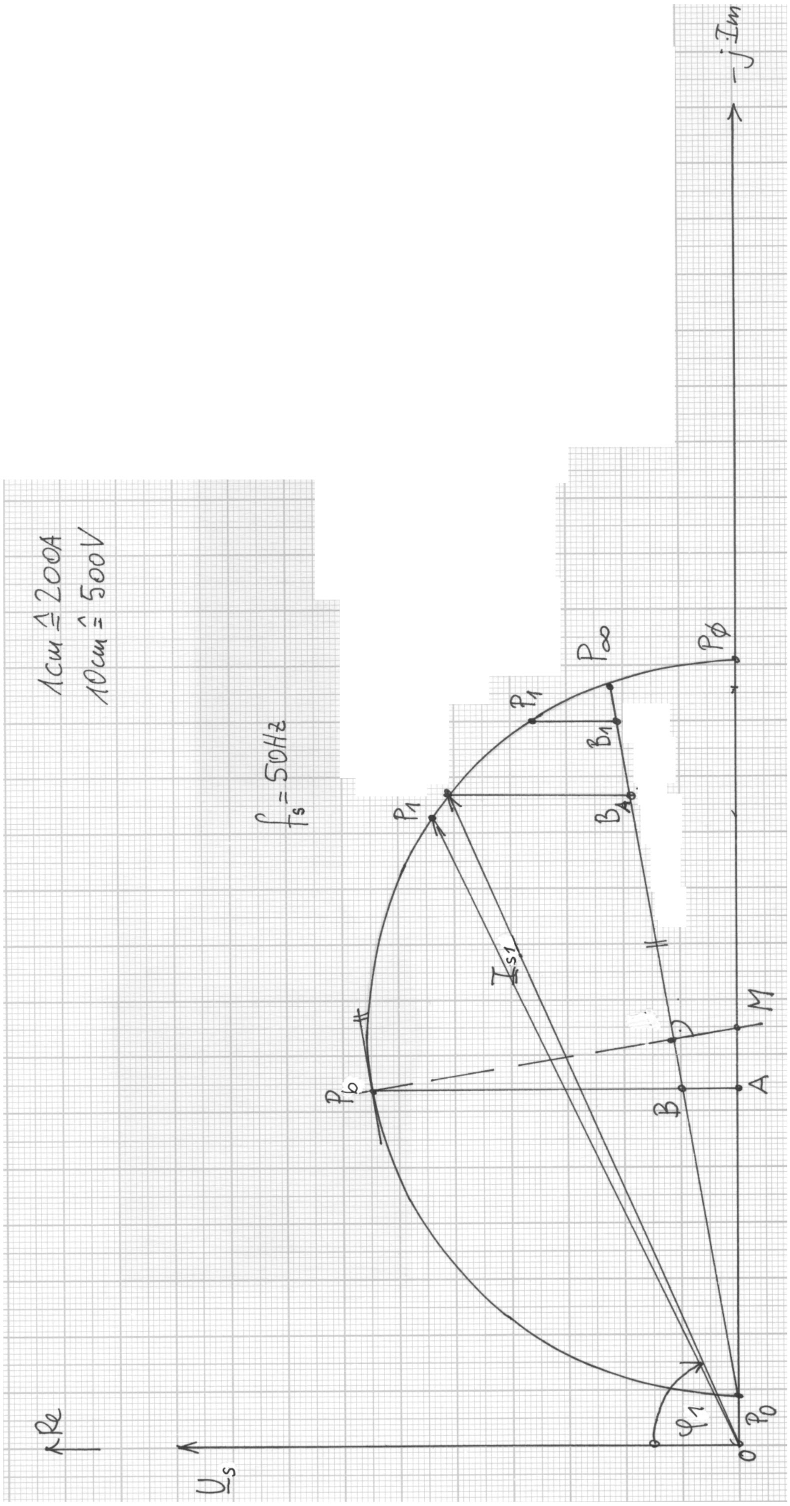
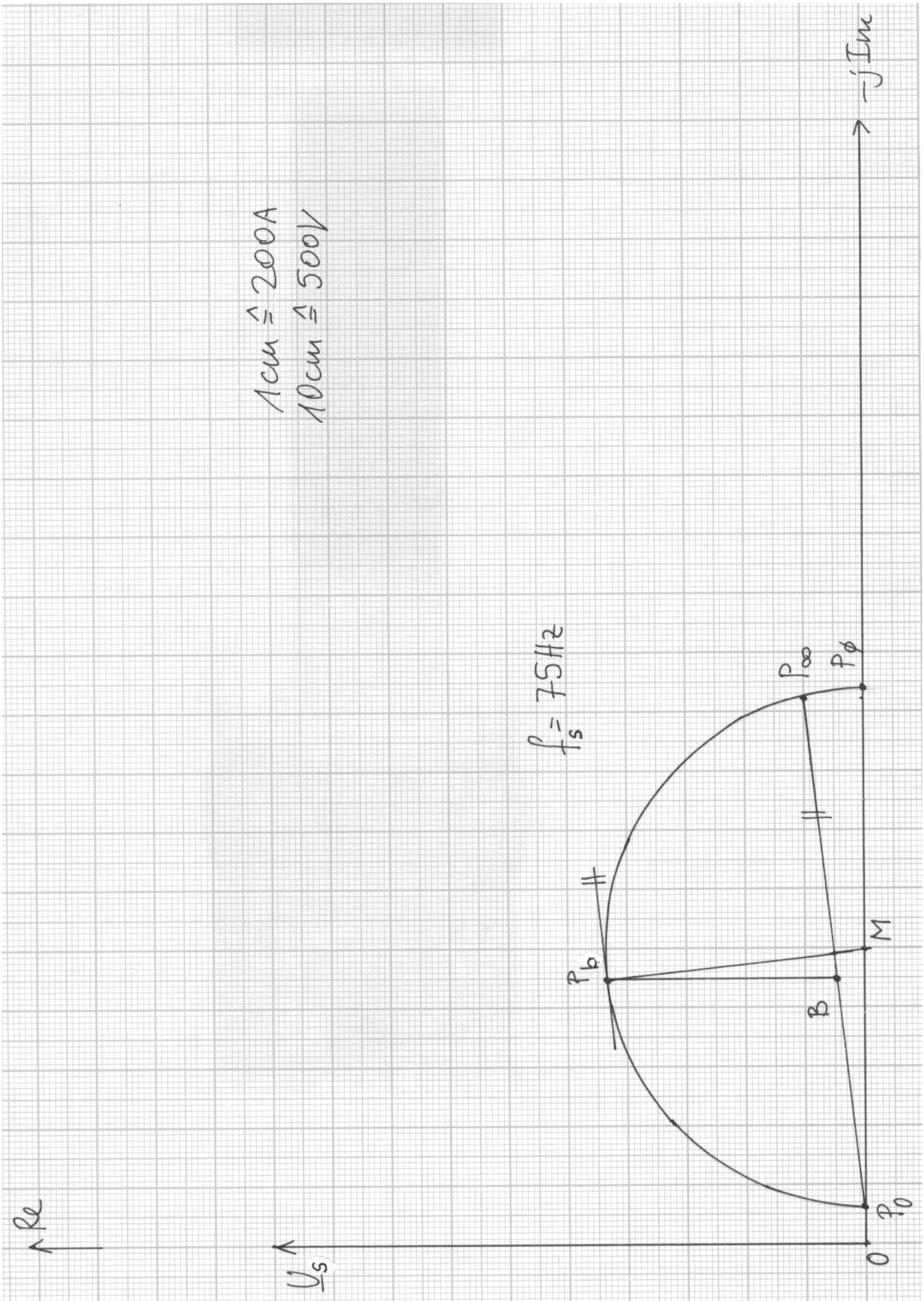


Fig. 2.7-3: Simplified OSSANNA-circle at 50 Hz



<u>Fig. 2.7-4:</u> Simplified OSSANNA-circle at 75 Hz

Exercise 2.8: Asynchronous motor for a wood grinding machine

An AC asynchronous motor is used for a wood grinding machine. The following line data of voltage, current and power factor were measured at $U_N = 380$ V Δ:
Rated power: $I_N = 80$ A, $2p = 8$, $f_N = 50$ Hz,
No load: $I_{s0} = 30$ A, $\cos\varphi_0 \approx 0$,
Locked rotor: $I_{s1} = 400$ A, $\cos\varphi_1 = 0.4$
Stator resistance $R_s = 0.2$ Ω/phase.

1) How big is starting torque for rated voltage ? All the losses except stator and rotor winding losses $P_{Cu,s}$ and $P_{Cu,r}$ shall be neglected.
2) How big is the starting current and torque for Y/Δ-start-up?
3) Draw the simplified *OSSANNA* circle diagram with recommended scale: 1 cm ≙ 10 A. Determine from circle diagram the power factor at rated load.

Solution:

1) $s = 1$: $M_1 = \frac{P_{Cu,r}}{\omega_{syn}} = \frac{P_{e,in} - P_{Cu,s}}{2\pi f_s / p}$, $P_{e,in} = \sqrt{3} \cdot U_N I_{s1} \cos\varphi_1 = \sqrt{3} \cdot 380 \cdot 400 \cdot 0.4 = 105.3\,\text{kW}$,

$$P_{Cu,s} = 3 \cdot R_s \cdot I_{s1,phase}^2 = 3 \cdot 0.2 \cdot (400/\sqrt{3})^2 = 32.0\,\text{kW},\; P_{Cu,r} = 105.3 - 32.0 = 73.3\,\text{kW}$$

$$\omega_{syn} = \frac{2 \cdot \pi \cdot f_s}{p} = \frac{2 \cdot \pi \cdot 50}{4} = 78.54\,\text{s}^{-1}$$

$$M_1 = \frac{73300}{78.54} = \underline{\underline{933\,\text{Nm}}}$$

2) The direct starting current at delta connection is $I_{s,1} = 400\text{A}$. For Y/D–start-up at Y-connection the current is only 1/3, because:
1. The phase voltage is only $380/\sqrt{3} = 220\,\text{V}$
2. For Y-connection the line current is the same as the phase current, therefore it is $1/\sqrt{3}$ smaller as the current for D.

$$I_{1Y} = \frac{400}{3} = \underline{\underline{133{,}3\,\text{A}}},\quad M_{1Y} = \frac{933}{3} = \underline{\underline{311\,\text{Nm}}}.$$

3) Simplified *OSSANNA* circle is drawn from points P_0 and P_1:
P_0 : $\cos\varphi_0 = 0 \Rightarrow \varphi_0 = 90°$, $I_{s0,phase} = 30/\sqrt{3} = 17.3\,\text{A}$
P_1 : $\cos\varphi_1 = 0.4 \Rightarrow \varphi_1 = 66.4°$, $I_{s1,phase} = 400/\sqrt{3} = 231\,\text{A}$.
Perpendicular line on distance $\overline{P_0P_1}$, which is intersecting it into two halves, gives circle centre point M with intersection with abscissa (-Im-axis). Determination of torque line $\overline{P_0P_\infty}$ is done by evaluating stator and rotor winding losses (values taken from 1)) at locked rotor (Fig. 2.8-1):

$$\left.\begin{aligned} \overline{P_1B_1} \cdot 3 \cdot U_{s1} = P_{Cu\,r} \Rightarrow \overline{P_1B_1} = \frac{73300}{3 \cdot 380} = 64.3\,\text{A} \\ \overline{B_1A_1} \cdot 3 \cdot U_{s1} = P_{Cu\,s} \Rightarrow \overline{B_1A_1} = \frac{32000}{3 \cdot 380} = 28.0\,\text{A} \end{aligned}\right\} \Rightarrow P_\infty$$

Nominal current/phase: $80/\sqrt{3} = 46.1\,\text{A}$. With this value we get from circle diagram:
$\varphi_N = 32^o$, $\underline{\underline{\cos\varphi_N = 0.848}}$

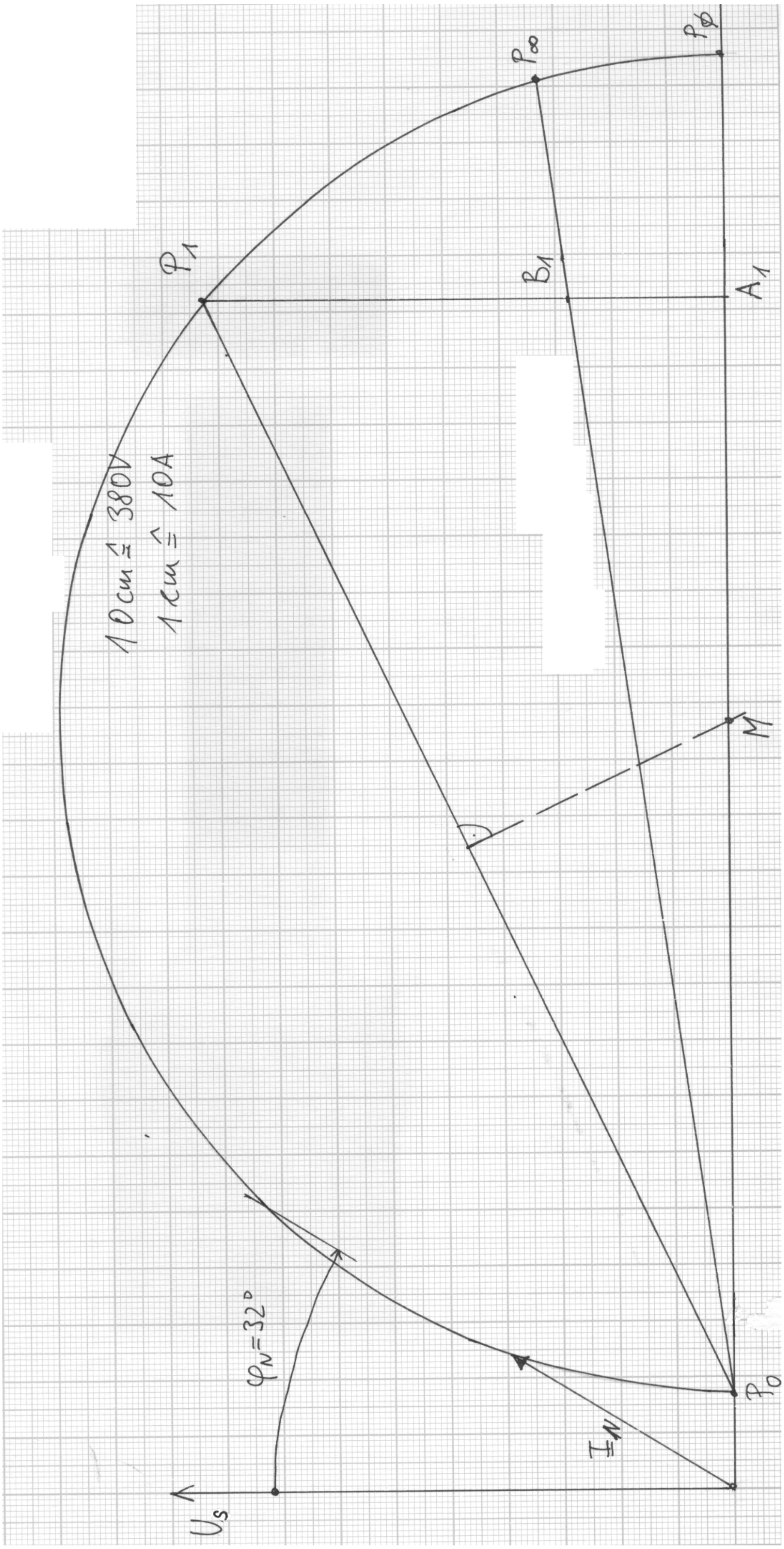


Fig. 2.8-1: Simplified OSSANNA circle for wood grinder motor

Exercise 2.9: Slip-ring motor for heavy duty starting

For starting loads with big moment of inertia (= Heavy duty starting) an alternating current induction motor with slip ring rotor was selected. The motor has the following data for 50 Hz:

Stator winding:	Rotor winding:
U_{sN} = 380 V Y (line to line)	U_{r0} = 280 V Y (line to line, at stand-still, f_r = 50 Hz)
I_{sN} = 264 A	I_{rN} = 350 A
P_{sN} = 150 kW (electrical input)	R_r = 0.0143 Ω/phase
s_b = 0.19	

The measured characteristics M = f (n/n_{syn}), I_s = f (n/n_{syn}) for short-circuited rotor are shown in Fig. 2.9-3.

1) Choose the value of rotor external starting resistor (in Ω/phase), so that the machine will have a starting torque of 2500 Nm?
2) How big is the starting current in this case?
3) Determine rated slip s_{N}!
4) Calculate the braking torque for counter-current braking with external rotor resistor from 1) for a speed $n = 95\ \%\ n_N$.
5) What number of poles has the motor ? How big is rated efficiency ?

Solution:

1) $M = 2.5\,\text{kNm}$ occurs according to Fig. 2.9-3 at slip $s_1 = 0.12$ and $s_2 = 0.3$. With $s_1 = 0.12$ as selected slip/Torque for starting, the accelerating torque for 0 < s < 1 is steadily decreasing from starting value $M_1 = 2.5\,\text{kNm}$, while for $s_2 = 0.3$ torque increases up to M_b! So starting is quicker and more powerful. So $s_2 = 0.3$ is chosen.

$$s = 1 - \frac{n}{n_{syn}},\quad \frac{R_r}{s} = \frac{R_r + R_V}{1}\quad (s\text{=}1)$$

$$R_V = \left(\frac{1}{s_2} - 1\right) R_r = \left(\frac{1}{0.3} - 1\right) \underbrace{R_r}_{0.0143} = \underline{\underline{0.033\Omega}}$$

2) Starting current I_{s1}: According to graph in Fig. 2.9-3 we get $I_s(s_2 = 0.3) = \underline{\underline{1250\ \text{A}}}$

Here $s_1 = 0.12$ ($R_V = 0.105\,\Omega$) would be adequate, because $I_s(s_1 = 0.12) = 550\ \text{A}$ is much lower.

3) $I_{sN} = 264\ \text{A} \rightarrow s_N = 3\%$ according to graph (Fig. 2.9-3).

4) Counter-current braking (Fig. 2.9-1) at: $n = 0.95 \cdot n_N = 0.95 \cdot (1 - s_N) \cdot n_{syn} = 0.92 \cdot n_{syn}$

$$n_N = (1 - s_N) \cdot n_{syn} = 0.97 \cdot n_{syn}$$

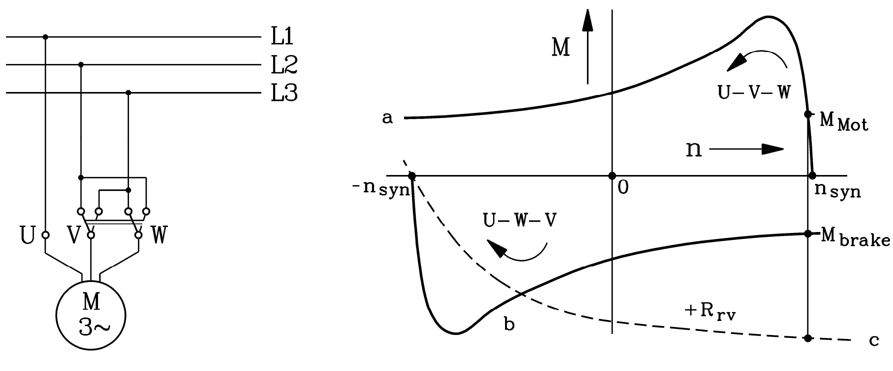


Fig. 2.9-1: Counter-current braking by exchanging of two motor terminals (dashed: with external rotor resistance R_{rv})

2 terminals exchanged: Direction of rotating field is reversed: $n_{syn} \rightarrow -n_{syn} \rightarrow$ new slip

$$s_{new} = \frac{-n_{syn} - 0{,}92 \cdot n_{syn}}{-n_{syn}} = 1.92$$

Determination of braking torque according to Fig. 2.9-1 with consideration of external rotor resistance (Fig. 2.9-2):

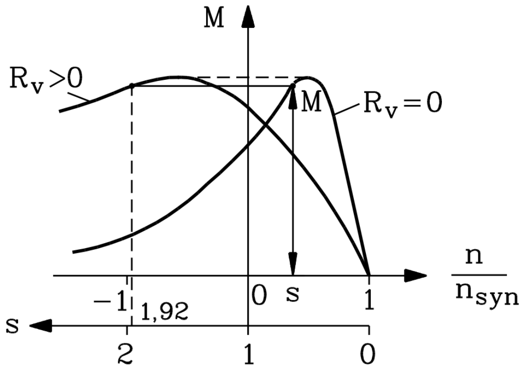


Fig. 2.9-2: Shearing of torque-speed characteristic due to external rotor R_v

$$\frac{R_r + R_v}{s_{new}} = \frac{R_r}{s} \quad \Rightarrow \quad \text{with } R_V = 0.033\,\Omega: \quad s = 0.58$$

Braking torque: $M(s = 0.58) = M_{brake} = \underline{\underline{1700\text{ Nm}}}$ according to the characteristic Fig. 2.9-3.

5) Fig. 2.9-3 gives at $s_N = 3\% \rightarrow M_N \approx 900\,\text{Nm}$! Mechanical output power is only smaller than electrical input power due to losses.

$P_m = 2\pi n_N \cdot M_N = 2\pi \cdot 0.97 \cdot M_N \cdot n_{syn} = 137\text{ kW}$ for $n_{syn} = 1500/\text{min}$

$\Rightarrow \underline{\underline{2p = 4}}$

Rated efficiency: $\eta_N = \frac{P_{m,out}}{P_{e,in}} = \frac{P_m}{P_{sN}} = \frac{137}{150} = \underline{\underline{0.913}}$

<u>Exercise 2.10</u>: Replacement of central drive for a spinning machine

A central drive for a spinning machine, consisting of one mains operated standard induction motor with a power of 11 kW, 50 Hz, supplies via a belt transmission with transmission shafts 10 spinning stations. It shall be replaced by a group drive of 10 decentralised motors, each consisting of a motor and gear with data per drive: 1.1 kW motor power, 50 Hz, 1390/min, 1:53 gear ratio for low speed operation. These new drives are separately controllable, thus enhancing spinning machine performance. Is possible to chose the new 1.1 kW drives as:

(a) Standard induction motors with a standard gear or
(b) Energy-saving induction motors with energy-saving gear.

The energy saving motors are made of low-loss magnetic steel sheet, having increased length of laminated core. The energy-saving gear operates with synthetic oil instead of standard mineral oil. Option (b) is therefore more expensive.

		Motor efficiency	Gear efficiency	Price
(a)	Standard drive	0.71	0.66	€ 415,--
(b)	Energy-saving drive	0.84	0.70	€ 435,--

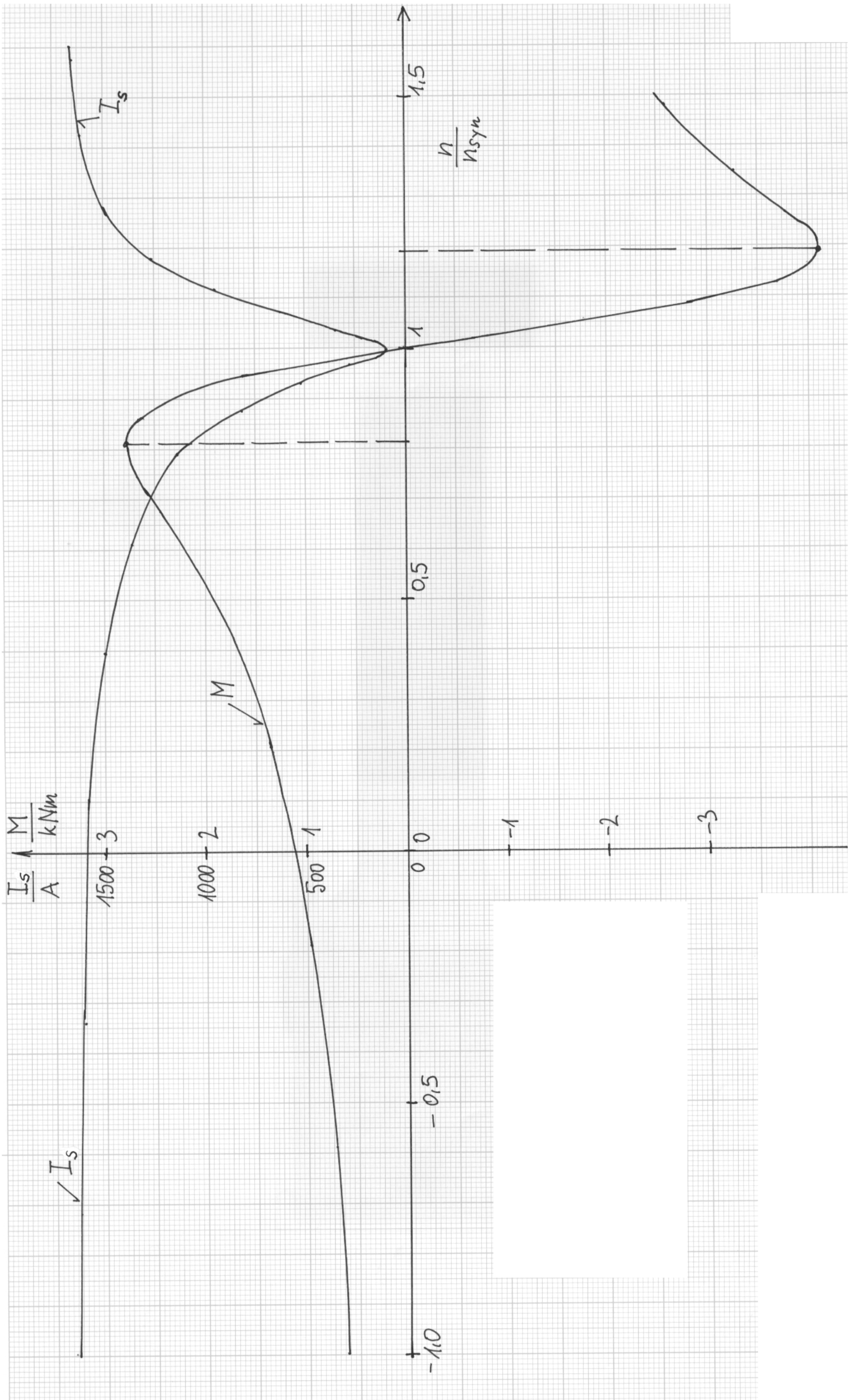


Fig. 2.9-3: Measured characteristics $M = \mathrm{f}\,(n/n_{syn})$ and $I_s = \mathrm{f}\,(n/n_{syn})$ for short-circuited rotor at rated voltage

1) How big is speed, power and torque for the cases a) and b) at gear output for 1.1 kW motor output power ?

2) How big is the number of poles for the decentralised motors?

3) How big is for a required torque of 255 Nm per drive at gear output and rated speed according to 1) the active power input for a) and b) ?

4) Decentralized motors are operating according to 3) continuously. For which time of operation, calculated in working days, is the increased price of the energy-saving drives paying off due to lower energy costs ? Drives are operated in 2 shifts (2x8 = 16 h/day) Total costs for electric energy are 9.5 cent/kWh.

5) Assuming that option a) has chosen for replacing the central drive. In how many working days replacing of a) by option b) will pay off due to lower energy costs? (Costs for the replacement are not calculated).

6) How big are for group drive option b) the saved energy costs per year compared to option a) (2 shifts operation, 5 days per week, 52 weeks per year) ?

Solution:

1) $n = \frac{n_{mot}}{i} = \frac{1390}{53} = \underline{\underline{26.2/\min}}$

Option a): $P = \eta_{gear} \cdot P_{mot,N} = 0.66 \cdot 1100 = \underline{\underline{726}}\ \mathrm{W}$, $M = \frac{P}{2\pi n} = \frac{726}{2\pi \frac{26.2}{60}} = \underline{\underline{264.6\ \mathrm{Nm}}}$

Option b): $P = \eta_{gear} \cdot P_{mot,N} = 0.7 \cdot 1100 = \underline{\underline{770}}\ \mathrm{W}$, $M = \frac{770}{2\pi \frac{26.2}{60}} = \underline{\underline{280.6\ \mathrm{Nm}}}$

2) 1390 /min = $n_{mot,N}$, $f_{sN} = 50\ \mathrm{Hz} \Rightarrow n_{syn} = 1500\ /\mathrm{min}$ to get a nominal slip, which is small enough to be a realistic value. $s_N = \frac{n_{syn} - n_{mot,N}}{n_{syn}} = 7.3\%$ is the smallest possible value,

$$\Rightarrow 2p = 2\frac{f_{sN}}{n_{syn}} = 2\frac{50}{\frac{1500}{60}} = \underline{\underline{4}}$$

3) $P = 2\pi n M = 2\pi \cdot 26.2 \frac{1}{60} 255 = 700\ \mathrm{W} = P_{out}$

Option a) $P_{e,in} = \frac{P_{out}}{\eta_{gear} \cdot \eta_{mot}} = \frac{700}{0.66 \cdot 0.71} = \underline{\underline{1494\ \mathrm{W}}}$

Option b) $P_{e,in} = \frac{P_{out}}{\eta_{gear} \cdot \eta_{mot}} = \frac{700}{0.70 \cdot 0.84} = \underline{\underline{1190\ \mathrm{W}}}$

4) Electrical energy costs:

Option a) $0.095 \frac{\mathrm{Euro}}{\mathrm{kWh}} \cdot 16 \frac{\mathrm{h}}{\mathrm{day}} \cdot 1.494\ \mathrm{kW} = 2.27 \frac{\mathrm{Euro}}{\mathrm{day}}$

Option b) $0.095 \frac{\mathrm{Euro}}{\mathrm{kWh}} \cdot 16 \frac{\mathrm{h}}{\mathrm{day}} \cdot 1.190\ \mathrm{kW} = 1.81 \frac{\mathrm{Euro}}{\mathrm{day}}$

Electric energy costs difference is 0.462 Euro/day for one drive. The difference in investment cost per drive is $435-415=20\,\text{Euro}$. This is paid off by lower energy costs in $\frac{20}{0.462}=\underline{\underline{43.28\text{ working days}}}$

5) For one energy-efficient drive (option b)) 0.462 Euro/day will be saved. When the energy saving drive will be purchased with 435 Euro, the amortisation time would be:
$\frac{435}{0.462}=\underline{\underline{941.56\text{ working days}}}\approx 3.6\text{ working years } (=52\text{ weeks per year}, 5\text{ days per week})$

6) 52 weeks, 5 working days per week = 260 working days per year; saving per working day: 0.462 Euro: 10 drives: Saved energy costs for option b) per year:
$10\cdot 0.462\cdot 260=\underline{\underline{1201.2\text{ Euro}}}$

Exercise 2.11: Electric propulsion system for electric vehicle

An inverter fed cage-induction motor with water jacket cooling is used as central drive for an electric automobile. The propulsion system consists of:

- PEM-fuel cell as voltage source: DC voltage 250 V
- PWM voltage source dc-link inverter: max. output voltage: 100 V (line-to-line, r.m.s.)
- Four pole, cage induction motor: nominal frequency f_{sN} = 135 Hz, delta connected winding $R_s \approx 0$, X_h = 1.86 Ω, $X_{s\sigma}$ = 0.063 Ω, $X'_{r\sigma}$ = 0.089 Ω, R'_r = 0.013 Ω (phase values).
- Single stage gear: gear ratio i = 8 for speed reduction, gear efficiency: 97 %
- Drive wheels of automobile: Wheel diameter: 0.7 m, rated wheel torque: 590 Nm at induction motor frequency 135 Hz.

1) How big is rated induction motor torque?

2) Calculate *BLONDEL´s* motor leakage coefficient σ, motor break-down torque M_b, motor break-down slip s_b and corresponding rotor frequency f_{rb} for $U_{s,LL}$ = 100 V, f_s = 135 Hz !

3) Determine motor nominal slip and speed, as well as output power for $U_{s,LL}$ = 100 V and f_s = 135 Hz. Consider only rotor cage losses, so you can use *KLOSS* formula !

4) Which rotor frequency f_r appears at nominal point or operation acc. to 3) ?

5) The motor will be operated with field weakening for f_s > 135 Hz at $U_{s,LL}$ = 100 V = const. to supply constant power output! Which maximum synchronous speed and maximum motor operating speed are possible ?

6) How big is for maximum motor speed from 5) the maximum speed of the automobile?

Solution:

1) $M_{mot}=M_{wheel}\frac{1}{\eta_{gear}}\frac{1}{i}=590\frac{1}{0.97}\frac{1}{8}=\underline{\underline{76}}\,\text{Nm}\quad \eta_{gear}=0.97\quad i=8\quad M_{wheel}=590\,\text{Nm}$

2) $X_s=X_{s\sigma}+X_h=0.063+1.86=1.923\,\Omega$
$X'_r=X'_{r\sigma}+X_h=0.089+1.86=1.949\,\Omega$

$$\sigma = 1 - \frac{X_h^2}{X_s X_r'} = 1 - \frac{1.86^2}{1.923 \cdot 1.949} = \underline{\underline{0.077}}$$

M_b for $R_s \approx 0$ from *KLOSS* formula: $M_b = \frac{m_s \cdot p}{2}\left(\frac{U_s}{\omega_s}\right)^2 \frac{1-\sigma}{\sigma \cdot L_s}$

$m_s = 3,\ p = 2,$ phase voltage $U_s = U_{s,LL} = 100V,\ \omega_s = 2\pi \cdot f_s = 2\pi \cdot 135 = 848.2\ \mathrm{s}^{-1}$

$L_s = X_s / \omega_s = 1.923/848.2 = 2.267\ \mathrm{mH}$

$$M_b = \frac{3 \cdot 2}{2}\left(\frac{100}{848.2}\right)^2 \frac{1-0.077}{0.077 \cdot \frac{2.267}{10^3}} = \underline{\underline{220.5\ \mathrm{Nm}}}$$

$$s_b(R_s \approx 0) \cong \frac{R_r'}{\sigma \cdot X_r'} = \frac{0.013}{0{,}077 \cdot 1.949} = \underline{\underline{0.0866}}$$

$$f_{rb} = s_b \cdot f_s = 0.0866 \cdot 135 = \underline{\underline{11.7\ \mathrm{Hz}}}$$

3) $R_s \approx 0 \Rightarrow$ K*LOSS* formula: $\frac{M_e}{M_b} = \frac{2}{\frac{s}{s_b} + \frac{s_b}{s}}$ $x = \frac{s}{s_b},\ y = \frac{M_b}{M_e} \Rightarrow \frac{1}{y} = \frac{2}{x + \frac{1}{x}}$

$x + \frac{1}{x} - 2y = 0 \Rightarrow x^2 - 2 \cdot y \cdot x + 1 = 0$ $x = y \pm \sqrt{y^2 - 1}$

$\frac{s}{s_b} = \frac{M_b}{M_e} - \sqrt{\left(\frac{M_b}{M_e}\right)^2 - 1}$ $\frac{M_b}{M_N} = \frac{220.5}{76} = 2.9$

$$\frac{s_N}{s_b} = 2.9 - \sqrt{2.9^2 - 1} = 0.178 \Rightarrow s_N = 0.178 \cdot 0.0866 = \underline{\underline{0.0154}}$$

$$n_N = (1 - s_N) \cdot n_{syn} = (1 - 0.0154) \cdot 4050 = \underline{\underline{3988/\mathrm{min}}}$$

$$n_{syn} = f_s / p = 135/2 = 67.5\ /\mathrm{s} = 4050\ /\mathrm{min}$$

$$P_{out} = P_m = 2\pi \cdot n_N \cdot M_N = 2\pi \frac{3988}{60} \cdot 76 = \underline{\underline{31736\ \mathrm{W}}}$$

4) $f_{rN} = s_N \cdot f_{sN} = 0.0154 \cdot 135 = \underline{\underline{2.08\ \mathrm{Hz}}}$

5) Constant power operation (Fig. 2.11-1):

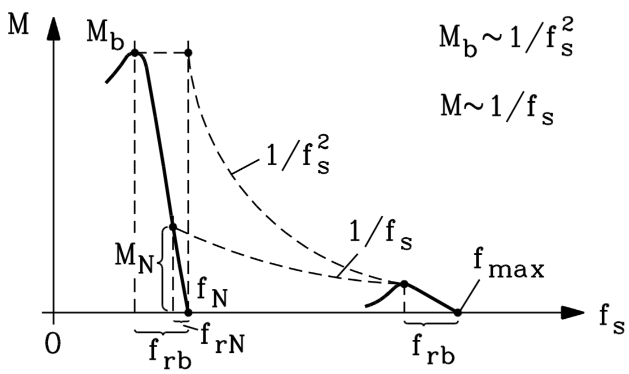


Fig. 2.11-1: Decrease of torque at constant power and decrease of break down torque due to flux weakening

Decrease of break down torque with inverse square of increasing stator frequency, whereas torque at constant power decreases only with inverse of frequency. Intersection of both curves marks end of constant power range:

$$M_{bN}\left(\frac{f_{sN}}{f_{s,\max}}\right)^2 = M_N\left(\frac{f_{sN}}{f_{s,\max}}\right) \qquad \Rightarrow \qquad f_{s,\max} = f_{s,N}\frac{M_{bN}}{M_N}$$

With $M_{bN} = 220.5$ Nm, $M_N = 76$ Nm, $f_{sN} = 135\ \text{Hz} \Rightarrow f_{s,\max} = 135\frac{220.5}{76} = \underline{\underline{391.7\ \text{Hz}}}$

$$n_{syn,\max} = f_{s,\max}/p = 391.7/2 = 195.8\ /\text{s} = \underline{\underline{11750\ /\text{min}}}$$

$$f_{s,\max} - f_{r,b} = n_{\max}\cdot p \Rightarrow n_{\max} = \frac{f_{s,\max}}{p} - \frac{f_{r,b}}{p} = \frac{391.7}{2} - \frac{11.7}{2} = 190\ /\text{s} = \underline{\underline{11400\ /\text{min}}}$$

6) $d_{wheel} = 0.7$ m

$$n_{wheel,\max} = n_{mot,\max}/i = 11400/8 = 1422.8\ /\text{min}$$

$$v_{a,\max} = d_{wheel}\cdot\pi\cdot n_{wheel,\max} = 0.7\cdot\pi\cdot\frac{1422.8}{60} = 52.14\ \text{m/s} = \underline{\underline{187.7\ \text{km/h}}}$$

Exercise 2.12: Tunnel fan motor

In *Arlberg* tunnel between *Tyrol* and *Vorarlberg*, *Austria*, pole-changing squirrel-cage induction motors are used for driving axial fans for tunnel ventilation to expel exhaust gases ! Motors are equipped with *DAHLANDER*-winding, 6 kV nominal voltage, 50 Hz line frequency. This winding allows two speed stages to change air flow.

Motor data:
High speed stage: 777 kW, 984 /min, 6 kV, delta connection
breakdown torque: $3M_N$, starting torque (s = 1): $0.8M_N$, breakdown slip: $s_b = 0.15$
Low speed stage: 6 kV, star connection

1) Which two synchronous speed stages can be operated with the motor?
2) The nominal slip for the low speed stage amounts 2.4 %. How big is nominal speed ?
3) Determine power demand of fan for low speed stage !
4) Assuming that the product of motor efficiency and power factor is 0.8 for both speed stages: How big is stator line current for both speed stages ? How big is the corresponding phase current in motor winding?
5) Stator current at locked rotor for nominal voltage and frequency is 7.5-times nominal current for high speed stage. How big is the current consumption of the motor at start-up in the high speed step, when the Y-D-start-up is used ?
6) Sketch for the Y-D-start-up from 5)
   a) motor torque-speed curve for Y and D connection,
   b) fan torque-speed curve !
   Is Y-D-start-up possible for the motor against load torque due to fan ?

Solution:

1) High speed stage: $984\ /\text{min} \Rightarrow \underline{\underline{n_{syn} = 1000\ /\text{min}}}$ for 50 Hz ($2p$ = 6).

Low speed stage with *DAHLANDER*-winding: $p^* = 2p : 2p^* = 12$

$$n_{syn}^* = \frac{f_s}{p^*} = \frac{50}{6} = 8.33\ /\text{s} = \underline{\underline{500\ /\text{min}}}$$

2) $s_N^* = 2.4\% = \frac{n_{syn}^* - n^*}{n_{syn}^*} \Rightarrow n^* = (1 - s_N^*)\cdot n_{syn}^* = (1 - 0.024)\cdot 500 = \underline{\underline{488\ /\text{min}}}$

3) Fan: $M_{fan} \approx n^2$, $P_{fan} = 2\pi \cdot n \cdot M \approx n^3$

$\Rightarrow P_{fan} = 777$ kW in high speed stage. At low speed stage fan power is:

$$P_{fan}* = \left(\frac{n*}{n}\right)^3 \cdot P_{fan} = \left(\frac{488}{984}\right)^3 \cdot 777 = \underline{\underline{94.8\,\text{kW}}}$$

4) $P_{fan} = \sqrt{3} \cdot U_N \cdot I_N \cdot \underbrace{\cos\varphi_N \cdot \eta_N}_{0,8}$

$U_N = 6\,\text{kV}$ $\Delta : I_{phase} = \frac{I_{line}}{\sqrt{3}}$ $\text{Y} : I_{phase} = I_{line}$

$2p$	6	12
Winding connection	Δ	Y
I_{line} /A	93.5	11.4
I_{phase} /A	54.0	11.4
P_{fan} /kW	777	94.8

5) High speed stage: $I_1 = I_s(s=1) = 7.5 \cdot I_N = 7.5 \cdot 93.5 = 701.25$ A

For Y-D–start-up: I_1 is reduced to $I_1/3 : I_{1Y} = \frac{701.25}{3} = \underline{\underline{233.75\text{ A}}}$

6) Y-D-Start-up:

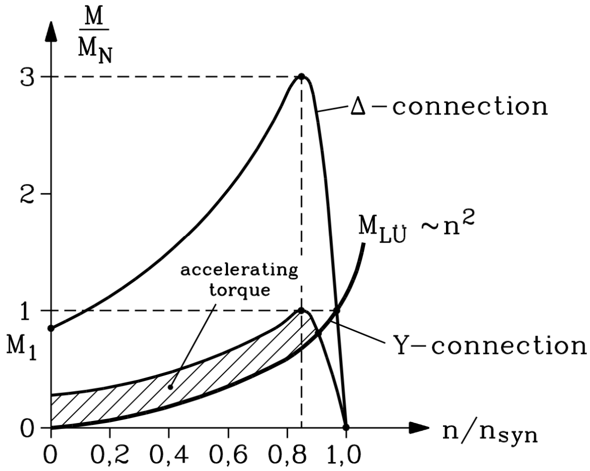


Fig. 2.12-1: Torque-speed curve for Y and D winding connection, fan load torque curve $M_{\text{fan}} = M_{\text{Lü}}$

D-connection: $M_1 = 0.8 \cdot M_N$, $M_b = 3 \cdot M_N$, $s_b = 0.15$, $s_N = \frac{1000-984}{1000} = 1.6\%$

Y-connection: $M = M/3$

Motor torque is bigger than load torque, so start-up also at Y-connection is possible !

Exercise 2.13: Traction drive for high speed train

A **modern high speed train (similar to ICE3)** has following main propulsion data:
$0 \le v \le v_{\text{N}} = 130\text{km/h}$: propulsion force F_{Z} is constant = F_{ZN}
$v_{\text{N}} \le v \le v_{\max} = 330\text{km/h}$: propulsion power P_{Z} is constant = P_{ZN} = 8 MW
At rated speed v_{N} the rotational speed of the wheels is $n_{\text{Wheel,N}} = 850/\min$.

1. Determine propulsion force F_{ZN} at rated speed ! Calculate wheel rotational speed n_{Wheel} at maximum speed v_{max} ! Sketch the diagram of propulsion power versus speed $P_Z(v)$, propulsion force versus speed $F_Z(v)$. Give the same curves as function of rotational speed of wheels $P_Z(n_{Wheel})$ und $F_Z(n_{Wheel})$!

2. The high speed train is propelled by 16 four-pole ($2p = 4$) inverter-fed induction motors. Each motor transmits torque via a gear with ratio $i = 2.5$ to the wheel-set shaft. The motors are at high speed, the wheel-set spins at low speed. Give the relationship between motor rotational speed n and wheel rotational speed n_{Wheel} !

3. Determine motor rated power P_N , motor rated rotational speed n_N, motor rated torque M_N (all at v_N) ! Give in addition the values of P, M and n_{max} at v_{max}!

4. Sketch for the demand given by diagram of 1) the motor power curve $P(n)$ and motor torque curve $M(n)$ of one motor with correct scale for speed range $0 \le n \le n_{max}$!

5. At v_N the rated slip of the induction motors is $s_N = 1$ %. Evaluate corresponding stator frequency of motors !

6. How big is maximum stator frequency $f_{s,max}$ at v_{max} and motor no-load operation ($s = 0$)?

7. Determine break down torque of motors at maximum speed $M_b(v_{max})$, hence at maximum frequency $f_{s,max}$, if stator voltage U_S is kept constant $U_S = U_{SN}$ for $v > v_N$. Break down torque at rated frequency is $M_b(v_N) = 3\ M_N$. Neglect stator resistance R_s ! Will motors break down at maximum frequency $f_s = f_{s,max}$, if they are loaded with torque $M(n_{max})$ according to 4) ?

Solution:

1) $v_N = 130 km/h = 36.11 m/s: \quad F_{ZN} = P_{ZN} / v_N \quad \Rightarrow \quad F_{ZN} = \frac{8000000}{36.11} = \underline{\underline{221545}}\ \text{N}$

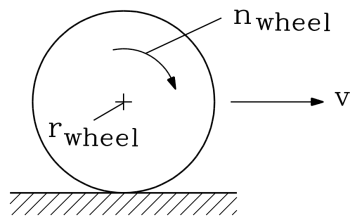


Fig. 2.13-1: Wheel speed and rotational speed

$$n_{Wheel} \cdot 2\pi \cdot r_{Wheel} = v \quad \Rightarrow \quad n_{Wheel,\max} = n_{Wheel,N} \cdot \frac{v_{\max}}{v_N} = 850 \cdot \frac{330}{130} = \underline{\underline{2157.7}}\ /\text{min}$$

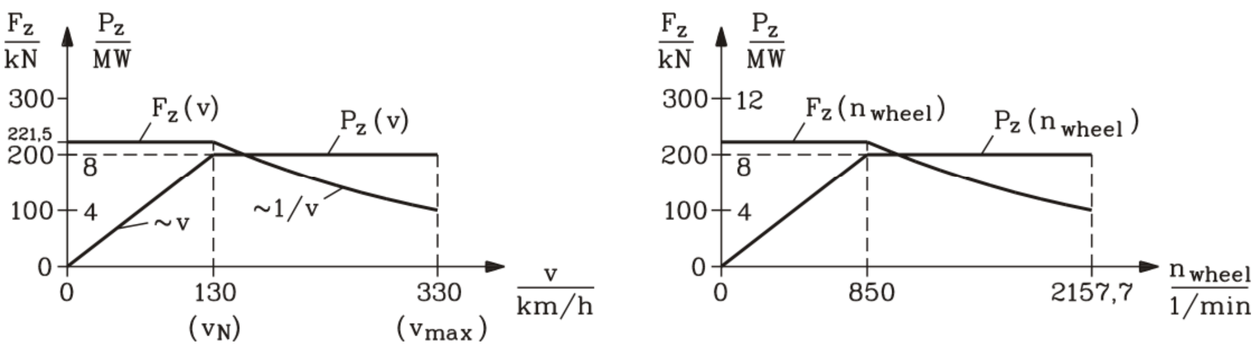


Fig. 2.13-2: Propulsion force and power versus train speed and versus wheel speed

2) $\underline{\underline{n = i \cdot n_{Wheel}}}$

3) $P_N = P_{ZN}/16 = 8000000/16 = 500000W = \underline{\underline{500}}\,\text{kW}$

$n_N = 2.5 \cdot 850 = \underline{\underline{2125}}\,/\text{min} \qquad M_N = \dfrac{P_N}{2\pi n_N} = \dfrac{500000}{2\pi \cdot (2125/60)} = \underline{\underline{2246.9}}\,\text{Nm}$

$P(v_{\max}) = P_{ZN}/16 = \underline{\underline{500}}\,\text{kW}$

$n_{\max} = 2.5 \cdot 2157.7 = \underline{\underline{5394.2}}\,/\text{min} \quad M(n_{\max}) = \dfrac{P_N}{2\pi n_{\max}} = \dfrac{500000}{2\pi \cdot (5394.2/60)} = \underline{\underline{885.1}}\,\text{Nm}$

4)

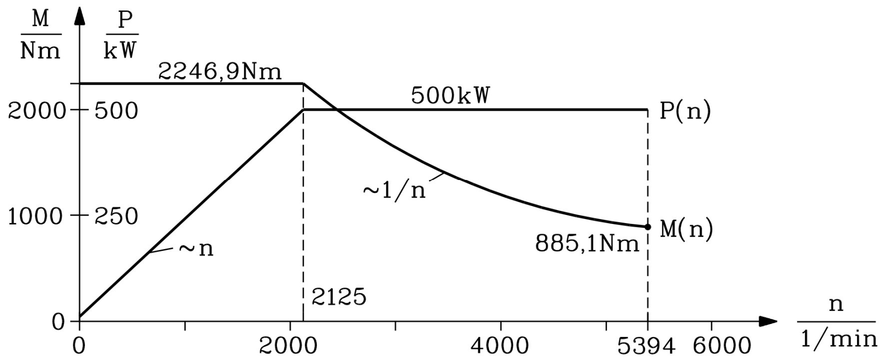


Fig. 2.13-3: Motor torque and power for the corresponding power demand of Fig. 2.13-1, versus rotor speed

5) $f_{sN} = \dfrac{n_N \cdot p}{1 - s_N} = \dfrac{(2125/60)\cdot 2}{1 - 0.01} = \underline{\underline{71.55}}\,\text{Hz}$

6) $f_{s,\max}(s=0) = \dfrac{n_{\max} \cdot p}{1 - s} = n_{\max} \cdot p = (5394.2/60)\cdot 2 = \underline{\underline{179.8}}\,\text{Hz}$

7) *KLOSS* formula: $M_b \sim (U_s / f_s)^2 : \dfrac{M_b(v_{\max})}{M_b(v_N)} = \left(\dfrac{f_{sN}}{f_{s,\max}}\right)^2 = \left(\dfrac{71.55}{179.8}\right)^2 = 0.158$

$M_b(v_{\max}) = 0.158 \cdot 3M_N = 0.158 \cdot 3 \cdot 2246.9 = \underline{\underline{1067.4}}\,\text{Nm}$

Load torque at maximum speed: acc. to Fig. 2.13-3: 885.1 Nm. This is smaller than break down torque at maximum speed 1067.4 Nm by 17%, so motors will not break down at maximum speed.