<u>Exercise 3.1</u>: Synchronous generator of a hydropower station

Synchronous hydropower generators, mounted vertically at a hydropower station on the river *Euphrat / Turkey*, are driven by *Kaplan* turbines. They have the following rated data:

$S_N = 45$ MVA; $U_N = 10.3$ kV Y (line-to-line voltage)
$2p = 72$; $f_N = 50$ Hz; $\cos\varphi_N = 0.8$ over-excited; $\eta_N = 97$ %

The manufacturer supplied the following data from measurements:

a) no-load characteristic (line-to-line voltage):

$U_{s,LL,0}$	7 725	10 300	11 330	12 360	13 390	V
I_f	530	780	920	1 150	1 600	A

b) short-circuit characteristic

I_{sk}	1 000	2 000	A
I_f	358	716	A

1) Draw the no-load characteristic and the short-circuit characteristic in one diagram (ordinate in per-unit voltage $U_{s,LL,0}/U_N$ and per-unit current I_{sk}/I_N; abscissa scale: 1 cm ≙ 125 A).
2) Determine the no-load/short-circuit ratio k_K from diagram of 1), the per-unit synchronous reactance x_d and the synchronous reactance in Ω !
3) The measured stator leakage voltage drop is given as $u_{s\sigma} = 18$ %. Draw the voltage phasor diagram of generator operation for $x_d = x_q$ and $U = U_N$, $I = I_N$, $\cos\varphi = 1$ in per-unit values with $R_s = 0$ in generator arrow system (recommended scale factor: 1 p.u. ≙ 10 cm).
4) Calculate the values of u_p, u_h, U_p and U_h analytically from the diagram 3).
5) The polar moment of inertia is given with $J = 14.8 \cdot 10^6$ kgm^2. Determine the nominal acceleration time T_J !

<u>Solution:</u>

1)
No-load and short-circuit characteristic in per unit (Fig. 3.1-2):

U_{s0}/U_N	*p. u.*	0.75	1	1.1	1.2	1.3
I_f	A	530	780	920	1150	1600

I_{sk}/I_N	*p. u.*	0.4	0.8
I_f	A	358	716

Rated current: $I_N = S_N/(\sqrt{3}\cdot U_N) = 45000/(\sqrt{3}\cdot 10.3) = 2522 A$

2)
$$k_K = I_{f0}/I_{fk} = \underline{\underline{0.87}} \quad \text{according to Fig. 3.1-2}$$
$$x_d = 1/k_K = 1/0.87 = \underline{\underline{1.15 p.u.}}$$
$$Z_N = U_{N,ph}/I_N = 10300V/(\sqrt{3}\cdot 2522A) = 2.36\Omega$$
$$X_d = x_d \cdot Z_N = 1.15\cdot 2.36\,\Omega = \underline{\underline{2.71\Omega}}$$

3) $$u_{s\sigma} = 18\% = \frac{X_{s\sigma}\cdot I_N}{U_{N,ph}} \Rightarrow X_{s\sigma}/Z_N = u_{s\sigma} = 0.18 p.u.$$

Voltage diagram at unity power factor and neglected stator resistance: Fig. 3.1-1, Fig. 3.1-3.

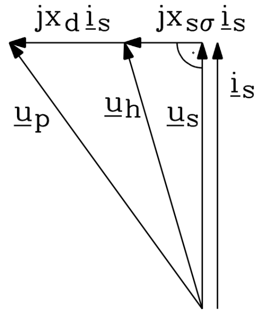


Fig. 3.1-1: Voltage diagram at unity power factor, generator arrow system

Generator arrow system: Active component of current IN PHASE with voltage at generator operation.

4) From Fig. 3.1-1 we calculate:

$$u_p = \sqrt{(x_d \cdot i_s)^2 + u_s^2} = \sqrt{(1.15 \cdot 1)^2 + 1^2} = \underline{\underline{1.524\,p.u.}}$$

$$u_h = \sqrt{(x_{s\sigma} \cdot i_s)^2 + u_s^2} = \sqrt{(0.18 \cdot 1)^2 + 1^2} = \underline{\underline{1.016\,p.u.}}$$

$$U_p = u_p \cdot U_{N,ph} = 1.524 \cdot \frac{10300}{\sqrt{3}} = \underline{\underline{9063V}}, \qquad U_h = u_h \cdot U_{N,ph} = 1.016 \cdot \frac{10300}{\sqrt{3}} = \underline{\underline{6042V}}$$

5) $$P_{Nm} = \frac{P_{Nel}}{\eta_N} = \frac{S_N \cdot \cos\varphi_N}{\eta_N} = \frac{45 \cdot 10^6 \cdot 0.8}{0.97} W = 37.11 MW$$

$$\Omega_{mN} = 2\pi \cdot \frac{f_N}{p} = 2\pi \cdot \frac{50}{36} = 8.727/s, \quad M_N = \frac{P_{Nm}}{\Omega_{mN}} = \frac{37.11 \cdot 10^6}{8.727} Nm = 4252.7 kNm$$

$$T_J = J \cdot \frac{\Omega_{mN}}{M_N} = 14.8 \cdot 10^6 \cdot \frac{8.727}{4.257 \cdot 10^6} = \underline{\underline{30.37s}}$$

Exercise 3.2: Industrial generator

In a paper mill in *Pöls / Austria*, the excess steam of the manufacturing process is used to generate electric power by an industrial steam turbine and a synchronous cylindrical-rotor generator. No-load and short-circuit characteristic of the synchronous machine are given.

U_{s0}/U_N	0.5	0.75	1	1.1	1.2	1.3
I_f/A	30	54	100	140	200	300

I_{sk}/I_N	0.5	1.0
I_f/A	45	90

The per-unit stator leakage voltage drop is $u_{s\sigma} = 17$ %. The excitation current for U_N, I_N, $\cos\varphi = 0$ overexcited, is $I_f = 260$ A.

1) Draw both characteristics in one diagram (scale: 1 p.u. ≙ 1cm, abscissa: 20 A ≙ 1cm)!
2) Determine per-unit synchronous reactance x_d !
3) In generator operation at fixed excitation current $I_f = 260$ A, the machine is operated at power factor $\cos\varphi = 1$. Calculate the corresponding stator current I_s / I_N. Assume constant iron saturation = consider x_d to be constant according to 1). Neglect stator resistance. Use consumer arrow system !
4) Calculate the p.u. static pull-out power for motor operation, rated current and rated voltage, power factor 0.75 (overexcited). Stator resistance is neglected.

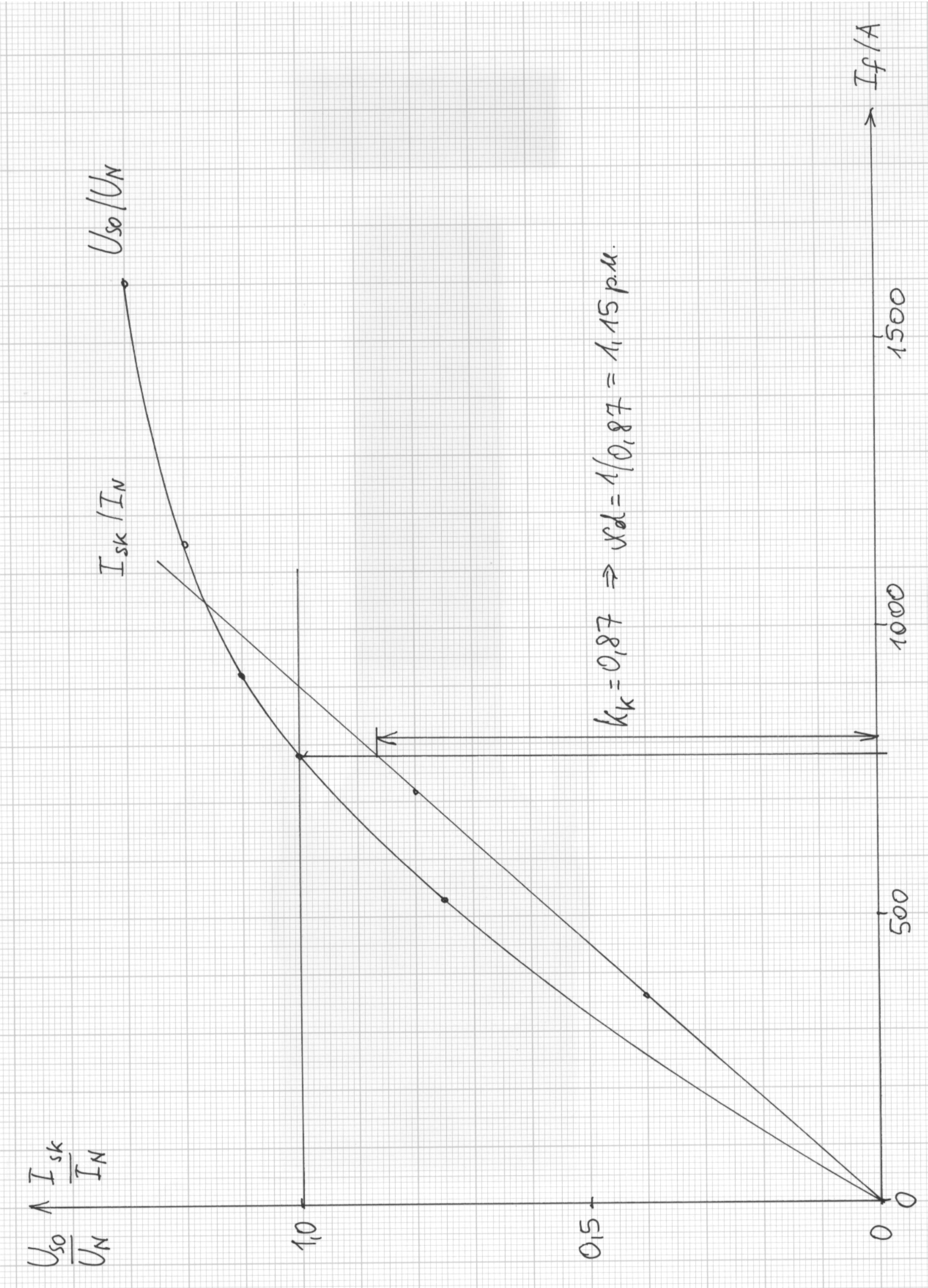


Fig. 3.1-2: No-load and short-circuit characteristic of hydro-generator

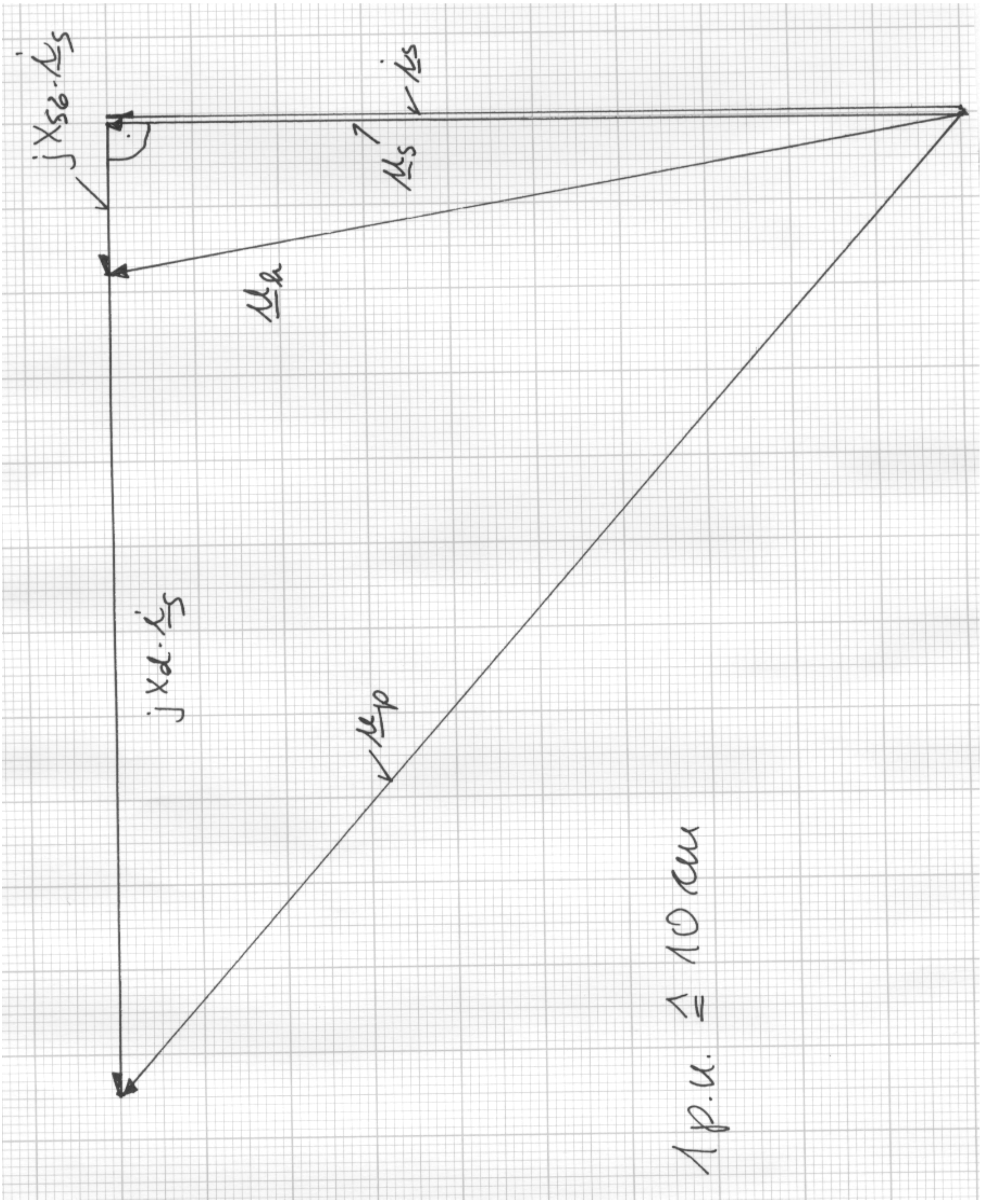


Fig. 3.1-3: Voltage diagram for hydro generator at unity power factor, generator arrow system

Solution for Exercise 3.2:

1) No-load and short-circuit characteristic drawn in Fig. 3.2-3. From there we get: $k_K = \underline{\underline{1.12}}$
2) $k_K = 1.12 \Rightarrow x_d = 1/k_K = 1/1.12 = \underline{\underline{0.89\,p.u.}}$
3) Constant saturation means constant X_d !

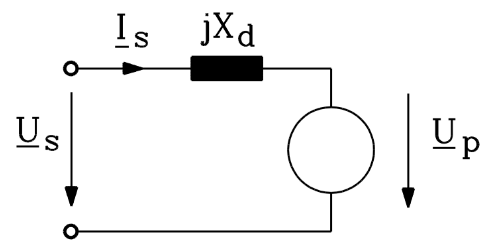


Fig. 3.2-1: Equivalent circuit of cylindrical-rotor synchronous machine

Calculation of stator current at unity power factor (b) for excitation current of overexcited power factor zero (a) according to equivalent circuit Fig. 3.2-1 and corresponding phasor diagram Fig. 3.2-2. Consumer arrow system demands, that active current is in phase opposition to voltage in generator mode.

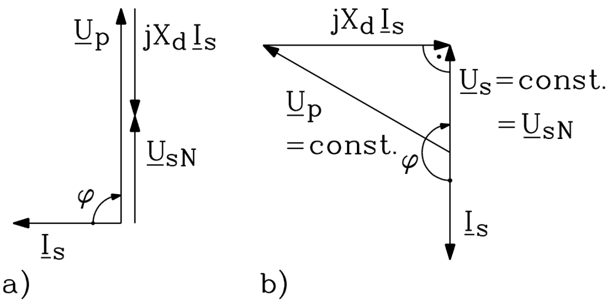


Fig. 3.2-2: Phasor diagram a) for zero power factor over-excited, b) for unity power factor at the same stator voltage and excitation current

Constant excitation current means constant back EMF:
$I_f = 260A = const. \Rightarrow U_p = const. \quad (U_p = X_{hd} \cdot I_f' \;!)$

From Fig. 3.2-2 we get:
a): $\cos\varphi = 0$ (overexcited) , $R_s \cong 0$: $\underline{U}_s = jX_d\underline{I}_s + \underline{U}_p$. Rated current and voltage yield:

$$U_P = U_{sN} + X_d I_{sN} \quad \left| \cdot \frac{1}{U_{sN}} \right. \qquad \Rightarrow \qquad u_P = 1 + x_d$$

b): $\varphi = \pi \quad |\cos\varphi| = 1$, $R_s \cong 0$: $P = 3 \cdot U_{sN} \cdot I_s \cdot \cos\varphi = -3U_{sN}I_s$
Rated voltage and back EMF from a) yield:

$$U_p^2 = X_d^2 I_s^2 + U_{sN}^2 \quad \left| \cdot \frac{1}{U_{sN}^2} \right. \qquad \Rightarrow \qquad u_p^2 = x_d^2 i_s^2 + 1$$

From a) and b) follows:

$$1 + x_d = \sqrt{1 + x_d^2 i_s^2} \;\rightarrow\; \underbrace{\sqrt{\frac{(1+x_d)^2 - 1}{x_d^2}}} = i_s$$

$$\frac{2x_d + x_d^2}{x_d^2} = 1 + \frac{2}{x_d}$$

$$i_s = \sqrt{1 + \frac{2}{x_d}} = \sqrt{1 + \frac{2}{0.89}} = 1.8$$

The current rises in case b) due to the big back EMF up to <u>180 %</u> of rated current I_{sN}. This overload may be operated only for short time in order to avoid over-heating of generator winding.

4) Motor operation: Rated voltage and current, from voltage phasor diagram for $\cos\varphi = 0.75 \Rightarrow \varphi = 41.4°$ we get a back EMF $u_P = 1.72\,p.u.$! Pull-out power for that back EMF occurs in cylindrical-rotor motor at load angle ϑ = -90°. In per unit it is:

$$\frac{P_{e,po}}{3 \cdot U_{N,ph} I_N} = -\frac{u_s \cdot u_p}{x_d} \cdot \sin(-90°) = \frac{1 \cdot 1.72}{0.89} = \underline{\underline{1.936\,p.u.}}$$

Pull-out power with excitation according to 0.75 power factor at rated current and voltage is <u>94%</u> higher than rated apparent power.

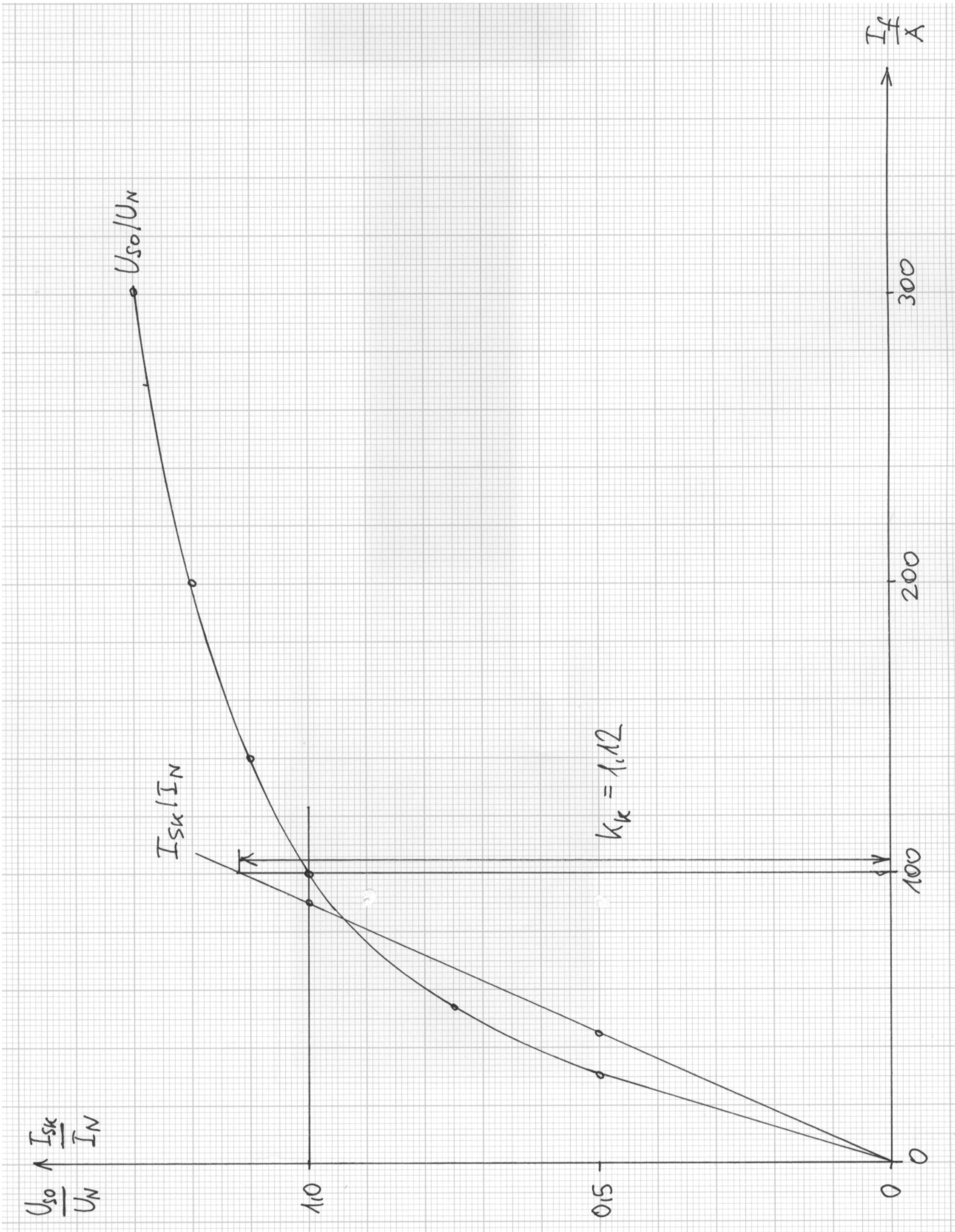


Fig. 3.2-3: No-load and short-circuit characteristic of industrial generator

<u>Excercise 3.3</u>: *Diesel*-Generator

In a power station in *Jordan*, *diesel* motor driven synchronous salient-pole generators are in operation. One of them will be dismounted to be used as a motor of a big fan blower drive in a plant nearby.

Machine data: $U_N = 6.3$ kV Y (line-to-line voltage), $S_N = 2.5$ MVA, $f_N = 50$ Hz, $2p = 20$,
$x_d = 1.1$ p.u., $x_q = 0{,}6 \cdot x_d$. The stator resistance can be neglected.
The stator leakage reactance $x_{s\sigma}$ was determined by the standardized *POTIER* measurement method as $x_{s\sigma} = x_P = 0.17$ p.u.

1. Draw the voltage phasor diagram of the salient-pole machine in <u>motor operation</u> with
   $U = U_N$, $I = 1.2\, I_N$, $\cos\varphi = 1$.
   Determine with it the p.u. value of the synchronous internal voltage u_p. (Scale: 1p.u. ≙ 1cm)

2. The mechanical power based on the consumer reference arrow system is given by
   $$P_m = -m_s \cdot \frac{U_s U_p}{X_d} \cdot \sin\vartheta - m_s \cdot \frac{U_s^2}{2} \cdot \left(\frac{1}{X_q} - \frac{1}{X_d}\right) \cdot \sin(2\vartheta).$$
   Calculate the pull-out load angle and torque in generator mode at fixed excitation current according to point 1.

3. a) Determine the spring stiffness $c_\vartheta = \mathrm{d}M/\mathrm{d}\vartheta$ (Nm/rad) for oscillations at the operating point $\vartheta_0 = 0$ with excitation current of point 1) !
   b) Calculate the corresponding natural angular frequency $\omega_e = \sqrt{|c_\vartheta| \cdot p/J}$ and f_e without influence of damper cage, based on the polar moment of inertia $J = 30\,000$ kgm^2.

<u>Solution:</u>

1. $u_s = 1,\ i_s = 1.2,\ x_d = 1.1,\ x_{s\sigma} = 0.17,\ \cos\varphi = 1,\ x_q = 0.6 \cdot x_d = 0.66$
   Motor operation: u is leading with respect to u_p
   $x_d \cdot i_s = 1.1 \cdot 1.2 = 1.32, \quad x_q \cdot i_s = 0.66 \cdot 1.2 = 0.80, \quad x_{s\sigma} \cdot i_s = 0.17 \cdot 1.2 = 0.20$
   Diagram is given in Fig. 3.3-1. From that we get u_p = <u>1.6 p.u.</u>
2. Pull-out power is evaluated by calculating maximum active power, by derivation of power with load angle:
$$-P_m = \underbrace{m_s \frac{U_s \cdot U_p}{X_d}}_{\mathrm{A}} \sin\vartheta + \underbrace{m_s \cdot \frac{U_s^{\,2}}{2} \left(\frac{1}{X_q} - \frac{1}{X_d}\right)}_{\mathrm{B}} \sin(2\vartheta) = -M \cdot \Omega_{syn}$$
$$-\frac{dP_m}{d\vartheta} = 0 = A\cos\vartheta + 2B \cdot \underbrace{\cos(2\vartheta)}_{2\cos^2\vartheta - 1}$$

With abbreviation $x = \cos\vartheta_{p0}$: $\quad 4Bx^2 + Ax - 2B = 0, \qquad x^2 + \frac{A}{4B}x - \frac{1}{2} = 0$

Solution: $x_{1,2} = -\frac{A}{8B} \pm \sqrt{\left(\frac{A}{8B}\right)^2 + \frac{1}{2}} = \cos\vartheta$, hence we get:

$$\cos\vartheta_{p0} = -\frac{U_p/U_s}{4\cdot\left(\frac{X_d}{X_q}-1\right)} \pm \sqrt{\frac{(U_p/U_s)^2}{4^2\cdot\left(\frac{X_d}{X_q}-1\right)^2}+\frac{1}{2}}$$

With data $U_p/U_s = U_p/U_{sN} = u_p = 1.6, \quad X_d/X_q = 1/0.6$ we get:

$$\frac{U_p}{U_s}\cdot\frac{1}{4}\cdot\frac{1}{\frac{X_d}{X_q}-1} = 1.6\cdot\frac{1}{4}\cdot\frac{1}{\frac{1}{0.6}-1} = 0.6, \quad \cos\vartheta_{p0} = -0.6+\sqrt{0.6^2+\frac{1}{2}} = 0.32$$

Second solution $\cos\vartheta_{p0} = -0.6-\sqrt{0.6^2+\frac{1}{2}} = -1.52$ does not yield a solution for ϑ_{p0}, because $\left|\cos\vartheta_{p0}\right| \le 1$ is not fulfilled.

Pull-out load angle: $\vartheta_{p0} = \arccos 0.32 = \underline{\underline{71.3°}}$ (positive in generator mode)

Pull-out torque: $M_{p0} = \Omega_{syn}^{-1}\cdot P_m \quad (\vartheta = \vartheta_{p0})$ (negative in generator mode)

With $\Omega_{syn} = 2\pi\frac{f}{p} = 2\pi\frac{50}{10}s^{-1} = 31.4\,s^{-1}, \quad U_{sN} = U_N/\sqrt{3} = 6.3kV/\sqrt{3} = 3637V$,

$Z_N = \frac{U_{sN}}{I_{sN}} = \frac{3637}{229}\Omega = 15.87\,\Omega$, $I_{sN} = \frac{S_N}{\sqrt{3}\cdot U_N} = \frac{2500\cdot 10^3}{\sqrt{3}\cdot 6300}A = 229A$ we get

$$M_{p0} = \frac{1}{31.4}\cdot\left(-\frac{3\cdot 3637\cdot 1.6\cdot 3637}{1.1\cdot 15.87}\cdot\sin 71.3° - 3\cdot\frac{3637^2}{2}\cdot\frac{1}{15.87}\left(\frac{1}{0.66}-\frac{1}{1.1}\right)\cdot\sin(2\cdot 71.3°)\right) =$$
$$= \underline{\underline{-124374\,Nm}}$$

3. a) Spring stiffness at no-load ($\vartheta_0 = 0$):

$$c_\vartheta = \left.\frac{dM}{d\vartheta}\right|_{\vartheta_0=0} = -\left.\left(A\cos\vartheta_0 + 2B\cdot\cos(2\vartheta_0)\right)\cdot\frac{1}{\Omega_{syn}}\right|_{\vartheta_0=0} = -(A+2B)\frac{1}{\Omega_{syn}}$$
$$= -\frac{1}{\Omega_{syn}}\cdot\left(m_s\frac{U_sU_p}{X_d} + m_s\frac{U_s^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\cdot 2\right) =$$
$$= -\frac{1}{31.4}\left(3\cdot\frac{3637\cdot 1.6\cdot 3637}{1.1\cdot 15.87} + 3\cdot\frac{3637^2}{2}\cdot\left(\frac{1}{0.66}-\frac{1}{1.1}\right)\cdot\frac{2}{15.87}\right) =$$
$$= \underline{\underline{-164095\,Nm/rad}}$$

b) Natural angular frequency (without influence of the damper cage)

$$\omega_e = \sqrt{\frac{|c_\vartheta|p}{J}} = \sqrt{\frac{164095\cdot 10}{30000}} = \underline{\underline{7.39\,s^{-1}}}$$

natural frequency: $f_e = \frac{\omega_e}{2\pi} = \underline{\underline{1.18Hz}}$

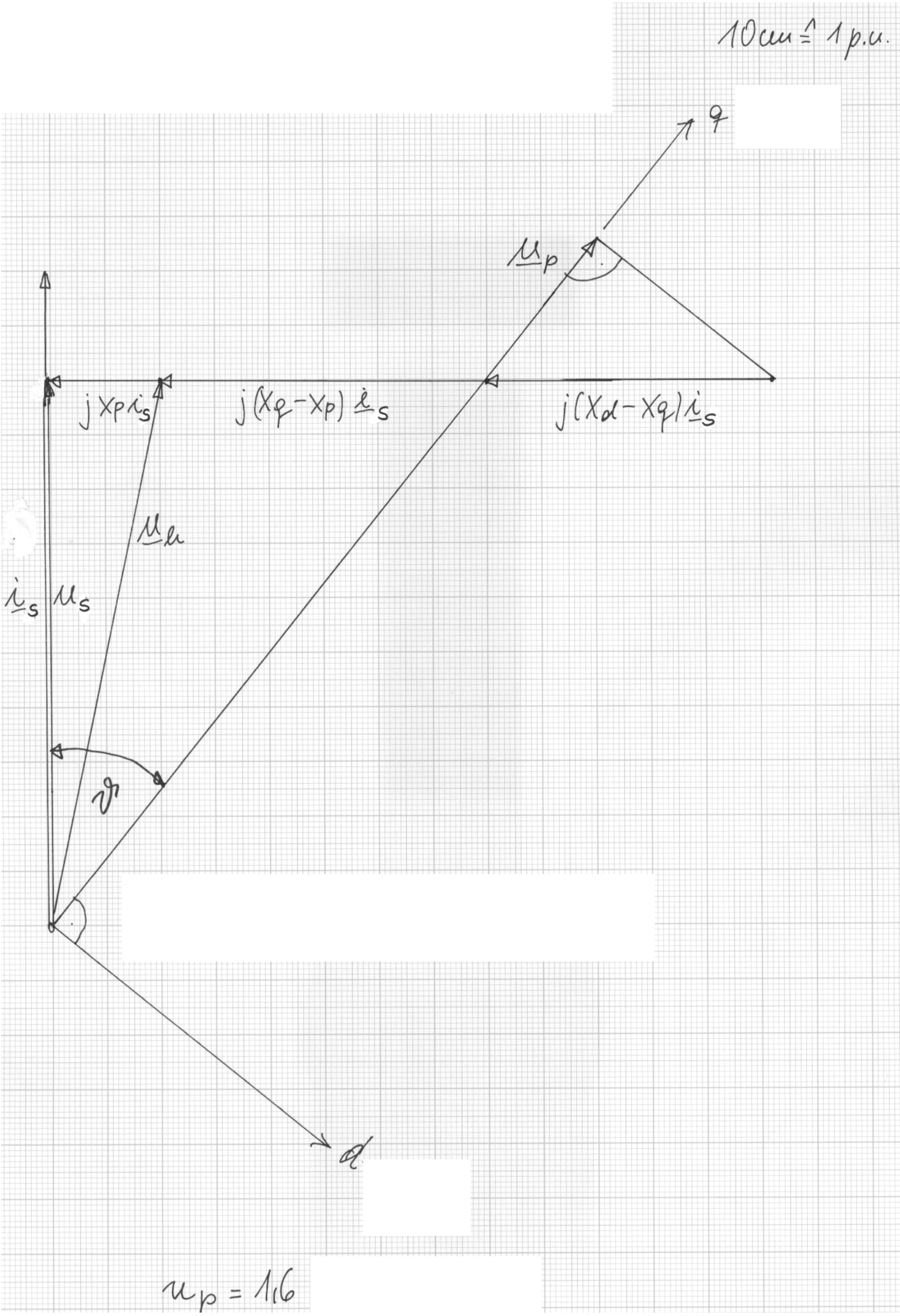


Fig. 3.3-1: Voltage phasor diagram of salient pole synchronous motor

<u>Exercise 3.4</u>: All-Electric-Ship Generator

For a big cruising vessel the *diesel* powered synchronous ship generators have the following data:

$U_N = 10$ kV Y
$I_N = 400$ A
$2p = 10$, $f_N = 50$ Hz, $x_d = 0.9$ p.u..

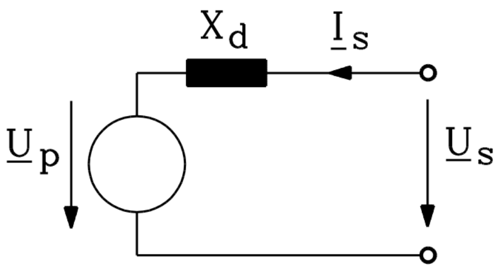
The stator resistance can be neglected, and $x_d = x_q$ may be assumed.

1. Determine apparent rated power and synchronous speed.
2. Calculate per-unit back EMF $\underline{u}_p$ from equivalent circuit diagram for operation at $U_s = 90\,\%$ of U_{sN}, $I_s = 110\,\%$ of I_N for the following three load cases:
   a) $\cos\varphi = 0.8$ overexcited = inductive load, machine itself is capacitve
   b) $\cos\varphi = 1$ = resistive load
   c) $\cos\varphi = 0.9$ under-excited = capacitive load, machine itself is inductive.

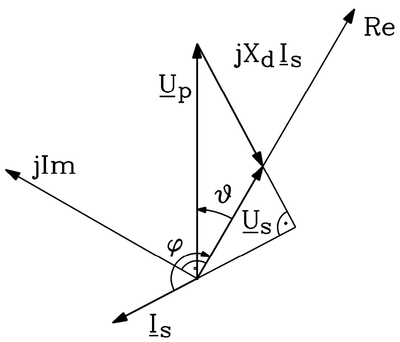
3. Evaluate the static pull-out torque M_{p0} for the given operating points a), b), c) in 2).

<u>Solution:</u>

1. $S_N = \sqrt{3}\cdot U_N \cdot I_N = \sqrt{3}\cdot 10000\cdot 400 = 6928203 = \underline{\underline{6.93}}\,\mathrm{MVA}$,
   $n_{syn} = f_s / p = 50/5 = 10/s = \underline{\underline{600}}\,/\mathrm{min}$
2. Linear equivalent circuit diagram Fig. 3.4-1 of a cylindrical-rotor machine:
   $\underline{U}_p + jX_d \underline{I}_s = \underline{U}_s \qquad x_d = const. = 0.9\,p.u. \quad (r_s = 0)$



<u>Fig. 3.4-1:</u> Equivalent circuit per phase of round-rotor synchronous machine



<u>Fig. 3.4-2:</u> Voltage phasor diagram for generator operation, overexcited

Phasor diagram for generator operation Fig. 3.4-2 gives $\underline{U}_p$ is leading with respect to $\underline{U}_s$. We chose voltage phasor to be real: $\underline{U}_s = U_s$. So we get for back EMF:

$\underline{U}_p = U_p \cos\vartheta + jU_p \sin\vartheta$ or $\underline{U}_p = \underline{U}_s - jX_d \underline{I}_s$

$\underline{I}_s = I_s \cos\varphi - jI_s \sin\varphi$

In per unit values we get:

$$u_p = \frac{U_p}{U_{sN}},\quad u_s = \frac{U_s}{U_{sN}} = 0.9,\quad i_s = \frac{I_s}{I_{sN}} = 1.1,\quad X_d / Z_N = x_d,\quad Z_N = \frac{U_{sN}}{I_{sN}} = 14.43\,\Omega$$

$$U_{sN} = U_{N,ph} = U_N/\sqrt{3} = 10kV/\sqrt{3} = 5773V,\quad I_{sN} = 400\,A$$

$\underline{u}_p = u_s - jx_d\left(i_s\cos\varphi - j\cdot i_s\sin\varphi\right) = u_s - j\cdot x_d i_s\cdot\cos\varphi - x_d i_s\cdot\sin\varphi$

$= 0.9 - 0.9\cdot 1.1\cdot\sin\varphi - j\cdot 0.9\cdot 1.1\cdot\cos\varphi = 0.9 - 0.99\cdot\sin\varphi - j\cdot 0.99\cdot\cos\varphi$

Consumer reference frame: $\cos\varphi$ negativ in generator mode !

a)	$\cos\varphi = -0.8 \rightarrow \sin\varphi = -0.6$	overexcited:	current leading:	$\varphi < 0$
b)	$\cos\varphi = -1 \rightarrow \sin\varphi = 0$	resistive:	current in phase opposition:	$\varphi = \pi$
c)	$\cos\varphi = -0.9 \rightarrow \sin\varphi = 0.43$	underexcited:	current lagging:	$\varphi > 0$

a) $\underline{u}_p = 0.9 + 0.99\cdot 0.6 + j\cdot 0.99\cdot 0.8 = 1.494 + j\cdot 0.792,\quad u_p = \underline{\underline{1.69\,p.u.}}$

b) $\underline{u}_p = 0.9 + 0 + j\cdot 0.99 = 0.9 + j\cdot 0.99,\quad u_p = \underline{\underline{1.34\,p.u.}}$

c) $\underline{u}_p = 0.9 - 0.99\cdot 0.43 + j\cdot 0.99\cdot 0.9 = 0.474 + j\cdot 0.89,\quad u_p = \underline{\underline{1.01\,p.u.}}$

3. $P_m = -m_s\dfrac{U_sU_p}{X_d}\sin\vartheta,\quad P_{m,p0} = -\dfrac{m_s\cdot U_s\cdot U_p}{X_d}\quad\left(\vartheta_{p0} = \dfrac{\pi}{2}\right)$ generator mode

$M_{p0} = -\dfrac{1}{\Omega_{syn}}\cdot m_s\dfrac{U_sU_p}{X_d}\qquad \Omega_{syn} = 2\pi\dfrac{f}{p} = 2\pi\cdot\dfrac{50}{5}s^{-1} = 62.8\,s^{-1}$

a) $M_{p0} = -\dfrac{1}{62.8}\cdot 3\cdot 0.9\cdot 5773\cdot 1.69\cdot 5773\cdot\dfrac{1}{0.9\cdot 14.43} = \underline{\underline{-186.4\,kNm}}$

b) $M_{p0} = -\dfrac{1}{\Omega_{syn}}\cdot u_s\cdot\dfrac{u_p}{x_d}\cdot S_N = -\dfrac{1}{62.8}\cdot 0.9\cdot 1.34\cdot\dfrac{1}{0.9}\cdot 6927600 = \underline{\underline{-147.8\,kNm}}$

c) $M_{p0} = -186.4\cdot\dfrac{1.01}{1.69}Nm = \underline{\underline{-111.4\,kNm}}$

With decreasing excitation the pull-out torque decreases !

<u>Exercise 3.5</u>: Hydro Generator for a river hydro power-plant

Synchronous salient-pole generators at a *Don* river (*Ucraine*) hydro-power plant have the following data:

$U_N = 10$ kV Y	(line-to-line)	$I_N = 2$ kA	$f_N = 50$ Hz;	$2p = 40$
$x_d = 0.9$ p.u.;	$x_q = 0.55$ p.u.;	$R_s \approx 0$.		

1) Draw the phasor diagram in per unit values at generator operation with following data: $U_s = U_N$ $\quad I_s = 0.8\cdot I_N$ $\quad\cos\varphi = 0.7$ overexcited.
   How big is the back EMF $U_p = |\underline{U}_p|$, taken from diagram? Scale: 1 p.u. $\hat{=}$ 5 cm. Determine synchronous speed.
2) Starting from the operation point of 1) the driving torque and therefore the load angle ϑ is changed at constant values of $U_s = U_N$ and U_p.
   a) Draw current phasor diagram for load angles $\vartheta = 0°, 15°, 30°, ..., 180°$.
   b) Derive from that diagram the torque-load angle-curve $M_e = f(\vartheta)$ with a correct scale.
3) Calculate the torque for the operation point of 1) from the curve $M_e = f(\vartheta)$ from 2b). How big is load angle ? Compare it with load angle of phasor diagram 1). How big is the generator pull-out torque M_{p0} ?

<u>Solution :</u>

1) p.u. values for phasor diagram: $I_s = 0.8\cdot I_{sN} \Rightarrow i_s = 0.8,\ U_s = U_{sN} \Rightarrow u_s = 1$

$x_d = 0.9$, $x_q = 0.55$, $\cos\varphi = 0.7$ overexcited. In load reference arrow system active power is < 0 at generator operation, so power factor is negative, corresponding with phase angle $\varphi = 134.4°$.
$x_d \cdot i_s = 0.9 \cdot 0.8 = 0.72$ p.u., $x_q \cdot i_s = 0.55 \cdot 0.8 = 0.44$ p.u.; $n_{syn} = f / p = 50/20 = 2.5/s = \underline{\underline{150/\min}}$

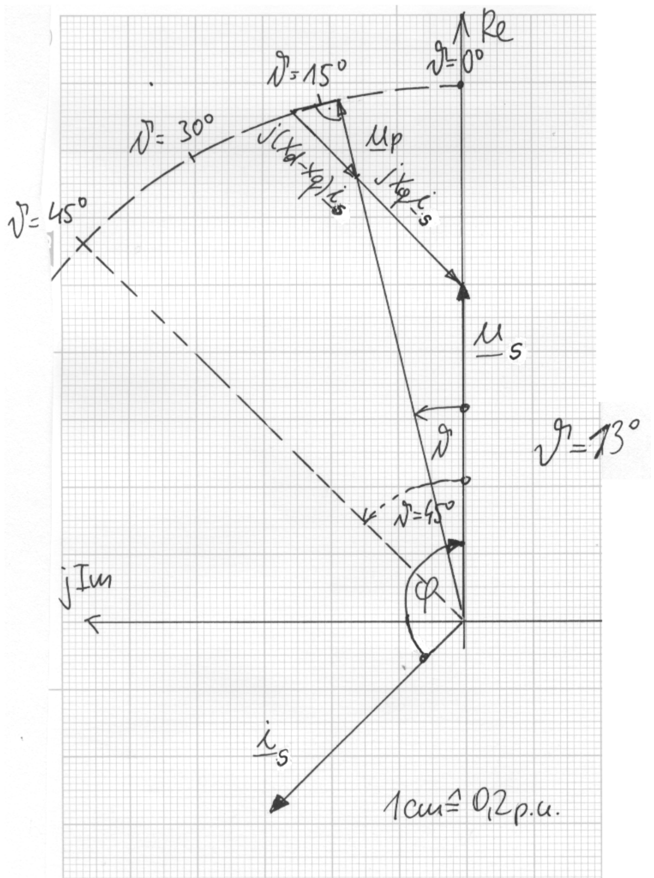


Fig. 3.5-1: Voltage phasor diagram

From Fig. 3.5-1 we get:
$\Rightarrow u_p \mathrel{\hat{=}} 8$ cm, $\lambda = 5$ cm/p.u. $\Rightarrow u_p = 1.6$ p.u. $\Rightarrow U_p = 1.6 \cdot 5773.5$ V $= \underline{\underline{9237.6 \text{ V}}}$ (rms-value)

2) a) By keeping U_s, U_p constant, but varying load angle between them, we get diagram Fig. 3.5-3 according to the rules shown in Fig. 3.5-2.

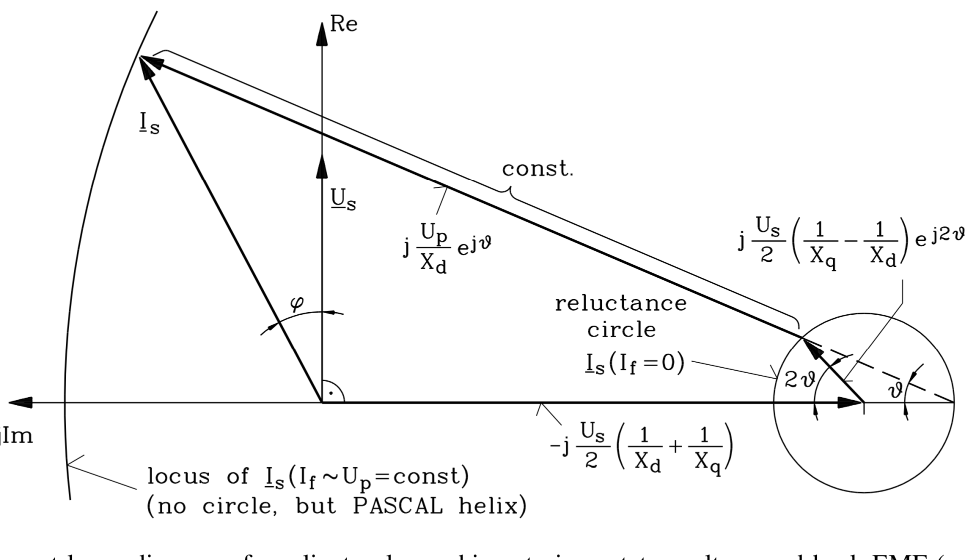


Fig. 3.5-2: Current locus diagram of a salient-pole machine at given stator voltage and back EMF (= synchronous generated voltage) as a function of the load angle

b) $M_{\mathrm{e}} = \dfrac{P}{\Omega_{\mathrm{syn}}} \overset{R_{\mathrm{s}}=0}{=} \dfrac{3 \cdot U_{\mathrm{s}} \cdot \mathrm{Re}\{\underline{I}_{\mathrm{s}}\}}{\Omega_{\mathrm{syn}}}$, $\quad \dfrac{u_{\mathrm{s}}}{x_{\mathrm{d}}} = \dfrac{1}{0.9} = 1.11\,; \dfrac{u_{\mathrm{s}}}{x_{\mathrm{q}}} = \dfrac{1}{0.55} = 1.82\,; \dfrac{u_{\mathrm{p}}}{x_{\mathrm{d}}} = \dfrac{1.6}{0.9} = 1.78$ p.u.

$\mathrm{Re}\{\underline{I}_{\mathrm{s}}\} = I_{\mathrm{s,active}}$ to determine active power

ϑ / °	0	15	30	45	60	75	90	105	120	135	150	165	180
i_{s}/p.u.	0.67**)	0.86	1.22	1.62	2.0	2.3	2.52	2.7	2.76	2.86	2.88	2.89	2.89*)
$i_{\mathrm{s,active}}$/p.u.	0	0.66	1.2	1.6	1.84	1.94	1.78	1.55	1.22	0.92	0.6	0.3	0
$-M_{\mathrm{e}}$/kNm	0	1455	2646	3528	4057	4277	3925	3418	2690	2028	1323	661	0

*): $i_{\mathrm{s}} = \left|\dfrac{u_{\mathrm{s}} + u_{\mathrm{p}}}{x_{\mathrm{d}}}\right| = \left|\dfrac{1+1.6}{0.9}\right| = 2.89$ **): $i_{\mathrm{s}} = \left|\dfrac{u_{\mathrm{s}} - u_{\mathrm{p}}}{x_{\mathrm{d}}}\right| = \left|\dfrac{1-1.6}{0.9}\right| = |-0.67| = 0.67$

In load reference system all torque values in generator operation for $\vartheta > 0$ are to be counted negative!

With

$S_{\mathrm{N}} = 3 \cdot U_{\mathrm{sN}} \cdot I_{\mathrm{N}} = \sqrt{3} \cdot 10000 \cdot 2000$ VA $= 34640$ MVA

$\Omega_{\mathrm{syn}} = 2 \cdot \pi \cdot \dfrac{f}{p} = 2 \cdot \pi \cdot \dfrac{50}{20}\, s^{-1} = 15{,}7\, s^{-1}$

we get the value for the torque, taken from the active current component of Fig. 3.5-3:

$M_{\mathrm{e}} = \dfrac{u_{\mathrm{s}} \cdot i_{\mathrm{s,active}}}{\Omega_{\mathrm{syn}}} \cdot S_{\mathrm{N}} = i_{\mathrm{s,active}} \cdot \dfrac{1}{15{,}7} \cdot 34640 \text{ kNm} = \underline{2205\text{kNm} \cdot i_{\mathrm{s,active}}}$

The corresponding torque-load angle-curve is given in Fig. 3.5-4.

$\Rightarrow M_{\mathrm{p0}} = 4277$ kNm at $\vartheta_{\mathrm{p0}} = 75°$ exact calculation (compare to exercise 3.3.) gives:
$M_{\mathrm{p0}} = 4189$ kNm at $\vartheta_{\mathrm{p0}} = 71{,}5°$

Remark: load reference-arrow system: $M_{\mathrm{p0}} < 0$ in generator operation
generator reference-arrow system: $M_{\mathrm{p0}} > 0$ in generator operation

3) Rated torque: $M_{\mathrm{N}} = \dfrac{P_{\mathrm{N}}}{\Omega_{\mathrm{syn}}} = \dfrac{S_{\mathrm{N}} \cdot \cos\varphi_{\mathrm{N}}}{\Omega_{\mathrm{syn}}} = \dfrac{34640 \cdot 0.7}{15.7} = \underline{\underline{1544 \text{ kNm}}}$

At 80% rated current and rated power factor torque is reduced by 20%:

$M = \dfrac{0.8 \cdot P_{\mathrm{N}}}{\Omega_{\mathrm{syn}}} = 0.8 \cdot 1544 \text{ kNm} = \underline{\underline{1235 \text{ kNm}}}$. Load angle is acc. to Fig. 3.5-4: $\underline{13°}$! This conicides with the result from phasor diagram Fig. 3.5-1. Pull-out torque is about $\underline{4280}$ kNm.

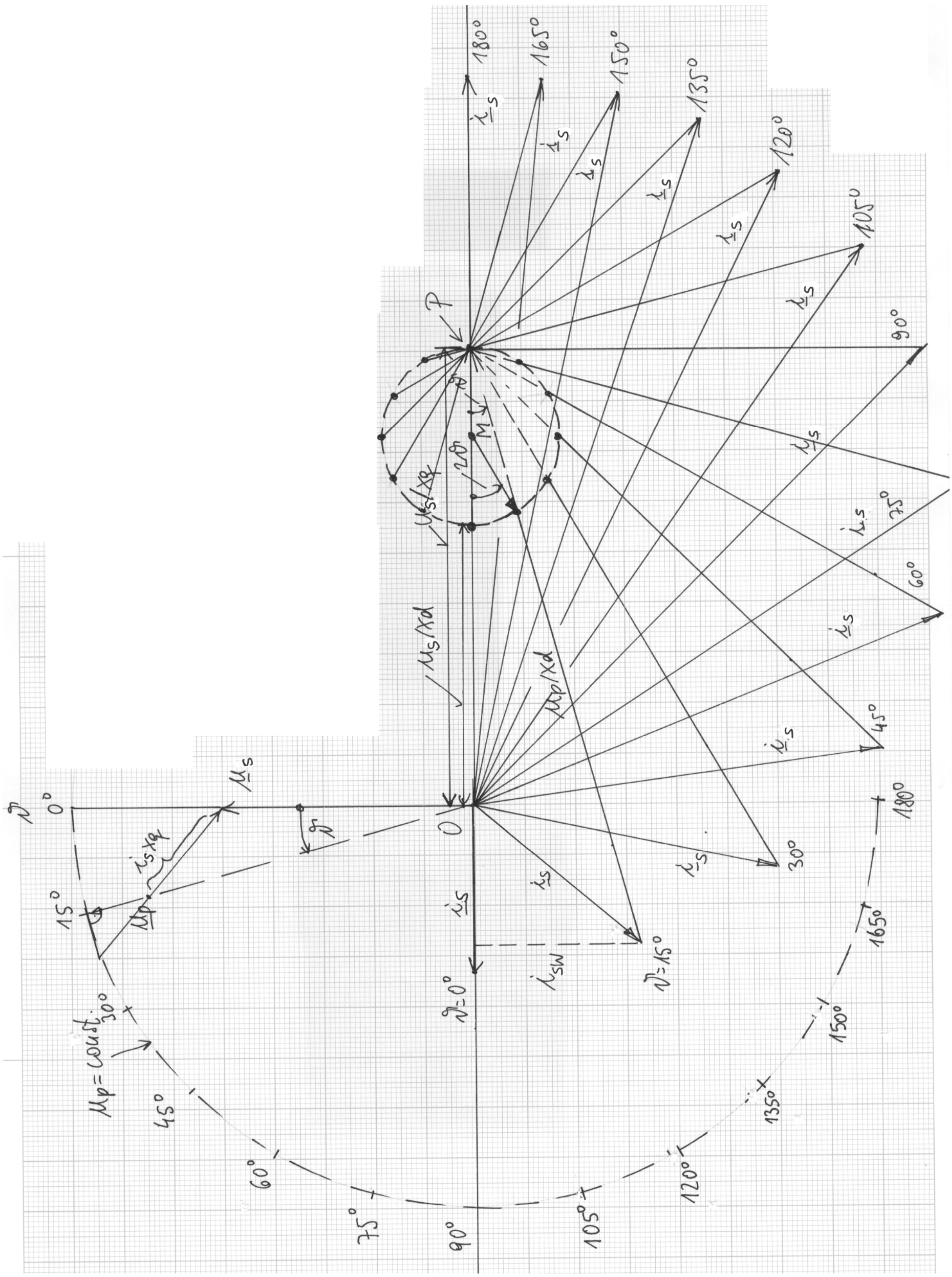


Fig. 3.5-3: Current phasor diagram (in per unit) as root locus in dependence of load angle at constant voltage and back EMF

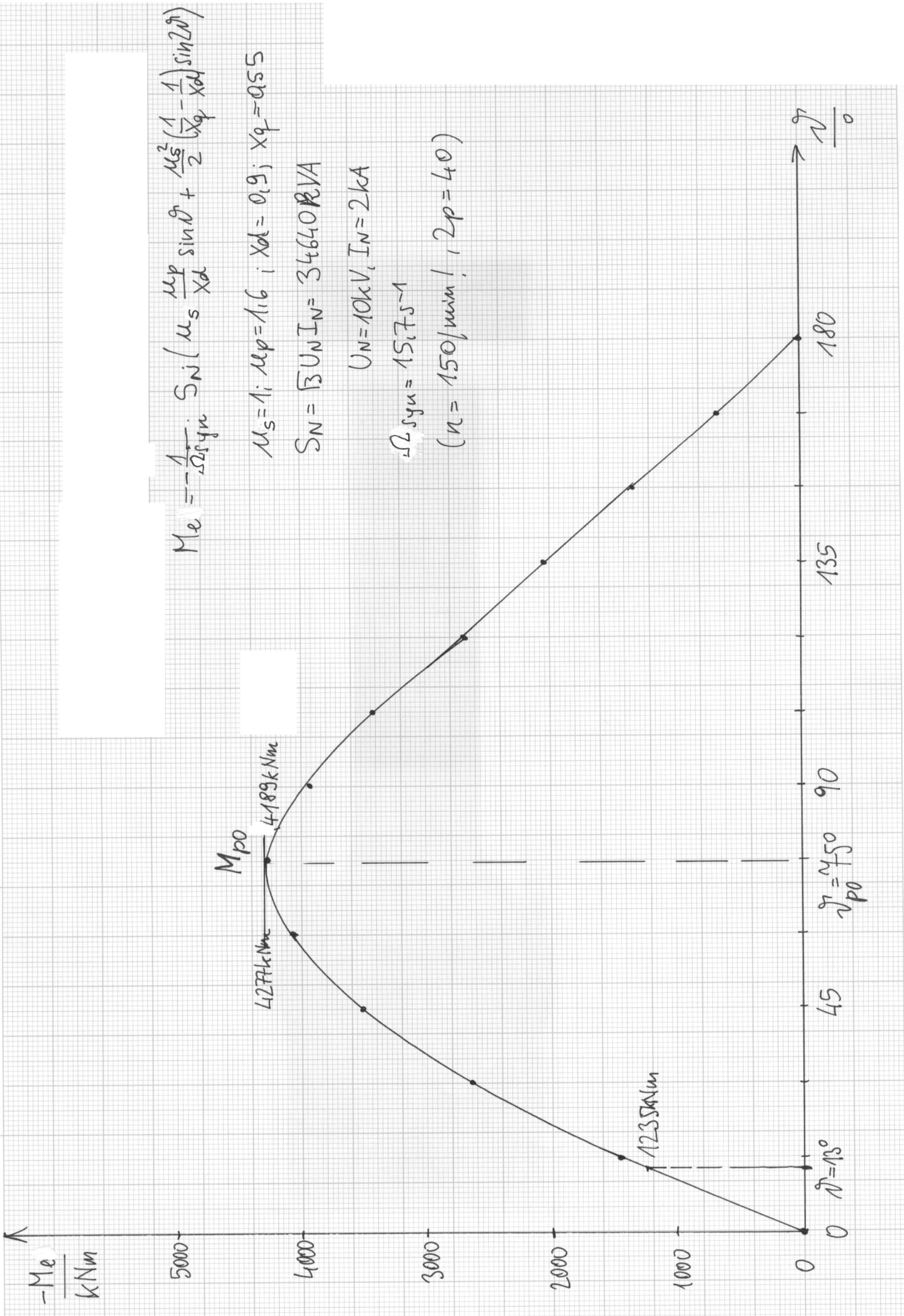


Fig. 3.5-4: Torque-load angle-curve of salient pole synchronous machine in generator mode

Exercise 3.6: Synchronous Motor for Big Fan Blower

In a chemical factory in *Japan* a synchronous machine with round rotor is used to drive a big air compressor for the chemical process. At the same time the machine is used to compensate for inductive load in the factory by over-excited operation. Following data of this motor is given (the stator resistance can be neglected.):

$P_N = 2500$ kW (output power) $U_N = 6300$ V Y (line-to-line) $\cos\varphi_N = 0.9$ (overexcited)
$2p = 8$ $f_N = 60$ Hz $x_d = 1.1$ p.u.

1) Draw the p.u. voltage phasor diagram at motor operation with following data (Scale: 5 cm $\hat{=}$ 1 p.u.) and determine back EMF u_p:
$P = P_N$ $U = 0.9 \cdot U_N$ $\cos\varphi = \cos\varphi_N$
2) How big is the torque for the operating point in 1) ?
3) At constant torque the excitation current I_f is reduced to 70 % of the original value. Draw the new voltage phasor diagram in the one of 1). How big is the new power factor $\cos\varphi$? Take for that the back EMF U_p as linear proportional to excitation current I_f, which corresponds to constant iron saturation.

Solution:

1) Phasor diagram in p.u. values (Fig. 3.6-2):

Rated current: $I_{sN} = I_N = \frac{S_N}{\sqrt{3}\cdot U_N} = \frac{P_N}{\sqrt{3}\cdot\cos\varphi_N\cdot U_N} = \frac{2500000}{\sqrt{3}\cdot 0.9\cdot 6300}\text{ A} = \underline{254.6}\text{ A}$

Overload current due to reduced voltage, but full power:

$$I_s(P_N, 0.9\cdot U_N, \cos\varphi_N) = \frac{P_N}{\sqrt{3}\cdot\cos\varphi_N\cdot 0.9\cdot U_N} = \frac{2500000}{\sqrt{3}\cdot 0.9\cdot 0.9\cdot 6300}\text{ A} = \underline{282.8}\text{ A}$$

$u_s = \underline{\underline{0.9\text{ p.u.}}}$ $i_s = \frac{I_s}{I_{sN}} = \underline{\underline{1.11\text{ p.u.}}}$

$x_d = 1.1$ p.u. $\Rightarrow$ $x_d \cdot i_s = 1.1\cdot 1.11\text{ p.u.} = \underline{\underline{1.22\text{ p.u.}}}$

motor operation: u_s leads with respect to u_p, Result: $\underline{u_p = 1.8\text{ p.u.}}$

2) $$M_N = \frac{P_N}{\Omega_{syn}} = \frac{P_N}{2\cdot\pi\cdot n_{syn}} = \frac{P_N\cdot p}{2\cdot\pi\cdot f_N} = \frac{2500000\cdot 4}{2\cdot\pi\cdot 60}\text{Nm} = \underline{\underline{26.53\text{ kNm}}}$$

3) In 1) $u_p = 1.8$ p.u., now reduced excitation $u_p^* = 0.7\cdot u_p = \underline{1.26\text{ p.u.}}$ ($U_p \sim I_f$!)

As M = const. (f_s, n = const.) $\Rightarrow \underline{P = \text{const.}}$

As U_s = const. $\Rightarrow P = 3\cdot U_s\cdot\underbrace{I_s\cdot\cos\varphi_s}_{I_{s,active}} = const. \Rightarrow \underline{I_{s,active} = const.} \Rightarrow \underline{x_d\cdot I_{s,active} = const.}$

So for constant torque and voltage $j\cdot x_d\cdot i_{s,active} = j\cdot x_d\cdot i_{s,w}$ is constant (dashed line in Fig. 3.6-2). In Fig. 3.6-2 the intersection of the circle of u_p^* = const. (centre: point of origin) and $j\cdot x_d\cdot i_{s,w} = const.$ delivers two operation points S and S'. Point S' is unstable, as $|\vartheta^*| > 90°$ (Fig. 3.6-1). So new operation point is S, which satisfies all conditions made above. We get from Fig. 3.6-2 at S: $x_d\cdot i_s^* = 1.14\text{ p.u.} \Rightarrow \underline{i_s^* = 1.04\text{ p.u.}} \Rightarrow \underline{I_s^* = 264.8\text{ A}}$

$P_N = 3\cdot 0.9\cdot U_{sN}\cdot I_s^*\cdot\cos\varphi^* \Rightarrow \underline{\underline{\cos\varphi^* = 0.96}} \Rightarrow \varphi^* = 16.3°$

By direct taking phase angle from diagram we get: $\varphi^* = 14.5°$, due to drawing inaccuracy.

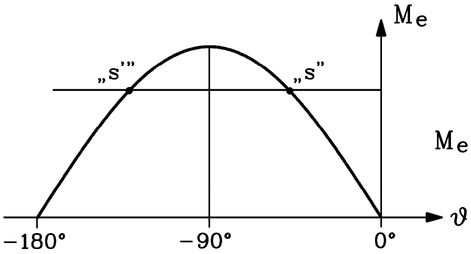


Fig. 3.6-1: Stable (*S*) and unstable (*S´*) operation pint for given motor torque

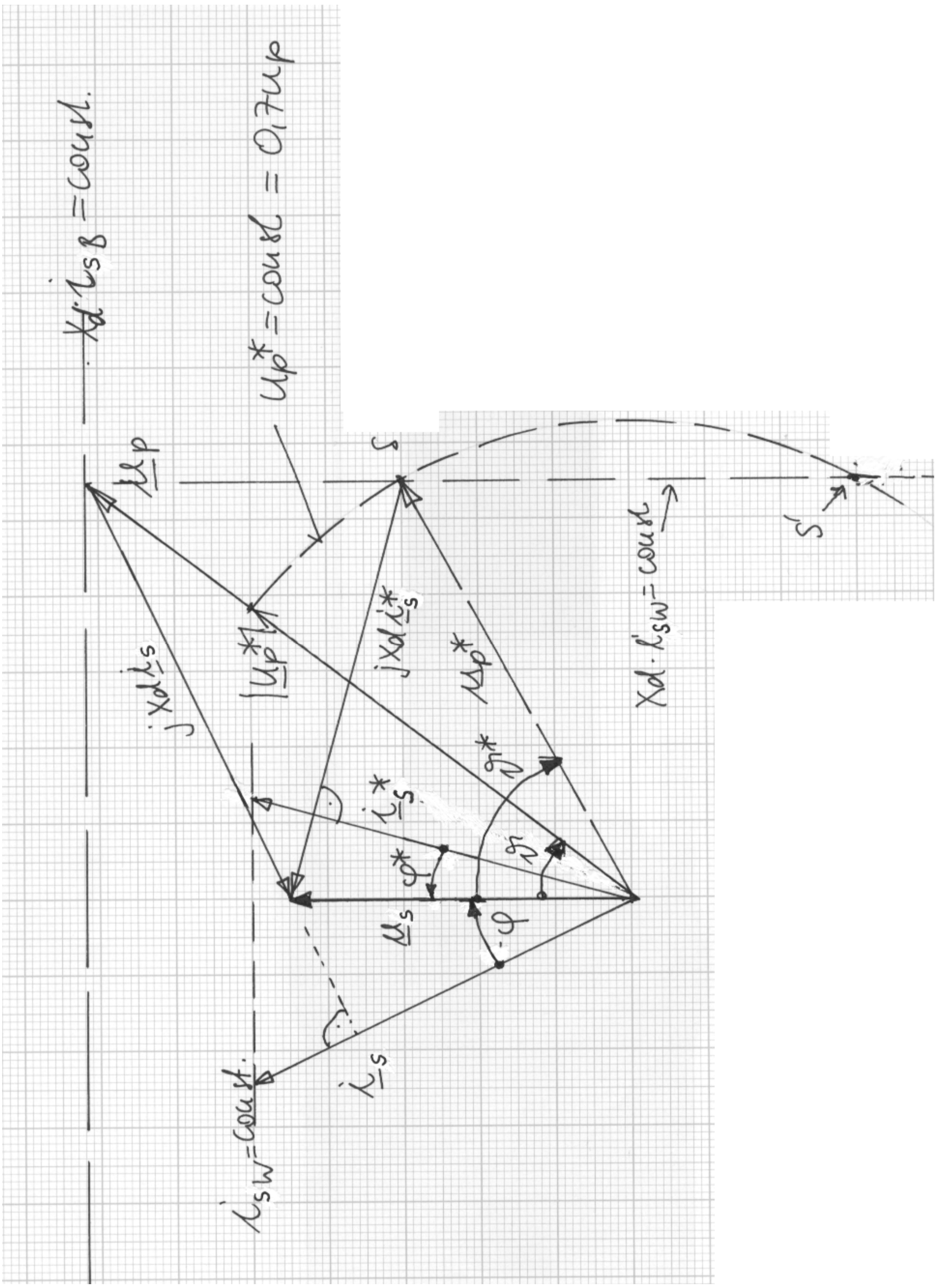


Fig. 3.6-2: Phasor diagram at 90% rated voltage for big and small back EMF, but constant power

<u>Exercise 3.7:</u> PM motor as High-speed compressor drive

A 4 pole synchronous motor with permanent magnet rotor, stator water jacket cooling and Y-connected three-phase stator winding is used to drive an air turbo-compressor at high Speed. The motor is fed by a voltage source d.c. link inverter. Maximum inverter voltage is $U_{max} = 230$ V, maximum inverter current is $I_{max} = 250$ A (fundamental phase values, r.m.s.).

Inverter data: $U_{\text{phase,k=1,max}} = 230\text{V}$, $I_{\text{phase,k=1,max}} = 250\text{A}$

Compressor data: $n_N = 24000/\min$, $P_N = 65\text{kW}$

Motor data: $2p = 4$, $L_d = L_q = 0{,}169\text{mH}$, $U_p = 6.25\text{V}$ (at 50 Hz, phase value, r.m.s.)

For the following, neglect all motor losses!

1. Determine rated motor frequency!
2. Calculate at rated speed back EMF U_{pN}!
3. Evaluate motor rated current for q-current operation ($I_s = I_{sq}$, $I_{sd} = 0$)!
4. Determine stator phase voltage for rated operation according to 3)! Is inverter voltage and current limit sufficient?
5. Draw voltage and current phasor diagram for rated operation 3) (Scale: 20 V/cm, 20 A/cm). Evaluate motor apparent power and power factor! Is the motor running over- or under-excited?

<u>Solution:</u>

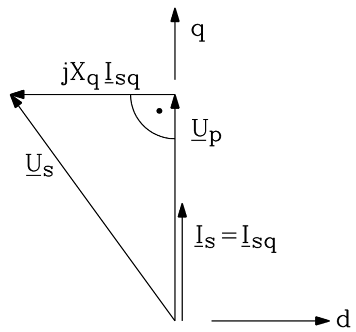
1) $f_{sN} = n_N \cdot p = (24000/60) \cdot 2 = \underline{\underline{800}}\,\text{Hz}$

2) $U_{pN} = \dfrac{f_{sN}}{f} \cdot U_p = \dfrac{800}{50} \cdot 6.25 = \underline{\underline{100}}\,\text{V}$

3) $P_{e,in} = P_{m,out} = P_N = 65000\,\text{W}$

q-current operation, no losses (Fig. 3.10-1): $U_{sN} \cos\varphi_N = U_{pN}$

$$P_{e,in} = 3 \cdot U_{sN} I_{sN} \cos\varphi_N = 3U_{pN} I_{sqN} \quad \Rightarrow \quad I_{sqN} = \frac{65000}{3 \cdot 100} = \underline{\underline{216.7}}\,\text{A}$$



<u>Fig. 3.7-1:</u> Phasor diagram at q-current operation

4) Acc. to Fig. 3.7-1 we get:

$$U_{sN} = \sqrt{U_{pN}^2 + (2\pi \cdot f_{sN} \cdot L_q \cdot I_{sqN})^2} = \sqrt{100^2 + (2\pi \cdot 800 \cdot 0.169 \cdot 10^{-3} \cdot 216.7)^2} = \underline{\underline{209.5}}\,\text{V}$$

$U_{sN} = 209.5V < 230V = U_{phase,k=1,\max}$, $I_{sN} = 216.7A < 250A = I_{phase,k=1,\max}$

The inverter rating is sufficient for that drive purpose.

5) $S_N = 3 \cdot U_{sN} \cdot I_{sN} = 3 \cdot 209.5 \cdot 216.7 = \underline{\underline{136.2}}\,\text{kVA}$

$\cos\varphi_N = P_N / S_N = 65000/136200 = \underline{\underline{0.477}}$

Motor current is lagging; motor is inductive consumer = under-excited operation.

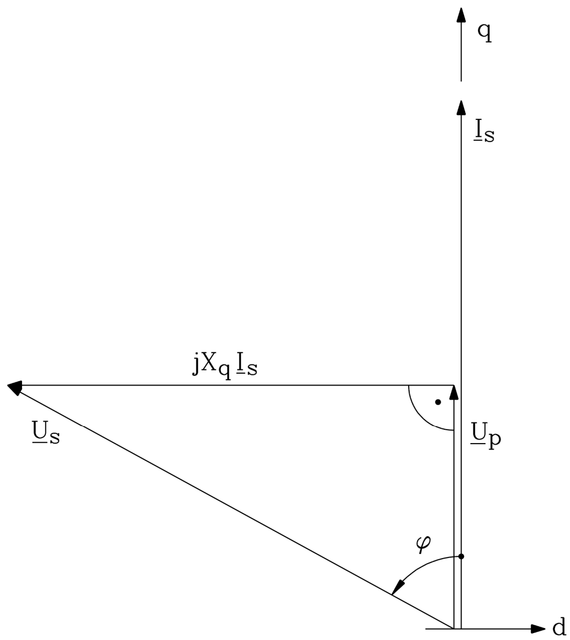


Fig. 3.7-2: Voltage and phasor diagram of PM motor at rated load and speed

Exercise 3.8: Permanent magnet motor as a machine tool drive

A 6 pole *permanent magnet motor* has surface mounted magnets on the rotor with a height of h_M = 3.5 mm. The stator core and winding data are:

d_{si} = 100 mm (bore diameter) l = 100 mm (iron length)

Q = 36 (number of stator slots) N_c = 20 (number of turns per coil)

a = 1 (all coils series connected)

The three phases of the single-layer winding are in Y-connection. The mechanical air gap and the thickness of the non-magnetic rotor bandage sum up to an overall height of δ= 1.4 mm.

Magnet data: Material NdFeB, at 20°C: B_R = 1.1 T (magnetic remanence flux density)
H_{CB} = 875 kA/m (coercive field strength)

1) How big is the magnetic flux density in the air gap at no load (stator current is zero) ? Assume infinite permeability for iron! Give a rough sketch of the air gap flux density field curve, if there are no gaps between the magnets!
2) How big is the amplitude of the fundamental flux density field wave of 1)?
3) Calculate the r.m.s. value of the fundamental harmonic of the back EMF at a stator frequency of f_s =100 Hz!
4) Verify with the magnetic data, that the rare earth magnets behave passively like air ($\mu_M = \mu_0$)!
5) How big is the unsaturated magnetizing reactance X_h ?

6) How big must be the r.m.s. phase value of the fundamental harmonic voltage U_s of the inverter output voltage at 100 Hz, so that maximum torque can be reached at a phase current of 10 A r.m.s.? Neglect R_s and assume $X_{s\sigma} = 1.4\ \Omega$.
7) Draw a voltage phasor diagram for operation point 6) with scale 25 V/cm and 2 A/cm!

Solution:

1)

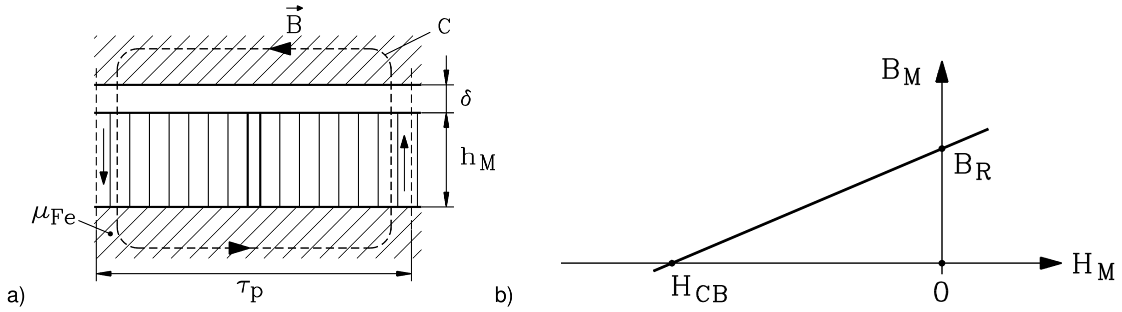


Fig. 3.8-1: a) *Ampere´s* law at no-load for surface mounted permanent magnets, b) Permanent magnet characteristic

$$\oint_l \vec{H} \bullet d\vec{s} = 2 \cdot H_\delta \cdot \delta + 2 \cdot H_M \cdot h_M = \Theta = 0 \qquad \text{(infinite iron permeability: } H_{Fe} = 0\text{)}$$

$$\Rightarrow B_\delta = \mu_0 \cdot H_\delta = -\frac{h_M}{\delta} \cdot \mu_0 \cdot H_M$$

Permanent magnet characteristic in second quadrant of $B_M(H_M)$-plane: $B_M = \mu_M \cdot H_M + B_R$

$$\mu_M = \frac{B_R}{H_{CB}} = \frac{1.1}{875000} = 12.57 \cdot 10^{-6} \cong 4 \cdot \pi \cdot 10^{-7} Vs/(Am) = \mu_0$$

Constant magnetic flux: $\Phi_M = \Phi_\delta$: $B_M = B_\delta$:

$$\Rightarrow B_\delta = B_M = \mu_M \cdot H_M + B_R = \mu_M \cdot \left(-\frac{\delta}{\mu_0 \cdot h_M} \right) \cdot B_\delta + B_R$$

$$B_\delta = \frac{B_R}{1 + \frac{\mu_M \cdot \delta}{\mu_0 \cdot h_M}} = \frac{1.1}{1 + \frac{1.4}{3.5}} = \underline{\underline{0.786 T}} = B_p$$

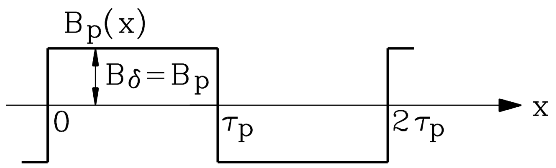


Fig. 3.8-2: No-load permanent magnet air gap flux density curve at 100% pole coverage ratio of magnets

2) *FOURIER* analysis leads to fundamental harmonic of a rectangular field curve of Fig. 3.8-2:

$$B_{\delta,\mu=1} = B_\delta \cdot \frac{4}{\pi} \cdot \sin\left(\frac{\pi}{2}\right) = 0.786 \cdot \frac{4}{\pi}\ \mathrm{T} = \underline{\underline{1.0\ \mathrm{T}}}$$

3) With following motor data:

$$2p = 6,\ \mathrm{m} = 3,\ q = \frac{Q}{2 \cdot p \cdot m} = \frac{36}{6 \cdot 3} = \underline{2},\ N_s = p \cdot q \cdot \frac{N_c}{a} = 2 \cdot 3 \cdot \frac{20}{1} = \underline{120}$$

$$\tau_p = \frac{d_{si} \cdot \pi}{2 \cdot p} = \frac{100 \cdot \pi}{6}\ \mathrm{mm} = \underline{52.4\ \mathrm{mm}} \quad , f_s = 100\ \mathrm{Hz},$$

fundamental winding factor: $k_{w,1} = k_{d,1} \cdot k_{p,1}$

single layer winding has full-pitched coils: $k_{p,1} = 1$

$$k_{d,1} = \frac{\sin\left(\frac{\pi}{2 \cdot m}\right)}{q \cdot \sin\left(\frac{\pi}{2 \cdot m \cdot q}\right)} = \frac{\sin\left(\frac{\pi}{6}\right)}{2 \cdot \sin\left(\frac{\pi}{6 \cdot 2}\right)} = \underline{0.9659} = k_{w,1}$$

we get:

$$U_{p,1} = \sqrt{2} \cdot \pi \cdot f_s \cdot N_s k_{w,1} \cdot \frac{2}{\pi} \cdot \tau_p \cdot l \cdot B_{\delta,\mu=1} = 2\pi \cdot 100 \cdot 120 \cdot 0.9659 \cdot \frac{2}{\pi} \cdot 0.0524 \cdot 0.1 \cdot 1.0 = \underline{\underline{171.5\ \text{V}}}$$

4) Permeability of magnets:

$$\mu_M = \frac{B_R}{H_{CB}} = \frac{1.1}{875000} = 12.57 \cdot 10^{-6} \cong 4 \cdot \pi \cdot 10^{-7} Vs/(Am) = \mu_0$$

Rare earth permanent magnets have the same permeability as air.

5)

$$X_{h,\nu=1} = 2 \cdot \pi \cdot f_s \cdot \mu_0 \cdot N_s^2 \cdot k_{w,1}^2 \cdot \frac{2 \cdot m}{\pi^2} \cdot \frac{l \cdot \tau_p}{p \cdot (\delta + h_M)} =$$

$$= 2 \cdot \pi \cdot 100 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 120^2 \cdot 0.9659^2 \cdot \frac{2 \cdot 3}{\pi^2} \cdot \frac{0.1 \cdot 0.0524}{3 \cdot (1.4 + 3.5) \cdot 10^{-3}} = \underline{\underline{2.3\ \Omega}}$$

$\mu_M \approx \mu_0 \quad \Rightarrow \quad$ resulting magnetic effective air gap is $\delta + h_M$.

5) Maximum torque at given current is reached, if stator current is only q-axis current: $I_s = I_q$

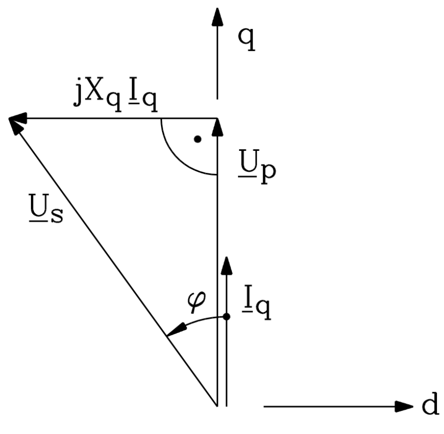


Fig. 3.8-3: Voltage diagram for q-current, motor operation, R_s neglected

Due to constant air gap and neglected iron saturation it is $X_d = X_q$.
With $X_{s\sigma} = 1.4\ \Omega$, $\quad R_s \cong 0$, $\quad$ we get acc. to Fig. 3.8-3:

$$U_s^2 = (X_q \cdot I_q)^2 + U_p^2, \quad I_q = 10\ \text{A}, \quad X_q = X_{s\sigma} + X_h = (1.4 + 2.3)\,\Omega = \underline{3.7\ \Omega}$$

$$\Rightarrow \quad U_s = \sqrt{(X_q \cdot I_q)^2 + U_p^2} = \sqrt{(3.7 \cdot 10)^2 + 171.5^2}\ \text{V} = \underline{\underline{175.4\ \text{V}}}$$

6) Voltage phasor diagram acc. to Fig. 3.8-3 is given in Fig. 3.8-4.

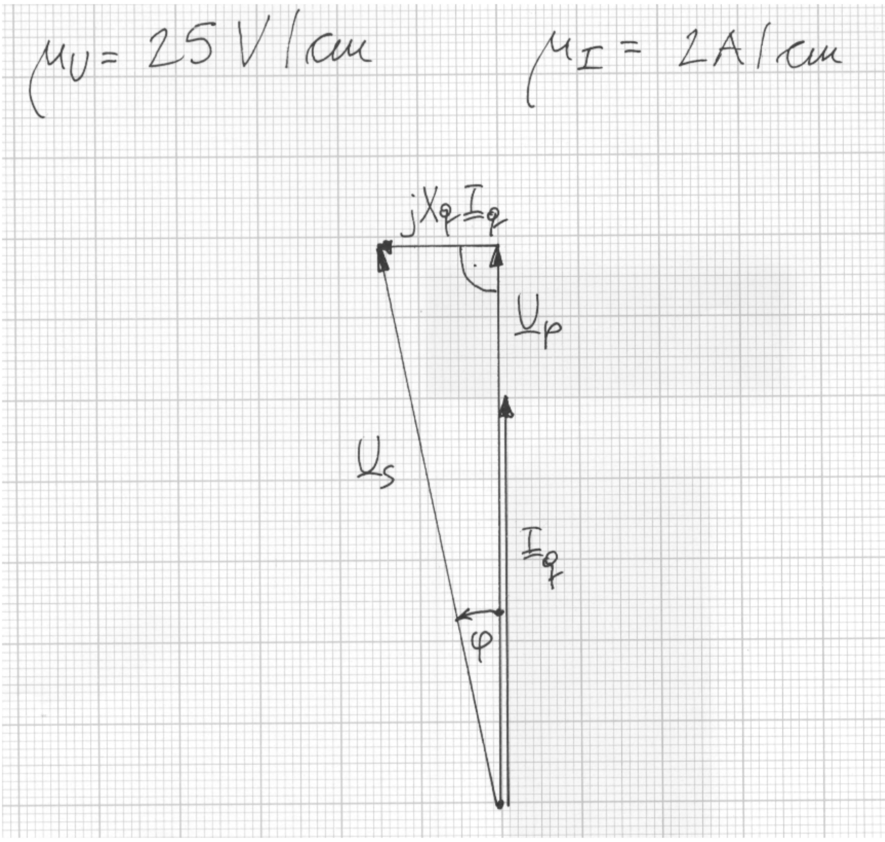


Fig. 3.8-4: Voltage diagram for q-current, motor operation, R_s neglected

Exercise 3.9: Permanent magnet synchronous motor as robot drive

An 8-pole permanent magnet synchronous machine is fed by an inverter and used as drive for a robot arm of a **welding robot**. By rotor position control the motor is operated with q-current to get maximum torque. The maximum thermal continuous current is 12 A r.m.s.. For a short time of a few seconds the motor winding can be overloaded with 4 times rated current. It was measured $U_s = 90$ V Y phase no-load voltage r.m.s. at $f_s = 50$ Hz stator frequency. The maximum inverter output phase voltage (fundamental) is $U_s = 230$ V r.m.s.

Motor data:
$f_{sN} = 100$ Hz (rated frequency), $L_d = 4.5$ mH (synchronous inductance)
The stator resistance is neglected.

1.) The machine is tested in the test bay of the manufacturer. It is driven as generator with another motor between 0 and 6000/min. At open stator winding terminals no-load voltage is measured. Draw the function of no-load voltage (line-to-line, r.m.s.) versus speed (Scale: abscissa: 1000/min ≙ 2 cm, ordinate: 500 V ≙ 2 cm).

2.) Is it possible to determine by measurement acc. to 1), if the magnets are properly magnetized? Do we need to know magnet temperature to draw a correct conclusion?

3.) Draw the voltage phasor diagram for rated frequency
a) at nominal current, b) at maximum current (Scale: 25 V/cm, 5 A/cm)

4.) Determine nominal speed of the motor !

5.) What is the motor torque for 3 a.) and 3 b.), if the motors losses are neglected ?

6.) Draw the $M(n)$-curve of the PM motor Fig. 3.9-1 due to current and voltage limit. Give further $M(n)$-curve for continuous operation (Scale: Abscissa: 100/min per cm, ordinate: 10 Nm per cm).

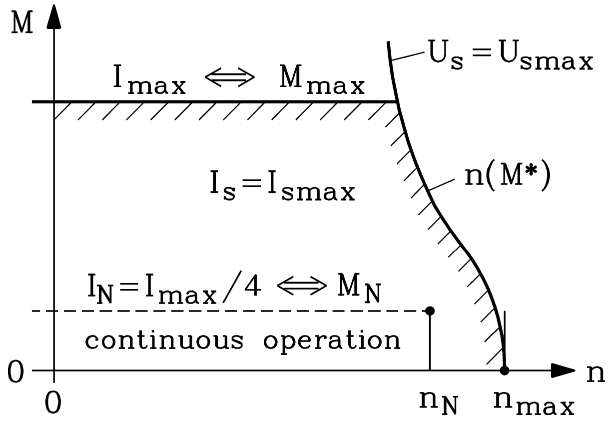


Fig. 3.9-1: Voltage, current and thermal limit of PM drive

Solution:

1.) $U_p = 90$ V, $f_s = 50$ Hz, $n = f_s / p = 50 / 4 = 12.5$ /s $= 750$ / min

$U_p = \omega_s \cdot \Psi_p / \sqrt{2} = 2 \cdot \pi \cdot f_s \cdot \Psi_p / \sqrt{2} = 2 \cdot \pi \cdot n \cdot p \cdot \Psi_p / \sqrt{2} \Rightarrow \quad U_p \sim n \cdot \Psi_p$

Open circuit no-load phase voltage = back EMF:

$$U_{p,\max} = \frac{n_{\max}}{n} \cdot U_p = \frac{6000}{750} \cdot 90 = 720\,\text{V},$$

Line-to-line voltage (Y): $U_{p,LL} = \sqrt{3} \cdot 720 = 1247$ V ... Diagram $U_{p,LL}(n)$: Fig. 3.9-2

2.) U_p is proportional to speed and permanent magnet flux $U_p \sim \Psi_p$

Due to $\Psi_p = N \cdot k_w \dfrac{2}{\pi} \cdot \tau_p \cdot l \cdot B_{p,\mu=1}$, $B_{p,\mu=1} = \dfrac{4}{\pi} \cdot B_p$ and $B_p = \dfrac{B_R}{1 + \dfrac{\mu_M}{\mu_0} \cdot \dfrac{\delta}{h_M}}$ the back EMF is directly proportional to remanence flux density B_R. Therefore it is possible to determine by back EMF measurement, if the magnets are properly magnetized. For that, the magnet temperature must be known, as remanence depends on temperature: $B_R = B_R\,(\vartheta)$!

3.) $L_d = 4.5mH$, $R_s \cong 0$, Due to constant magnetic air-gap and neglected iron saturation we may assume: $L_d = L_q$

$X_{dN} = \omega_{sN} \cdot L_d = 2\pi \cdot f_{sN} \cdot L_d = 2\pi \cdot 100 \cdot 4.5 \cdot 10^{-3} = 2.83\Omega$

a.) $I_q = I_s = 12A = I_N$: $X_{dN} \cdot I_N = 2.83 \cdot 12 = 33.9V = X_{qN} \cdot I_N$

b.) $I_q = I_s = 4 \cdot I_N = 48A$: $X_{dN} \cdot 4 \cdot I_N = 4 \cdot 33.9 = 135.7V = X_{qN} \cdot 4 \cdot I_N$

$$U_{pN} = \frac{f_{sN}}{f_s} \cdot U_p = \frac{100}{50} \cdot 90 = 180\,\text{V}$$

a.) $U_s = \sqrt{U_p^2 + (X_{qN} \cdot I_N)^2} = \sqrt{180^2 + 33.9^2} = 183.2V$

b.) $U_s = \sqrt{U_p^2 + (X_{qN} \cdot 4I_N)^2} = \sqrt{180^2 + 135.7^2} = 225.4V$

Voltage phasor diagram Fig. 3.9-3.

4) $n_N = \frac{f_{sN}}{p} = \frac{100}{4} = 25/s = \underline{\underline{1500/\min}}$

5) No losses considered: $\eta = 1$: $P_{out} = P_m = P_{in} = P_e \Rightarrow \quad P_e = 3 \cdot U_s \cdot I_s \cdot \cos\varphi = 3 \cdot U_p \cdot I_s$

$$U_s \cdot \cos\varphi = U_p \,,\; M_e \cdot \Omega_{syn} = P_m \Rightarrow \quad M_e = \frac{3 \cdot U_p \cdot I_s}{\Omega_{syn}} \,,\; \Omega_{syn} = 2\pi \cdot \frac{f_{sN}}{p} = 2\pi \cdot 25 = 157.1/\mathrm{s}$$

a.) $M_e = \frac{3 \cdot U_p \cdot I_s}{\Omega_{syn}} = \frac{3 \cdot 180 \cdot 12}{157.1} = \underline{\underline{41.25Nm}}$, b.) $M_e = \frac{3 \cdot U_p \cdot I_s}{\Omega_{syn}} = \frac{3 \cdot 180 \cdot 48}{157.1} = \underline{\underline{165Nm}}$

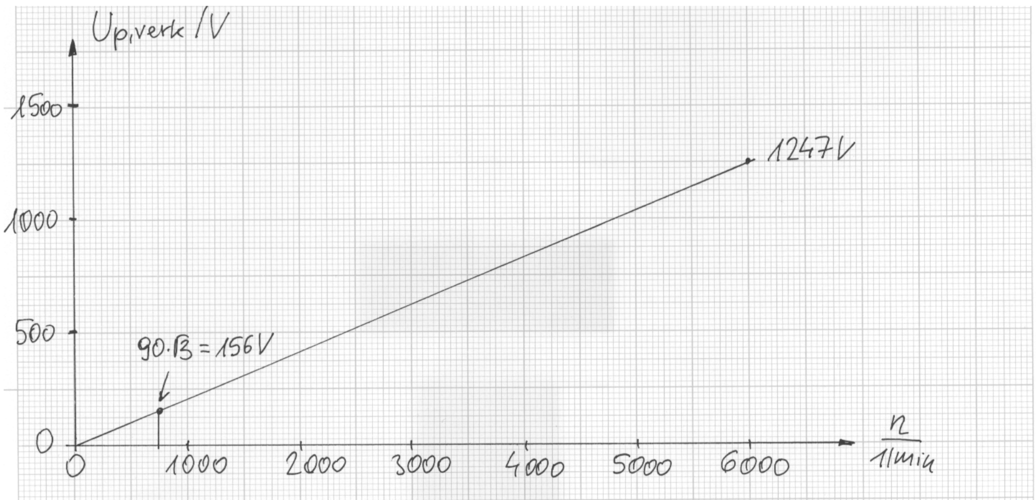


Fig. 3.9-2: Back EMF (r.m.s., line-to-line) versus speed, measured at open circuit, no-load generator operation

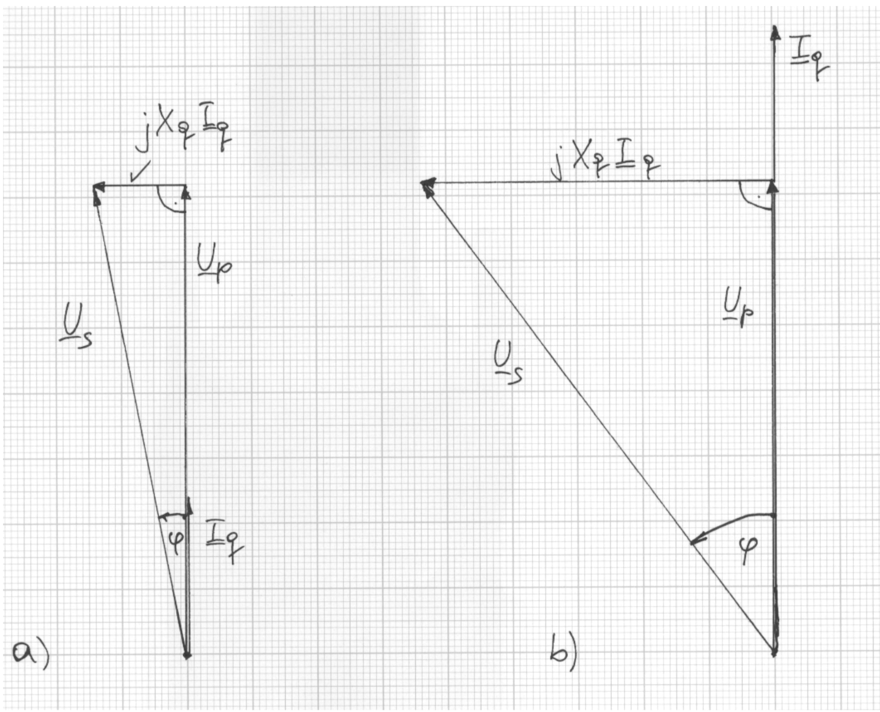


Fig. 3.9-3: Voltage phasor diagram for q-current operation: a) rated current, b) 4 times overload

6.) a) Voltage limit: $U_{s,max} = 230\mathrm{V}$: $U_{s\max}^2 = \left(\frac{n}{n_N} \cdot X_{qN} \cdot I_q\right)^2 + \left(\frac{n}{n_N}\right)^2 \cdot U_{pN}^2$

$n_N = 1500/\text{min}$, $X_{q\mathrm{N}} = 2.83\ \Omega$, $U_{pN} = 180\text{V}$

Speed ratio: $\dfrac{n}{n_N} = \nu:\ I_q = \dfrac{\sqrt{U_{s\max}^2 - \nu^2 \cdot U_{pN}^2}}{\nu \cdot X_{qN}} = I_s^*$

Maximum speed: $n_{\max}$: $I_q = 0$: ideal motor no-load, $n_{\max} = n_N \cdot \dfrac{U_{s\max}}{U_{pN}} = 1500 \cdot \dfrac{230}{180} = 1917/\min$

$$M_e = \frac{3 \cdot U_p \cdot I_q}{2\pi \cdot n} = \frac{3 \cdot 2\pi(p \cdot n) \cdot \Psi_p \cdot I_q}{2\pi \cdot n \cdot \sqrt{2}} = \frac{3 \cdot p \cdot \Psi_p \cdot I_q}{\sqrt{2}}, \quad \Psi_p = \sqrt{2} \cdot \frac{U_p}{\omega_s} = \frac{\sqrt{2} \cdot 180}{2\pi \cdot 100} = 0.405Vs$$

$\nu = n/n_N$	I_s^*	n	M^*
	A	1/min	Nm
1.2780	0	1917	0
1.16	28.5	1750	98
1	50.8	1500	174.6
0.9	64.3	1350	221

Table 3.9-1: Operation data at the voltage limit

b) Current limit: $I_{s,\max} = 48$ A: $M_{\max} = 165$ Nm
c) Thermal limit: $I_{\mathrm{N}} = 12$ A: $M_{\mathrm{N}} = 41.25$ Nm

Diagram of voltage, current and thermal limit for *M(n)* is given in Fig. 3.9-4.

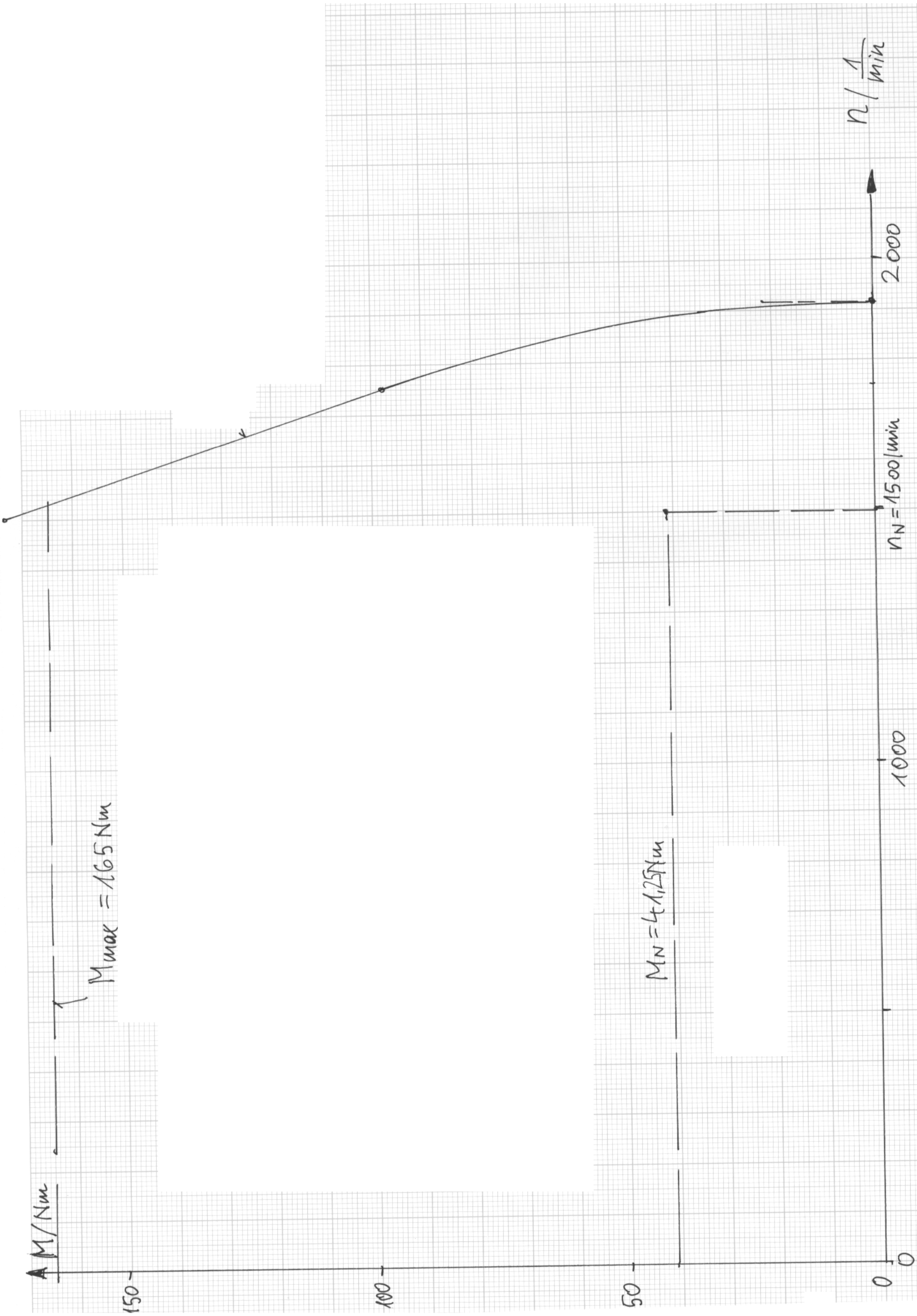


Fig. 3.9-4: Voltage, current and thermal limit according to Fig. 3.9-1