5. The Slipring Induction Machine

In this chapter, the operation principle of electric machines with rotating fields supplied with a.c. sine-wave voltages at **steady state** is discussed. **Transient incidences**, e.g. running up, sudden short circuits and sudden blockade of the rotor are beyond the scope. The electric machines are mathematically described in the **consumer reference-arrow system**.

Motor operation:
At **motor operation**, the electric (absorbed, consumed) power P_e and the power factor $\cos\varphi$ are positive, as well as the mechanic (delivered) power P_m. The electro-magnetically generated torque M_e is also positive and acts **in a propelling way**. The load torque M_s of the driven load has to be overcome to maintain the rotation of the electric machine.

Generator operation:
At **generator operation**, the power factor is negative, the absolute value of the phase angle between current and voltage is **larger** than 90° and **smaller** than 270°. Therefore, the electric (delivered) power is negative, as well as the mechanically absorbed power. It is taken from the mechanical system (e.g. a steam turbine plant). The electro-magnetically generated torque of the machine is also negative, it **breaks** and has to be overcome by the driving mechanical torque to maintain the rotation of the electric machine (Table 5.1).

	active power $P = mUI\cos\varphi$	**reactive power** $Q = mUI\sin\varphi$
$0 \le \varphi < 90°$	$P > 0$, motor	$Q > 0$, inductive load
$90° \le \varphi < 180°$	$P < 0$, generator	$Q > 0$, inductive load
$180° \le \varphi < 270°$	$P < 0$, generator	$Q < 0$, capacitive load
$270° \le \varphi < 360°$	$P > 0$, motor	$Q < 0$, capacitive load

Table 5.1: Correlation between phase angle and sign of active and reactive power flow in **consumer reference-arrow system**

Within the considered overall system, the **power must be balanced (energy conservation)**.
***Balance of active power**:*
The electric and mechanical systems have to be considered together, because, with electric ma-chines, the energy conversion takes place between these two systems. The sum of mechanical, electric and thermal energy (due to losses) is constant.
Balance of reactive power:
Here, consideration of the electric system is sufficient. No surplus of apparent power is gene-rated within a closed electric system, **capacitive and inductive apparent power counterba-lance each other**.

5.1 Operation Principle of Slipring Induction Machines

In the following, only the **fundamentals of the stator and the rotor field** ($\nu = \mu = 1$) are considered.

a) Torque Generation and Slip:
The rotary transformer (Chapter 4) which has a three-phase winding in both the stator and the rotor is used as a starting point and the phases of the rotor winding are shorted (Fig. 5.1). The three phases of the stator are supplied with a symmetrical three-phase system (current I_s, fre-quency f_s). The sinusoidally distributed fundamental of the air gap field $B_{\delta s}$, which is genera-ted by the stator winding, induces voltages with stator frequency into the **rotor winding at stand-still**. These voltages result in large short-circuit currents in the shorted rotor phases $I_{c,r}$.

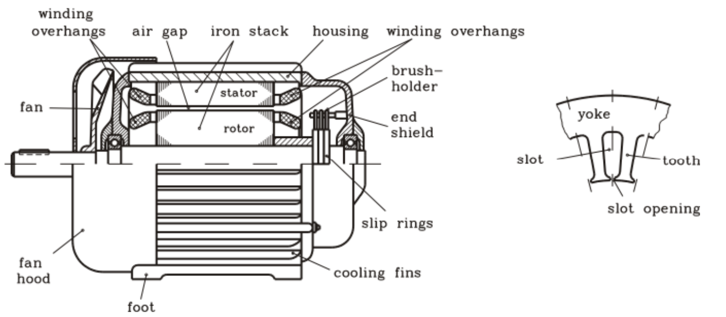


Fig. 5.1: Longitudinal cross-section of a fan-cooled slipring induction motor and detailed drawing of the stator lamination with slots for the winding

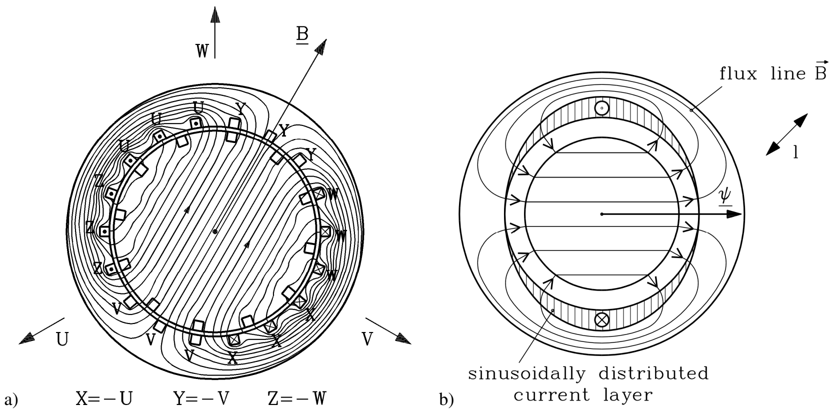


Fig. 5.2: Two-pole slipring induction motor, supplied with three-phase current, rotor at zero current:
a) numerically calculated stator field for $m_s = 3$, $q_s = 3$ at the time where $i_U = - i_W$ and $i_V = 0$
b) sketch of the fundamental of the stator magnetic field

Fig. 5.2 shows the stator field with the rotor at zero current. The current distribution that is concentrated in the stator slots (Fig. 5.2a) is represented by a sinusoidally distributed fundamental of the electric loading (Fig. 5.2b). The air gap flux density (number of field lines per unit circumference) is also sinusoidally distributed and is represented by a **"space vector"** $\underline{B}$ or $\underline{\Psi} = Nk_w \frac{2}{\pi}\tau_p l \hat{\underline{B}}_\delta$ (Fig. 5.3). The space vector has the attributes of a vector and is – to avoid confusion with the vector field $\vec{B}(x, y, z)$ – described by **complex pointer** with the two components "real part" and "imaginary part" in the **cross sectional plane** of the machine.

Space vector:
The **amplitude** of the space vector is proportional to the field amplitude of the radial component of $\vec{B}$, the **direction** is aligned with the maximum of the sinusoidal distribution and the **orientation** is in the direction of the north pole.

The *LORENTZ* force acts upon any current-carrying conductor of the rotor (conductor current = rotor coil current $I_{c,r}$)

$$\vec{F}_{conductor} = I_{c,r} \cdot (l\vec{e}_l \times \vec{B}_{\delta,s}) = I_{c,r} \cdot l \cdot B_{\delta,s} \cdot \vec{e}_t \quad , \qquad (5.1)$$

where $\vec{e}_l$ is the unit vector in the direction of the slot conductor and l is the axial length of the coil sides in the rotor slots. As the field lines cross the air gap perpendicularly, hence perpendicularly to the currents in the conductors, the forces act in circumferential direction (tangential unit vector $\vec{e}_t$) and are maximal. All conductor forces add to each other to give the total force F_e and – together with half of the rotor diameter $d_r/2$ acting as a lever – an **electromagnetic torque M_e**.

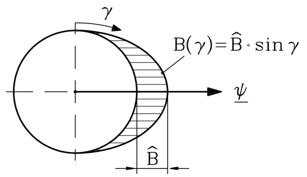


Fig. 5.3: In the air gap, the fundamental field is sinusoidally distributed along the circumference and can be represented by a "space vector" $\underline{\Psi}$.

$$M_e = F_e \cdot d_r / 2 \qquad (5.2)$$

The torque causes the rotation of the rotor into the direction of the rotating field of the stator. As soon as the rotor – accelerated from stand-still – turns at the **speed** n, respectively at mechanical angular speed $\Omega_m = 2\pi n$, the speed difference Δn between rotating field (speed n_{syn}) and rotor decreases.

$$\Delta n = n_{syn} - n \qquad (5.3)$$

The **"slip"** is defined according to (5.4).

$$s = \frac{n_{syn} - n}{n_{syn}} \quad \Rightarrow \quad f_r = s \cdot f_s \quad \text{respectively} \quad \omega_r = s \cdot \omega_s \ , \qquad (5.4)$$

At stand-still, it is $s = 1$. The slip decreases when the motor accelerates and is zero at $n = n_{syn}$, because the rotor accelerates as long as it is submitted to an electromagnetic torque which is caused by current flow in the rotor phases. Rotor currents flow as long as voltage is induced in the rotor phases, which is only the case if the speed difference Δn is different from zero. In this case, an observer that travels with the rotor can experience a change of the flux linkage of the rotor phases with the **frequency** f_r (Fig. 5.4).

As soon as the rotor rotates **as fast** as the stator field, the **slip becomes zero**. No voltage is induced any more ($f_r = 0$) and the rotor is at zero current ($I_{c,r} = 0$). Hence, no torque is produced, and the rotor does not accelerate any longer. In reality, **torque due to losses** (e.g. frictional torque) acts in a braking way so that the rotor needs a minimum electro-magnetic torque to maintain its speed. Therefore, some, even though small torque, has to exist. Hence, the rotor always rotates **asynchronously** (slower) when compared with the stator field which keeps overtaking the rotor field (**"asynchronous machine", "induction machine"**).

The current generation in the rotor is **without contact**. No sliding contacts are necessary: The construction of an induction machine is robust. However, the phase windings of the rotor can

be connected to sliprings (**slipring induction machine**), to allow additional external current supply of the rotor if required (**doubly-fed induction machine**) or additional external resistances in the rotor circuit to limit the high currents at start-up (slip $s = 1$) (**starting resistors**).

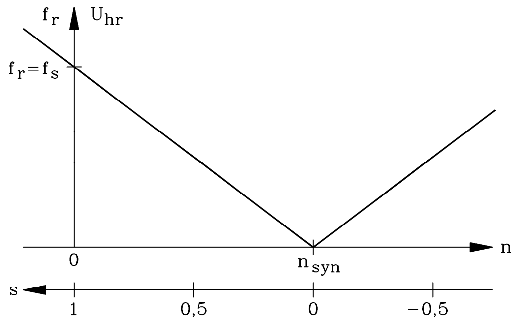


Fig. 5.4: Reduction of the rotor frequency f_r with increasing speed n up to synchronous speed

b) Equation of the Rotor Voltage:
Only parameters varying sinusoidally with respect to time are used, which are described using complex calculus for sine wave ac systems:

$$i_s(t) = \sqrt{2} I_s \cos(\omega_s t) = \mathrm{Re}\left(\sqrt{2} \underline{I}_s e^{j\omega_s t}\right) \quad \Rightarrow \quad i_s(t) \leftrightarrow \underline{I}_s \quad . \qquad (5.5a)$$

$$\text{e.g.: } R \cdot i + L \cdot \frac{di}{dt} = u \quad \Rightarrow \quad R \cdot \underline{I} + j\omega L \cdot \underline{I} = \underline{U} \qquad (5.5b)$$

Using complex notations, the **rotor voltage** per phase (with frequency f_r) that is induced via **mutual induction** by the rotating stator field according to Section 4.5 is described by (5.6).

$$\underline{U}_{i,r} = j\omega_r M_{sr} \underline{I}_s = j2\pi f_r M_{sr} \underline{I}_s \qquad (5.6)$$

Because of (5.6), an ac-rotor current I_r (with frequency f_r) flows in each phase. The voltages and the currents are phase shifted by $2\pi/3$ respectively between the rotor phases U, V, W, thereby forming a symmetrical three-phase system and generating via the rotor three-phase winding a rotor air gap rotating field.

$$B_{\delta,r}(x_r, t) = \hat{B}_{\delta,r} \cos(\gamma_r - \omega_r t) \ , \quad B_{\delta,r} \sim I_r \qquad (5.7)$$

This rotating field rotates with respect to the rotor (rotor circumferential coordinate x_r, respectively rotor circumferential angle $\gamma_r = x_r \pi / \tau_p$) with the circumferential speed

$$v_{r,syn} = \frac{dx_r}{dt} = 2 f_r \tau_p \ , \qquad (5.8)$$

thereby inducing a voltage into the rotor winding by self-induction

$$\underline{U}_{i,r} = j\omega_r L_{rh} \underline{I}_r = j2\pi f_r L_{rh} \underline{I}_r \quad . \qquad (5.9)$$

As the mechanical circumferential speed given here related to the stator bore diameter d_{si}

$$v_m = d_{si}\pi n = 2p\tau_p n \qquad (5.10)$$

adds to this relative speed $v_{r,syn}$, the rotating field generated by the rotor rotates with synchronous speed with respect to the stator (5.11). Hence, it rotates **as fast as** the rotating field generated by the stator winding.

$$v = v_m + v_{r,syn} = 2pn\tau_p + 2f_r\tau_p = 2p \cdot n_{syn}(1-s)\cdot\tau_p + 2\cdot sf_s\cdot\tau_p = \\ = 2p\cdot\frac{f_s}{p}\cdot(1-s)\cdot\tau_p + 2\cdot sf_s\cdot\tau_p = 2f_s\tau_p = v_{syn} \qquad (5.11)$$

Result:
The sinusoidal rotating fields of stator and rotor can be added to each other spatially to obtain the total sinusoidal rotating field which is the total air gap field of the induction machine.

The **rotor harmonics** increase the self-induced voltage according to Chapter 4 by the voltage $\underline{U}_{i,r,\sigma_o} = j\omega_r\sigma_{r,o}L_{rh}\underline{I}_r$. The *OHM*ic voltage drop at the **rotor phase resistance** R_r and another self-induced voltage generated by the **rotor slot leakage field** $L_{r,\sigma Q}$ and the **rotor end-winding leakage field** $L_{r,\sigma b}$ add to this. The sum of all rotor voltage components is zero, because of the short-circuit of the phases.

$$j\omega_r M_{sr}\underline{I}_s + j\omega_r L_{rh}\underline{I}_r + j\omega_r(\sigma_{r,o}L_{rh} + L_{r,\sigma Q} + L_{r,\sigma b})\underline{I}_r + R_r\underline{I}_r = 0 \qquad (5.12)$$

The phase **rotor leakage inductance** is defined as follows:

$$\boxed{L_{r\sigma} = L_{r,\sigma Q} + L_{r,\sigma b} + \sigma_{r,o}L_{rh}} \qquad (5.13)$$

c) Transfer Ratio:
Using a **transfer ratio *ü*** between stator and rotor winding,

$$\boxed{ü = \frac{k_{w,s}N_s}{k_{w,r}N_r}} \qquad (5.14)$$

that considers the number of turns of the stator and the rotor winding, as well as the winding factors of the distributed three-phase winding, the self-inductance and the mutual inductance can be replaced by a single **magnetising inductance L_h**, which is given by (5.17) and (5.18).

$$L_{sh} = üM_{sr} = ü^2L_{r,h} = \underline{\underline{L_h}} \qquad (5.15)$$

Using this transfer ratio *ü*, (5.12) can be re-written, giving (5.16).

$$j\omega_r üM_{sr}\underline{I}_s + j\omega_r ü^2L_{r,h}\frac{\underline{I}_r}{ü} + j\omega_r ü^2L_{r\sigma}\frac{\underline{I}_r}{ü} + ü^2R_r\frac{\underline{I}_r}{ü} = 0 \qquad (5.16)$$

With $m_r = m_s = m$ (= 3), (5.16) become (5.17) and (5.18):

$$ü^2L_{rh} = \left(\frac{k_{w,s}N_s}{k_{w,r}N_r}\right)^2\cdot\mu_0N_r^2k_{w,r}^2\cdot\frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = \mu_0N_s^2k_{w,s}^2\cdot\frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = L_{sh} \qquad (5.17)$$

$$ü\cdot M_{sr} = \frac{k_{w,s}N_s}{k_{w,r}N_r}\cdot\mu_0\cdot N_sk_{w,s}\cdot N_rk_{w,r}\cdot\frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = \mu_0N_s^2k_{w,s}^2\cdot\frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = L_{sh} \qquad (5.18)$$

In (5.19), the **superscript "'"** indicates that the **rotor parameters have been converted to the stator side by the transfer ratio *ü***. The advantage of this conversion is that the parameters of stator and rotor (currents, voltages as well as impedances) have the same orders of magnitudes in spite of very different winding data of stator and rotor.

$$R'_r = ü^2R_r, \quad L'_{r\sigma} = ü^2L_{r\sigma}, \quad \frac{\underline{I}_r}{ü} = \underline{I}'_r, \quad ü\underline{U}_r = \underline{U}'_r \qquad (5.19)$$

<u>*Example 5.1:*</u>
Stator resistance R_s = 1 Ω, rotor resistance R_r = 0.003 Ω, *ü* = 17.3, rotor resistance converted to stator winding parameters $R'_r = 7.3^2 0.003 = 0.9$ Ω.

Thereby, the **equation** of the **rotor voltage** simplifies to (5.20), using $\omega_r = s\cdot\omega_s$:

$$\boxed{js\omega_sL_h\underline{I}_s + js\omega_sL_h\underline{I}'_r + js\omega_sL'_{r\sigma}\underline{I}'_r + R'_r\underline{I}'_r = 0} \qquad (5.20)$$

d) Stator Voltage Equation:
The rotor field $B_{\delta r}$ (5.8) rotates relatively to the stator with the synchronous speed and induces the stator winding with the stator frequency f_s due to **mutual induction**.

$$\underline{U}_{i,s} = j\omega_sM_{sr}\underline{I}_r = j2\pi f_sM_{sr}\underline{I}_r \qquad (5.21)$$

Due to **self-induction**, the stator field $B_{\delta s}$ self-induces a voltage in the stator winding

$$\underline{U}_{i,s} = j\omega_sL_{sh}\underline{I}_s\,. \qquad (5.22)$$

The **stator harmonics** increase the self-induced voltage by voltage $\underline{U}_{i,s,\sigma_o} = j\omega_s\sigma_{s,o}L_{sh}\underline{I}_s$ (Chapter 4). The *OHM*ic voltage drop at the **stator phase resistance** R_s and another self-induced voltage generated by the **stator slot leakage field** $L_{s,\sigma Q}$ and the **stator end-winding leakage field** $L_{s,\sigma b}$ add to it. The sum of all components of the stator voltage have to equal the voltage at the terminals, which is determined by the phase voltage of the line $\underline{U}_s$.

$$\underline{U}_s = j\omega_sM_{sr}\underline{I}_r + j\omega_sL_{sh}\underline{I}_s + j\omega_s(\sigma_{s,o}L_{sh} + L_{s,\sigma Q} + L_{s,\sigma b})\underline{I}_s + R_s\underline{I}_s \qquad (5.23)$$

The **stator leakage inductance** per phase is defined as follows:

$$\boxed{L_{s\sigma} = L_{s,\sigma Q} + L_{s,\sigma b} + \sigma_{s,o}L_{sh}} \qquad (5.24)$$

Using the transfer ratio *ü*, the **equation of the stator voltage** (5.23) becomes (5.25).

$$\underline{U}_s = j\omega_s \cdot \ddot{u} M_{sr} \cdot \frac{\underline{I}_r}{\ddot{u}} + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s$$

$$\boxed{\underline{U}_s = j\omega_s L_h \underline{I}'_r + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s} \tag{5.25}$$

Result:
The two equations (5.20), (5.25) describe the behaviour of an induction machine concerning the fundamental field, where harmonics are included as harmonic leakage of stator and rotor field in both leakage inductances.

Both equations are very similar to the equations of the phase voltages of the high and low voltage sides of transformers (see text book: *"Basics of Electrical Engineering, Part: Electric Energy Conversion").* However, there are some significant differences:

- The **magnetising inductance** L_h is determined by **all three** stator and rotor phases, whereas with ac-transformers, the three phases are de-coupled from each other. Hence, in the case of the induction machine, three phases of stator and rotor generate ONE total field.
- The frequencies of the stator f_s and the rotor f_r are – except at stand-still – **different**.
- The windings and the magnetic fields are **distributed** along the circumference, whereas a transformer generally has concentric coils and bundled fluxes.

5.2 Steady State Behaviour of an Induction Machine

a) Equations for Stator and Rotor Voltage, T-Equivalent Circuit:
The stator and rotor voltages are described per phase. The equations form a linear, complex set of equations, where the unknowns are the stator and rotor currents $\underline{I}_s$, $\underline{I}_r'$. If (5.20) is divided by s, only expressions with stator frequency $\omega_s L = X$ occur beside the *OHM*ic parameters. Using these **reactances $X = \omega_s L$**, (5.20) and (5.25) become (5.26) and (5.27):

$$\boxed{\underline{U}_s = R_s \underline{I}_s + jX_{s\sigma} \underline{I}_s + jX_h (\underline{I}_s + \underline{I}'_r)} \tag{5.26}$$

$$\boxed{0 = \frac{R'_r}{s} \underline{I}'_r + jX'_{r\sigma} \underline{I}'_r + jX_h (\underline{I}_s + \underline{I}'_r)} \tag{5.27}$$

Stator leakage reactance: $X_{s\sigma} = \omega_s L_{s\sigma}$
Magnetising reactance: $X_h = \omega_s L_h$
Rotor leakage reactance: $X'_{r\sigma} = \omega_s L'_{r\sigma}$

Fig. 5.5 shows the **T-equivalent circuit** for the fundamental of the induction machine derived from (5.26) and (5.27), which can be verified by the mesh equations for stator and rotor circuit.

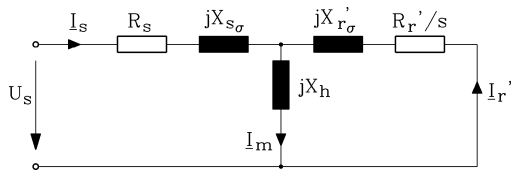


Fig. 5.5: T-equivalent circuit per phase for the field fundamental of an induction machine for currents and voltages changing sinusoidally with time (iron losses P_{Fe} neglected, U_s assumed as real)

The **"magnetising current" I_m** can only be measured as a "real" current at **no-load**, where it equals the stator current, because the rotor current is zero.

$$\boxed{\underline{I}_m = \underline{I}_s + \underline{I}'_r} \tag{5.28a}$$

At **load**, it represents the combined effect of stator and rotor field. It corresponds to the phase current that would be needed to excite the total air gap field at load. This total field induces voltages in the stator and the rotor winding via self as well as mutual induction. The total effect is called **internal voltage** $\underline{U}_h$.

$$\boxed{\underline{U}_h = j\omega_s L_h \cdot \underline{I}_m} \tag{5.28b}$$

At the rotor side, the rotor frequency is effective, hence the **internal voltage on the rotor side** U_{hr} is given by (5.28c).

$$\boxed{\underline{U}_{hr} = j \cdot s \cdot \omega_s L_h \cdot \underline{I}_m} \tag{5.28c}$$

If I_m is assumed constant independently of the slip (rough approximation!), U_{hr} decreases linear with decreasing slip (Fig. 5.4). In practice, I_m decreases with increasing slip. Main and leakage fields can only be separated under strong simplifications. Strictly speaking, magnetising and leakage inductances cannot be distinguished experimentally. Therefore, the measurable parameters **"stator phase inductance"** L_s and **"rotor phase inductance"** L'_r, respectively **stator** and **rotor reactance** are used, because they can be measured.

$$L_s = L_h + L_{s\sigma} \qquad X_s = X_h + X_{s\sigma} \tag{5.29}$$
$$L'_r = L_h + L'_{r\sigma} \qquad X'_r = X_h + X'_{r\sigma} \tag{5.30}$$

According to *BLONDEL*, the leakage flux is described by the **leakage coefficient** σ, which can be measured directly (5.41).

$$\boxed{\sigma = 1 - \frac{L_h^2}{L_s L'_r} = 1 - \frac{X_h^2}{X_s X'_r}} \tag{5.31}$$

The **rotor current** is derived from the rotor voltage equation (5.27) .

$$\boxed{\underline{I}'_r = -\underline{I}_s \frac{jX_h}{\frac{R'_r}{s} + jX'_r}} \tag{5.32}$$

Combining (5.26) and (5.27), the **stator current** is obtained as a function of the line-supplied stator phase voltage and the slip:

$$\boxed{\underline{I}_s = \underline{U}_s \frac{R'_r + jsX'_r}{(R_s R'_r - s \cdot \sigma \cdot X_s X'_r) + j(s \cdot R_s X'_r + X_s R'_r)}} \tag{5.33}$$

For $s = 0$, the rotor current is zero and the stator current (**no-load current**) is only limited by the stator phase impedance, as can be derived from (5.32) to (5.34). The current changes linear with the voltage, because the T-equivalent circuit (where iron losses are neglected) has constant parameters.

$$\underline{I}_s(s = 0) = \frac{\underline{U}_s}{R_s + jX_s} \tag{5.34}$$

b) Rated Parameters and Per-Unit Values:
In order to estimate magnitudes independently of the parameters of a machine, the **"per-unit"-system** has been introduced. As every machine is designed for a certain **rated voltage** U_N and **rated current** I_N, (these data are given on the nameplate), these data are very well suited as reference values. The rated voltage on the nameplate is the **line-to-line value** (e.g. 400 V between terminals U-V etc.). At *delta-connection*, it equals the rated phase voltage, at *Star-connection*, the rated phase voltage is by $1/\sqrt{3}$ smaller than the rated voltage.

Example 5.2-1:
Rated voltage U_N = 400 V, rated current I_N = 100 A
a) Star-connection:
Rated phase voltage $U_{str,N} = U_N / \sqrt{3} = \underline{\underline{231}}$ V, rated phase current $I_{str,N} = I_N = \underline{\underline{100}}$ A
b) Delta-connection:
Rated phase voltage $U_{str,N} = U_N = \underline{\underline{400}}$ V, Rated phase current $I_{str,N} = I_N = 100/\sqrt{3} = \underline{\underline{58}}$ A
The rated apparent power is the same in both cases:
a) $S_N = 3U_{str,N} I_{str,N} = 3 \cdot 231 \cdot 100 = 69.3\,\mathrm{kVA}$
b) $S_N = 3U_{str,N} I_{str,N} = 3 \cdot 400 \cdot 58 = 69.3\,\mathrm{kVA}$

In the following, phase values are always used for the equivalent circuit of Fig. 5.5, therefore, the index *"ph"* is suppressed. These values (rated values U_N, I_N) are taken as **reference** for **"per-unit"**-values. Using per-unit values for current and voltage (5.35), every impedance Z can also be expressed as a "per-unit" impedance z:

$$u = U / U_N\,,\; i = I / I_N \tag{5.35}$$

$$Z = \frac{U}{I} = \frac{U/U_N}{I/I_N} \cdot \frac{U_N}{I_N} = \frac{u}{i} \cdot Z_N \quad \text{or} \quad z = \frac{Z}{Z_N} = \frac{u}{i} \tag{5.36}$$

The reference impedance (**rated impedance**) of an electric three-phase machine is given by the ratio of rated voltage and current.

Leakage coefficient σ:
Modern induction machines for a high breakdown torque are designed for the leakage to be as small as possible, because leakage fields are "useless" as they do not contribute to the energy conversion. The leakage coefficient σ values generally 0.08 to 0.1.

Phase resistances:
The phase resistances are also small. The values of $r_s = R_s / Z_N$, $r'_r = R'_r / Z_N$ are only some percent of the rated impedance to avoid too large copper losses.

Magnetising inductance:
The magnetising inductance shall be large, because it couples the stator and the rotor, thereby determining the energy conversion. It has been shown in Section 4.4 that the reciprocal value of the air gap is an important parameter for the magnitude of L_h. Hence, the air gap is aimed to be as small as possible. The minimum thickness for mechanic stability (touching of the rotor due to eccentricities, bearing play, rotor deflection, ...) is about 0.28 mm at small motors (typically 500 W rated power), it increases with increasing rated power and shaft height. The magnetising inductance is about 2.5 to 3 times the rated impedance: $X_h / Z_N = 2.5 \ldots 3.0$.

Leakage inductances:
The value of the sum of the leakage reactances is derived from (5.37).

$$\sigma = 1 - \frac{1}{\left(1 + \frac{X_{s\sigma}}{X_h}\right)\left(1 + \frac{X'_{r\sigma}}{X_h}\right)} \approx 1 - \left(1 - \frac{X_{s\sigma}}{X_h}\right)\left(1 - \frac{X'_{r\sigma}}{X_h}\right) \approx \frac{X_{s\sigma} + X'_{r\sigma}}{X_h} \tag{5.37}$$

$$X_{s\sigma} + X'_{r\sigma} \approx \sigma X_h \approx \sigma X_s \tag{5.38}$$

This sum is – with respect to the rated impedance – about (0.08...0.1)·(2.5...3) = 0.2...0.3. Decomposition into separate values for stator and rotor is – as mentioned above – problematical, but is often approximated by a "half-half" separation:

$$X_{s\sigma} / X_h \approx X'_{r\sigma} / X_h \cong 0.10 \ldots 0.15$$

c) Current Consumption of Induction Machines:
No-load:
The no-load speed equals the synchronous speed. Operation with rated voltage: u = 1: R_s is negligible when compared with X_s:

$$\underline{I}_s(s=0) = \frac{\underline{U}_s}{R_s + jX_s} \approx -j\frac{\underline{U}_s}{X_s} \quad \Rightarrow \quad \frac{\underline{I}_s}{I_N}(s=0) \approx -j\frac{\underline{U}_s / U_N}{X_s / Z_N} = -j\frac{1}{x_s} \tag{5.39}$$

Example 5.2-2:
As it is $x_s = x_h + x_{s\sigma} \cong 3.0 + 0.15 = 3.15$, the **no-load current** values about 1/3 of the rated current. At 100 A rated current, the no-load current of the induction machine is about 33 A.

Short-circuit (stand-still):
If the rotor is blocked, the slip becomes 1. If the resistances are neglected compared to the reactances in (5.33), the current consumption of the motors at $s = 1$ is given by (5.40) (**"starting current", "locked rotor current"**):

$$\underline{I}_s(s=1) \approx -j\underline{U}_s \frac{1}{\sigma \cdot X_s} \quad \Rightarrow \quad \frac{\underline{I}_s}{I_N}(s=1) \approx -j\frac{\underline{U}_s / U_N}{\sigma X_s / Z_N} = -j\frac{1}{\sigma \cdot x_s} \tag{5.40}$$

Result:
*The stator current at short-circuit is by $1/\sigma$ larger than at no-load (about 10 to 12 times as large). Therefore, the point of operation s = 1 is called **short circuit point**, because the current has the magnitude of a short-circuit current.*

Example 5.2-3:
Short-circuit current: σ = 0.08, u = 1, x_s = 2.6: $i(s = 1) = 1/(2.6 \cdot 0.08) = \underline{4.8}$
The short-circuit current is 4.8 times the rated current. As the leakage decreases with increasing motor size, it is generally 5 to 7 times the rated current in the case of large motors (Fig. 5.8).

The instruction for measurement of *BLONDEL*'s leakage coefficient is derived from (5.39) and (5.40):

$$\boxed{\sigma \cong \frac{U_s / I_s(s=1)}{U_s / I_s(s=0)}} \qquad (5.41)$$

Flux compensation:
The rotor current converted to stator winding parameter at $s = 1$ is not much smaller than the stator current, but of **opposite sign**:

$$\underline{I}'_r(s=1) = -\underline{I}_s \frac{jX_h}{R'_r + jX'_r} \approx -\underline{I}_s \frac{X_h}{X'_r} \approx -\underline{I}_s \qquad (5.42)$$

Hence, the rotating fields of stator and rotor are spatially oriented in opposite direction and almost cancel each other (**"Flux compensation"**, similar as with transformers). The magnetising current $\underline{I}_m = \underline{I}_s + \underline{I}'_r \approx 0$ is very small and may be neglected.

Rated operating point:
At rated slip s_N, the currents in the stator and rotor windings shall generate rated torque. Generally, the rated slip is only some percent. Hence, the rated motor speed is only slightly smaller than the synchronous speed. At this operating point, the rotor and stator current have nearly opposite signs. Fig. 5.6 shows a typical current phasor-diagram at rated operation, as well as the corresponding fundamental flux distribution.

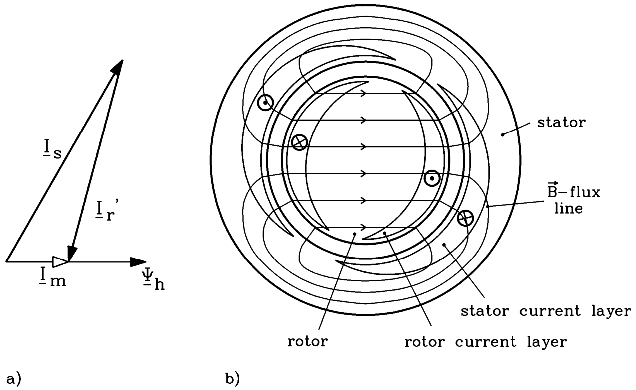


Fig. 5.6: Rated operating point of an induction machine:
a) Time phasor-diagram of stator, rotor and magnetising current as well as magnetising flux linkage
b) Cross sectional area of an induction machine: The fundamental of the electric loading of stator and rotor are drawn with a spatial displacement according to the time phase shift of the phasors of a). The resultant field in the air gap is represented by the space vector $\underline{\Psi}_h$. The **"flux compensation"** is clearly to see. Each electric loading for itself would generate a significantly stronger field (iron saturation neglected) along their spatial orientation (see Fig. 5.2).

Example 5.2-4:
Four pole induction machine at 50 Hz-line: synchronous speed $n_{syn} = 1500$/min,
rated speed $n_N = 1450$/min, hence $s_N = 0.033 = 3.3\%$.

d) Phasor Diagram of an Induction Machine:
The time phasor-diagram per phase is derived from the T-equivalent circuit. In Fig. 5.7, it is shown for the rated operating point. The stator current always **lags** behind the stator voltage.

Hence, the induction machine is an **inductive consumer**. At every working point, needs inductive reactive power from the grid to excite the magnetic field in the air gap. The voltage drop at the "fictitious" resistance R'_r / s is composed by the "real" voltage drop $R'_r \underline{I}'_r$ and the value $R'_r \underline{I}'_r (1/s - 1)$ which describes the electromechanical power conversion.

e) Torque Speed Characteristic:
The electromagnetic torque produced by the machine can be determined via the *LORENTZ* force (5.1) or – which is much easier – from the balance of power. The power that is transferred from the stator via the air gap to the rotor without electric contact is given by the consumed line power P_{in} minus the copper losses in the stator winding, as can be derived from the T-equivalent circuit (iron losses neglected). This power is named **"air gap power"** or "**rotor power input"** P_δ. All m_s stator phases have to be considered for the balance!

$$P_{in} = m_s U_s I_s \cos\varphi = m_s \operatorname{Re}\{\underline{U}_s \underline{I}_s^*\} \qquad (5.43)$$

where $\underline{I}_s^* = (I_{s,Re} + jI_{s,Im})^* = I_{s,Re} - jI_{s,Im} = I_s \cos\varphi - jI_s \sin\varphi$.
The resultant rotating field rotates, as we know, with synchronous speed $\Omega_{syn} = 2\pi n_{syn}$. The rotor input torque M_e transferred to the rotor by the rotating field is given by the correlation "power = torque x angular speed" and (5.44).

$$P_\delta = P_{in} - m_s R_s I_s^2 \qquad (5.44)$$

$$\boxed{M_e = \frac{P_\delta}{\Omega_{syn}} = p\frac{P_\delta}{\omega_s}} \qquad (5.45)$$

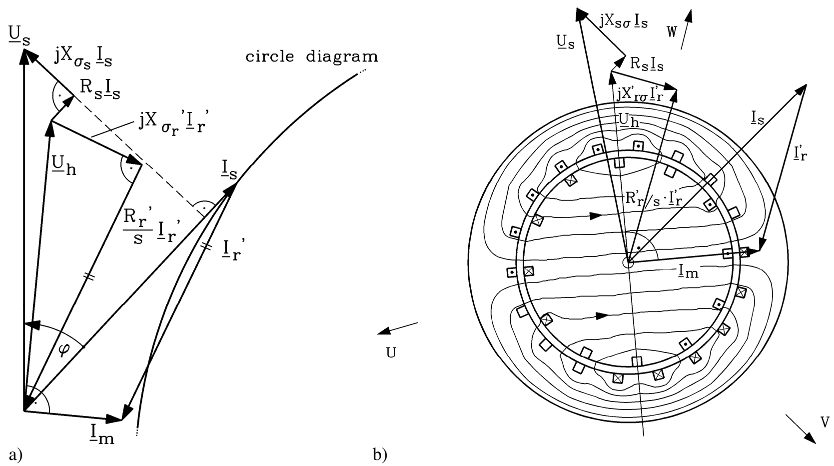


Fig. 5.7: Typical phasor diagram per phase of an induction machine: rated operating point, rated slip.
a) At constant voltage, the peak of the stator current moves along a circle if the slip (the load) changes (circle diagram, *OSSANNA*'s circle).
b) Numerically calculated flux distribution for $2p = 2$, $q_s = 3$, $q_r = 2$ and corresponding phasor diagram.

Combining (5.33) with (5.43) and (5.44), the air gap power is obtained as a function of U_s and s. Hence, using (5.45), the **torque** is (please verify by yourself!).

$$M_e = m_s \frac{p}{\omega_s} U_s^2 \frac{s(1-\sigma) X_s X_r' R_r'}{(R_s R_r' - s\sigma X_s X_r')^2 + (sR_s X_r' + X_s R_r')^2} \qquad (5.46)$$

Result:
- The torque changes with the square of the voltage.
- At no-load (s = 0), the torque is zero.
- At infinitely high positive or negative slip, the torque is also zero.
- Therefore, the torque has an extremum between s = 0 and +∞ as well as 0 and -∞. These extrema are called ***motor*** *and* ***generator breakdown torque*** *M_b.*
If the load torque surpasses the breakdown torque, the machine is slowed down to zero speed. We say, the machine ***"breaks down"*** *and* ***"stalls"****.*

Example 5.2-5:
A perturbation in the grid causes a 10 % voltage drop. How does the torque change?
As it is $M_e \sim U_s^2$, the torque decreases by about 20 %.

Example 5.2-6:
Torque speed characteristic and **stator current speed characteristic**:
As it is $n = (1-s) \cdot f_s / p$, M_e and I_s can be drawn as functions of s or n according to (5.33) and (5.46) (Fig. 5.8).
Parameters of the induction machine:
R_s/X_s =1/100, R_r/X_r = 1.3/100, σ = 0.067, $X_s = X'_r = 3Z_N$, $Z_N = U_N/I_N$ (phase values)

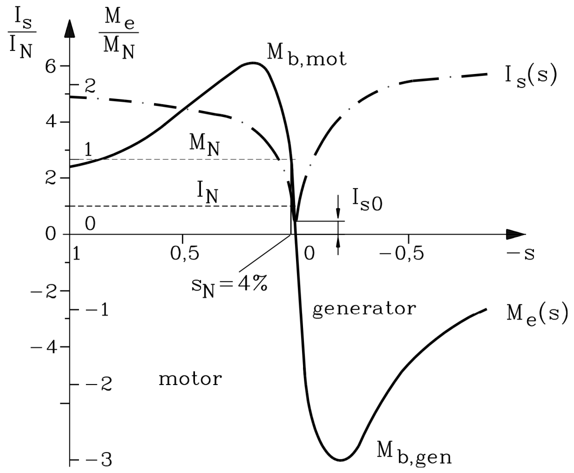


Fig. 5.8: Torque M_e and stator current I_s as functions of the slip s (R_s/X_s =1/100, R_r/X_r = 1.3/100, σ = 0.067, $X_s = X'_r = 3Z_N$, $Z_N = U_{N,ph}/I_{N,ph}$)

f) Motor and Generator Operation of Induction Machines:
If the rotor rotates slower that the rotating field, the torque is positive. The rotor is pulled by the torque ($s > 0$, **motor operation**).

The rotor can only rotate faster than the rotating field if it is externally driven. Then, the slip is negative: $n > n_{syn}$ $(s < 0)$. The electromagnetic torque becomes negative and brakes. The machine becomes a **generator** that needs reactive power from the grid to excite the air gap magnetic field but supplies electric power to the grid which it consumes in the form of mechanical energy at the shaft e.g. via a wind turbine.

At **isolated operation** without a supplying grid, the induction machine can only work as a generator if a voltage source supplies inductive reactive power. This can be realised by a capacitor bank connected in parallel to the terminals of the machine. It consumes capacitive reactive power, hence, it supplies inductive reactive power; therefore, the reactive power of the system capacitor bank and induction machine is balanced.

g) Braking by reversal of the $M_e(s)$-characteristic:
At operation with $s > 1$, the rotor is driven against the direction of the rotating field: $n < 0$. Yet, the electromagnetic torque is positive. So, the mechanic power at the shaft is negative.

$$P_m(s > 1) = 2\pi \cdot n \cdot M < 0$$

At the same time, mechanic power is supplied to the shaft and electric power is supplied to the motor terminals. The total power is dissipated as generated heat in the windings by copper losses, in the laminated stator and rotor core by iron losses and as frictional losses in the bearings. The machine works as a **brake**.

h) Breakdown Torque:
A maximum of the electromagnetic torque, the breakdown torque, exists at generator and at motor operation. Using

$$\frac{dM_e}{ds} = 0 \qquad (5.47)$$

and (5.46), the **breakdown slip** s_b at motor operation ($s_{b,mot} = s_b > 0$) and generator operation ($s_{b,gen} = -s_b < 0$) is obtained. The **absolute value** of the breakdown slip is the same for generator and for motor operation.

$$s_b = \frac{R_r'}{X_r'} \cdot \sqrt{\frac{R_s^2 + X_s^2}{R_s^2 + \sigma^2 X_s^2}} \approx \frac{R_r'}{\sigma X_r'} \quad , \quad s_{b,mot} = s_b \quad , \quad s_{b,gen} = -s_b \qquad (5.48a)$$

$$R_s = 0: \quad s_b = \frac{R_r'}{\sigma X_r'} \qquad (5.48b)$$

As the stator resistance R_s is much smaller than the stator reactance X_s at 50 Hz / 60 Hz operation, the simplification (5.48b) is acceptable. A large rotor resistance and / or a small leakage coefficient increase the breakdown slip.

Using (5.48a), (5.46) gives the **motor and generator breakdown torque** (5.49), where the plus and the minus are valid for the motor and the generator breakdown torque respectively.

$$M_{b,\mathrm{mot/gen}} = \pm \frac{m_s}{2} \frac{p}{\omega_s^2} U_s^2 \frac{1}{\pm \frac{R_s}{\omega_s} + \frac{1}{(1-\sigma)\omega_s X_s} \cdot \sqrt{(R_s^2 + X_s^2)(R_s^2 + \sigma^2 X_s^2)}} \qquad (5.49)$$

At generator operation, the denominator of the ratio in (5.49) is smaller than at motor operation. Therefore, the **generator breakdown torque is larger than the motor breakdown torque** (Fig. 5.8).

Descriptive explanation:
At motor operation, a part of the consumed electric power is dissipated as copper losses prior to the conversion into mechanical energy in the air gap. This dissipated energy does not contribute to the torque generation, thus torque is smaller. At generator operation, all losses – including the stator copper losses – have to be supplied by the consumed mechanical power, respectively by the air gap torque, hence the torque is bigger.

If the stator copper losses are neglected, generator and motor breakdown torque have the same absolute value.

$$\boxed{R_s = 0: \quad M_b = \frac{m_s}{2}\frac{p}{\omega_s}U_s^2\frac{1-\sigma}{\sigma X_s}} \quad , \quad M_{b,mot} = M_b \quad , \quad M_{b,gen} = -M_b \qquad (5.50)$$

i) KLOSS' Formula:

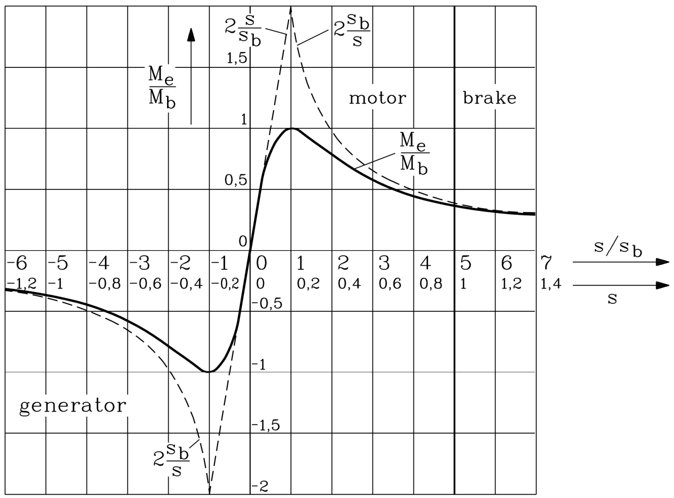


Fig. 5.9: Electromagnetic torque of an induction machine depending on the slip, stator resistance $R_s = 0$ (*KLOSS*' formula), at the example of breakdown slip $s_b = 0.2$.

If the stator resistance is neglected ($R_s = 0$) (5.46) simplifies significantly:

$$M_e \approx m_s \frac{p}{\omega_s}U_s^2\frac{1-\sigma}{X_s}\frac{sR_r'X_r'}{R_r'^2 + (s\sigma X_r')^2} \qquad (5.51)$$

Using (5.48b) and (5.50), (5.51) becomes very clear (***KLOSS*' formula**):

$$\boxed{R_s = 0: \quad \frac{M_e}{M_b} = \frac{2}{\frac{s_b}{s} + \frac{s}{s_b}}} \qquad (5.52)$$

Result:
- *At small slip, the torque increases linear* $(M_e / M_b \approx 2s / s_b)$.
- *At large slip, the torque decreases hyperbolically* $(M_e / M_b \approx 2s_b / s)$.

j) Why do Induction Machines Show a Breakdown Torque?
At **no-load**, only magnetising current is consumed. The voltage induced by the current balances the line-voltage, and the current amplitude is only determined by this correlation. The rotor is at zero-current and the magnetic field cannot generate any torque.
At **load**, the slip increases. Voltage is induced in the rotor, but only with very small rotor frequency f_r. Therefore, the rotor resistance R_r dominates when compared with the rotor reactance $\omega_r L_r = s\omega_s L_r$. The rotor circuit is mainly *OHM*ic. The rotor current I_r is almost in phase with the induced voltage and generates torque together with the magnetic field.
If the slip increases, the induced rotor voltage, the rotor current and – at first – the torque increase also. However, with increasing rotor frequency, the rotor reactance X_r increases also and the **phase shift between rotor current and voltage increases.** The rotor current increases with increasing slip, mainly due to its imaginary part, whereas the real part decreases. As only the real part of the rotor current generates torque with the stator field, the torque **decreases** while the total current increases.
At **very large slip**, the rotor resistance can be neglected when compared with the rotor reactance. The rotor current is a pure reactive current. No active power is transferred to the rotor. The torque decreases towards zero torque.

5.3 Asynchronous Energy Conversion

a) Power flow in Induction Machines
As currents in the rotor windings are needed to generate a torque, the corresponding rotor copper losses cannot be avoided.

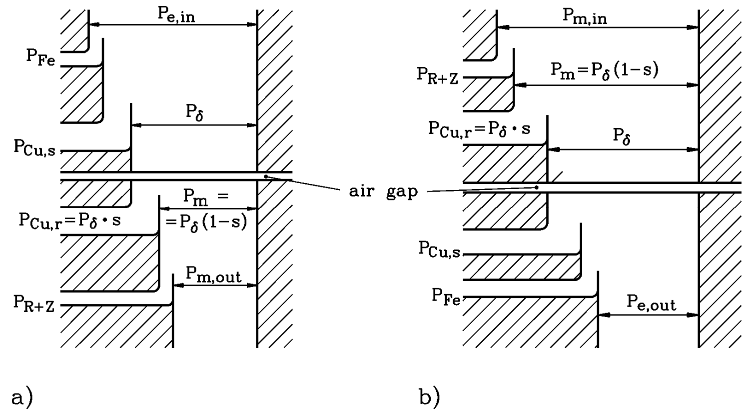


Fig. 5.10: Power flow in an induction machine a) motor operation, b) generator operation

Power flow at motor operation:
The **air gap power** P_δ equals the **consumed electric power** $P_{e,in}$ minus the **stator copper losses** in the **stator winding** $P_{Cu,s}$ and the **iron losses** (eddy current and hysteresis losses) in

the stator lamination P_{Fe} that are caused by the pulsation of the magnetic field with stator frequency in the stator iron core. Additional losses in the stator winding and in the laminated core due to field harmonics are neglected here.

$$P_\delta = P_{e,in} - P_{Cu,s} - P_{Fe} \tag{5.53}$$

The air gap power is given by the electromagnetic torque in the air gap and the speed of the air gap field Ω_{syn}.

$$\boxed{M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{P_\delta}{\Omega_m/(1-s)}} \tag{5.54}$$

The electromagnetic torque is directly linked with the **copper losses** in the **rotor winding** $P_{Cu,r}$ via the slip.

$$P_\delta = \frac{M_e \Omega_m}{1-s} = \frac{P_m}{1-s} = P_m + P_m \frac{s}{1-s} = P_m + P_{Cu,r} \tag{5.55}$$

$$\boxed{P_m = M_e \Omega_m = (1-s) P_\delta} \tag{5.56}$$

$$\boxed{P_{Cu,r} = P_\delta - P_m = s P_\delta} \tag{5.57}$$

At rated slip operation, the iron losses in the rotor lamination can be neglected, because of the low frequency of the magnetic field in the rotor. The **mechanic power** P_m is reduced by the **frictional and windage losses** P_R and **additional losses** P_Z that are caused by field harmonics in the rotor. The output power at the shaft is only $P_{m,out}$ (Fig. 5.10a). Hence, the **shaft torque (coupling torque)** $\boldsymbol{M_s}$ is by the **rotor loss torque** $\boldsymbol{M_d}$ smaller than the electromagnetic torque M_e in the air gap.

$$\boxed{P_{m,out} = P_m - P_R - P_Z} \tag{5.58}$$

$$M_s = \frac{P_{m,out}}{\Omega_m} \quad < \quad M_e = \frac{P_m}{\Omega_m} \quad \Leftrightarrow \quad M_d = M_e - M_s \tag{5.59}$$

<u>Power flow at generator operation:</u>
At generator operation, the power flow of Fig. 5.10a reverses to Fig. 5.10b. The **efficiency** of the induction machine is given by (5.60).

$$\text{Motor operation: } \boxed{\eta = \frac{P_{m,out}}{P_{e,in}}}, \quad \text{generator operation: } \boxed{\eta = \frac{P_{e,out}}{P_{m,in}}}. \tag{5.60}$$

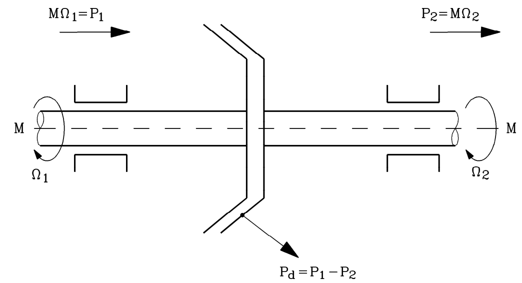
b) Consideration of a Slip Clutch as Mechanical Analogy to an Induction Machine:
The mechanical analogy to an induction machine is the **slip clutch** (Fig. 5.11). The driving torque M at the propelling shaft 1 equals the torque at the propelled shaft 2. Torque transmission is only possible, if the friction disc 2 slips with respect to the friction disc 1. Then, the speed Ω_2 is by the slip s smaller than the speed of the propelling shaft Ω_1 and it is

$\Omega_2 = (1 - s)\Omega_1$. The delivered power $P_2 = M\Omega_2$ is by the slip losses $P_d = s\Omega_1 M$ smaller than the supplied power $P_1 = M\Omega_1$.

Induction machine	Slip clutch
Ω_{syn} , Ω_m	Ω_1 , Ω_2
P_δ , $P_{Cu,r}$, P_m	P_1 , P_d , P_2
M_e	M

<u>Table 5.1:</u> Correspondences between induction machine and slip clutch

Another mechanical analogy: force transmission between wheel and rail at railways
The force transmission from the wheels to the rail at railway locomotives also results from slipping. The driving wheels of a locomotive have to slip to transmit the driving torque to the rail, while the wheels of the non driven axes, e.g. of the pulled wagons, are rolling slipless ("mere" rolling).



<u>Fig. 5.11:</u> A slip clutch is a mechanical analogy for an induction machine

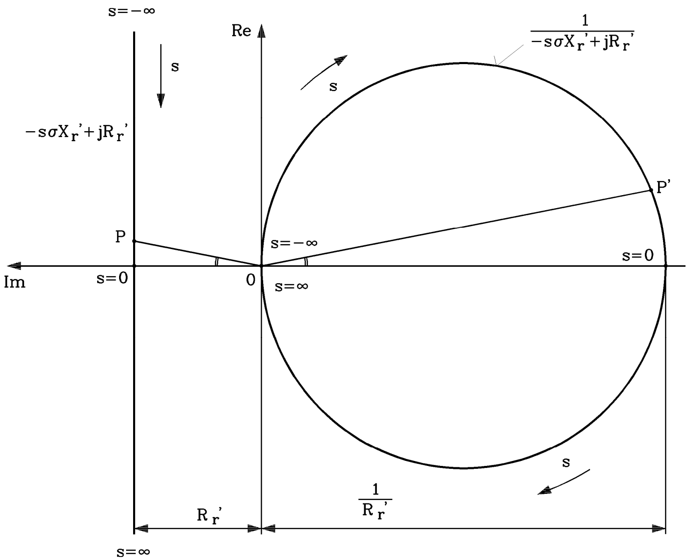
5.4 Circle Diagram of an Induction Machine

a) The Locus Diagram of the Stator Current as Function of the Slip is a Circle:
The phasor diagram shown in Fig. 5.7 is given for a certain **value** of the slip. If the induction machine is line-operated, the slip changes with the load, whereas the amplitude and the phase of the feeding voltage remain constant. The amplitude and the phase of the stator current phasor change with changing load (changing slip). If the other parameters of the machine remain constant, the locus diagram of the stator current phasor describes a circle as the slip changes (***OSSANNA*'s circle**).

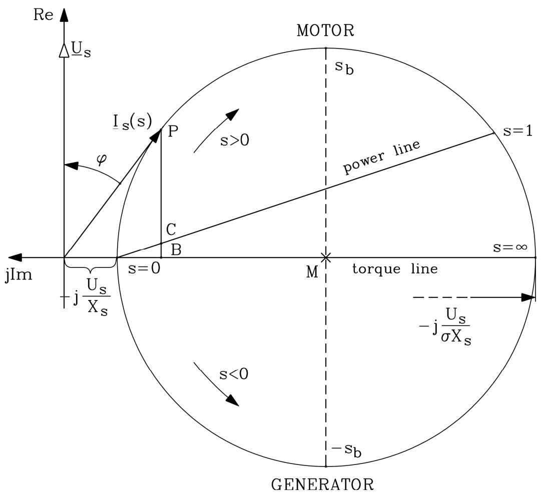
b) HEYLAND's Circle
If the stator resistance is neglected ($\boldsymbol{R_s = 0}$), a simplified circle diagram is obtained (***HEY-LAND*'s circle**). If the stator voltage phasor is put in the real axis, it is $\underline{U}_s = U_s$, and (5.61) is derived from (5.33).

$$\underline{I}_s = U_s \frac{1}{X_s} \cdot \frac{R_r' + jsX_r'}{-s\cdot\sigma\cdot X_r' + jR_r'} = \frac{U_s}{X_s} \cdot \left(\frac{(1-1/\sigma)R_r'}{-s\cdot\sigma\cdot X_r' + jR_r'} - j\frac{1}{\sigma} \right) \tag{5.61}$$

The expression $\underline{G}(s) = -s\sigma X_r' + jR_r'$ describes a straight line in the complex plane that is parallel to the real axis (Fig. 5.12a).



a)



b)

Fig. 5.12: Locus diagram of the stator current of an induction machine at R_s = 0: a) derivation, b) *HEYLAND*'s circle

The reciprocal value (inversion) of $\underline{G}(s)$ is a circle $\underline{K}(s)$ (see Appendix A1). By the inversion, each point P of the straight line becomes a point P' of a circle, because of (5.62).

$$\underline{K}(s)=\frac{1}{\underline{G}(s)}=\frac{1}{Z(s)\cdot e^{j\varphi(s)}}=\frac{e^{-j\varphi(s)}}{Z(s)} \qquad (5.62)$$

Thereby, the distance $\overline{0P}=Z(s)$ equals $\overline{0P'}=1/Z(s)$ and the centre of the circle is on the imaginary axis. Then, the circle is mirrored at the negative imaginary axis by multiplication with the negative real number $(1-1/\sigma)\cdot R_r'$ and displaced from the point of origin to the right along the imaginary axis by adding $-j/\sigma$. The multiplication with U_s/X_s does only change the size, but not the location of the circle (Fig. 5.12b).

The points

$$\underline{I}_s(s=0)=-j\frac{U_s}{X_s} \text{ (no-load current)} \qquad (5.63)$$

$$\underline{I}_s(s=\infty)=-j\frac{U_s}{\sigma X_s} \text{ ("ideal" short-circuit current)} \qquad (5.64)$$

lie also on the negative imaginary axis (points P_0 and P_∞). The centre M is at half distance in-between. The electric real power at the **motor operating point *P*** at slip s is

$$P_{e,in}=m_s U_s I_{s,w} \quad , \qquad (5.65)$$

where

$$I_{s,w}=I_s\cos\varphi=\overline{PB}=\overline{PC}+\overline{CB} \qquad (5.66)$$

is the **real part of the stator current**. Because of $R_s = 0$, only the copper losses in the rotor winding remain to be considered. For the power balance, it is

$$P_{e,in}=P_m+P_{Cu,r}=m_s U_s(\overline{PC}+\overline{CB}). \qquad (5.67)$$

At s = 1, it is n = 0; and at s = 0, it is M_e = 0. Therefore, P_m is zero at these two working points. Accordingly, the straight line $\overline{P_0P_1}$ is the **"power line"** that separates the real current into the two components $\overline{PC}$ and $\overline{CB}$. The section $\overline{PC}$ above the power line is proportional to the mechanical power. The electric real power equals the air gap power P_δ, as the stator losses are neglected.

$$P_{e,in}=P_\delta=M_e\Omega_{syn}=m_s U_s\overline{PB} \qquad (5.68)$$

At the points of the circle P_0 and P_∞, the torque M_e is zero. Therefore, the distance $\overline{P_0P_\infty}$ is called **"torque line"**. It is lying on the negative imaginary axis and the distance $\overline{PB}$ is measured from this straight line to the operating point P on the circle.

Operating points on the **lower half of the circle** are generator operating points, because the real power is negative ($\cos\varphi<0$). The breakdown points (maximum torque) $s_{b,mot}$ and $s_{b,gen}$ are given by the maximum length of the distance $\overline{PB}$. Generator and motor breakdown torque have the same absolute value. This corresponds to *KLOSS*' formula. The current **always lags the voltage**; the induction machine always – independently of generator or motor operation – operates at inductive power.

c) OSSANNA's Circle

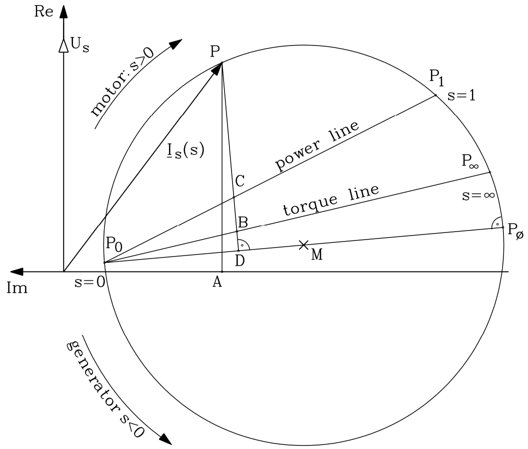


Fig. 5.13: Circle diagram of an induction machine at $R_s > 0$ (*OSSANNA*'s circle)

OSSANNA's circle that corresponds to (5.33) (Fig. 5.13) is the locus diagram of the stator current for $R_s > 0$. It has the following important differences when compared with *HEYLAND*'s circle:

- The centre M is somewhat **above** the negative imaginary axis.
- The distance $\overline{P_0P_\infty}$ is **no longer** the diameter of the circle. The point P_∞ is placed above the negative imaginary axis.
- Therefore, the torque line lies above the imaginary axis, as well as the circle diameter that passes the points $s = 0$, M and the **"diameter-point" $P_\varnothing$**.

- The electric real power at the motor terminals is proportional to the motor real current (distance $\overline{PA}$).

$$P_e = m_s U_s \overline{PA} = m_s U_s I_s \cos\varphi \tag{5.69}$$

- The losses are determined as follows: The line section perpendicular on the circle diameter (that passes $s = 0$) between the chosen **operating point** P and the root point D is determined. The intersections of the power and torque lines are called – as in the case of *HEYLAND*'s circle – points C and B.

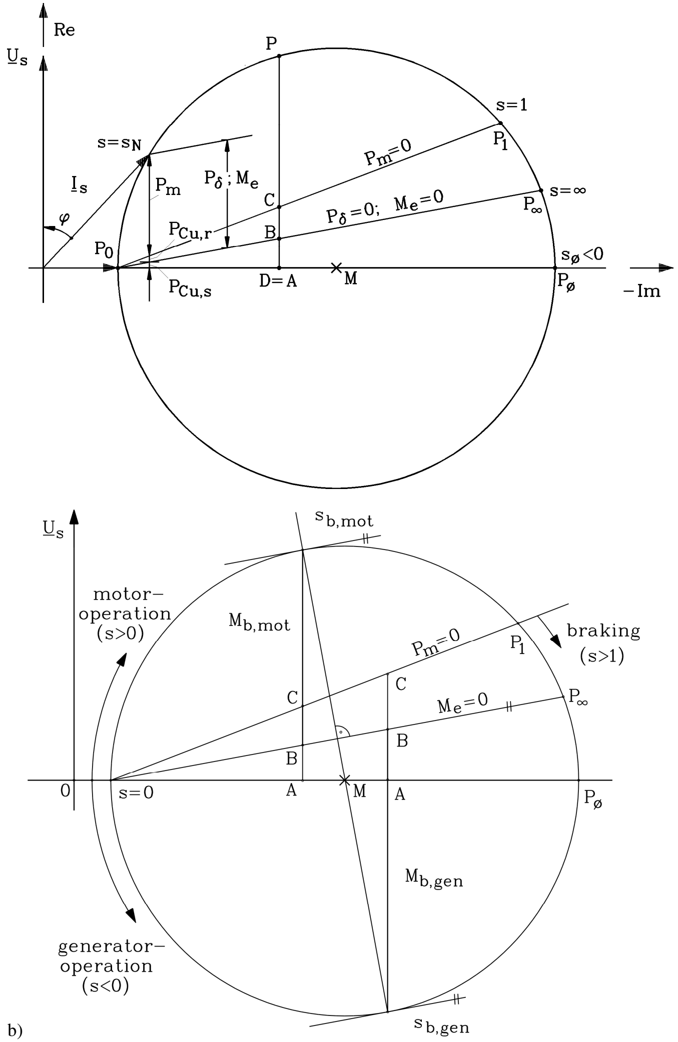
$$P_\delta = m_s U_s \overline{PB} \quad \Rightarrow \quad M_e = \frac{P_\delta}{\Omega_{syn}} \tag{5.70}$$

$$P_m = m_s U_s \overline{PC} \tag{5.71}$$

$$P_{Cu,r} = m_s U_s \overline{BC} = P_\delta - P_m \tag{5.72}$$

$$P_{Cu,s} = P_e - P_\delta \tag{5.73}$$

The copper losses in the stator winding cannot be seen directly in the form of a distance, but are given by the difference of the two distances $\overline{PA}$ and $\overline{PB}$ according to (5.73).



a) b)

Fig. 5.14: Simplified *OSSANNA*'s circle: The centre M of the circle is put on the negative imaginary axis as in the case of *HEYLAND*'s circle, but $R_s > 0$ is maintained. Therefore, the points $P_\varnothing$ and P_∞ are different. a) power spreading, b) motor and generator breakdown torque

d) Simplified OSSANNA's Circle: $R_s > 0$, but centre M on the negative imaginary axis:
Except in the case of machines with small rated power (typically < 1 kW), the influence of the *OHM*ic stator resistance is generally so small that the centre M of *OSSANNA*'s circle is almost

on the imaginary axis. Therefore, the circle diagram is often drawn with the centre M on the negative imaginary axis, but without turning the torque line into the negative imaginary axis. Thereby, ***OSSANNA***'s **simplified circle** (Fig. 5.14) is obtained, from which – as in the case of the exact circle diagram – the working characteristic of the machine is derived.

e) Straight Line of the Slip

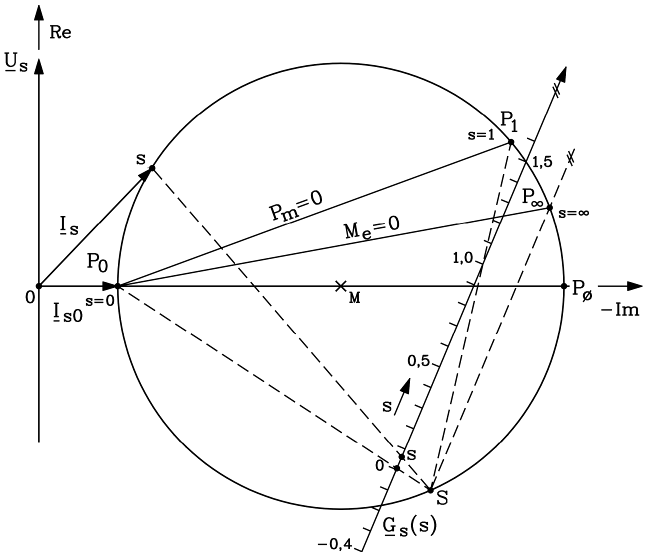


Fig. 5.15: Scaling the circle diagram with the values of the slip s via the straight line of the slip $\underline{G}_s(s)$: this straight line is determined by three known operating points (here: P_0, P_1 and P_∞) and a centre of inversion S that is chosen arbitrarily on the circle. It is scaled linear in s (here: $-0.4 \le s \le 1.5$). The point of intersection with the circle diagram of the connecting lines between S and the chosen point of value s on the slip line give the corresponding locus of the slip s on the circle diagram.

The circle diagram has been derived by an inversion of a complex straight line $\underline{G}(s)$ that was linear scaled with the slip s. Therefore, the denotations of the operating points P on the circle and the corresponding values of s can be determined in the same way. Thereby, any straight line $\underline{G}_s(s)$ (**"slip line"**) that is scaled linear with s and that has its centre of inversion S on the circle can be used, not only $\underline{G}(s)$. Then, the straight line is transformed into the circle via inversion.

Mostly, the stator current data "r.m.s. value" and "phase" with reference to the voltage of **three working points** are known from measurements, e.g. no-load point P_0, rated operating point P_N and short-circuit point P_1. In Fig. 5.15 it is assumed that we know P_0, P_1 and P_∞. The centre of inversion S on the circle is chosen arbitrarily and is connected with the points P_0, P_1 and P_∞ (dotted lines in Fig. 5.15). As the image point of P_∞ on the line $\underline{G}_s(s)$ is at infinite distance ($s = \pm\infty$), $\underline{G}_s(s)$ must be – according to *EUKLID*'s postulate on parallels – in parallel to $\overline{SP_\infty}$. The point of intersection of $\underline{G}_s(s)$ with the line through $\overline{SP_\infty}$ is also at infinite distance. The points of intersection of $\overline{SP_0}$ and $\overline{SP_1}$ with the slip line $\underline{G}_s(s)$ give the points $s = 0$ and $s = 1$ on the slip line. Then, the slip line can be scaled by linear subdivision

e.g. from –0.4 to 1.5. The point of intersection on the circle, obtained by elongation of the connecting line from S to the individual chosen value of s on the slip line, gives the locus of the stator current phasor for an operation point at that slip s.

5.5 Start-up of Slipring Induction Machines using Starting Resistors

a) Increase of the Starting Torque:
The start-up of an induction machine causes the following problem: Fig. 5.8 shows that the starting current ($s = 1$) values about 5-times the rated current, but the starting-torque that is available to accelerate the drive values only about 90% of the rated torque. The reason for this is the large reactive current at high values of the slip s, where s is much larger that the break-down slip. Notably motors that are required to
- start against high **"break-off torques"** of the driven load or
- accelerate high inertias (**"heavy duty"**, e.g. centrifuges),

require an **increase of the starting torque**.

b) Reduction of the Starting-Current:
The high starting current has to be supplied by the grid and can lead to transient decrease of the terminal voltage due to the high voltage drop at the impedances of the supply. Furthermore, the high starting current wears the motor thermally (Note that 5-times rated current results in 25-times rated copper losses!).

c) Starting Resistances:
The starting torque can be increased up to maximum torque (breakdown torque) by use of external **starting resistors** that are connected to the rotor phases via sliprings. At the same time, these resistors decrease the current down to the current at breakdown torque.

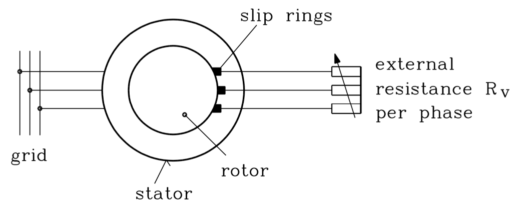


Fig. 5.16: Using external resistances via sliprings in the rotor circuit of an induction machine

Example 5.5-1:
Motor with $M(s)$-characteristic according to Fig. 5.8: The stator current at breakdown values only about $3.5I_N$, whereas it is about $5I_N$ at locked rotor.
The T-equivalent circuit shows that a change of the rotor resistance R_r via additional external resistances does not change the circuit diagram as long as the ratio "resistance/slip" remains unchanged.

$$\frac{R_r'}{s} = konst. \qquad (5.74)$$

If an external starting resistor R_v is connected to each rotor phase, (5.75) has to be met to keep the current unchanged.

$$\frac{R_r + R_v}{s} = \frac{R_r}{s^*} = const. \qquad (5.75)$$

Result:
With use of an external rotor resistance R_v per phase, the motor behaves at slip s as it behaves at the slip s^* without the use of R_v.

Therefore, if the motor shall start ($s = 1$) with the motor breakdown torque, R_v must be chosen according to (5.75), so that the slip s^* equals the breakdown slip s_b.

$$\frac{R_r + R_v}{1} = \frac{R_r}{s_b} \quad \Rightarrow \quad R_v = R_r\left(\frac{1}{s_b} - 1\right) \qquad (5.76)$$

Graphically, (5.75) corresponds to a **"shear"** (linear dilation) of the $M(n)$-, respectively the $M(s)$-characteristic. For each chosen value of s^*, the corresponding torque $M_e(s^*)$ occurs at a new value of s which is determined according to (5.75).

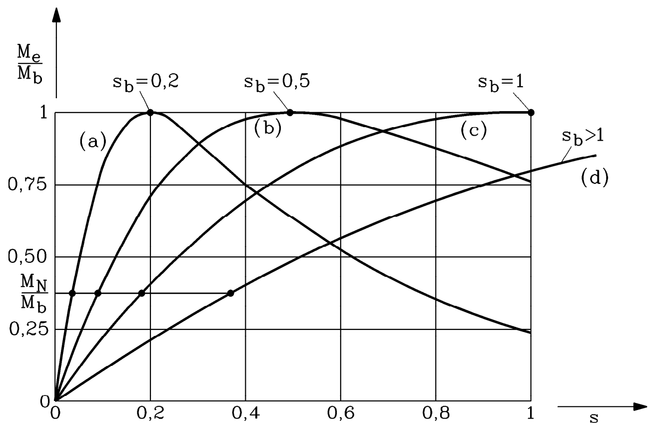


Fig. 5.17: Torque characteristic of a slipring-induction motor ($M_b/M_N = 2.65$) with additional rotor resistances. Without external resistances (a), the breakdown torque occurs at the slip 0.2, the starting torque is as small as $0.24M_b$. However, with use of an external resistance per phase $R_v = 4R_r$, we get "starting torque" = "breakdown torque" (c).

Example 5.5-2:
Slipring-induction motor with $M_b/M_N = 2.65$, breakdown slip 0.2 (Fig. 5.17):
- Without external resistances, the starting torque is only $0.65M_N$, resp. $0.24M_b$ (curve a).
- In the case of $R_v/R_r = 4$, the motor starts with the breakdown torque (curve c). Please verify this by yourself (mathematically as well as graphically) by means of the simplified $M(s)$-curve for $R_s = 0$ (*KLOSS*' formula).
- Can you estimate the ratios R_v/R_r for the other two $M(s)$-graphs (b) and (d)? (Solution: $R_v/R_r = 1.5$ (b) resp. 8 (d)). After starting with curve c), the torque decreases with decreasing slip. R_v has to be continually decreased (e.g. by using **water resistances for R_v**) if the electromagnetic torque shall remain the breakdown torque (e.g. in Fig. 5.17 at $s = 0.5$, $R_v/R_r = 1.5$ is required). The improved starting behaviour has to be paid for by copper losses in the external starting resistances. However, these occur **outside** of the machine. The machine itself is protected from overheating.

5.6 Variable Speed Operation of Slipring Induction Machines

The external rotor resistances allow variable speed of slipring induction machines. For example, the torque of an elevator is independent from the speed (Chapter 7.1): M_s = const. The points of intersection for $M_s = M_N$ = const. with the torque curves (a) – (d) of Fig. 5.17 give values for the slip between 4% and 37%, hence motor speed varies: $n/n_{syn} = 1 - s$ = 63%...96%.

Result:
Variation of external rotor resistances allows speed variable operation of slipring-induction machines.

Drawbacks:
- At no load ($M_s = 0$), the motor always accelerates up to synchronous speed.
- Due to the losses in the external rotor resistances, the efficiency of the motor is smaller at low motor speed.

Small motor speed means high slip and hence high slip losses sP_δ. However, if the driven torque M_s decreases strongly with decreasing motor speed, P_δ decreases also and sP_δ remains small enough. This is the case with fan and pump applications Therefore, slipring-induction machines are used e.g. for variable speed operation of large boiler feed pumps in thermal power plants. The flow rate $\dot{V}$ of pumps, turbines, fans or ventilators is proportional to the speed n, the generated pressure Δp is proportional to n^2 (**"*EULER's* turbine equation"**). Therefore, the demand of mechanical power P of a pump increases with n^3.

$$\dot{V} \sim n, \quad \Delta p \sim n^2 \quad \Rightarrow \quad P = \dot{V} \cdot \Delta p \sim n^3 \qquad (5.77)$$

The driving motor has to supply an electromagnetic torque that is – if the motor's loss torque M_d is neglected – proportional to n^2.

$$P_{m,out} = \Omega_m M_s = 2\pi n M_s \approx 2\pi n M_e \quad \Rightarrow \quad M_e = \frac{P}{2\pi n} \sim \frac{n^3}{n} = n^2 \qquad (5.78)$$

Example 5.6-1:
Speed variable slipring-induction motor with external rotor resistances R_v: Comparison (1) "elevator drive" and (2) "pump drive": Reduction of the speed down to 60% of n_{syn} for the run-in of the cabin into the station (1), resp. for the reduction of the flow rate (2) (Fig. 5.18).

Power balance with neglection of R_s and P_{Fe}:
$P_{e,in} \cong P_\delta = P_{Cu,r} + 3R_v I_r^2 + P_m \qquad P_m = 2\pi n M_e \qquad P_\delta = 2\pi n_{syn} M_e$

	Elevator drive	Pump drive
Load torque	$M_s = M_N = konst.$	$M_s = (n/n_{syn})^2 \cdot M_N$
Load torque at $n/n_{syn} = 0.6$	$M_s = M_N$	$M_s = 0.36 \cdot M_N$
$P_\delta(n)/P_{\delta N} = P_\delta(n)/(2\pi n_{syn} M_N)$	1	0.36
$P_m(n)/P_{\delta N}$	0.6	0.22
$(P_{Cu,r} + 3R_v I_r^2)/P_{\delta N}$	**0.4 (!)**	**0.14**

Table 5.2: Variable speed operation of a slipring-induction motor at elevator and pump loads

Result:
At constant load torque, a 40 % reduction of the motor speed requires rotor losses of 40% rated power. This is not a useful solution. With pump drives, the power loss of only 14% rated power is tolerable. Note that the power demand of the pump at 60% rated speed is only 22% rated power.

Modern large variable speed drives with slipring induction machines do not use external rotor resistances but an external voltage with rotor frequency, that can be varied by a converter. Here, no copper losses in external rotor resistances occur (see Chapter 7).

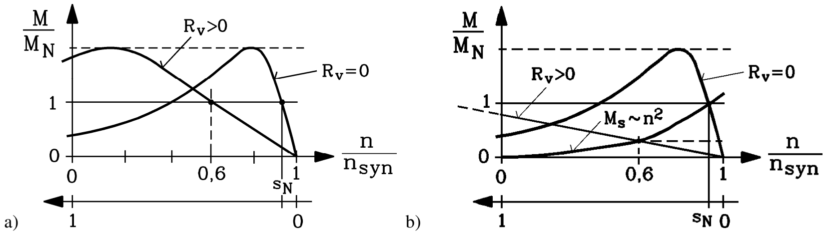


Fig. 5.18: *M(n)*-characteristic of a slipring-induction motor with external resistances as variable speed drive a) constant load torque, b) load torque increases with the square of the motor speed (pump, variable flow rate)

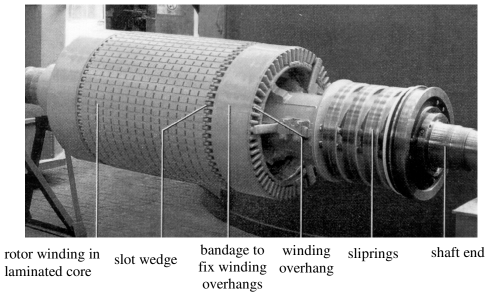


Fig. 5.19: Slipring rotor: rotor winding in laminated core, right: sliprings