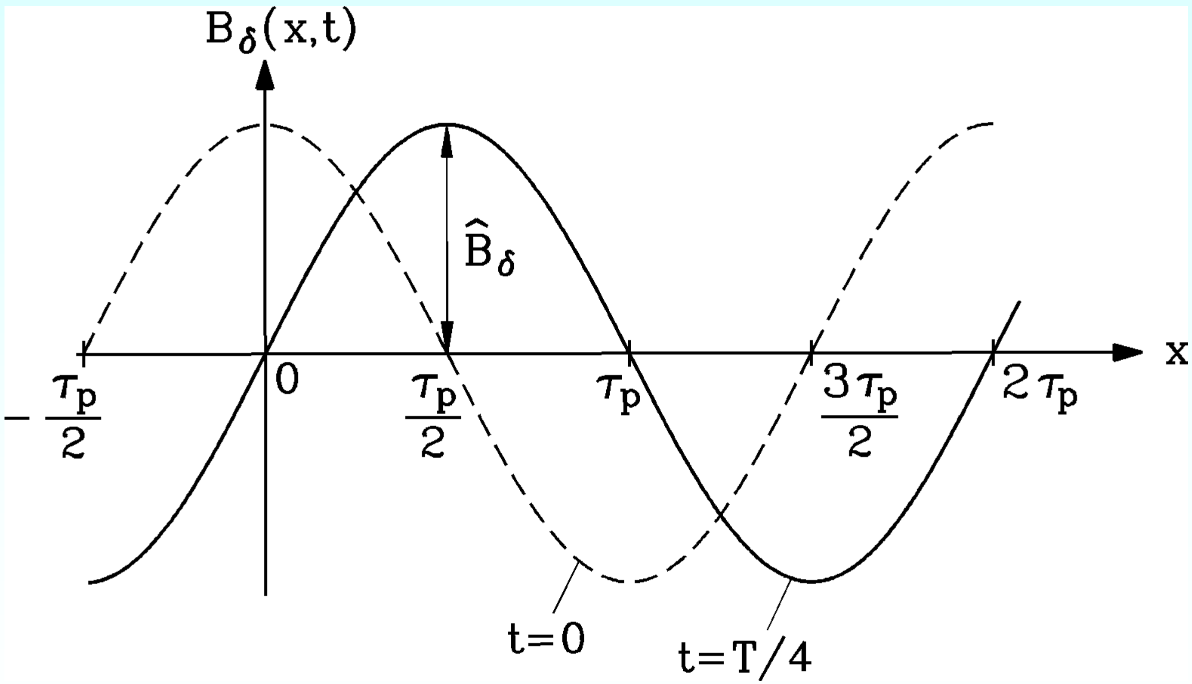
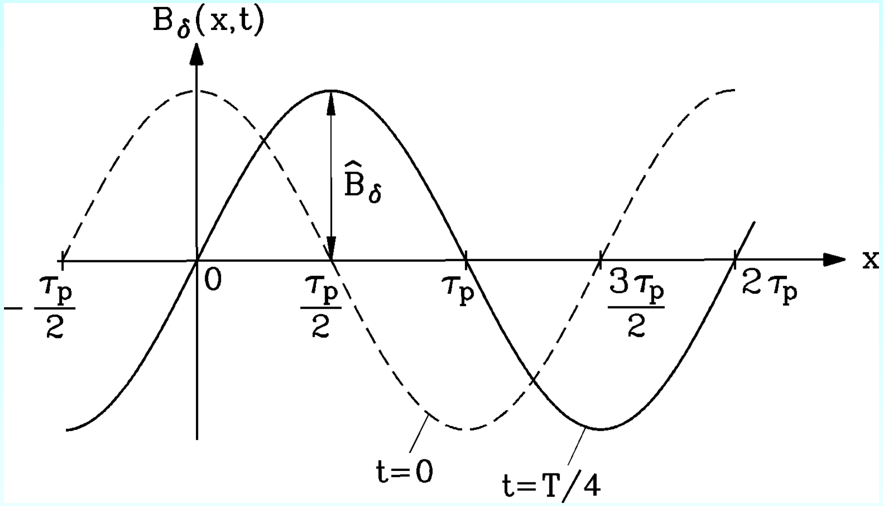
3. Mathematical Analysis of Air-Gap Fields



Rotating waves - Travelling waves

- **Rotating wave:** *x* is stator circumference co-ordinate (*rotating machine*)
- **Travelling wave:** *x* is stator linear co-ordinate (*linear machine*)



$$B_{\delta 1}(x,t) = \hat{B}_{\delta 1} \cos\left(\frac{x \cdot \pi}{\tau_p} - 2\pi f \cdot t\right)$$

Wave velocity: Observer, who is moving with the wave, sees constant argument of cos() = *const.*

$$v_{syn} = \frac{dx}{dt} = \frac{d}{dt}(const. + 2\pi f t)\frac{\tau_p}{\pi} = \underline{\underline{2 f \tau_p}}$$

- **Wave in opposite direction:** $B_{\delta 1}(x,t) = \hat{B}_{\delta 1} \cos(\frac{x \cdot \pi}{\tau_p} + 2\pi f \cdot t) \quad \Rightarrow \quad v_{syn} = -2 f \tau_p$
- *Example:*

At frequency f = 50 Hz: v_{syn} in *m/s* is SAME number as pole pitch in *cm*: $v_{syn}^{[m/s]} = \tau_p^{[cm]}$

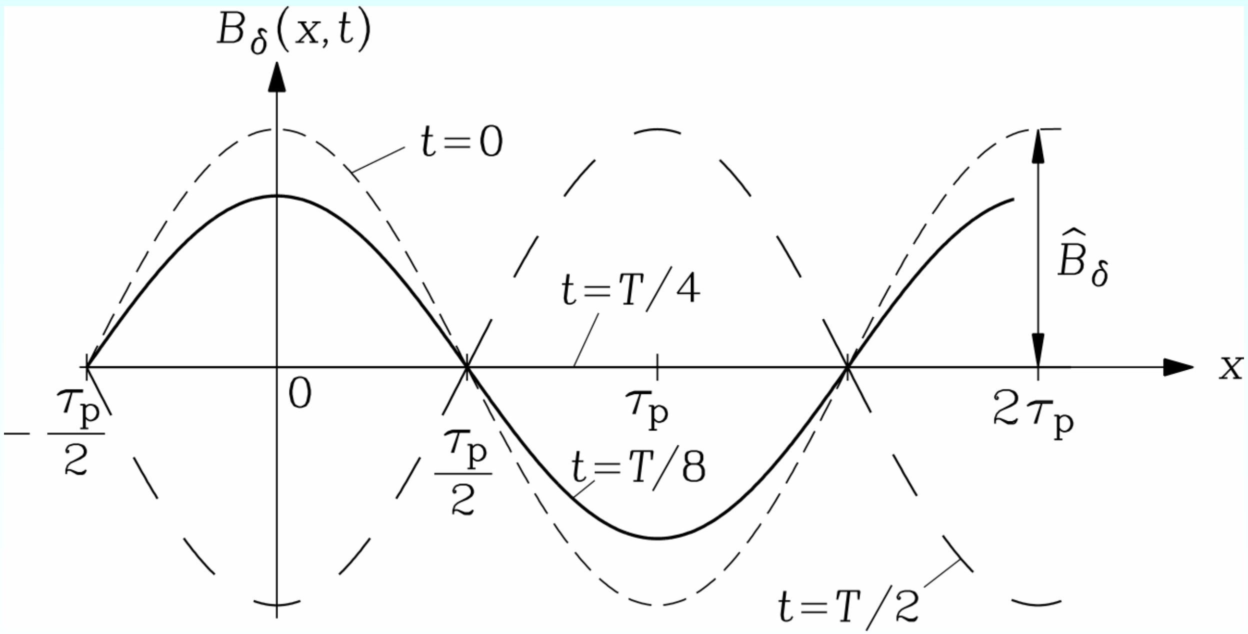
e.g. 2-pole turbine generator (2p = 2) in thermal power plant: n_{syn} = 3000/min:

- stator bore diameter d_{si} = 1.2 m, pole pitch τ_p = 1.2π/2 = 1.88 m = 188 cm
- v_{syn} = 188 m/s = 676 km/h = rotor surface velocity, as rotor is spinning synchronously with rotating stator magnetic field wave (synchronous machine !)

Pulsating fields – standing waves

- **Standing wave is pulsating field:** The location of wave nodes and wave amplitudes is fixed, but the amplitudes are pulsating.
- At time t = 0: Maximum of wave at x = 0 has amplitude $\hat{B}_{\delta 1}$
  at t = T/8: $\hat{B}_{\delta 1}/\sqrt{2}$,
  at t = T/4: amplitude is zero !
  at t = T/2: $-\hat{B}_{\delta 1}$ and so on.

$$B_{\delta 1}(x,t) = \hat{B}_{\delta 1}\cos\left(\frac{x\cdot\pi}{\tau_p}\right)\cdot\cos(2\pi f\cdot t)$$



FOURIER-Analysis: Determining fundamental & harmonic waves

- ***FOURIER*-series:** A periodical function $V(\gamma)$ with period 2π may be described by an infinite sum of sine & co-sine functions with decreasing wave length.

$$V(\gamma) = V_0 + \sum_{\nu=1,2,3,\ldots}^{\infty} \left[\hat{V}_{\nu,a} \cdot \cos(\nu \cdot \gamma) + \hat{V}_{\nu,b} \cdot \sin(\nu \cdot \gamma) \right]$$

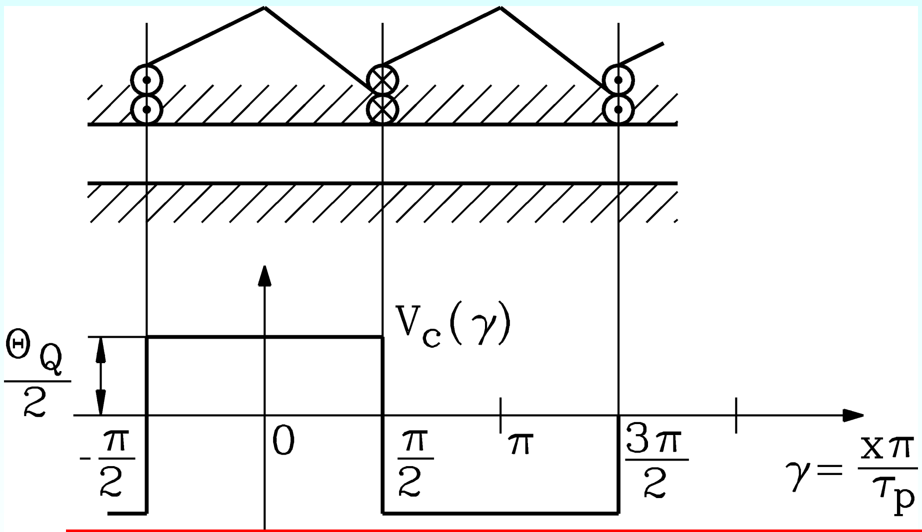
- **Ordinal numbers**: $\nu = 1, 2, 3, \ldots$
- **Amplitudes:** $\hat{V}_{\nu,a} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \cos(\nu \cdot \gamma) \cdot d\gamma$, $\hat{V}_{\nu,b} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \sin(\nu \cdot \gamma) \cdot d\gamma$
- **Average value:** $V_0 = \frac{1}{2\pi} \int_0^{2\pi} V(\gamma) \cdot d\gamma$
- **Magnetomotive force (MMF) of air gap field:**
  - **a) NO UNIPOLAR flux: $V_0 = 0$**
  - **b) MMF *V* <u>*symmetrical to abscissa*</u>: NO even ordinal numbers**
  - **c) By choosing origin so, that MMF *V* is <u>even</u> function $V(\gamma) = V(-\gamma)$:**
    NO sine-wave functions occur in *FOURIER* sum.

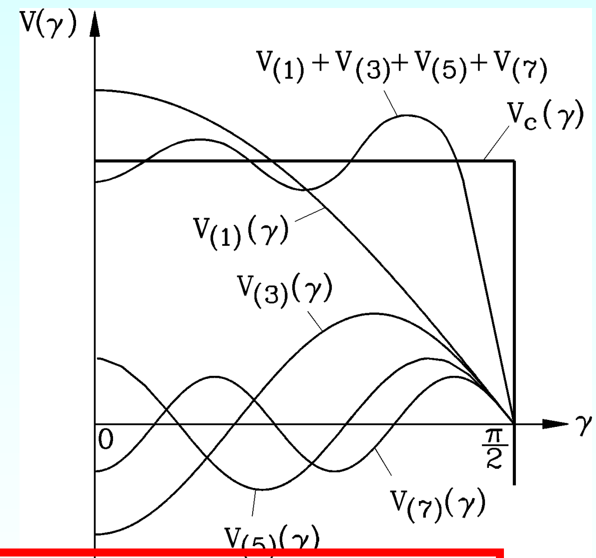
FOURIER-Analysis of field of one full-pitched coil ($q = 1$)

- Magnetomotive force $V_c(x)$ is block-shaped function, slot-Ampere-turns $\Theta_Q = 2N_c i_c$
- Circumference angle : $\gamma = x \cdot \pi / \tau_p$:

$$V_c(\gamma) = \sum_{\nu=1,3,5,\ldots}^{\infty} \hat{V}_{c,\nu} \cos(\nu\gamma)$$

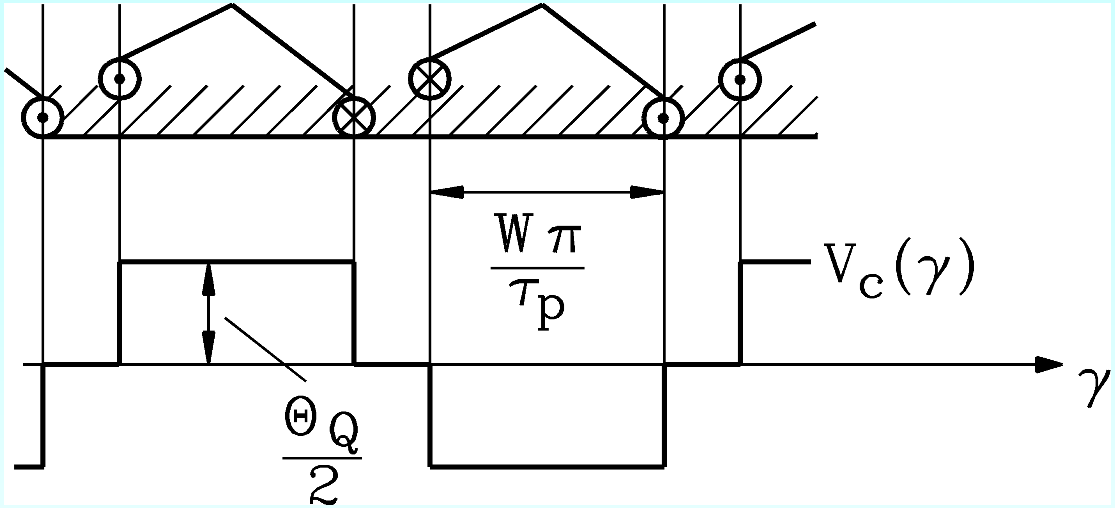
Example: **Four pole machine**: Half circumference = π in **"mechanical degrees"**, equals two pole pitches $x = 2\tau_p$, thus $\gamma = 2\pi$, in **"electrical degrees"**





$$\hat{V}_{c,\nu} = \frac{2}{\pi} \int_0^{\pi} V_c(\gamma) \cos(\nu\gamma) d\gamma = \frac{\Theta_Q}{2} \cdot \frac{4}{\nu\pi} \cdot \sin\left(\frac{\nu\pi}{2}\right), \qquad \nu = 1, 3, 5, \ldots.$$

FOURIER-Analysis of field of one pitched coil (q = 1)



- $$\hat{V}_{c,\nu} = \frac{2}{\pi}\int_0^{\pi} V_c(\gamma)\cos(\nu\gamma)d\gamma = \frac{\Theta_Q}{2}\cdot\frac{4}{\nu\pi}\cdot\sin\left(\frac{W}{\tau_p}\cdot\frac{\nu\pi}{2}\right),\quad \nu = 1,\ 3, 5, \ldots$$
- Compare MMF function´s *FOURIER* series with that of full-pitched coils: Amplitudes are <u>smaller</u> by the **"pitch coefficient" $k_{p,\nu}$** :

$$k_{p,\nu} = \sin\left(\frac{W}{\tau_p}\cdot\frac{\nu\pi}{2}\right)$$

<u>Example:</u> W/τ_p = 5/6 (q = 2): ν = 1: k_{p1} = 0.966, ν = 5: k_{p5} = 0.259

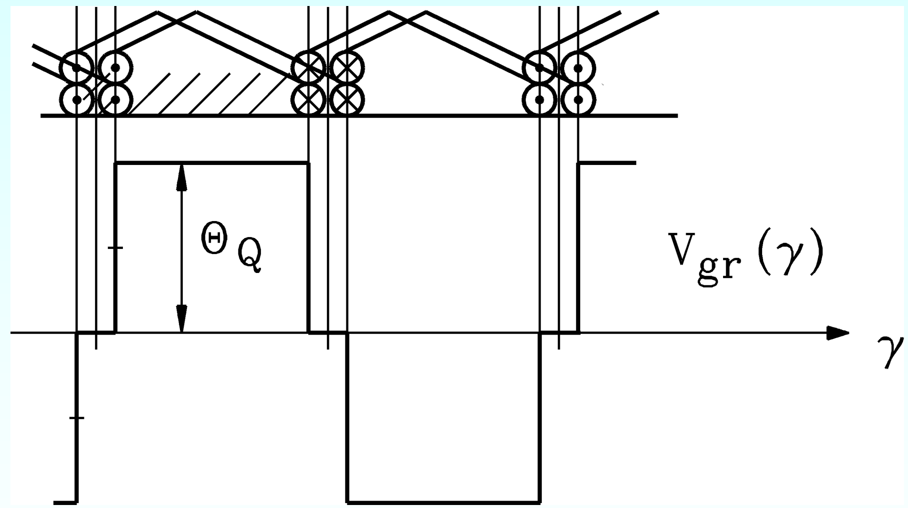
FOURIER-Analysis of field of a full-pitched coil group *q* > 1

- Magnetomotive force of field of full-pitched coil group: $\nu = 1, 3, 5, ...$

$$\hat{V}_{gr,\nu} = \frac{2}{\pi}\int_0^{\pi} V_{gr}(\gamma)\cos(\nu\gamma)d\gamma = \frac{q\Theta_Q}{2}\cdot\frac{4}{\nu\pi}\cdot\sin\left(\frac{\nu\pi}{2}\right)\cdot\frac{\sin(\frac{\nu\pi}{2m})}{q\cdot\sin(\frac{\nu\pi}{2mq})}$$

- **"Distribution coefficient"**

$$k_{d,\nu} = \frac{\sin(\frac{\nu\pi}{2m})}{q\cdot\sin(\frac{\nu\pi}{2mq})}$$



Example: $q = 2$

MMF amplitude: $q\Theta_Q/2 = \Theta_Q$

- With relationship we get

$$\frac{q\Theta_Q}{2} = qN_c i_c = \frac{2pqN_c}{a}\cdot\frac{ai_c}{2p} = N\cdot\frac{i}{2p} \quad\Rightarrow\quad \hat{V}_{gr,\nu} = N\cdot\frac{i}{2p}\cdot\frac{4}{\nu\pi}\cdot\sin\left(\frac{\nu\pi}{2}\right)\cdot k_{d,\nu}$$

FOURIER-series of field of one phase with pitched coil groups $q > 1$, $W/\tau_p < 1$

- *FOURIER*-analysis of MMF of one phase $V_{phase}(\gamma)$:

$$\hat{V}_{phase,\nu} = N \cdot \frac{i}{2p} \cdot \frac{4}{\nu\pi} \cdot k_{p,\nu} \cdot k_{d,\nu} \, , \qquad \nu = 1, 3, 5, \ldots$$

- Phase current $i = I \cdot \sqrt{2} \cdot \cos(\omega t)$

- The MMF distribution (and hence the air gap field) is a sum of pulsating, standing waves **(„Pulsating field“).**

$$\hat{V}_{phase,\nu} = \frac{2 \cdot \sqrt{2}}{\pi} \cdot \frac{N}{p} \cdot \frac{1}{\nu} \cdot k_{p,\nu} \cdot k_{d,\nu} \cdot I \, , \qquad \nu = 1, 3, 5, \ldots$$

$$V_{phase}(\gamma, t) = \sum_{\nu=1,3,5,\ldots}^{\infty} \hat{V}_{phase,\nu} \cdot \cos(\nu\gamma) \cdot \cos(\omega t)$$

- **"Winding coefficient"** $k_{w,\nu} = k_{p,\nu} \cdot k_{d,\nu}$

Example of typical three phase windings A, B, C

- **Pitch coefficient, distribution coefficient** and **winding coefficient** for pitched winding with 6, 12 and 18 slots per pole pair Q/p

	A			**B**			**C**		
	$q = 1, W/\tau_p = 2/3$			$q = 2, W/\tau_p = 5/6$			$q = 3, W/\tau_p = 7/9$		
	$Q/p = 6$			$Q/p = 12$			$Q/p = 18$		
ν	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$
1	0.866	1	0.866	**0.966**	**0.966**	**0.933**	**0.940**	**0.960**	**0.902**
5	-0.866	1	-0.866	0.259	0.259	0.067	-0.174	0.218	-0.038
7	0.866	1	0.866	0.259	-0.259	-0.067	0.766	-0.177	-0.136
11	-0.866	1	-0.866	**0.966**	**-0.966**	**-0.933**	0.766	-0.177	-0.136
13	0.866	1	0.866	**-0.966**	**-0.966**	**0.933**	-0.174	0.218	-0.038
17	-0.866	1	-0.866	-0.259	-0.259	0.067	**0.940**	**0.960**	**0.902**
19	0.866	1	0.866	-0.259	0.259	-0.067	**-0.940**	**0.960**	**-0.902**

- Pitch and distribution coefficient REDUCE certain field harmonic amplitudes.
- Winding coefficient is a PERIODICAL function of ordinal number ν.

Inserting form-wound two-layer winding in induction generator stator

Winding overhang

Q = 72 slots

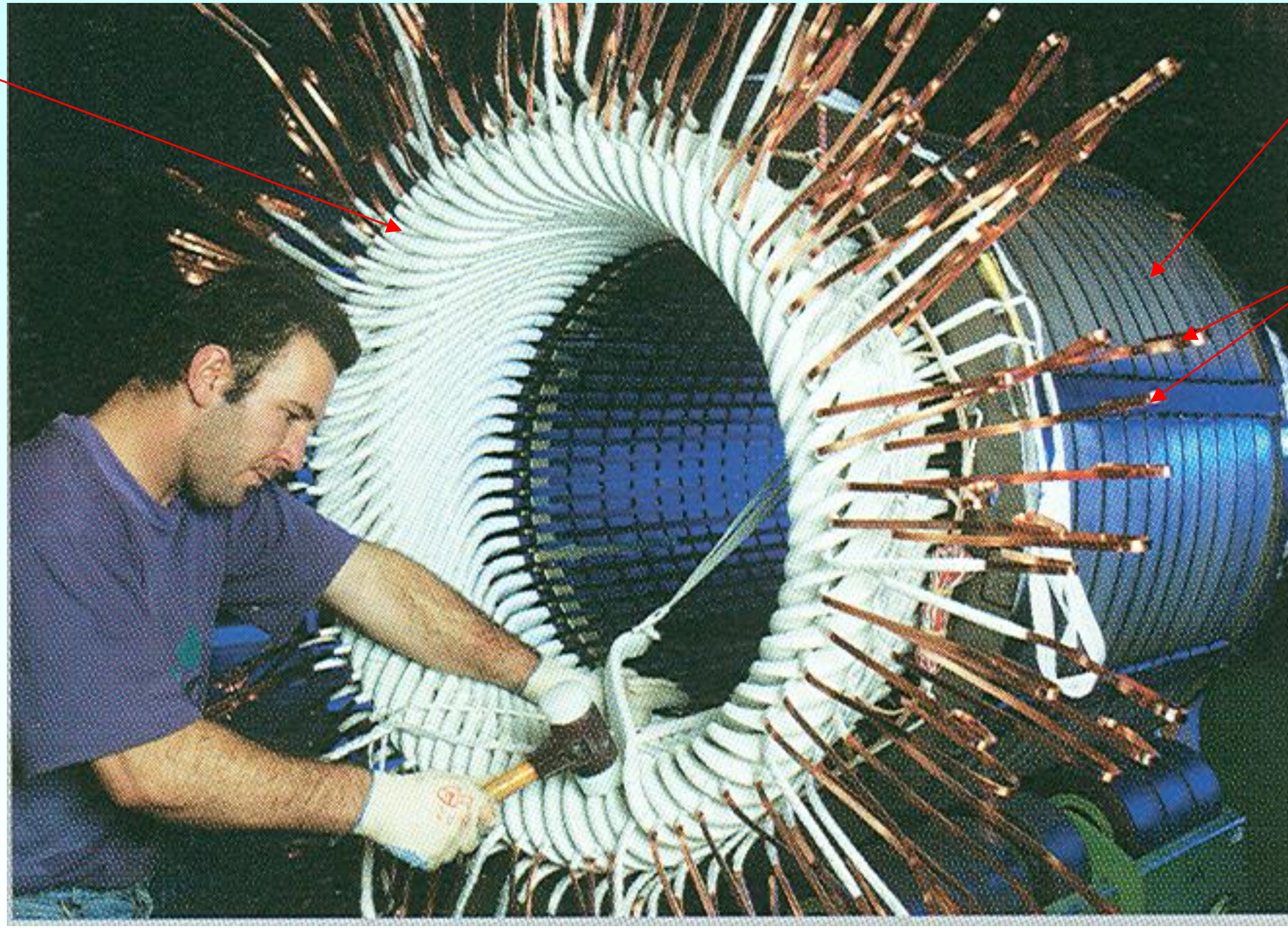
72 coils

m = 3 phases

$2p$ = 4 poles

q = 6 slots per pole and phase

$$Q = 2p \cdot m \cdot q =$$
$$= 4 \cdot 3 \cdot 6 = 72$$



Stator iron stack

coil ends

Source: Winergy Germany

Inserting form-wound two-layer winding in stator slots

Winding overhang

Stator iron stack

60 slots for a 4-pole induction machine

q = 5:

$$Q = 2p \cdot m \cdot q =$$
$$= 4 \cdot 3 \cdot 5 = 60$$

Source:

ABB, Switzerland

Stator three phase two-layer winding of induction

Q = 72 slots

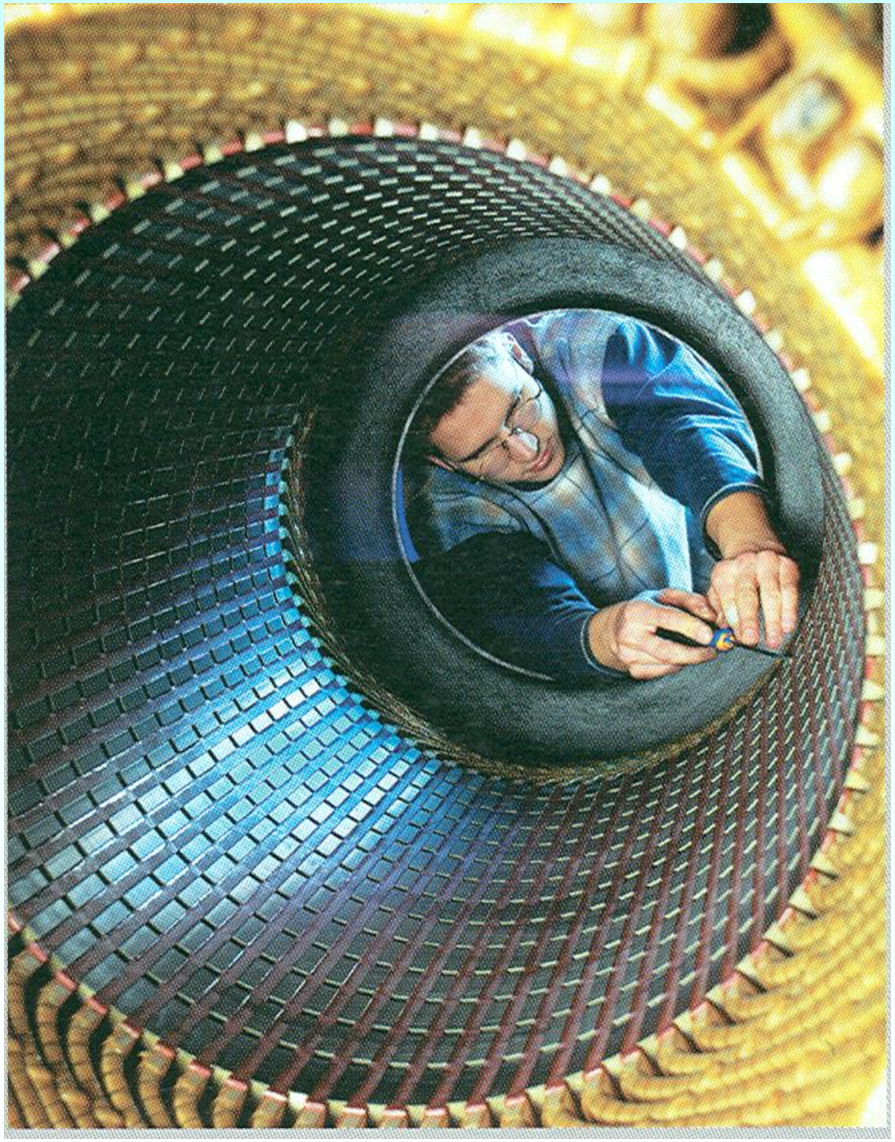
72 coils

m = 3 phases

$2p$ = 4 poles

q = 6 slots per pole and phase

$$Q = 2p \cdot m \cdot q =$$
$$= 4 \cdot 3 \cdot 6 = 72$$



- Completed two-layer winding after the impregnation with resin
- Some resin "noses" are removed after the impregnation to get a smooth air gap

Source:

Winergy

Germany

FOURIER-analysis of field of a 3-phase winding

- **Pulsating fields per phase** are shifted spatially by $2\tau_p/3$ as phases U, V, W. These 3 phases are fed by sinusoidal AC current with phase shift of $T/3$.

$$V_{U\nu}(\gamma,t) = \hat{V}_{phase,\nu} \cdot \cos(\nu\gamma) \cdot \cos(\omega t)$$

$$V_{V\nu}(\gamma,t) = \hat{V}_{phase,\nu} \cdot \cos(\nu(\gamma - 2\pi/3)) \cdot \cos(\omega t - 2\pi/3)$$

$$V_{W\nu}(\gamma,t) = \hat{V}_{phase,\nu} \cdot \cos(\nu(\gamma - 4\pi/3)) \cdot \cos(\omega t - 4\pi/3)$$

- **Pulsating field per phase** is decomposed in **negative** and **positive sequence** rotating field waves (clockwise and counter-clockwise rotation) by use of trigonometric summation law: $\cos\alpha \cdot \cos\beta = \frac{1}{2} \cdot [\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$$V_{U\nu}(\gamma,t) = \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma + \omega t) + \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma - \omega t)$$

$$V_{V\nu}(\gamma,t) = \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma - \frac{\nu 2\pi}{3} + \omega t - \frac{2\pi}{3}) + \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma - \frac{\nu 2\pi}{3} - \omega t + \frac{2\pi}{3})$$

$$V_{W\nu}(\gamma,t) = \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma - \frac{\nu 4\pi}{3} + \omega t - \frac{4\pi}{3}) + \frac{\hat{V}_{phase,\nu}}{2}\cos(\nu\gamma - \frac{\nu 4\pi}{3} - \omega t + \frac{4\pi}{3})$$

Fundamental of air gap MMF (and field)

- **Resulting effect (summation)** of all 3 pulsating phase fields U, V, W:

$$V(\gamma,t)=\sum_{\nu=1,3,5,...}^{\infty}(V_{U\nu}(\gamma,t)+V_{V\nu}(\gamma,t)+V_{W\nu}(\gamma,t))=\sum_{\nu=1,3,5,...}^{\infty}V_{\nu}(\gamma,t)$$

- Summation for ordinal number $\nu = 1$ ("Fundamental"):

$$V_{U1}(\gamma,t)=\frac{\hat{V}_{phase,1}}{2}\cos(\gamma+\omega t)+\frac{\hat{V}_{phase,1}}{2}\cos(\gamma-\omega t)$$

$$V_{V1}(\gamma,t)=\frac{\hat{V}_{phase,1}}{2}\cos(\gamma+\omega t-\frac{4\pi}{3})+\frac{\hat{V}_{phase,1}}{2}\cos(\gamma-\omega t)$$

$$V_{W1}(\gamma,t)=\frac{\hat{V}_{phase,1}}{2}\cos(\gamma+\omega t-\frac{8\pi}{3})+\frac{\hat{V}_{phase,1}}{2}\cos(\gamma-\omega t)$$

With $\cos(\lambda)+\cos(\lambda-\frac{4\pi}{3})+\cos(\lambda-\frac{8\pi}{3})=0$ we get :

- Result: **rotating wave:**

$$V_1(x,t)=\frac{3}{2}\hat{V}_{phase,1}\cos(\gamma-\omega t)\quad\Rightarrow\quad B_\delta=\mu_0 V/\delta$$

Harmonic waves of air gap MMF and field

- Summation of pulsating phase fields for $\nu > 1$:

$$V_3(\gamma,t) = 0 + 0 = 0 \qquad V_5(\gamma,t) = \frac{3}{2}\hat{V}_{phase,5}\cos(5\gamma + \omega t)$$

$$V_7(\gamma,t) = \frac{3}{2}\hat{V}_{phase,7}\cos(7\gamma - \omega t) \qquad V_9(\gamma,t) = 0 + 0 = 0$$

- For ordinal numbers ν, **which are divisible by 3**, the summation of harmonic phase fields U, V, W is **zero**: The 3 positive and negative sequence phase waves cancel.

  For **ν = 7, 13, 19, ...** cancel the 3 negative sequence phase waves. The 3 positive sequence phase waves add with the SAME phase angle and rotate WITH the fundamental wave.

  For **ν = 5, 11, 17, ...** cancel the 3 positive sequence phase waves. The 3 negative sequence phase waves add with the SAME phase angle, rotating inverse to fundamental.

- ***Facit:*** *A 3-phase winding, fed by symmetrical 3-phase sinusoidal current system, excites a step-like MMF distribution in air gap V(x,t), which may be decomposed into fundamental and harmonic rotating waves. Wave lengths are determined by ordinal numbers. Only odd ordinal numbers exist ν = 1, 5, 7, 11, 13, 17, 19, ..., which are indivisible by 3.*

Fundamental and harmonic air gap field waves

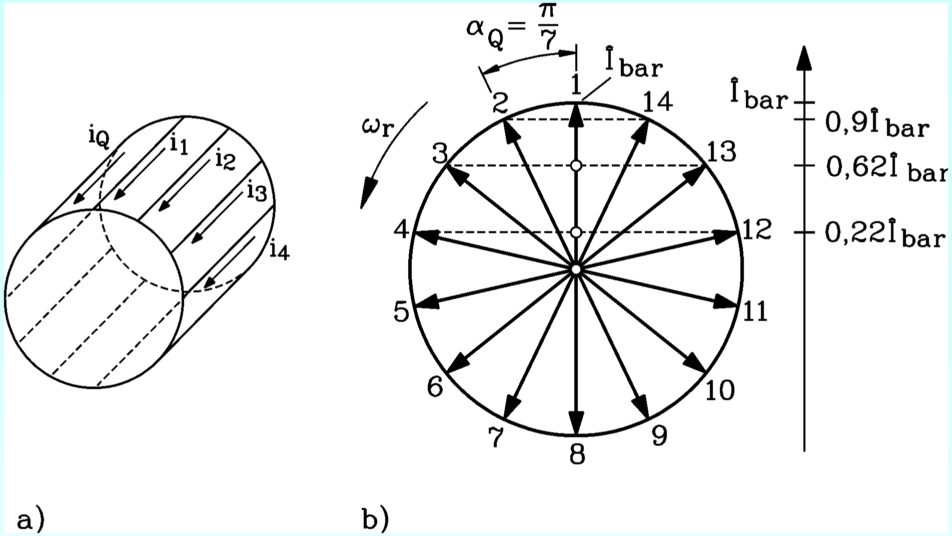
$$V(x,t)=\sum_{\nu} V_\nu(x,t)=\sum_{\nu}\frac{\sqrt{2}}{\pi}\frac{m}{p}N\frac{k_{w,\nu}}{\nu}I\cdot\cos\left(\frac{\nu\pi x}{\tau_p}-\omega t\right)$$

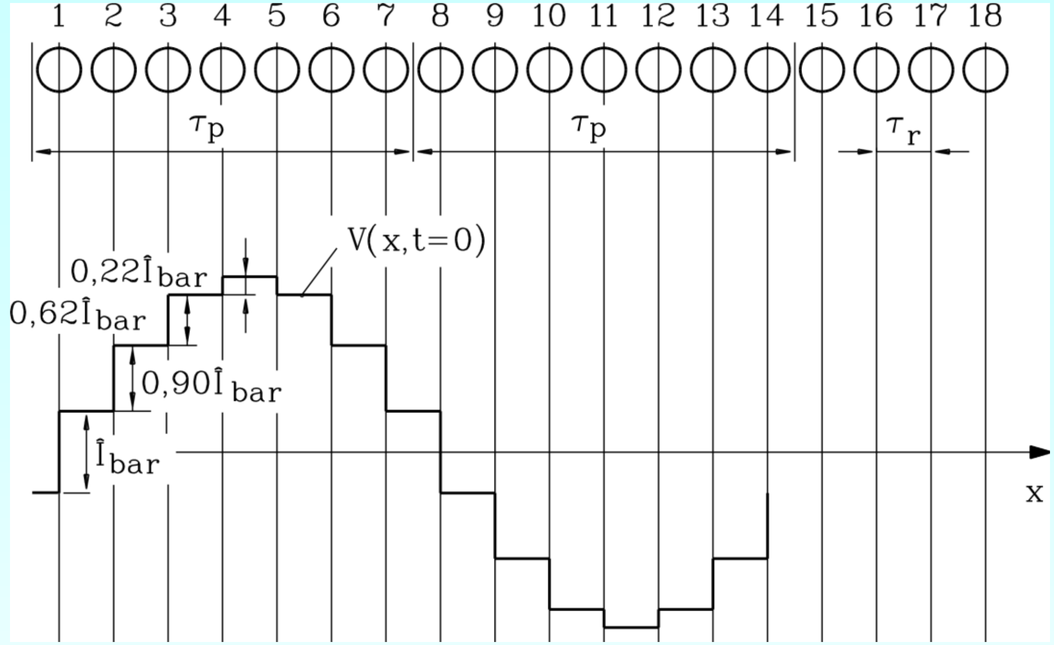
ν = 1, -5, 7, -11, 13, -17,... Phase number m: is usually 3

- **Positive and negative ordinal numbers:** $\nu = 1 + 2mg \qquad g = 0, \pm 1, \pm 2, \pm 3, \ldots$
- Velocity of harmonic waves decreases with 1/ ν : $v_{syn,\nu} = 2 f \tau_p / \nu$
- Wave amplitudes (% of fundamental): $\hat{B}_{\delta\nu} / \hat{B}_{\delta 1}$ (%) **Underlined: "slot harmonics" !**

ν	q = 1, W/τ_p = 2/3, Q/p = 6	q = 2, W/τ_p = 5/6, Q/p = 12	q = 3, W/τ_p = 7/9, Q/p = 18
1	**100**	**100**	**100**
-5	-20	1.4	-0.8
7	14.3	-1.0	-2.2
-11	-9.1	**-9.1**	-1.4
13	7.7	**7.7**	-0.3
-17	-5.6	-0.4	**5.9**
19	5.3	0.38	**-5.3**

Magnetomotive force and air gap field of squirrel cage winding





a) Rotor: Squirrel cage: Q_r *conductive* bars (copper, aluminium) in Q_r slots. Bars are short-circuited by 2 *conductive* rings an the front ends.

b) Symmetrical rotor current system: In each bar flows a **sinusoidal bar current** with a constant phase shift to the current of the adjacent bar**.** Thus each bar **is a phase of a Q_r-phase system.**

Example: Q_r = 28 bars, $2p$ = 4: Bar current system repeats after Q_r/p = 14 bars. Phase shift is **"slot angle"** $\alpha_Q = 2\pi p/Q_r = \pi/7$.

FOURIER-Analysis of MMF (and field) of squirrel cage

- Each bar = 1 phase AND each bar = half turn of a coil. Hence:
  Pitch and distribution coefficient = 1.
  μ : Ordinal number of harmonic waves of rotor field of cage winding

- Relationships: $N \to 1/2, \quad m \to Q_r, \quad k_{w,\nu} \to 1, \quad I \to I_{bar}, \quad \nu \to \mu$

- *FOURIER*-series:

$$V(x,t) = \sum_{\mu=1,\ldots}^{\infty} \frac{\sqrt{2}}{\pi} \frac{Q_r}{p} \frac{1}{2} \frac{1}{\mu} I_{bar} \cdot \cos\left(\frac{\mu \pi x}{\tau_p} - 2\pi f_r t\right)$$

with ordinal numbers

$$\mu = 1 + \frac{Q_r}{p} g_r \qquad g_r = 0, \pm 1, \pm 2, \ldots$$

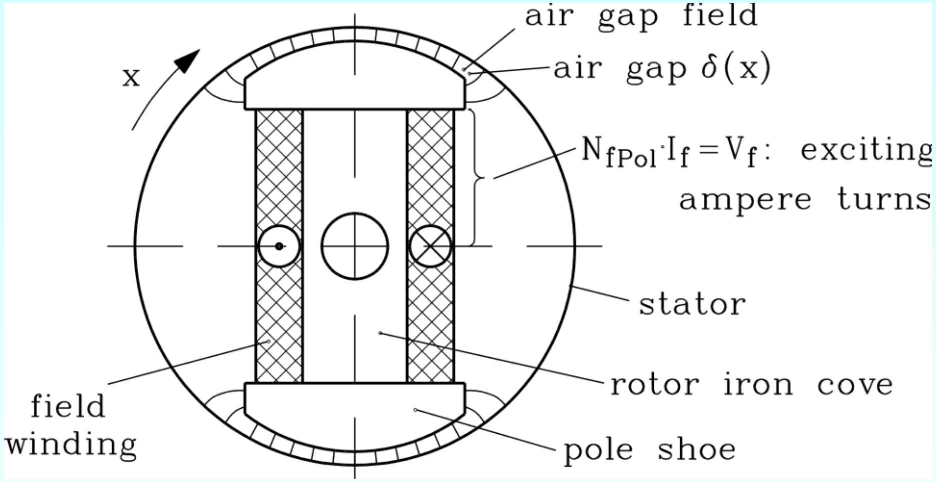
- *Example:* Q_r = 28 bars, $2p$ = 4: Ordinal numbers μ = 1, -13, 15, -27, 29, ...
  Wave amplitudes decrease with $1/\mu$!

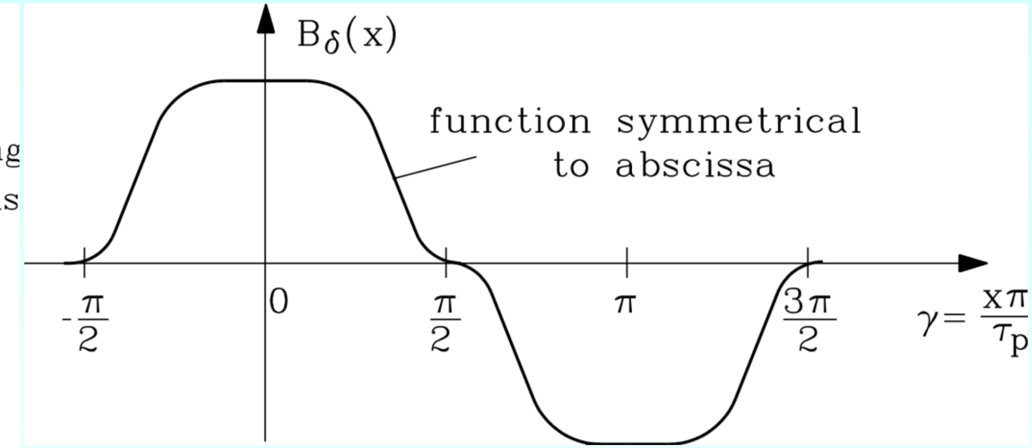
Copper cage rotor of an induction machines

Source:
Breuer Motors,
Germany

Salient pole air gap field, excited by DC current

- **Air gap field** H_δ : e.g. 2-pole rotor of salient pole synchronous machine: Rotor winding ampere-turns $N_{fPol} \cdot I_f$, (N_{fPol} : number of turns per pole)
- **Air gap** enlarged at inter-pole gaps, hence air gap is function of circumference co-ordinate $\delta(x)$.





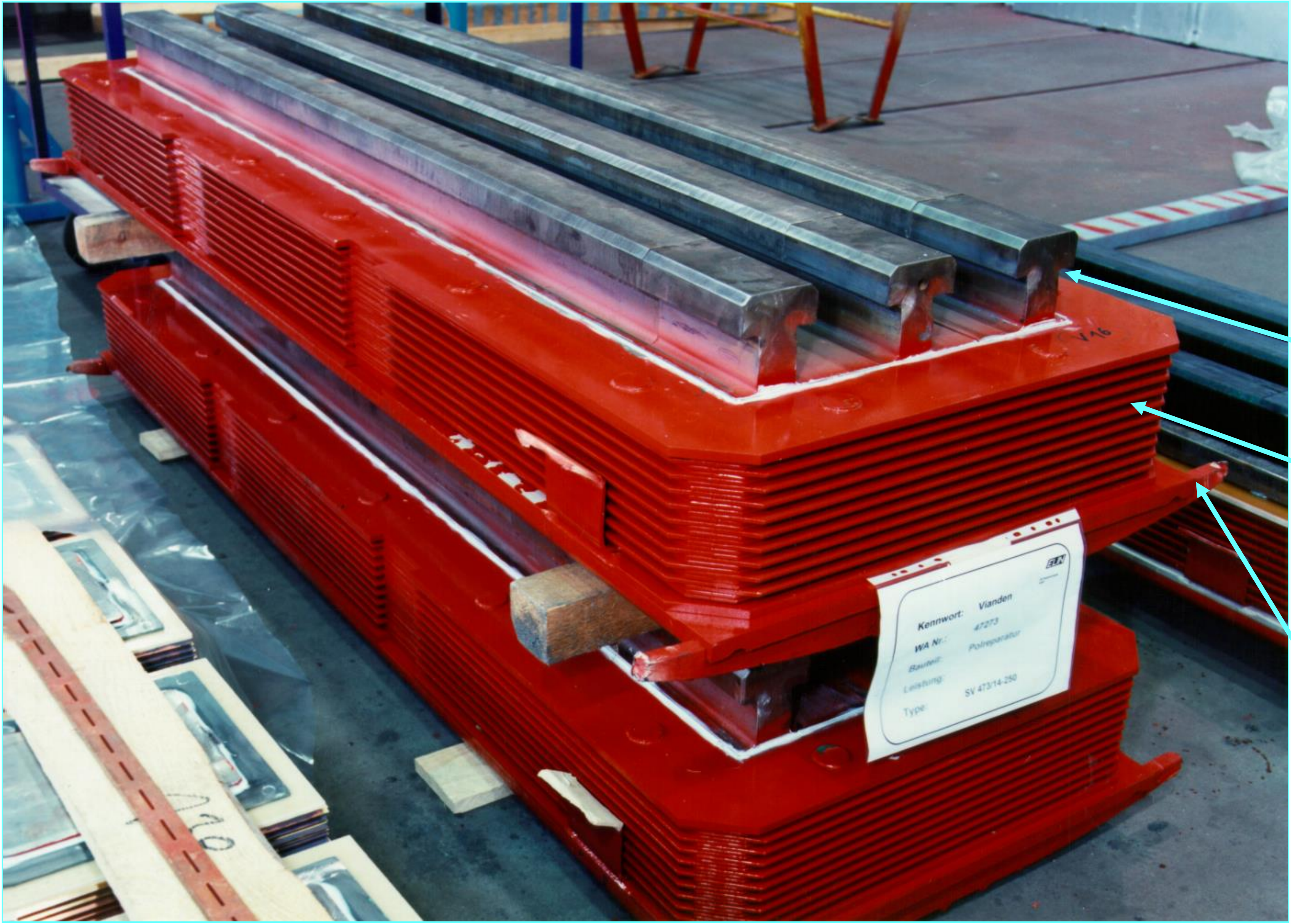
- Calculation of **radial component B_δ** of rotor air gap field by *AMPERE´*s law:

$$\oint_C \vec{H} \cdot d\vec{s} = 2N_{fPol} I_f = 2V_f = 2H_\delta \delta(x) + 2H_{Fe} \Delta_{Fe} = 2H_\delta \delta(x)$$

$$B_\delta(x) = \mu_0 H_\delta(x) = \mu_0 \frac{V_f}{\delta(x)}$$

(Iron: $\mu_{Fe} \to \infty$)

Completed salient pole before mounting



Pump storage hydro power plant *Vianden/Belgium*

Refurbishment

Three-fold hammer head fixation

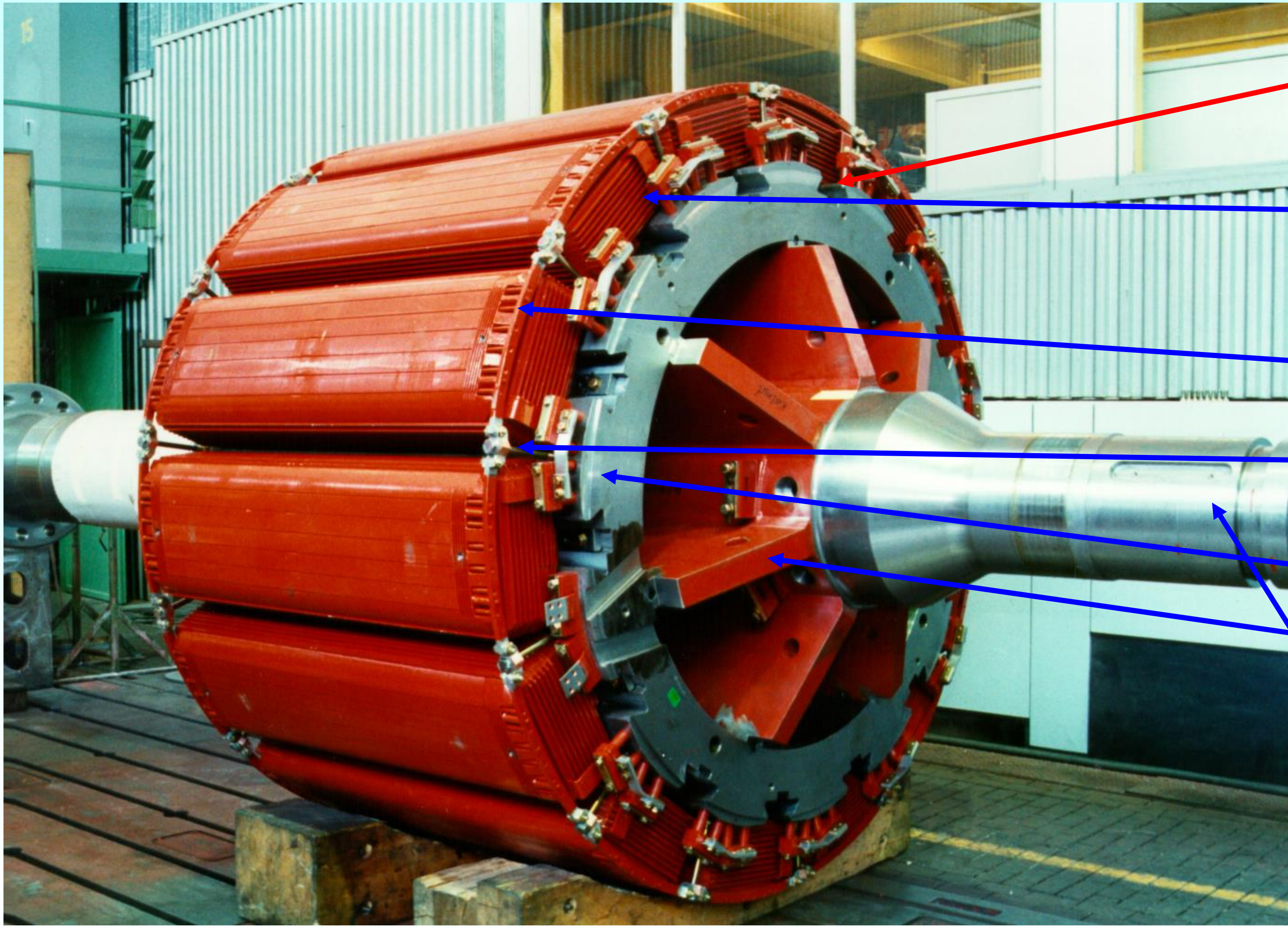
"Cooling fins" by increased copper width

Damper ring segments

Source:

Andritz Hydro, Austria

Completed "big hydro" salient pole synchronous rotor for high centrifugal force at over-speed, 14 poles



Dove tail fixation of rotor poles

"Cooling fins" by increased copper width

Damper ring

Damper retaining bolts

Rotor back iron

Rotor spider

Generator shaft

Source:

Andritz Hydro, Austria

FOURIER-Analysis of air gap field of DC-excited salient poles

- Field curve is symmetrical to abscissa: $B_\delta(\gamma) = -B_\delta(\gamma + \pi)$
  only harmonics with odd ordinal numbers
- Field curve as *FOURIER*-Cosine-series: $B_\delta(\gamma) = \sum_{\mu=1,3,5,\ldots}^{\infty} \hat{B}_{\delta\mu} \cos(\mu\gamma) \quad \mu = 1, 3, 5, 7, 9, \ldots$

- **Shape of field curve** is determined by air gap function $\delta(x)$ (= contour of pole shoe);
  hence also ordinal numbers, divisible by 3, are existing !

- Poles rotate with **constant rotational speed *n*:** $\Omega_m = 2\pi n = 2v_m/d_{si}$ Hence all rotor waves (fundamental and harmonics) B_μ move with same speed v_m with respect to stator winding. Stator induced voltage of μ-th harmonic has μ-times higher frequency than that of fundamental $\mu = 1$.

$$f_\mu = \frac{\omega_\mu}{2\pi} = \frac{\mu p \Omega_m}{2\pi} = \mu \cdot p \cdot n \qquad \gamma_s(t) = \gamma_r + \gamma(t) = \gamma_r + p \cdot \Omega_m \cdot t$$

$$B_{\delta\mu}(\gamma_r) = \hat{B}_{\delta\mu} \cdot \cos(\mu\gamma_r) = \hat{B}_{\delta\mu} \cdot \cos(\mu\gamma_s - \mu p \Omega_m t) = \hat{B}_{\delta\mu} \cdot \cos(\frac{\mu\pi x}{\tau_p} - \omega_\mu t)$$

Facit: In stator winding sinusoidal voltage due to fundamental rotor wave ($\mu = 1$) is induced, but ALSO harmonic voltages with higher frequencies, whose harmonic stator currents, excite magnetic fields, which may interfere from power line to parallel telephone cables.