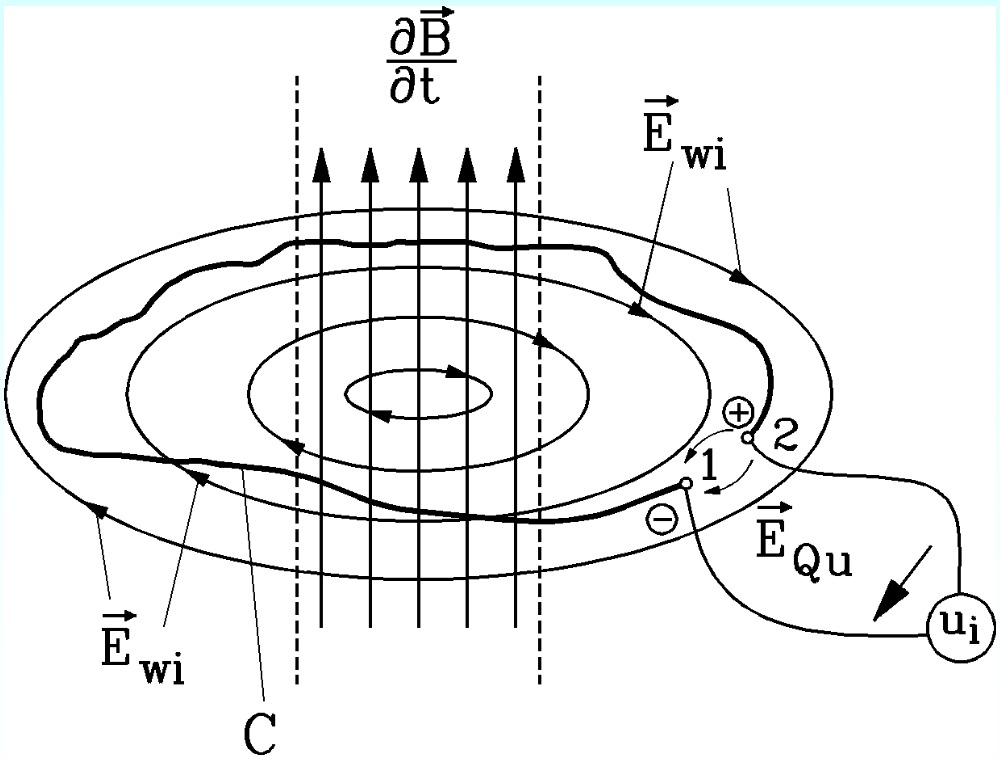
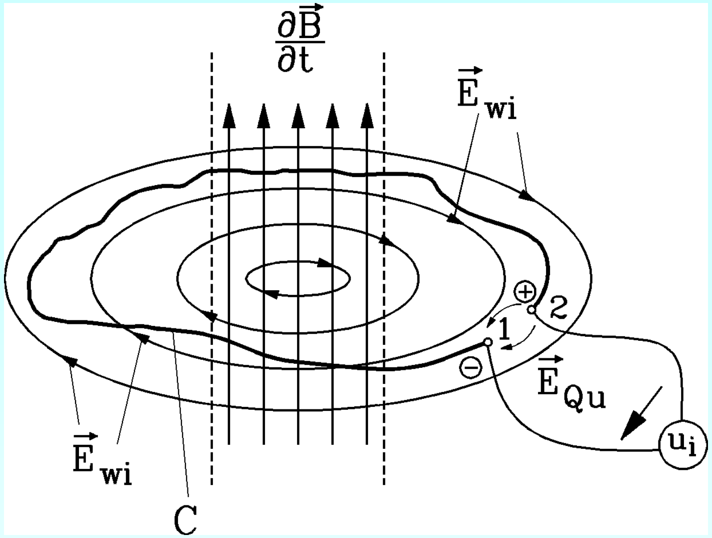
4. Voltage Induction in Three-Phase Machines

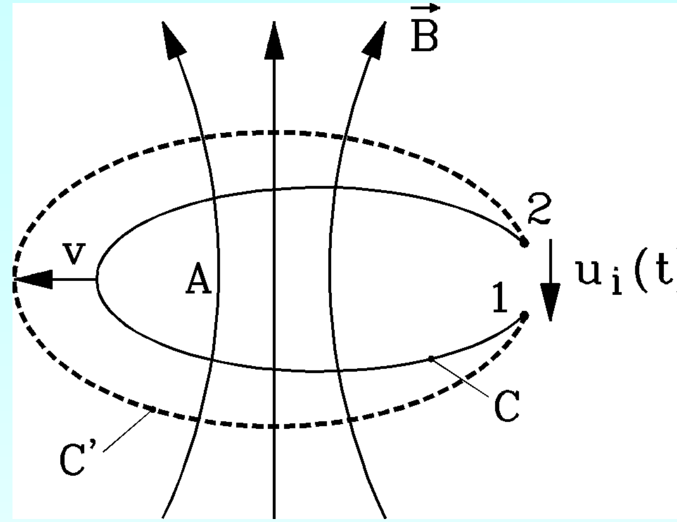


FARADAY´s law of induction

a)



b)



Each change of flux *Φ*, which is linked to conductor loop *C*, causes an induced voltage u_i in that loop; the induced voltage is the negative rate of change of the linked flux.

$$u_i = -d\Phi / dt \qquad \text{Fluß:}\ \Phi = \int_A \vec{B} \cdot d\vec{A}$$

- If coil is used instead of loop with *N* series connected turns, so u_i *is N*-times bigger:
  $$u_i = -N \cdot d\Phi / dt$$
- **"Flux linkage"** $\Psi = N \cdot \Phi \quad \Rightarrow \quad \boxed{u_i = -d\Psi / dt}$
- **Changing of *Ψ*:** a) *B* is changing, b) Area *A* is changing with velocity *v*

Induction in resting and moving coils

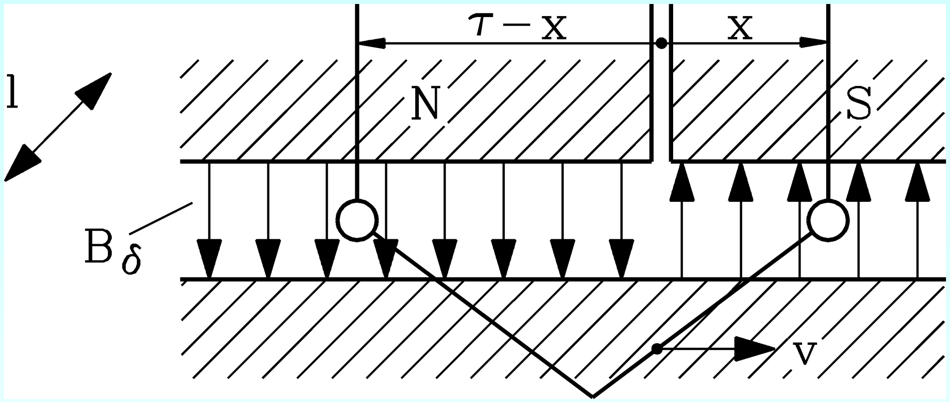
Resting coils	*Moving coils*
Flux density *B* is changing with time	Flux density *B* is constant with time
Coil at rest	Coil moving with velocity *v*
$u_i = -d\Psi / dt = -N \cdot d\Phi / dt$	
$u_i = -\partial \Psi / \partial t = \oint \vec{E}_{Wi} \cdot d\vec{s}$	$u_i = \oint \left(\vec{v} \times \vec{B}\right) \cdot d\vec{s} = \oint \vec{E}_b \cdot d\vec{s}$
Electric field strength $\vec{E}_{Wi}$ $(\vec{E}_{Wi} \Leftrightarrow -\partial \vec{B}/\partial t)$	Electric field strength $\vec{E}_b = \vec{v} \times \vec{B}$
Application of *FARADAY*´s law:	
• Transformer coils • Stator coils of AC machines	• Rotating armature of DC machines
Transformer induction	*Rotating induction*

- $$\frac{d\Phi}{dt} = \frac{d}{dt}\int_A \vec{B} \cdot d\vec{A} = \int_{A=const.} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} - \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{s} \quad \text{(Derivative of product !)}$$

$$u_i = \oint_{N \cdot C} \left(\vec{E}_{Wi} + \vec{E}_b\right) \cdot d\vec{s} = N \cdot \int_A -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} + N \cdot \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{s} = -\frac{d\Psi}{dt}$$

Example: Induced voltage in simple linear machine

- Coil (number of turns N_c, coil span τ) moves within air gap between iron yoke and permanent magnets (Poles N-S-N-S, Pole width $b_p = \tau$) with velocity v .



a) u_i induced in moving coil:

$\partial B / \partial t = 0$: no change of flux density.

Loop C only considered along length $2l$, as winding overhang outside of magnetic field.

$\vec{v}, \vec{B}, \vec{s}$ perpendicular to each other:

$$u_i = N_c \cdot 2\int_0^l (\vec{v} \times \vec{B}) \cdot d\vec{s} = \underline{2N_c vBl}$$

b) u_i derived from change of total flux linkage: observer rests with coil: $u_i = -d\Psi/dt$:

(ALTERNATIVE CALCULATION TO a) !)

Flux linkage changes $d\Psi/dt$, because coil moves, giving change of coil co-ordinate $x = vt$!

Coil flux linkage: $\Psi = N_c \int_A \vec{B} \cdot d\vec{A} = N_c \cdot l \cdot [(\tau - x)B_\delta - xB_\delta] = N_c l B_\delta (\tau - 2x)$

Induced voltage: $u_i = -d\Psi / dt = -N_c l B_\delta \cdot d(\tau - 2 \cdot v \cdot t) / dt = \underline{2N_c v B_\delta l}$

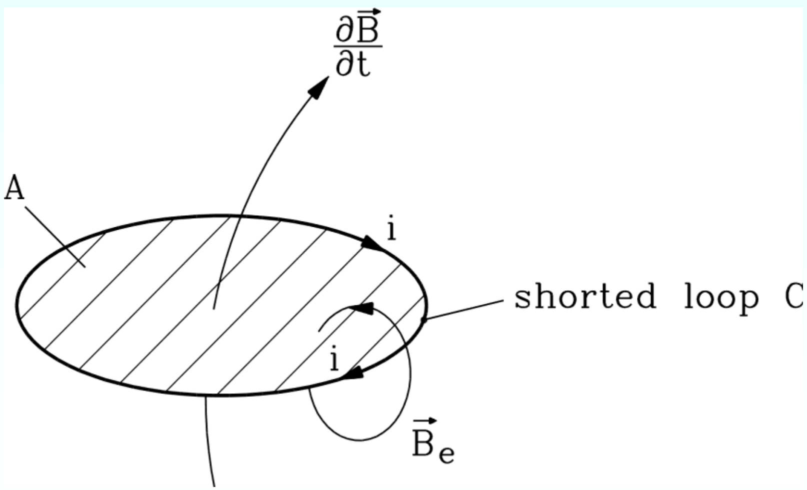
Facit : Induced voltage u_i may be ALWAYS derived from change of total flux linkage.

Law of induction: also called: "*LENZ´s* rule"

Lenz´s rule: A change of flux linkage induces voltage u_i, which drives a current i in the loop, which excites a magnetic field B_e, whose direction is opposite to the original change of flux linkage.

- ***Example: Induction in short circuited loop at rest***:

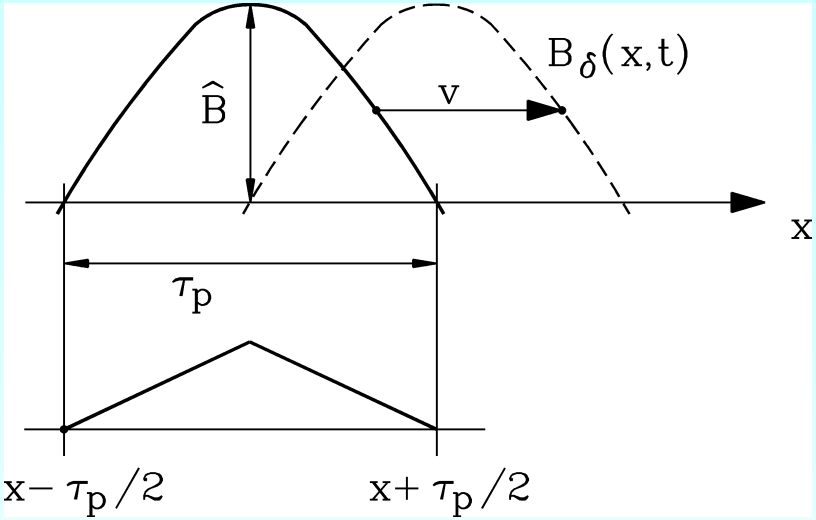
- The change of external field B causes an increase of flux density with orientation from bottom to top. This causes increase of flux in loop area A and **induces electrical field** E_{Wi}.
- E_{Wi} is left hand oriented to $\partial\vec{B}/\partial t$ and drives in loop C a **current** i.
- Current i excites (***Ampere´s* law !**) a **right hand oriented** magnetic field B_e .
- Orientation of B_e is opposite to change of original flux density $\partial\vec{B}/\partial t$.

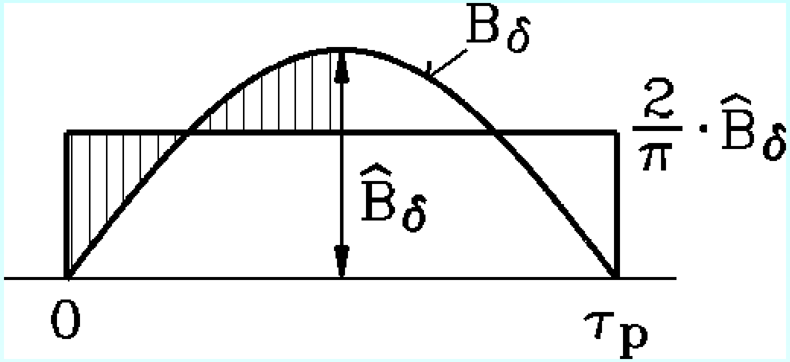


Facit:

The „reaction field" B_e acts AGAINST the original flux density change !

Induction of voltage in stator coil





- Sinusoidal moving wave $B_{\delta 1}(x,t)=\hat{B}_{\delta 1}\cos(x\pi/\tau_p-\omega t)$ causes changing coil flux $\Phi(t)$

$$\Phi(t)=l\int_{-\tau_p/2}^{\tau_p/2} B_{\delta 1}(x,t)dx=\frac{2}{\pi}\tau_p l\hat{B}_{\delta 1}\cdot\cos\omega t \;\Rightarrow\; \text{flux linkage } \Psi(t)=N_c\Phi(t)$$

- **Induced AC voltage in coil is sinusoidal**: $u_{i,c}(t)=-d\Psi_c(t)/dt=\hat{U}_{i,c}\sin\omega t$

Voltage amplitude:

$$\hat{U}_{i,c}=\omega N_c\Phi_c=2\pi f N_c\frac{2}{\pi}\tau_p l\hat{B}_{\delta 1}$$

(full-pitched coil)

Induced voltage by fundamental and harmonic waves

- Rotating rotor field (speed *n*): is a *FOURIER*-sum of **fundamental** and **harmonic waves**:

$$B_{\delta,\mu}(x,t) = \hat{B}_{\delta\mu}\cos\left(\frac{\mu x\pi}{\tau_p} - \mu\cdot\omega\cdot t\right), \quad \mu = 1, 3, 5, 7, \ldots \quad \omega = 2\pi\cdot n\cdot p$$

- AC coil flux: $\Phi_{c\mu}(t) = l\int_{-\tau_p/2}^{\tau_p/2} B_{\delta,\mu}(x,t)dx = \frac{2}{\pi}\cdot\frac{\tau_p}{\mu} l\hat{B}_{\delta\mu}\cdot\sin\left(\frac{\mu\pi}{2}\right)\cdot\cos(\mu\omega t)$

- Induced voltage: $u_{i,c,\mu} = -N_c\frac{d\Phi_{c\mu}}{dt} = \mu\omega\cdot N_c\cdot\frac{2}{\pi}\frac{\tau_p}{\mu} l\hat{B}_{\delta\mu}\cdot\sin\left(\frac{\mu\pi}{2}\right)\cdot\sin(\mu\omega t)$

Facit:

In stator coil not only "useful" voltage due to fundamental (frequency $f = n\cdot p$) is induced, but also harmonic AC voltages with smaller amplitudes, but increased frequencies.

- **Smaller** voltage amplitudes proportional $\hat{B}_{\delta\mu}$, **harmonic frequencies** $f_\mu = \mu\omega/(2\pi)$.

Note: $\sin(\mu\pi/2) = (-1)^{(\mu-1)/2}$ with μ = 1, 3, 5, ... gives *only 1, -1, 1, -1, ... Expression changes only sign, but not amplitude.*

Example: No-load voltage in full-pitched coils

- 12-pole synchronous generator: n = 500/min, $2p$ = 12, full-pitched coils,
  stator coil data: $N_c = 2$, $W = \tau_p = 0.5$ m, l = 1 m

  Fundamental frequency of induced voltage: $f = n \cdot p = (500/60) \cdot 6 = 50\text{Hz}$

- Induced harmonic voltage amplitudes depend on rotor air gap field amplitudes $\hat{B}_{\delta\mu}$:

μ	$\hat{B}_{\delta\mu}$	$\hat{B}_{\delta\mu} / \hat{B}_{\delta 1}$	f_μ	$\Phi_{c\mu}$	$U_{i,c\mu} = \hat{U}_{i,c\mu} / \sqrt{2}$	$U_{i,c\mu} / U_{i,c1}$
-	T	%	Hz	mWb	V	%
1	0.9	**100**	50	286.5	127.2	**100**
3	0.15	**16.7**	150	-15.9	-21.2	**16.7**
5	0.05	**5.6**	250	3.3	7.1	**5.6**
7	0.05	**5.6**	350	-2.3	-7.1	**5.6**

Facit: *Amplitude spectra of inducing field and induced voltage are identical: For a full-pitched coil the spatial field distribution and the time function of voltage are identical !*

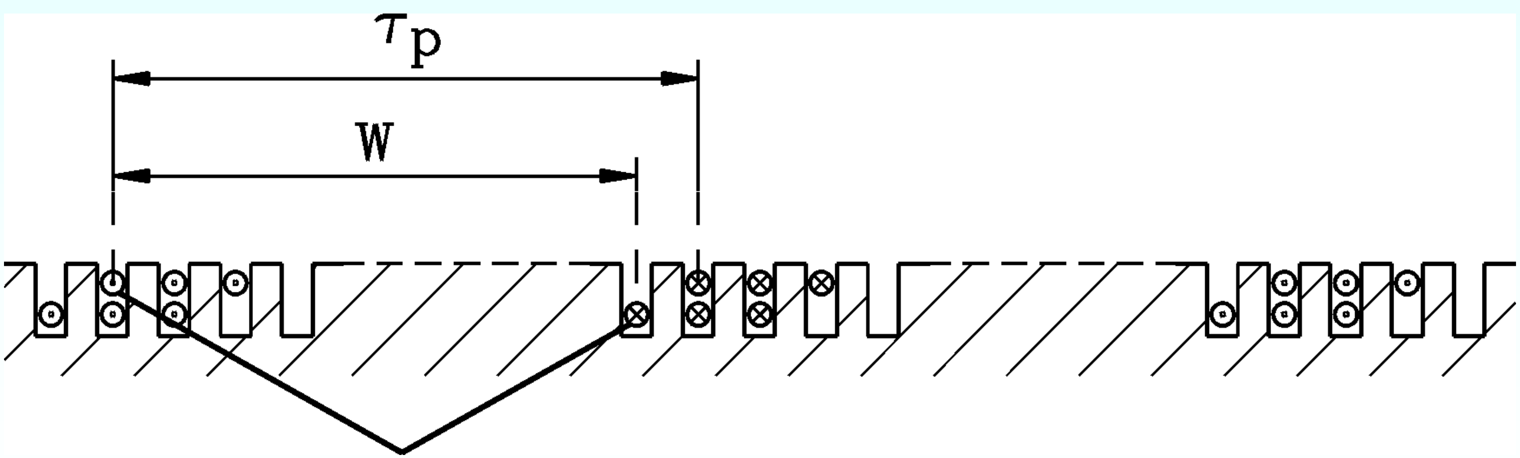
Induction of voltage in pitched coil

- Pitched coil: Coil span is only *W* instead of τ_p :

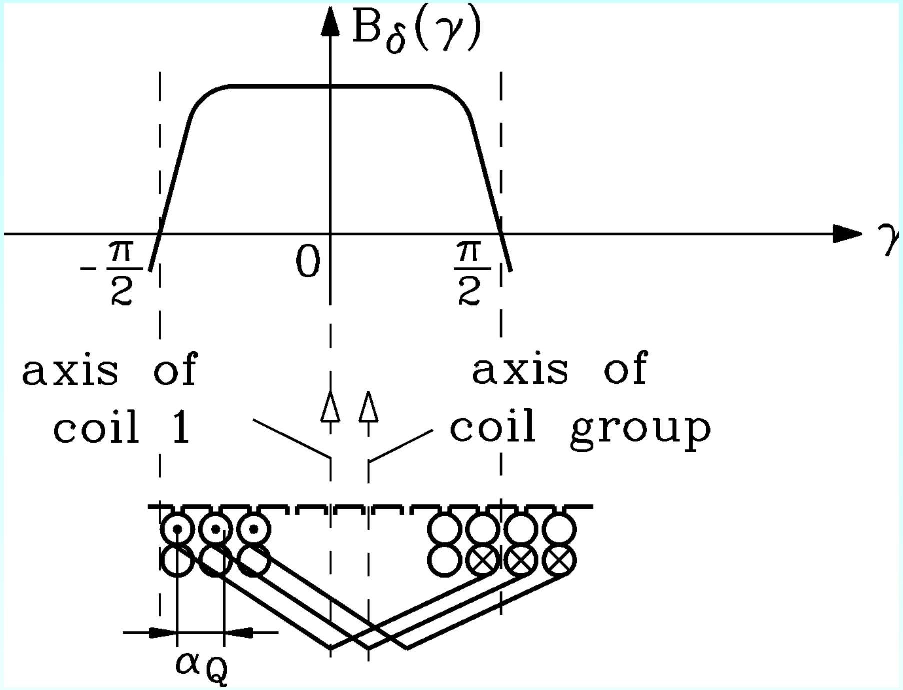
$$\Phi_{c\mu}(t) = l\int_{-W/2}^{W/2} \hat{B}_{\delta\mu}\cos\left(\frac{\mu\pi x}{\tau_p} - \mu\omega t\right)dx = \frac{2}{\pi}\frac{\tau_p}{\mu} l\hat{B}_{\delta\mu}\cdot\sin\left(\mu\frac{\pi}{2}\frac{W}{\tau_p}\right)\cdot\cos\omega t$$

Linked coil flux is smaller by **pitch coefficient** $k_{p,\mu}$, compared to full-pitched coil.

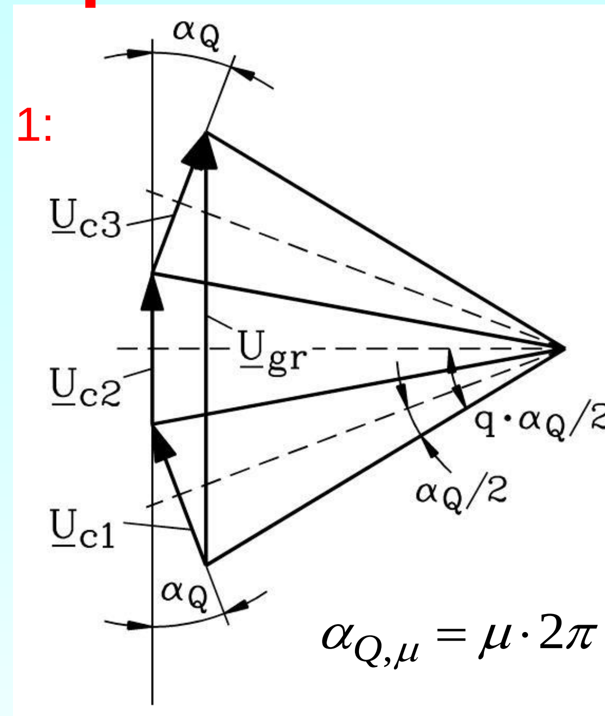
$$k_{p,\mu} = \sin\left(\mu\frac{\pi}{2}\cdot\frac{W}{\tau_p}\right)$$



Induction of voltage in group of coils



$\mu = 1$:



$$\alpha_{Q,\mu} = \mu \cdot 2\pi/(2mq)$$

• The induced sinusoidal AC voltage per coil group is the sum of complex phasors of the q coils. The coil voltage phasors are phase shifted by angle $\alpha_{Q,\mu}$ between adjacent coils:

• **Distribution coefficient:**

$$k_{d,\mu} = \frac{\hat{U}_{i,gr,\mu}}{q\hat{U}_{i,c,\mu}} = \frac{2\sin\left(q\frac{\alpha_{Q,\mu}}{2}\right)}{q\cdot 2\sin\left(\frac{\alpha_{Q,\mu}}{2}\right)} = \frac{\sin\left(\mu\frac{\pi}{2m}\right)}{q\cdot\sin\left(\mu\frac{\pi}{2mq}\right)}$$

Induced voltage per phase

- Machine with $2p$ poles, **two-layer winding: One phase consists of** $2p$ coil groups with q pitched coils per group.
- Induced voltage per phase (r.m.s. value):

Fundamental:

$$U_{i1} = \sqrt{2}\pi f \cdot N \cdot k_{w1} \cdot \frac{2}{\pi} \tau_p l \hat{B}_{\delta 1} \qquad N = 2pqN_c / a \qquad k_{w1} = k_{d1} \cdot k_{p1}$$

μ-th harmonic:

$$U_{i,\mu} = \sqrt{2}\pi \mu f \cdot N \cdot k_{w,\mu} \cdot \frac{2}{\pi} \frac{\tau_p}{\mu} l \hat{B}_{\delta\mu}$$

Example: 12-pole synchronous generator: $n = 500/\text{min}$, $2p = 12$, $f = 50$ Hz

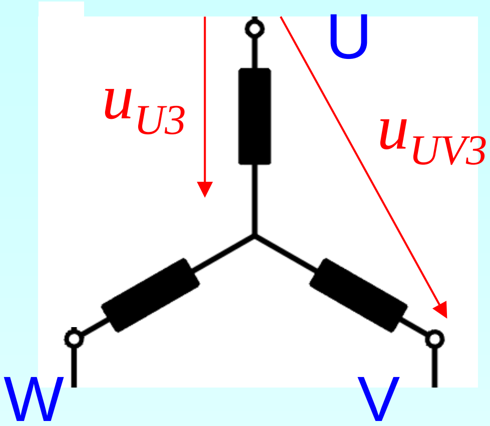
- Stator winding: $N_c = 2$, $q = 2$, $W = 5/6\tau_p$, $a = 1$, $\tau_p = 0.5$ m, $l = 1$ m

- Number of turns per phase: $N = 2pqN_c / a = 12 \cdot 2 \cdot 2 / 1 = \underline{\underline{48}}$

μ	$\hat{B}_{\delta\mu}$	$\hat{B}_{\delta\mu} / \hat{B}_{\delta 1}$	f_μ	$\Phi_{c\mu}$	$U_{i,\mu}$	$U_{i,\mu} / U_{i,1}$
-	T	%	Hz	mWb	V	%
1	0.9	**100**	50	276.7	2850.1	**100**
3	0.15	**16.7**	150	-11.3	-254.6	**8.9**
5	0.05	**5.6**	250	0.8	11.4	**0.4**
7	0.05	**5.6**	350	-0.6	-11.4	**0.4**

Facit: *By pitching and by coil group arrangement voltage harmonics are reduced drastically.*

Star connection: no „third“ voltage harmonic



$$u_{U3}(t) = \hat{U}_3 \cdot \cos(3\omega t)$$

$$u_{V3}(t) = \hat{U}_3 \cdot \cos(3(\omega t - 2\pi/3)) = \hat{U}_3 \cdot \cos(3\omega t) = u_{U3}(t)$$

$$u_{W3}(t) = \hat{U}_3 \cdot \cos(3(\omega t - 4\pi/3)) = \hat{U}_3 \cdot \cos(3\omega t) = u_{U3}(t)$$

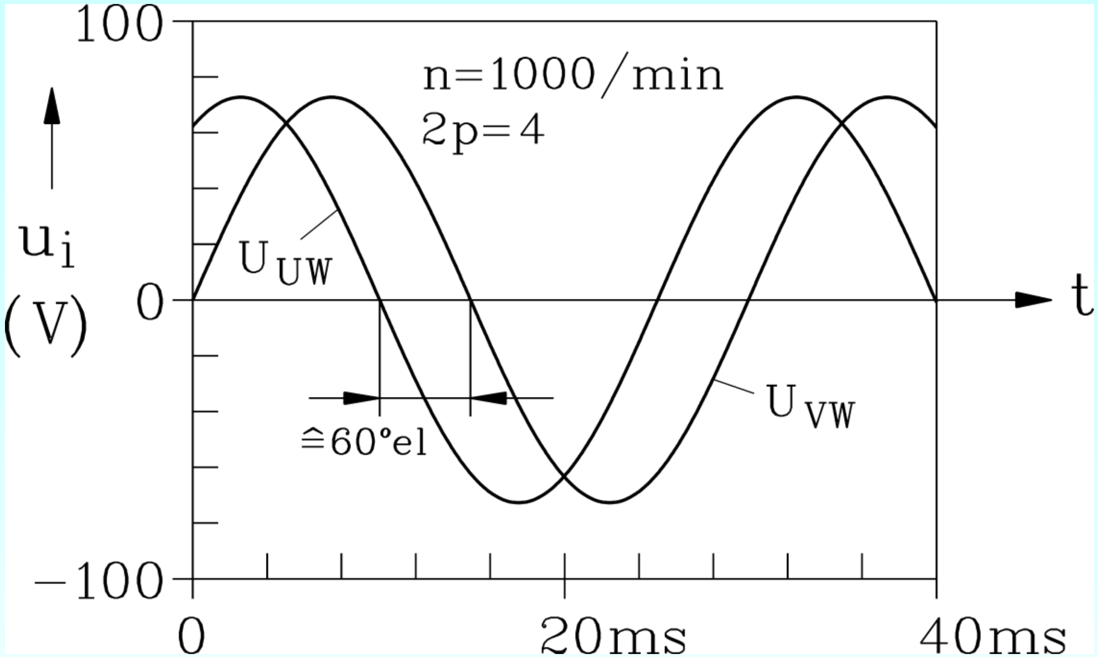
$$u_{UV3}(t) = u_{U3}(t) - u_{V3}(t) = u_{U3}(t) - u_{U3}(t) = 0$$

If the stator winding is **star connected**, the third harmonic voltages in all three phases U, V, W are IN phase and IDENTICAL !

Therefore the line-to-line voltages do not show 3rd harmonic voltage component. Phase voltages in phase cause IN PHASE 3rd harmonic currents, which CANNOT flow at isolated star point (due to 2nd *Kirchhoff*'s law)

$$\underline{I}_3 = \underline{U}_3 / \underline{Z}_3 \quad \Rightarrow \quad \underline{I}_{U3} + \underline{I}_{V3} + \underline{I}_{W3} = 3\underline{I}_3 = 0 \quad \Rightarrow \quad \underline{I}_3 = 0$$

Star connection: no „third“ voltage harmonic



Measured no-load voltage line-to-line of a 4 pole PM synchronous generator at 1000/min, *q* = 3, skewed slots, star connection, **showing nearly ideal sine wave back EMF**

Fourier-Analysis of no-load voltage: $\mu = 1$: 33.5 Hz, 74.8 V

$\mu = 5$: 167 Hz, 0.34 V

Other amplitudes $\mu > 5$ are negligible !

Three phase winding: Self induction leads to magnetizing inductance

- Stator air gap field waves, excited by stator current *I*, induce in stator winding by **self induction the voltage *u_i* !**

$$B_{\delta\nu}(x,t)=\hat{B}_{\delta\nu}\cdot\cos\left(\frac{\nu\pi x}{\tau_p}-\omega t\right)\qquad \hat{B}_{\delta\nu}=\frac{\mu_0}{\delta}\frac{\sqrt{2}}{\pi}\frac{m}{p}N\frac{k_{w,\nu}}{\nu}I\qquad \nu=1,-5,7,-11,13,\cdots$$

- Stator air gap field waves $B_{\delta\nu}(x,t)$: Speed n_ν is n_{syn}/ν. Hence stator field fundamental and field harmonics induce in stator coils **ALL with the same frequency *f* .**

$$f_\nu=\nu\cdot p\cdot(n_{syn}/\nu)=p\cdot n_{syn}=f$$

- r.m.s. of self-induced voltage per phase for each ν-th field harmonic:

$$U_{i,\nu}=\sqrt{2}\pi f\cdot N\cdot k_{w,\nu}\cdot\frac{2}{\pi}\frac{\tau_p}{\nu}l\hat{B}_{\delta\nu}$$

- **Magnetizing inductance per phase**: $L_{h\nu}$ for ν-th air gap field harmonic wave.

$$U_{i,\nu}=\omega L_{h\nu}I$$

$$\boxed{L_{h\nu}=\mu_0 N^2\frac{k_{w,\nu}^2}{\nu^2}\frac{2m}{\pi^2}\frac{l\tau_p}{p\cdot\delta}}$$

Stray inductance of stator winding per phase

$$L_\sigma = L_{\sigma,Q} + L_{\sigma,b} + L_{\sigma,o}$$

- Air gap field: Fundamental wave = **Magnetizing field (subscript h):** $L_h = L_{h,\nu=1}$

  Magnetizing inductance L_h

- Magnetic field in **slots** (slot stray field) and around the **winding overhang** is NOT linked with rotor winding. It does NOT produce any forces with rotor current. Hence it does NOT contribute to electromechanical energy conversion, and is thus called **stray field (subscript σ).**

- Stray flux induces in stator winding additional voltage by self induction. Hence we define:

  Slot stray inductance $L_{\sigma Q}$, overhang stray inductance $L_{\sigma b}$: $U_{i\sigma,Q+b} = \omega(L_{\sigma Q} + L_{\sigma b})I$

- Air gap field harmonic waves induce stator winding with voltage $U_{i,\nu}$ with the same frequency *f*. So they are summarized as total harmonic voltage : $\sum_{\nu=1,-5,7,...}^{\infty} U_{i,\nu}$

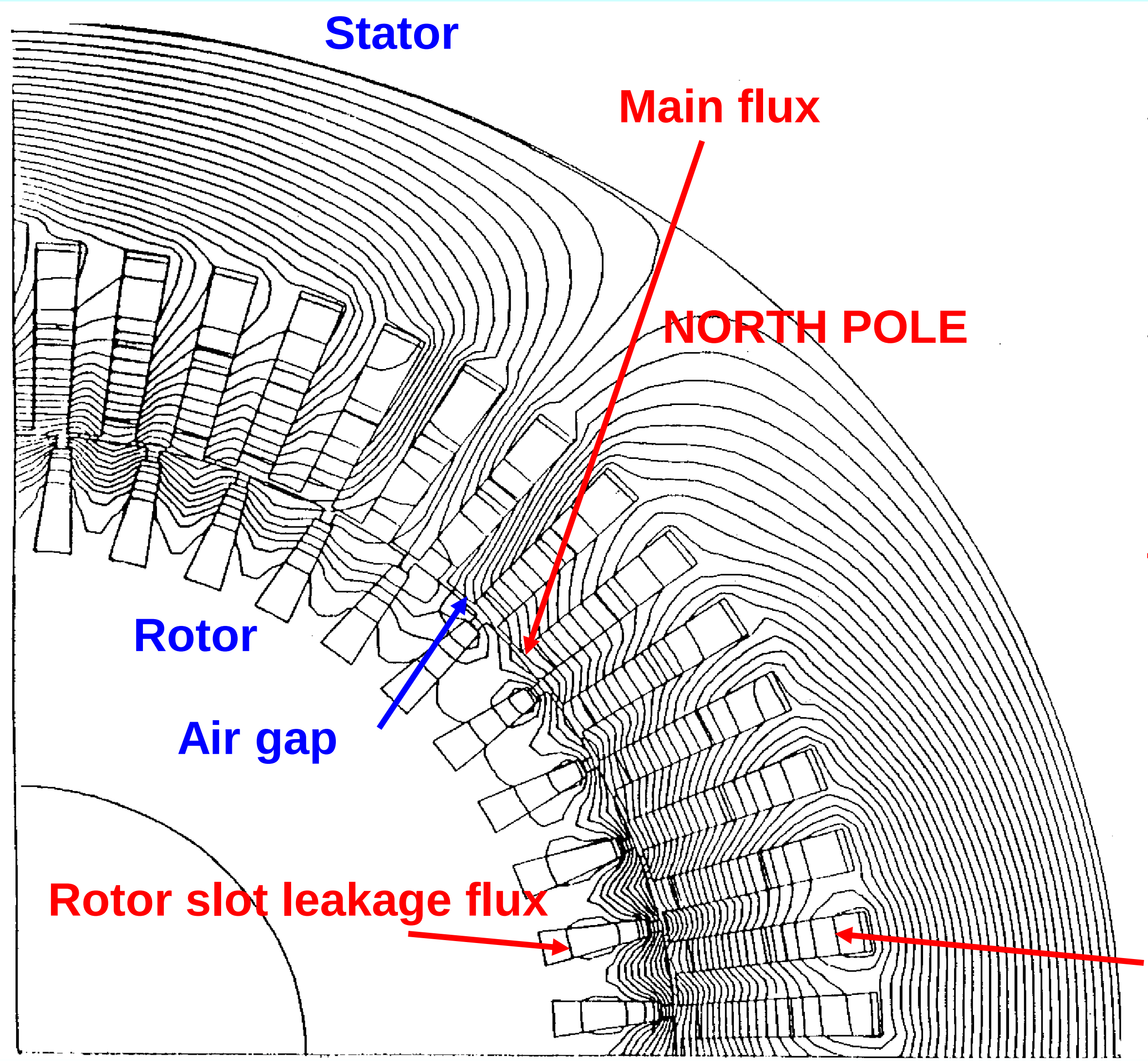
$$L_{h,total} = \frac{\sum_{\nu=1,-5,7,...}^{\infty} U_{i,\nu}}{\omega I} = \sum_{\nu=1,-5,7,...}^{\infty} L_{h\nu} = (1+\sigma_o)L_{h,\nu=1}$$

$$\sigma_o = \sum_{\nu=1,-5,7,..}^{\infty} \left(\frac{k_{w,\nu}}{\nu \cdot k_{w,1}} \right)^2 - 1$$

σ_o : **harmonic stray coefficient** (is small: ca. 0.03 ... 0.09).

- Harmonic field waves are linked to rotor, but "disturbe" basic machine function; hence they are summarized in **harmonic stray inductance $L_{\sigma o}$** : $U_{i\sigma,o} = \omega L_{\sigma,o} I, \quad L_{\sigma,o} = \sigma_o L_h$

Field lines *B* of a cage induction machine



Main flux: Links stator and rotor winding; field lines cross the air gap

Leakage flux (stray flux): Is only linked with either stator or rotor winding; field lines DO NOT cross the air gap

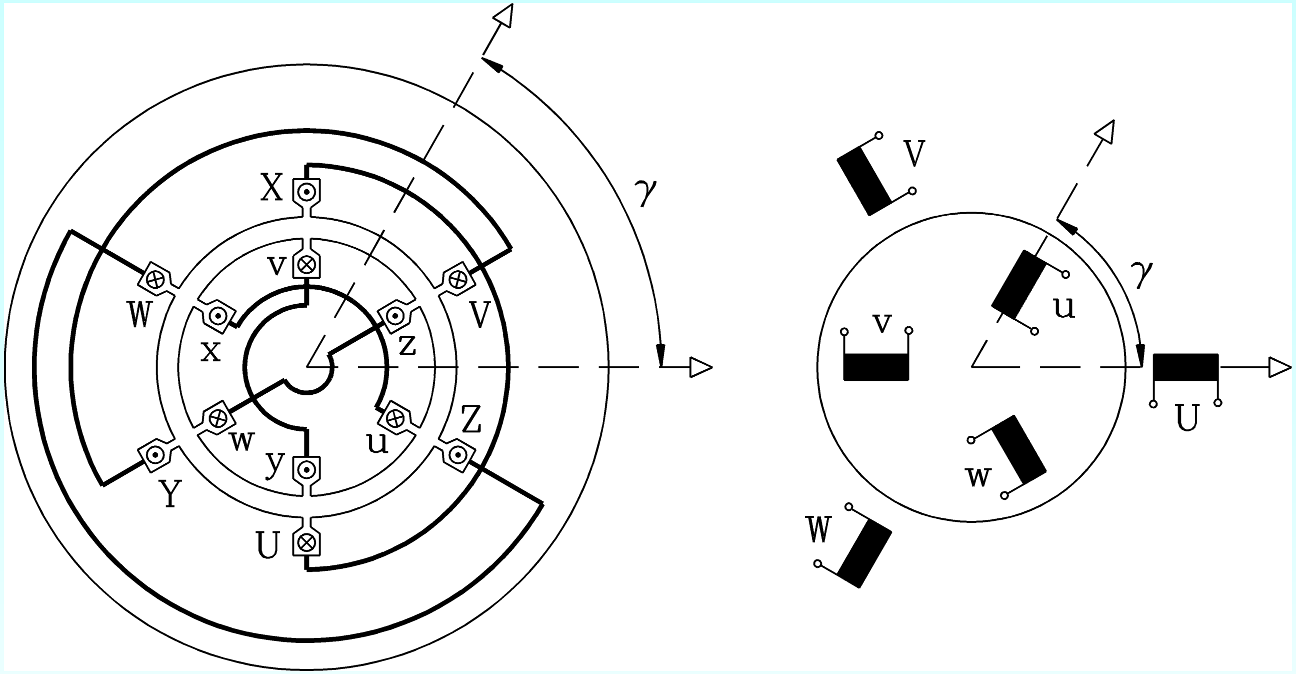
Example:

Four-pole wedge bar rotor:

Field lines at stand still ($n = 0$)

- Rotor frequency = Stator frequency
- Rotor current is NEARLY in phase opposition to stator current

Three phase winding in stator and rotor



- In stator and in rotor each a three-phase winding is arranged:

- in stator: 3 phases between terminals U-X, V-Y, W-Z, subscript *s*,
- in rotor: 3 phases between terminals u-x, v-y, w-z, subscript *r*.

- We assume: **Rotor** is at rest (stand still), and is turned by angle γ with respect to **stator.**

γ = angle between winding axis of stator and rotor winding (= centre of coils).

NOTE: $\gamma = 2\pi$, if rotor is shifted to stator by 2 poles: $2\tau_p$.

- Pole numbers of stator and rotor winding must be identical $2p$!

Mutual inductance between stator and rotor phase

	Stator	Rotor
Pole count	$2p$	$2p$
Phase count	m_s	m_r
Turns/Phase	N_s	N_r
Pitching	W_s/τ_p	W_r/τ_p
Coils/group	q_s	q_r
Slot count	Q_s	Q_r

From now on only fundamental field waves considered !

$Q_r \neq Q_s$

- **Mutual inductance:** e. g.: Stator air gap wave $B_\delta(x,t)$ induces voltage in rotor winding:

$B_\delta(x,t) = \hat{B}_\delta \cdot \cos(\frac{\pi x}{\tau_p} - \omega_s t)$ with amplitudes $\hat{B}_\delta = \frac{\mu_0}{\delta}\frac{\sqrt{2}}{\pi}\frac{m_s}{p} N_s k_{ws} I_s$

- Amplitudes of induced voltages in rotor winding:

$U_{i,r} = \sqrt{2}\pi f_s \cdot N_r \cdot k_{wr} \cdot \frac{2}{\pi}\tau_p l \hat{B}_\delta$ Rotor frequency f_r (at **locked** rotor = stand still): $f_r = f_s$.

- *Fundamental* wave: **Mutual inductance per phase M_{sr} :** $U_{i,r} = \omega_s M_{sr} I_s$

$$M_{sr} = \mu_0 N_s k_{w,s} N_r k_{w,r} \frac{2m_s}{\pi^2}\frac{1}{p}\frac{\tau_p l}{\delta}$$

Note: $M_{sr} = M_{rs}$ at $m_s = m_r$!

Rotary transformer

- Induced rotor voltages are **phase shifted** by angle γ with respect to stator voltages, as rotor is shifted by that angle γ mechanically.

- Series connection of stator and rotor winding U and u (in the same way: V and v; W and w)

$\Rightarrow$ The following resulting voltage occurs between FIRST terminal of stator winding and SECOND terminal of rotor winding (per phase):

$$\underline{U} = \underline{U}_s + \underline{U}_r = U_s + U_r e^{-j\gamma}, \quad \text{e. g. } U_r = U_s: \quad \underline{U} = U_s + U_s e^{-j\gamma} = U_s \cdot \left(1 + e^{-j\gamma}\right)$$

- By turning the rotor we get a continuous change of angle γ.

Facit: *With rotary transformer a **continuous change** of output voltage between 0 and 2U_s is possible at constant line frequency, which is used in test facilities as variable voltage source.*