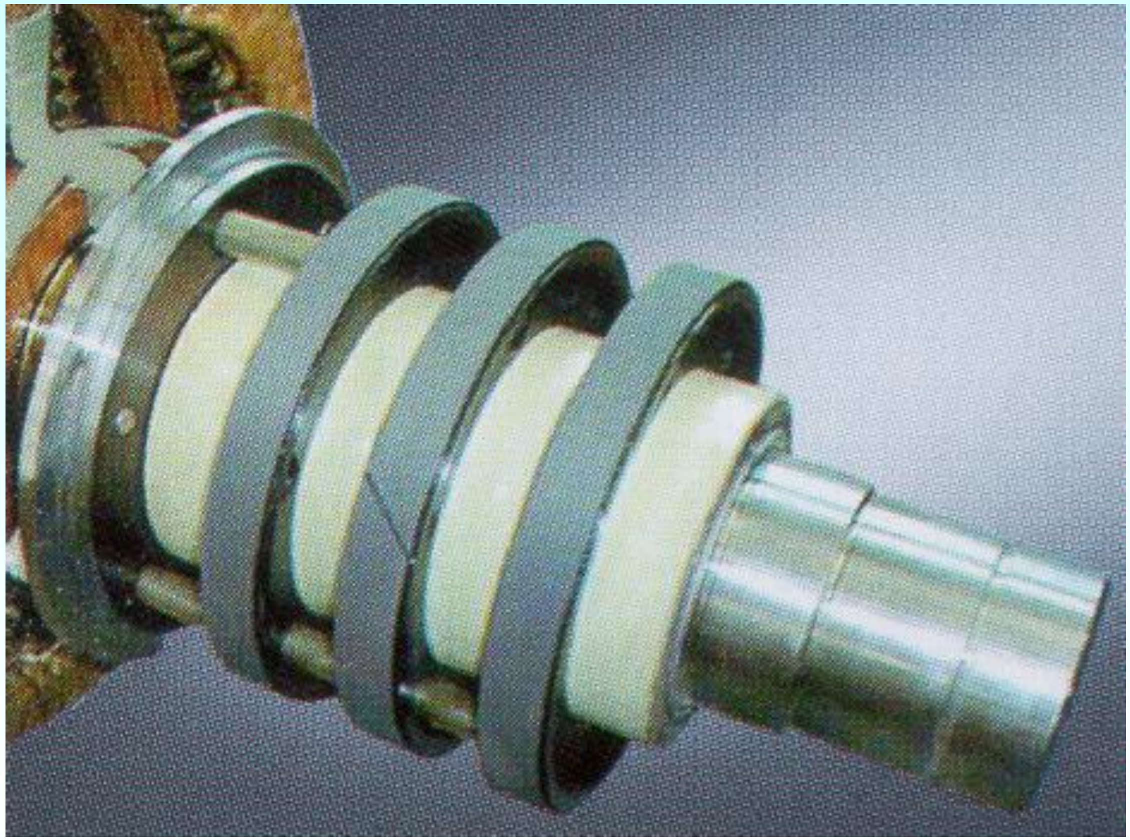
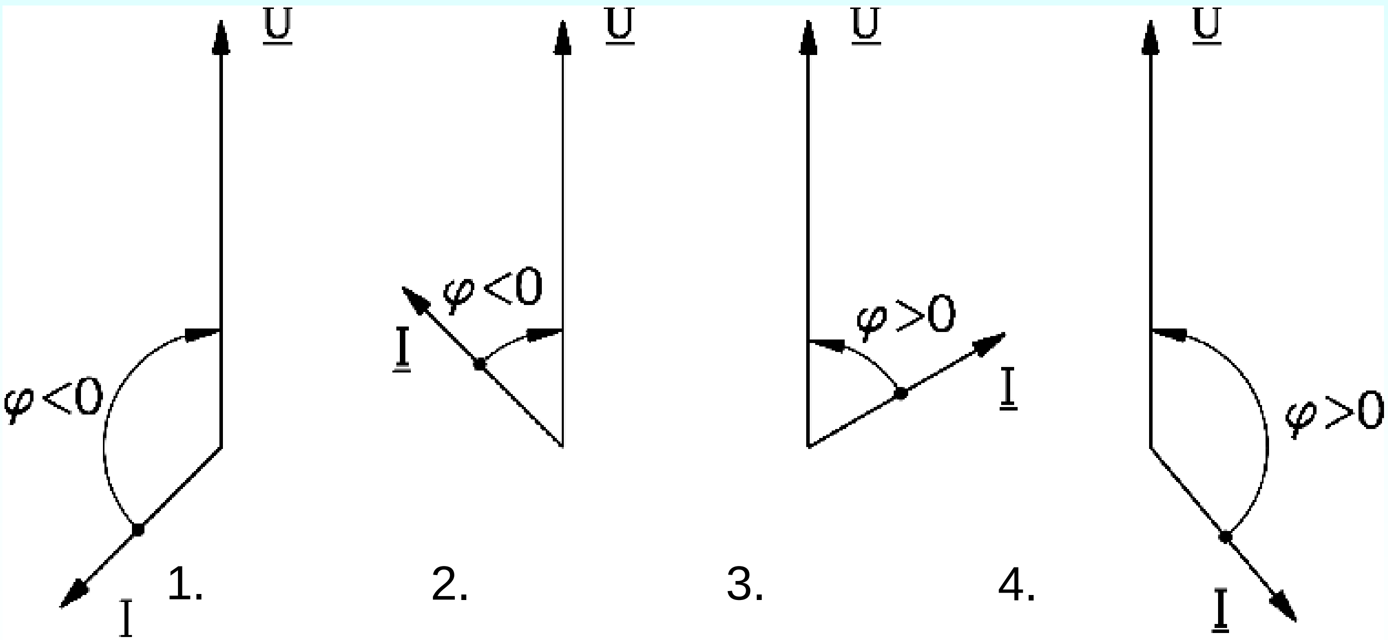
5. The Slip-Ring Induction Machine



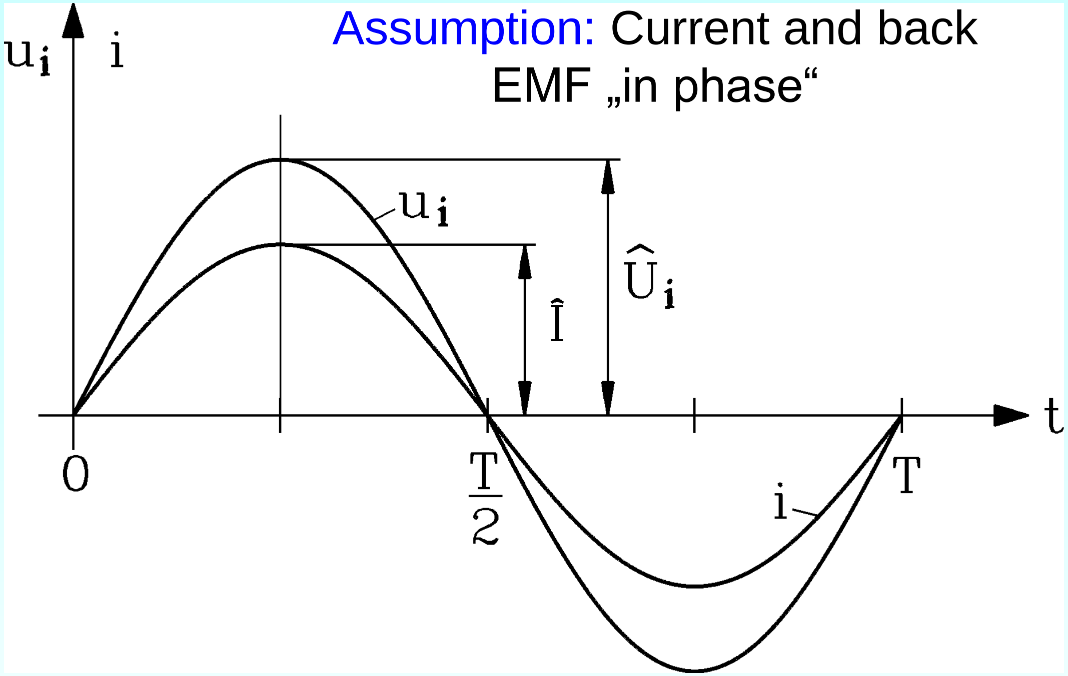
Source: Siemens AG

Active and reactive power in consumer arrow system

		Active power $P = mUI\cos\varphi$	Reactive power $Q = mUI\sin\varphi$
1.	$-180° \le \varphi < -90°$	$P < 0$, Generator	$Q < 0$, capacitive consumer
2.	$-90° \le \varphi < 0°$	$P > 0$, Motor	$Q < 0$, capacitive consumer
3.	$0 \le \varphi < 90°$	$P > 0$, Motor	$Q > 0$, inductive consumer
4.	$90° \le \varphi < 180°$	$P < 0$, Generator	$Q > 0$, inductive consumer



Torque generation with sine wave current feeding



Sinusoidal phase current
Sinusoidal back EMF, resulting in

a) pulsating power per phase, but

b) smooth constant power and constant torque for all three phases.

Using internal power per phase we get constant resulting power:

$$p_\delta(t) = \hat{U}_p \cos(\omega t)\cdot \hat{I}\cos(\omega t) + \hat{U}_p \cos(\omega t - 2\pi/3)\cdot \hat{I}\cos(\omega t - 2\pi/3) + \hat{U}_p \cos(\omega t - 4\pi/3)\cdot \hat{I}\cos(\omega t - 4\pi/3)$$

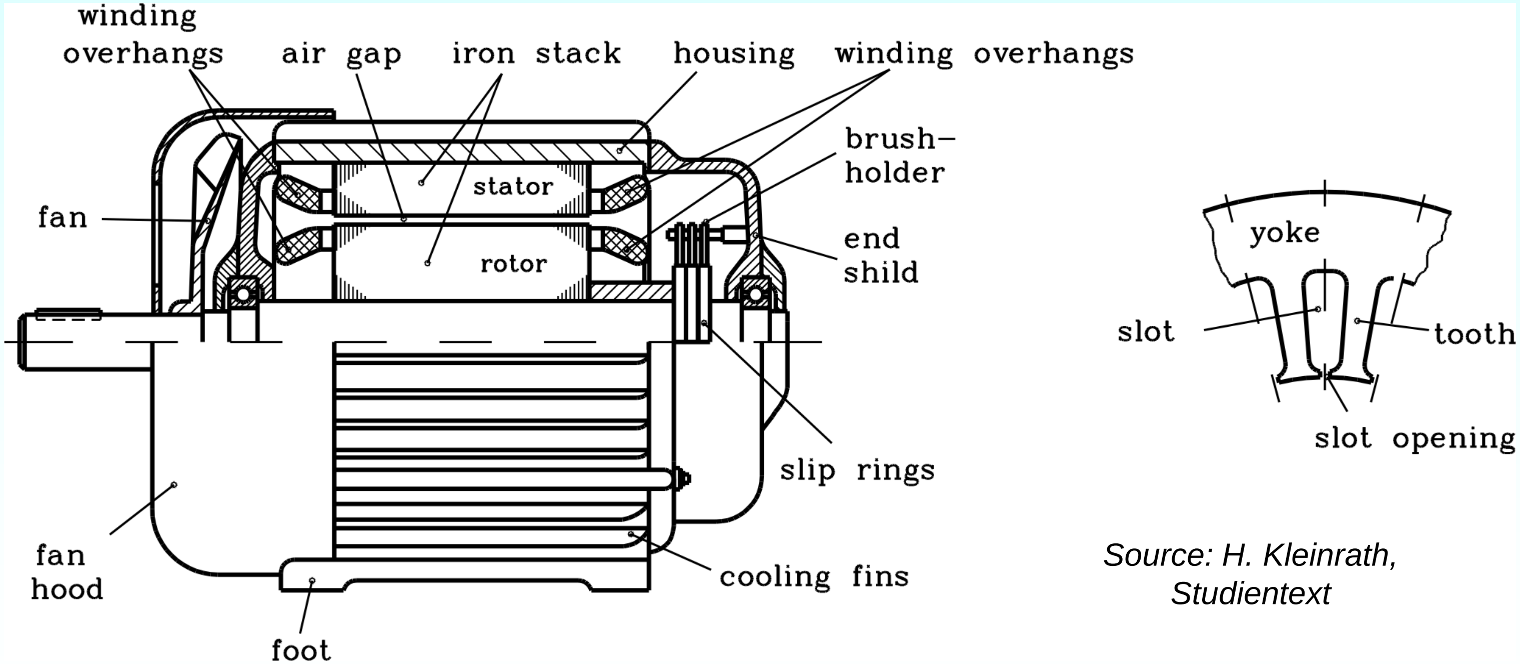
$$p_\delta(t) = \frac{\hat{U}_p \hat{I}}{2}\cdot[\cos(2\omega t)+1] + \frac{\hat{U}_p \hat{I}}{2}\cdot\left[\cos(2\omega t - \frac{4\pi}{3})+1\right] + \frac{\hat{U}_p \hat{I}}{2}\cdot\left[\cos(2\omega t - \frac{8\pi}{3})+1\right]$$

$$p_\delta(t) = m\frac{\hat{U}_p \hat{I}}{2} = const.$$

$$M_e = \frac{(3/2)\cdot \hat{U}_p \cdot \hat{I}}{2\cdot \pi \cdot n}$$

Slip ring Induction machine

- Stator and rotor house a three phase distributed AC winding
- The three rotor phases are **short circuited**
- The 3 stator phases are fed by three phase voltage & current system (I_s, frequency f_s), and excite fundamental air gap field (amplitude $B_{\delta,s}$), which rotates with speed n_{syn} .
- If **rotor turns with $n \neq n_{syn}$** (= **ASYNCHRONOUSLY)**, then $B_{\delta,s}$ induces in rotor winding the voltage U_{rh} per phase, which drives rotor phase current $I_{c,r}$.
- Rotor phase current and stator air gap field produce via *LORENTZ*-force the **torque M_e.**

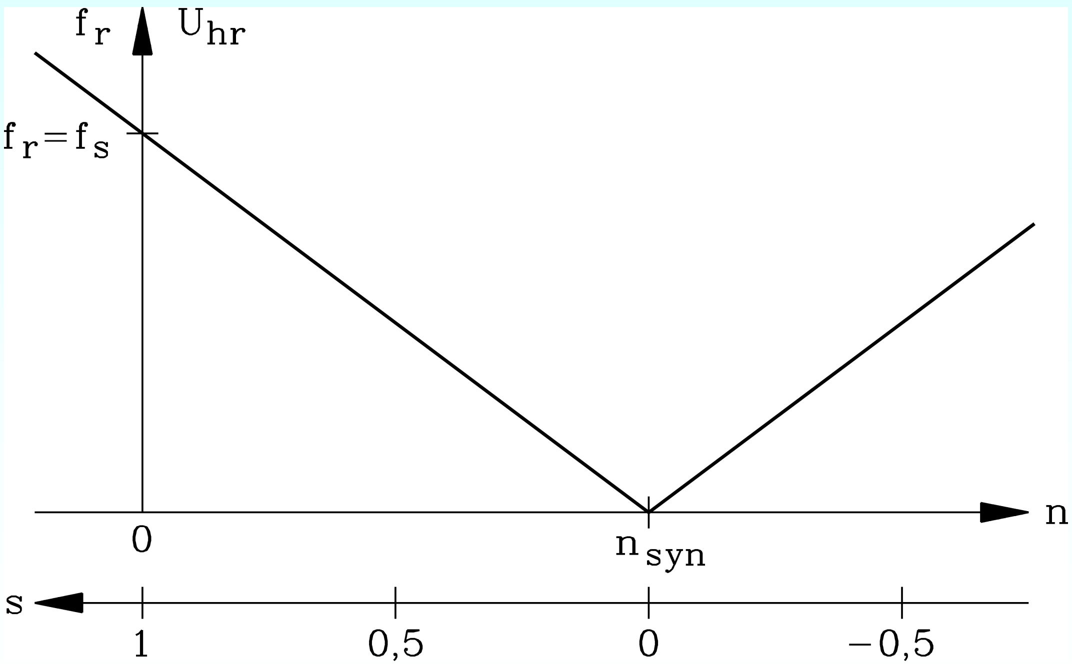


Source: H. Kleinrath, Studientext

Rotor frequency and slip

- In rotary transformer (n = 0) rotor voltage is phase shifted to stator voltage, depending on rotor position angle γ_r : $\underline{U}_{rh} e^{j\omega_s t} = U_{rh} \cdot e^{-j\gamma_r} \cdot e^{j\omega_s t}$
- When rotor turns with n = const. > 0, rotor position angle will increase continuously:
- $\gamma_r = p \cdot 2\pi n \cdot t + \gamma_{r0} \Rightarrow \underline{U}_{rh} e^{j\omega_r t} = U_{rh} \cdot e^{j(-2\pi \cdot n \cdot p + \omega_s)t} \cdot e^{-j\gamma_{r0}}$

Rotor frequency $$f_r = f_s - n \cdot p$$



- **Slip *s*** (Definition):

$$f_r = s \cdot f_s$$

$$s = \frac{f_s / p - n}{f_s / p}$$

$$s = \frac{n_{syn} - n}{n_{syn}}$$

Rotor voltage equation

- CONSTANT speed = CONSTANT frequencies = STATIONARY machine performance = only sinusoidal time functions of current and voltage = complex phasor calculus is used

$$i_s(t) = \sqrt{2} I_s \cos(\omega_s t) = \mathrm{Re}\left(\sqrt{2}\underline{I}_s e^{j\omega_s t}\right) \quad \Rightarrow \quad i_s(t) \leftrightarrow \underline{I}_s$$

- Rotor winding short circuited: $u_r = 0$: (R_r: winding resistance per rotor phase)

$$R_r \cdot i_r = u_r + u_{i,r} = u_r - d\Psi_r / dt \quad \Rightarrow \quad R_r \cdot i_r + d\Psi_r / dt = u_r = 0$$

Ψ_r : total flux linkage of one rotor phase:

a) **Mutual induction** of **stator rotating field** into rotor winding: $j\omega_r M_{sr} \underline{I}_s$

b) **Self induction** by **rotor rotating air gap field**: Rotor AC currents per phase I_r , oscillating with rotor frequency f_r , excite rotor rotating field !

$$B_{\delta,r}(x_r, t) = \hat{B}_{\delta,r} \cos(\gamma_r - \omega_r t), \quad B_{\delta,r} \sim I_r$$

and induce a voltage into rotor phase winding $j\omega_r L_{rh} \underline{I}_r$

c) **Self induction** by rotor stray field with 3 components: $L_{r\sigma} = L_{r,\sigma Q} + L_{r,\sigma b} + \sigma_{r,o} L_{rh}$

- **harmonic fields:** $j\omega_r \sigma_{r,o} L_{rh} \underline{I}_r$,**slot stray field** $L_{r,\sigma Q}$, **winding overhang stray field** $L_{r,\sigma b}$

$$j\omega_r M_{sr} \underline{I}_s + j\omega_r L_{rh} \underline{I}_r + j\omega_r (\sigma_{r,o} L_{rh} + L_{r,\sigma Q} + L_{r,\sigma b}) \underline{I}_r + R_r \underline{I}_r = 0$$

Stator voltage equation

- **Mutual induction:** Rotor field $B_{\delta,r}$ rotates relatively to stator with synchronous speed:

$$v = v_m + v_{r,syn} = 2pn\tau_p + 2f_r\tau_p = 2p \cdot n_{syn}(1-s) \cdot \tau_p + 2 \cdot sf_s \cdot \tau_p$$

$$v = 2p \cdot \frac{f_s}{p} \cdot (1-s) \cdot \tau_p + 2 \cdot sf_s \cdot \tau_p = 2f_s\tau_p = v_{syn}$$

Hence it induces the stator winding with stator frequency f_s: $j\omega_s M_{rs} \underline{I}_r$

- **Self induction:** Stator air gap field $B_{\delta,s} \Rightarrow$ leads to self induced stator voltage: $j\omega_s L_{sh} \underline{I}_s$
- **Self induction** by stator stray fields (3 components): $L_{s\sigma} = L_{s,\sigma Q} + L_{s,\sigma b} + \sigma_{s,o} L_{sh}$

  Harmonic fields: $j\omega_s \sigma_{s,o} L_{sh} \underline{I}_s$, **slot stray field** $L_{s,\sigma Q}$, **winding overhang stray field** $L_{s,\sigma b}$
- resistive voltage drop at **stator winding resistance** R_s

- Sum of all stator voltage components must balance the voltage at the winding terminals $\underline{U}_s$ (voltage per phase), which is impressed by the feeding grid !

Stator voltage equation:

$$\underline{U}_s = j\omega_s M_{rs} \underline{I}_r + j\omega_s L_{sh} \underline{I}_s + j\omega_s (\sigma_{s,o} L_{sh} + L_{s,\sigma Q} + L_{s,\sigma b}) \underline{I}_s + R_s \underline{I}_s$$

Transfer ratio

- **Transfer ratio *ü*** from stator to rotor winding: $$\ddot{u} = \frac{k_{w,s}N_s}{k_{w,r}N_r}$$

  we get with $m_r = m_s = m$ (= 3) :

$$\ddot{u}^2 L_{rh} = \left(\frac{k_{w,s}N_s}{k_{w,r}N_r}\right)^2 \cdot \mu_0 N_r^2 k_{w,r}^2 \cdot \frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = \mu_0 N_s^2 k_{w,s}^2 \cdot \frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = L_{sh}$$

$$\ddot{u} \cdot M_{sr} = \frac{k_{w,s}N_s}{k_{w,r}N_r} \cdot \mu_0 \cdot N_s k_{w,s} \cdot N_r k_{w,r} \cdot \frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = \mu_0 N_s^2 k_{w,s}^2 \cdot \frac{2m}{\pi^2}\frac{l\tau_p}{p\delta} = L_{sh}$$

$$L_{sh} = \ddot{u}M_{sr} = \ddot{u}^2 L_{rh} = \underline{\underline{L_h}}$$

Magnetizing inductance L_h $(m_r = m_s : M_{rs} = M_{sr})$

- *ü* in rotor voltage equation:

$$j\omega_r \ddot{u} M_{sr}\underline{I}_s + j\omega_r \ddot{u}^2 L_{r,h} \cdot (\underline{I}_r/\ddot{u}) + j\omega_r \ddot{u}^2 L_{r\sigma} \cdot (\underline{I}_r/\ddot{u}) + \ddot{u}^2 R_r \cdot (\underline{I}_r/\ddot{u}) = 0$$

$$R'_r = \ddot{u}^2 R_r \qquad L'_{r\sigma} = \ddot{u}^2 L_{r\sigma} \qquad I_r/\ddot{u} = I'_r \qquad \ddot{u}U_r = U'_r$$

Rotor voltage equation with *ü*: $(\omega_r = s\omega_s)$

$$js\omega_s L_h \underline{I}_s + js\omega_s L_h \underline{I}'_r + js\omega_s L'_{r\sigma}\underline{I}'_r + R'_r \underline{I}'_r = 0$$

Equivalent circuit diagram

- Introducing transfer ratio *ü* in **stator voltage equation**:

$$\underline{U}_s = j\omega_s \cdot \ddot{u}M_{sr} \cdot (\underline{I}_r / \ddot{u}) + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s$$

- $\underline{U}_s = j\omega_s L_h \underline{I}'_r + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s$

$$0 = js\omega_s L_h \underline{I}_s + js\omega_s L_h \underline{I}'_r + js\omega_s L'_{r\sigma} \underline{I}'_r + R'_r \underline{I}'_r$$

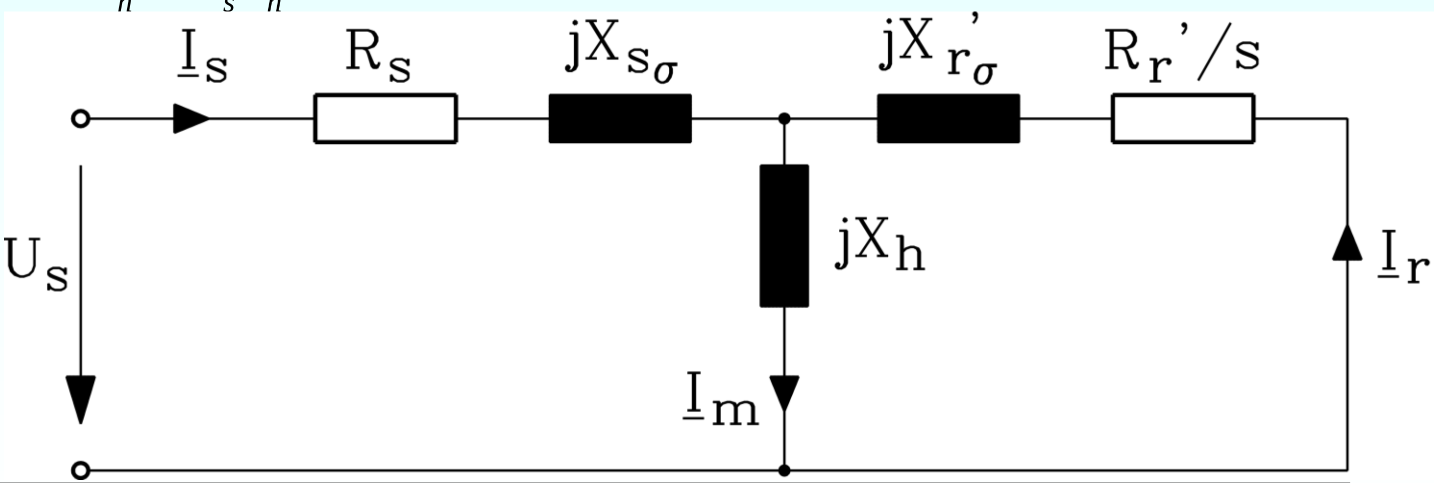
$$\boxed{\underline{U}_s = R_s \underline{I}_s + jX_{s\sigma} \underline{I}_s + jX_h(\underline{I}_s + \underline{I}'_r)}$$

$$\boxed{0 = \frac{R'_r}{s} \underline{I}'_r + jX'_{r\sigma} \underline{I}'_r + jX_h(\underline{I}_s + \underline{I}'_r)}$$

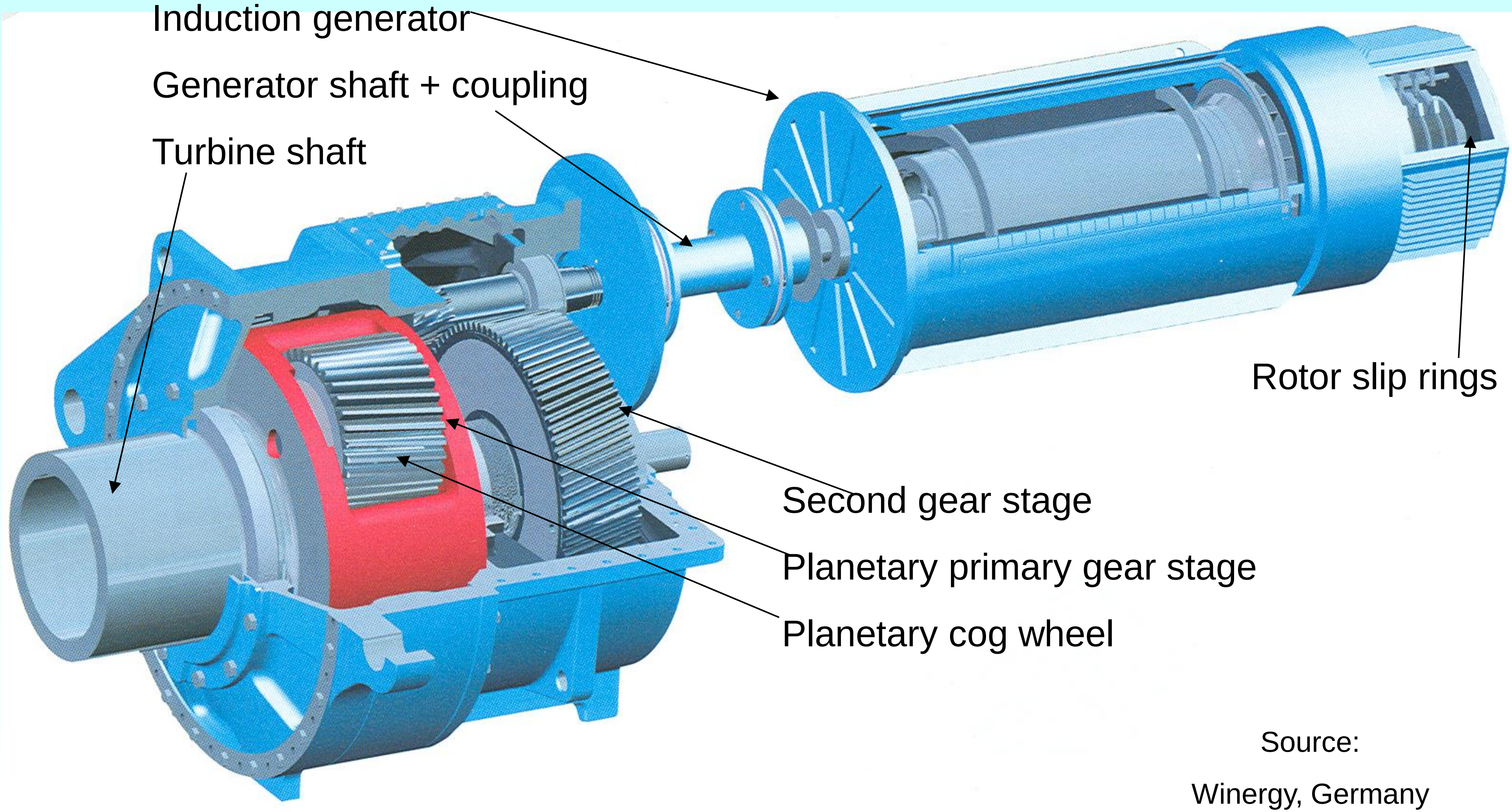
Stator leakage reactance: $X_{s\sigma} = \omega_s L_{s\sigma}$, **Rotor leakage reactance:** $X'_{r\sigma} = \omega_s L'_{r\sigma}$

Magnetizing reactance: $X_h = \omega_s L_h$

- **T-equivalent circuit:**



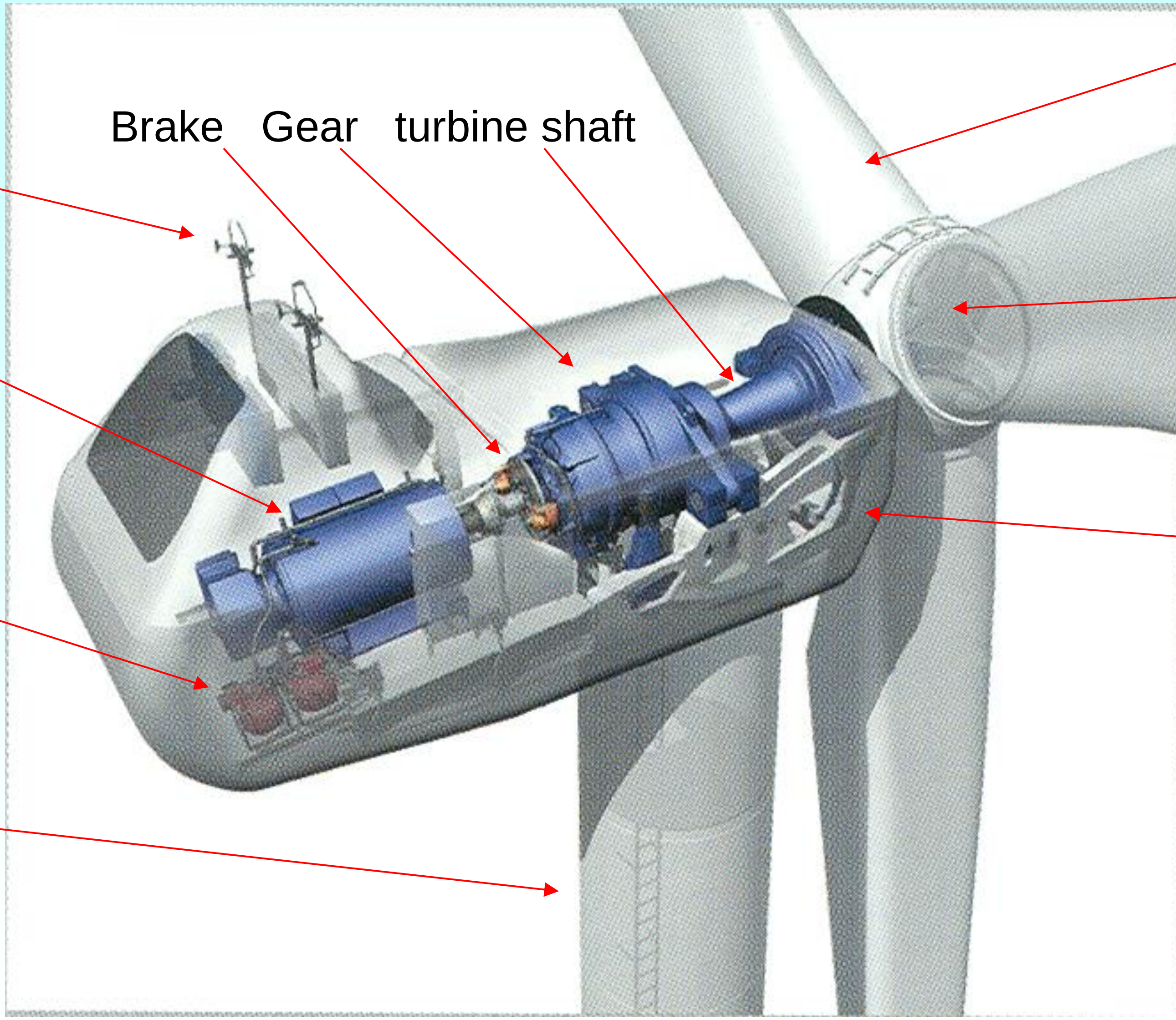
Geared doubly-fed induction wind generator



Source:

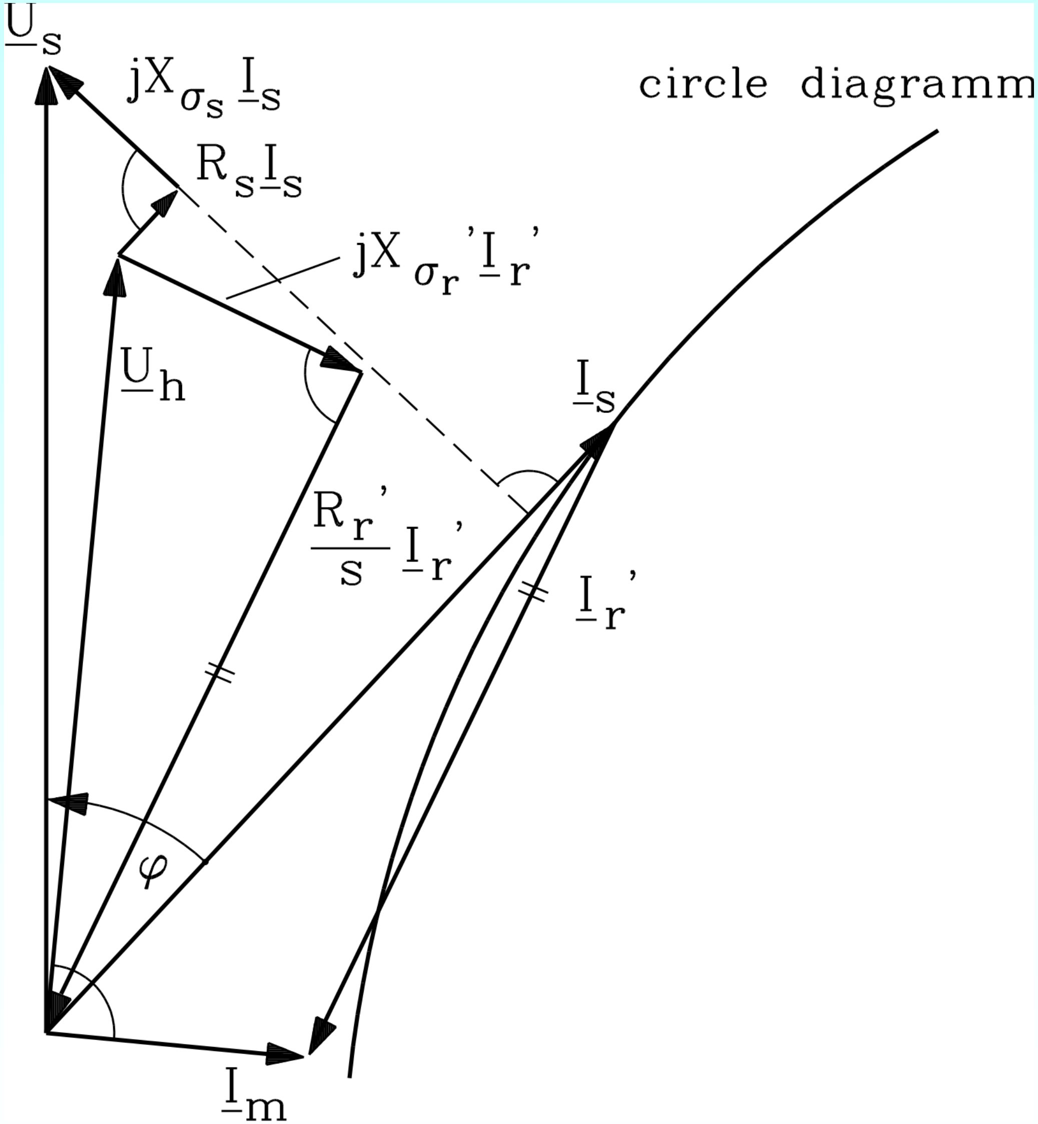
Winergy, Germany

Wind converter assembly



Source:
Winergy
Germany

Phasor diagram (per phase)



- **"Magnetizing current" :** represents the resulting effect of stator and rotor air gap field (= resulting magnetizing air gap field).

$$\underline{I}_m = \underline{I}_s + \underline{I}'_r$$

- **Internal voltage $\underline{U}_h$** : = Results from self and mutual induction in stator and rotor winding due to resulting magnetizing air gap field (“main flux”)

$$\underline{U}_h = j\omega_s L_h \cdot \underline{I}_m$$

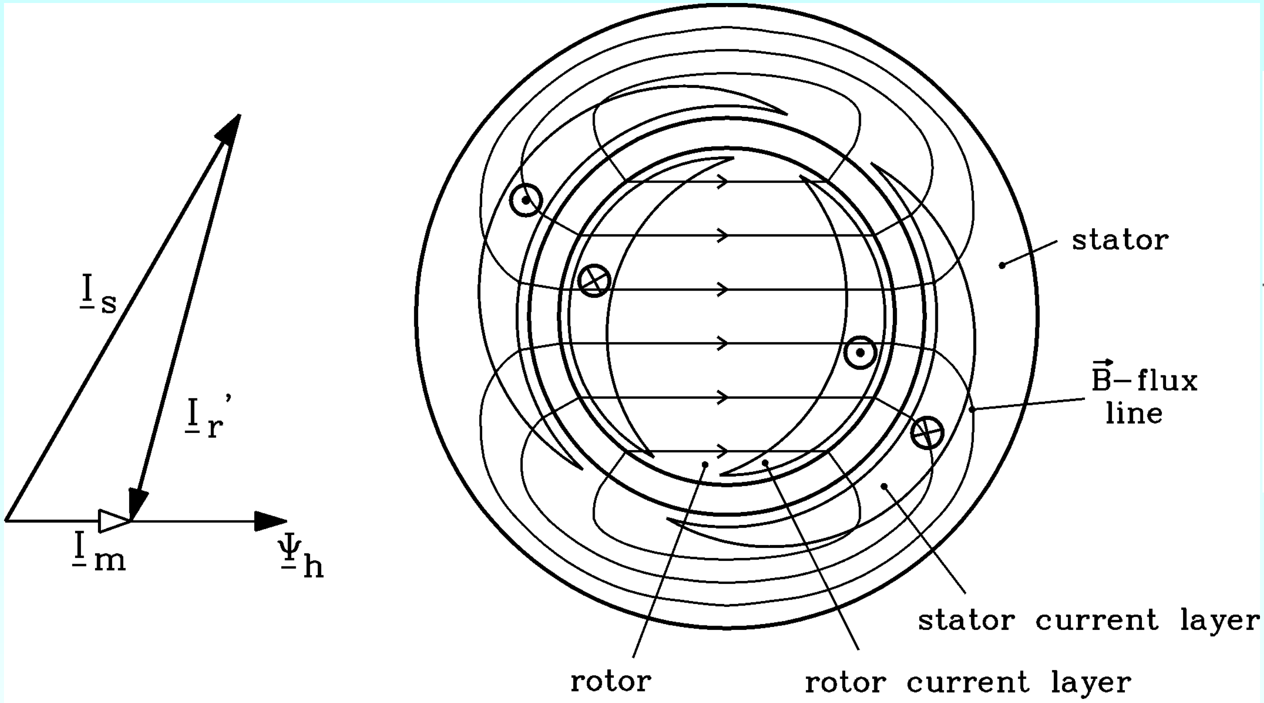
- Actually in rotor voltage & current change with rotor frequency. By dividing with slip *s*

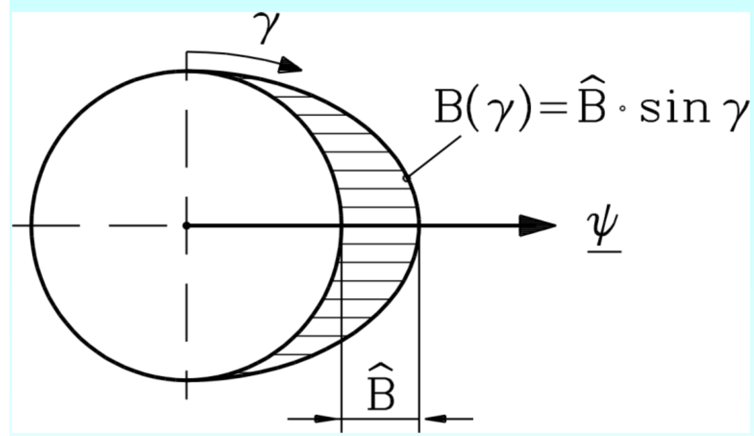
$$\underline{U}_{hr} / s = j \cdot s \cdot \omega_s L_h \cdot \underline{I}_m / s = \underline{U}_h$$

stator frequency appears in rotor equation.

- **Hence we get fictive rotor resistance** R'_r / s

Magnetizing air gap field and magnetizing current





- Stator and rotor fundamental air gap field may be regarded as excited by **sinusoidal distributed current stator and rotor load.** The superposition of both fundamentals yields the **resulting magnetizing fundamental air gap field wave.**
- Each sinusoidal distributed air gap field wave can be described by a **space vector** in the machine´s axial cross section plane. Length of the space vector = amplitude of the field wave, orientation of the space vector = position of north pole. **Alternatively the space vectors $\underline{B}$ or $\underline{\Psi}$ or $\underline{I}$ are used.**

Equivalent circuit parameters

- **Magnetizing and leakage inductance & -reactance:** $L_s = L_h + L_{s\sigma}$, $X_s = X_h + X_{s\sigma}$
  rotor side: $L'_r = L_h + L'_{r\sigma}$, $X'_r = X_h + X'_{r\sigma}$
- Leakage is quantified (*BLONDEL)* by **leakage coefficient** σ: $\sigma = 1 - \frac{L_h^2}{L_s L'_r} = 1 - \frac{X_h^2}{X_s X'_r}$
- ***Rated data and per-unit values of parameters**: Example:*
  rated voltage U_N = 400 V (line-to-line !), rated current I_N = 100 A, Star connection

Rated phase voltage $U_{ph,N} = U_N / \sqrt{3} = \underline{\underline{231}}$ V, rated phase current $I_{ph,N} = I_N = \underline{\underline{100}}$ A

Rated apparent power: $S_N = 3 U_{ph,N} I_{ph,N} = 3 \cdot 231 \cdot 100 = 69.3$ kVA

Rated impedance: $Z_N = U_{ph,N} / I_{ph,N} = 231/100 = 2.31$ Ohm

Leakage coefficient σ: *shall be small:* typically 0.08 ... 0.1.

Phase resistance: *shall be small:* $r_s = R_s / Z_N$, $r'_r = R'_r / Z_N$: only a few percent 3 ... 6% !

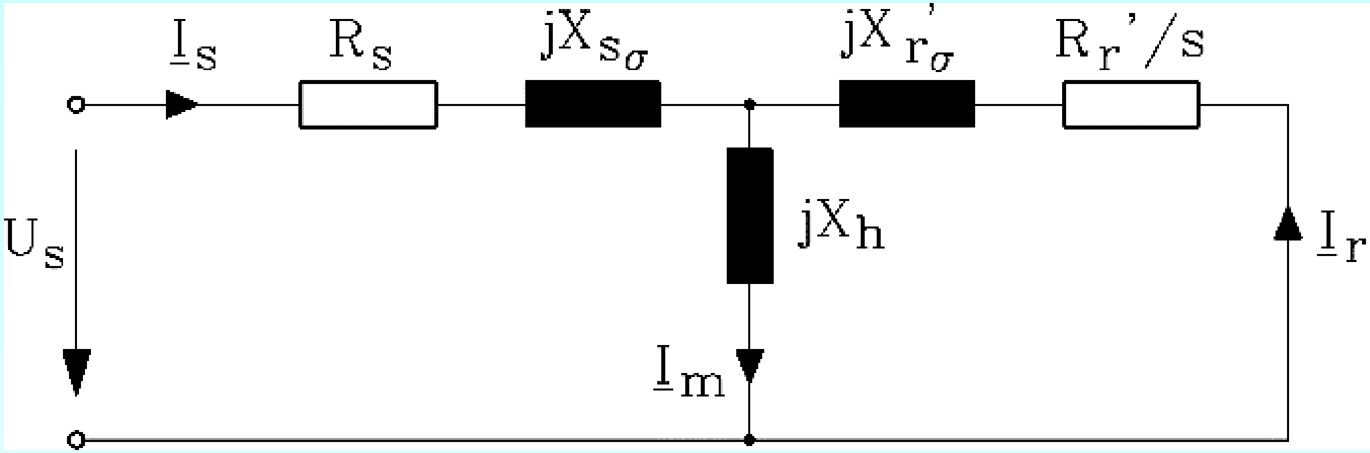
Magnetizing inductance: *shall be big* (= magnetic linkage of stator and rotor !): prop. $1/\delta$

$\Rightarrow$ SMALL air gap: mechanical lower limit ca. 0.28 mm in small motors:

$X_h / Z_N = 2.5 ... 3.0$ = 250% ... 300 %.

Leakage inductance: $X_{s\sigma} + X'_{r\sigma} \approx \sigma X_h \approx \sigma X_s$: $\sigma X_s / Z_N \approx$ (0.08...0.1)·(2.5...3) = 0.2 ... 0.3.

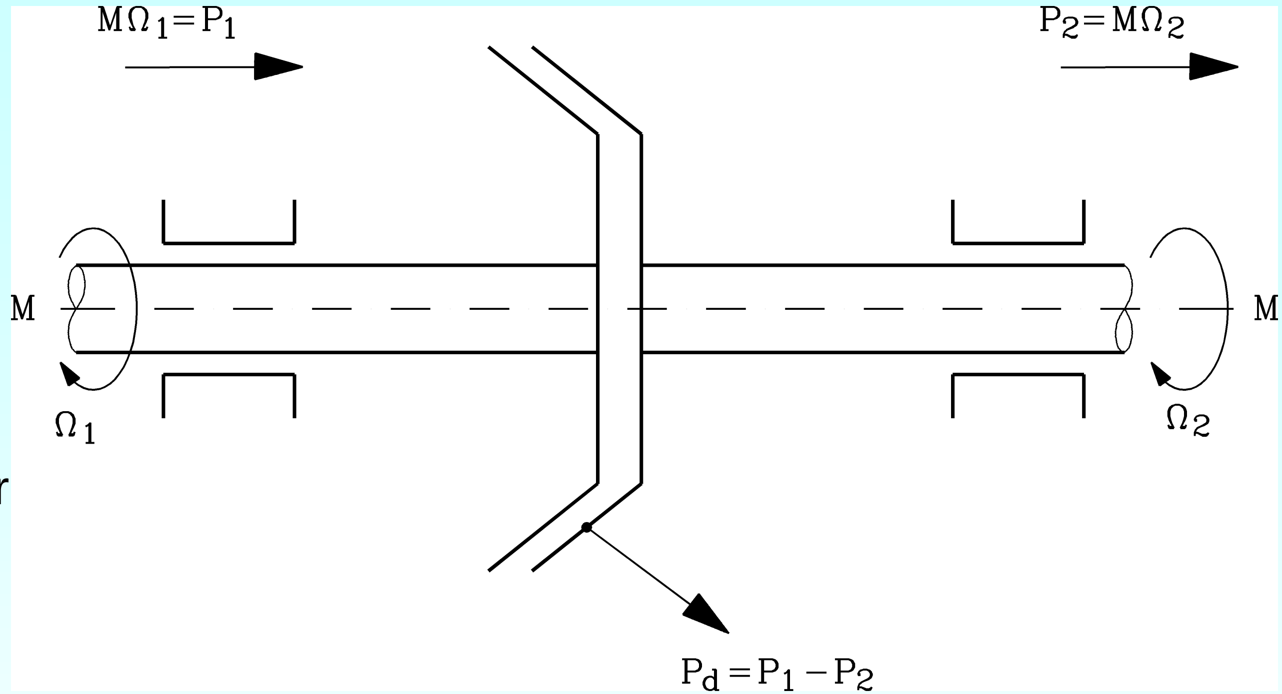
Asynchronous energy conversion



- **Electrical input power** $P_{e,in} = 3U_s I_s \cos\varphi$
- **Resistive losses** in **stator winding:** $P_{Cu,s} = 3R_s I_s^2$
- **Air gap power:** $P_\delta = P_{e,in} - P_{Cu,s} = 3\frac{R'_r}{s} I'^2_r$
- **Resistive losses** in **rotor winding:** $P_{Cu,r} = 3R_r I_r^2 = 3R'_r I'^2_r = sP_\delta$
- **Mechanical output power:** $P_{m,out} = P_\delta - P_{Cu,r} = (1-s)P_\delta$
- **Electromagnetic torque** $P_{m,out} = M_e \Omega_m = (1-s)P_\delta \quad \Rightarrow \quad M_e = \frac{1-s}{1-s} \cdot \frac{P_\delta}{\Omega_{syn}} = \frac{P_\delta}{\Omega_{syn}}$

The electromagnetic torque is proportional to air gap power.

Slip coupling – mechanical analogy to induction machine



- Driving input torque *M* at shat no. 1 = output torque at second shaft no.2.
- Transmission of torque only possible, if **friction disc 2** has a certain slip with respect to friction disc 1.

Hence output speed Ω_2 is smaller by the **slip *s*** than speed Ω_1 of input shaft.

$$\Omega_2 = (1 - s)\Omega_1.$$

- Output power: $P_2 = M\Omega_2$ is smaller by slip losses $P_d = s\Omega_1 M$ than input power $P_1 = M\Omega_1$.

Induction machine	Slip coupling
Ω_{syn} , Ω_m	Ω_1 , Ω_2
P_δ , $P_{Cu,r}$, P_m	P_1 , P_d , P_2
M_e	M

Stator and rotor current

- **Solution of the two linear equations** of T-equivalent circuit: Unknowns $\underline{I}_s, \underline{I}'_r$

$$\underline{U}_s = R_s \underline{I}_s + jX_s \underline{I}_s + jX_h \underline{I}'_r \qquad 0 = \frac{R_r}{s}\underline{I}'_r + jX'_r \underline{I}'_r + jX_h \underline{I}_s$$

$$\underline{I}_s = \underline{U}_s \frac{R'_r + jsX'_r}{(R_s R'_r - s\cdot\sigma\cdot X_s X'_r) + j(s\cdot R_s X'_r + X_s R'_r)}$$

$$\underline{I}'_r = -\underline{I}_s \frac{jX_h}{\frac{R'_r}{s} + jX'_r}$$

- **Solution for $R_s = 0$:**

$$\underline{I}_s = \frac{\underline{U}_s}{jX_s}\cdot\frac{R'_r + js\cdot X'_r}{R'_r + js\cdot\sigma X'_r} \qquad \underline{I}'_r = -\frac{\underline{U}_s}{X_s}\cdot\frac{s\cdot X_h}{R'_r + js\cdot\sigma X'_r}$$

- Derivation of electromagnetic **torque at $R_s = 0$:**

$$M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{m_s R'_r I'^2_r}{s\cdot\Omega_{syn}} = m_s \frac{p}{\omega_s} U_s^2 \frac{X_h^2}{X_s^2}\frac{sR'_r}{R'^2_r + (s\sigma X'_r)^2}$$

$$M_e = m_s \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{X_s}\frac{sR'_r X'_r}{R'^2_r + (s\sigma X'_r)^2}$$

KLOSS formula for torque (at R_s = 0)

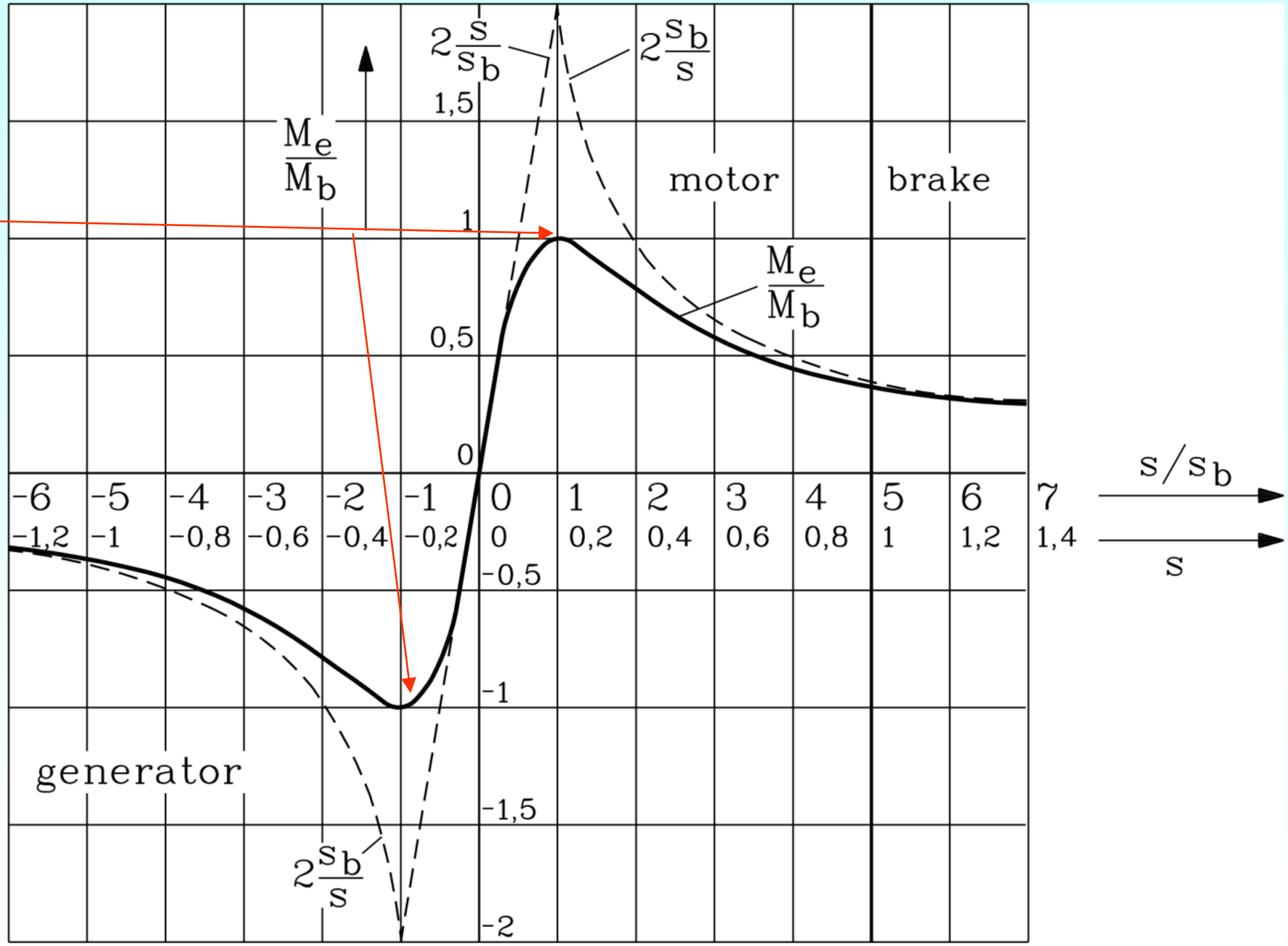
- **Breakdown torque:** Maximum of electromagnetic torque: $dM_e / ds = 0$

Breakdown slip s_b

$$R_s = 0: \quad \boxed{s_b = \frac{R'_r}{\sigma X'_r}}$$

Breakdown torque:

$$\boxed{M_b = \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{\sigma X_s}}$$



- ***KLOSS* formula:**

$$R_s = 0: \quad \boxed{\frac{M_e}{M_b} = \frac{2}{\frac{s_b}{s} + \frac{s}{s_b}}}$$

Example:

Breakdown slip $s_b = 0.2$.

Electromagnetic torque at $R_s > 0$

- From air gap power we derive electromagnetic **torque**:

$$M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{m_s R'_r I'^2_r}{s \cdot \Omega_{syn}}$$

$$M_e = m_s \frac{p}{\omega_s} U_s^2 \frac{s(1-\sigma) X_s X'_r R'_r}{(R_s R'_r - s\sigma X_s X'_r)^2 + (s R_s X'_r + X_s R'_r)^2}$$

- **Breakdown slip:** $\frac{dM_e}{ds} = 0$: **Breakdown slip** s_b in motor mode ($s_{b,mot} = s_b > 0$) and generator mode ($s_{b,gen} = -s_b < 0$) have the **same absolute value**:

$$s_b = \frac{R'_r}{X'_r} \cdot \sqrt{\frac{R_s^2 + X_s^2}{R_s^2 + \sigma^2 X_s^2}} \approx \frac{R'_r}{\sigma X'_r}$$

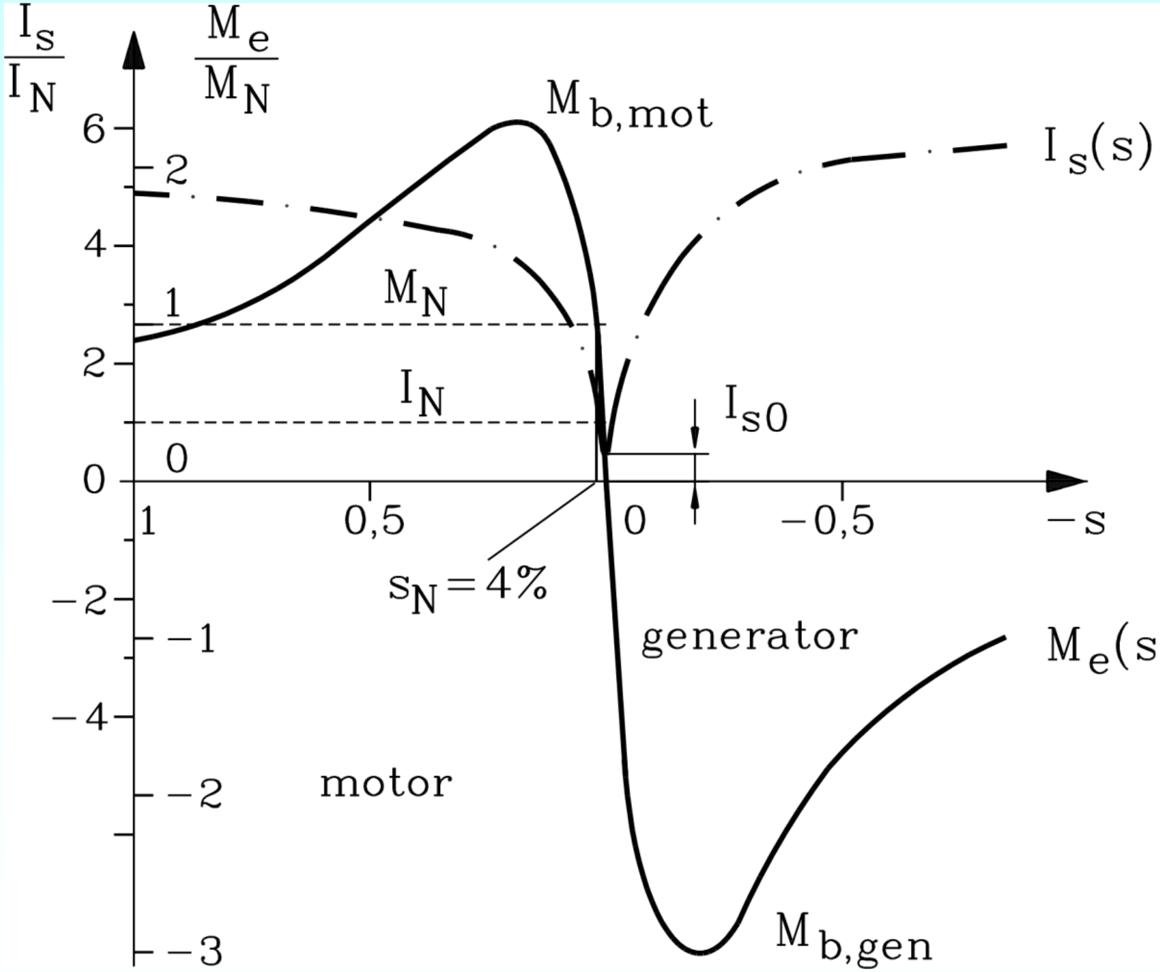
- **Breakdown torque:**

$$M_{\mathrm{b,mot/gen}} = \pm \frac{m_s}{2} \frac{p}{\omega_s^2} U_s^2 \frac{1}{\pm \frac{R_s}{\omega_s} + \frac{1}{(1-\sigma)\omega_s X_s} \cdot \sqrt{(R_s^2 + X_s^2)(R_s^2 + \sigma^2 X_s^2)}}$$

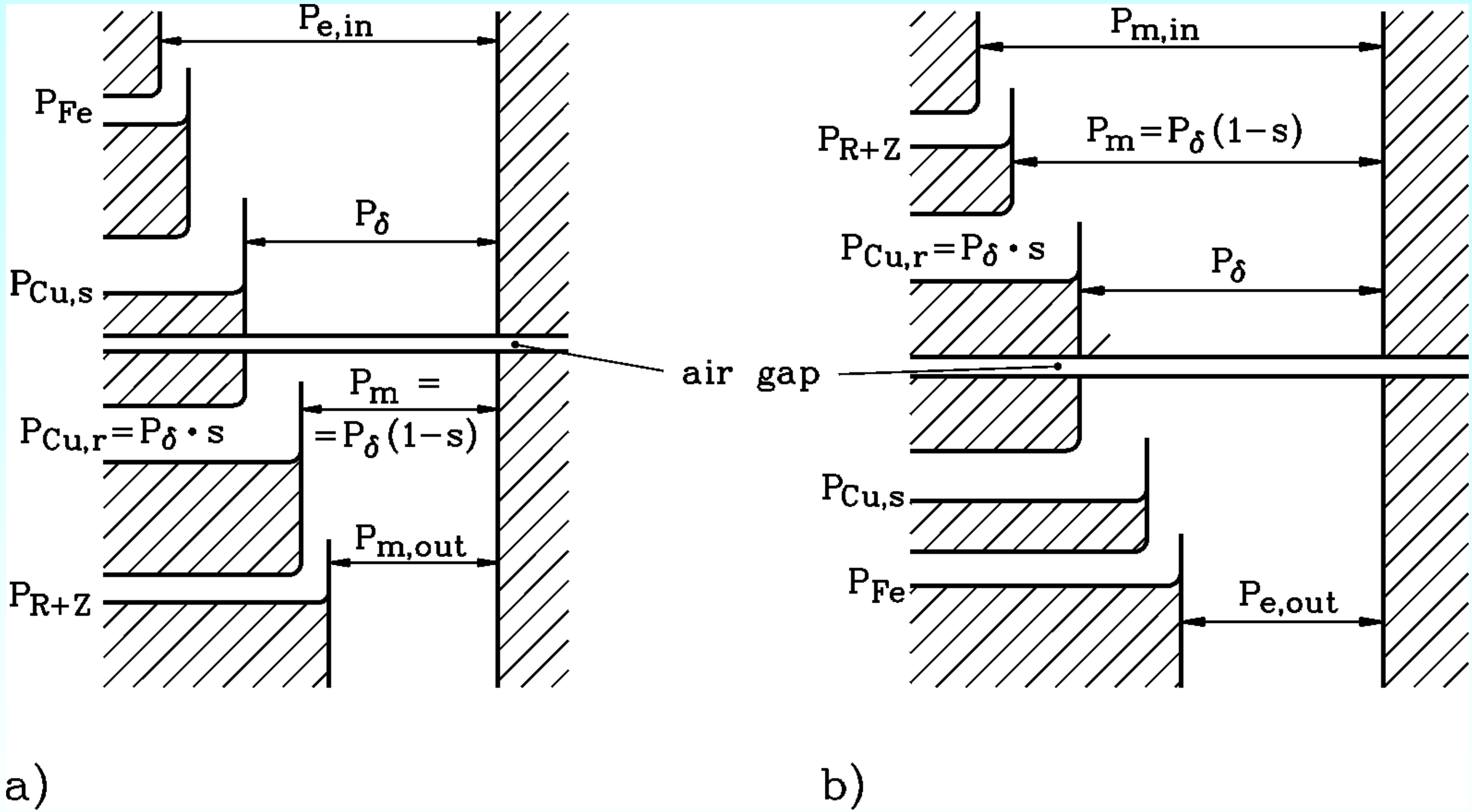
Motor breakdown torque is positive, generator breakdown torque is negative: In generator mode stator resistive losses must also be covered by air gap power, hence demanding a bigger air gap electromagnetic torque. **Hence generator breakdown torque is by that amount bigger than motor breakdown torque.**

Torque-speed and current-speed characteristic

- Due to $n=(1-s)\cdot f_s/p$: M_e and I_s can be described in dependence of s as well as of n !
- ***Example:*** R_s/X_s =1/100, R_r/X_r = 1.3/100, σ = 0.067, $X_s = X'_r = 3Z_N$, $Z_N = U_{ph,N}/I_{ph,N}$



Power flow in (a) motor- and (b) generator mode



- **Efficiency** of induction machine: motor: $\eta = \dfrac{P_{m,out}}{P_{e,in}}$, generator: $\eta = \dfrac{P_{e,out}}{P_{m,in}}$

Stator current magnitude of induction motors (1)

- **No-load:** No-load speed is synchronous speed: Slip is ZERO.

$$\underline{I}_s(s=0)=\frac{\underline{U}_s}{R_s+jX_s}\approx -j\frac{\underline{U}_s}{X_s} \qquad \frac{\underline{I}_s}{I_N}(s=0)\approx -j\frac{\underline{U}_s/U_N}{X_s/Z_N}=-j\frac{1}{x_s}$$

Example:

$x_s=x_h+x_{s\sigma}\cong 3.0+0.15=3.15:\ I_S/I_N\cong 1/X_S\cong 1/3$ **No-load current:** ca. 1/3 of rated current. At 100 A rated current the no-load current is ca. 33 A.

- **"Locked rotor" (Stand still):** Slip is 1: (**"Locked rotor current, starting current"**):

$$\underline{I}_s(s=1)\approx -j\underline{U}_s\frac{1}{\sigma\cdot X_s} \qquad \frac{\underline{I}_s}{I_N}(s=1)\approx -j\frac{\underline{U}_s/U_N}{\sigma X_s/Z_N}=-j\frac{1}{\sigma\cdot x_s}$$

Example:

σ = 0.08, x_s = 2.6: $i(s = 1)$ = 1/(2.6·0.08) = 4.8: **starting current** is 4.8-times rated current. Bigger motors have smaller leakage flux, so bigger starting current: typically 5 ... 7-times rated current.

Stator current magnitude of induction motors (2)

- **Rated operation:**

Rated slip s_N: **Rated current (Thermal continuous duty):**

We get rated torque !

- *Example:*

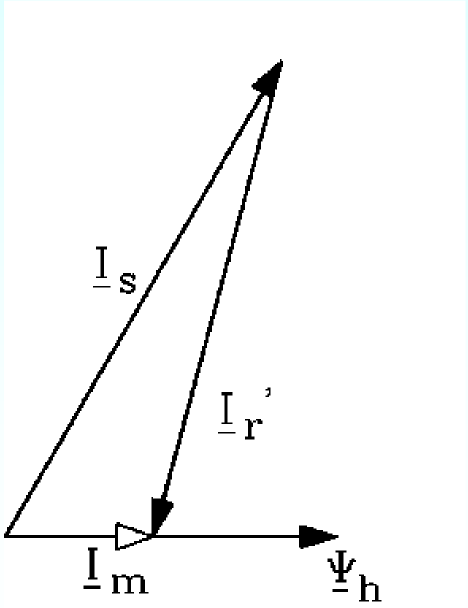
Four-pole machine, 50 Hz: No-load speed n_{syn} = 1500/min, Rated speed n_N = 1450/min, Rated slip s_N = 0.033 = 3.3%.

- **"Balance" of stator and rotor ampere turns:**

Rotor current nearly in opposite phase to stator current.

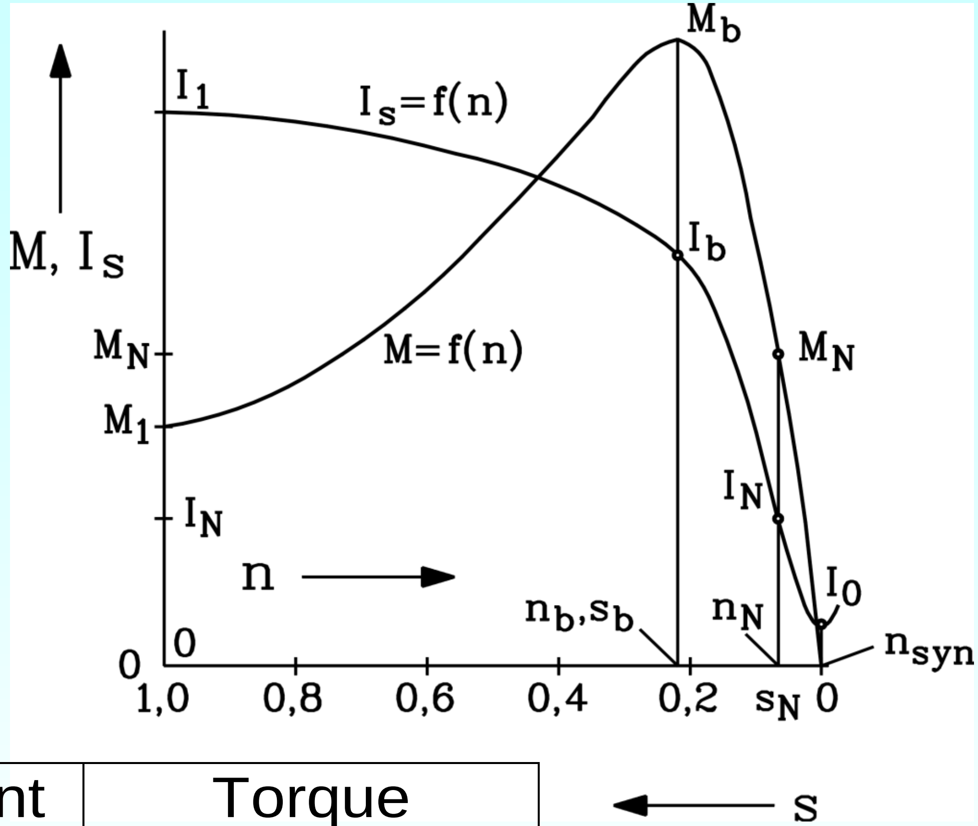
- *Example:*

$$\underline{I}'_{rN} = -\underline{I}_{sN}\frac{jx_h}{\frac{r'_r}{s_N}+jx'_r} = -\underline{I}_{sN}\frac{j2.9}{\frac{0.03}{0.03}+j3} = -\underline{I}_{sN}\frac{j2.9}{1+j3} \approx -\underline{I}_{sN}$$



Current and torque diagram

- Rather big no-load current for excitation of magnetic field, hence: air gap between stator and rotor should be as small as possible
- Very big starting current, but rather low starting torque
- Motor may be loaded at maximum only till breakdown torque



	Slip	Stator current	Torque
No load	$s = 0$	I_0 = ca. $0.3 I_N$	$M = 0$
Rated point	$s = s_N$	I_N	M_N
Break down	$s = s_b$	I_b = ca. $2.5 I_N$	M_b = ca. $2 M_N$
Starting	$s = 1$	I_1 = ca. $4 I_N$	M_1 = ca. $0.8 M_N$

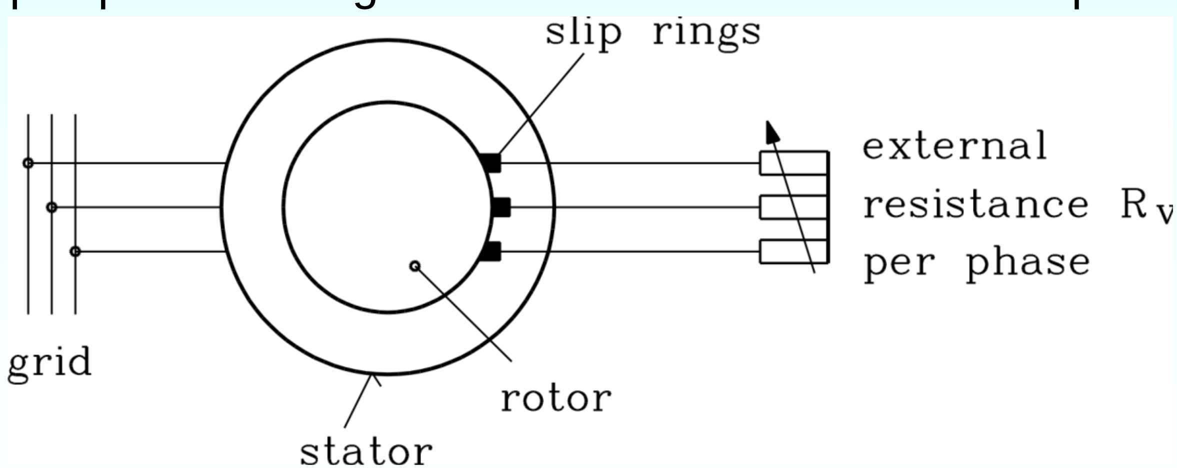
Starting of slip-ring induction machine with external rotor resistance

- External **resistance per phase are** connected to rotor phase via slip rings and carbon brush contacts. Hence rotor total resistance is increased. By that also the starting torque is increased. Maximum starting torque is motor breakdown torque. Starting slip s = 1 is then also breakdown slip. Starting current is reduced to breakdown current.

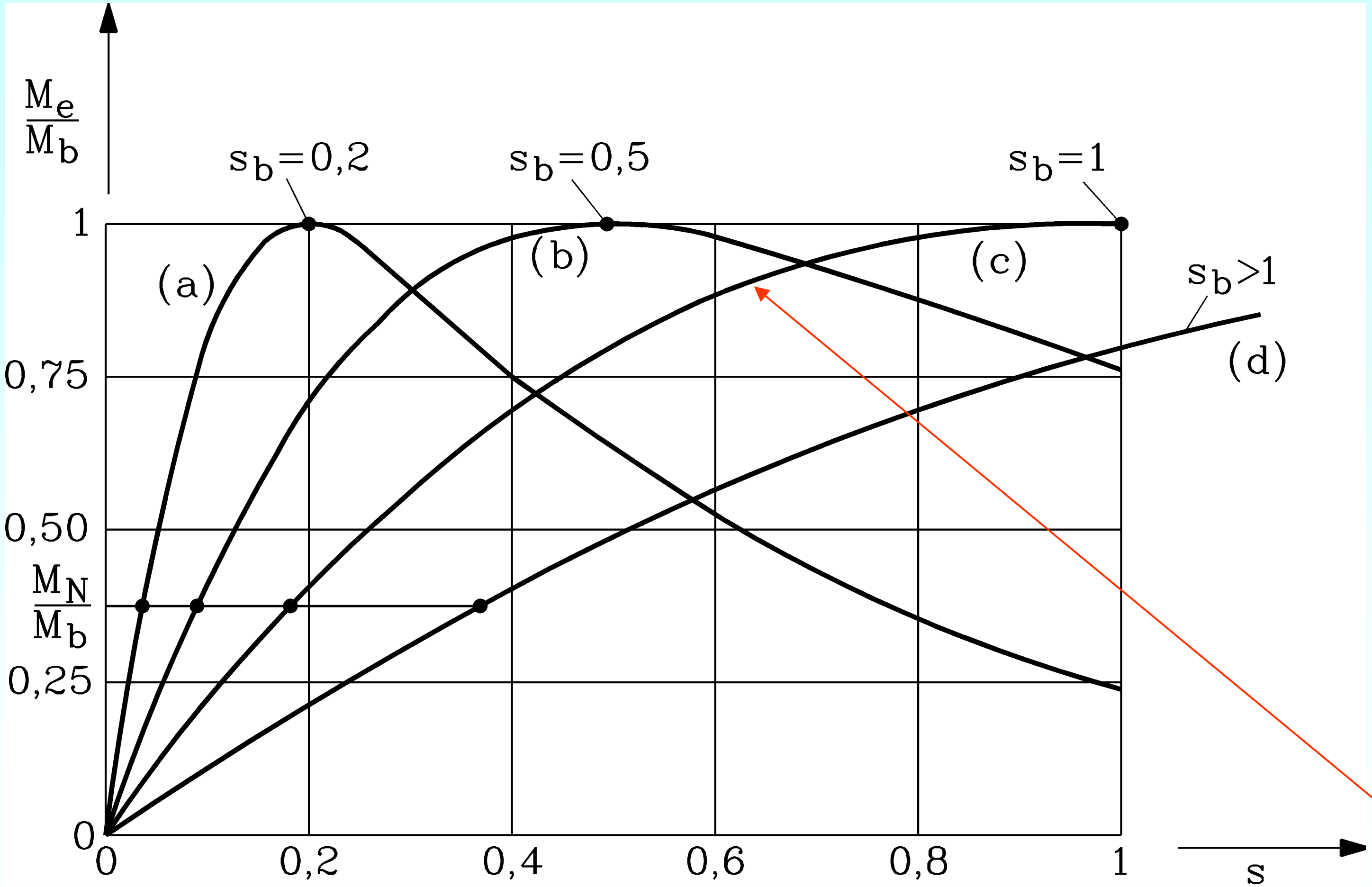
- By keeping $R_r' / s = const.$, the parameters of the equivalent circuit remain unchanged.

- External resistance R_v per phase: We get the same stator current at slip s as in case of $R_v = 0$ at s^*:

$$\frac{R_r + R_v}{s} = \frac{R_r}{s^*} = konst.$$



Torque-speed characteristic of slip-ring induction motor with external rotor resistance (M_b/M_N = 2.65)



Example: External rotor resistance $R_v = 4R_r$: Starting torque = Breakdown torque *(case c).*

How to define value of external rotor resistance ?

- ***Demand:*** Starting torque (at s = 1) shall be breakdown torque:

$$\frac{R_r + R_v}{1} = \frac{R_r}{s_b} \quad \Rightarrow \quad R_v = R_r\left(\frac{1}{s_b} - 1\right)$$

Example:

Slip-ring induction machine: Data: M_b/M_N = 2.65, Breakdown slip 0.2

- Without external rotor resistance we get: Starting torque $M_1 = 0.65 M_N = 0.24 M_b$ (case *a*).

$$R_v = R_r\left(\frac{1}{0.2} - 1\right) = 4R_r$$

- At R_v/R_r = 4 starting torque is breakdown torque ! (case *c*).

- **"Shear"** (linear dilation by R_v) **of *M*(*n*)- resp. *M*(*s*)-characteristic.** The torque value M_e at slip s^* (and R_v = 0) occurs at the new slip s !
- ***Result:*** Improved (quicker) starting due to increased torque and decreased current, BUT additional losses in external resistance. ***Advantage:*** These losses occur OUTSIDE of machine, hence they do not heat up the rotor winding.

Variable speed operation of slip-ring induction machines

- By changing the external rotor resistance, the *M(n)*-characteristic changes and allows variable speed operation of slip-ring induction machine.

- *Example :*

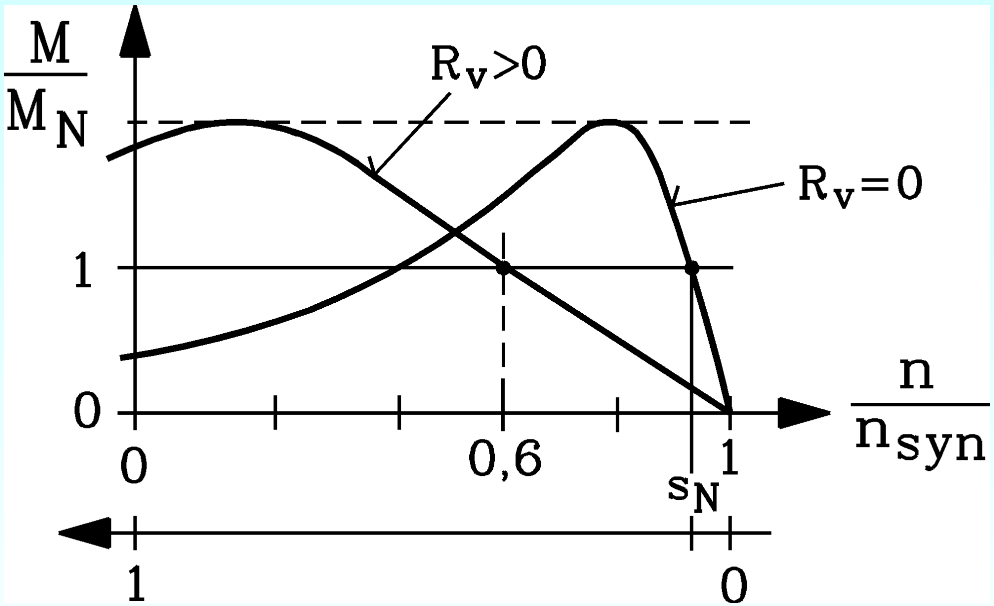
Compare "Motor for *elevator hoist*" and "Motor for *pump*":

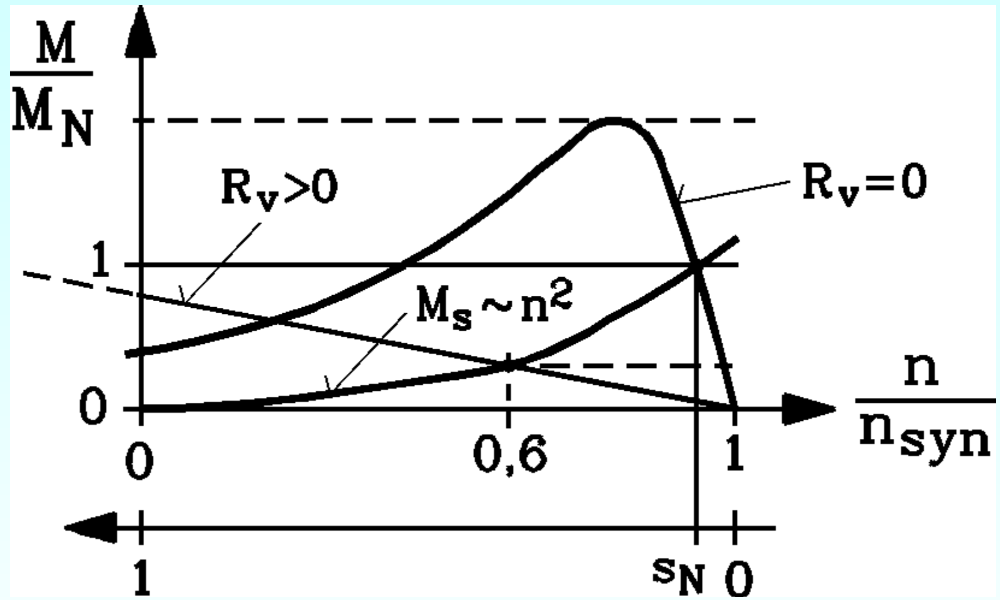
Demand: Reducing of speed from n_{syn} (100%) down to 60% !

Motor power balance (when neglecting stator resistive losses $3I^2R_s$ and iron losses P_{Fe}):

$$P_{e,in} \cong P_\delta = \Omega_{syn} M_e = P_{Cu,r} + 3R_v I_r^2 + P_m \qquad P_m = 2\pi n M_e \qquad P_\delta = 2\pi n_{syn} M_e$$

$M(n)$-characteristic of variable speed slip-ring induction machine





Constant load torque:

e.g. elevator

Big losses in motor: NOT USEFUL !

Square load torque:

e.g. pump

Small motor losses: USEFUL SOLUTION !

Variable speed operation of slip-ring induction machines

	Elevator	*Pump*
Load torque	$M_s = M_N = konst.$	$M_s = (n/n_{syn})^2 \cdot M_N$
Load torque at $n/n_{syn} = 0.6$	$M_s = M_N$	$M_s = 0.36 \cdot M_N$
$P_\delta(n)/P_{\delta N} = P_\delta(n)/(2\pi n_{syn} M_N)$	1	0.36
$P_m(n)/P_{\delta N}$	0.6	0.22
$(P_{Cu,r} + 3R_v I_r^2)/P_{\delta N}$	**0.4 (! BIG)**	**0.14 (SMALL)**

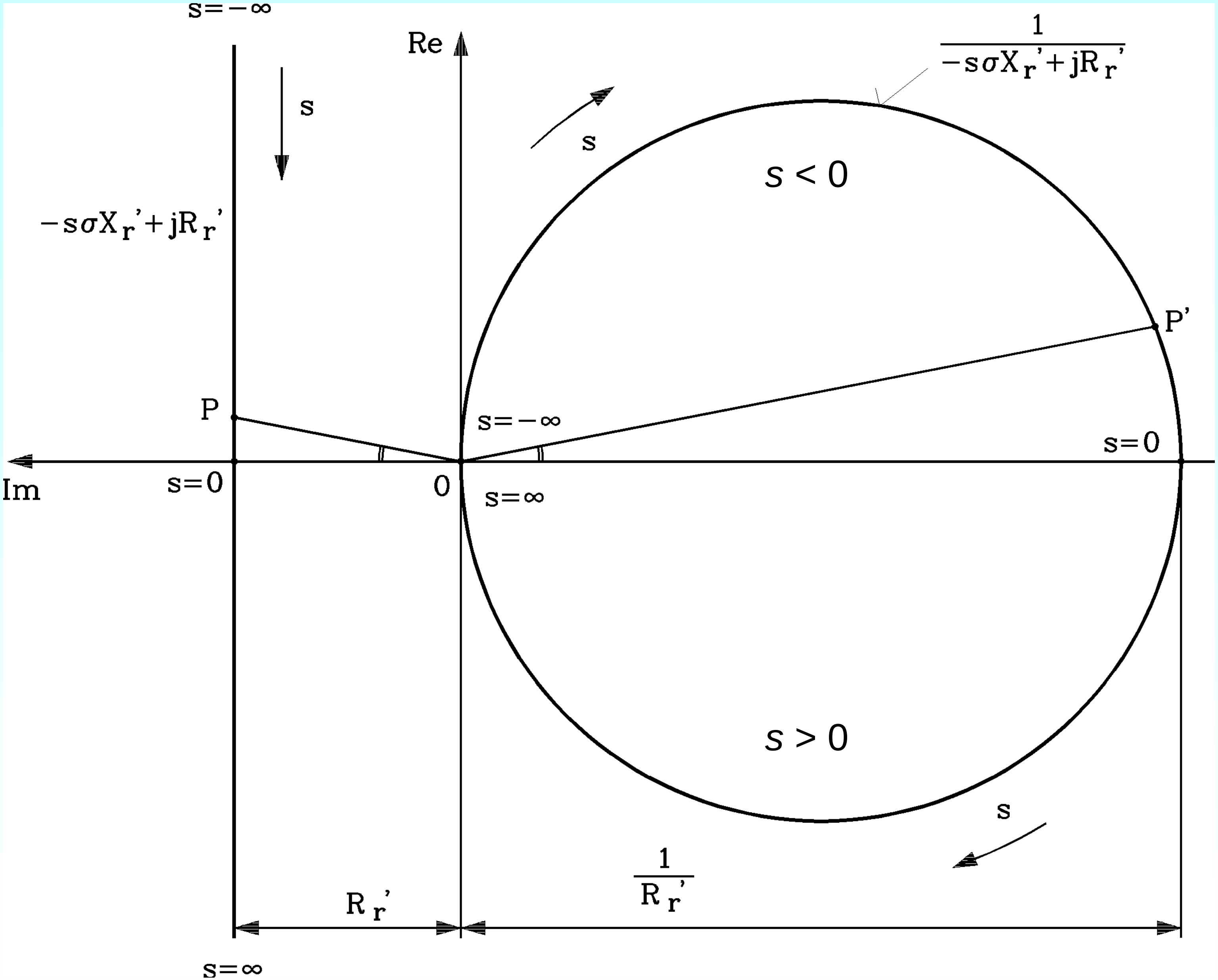
- *Elevator: Constant load torque:* Reduction of speed by 40% = at the cost of rotor losses of 40% of P_N = BIG LOSSES = NOT USEFUL !
- *Pump: Quadratic load torque:* Reduction of speed of 40 % = at the cost of only 14% of P_N = RATHER LOW LOSSES = USEFUL SOLUTION !

- *Still better: Inverter-fed induction machine: much lower losses (see later !)*

Stator current phasor locus: *HEYLAND*-circle ($R_s = 0$)

- **Stator current phasor:** $\underline{I}_s = U_s \frac{1}{X_s} \cdot \frac{R'_r + jsX'_r}{-s\cdot\sigma\cdot X'_r + jR'_r} = \frac{U_s}{X_s} \cdot \left(\frac{(1-1/\sigma)R'_r}{-s\cdot\sigma\cdot X'_r + jR'_r} - j\frac{1}{\sigma} \right)$
- $\underline{G}(s) = -s\sigma X'_r + jR'_r$: Straight line in complex plane, parallel to *Re*-axis
- Inverse = Inversion of $\underline{G}(s)$ yields a **circle** $\underline{K}(s)$: $\underline{K}(s) = \frac{1}{\underline{G}(s)} = \frac{1}{Z(s)\cdot e^{j\varphi(s)}} = \frac{e^{-j\varphi(s)}}{Z(s)}$
- Points *P* of straight line are transferred into points *P´* of circle: distance $\overline{0P} = Z(s) \;\Rightarrow\; \overline{0P'} = 1/Z(s)$. Centre of circle lies on -*Im*-axis.
- Multiplication with negative real number $(1-1/\sigma)\cdot R'_r$: Circle is mirrored at -*Im*-axis:
- Adding $-j/\sigma$: Circle is shifted to the right along –*Im*-axis. Multiplication with U_s / X_s does not change position of circle, but only its diameter.
- Circle points P_0 and P_∞: $\underline{I}_s(s=0) = -jU_s / X_s$ (**No-load current**)

$\underline{I}_s(s=\infty) = -jU_s/(\sigma X_s)$ (**"ideal" short-circuit current**)



Derivation of *HEYLAND*-circle ($R_s = 0$)

Torque and power line

- Electric real power at **motor operation: point *P*,** slip *s*: $P_{e,in} = m_s U_s I_{s,w}$
  Real component of stator current phasor $I_{s,w} = I_s \cos\varphi = \overline{PB} = \overline{PC} + \overline{CB}$

- **Power balance:** $P_{e,in} = P_m + P_{Cu,r} = m_s U_s (\overline{PC} + \overline{CB})$

- At *s* = 1 it is *n* = 0; at *s* = 0 it is $M_e = 0$. Hence mechanical power P_m is zero in both points.
  Connection line $\overline{P_0 P_1}$: **"Power line": Partitions** real current into $\overline{PC}$ and $\overline{CB}$.
  Section $\overline{PC}$ is proportional to mechanical power !

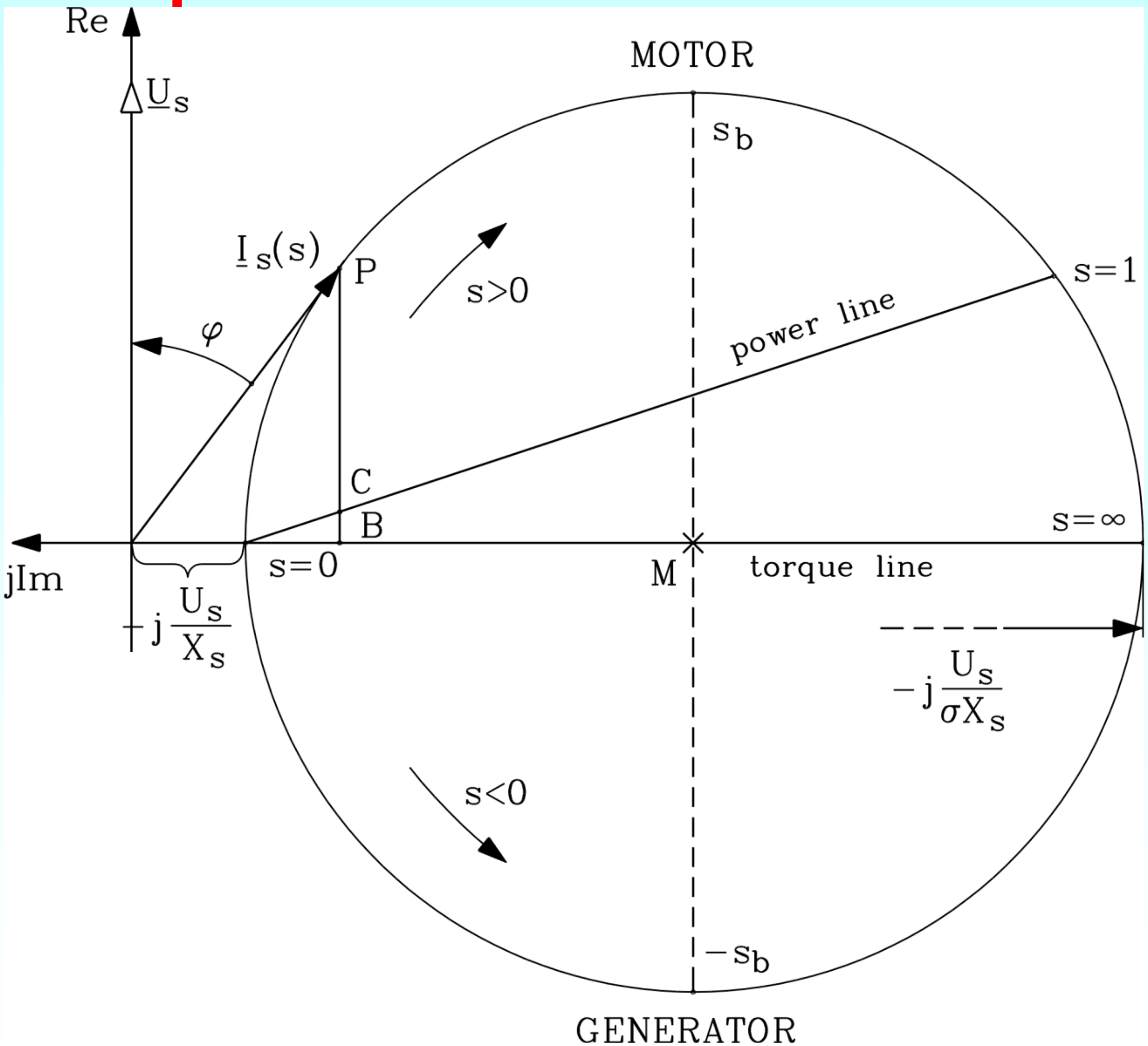
- $P_{e,in} = P_\delta = M_e \Omega_{syn} = m_s U_s \overline{PB}$: At the points P_0 and P_∞ torque M_e is zero.

  Connection line $\overline{P_0 P_\infty}$: **"Torque line".**
  Section $\overline{PB}$ is proportional to electromagnetic torque.

- Points at **lower semi-circle:** generator mode, real power is negative ($\cos\varphi < 0$).

- Break down points $s_{b,mot}$ and $s_{b,gen}$ (maximum torque) : $\overline{PB}$ **maximum.**
- Stator current **always lags behind stator voltage**; hence **induction machines** are **always inductive elements** – in motor as well as in generator mode !

Torque and power line in *HEYLAND*-circle ($R_s = 0$)



Stator current locus diagram for $R_s > 0$: *OSSANNA*-circle

- **Stator current locus diagram for $R_s > 0$:**
  - Centre of circle *M* lies a little bit **above** of negative *Im*-axis.
  - The distance $\overline{P_0 P_\infty}$ is **not** circle diameter. Point P_∞ is shifted slightly above the –*Im*-axis.

- Torque line lies above of -*Im*-axis.

- Circle diameter comprises points at *s* = 0, at *M* and the **"diameter-point" $P_\varnothing$**

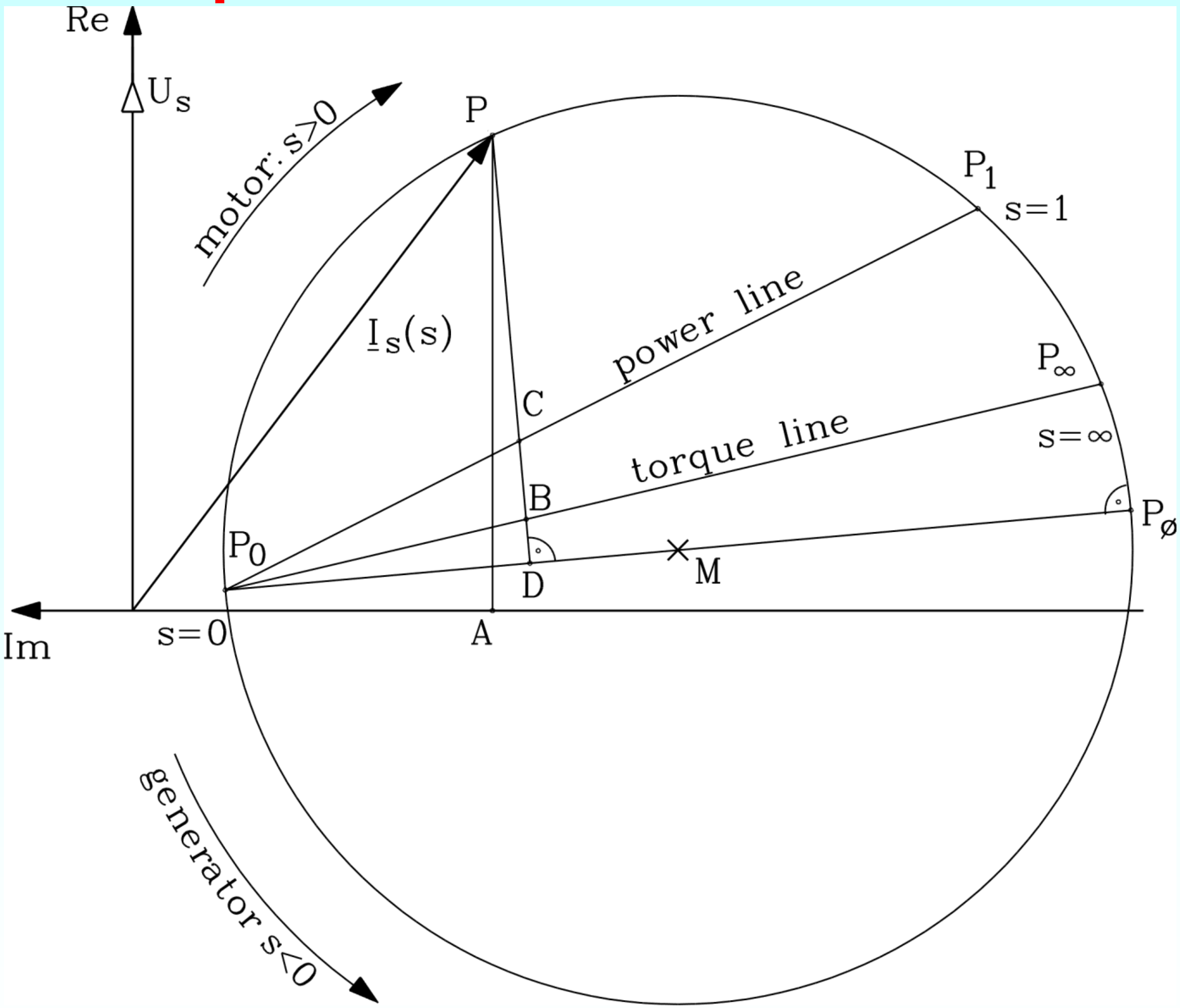
- Electrical real power is proportional to real motor current component (distance $\overline{PA}$):

$$P_e = m_s U_s \overline{PA} = m_s U_s I_s \cos\varphi$$

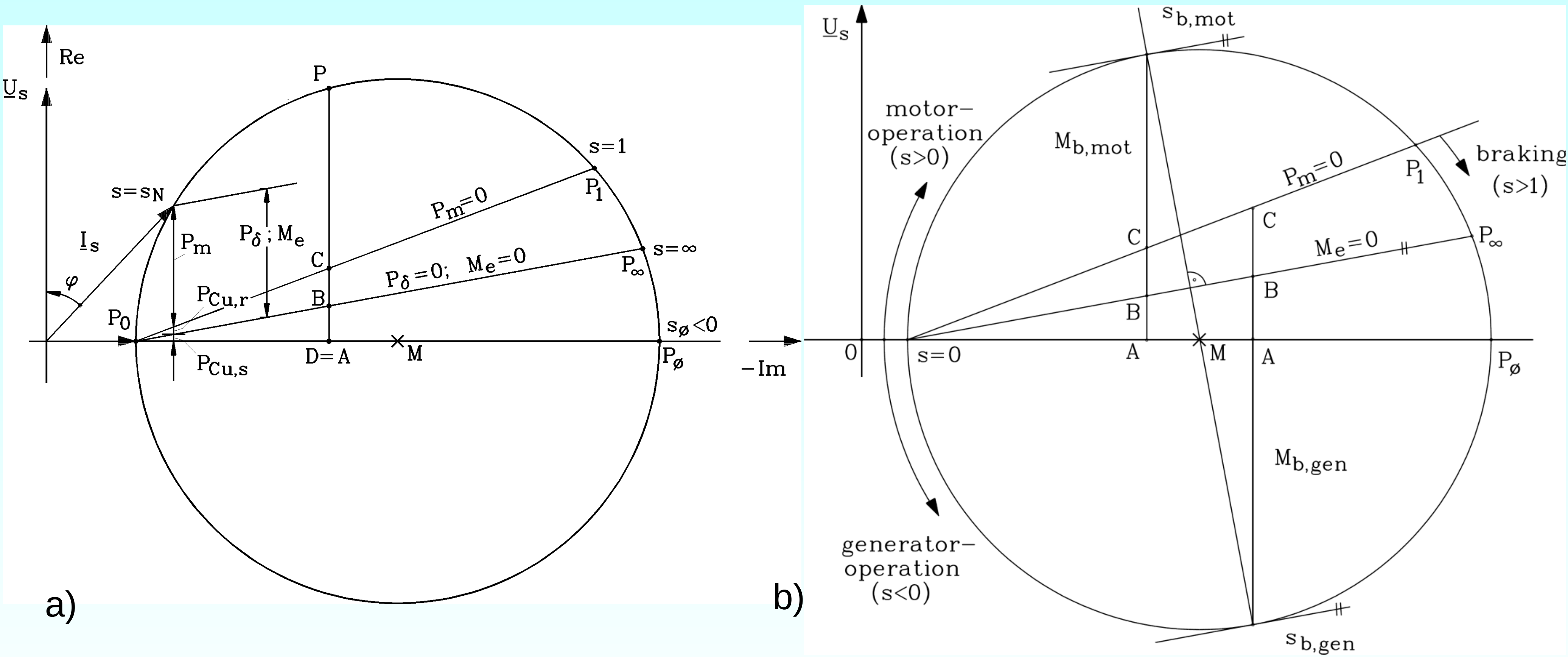
Loss balance in *OSSANNA*-circle ($R_s > 0$)

- Loss balance at **operation point** *P*: Vertical line from *P* to circle diameter yields new point *D*.
  Cross-over point of vertical line with power line and torque line yields points *C* and *B*.
- Air gap power gives torque: $P_\delta = m_s U_s \overline{PB} \quad \Rightarrow \quad M_e = \dfrac{m_s U_s \overline{PB}}{\Omega_{syn}}$
- Distance BC gives rotor losses: $P_{Cu,r} = m_s U_s \overline{BC} = P_\delta - P_m$
- Mechanical power: $P_m = m_s U_s \overline{PC}$
- Stator resistive losses (in stator winding) are not directly visible as distance.
  We get them as difference of $\overline{PA}$ and $\overline{PB}$: $P_{Cu,s} = P_e - P_\delta = m_s U_s (\overline{PA} - \overline{PB})$

Torque and power line in *OSSANNA*-circle ($R_s > 0$)



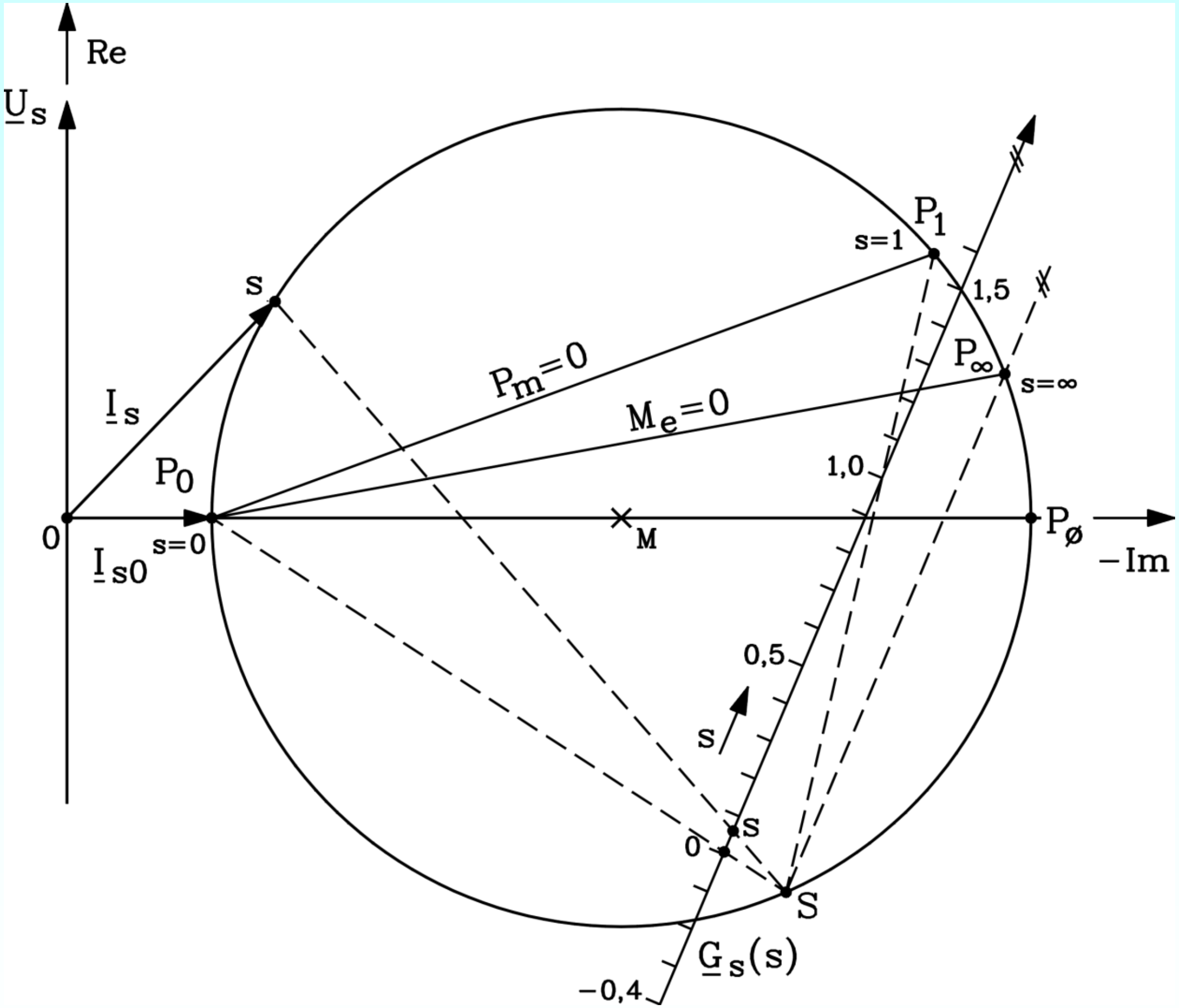
Simplified *OSSANNA*-circle: Circle centre *M* put on *–Im*-axis



- Centre of circle M assumed to be on -*Im*-axis (as it is with *HEYLAND*-circle), but assumption $R_s > 0$ still active !
- Hence we have to distinguish points $P_{\varnothing}$ and P_{∞} .

a) Torque and power line, b) Motor and generator break down torque.

Circle parameterization according to slip values, done with straight „slip line"



Complex slip line $\underline{G}_s(s)$ is generated from three known operation points (here: P_0, P_1 and P_∞) and an arbitrarily taken Centre of inversion *S*, lying on the circle.

- Slip line is linear parameterized in *s*.
- Slip line must be parallel to line S-P_∞ .

Intersection of connecting straight line (from *S* and selected slip value on slip line) with circle diagram delivers the operation point for selected slip *s*, hence the locus of the stator current phasor for that selected slip *s*.