6. The squirrel cage induction machine

Source: Breuer Motoren, Germany

Squirrel cage induction machine

- **Copper squirrel cage:**

Source: Breuer Motoren, Germany

for big power machines > 50 ... 100 kW and for traction machines:

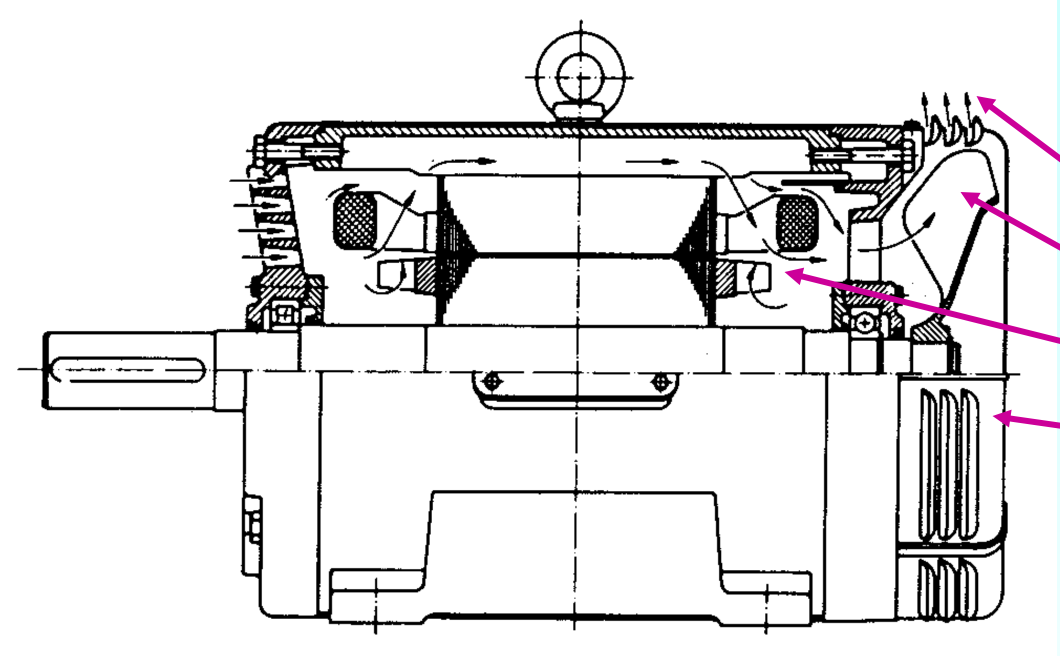
Massive, non-insulated copper bars in rotor slots. At both front ends short-circuited by two copper end rings by welding. Sometimes copper die cast rotors for smaller machines to increase efficiency.

- **Aluminium copper squirrel cage:**

Die cast cage for smaller machines < 50 .. 100 kW: The whole cage is cast as one piece with liquid aluminium. Additional fan blades for cooling at the end rings and balancing bolts are cast at the same time.

- **Two adjacent bars** form with the in between ring segments rotor loops, where stator rotating field induces the rotor voltage. This causes rotor bar current & end ring segment current. Rotor bar current together with stator field creates electromagnetic torque.

Aluminium die cast squirrel cage induction machine



Cage induction machine, open ventilated, air cooling

Air flow

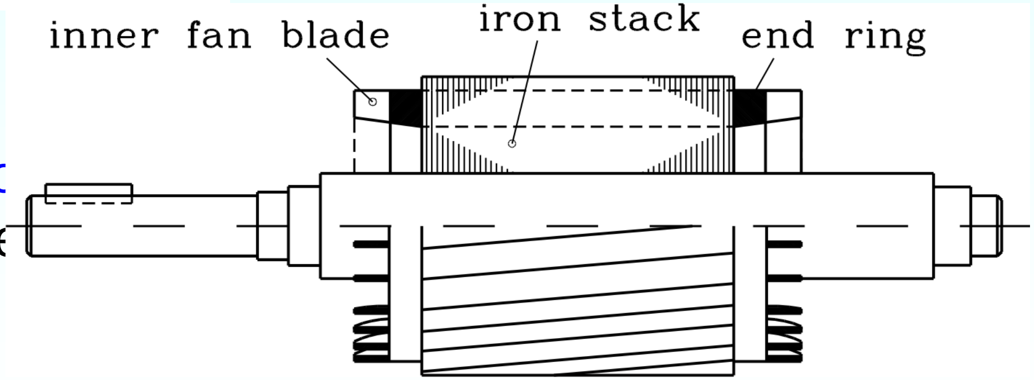
Shaft-mounted fan

Cage end ring with fan blades

Fan hood for guidance of air

Source: H. Kleinrath, Studientext

Source: H.-O. Seinsch, Teubner-Verlag



Aluminium die cast squirrel cage roto

skewed by one stator slot to reduce

losses, caused by slot harmonics

Induced rotor voltage per bar

- **Stator fundamental air gap wave** (amplitude $\hat{B}_{\delta,s}$) moves relatively to the rotor with speed $sv_{syn} = v_{syn} - v_m$. Two rotor bars, distanced by pole pitch τ_p = **"rotor loop"**. Magnetic flux per loop:

$$\Phi = \frac{2}{\pi}\tau_p l \hat{B}_{\delta,s}$$ **Magnetic flux per loop**

- **Induced voltage per loop**, induced with frequency $f_r = s f_s$:

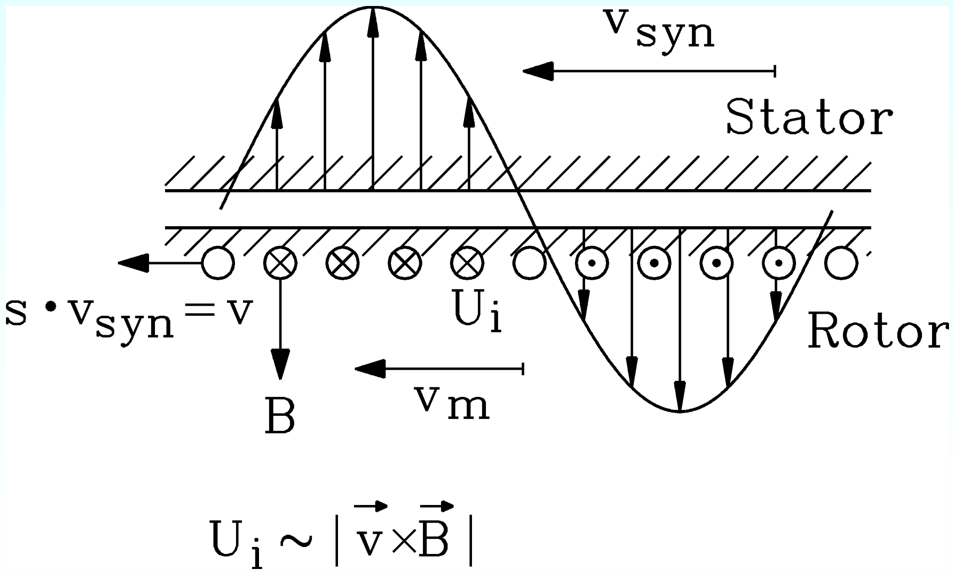
$$\hat{U}_{i,c} = 2\pi \cdot sf_s \cdot \frac{2}{\pi}\tau_p l \hat{B}_{\delta,s} = s \cdot 2(2 f_s \tau_p) \cdot l \cdot \hat{B}_{\delta,s} = s \cdot 2v_{syn} \cdot l \cdot \hat{B}_{\delta,s}$$

Per bar = half loop:

half voltage $\hat{U}_{i,bar} = \hat{U}_{i,c} / 2$

= **Rotor bar voltage**

$$\boxed{\hat{U}_{i,bar} = sv_{syn}\hat{B}_{\delta,s} l}$$



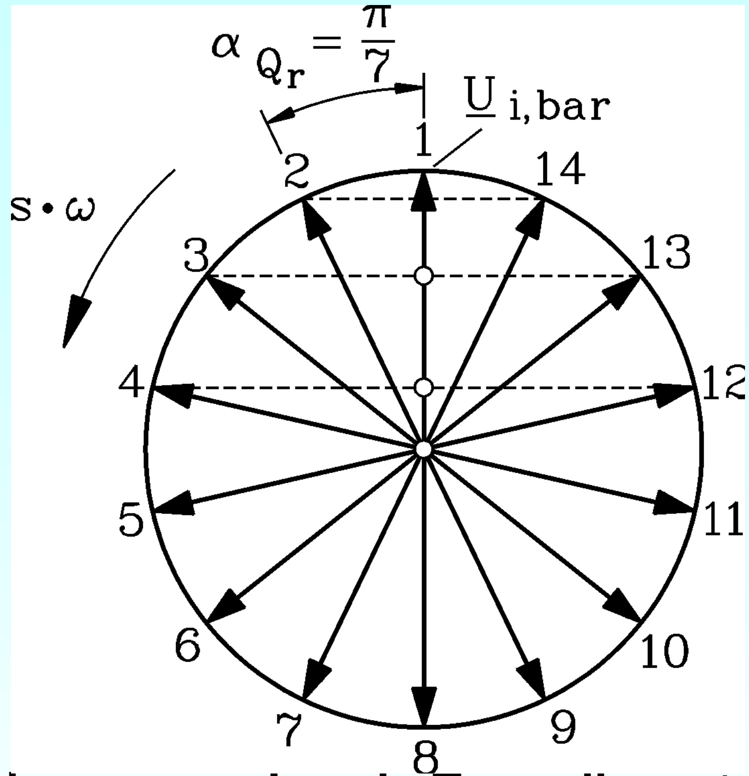
Rotor bar voltages form regular “bundle” of phasors

- **Distance between two bars = rotor slot pitch τ_{Qr}. It yields phase shift between adjacent bar voltages =**

= **Rotor slot angle** $\alpha_{Qr} = \frac{2\pi p}{Q_r}$

- ***Facit:***

Voltage phasors of all rotor bars form on complex plane a regular “bundle” of phasors.



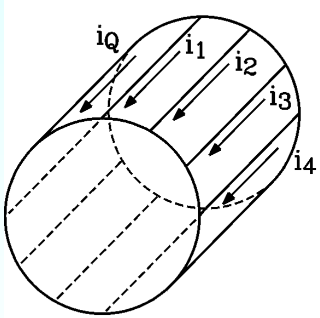
- ***Example:*** **Four pole cage rotor with Q_r/p= 14 bars per pole pair. Two adjacent bar voltage phasors are phase shifted by rotor slot angle** $\alpha_{Qr} = \frac{2\pi p}{Q_r} = \frac{2\pi \cdot 2}{28} = \pi / 7$
- **After 2 poles phase bundle is repeated: The bar voltages of bar 1 and 15, 2 and 16 etc. are in phase.**

Bar currents, bar forces, torque

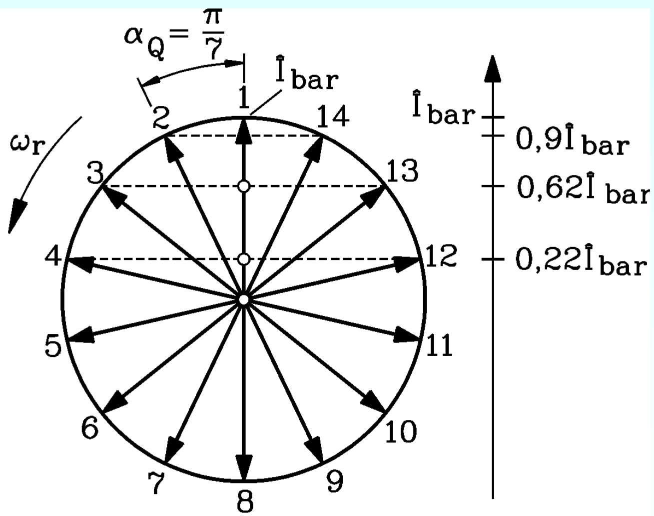
- **Rotor bar currents** form **regular current phasor bundle**, which excite a rotor air gap field wave. Only fundamental further considered. Together with stator fundamental field it forms the **resulting air gap magnetic field**.

- **The bar currents and the stator fundamental air gap field create per bar *per bar the tangential LORENTZ*-force:** $\boxed{\hat{F}_{bar} = \hat{I}_{bar} l \hat{B}_{\delta,s}}$

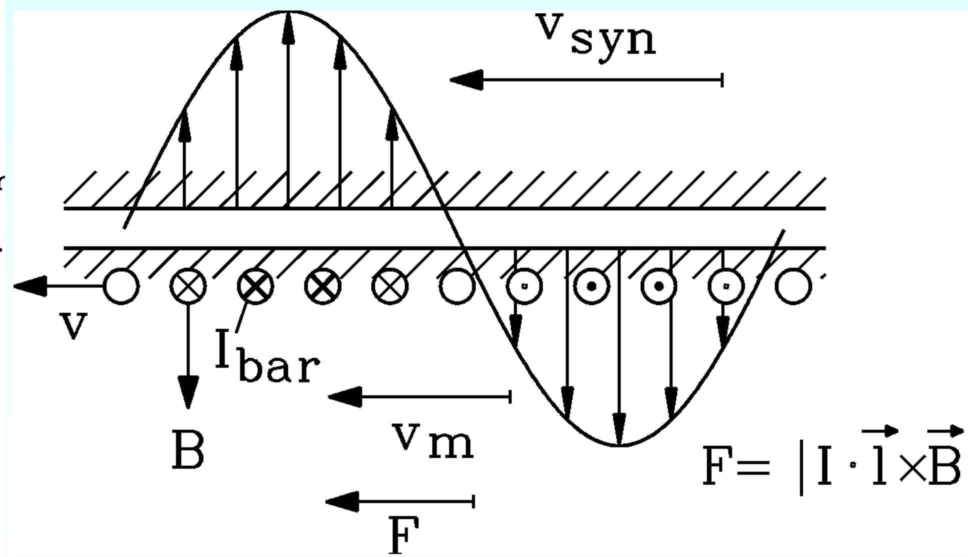
All bar forces form with the "lever" $d_r/2$ the electromagnetic torque M_e.



a)



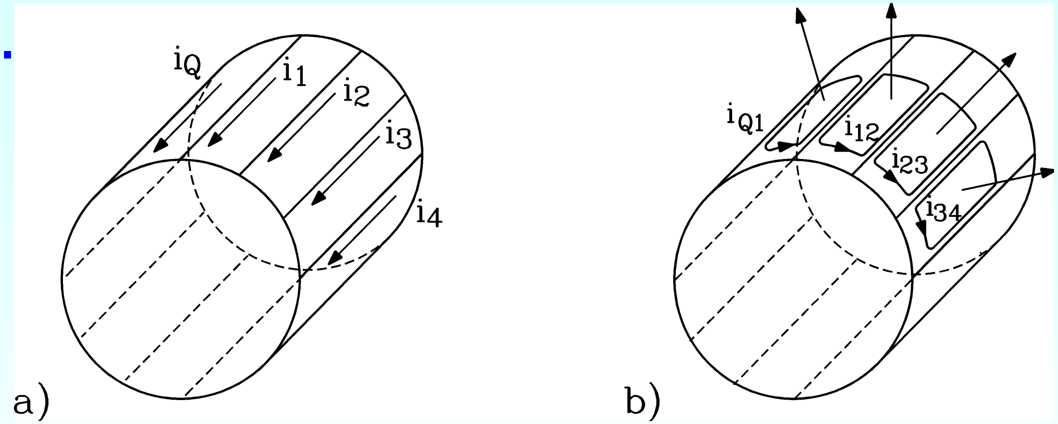
b)

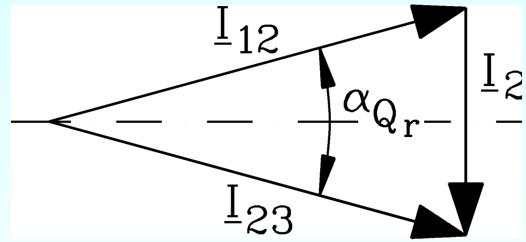


c)

Ring currents

- **Ring currents** flow in the **ring segments:** e. g. between bars No. 2 (bar current $\underline{I}_2$) and No. 3 (bar current $\underline{I}_3$) as ring section current $\underline{I}_{23}$.
- ***KIRCHHOFF´s node rule:*** $\underline{I}_{12} + \underline{I}_2 - \underline{I}_{23} = 0$. Hence the ring section currents are also phase shifted by rotor slot angle α_{Qr} and form a **regular bundle of ring section currents.**





$$I_2 = 2I_{12}\sin(\alpha_{Qr}/2) \quad \Rightarrow \quad I_{bar} = 2I_{Ring}\sin(p\pi/Q_r)$$

- **Resistance per ring section ΔR_{Ring} : the equivalent resistance** ΔR^*_{Ring} **is added to bar resistance R_{bar} :** $P_{Cu,r} = Q_r R_{bar} I^2_{bar} + 2Q_r \Delta R_{Ring} I^2_{Ring} = Q_r (R_{bar} + \Delta R^*_{Ring}) I^2_{bar}$

$$\boxed{\Delta R^*_{Ring} = \Delta R_{Ring} \cdot 1/(2\sin^2(\pi p/Q_r))}$$

Cage transfer ratio

- **Each bar may be regarded as a separate phase: number of windings N_r per phase: 1/2, number of rotor phases $m_r = Q_r$, winding coefficient $k_{wr} = 1$.**
- **Voltage and current transfer ratio are different:**

$$\boxed{\ddot{u}_U = \frac{k_{w,s} N_s}{k_{w,r} N_r}} \qquad \boxed{\ddot{u}_I = \frac{k_{w,s} N_s m_s}{k_{w,r} N_r m_r} = \frac{2 k_{w,s} N_s m_s}{Q_r}} \qquad \ddot{u}_U U_r = U_r' \qquad \frac{I_r}{\ddot{u}_I} = \frac{I_{bar}}{\ddot{u}_I} = I_r'$$

- **Rotor self and mutual inductance per phase (= per bar): with transfer ratio:**

$$\ddot{u}_U \ddot{u}_I L_{rh} = \left(\frac{k_{w,s} N_s}{k_{w,r} N_r}\right)^2 \frac{m_s}{m_r} \cdot \mu_0 N_r^2 k_{w,r}^2 \cdot \frac{2m_r}{\pi^2} \frac{l\tau_p}{p\delta} = \mu_0 N_s^2 k_{w,s}^2 \cdot \frac{2m_s}{\pi^2} \frac{l\tau_p}{p\delta} = L_{sh}$$

$$\ddot{u}_U \cdot M_{sr} = \frac{k_{w,s} N_s}{k_{w,r} N_r} \cdot \mu_0 \cdot N_r k_{w,r} \cdot N_s k_{w,s} \cdot \frac{2m_s}{\pi^2} \frac{l\tau_p}{p\delta} = \mu_0 N_s^2 k_{w,s}^2 \cdot \frac{2m_s}{\pi^2} \frac{l\tau_p}{p\delta} = L_{sh}$$

$$\ddot{u}_I \cdot M_{rs} = \frac{k_{w,s} N_s m_s}{k_{w,r} N_r m_r} \cdot \mu_0 \cdot N_s k_{w,s} \cdot N_r k_{w,r} \cdot \frac{2m_r}{\pi^2} \frac{l\tau_p}{p\delta} = \mu_0 N_s^2 k_{w,s}^2 \cdot \frac{2m_s}{\pi^2} \frac{l\tau_p}{p\delta} = L_{sh}$$

- **Result:** $\boxed{R_r' = \ddot{u}_U \ddot{u}_I R_r}$, $\boxed{L_{r\sigma}' = \ddot{u}_U \ddot{u}_I L_{r\sigma}}$, $\boxed{\ddot{u}_U M_{sr} = \ddot{u}_I M_{rs} = \ddot{u}_U \ddot{u}_I L_{rh} = \underline{\underline{L_h}}}$

Equivalent circuit for cage induction machine

- **Use of transfer ratios $\ddot{u}_U$, $\ddot{u}_I$ in the stator and rotor voltage equations:**

$$\underline{U}_s = j\omega_s \cdot \ddot{u}_I M_{rs} \cdot (\underline{I}_r / \ddot{u}_I) + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s$$

$$j\omega_r \ddot{u}_U M_{sr} \underline{I}_s + j\omega_r \ddot{u}_U \ddot{u}_I L_{r,h} \cdot (\underline{I}_r / \ddot{u}_I) + j\omega_r \ddot{u}_U \ddot{u}_I L_{r\sigma} \cdot (\underline{I}_r / \ddot{u}_I) + \ddot{u}_U \ddot{u}_I R_r \cdot (\underline{I}_r / \ddot{u}_I) = 0$$

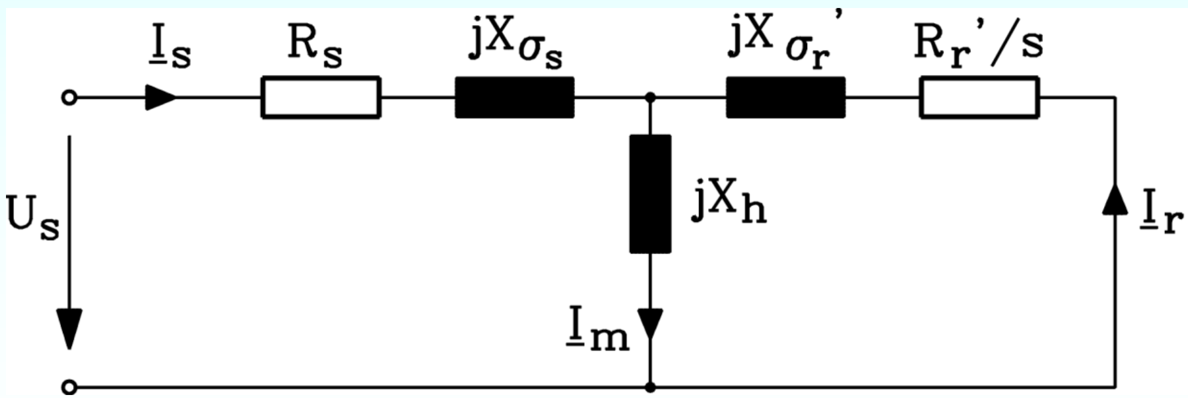
- $$\underline{U}_s = j\omega_s L_h \underline{I}'_r + j\omega_s L_h \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + R_s \underline{I}_s$$

$$0 = js\omega_s L_h \underline{I}_s + js\omega_s L_h \underline{I}'_r + js\omega_s L'_{r\sigma} \underline{I}'_r + R'_r \underline{I}'_r$$

$$\boxed{\underline{U}_s = R_s \underline{I}_s + jX_{s\sigma} \underline{I}_s + jX_h (\underline{I}_s + \underline{I}'_r)}$$

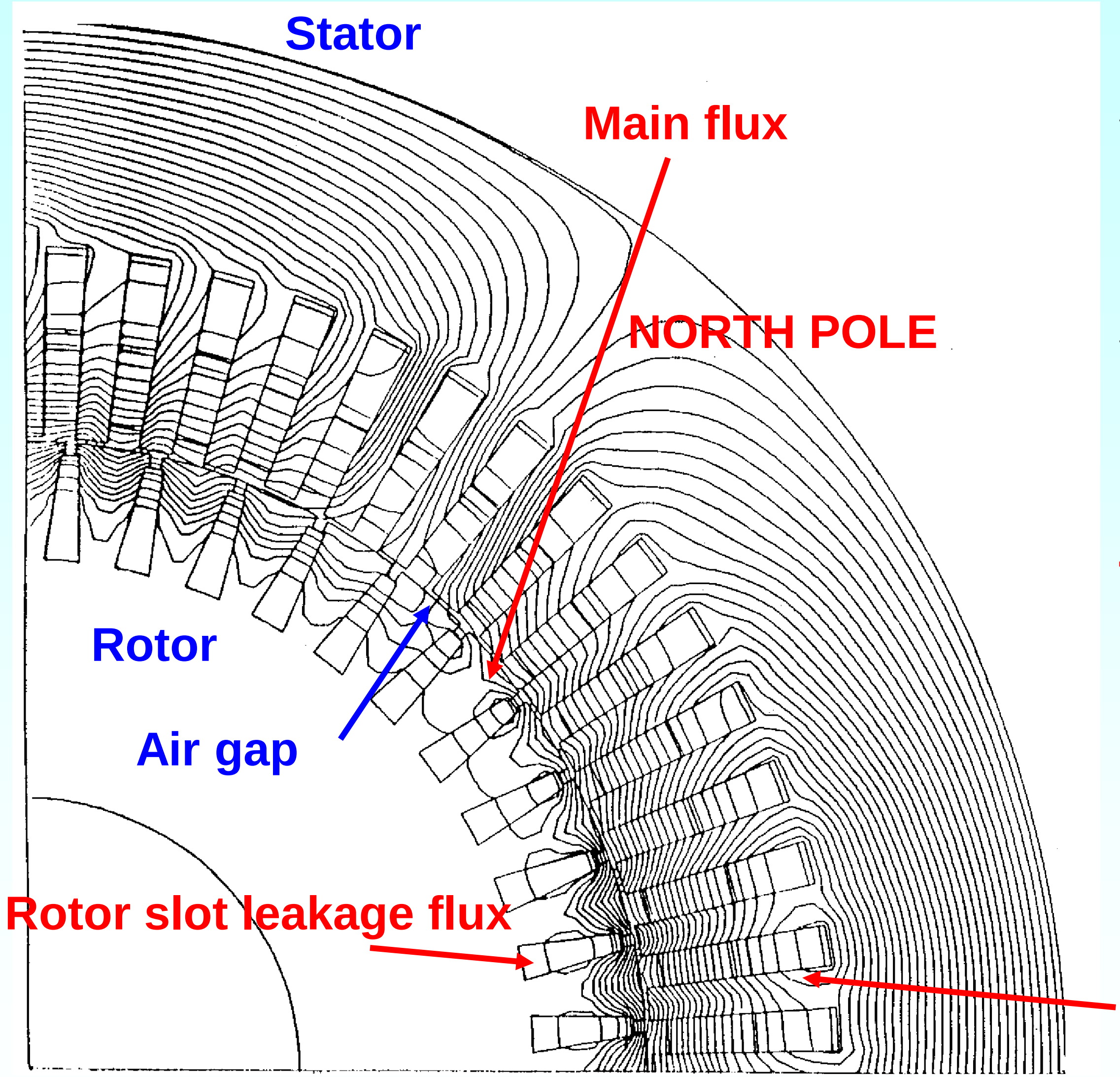
$$\boxed{0 = \frac{R'_r}{s} \underline{I}'_r + jX'_{r\sigma} \underline{I}'_r + jX_h (\underline{I}_s + \underline{I}'_r)}$$

- **T-Equivalent circuit per stator phase:**



- ***Facit:* We get the SAME equivalent circuit as with wound rotor induction machines.**

Field lines *B* of a cage induction machine



Main flux: Links stator and rotor winding; field lines cross the air gap

Leakage flux (stray flux): Is only linked with either stator or rotor winding; field lines DO NOT cross the air gap

Example:

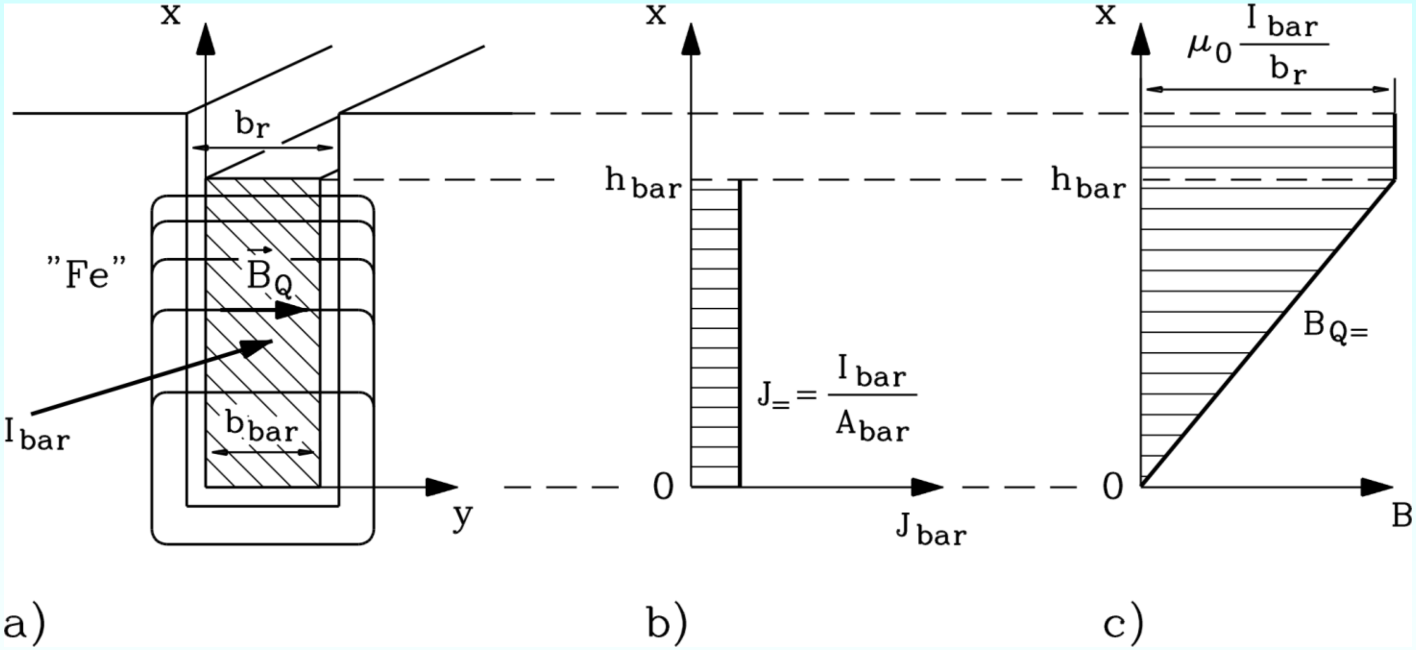
Four-pole wedge bar rotor:

Field lines at stand still (n = 0)

- Rotor frequency = Stator frequency
- Rotor current is NEARLY in phase opposition to stator current

Rotor slot stray flux

- **If rotor current density $J_{bar} = I_{bar} / A_{bar}$ is homogeneously distributed over bar cross section, then slot stray field, which crosses slot perpendicular to slot axis, increases linear with bar height *x* !**



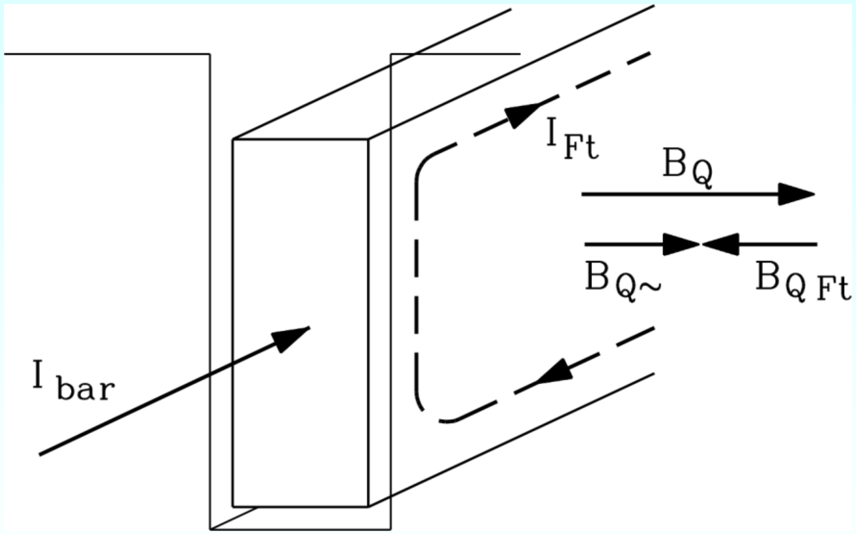
- **AMPERE´s law:** $\oint_C \vec{H} \cdot d\vec{s} = H_Q(x) \cdot b_r = J \cdot x \cdot b_{bar}$

$$B_Q(x) = \mu_0 J \frac{x \cdot b_{bar}}{b_r} = \mu_0 \frac{I_{bar}}{b_r} \cdot \frac{x}{h_{bar}}, 0 \le x \le h_{bar} \quad \text{or} \quad B_Q = \mu_0 \frac{I_{bar}}{b_r} \quad , x > h_{bar}$$

Current displacement in rotor bars

- **Slot flux density** is pulsating with rotor frequency, penetrating the rotor bar from the side. High rotor bars form a "massive short circuit loop". *FARADAY´s law yields:* $\boldsymbol{B_Q}$ induces voltage $u_i = -\mathrm{d}\Phi/\mathrm{d}t$ in bar, which causes **eddy current flow $\boldsymbol{I_{Ft.}}$** Self field of that eddy current B_{QFt} is directed opposite to $\boldsymbol{B_Q}$ due to *LENZ´s rule*.

- Hence the **eddy current** I_{Ft} flows in upper bar region IN direction of bar current I_{bar} , and in lower bar region OPPOSITE to bar current.



- ***Facit 1:***

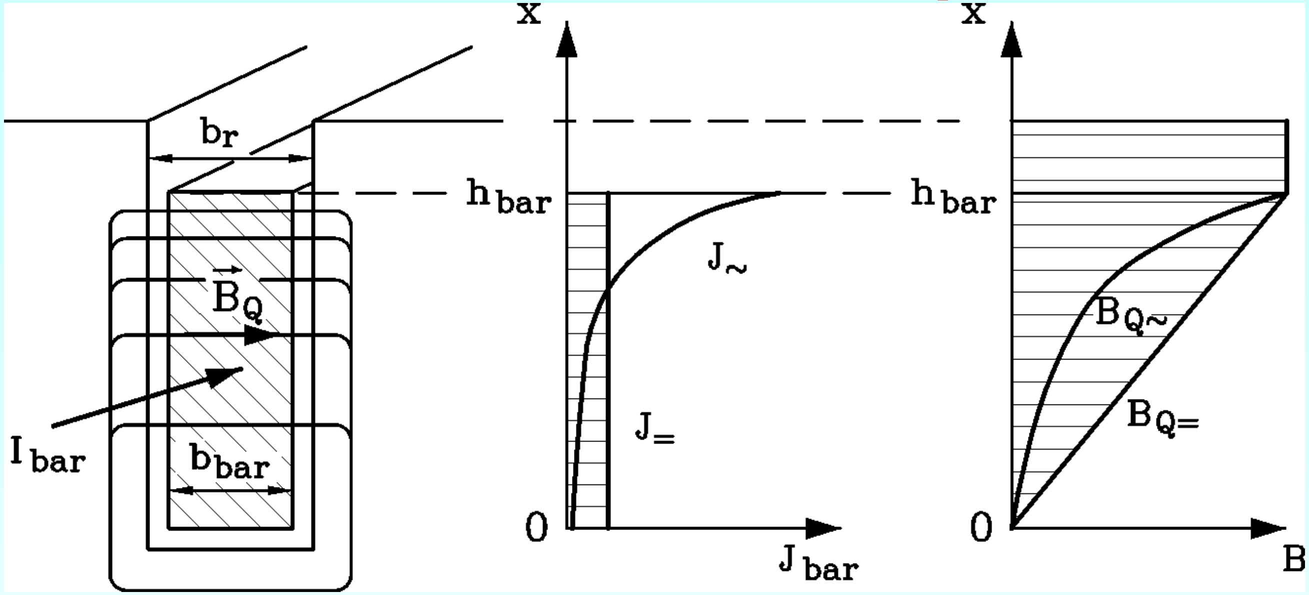
Due to I_{Ft} the resulting bar current density is HIGHER in upper bar region: **Current displacement towards upper bar region ("Skin effect")**.

- ***Facit 2:***

The **resulting** slot stray flux density $B_{Q\sim}$ is due to B_{QFt} **reduced.**

- **Current displacement INCREASES** with increasing rotor frequency f_r , with increasing electric bar-conductivity κ, with increasing bar height h_{bar} and with increasing permeability μ of conductor. (***Note:*** Copper and aluminium´s permeability is $\mu = \mu_0$!)

Effects of rotor current displacement



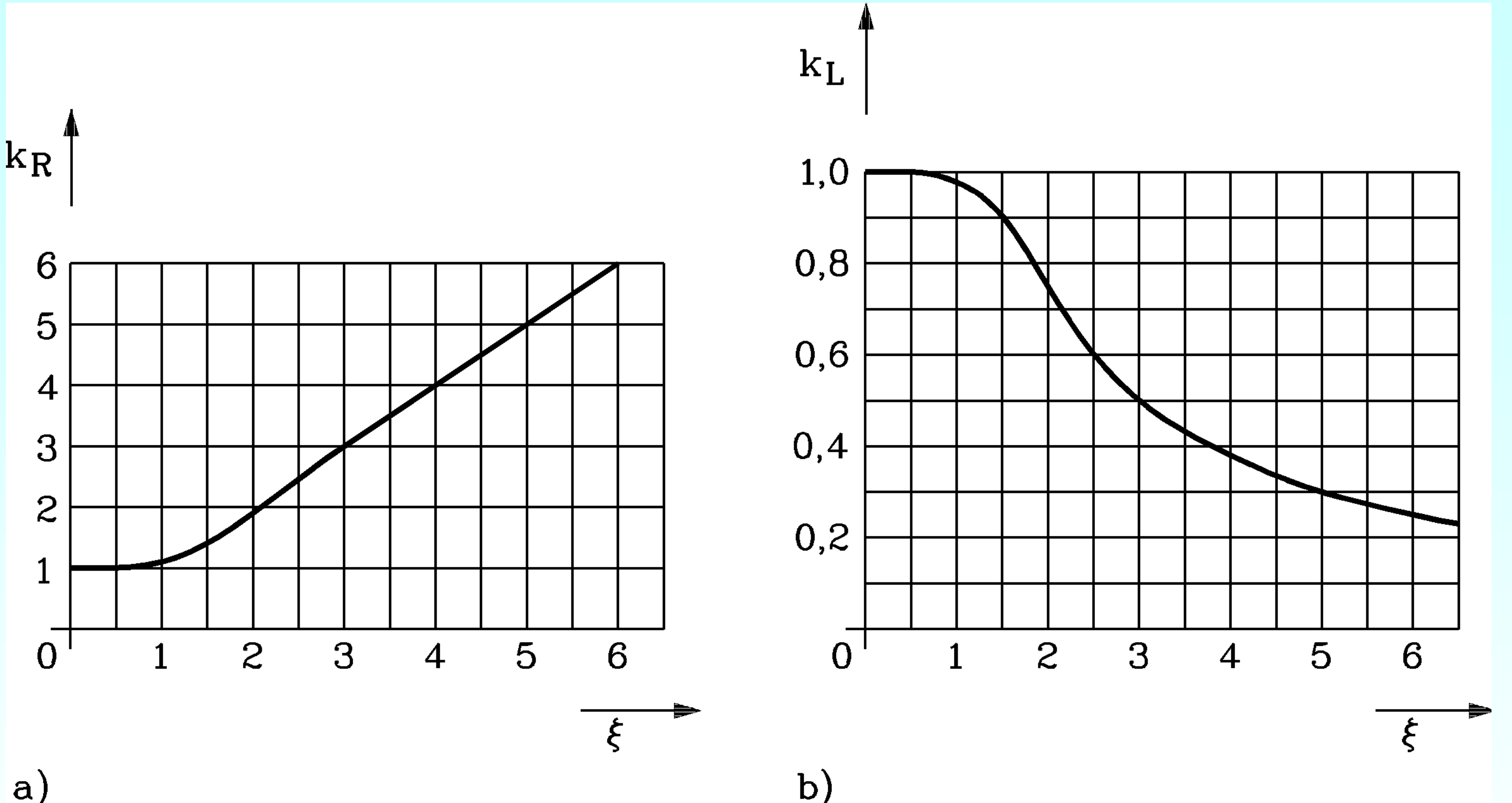
- **At high rotor frequency (e. g. $s = 1$) major part of bar current flows in upper bar region: so only reduced bar cross section is used for current flow. Thus "AC bar resistance" $R_{bar\sim}$ is higher than "DC bar resistance" $R_{bar=}$.**

- **Due to reduction of slot stray flux density the slot leakage flux is reduced. Hence the "AC bar inductance" $L_{bar\sim}$ is smaller than the "DC bar inductance" $L_{bar=}$.**

$$\boxed{R_{bar\sim} = k_R R_{bar=} > R_{bar=}} \qquad \boxed{L_{\sigma,bar\sim} = k_L L_{\sigma,bar=} < L_{\sigma,bar=}}$$

- **At low rotor frequency (e. g. $s = s_N$) <u>nearly NO current displacement occurs !</u>**

Resistance increase k_R and inductance decrease k_L



- **"Reduced" conductor height ξ : Per-unit value ξ, containing all relevant parameters:**

k_R, k_L for deep bar ("rectangular cross section") depend on: $\boxed{\xi = h_{bar}\sqrt{\pi f_r \mu \kappa \frac{b_{bar}}{b_r}}}$

Example: Current displacement in deep bar

- **Copper deep bar:**

- At 75°C bar temperature copper conductivity is κ_{Cu} = 50·10⁶ S/m.

- Bar width = slot width: $b_{bar} = b_r$ **,**

- Permeability: $\mu_{Cu} = \mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{Vs/(Am)}$

- Starting of induction machine: *s* = 1: Rotor frequency f_r = 50 Hz

- Bar height: h_{bar} = 3 cm

$$\xi = h_{bar}\sqrt{\pi f_r \mu\kappa \frac{b_{bar}}{b_r}} = 3\cdot 10^{-2}\cdot\sqrt{\pi\cdot 50\cdot 4\pi\cdot 10^{-7}\cdot 50\cdot 10^{6}\cdot 1} = 2.98 \approx 3$$

From curve $k_R(\xi)$ we get: $k_R(3) = 3$ and from $k_L(\xi)$ follows: $k_L(3) = 0.5$.

- ***Facit:***

- Rotor bar resistance increases up to 3-fold !

- Rotor bar inductance decreases down to 50%.

- ***Thumb rule:***

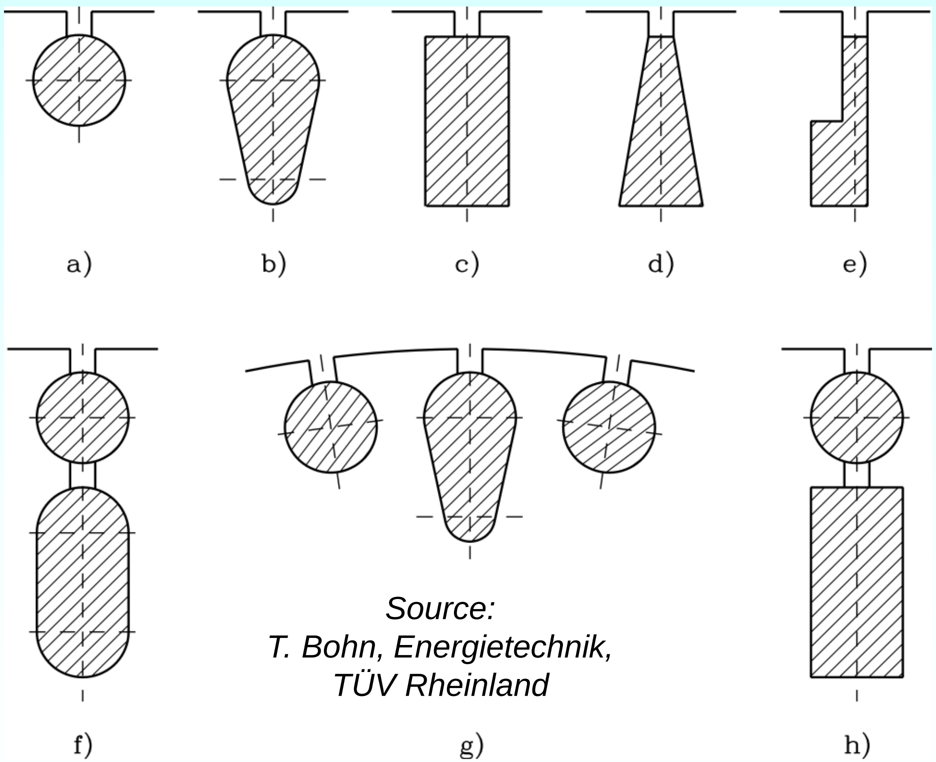
At 50 Hz the increase of resistance of copper deep bar is $k_R = h_{bar}[cm]$.

Increase of starting torque by current displacement

- **Increase of rotor losses** leads to **increase of starting torque *M_1*:**

$$M_e(s) = \frac{P_\delta}{\Omega_{syn}} = \frac{P_{Cu,r}/s}{\Omega_{syn}} \quad \Rightarrow \quad M_1 = M_e(s=1) = \frac{P_{Cu,r}}{\Omega_{syn}}$$

- **Special bar cross sections** for small and big starting torque:



Source:
T. Bohn, Energietechnik,
TÜV Rheinland

SMALL current displacement = *M_1* small:

a) Round bar, b) Oval bar,

BIG current displacement = *M_1* increased:

c) Deep bar, d) Wedge bar, e) L-bar,

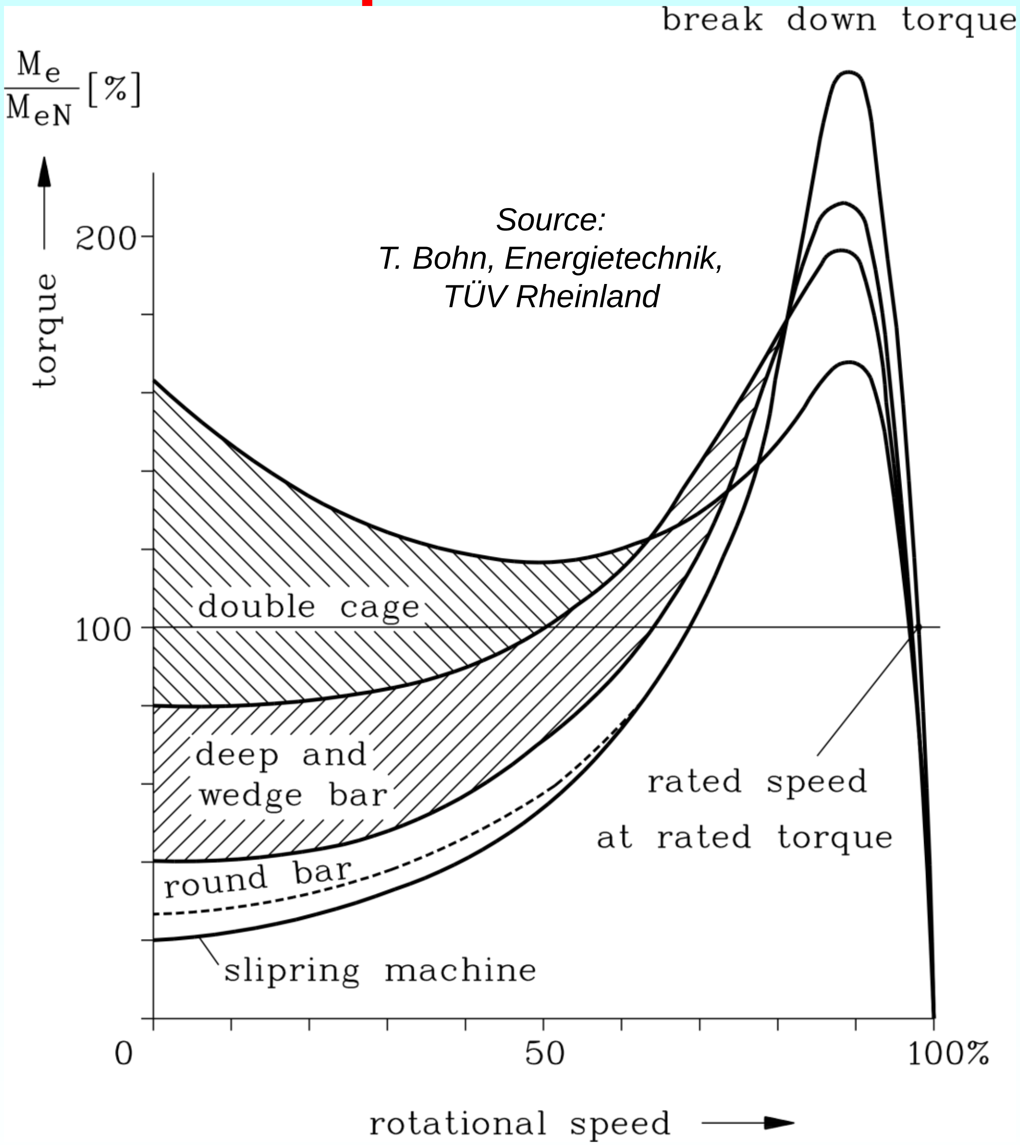
VERY BIG current displacement = *M_1* big:

f) and h): **Double bars**, g) alternating bars:

Round upper bronze bars (high resistance) cause – along with current displacement from lower in upper bar – high rotor losses, M_1 is big. Lower bar nearly without current (STARTING OF MOTOR, s = 1).

At rated slip small current displacement: Current flow mainly in lower bar: low losses !

Torque characteristics of induction machines



- $M(n)$-characteristics of induction machines with **different rotor bar cross sections**

- In the figure torque is given per unit of rated torque, speed per unit of synchronous speed !

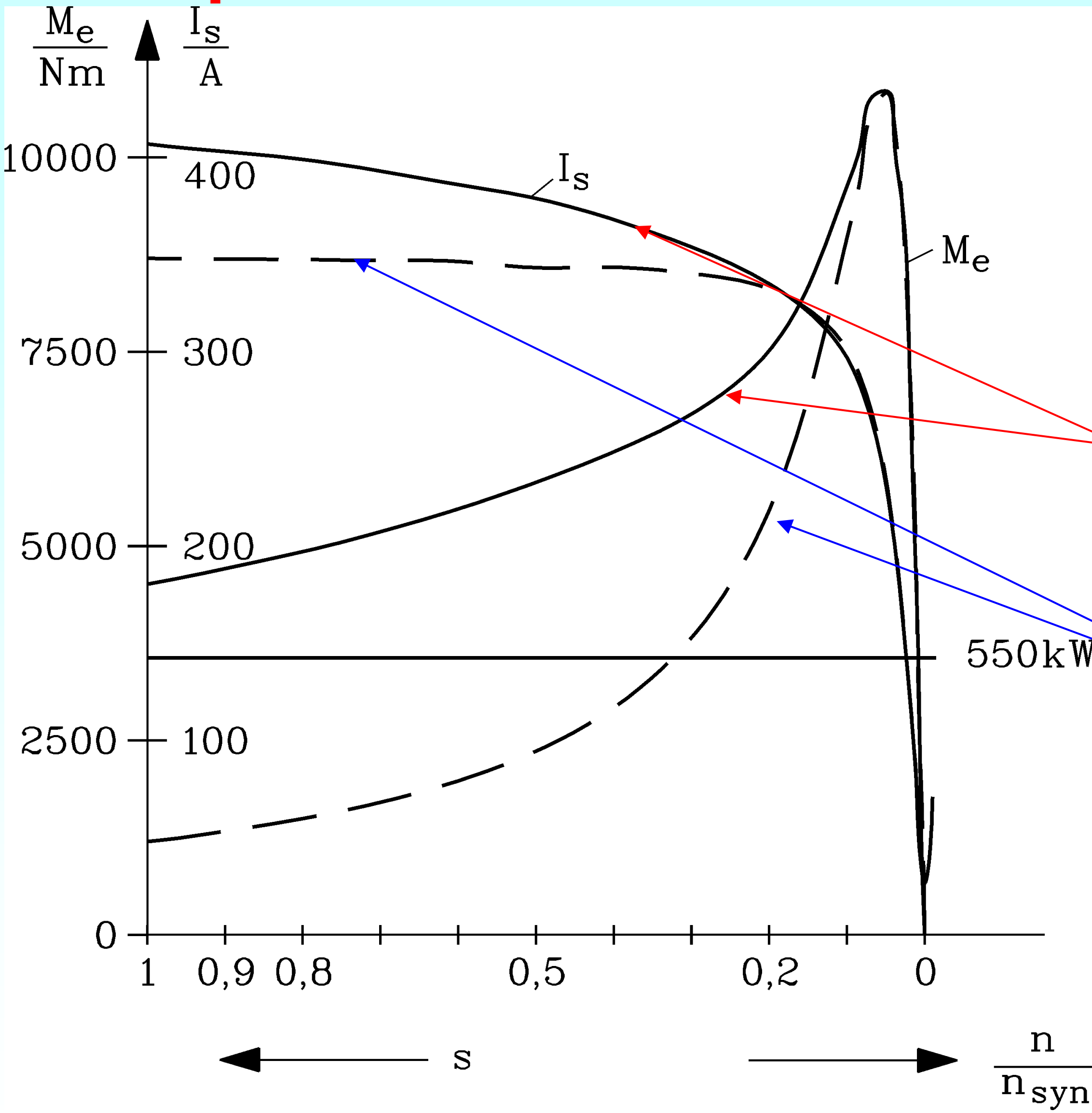
- **Wound rotor with round wire:** Rotor winding consists of many thin wires: no current displacement; similar: **Round bar rotor**

- **Wedge and deep bar rotor:** increased starting torque of about 40% ... 80%M_N; **Double cage rotor:** Starting torque reaches 160% M_N.

- Big current displacement needs deep bars = high dc bar inductance = big leakage coefficient σ. **Hence break down torque decreases.**

$$M_b = \pm \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1-\sigma}{\sigma X_s}$$

Deep bar rotor: Influence of current displacement

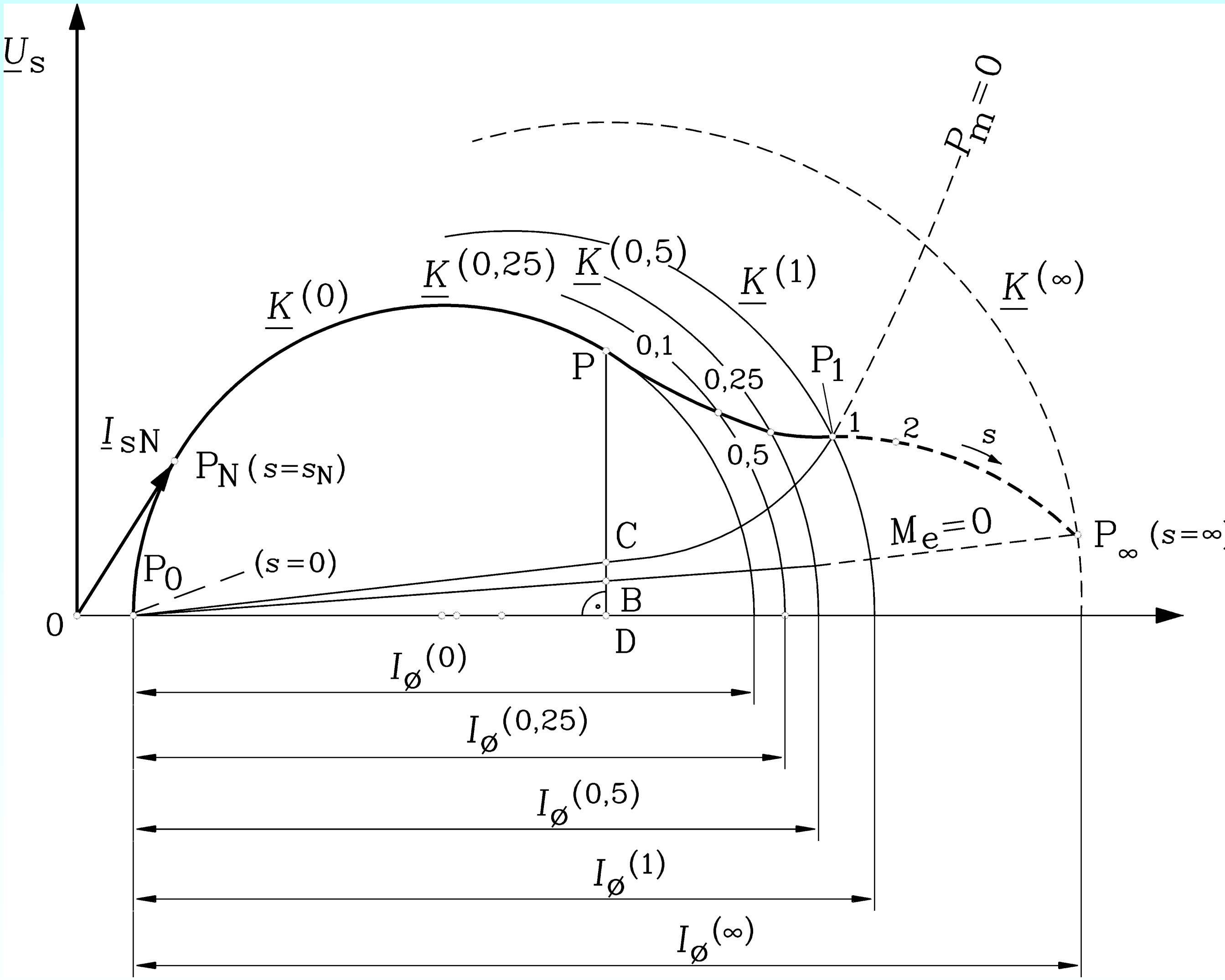


Calculated stator current and torque in dependence of speed:
550 kW, 4 poles, three phase, deep bar rotor cage
6.6 kV, 50 Hz

With current displacement
(k_L, k_R) in the rotor bars

Without current displacement
($k_L = 1$, $k_R = 1$): Torque and current are smaller at $s = 1$.

Influence of current displacement on "circle diagram"



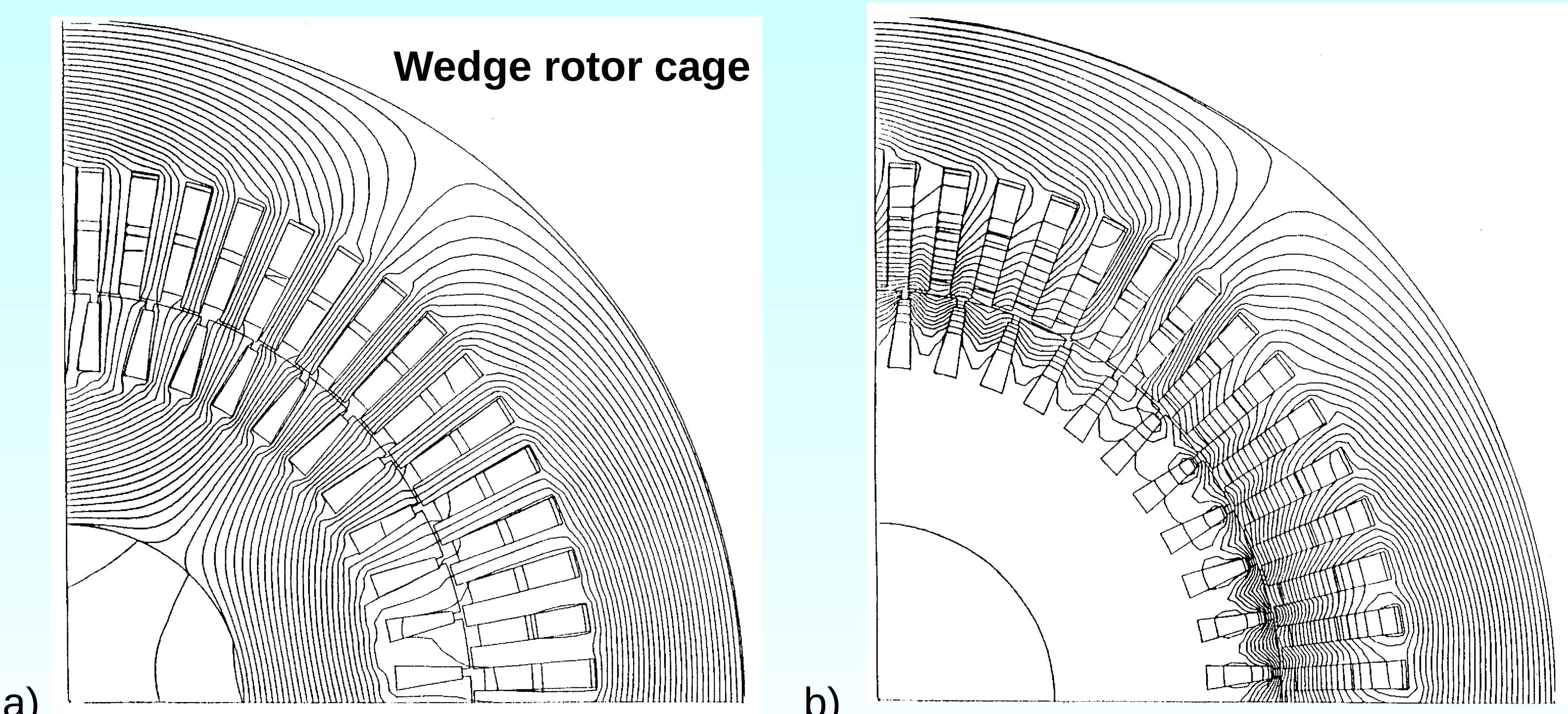
- **Increase of resistance:**

<u>**Coefficient k_R:**</u> **Shape of circle remains unchanged, but slip scale is shifted to the left towards no-load point P_0.**

- **Reduction of stray inductance: <u>Coefficient k_L:</u> increases circle diameter !**

- ***<u>Facit:</u> Each slip value s defines by $k_R(s)$, $k_L(s)$ a separate circle. Hence resulting current locus is <u>no longer a circle !</u>***

Flux density lines without / with current displacement



a) No-load: Rotor frequency zero: Nor rotor current, no current displacement.

b) Locked rotor (*s* = 1): Rotor frequency = stator frequency: Big current displacement. Rotor current phase opposite to stator current; flows mainly in upper part of rotor bars, repulses stator field to air gap.