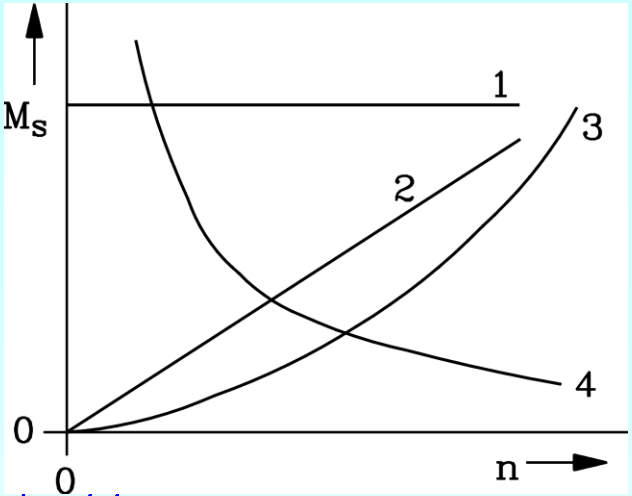
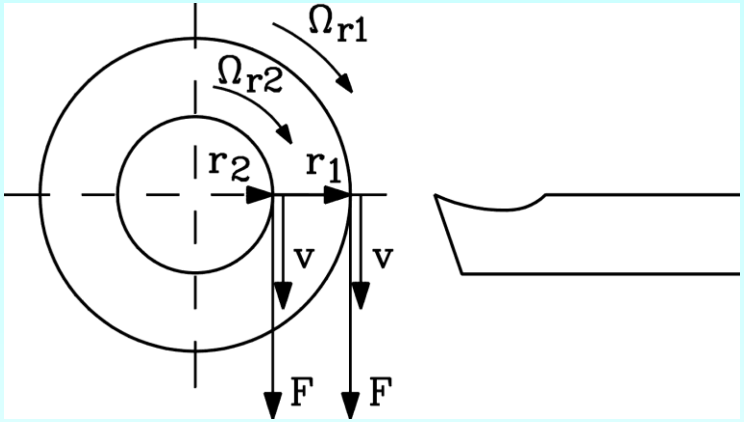
7. Induction Machine Based Drive Systems

Source: Siemens AG

Load characteristics of different machines





1) Constant torque: **- hoisting goods: elevators, cranes, ...** $M_s = m \cdot g \cdot (d/2)$

- piston compressors

2) Torque rises linear with speed: **- extrusion of plastics** $M_s \sim n$

3) Torque rises with square of speed: **rotating hydraulic machines**: pumps, fans, ventilators, turbo compressors, ship propulsion; *EULER*´s turbine equation ! $M_s \sim n^2$

4) Torque depends on inverse of speed ("constant power drives"): **Tooling machines: cutting, milling, drilling; winding machines; rolling e.g. steel sheets**

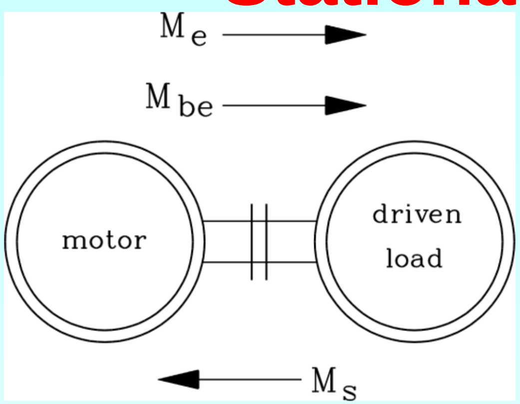
e. g. cutting: cutting speed *v* and cutting force *F* have to be constant at optimum values, independent of speed: $v = \Omega_{r1} \cdot r_1 = \Omega_{r2} \cdot r_2 = konst. \quad \Leftrightarrow \quad \Omega_r = 2\pi \cdot n \quad \Rightarrow \quad n = \frac{v}{2\pi \cdot r}$

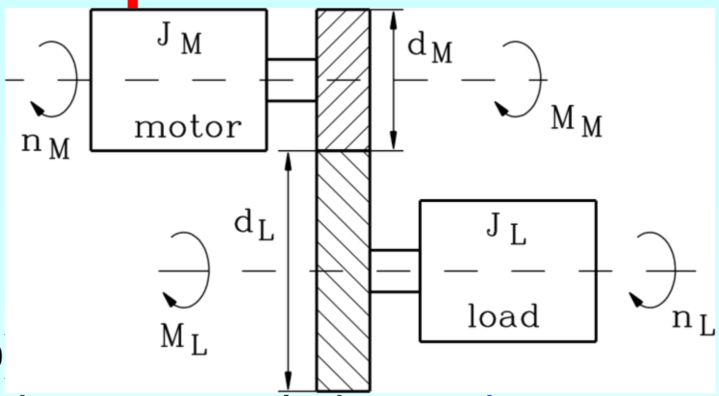
$$M_s = P/(2\pi \cdot n) \sim 1/n$$

Example: Drilling unit

Source: Aradex, Germany

Stationary point of operation

a) 

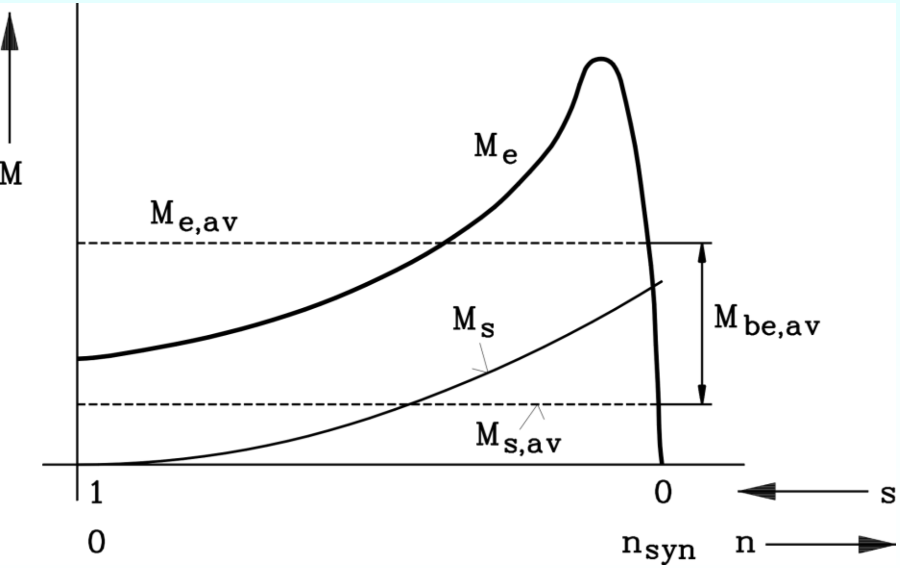
b) 

Directly coupled motor: $n_M = n_L$, $M_M = M_L$

via gear coupled: $n_M = i \cdot n_L$, $M_M = M_L / i$

$i = d_L / d_M$ gear transmission ratio

- Intersection of motor- and load characteristic defines stationary speed $n_M = n < n_{syn}$



Example: Fan drive

- Shaft torque $M_s = M_L$ (or in geared version M_L / i) brakes the motor. Motor has to come up with that torque continuously. $M_e - M_d = M_s$

If loss torque M_d in motor (friction, ...) is neglected, we calculate with air gap torque: $\boxed{M_e = M_s}$

- For acceleration we need: $M_e > M_s$

Starting (run-up) of induction motor

- **NEWTON´s law for acceleration:** $$J_{L+M} \cdot \frac{d(2\pi n)}{dt} = M_{be} = M_e - M_s$$

directly coupled motor: $J_{L+M} = J_L + J_M$, geared motor: $J_{M+L} = J_M + \frac{J_L}{i^2}$

- **"Starting time constant" T_J:** Induction machine runs up alone (= without coupled load) with rated torque $M_e = M_N$ from zero speed to rated speed. $M_s = 0, J_L = 0$

$$J_M \frac{d\Omega_m}{dt} = M_N \quad \Rightarrow \quad \int_0^{\Omega_{mN}} d\Omega_m = \int_0^{T_J} \frac{M_N}{J_M} dt \quad \Rightarrow \quad T_J = \frac{J_M}{M_N} \Omega_{mN}$$

<u>Small motors:</u> short starting time constant (< 1 second), <u>big motors:</u> up to > 10 s. The starting time constant is a measure for angular momentum J_M of rotor of machine.

- **Acceleration time t_a:** $$t_a = \int_0^{n_N} \frac{2\pi \cdot J}{M_e(n) - M_s(n)} \cdot dn \qquad t_a \approx \frac{2\pi n_N J}{M_{e,av} - M_{s,av}}$$

Estimate for t_a : Average values $M_{e,av}$ and $M_{s,av}$ for speed range 0...n_N are used !

Dissipated heat in rotor winding due to start-up

a) No-load start up (acceleration of masses / inertia): Motor runs up without load torque (M_s = 0). Only the rotating masses of motor and coupled load J are accelerated:

$$W_{Cu,r} = \int_0^{t_a} P_{Cu,r} \cdot dt = \int_0^{t_a} sP_\delta \cdot dt = \int_0^{t_a} s\Omega_{syn} M_e \cdot dt = \int_0^{t_a} s\Omega_{syn} J \frac{d\Omega_m}{dt} \cdot dt =$$

$$\int_0^{t_a} s\Omega_{syn}^2 J \frac{d(1-s)}{dt} \cdot dt = -\int_0^{t_a} s\Omega_{syn}^2 J \frac{ds}{dt} dt = -\int_1^0 s\Omega_{syn}^2 J \cdot ds = -J\Omega_{syn}^2 \left. \frac{s^2}{2} \right|_1^0 = \frac{J\Omega_{syn}^2}{2} = W_{kin}$$

$$\boxed{W_{Cu,r} = W_{kin}}$$

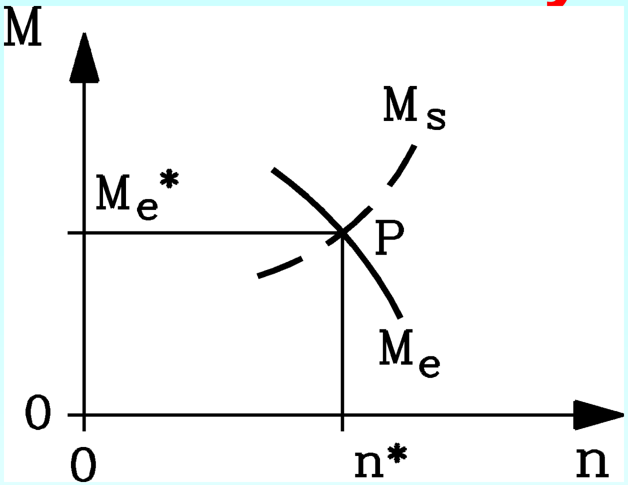
The heat $W_{Cu,r}$, dissipated in rotor winding of induction machine during start up, is of the same amount as the stored kinetic energy W_{kin} in the rotating masses J.

b) Loaded start up: Motor starts against load torque M_s :

- **Acceleration time increases by ratio** $M_{e,av} / (M_{e,av} - M_{s,av})$

- **Dissipated heat** in rotor winding **increases by**: $W_{Cu,r} = \frac{J\Omega_{syn}^2}{2} \cdot \frac{M_{e,av}}{M_{e,av} - M_{s,av}} > W_{kin}$

Stability of operation point $P = (M_e^*, n^*)$



- Linearization of characteristics M_e, M_s in point P: $\Omega_m^* = 2\pi n^*$

$$M_e(\Omega_m) \cong M_e(\Omega_m^*) + M_e' \cdot \Delta\Omega_m$$
$$M_s(\Omega_m) \cong M_s(\Omega_m^*) + M_s' \cdot \Delta\Omega_m$$

$M' = \mathrm{d}M / d\Omega_m^*$ at Ω_m^*

- Deviation of speed $\Delta\Omega_m = \Omega_m - \Omega_m^*$ in P at disturbance of equilibrium $M_e(\Omega_m^*) = M_s(\Omega_m^*)$ has to be calculated:

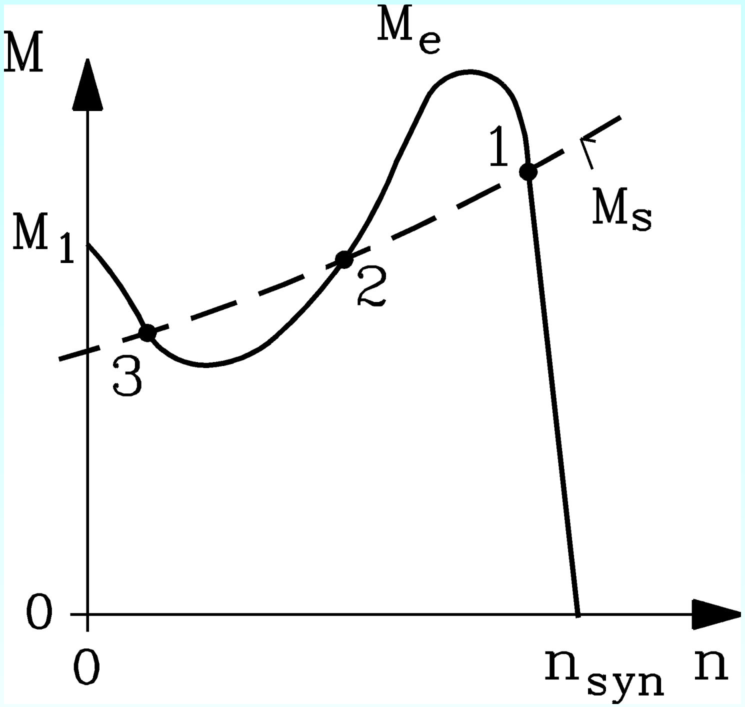
$$J \cdot \frac{d\Omega_m}{dt} = M_e(\Omega_m) - M_s(\Omega_m) \quad \Rightarrow \quad J \cdot \frac{d\Delta\Omega_m}{dt} - (M_e' - M_s') \cdot \Delta\Omega_m = 0$$

1st order linear differential equation has solution: $\Delta\Omega_m(t) \sim \exp\left(t \cdot \frac{M_e' - M_s'}{J}\right)$

- $dM_e / d\Omega_m - dM_s / d\Omega_m > 0$ *: Deviation of speed from steady state speed in operation point P increases with time; operating point P is* ***unstable****.*

- $dM_e / d\Omega_m - dM_s / d\Omega_m < 0$ *: Deviation of speed from steady state speed in operation point P decreases with time; operating point is* ***stable****.*

Example: Operating points of double cage motor

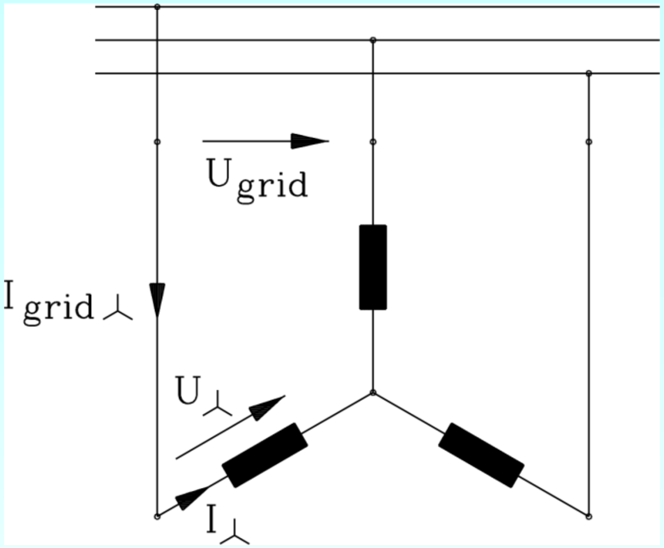


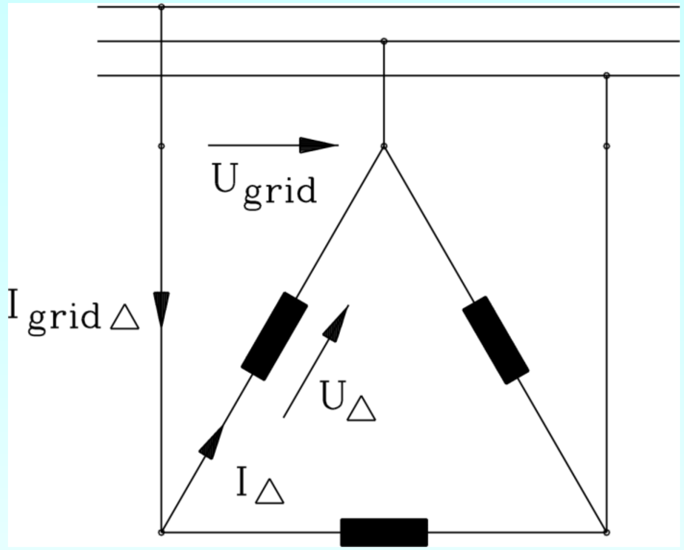
Operation points 1 and 3 are stable, point 2 is unstable.

During running up motor will stay in operation point 3. **The desired point 1 is NOT reached.**

No.	Operation points	$dM_e/d\Omega_m$	$dM_s/d\Omega_m$	$dM_e/d\Omega_m - dM_s/d\Omega_m$
1	**stable**	<0	>0	<0
2	**unstable**	>0	>0	>0
3	**stable**	<0	>0	<0

Y-D (star-delta) start-up to reduce starting current





Motor in Y-connection switched to the grid – starting with reduced current (one third !) - after start up switching to D-connection: torque increases to 3 times to get nominal power !

- **Star:** Phase voltage $U_Y = U_{grid}/\sqrt{3}$, phase current I_Y = line current $I_{grid,Y}$.
- **Delta:** Phase voltage U_Δ = Line-to-line voltage U_{grid}, Phase current $I_\Delta = I_{grid,\Delta}/\sqrt{3}$

$$U_Y = \frac{U_\Delta}{\sqrt{3}} \quad \Rightarrow \quad I_Y = \frac{I_\Delta}{\sqrt{3}} \quad \Rightarrow$$

Grid current: $I_{grid,Y} = \frac{I_{grid,\Delta}}{\sqrt{3}\cdot\sqrt{3}} = \frac{I_{grid,\Delta}}{3}$

$$\Rightarrow \quad M \sim U^2 \quad \Rightarrow$$

Torque: $\frac{M_{1Y}}{M_{1\Delta}} = \left(\frac{U_Y}{U_\Delta}\right)^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

Example: Y-D-Start-up

Starting of a double-cage induction machine:

Data of motor; P_N = 155 kW, f_N = 50 Hz, n_N = 974/min, U_N = 400 V, Y / D, $\cos\varphi_N$ = 0.85, η_N = 0.91

Calculate:

Rated torque:

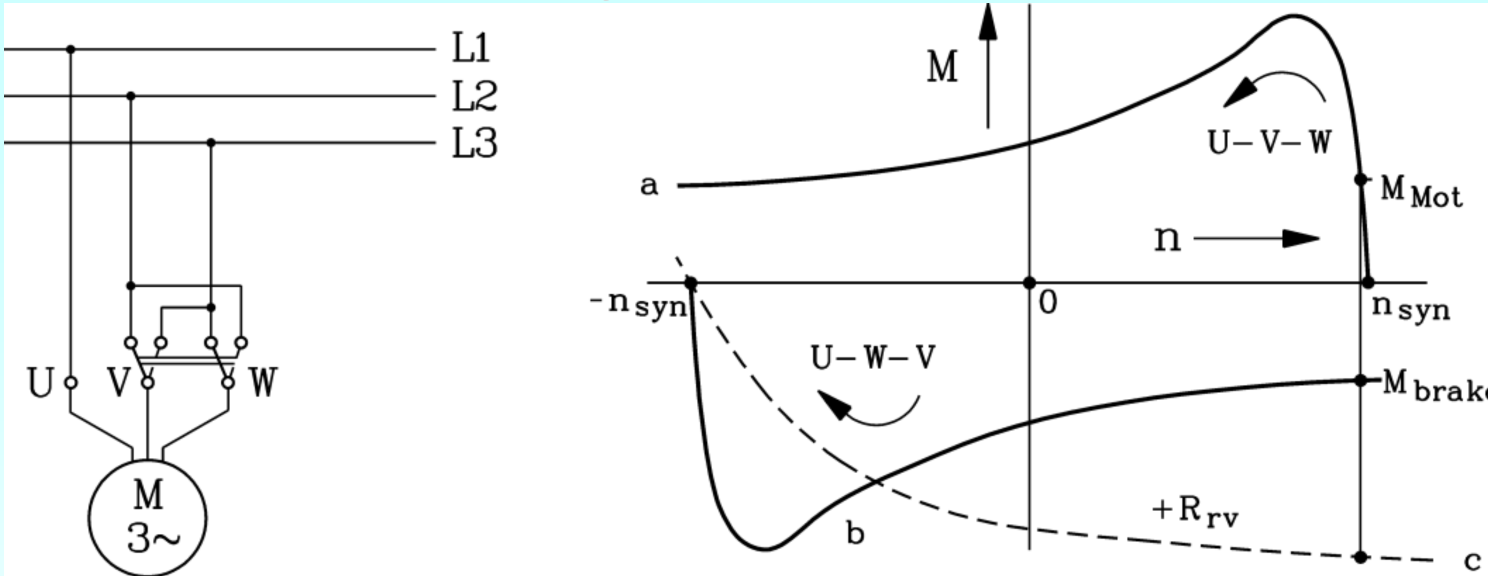
$$M_N = \frac{P_N}{2\pi n_N} = \frac{155000}{2\pi \cdot (974/60)} = 1520\,Nm$$

Rated current:

$$I_N = \frac{P_N}{\eta_N \cdot \cos\varphi_N \cdot \sqrt{3} U_N} = \frac{155000}{0.91 \cdot 0.85 \cdot \sqrt{3} \cdot 400} = 289\,A$$

	M_1/M_N	M_1/Nm	I_1/I_N	I_1/A
Δ-connection	2.1	3192	6	1735
Y-connection	0.7	1064	2	578

Braking by reversal



- **Change connection of 2 terminals (e.g. V and W):** Speed and torque are reversed (*M*-*n*-curve b instead of a), motor is braked, speed decreases. At n = 0 motor has to be disconnected from grid, otherwise it accelerates in opposite direction.
- **Slip-ring motor**: External resistances increase braking torque up to break down torque (curve c).
- Induction machine consumes electrical power via stator winding AND kinetic energy from rotating mass as mechanical power P_m via rotor winding. Neglecting stator resistance ($P_{in} \sim P_\delta$), both power components are dissipated as rotor winding heat: "rotor gets hot".

$$P_{Cu,r} = sP_\delta = -(1-s)P_\delta + P_\delta = |P_m| + |M_e \Omega_{syn}|$$

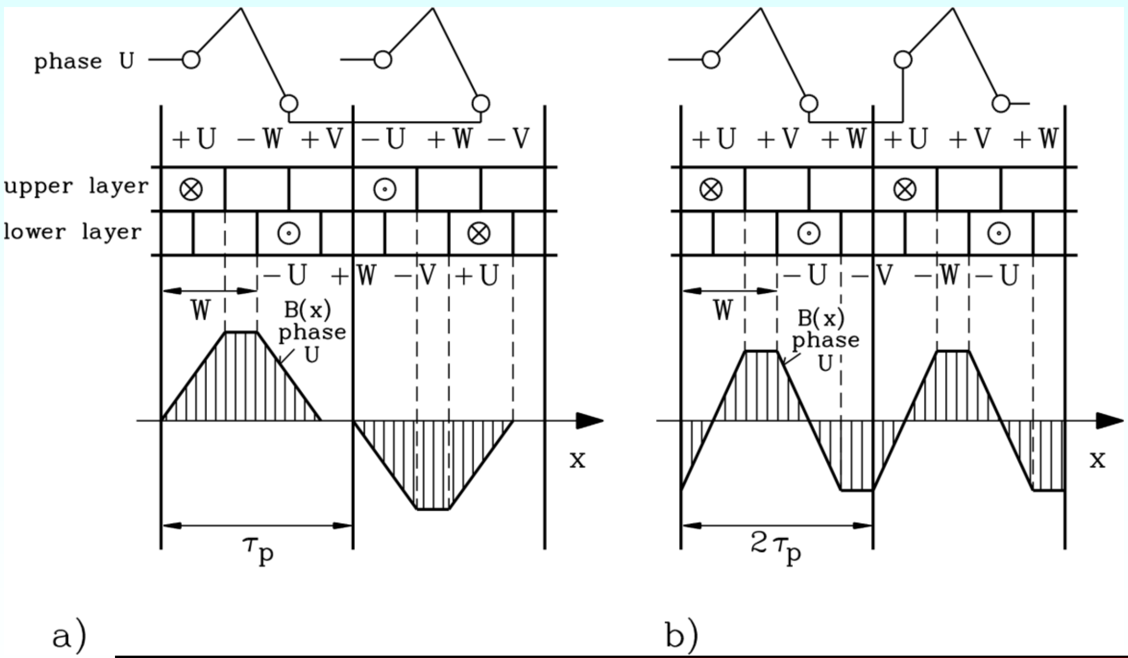
Pole changing cage induction motors

- **Several three-phase windings** with different pole count in stator slots: "step-wise" speed change through different synchronous speeds.

Example: Cage induction machine: 48 Stator slots
- 2-pole winding: q = 8, - 4-pole winding: q = 4, - 8-pole winding: q = 2.

Speed levels at 50 Hz-grid: 3000/min, 1500/min, 750/min.

Per winding system only 1/3 of slot cross section reduces nominal power per speed stage to 1/3. ***Note:*** **Rotor cage fits for each pole count of stator winding automatically !**



- **Special pole changing winding:**

ONE Winding system for 2 different pole numbers:

DAHLANDER-winding: $p_1 : p_2 = 1 : 2$

MMF of phase U depicted ($q \to \infty$)

a) 2-pole operation:

6-phase belt winding, pitching 0.5

b) 4-pole operation:

3-phase belt winding, fully pitched

Example: DAHLANDER-winding for tunnel ventilation

- Coarse, stepwise change of speed **in fan application** often sufficient !

Air flow per second $\dot{V} \sim n$

- **Pole changing tunnel fan ventilation motor: f_N = 50 Hz**
  (e. g. application in tunnels of *Alps*)

a) 4-pole operation:
n = 1500/min, $P_{Lü}$ = 800 kW, air flow rate 100 %

b) 8-pole operation:
n = 750/min, $P_{Lü}$ = 100 kW, air flow rate 50 %

c) switched off drive:
n = 0, P = 0, no air flow: 0 %

DAHLANDER-winding for tunnel drilling machine

Source: ELIN EBG Motoren GmbH, Austria

- Drive: Cage induction motor
- Power: 250/250 kW
- Voltage: 400 V
- Frequency: 50 Hz
- Speed: **738/1488 /min**
- Cooling: water jacket
- Number: 12 items

KAWASAKI HEAVY INDUSTRIES GMBH

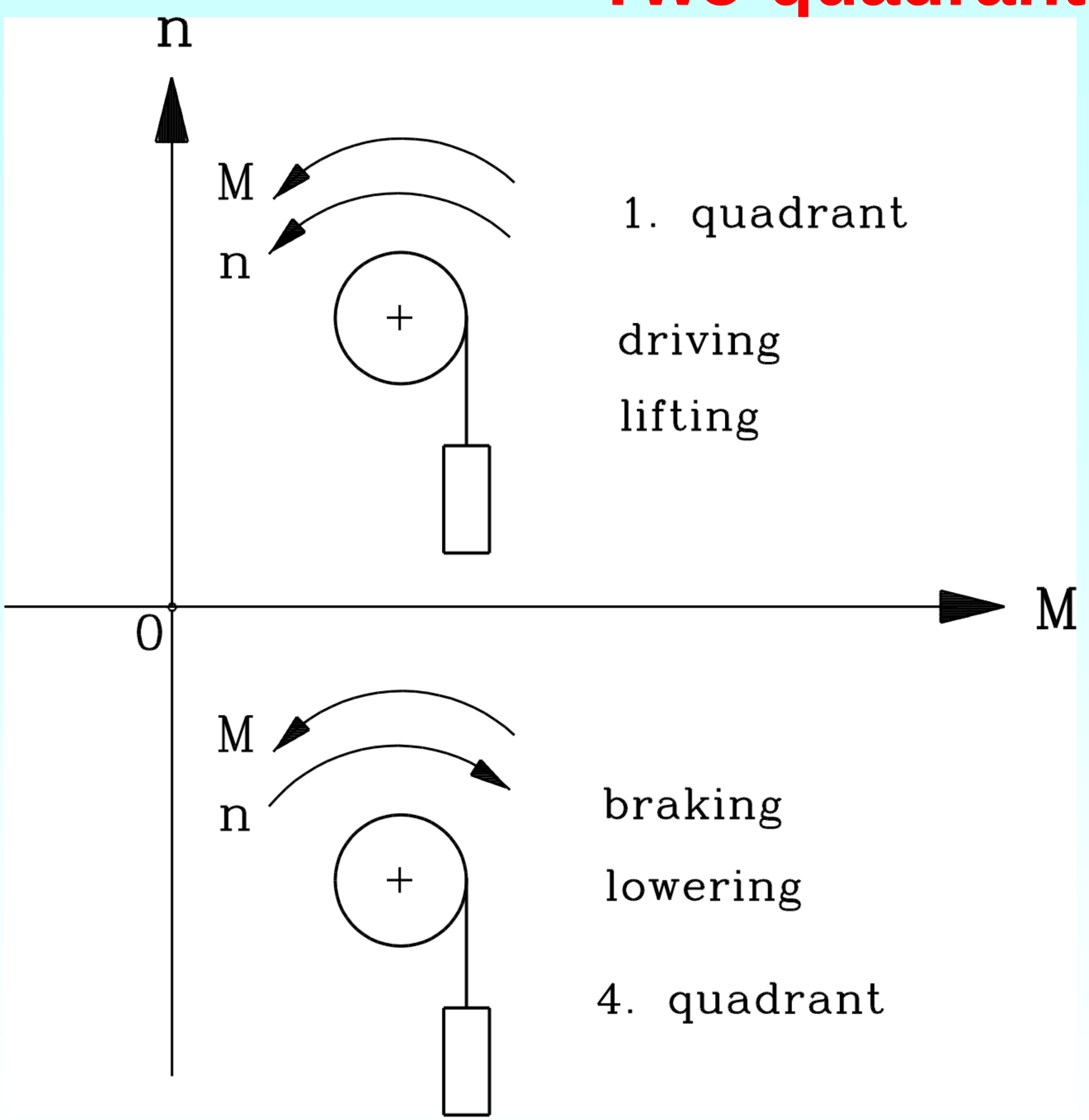
- Drilling head of tunnel drilling machine

  Used for CHUNNEL (UK-F)

Project: **2 tunnel drilling machines:** ***Channel Tunnel Rail Link***

Location: ***England***

Two-quadrant operation



Example: **Drive for elevators**

Demand: Continuously variable speed, smooth accelaration *a* and deceleration -*a* with limited jerk: d*a*/d*t* = small.

1st Quadrant:

Speed *n* and torque *M* positive:

- LIFTING
- **MOTOR operation**

$$P = 2\pi \cdot n \cdot M > 0$$

4th Quadrant:

Speed negative, torque positive to "hold" the load:

- LOWERING
- **GENERATOR operation**

$$P = 2\pi \cdot n \cdot M < 0$$

Single quadrant drive: Compressor motor

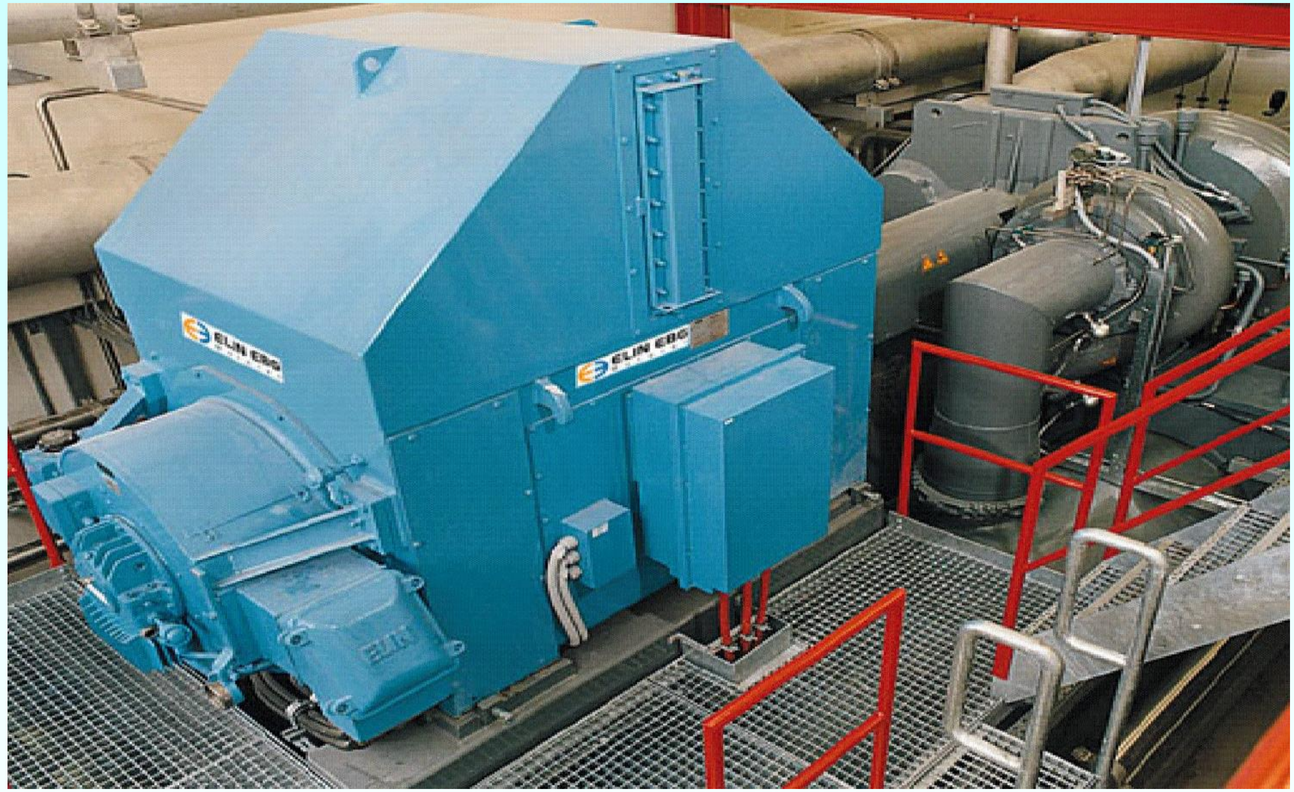
Source: ELIN EBG Motoren GmbH, Austria

- Motor: induction, four pole
- Power: 2250 kW
- Voltage: 6 kV/Grid operated
- Frequency: 50 Hz
- Speed: 1483 /min
- Cooling: water jacket
- Number: 1 item

Atlas Copco

- Application: Turbo compressor

- Efficiency 96,65%



Project:	***Biochemie Kundl /Tyrol***
Location:	***Austria***

Turbo compressor drives

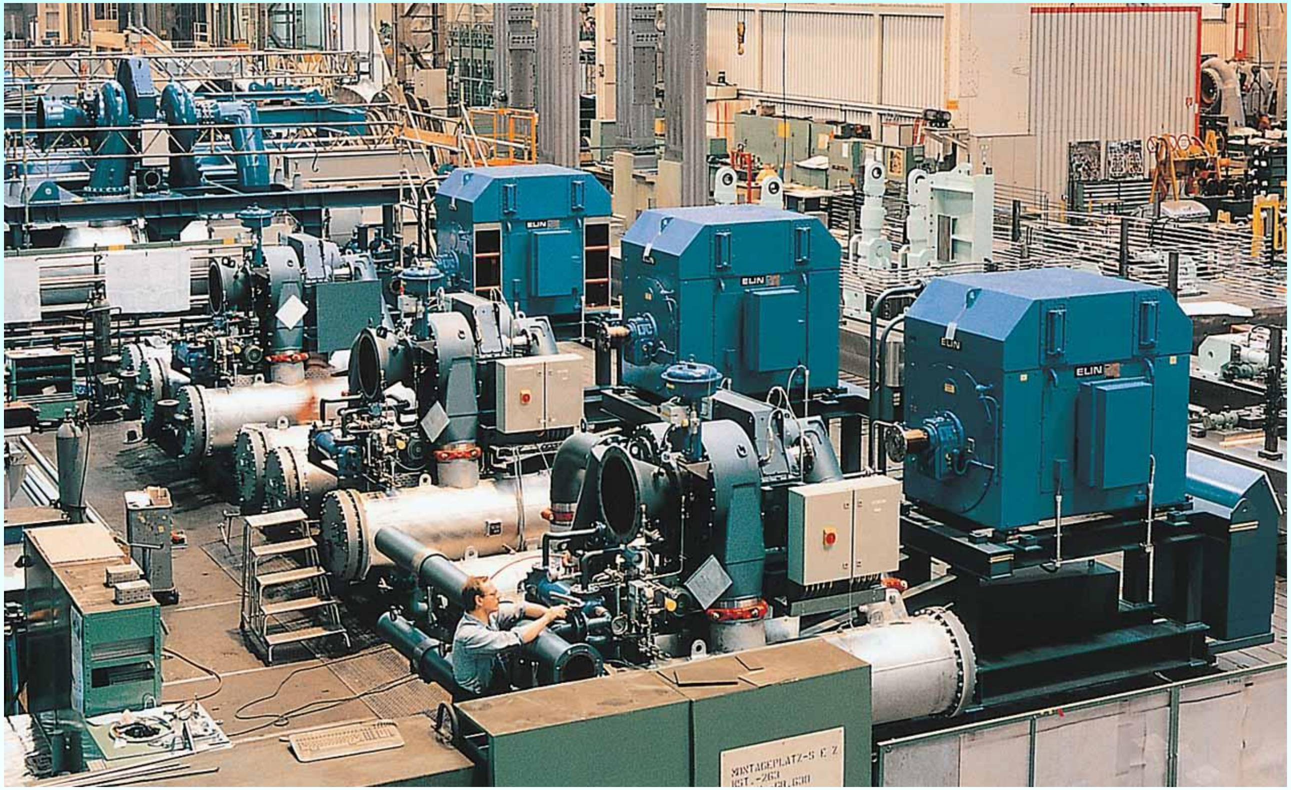
Source: ELIN EBG Motoren GmbH, Austria

- Motor: Cage induction two pole
- Power: 1850 kW
- Voltage: 6 kV/ Grid operated
- Frequency: 50 Hz
- Speed: 2975 /min
- Cooling: water jacket
- Number: 3 items

DEMAG DELAVAL
TURBOMACHINERY

- Turbo compressors in chemical plant
- Limited starting current

High efficiency



Project:	**„INFRA-LEUNA“**
Location:	***Germany***

Example: Cage induction motor drive

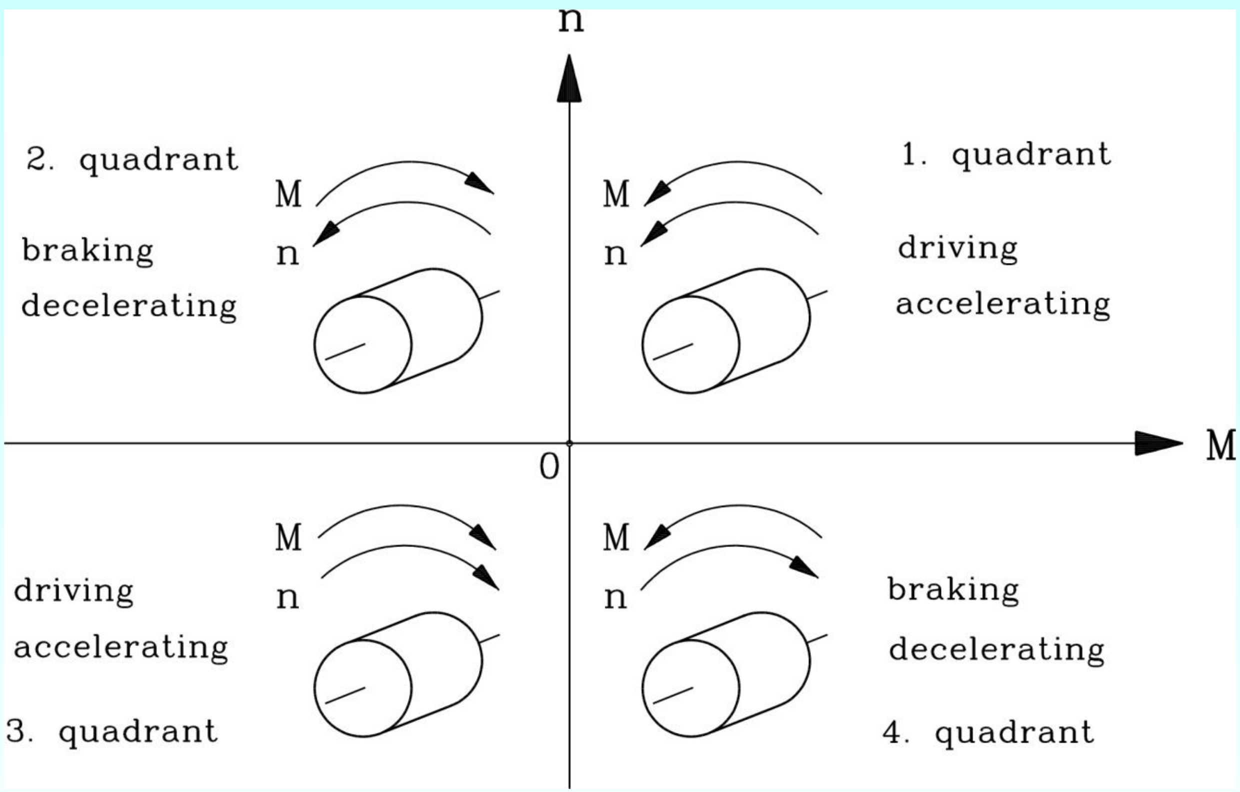
Source: ELIN EBG Motoren GmbH, Austria

Motor: Cage induction, four pole	
Power:	150 kW
Voltage:	400 V
Frequency:	50 Hz
Speed:	1480 /min
Cooling:	water jacket
Number:	~ 10 items/year

- **Propulsion of milling head for excavating coal in coal mines**

Project:	**Coal mining**
Location:	***India, Russia, Mexico***

Four quadrant operation



Example:
Drive system for electric vehicle

1st and 3rd quadrant:
Driving forward and backward:
MOTOR

2nd and 4th quadrant:
Generator braking in forward and backward direction:
GENERATOR

Example for 2nd and 4th quadrant at ELECTRIC TRACTION:
"Electrical brake": Feeding back into the grid via the overhead line and the catenary the kinetic energy of the decelerating train

Four-quadrant-operation: Street car (Tram)

Source: ELIN EBG Motoren GmbH, Austria

- Motor: Cage induction, four pole
- Power: 80 kW
- Max. voltage: 380 V Y
- Max. frequency: 140 Hz
- Speed: 2060/min
- Cooling: water jacket
- Number: 665 items

WIENER LINIEN

- Induction motor with die-cast Alu-cage rotor and stator round copper wire winding
- **Inverter operation**

Project:	**ULF – Ultra Low Floor street car**
Location:	***Vienna / Austria***

Inverter-fed induction machine

• **Frequency converter (inverter)** generates three-phase voltage system with variable frequency f_s and variable amplitude U_s (rms). Hence synchronous speed is continuously variable. With that induction machine is **continuously variable in speed.**

• **Reversal of speed =** Changing of two phases of stator winding. Changing of energy flow (motor / generator) by decreasing / increasing phase shift between voltage and current :

motor $\varphi < \pi / 2$ generator $\varphi > \pi / 2$

• Voltage amplitude U_s must be changed in proportion to f_s **to keep the flux in the machine constant**. Thus torque will stay constant, if the the same current is used.

For $R_s = 0$:

$$\underline{U}_s = j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

$$\frac{\underline{U}_s}{\omega_s} = jL_{s\sigma}\underline{I}_s + jL_h(\underline{I}_s + \underline{I}'_r) = j(\hat{\underline{\Psi}}_{s\sigma} + \hat{\underline{\Psi}}_h)/\sqrt{2} = j\hat{\underline{\Psi}}_s/\sqrt{2} = const.$$

Rule for controlling the inverter: $$\boxed{U_s \sim \omega_s}$$

• Slip: $s = f_r / f_s = \omega_r / \omega_s \quad \Rightarrow \quad \Omega_m = \frac{\omega_s}{p} - \frac{\omega_r}{p}$

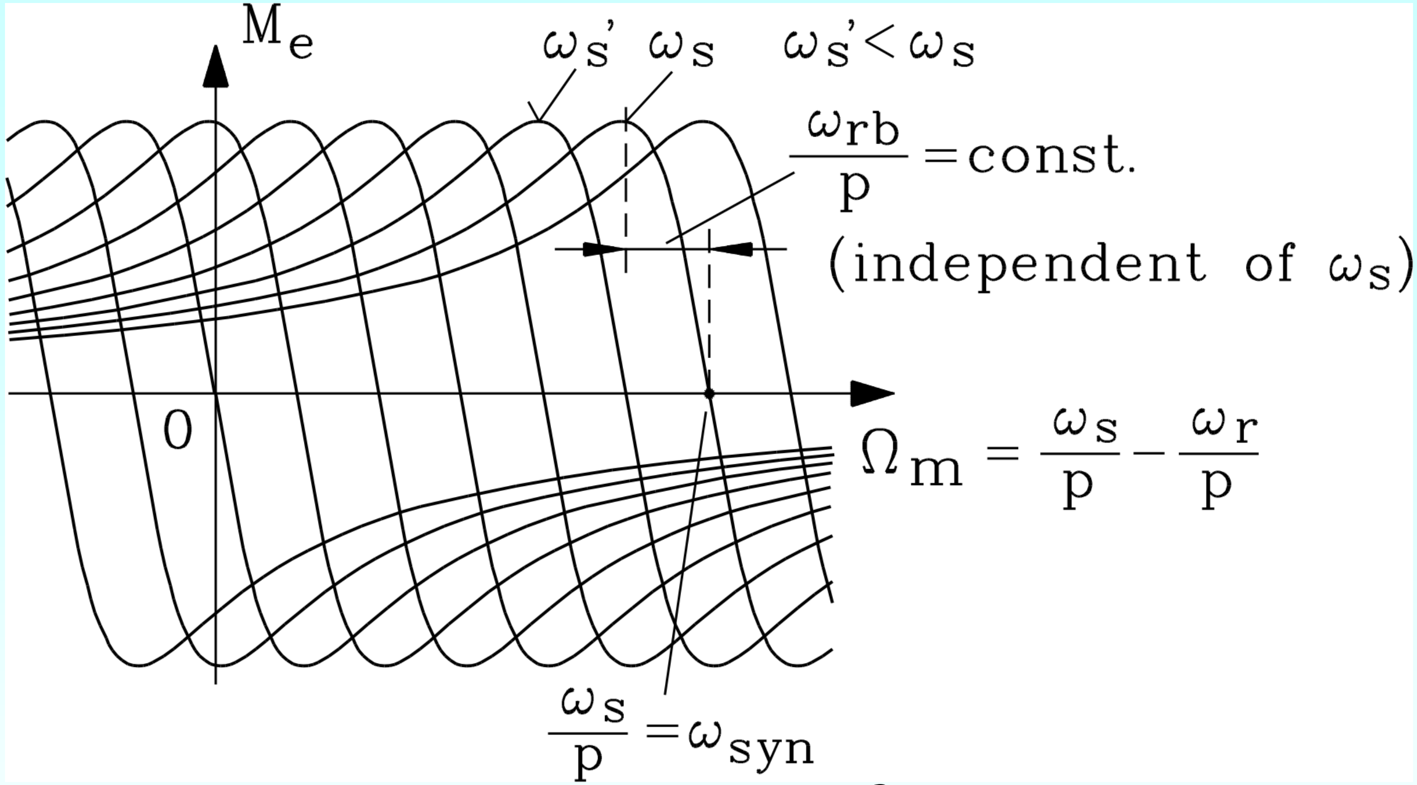
Curve $M_e(n) = M_e(\Omega_m)$ as **Curve** $M_e(\omega_r)$ for varying ω_s **is shifted in parallel !**

$M(n)$-Characteristic for inverter-fed induction machine

- $\underline{R_s = 0}$:

KLOSS formula:

$$M_e = \frac{2M_b}{\frac{s}{s_b} + \frac{s_b}{s}} = \frac{2M_b}{\frac{\omega_r}{\omega_{rb}} + \frac{\omega_{rb}}{\omega_r}}$$

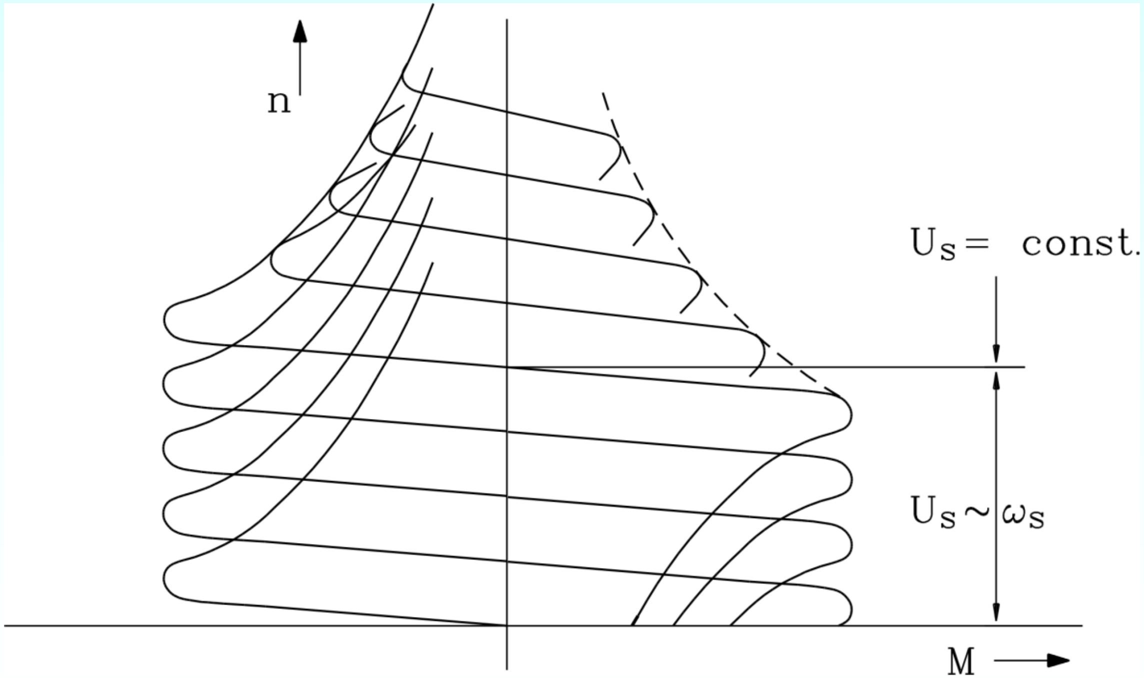


Break down torque M_b : $$M_b = \frac{m_s U_s^2}{\omega_s / p} \cdot \frac{1}{X_s} \cdot \frac{1-\sigma}{2\sigma} = \frac{m_s p}{2} \cdot \left(\frac{U_s}{\omega_s}\right)^2 \cdot \frac{1-\sigma}{\sigma L_s} = const.$$

Break down slip: $$s_b / s = \frac{R_r'}{s\sigma X_r'} = \frac{(\omega_s / \omega_r) \cdot R_r'}{\sigma \cdot \omega_s L_r'} = \frac{\omega_{rb}}{\omega_r}$$ with $$\omega_{rb} = \frac{R_r'}{\sigma L_r'}$$ **Slip frequency**

Flux weakening

- At maximum inverter output voltage $U_{s,max}$ magnetic flux DECREASES, when speed (and stator angular frequency ω_s) is raised further: $R_s = 0: \quad \hat{\Psi}_s = \sqrt{2} U_{s,\max} / \omega_s$ (**Flux weakening**).
- Break down torque decreases with the inverse of **square of frequency**: $M_b \sim U_{s,\max}^2 / \omega_s^2$
  Rotor break down frequency ω_{rb} remains constant: Hence inclination dM_e/ds of $M_e(n)$-characteristic in flux weakening range decreases with inverse of frequency



Influence of stator winding resistance R_s

- Voltage drop at stator resistance in stator voltage equation **MUST NOT be neglected at small angular frequency** ω_s

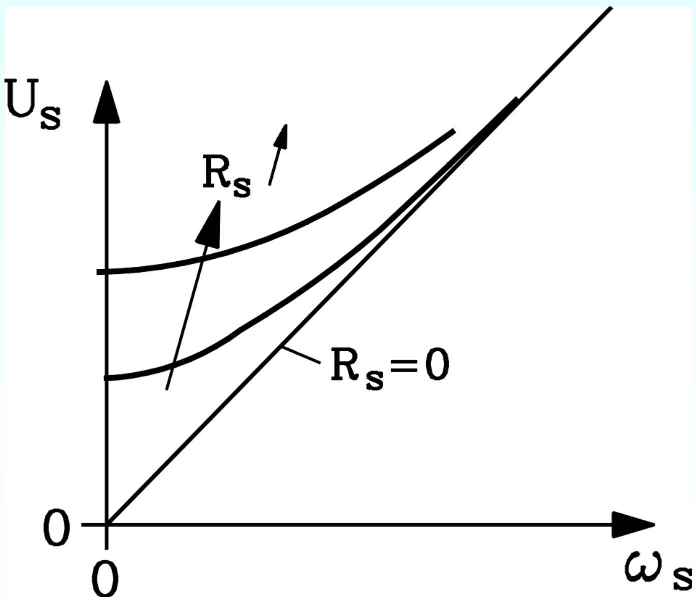
$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

- *Example:* Induction machine:
  Rated data: f_{sN} = 50 Hz, U_{sN} = 230 V:

$$f_s = 50 Hz : \frac{R_s}{\omega_s L_s} = \frac{0.06\Omega}{3.0\Omega} = 0.02$$

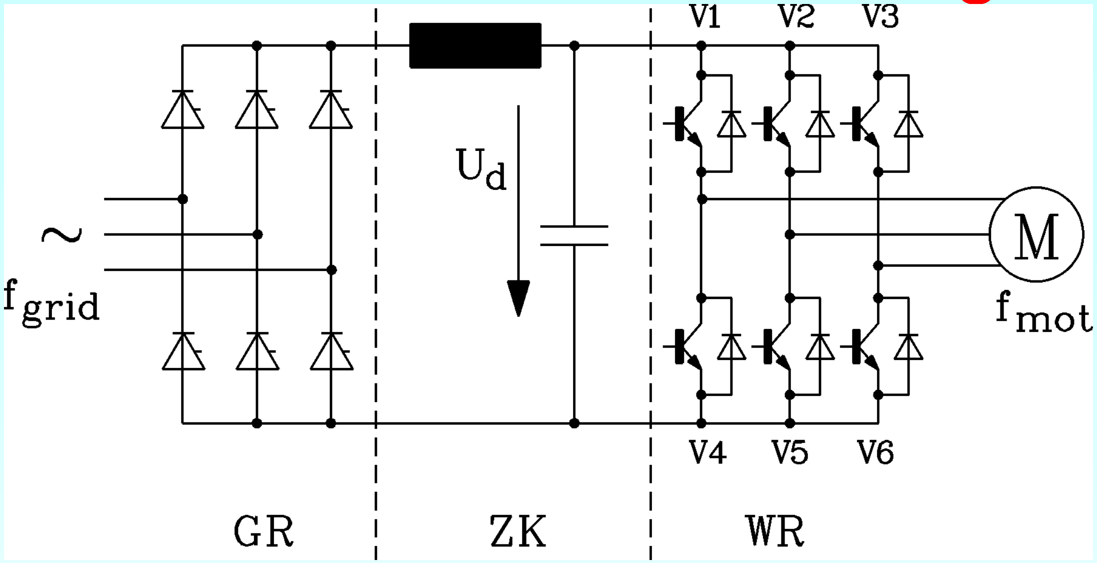
NOTE: At small f_s resistance R_s **must not be** neglected.

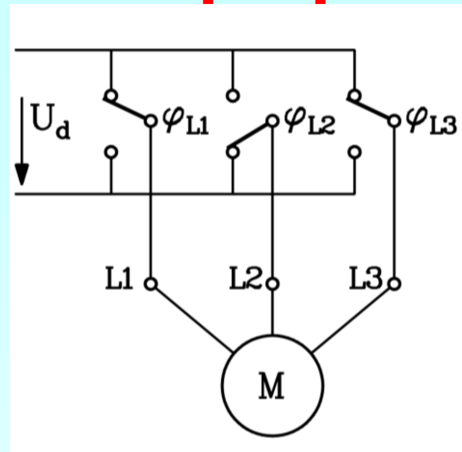
$$f_s = 5 Hz : \frac{R_s}{\omega_s L_s} = \frac{6}{\frac{5}{50} \cdot 300} = \underline{\underline{0.2}}$$

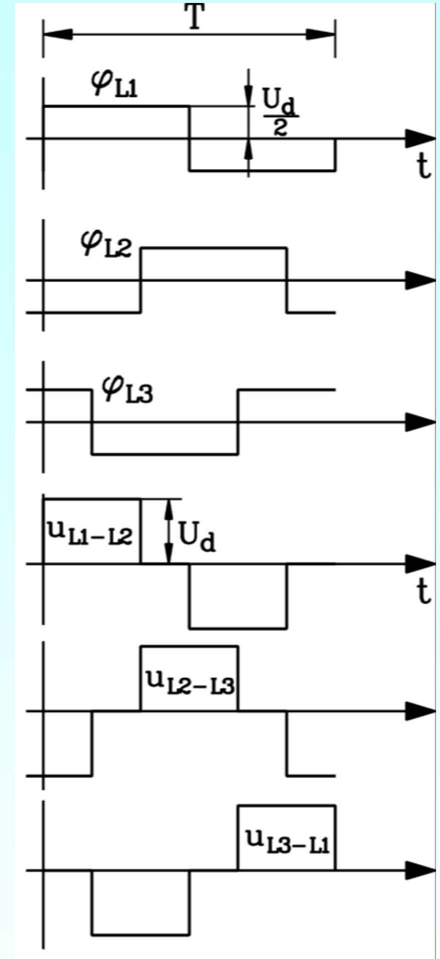


- Voltage drop at stator resistance reduces at constant stator phase voltage U_s the internal voltage U_h. Hence break down torque decreases with square of internal voltage !
- By **increasing of U_s by $R_s I_s$ internal voltage** U_h must be kept constant for constant M_b.

Inverter with voltage six step operation





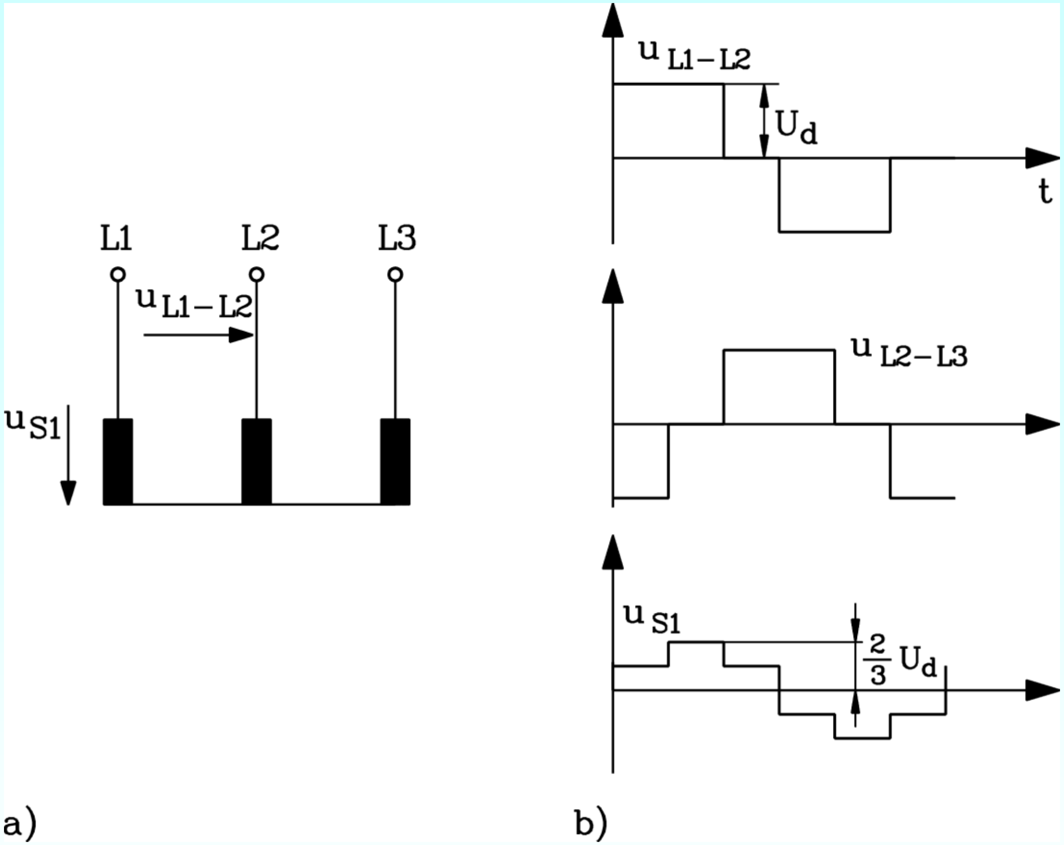


- Bridge rectifier with thyristors on grid side GR (firing angle α) generates variable DC voltage U_d in DC link ZK; voltage smoothed by capacitor.
- Inverter WR generates by six-step switching from U_d a **block shaped** line-to-line output voltage between terminals L1, L2, L3.

- DC link voltage U_d is changed by α **proportional** with output frequency f_{mot} .
- Grid side energy feed-back only possible with 2nd anti-parallel thyristor bridge: At $\alpha > 90°$ positive U_d and negative I_d give negative dc link power = power to the grid (gener. braking).

Voltage harmonics at six-step operation

- **Inverter output phase voltage:** $u_{S1} - u_{S2} = u_{L1-L2}; \quad u_{S2} - u_{S3} = u_{L2-L3}; \quad u_{S1} + u_{S2} + u_{S3} = 0;$



we get: $u_{S1} = \dfrac{2u_{L1-L2} + u_{L2-L3}}{3}$

- Block shaped line-to-line voltage, expanded as *FOURIER*-series:

$$u_L(t) = \sum_{k=1,-5,7,..}^{\infty} \hat{U}_{L,k} \cdot \cos(k \cdot \omega_s t)$$

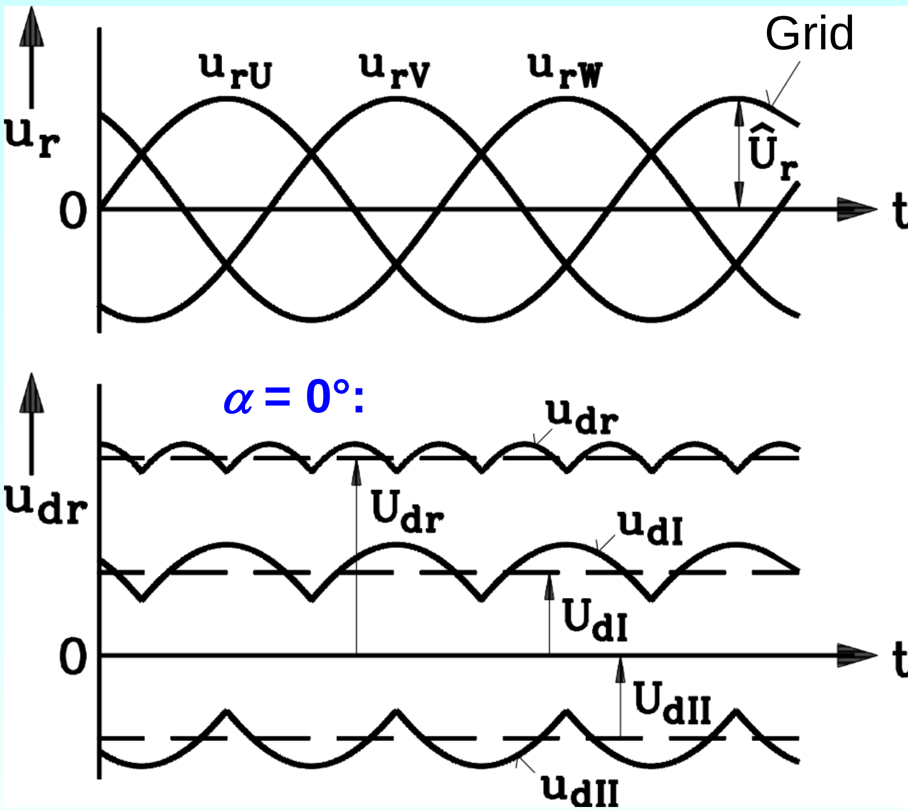
$$k = 1 + 6g, \qquad g = 0, \pm 1, \pm 2, \ldots$$

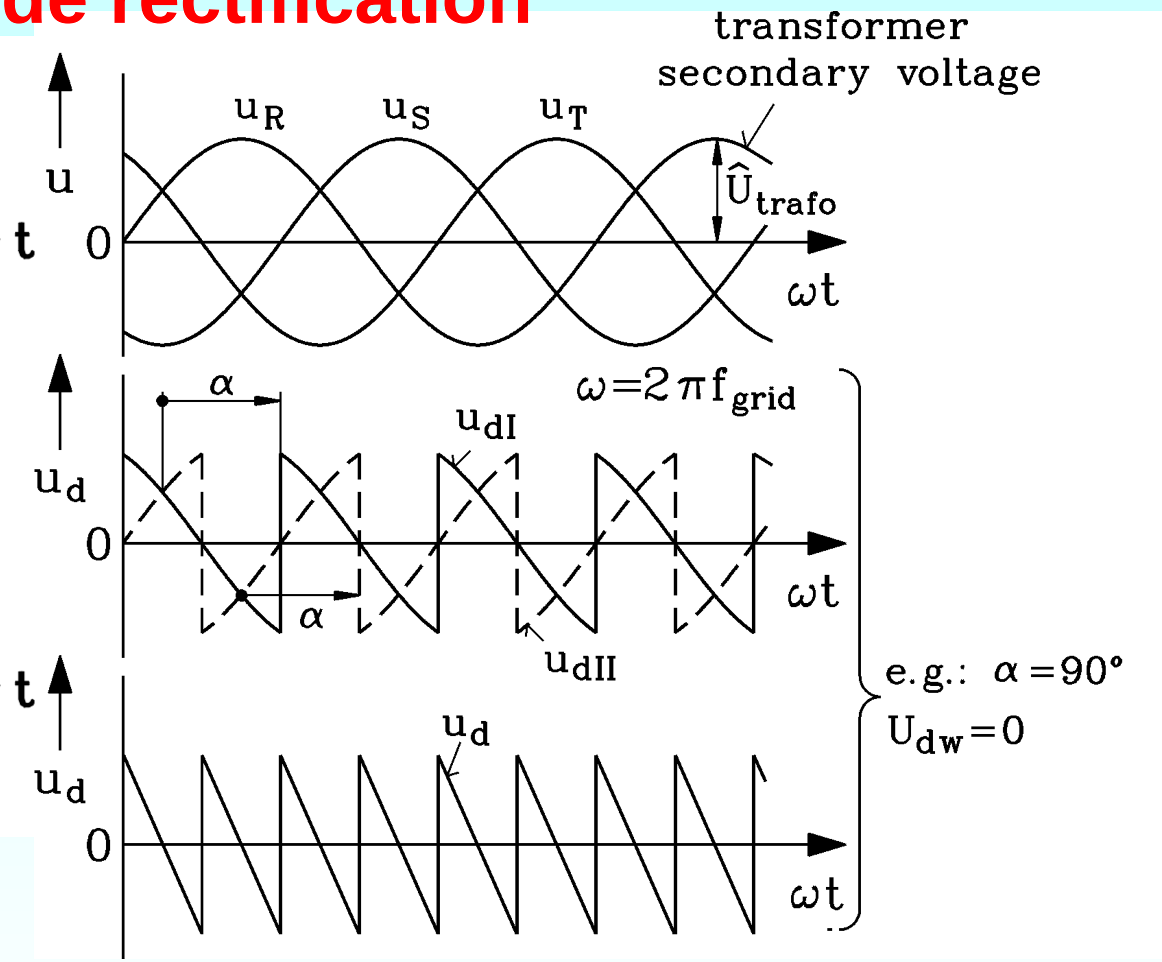
⇒ k = 1, -5, 7, -11, 13, ...

$$\hat{U}_{L,k} = \frac{2}{\pi}\sqrt{3}\frac{U_d}{k}$$

Electrical machine is fed with a blend of harmonic voltages of different amplitude, frequency and phase angle. Only fundamental (ordinal number k = 1) is desired. Voltage harmonics (|k| > 1) cause harmonic currents in electric machine with additional losses, torque pulsation, vibrations and acoustic noise.

Grid-side rectification

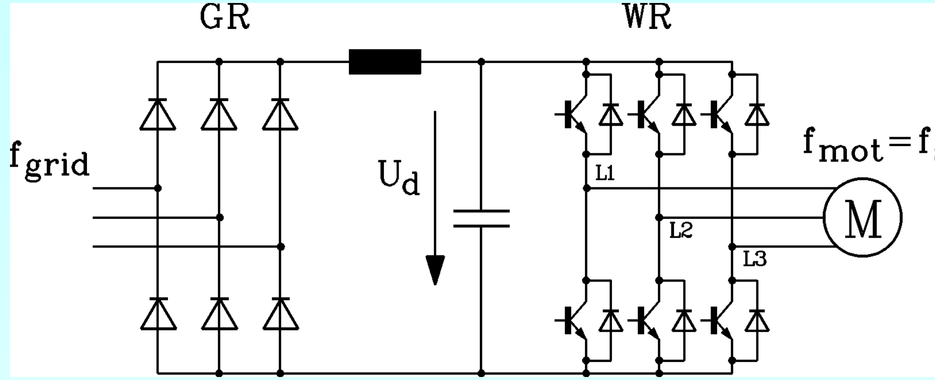




- Bridge rectifier on grid side GR:
  Maximum rectified voltage U_{d0} at firing angle $\alpha = 0$ (i.e. uncontrolled rectifying).
- Variable α: Controlled rectifying: e.g.: Zero rectified voltage U_d at firing angle $\alpha = 90°$!

$$U_{dw} = \frac{3}{\pi}\sqrt{3}\cdot\hat{U}_{Trafo}\cdot\cos\alpha = U_{dw,\max}\cos\alpha$$

Pulse width modulation (PWM)



- At grid side: Diode rectifier GR (= firing angle $\alpha = 0$): generates constant DC link voltage U_d , which is smoothed by capacitor: $U_d \sim U_{grid} = const.$

- Motor side inverter WR generates from U_d **by pulse width modulation** a line-to-line voltage between L1, L2, L3. Width of pulses is defined by comparison of **saw tooth signal** u_{SZ} (switching frequency f_{sch}) with AC reference signal u_{ref} , which pulsates with desired stator frequency f_s . With comparator a **PWM-signal** is generated to control power switches. *Reference signal is most often sine wave.*

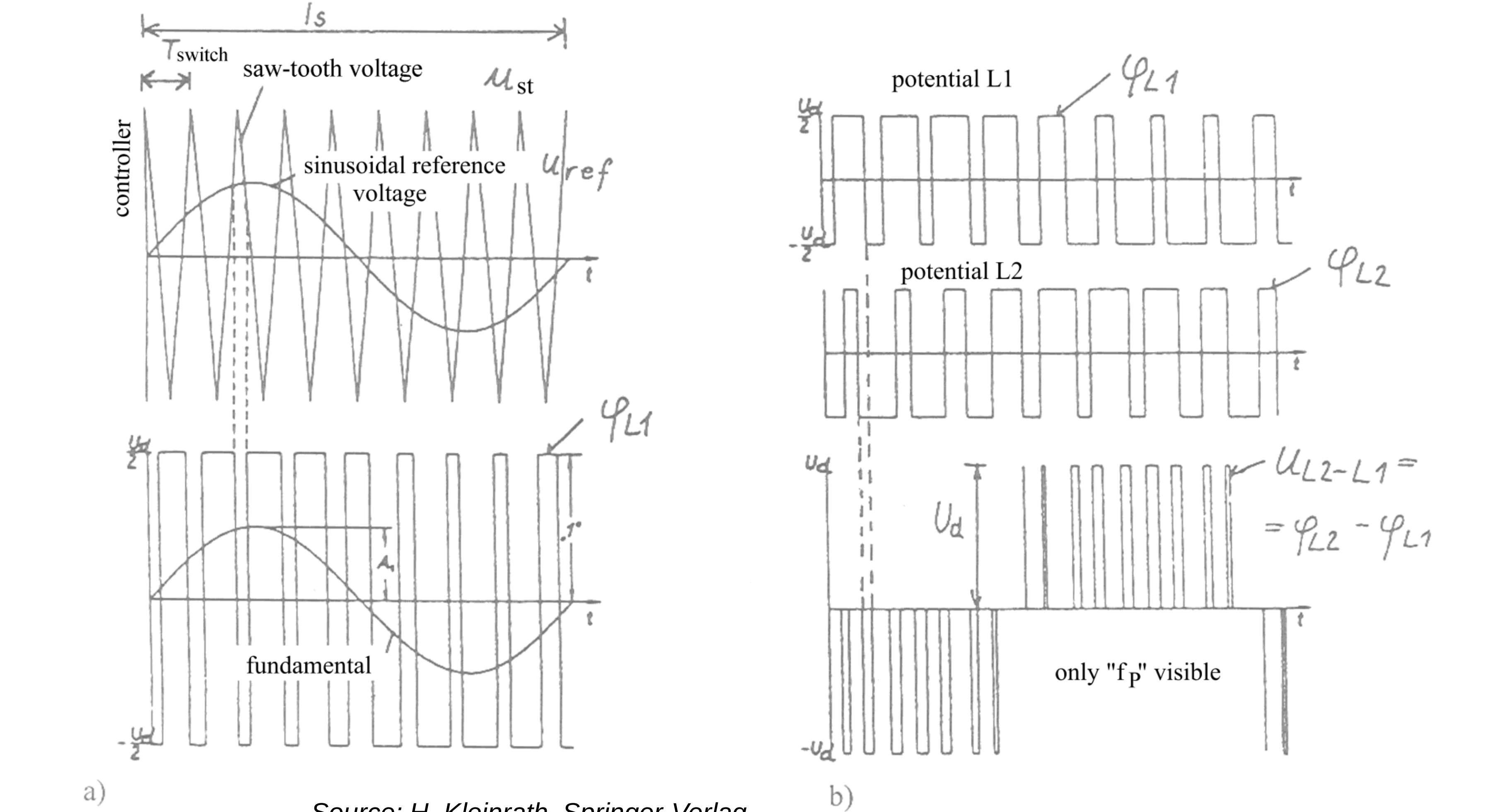
- Amplitude A1 of u_{ref} defines amplitude of fundamental of PWM voltage at motor terminal. So it is varied **proportional** to f_{mot} .

- Grid side: $\cos\varphi = 1$ No power flow back into grid possible. (For that a grid-side inverter and a grid-side inductance is necessary !). Therefore generator braking power has to be dissipated in ”brake"-resistors, which are connected in parallel with capacitor in DC link.

Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\varphi_{L1}(t)$ at terminal L1 varies with that PWM signal

b) Difference of two terminal potentials delivers line-to-line voltage $u_{L2-L1}(t)$



Source: H. Kleinrath, Springer-Verlag

Voltage harmonics: Six-step and PWM

- ***Six-step modulation:*** FOURIER spectrum of line-to-line inverter output voltage:

k	1	-5	7	-11	13
$\hat{U}_{Lk}/\hat{U}_{L1}$	1	-0.2	0.14	-0.1	0.08

- ***PWM:*** FOURIER spectrum of terminal electric potential $\varphi_{L1}(t)$ and of line-to-line voltage $u_{L2\text{-}L1}(t)$ (at modulation degree A_1 = 0.5 and switching frequency ration $f_{sch}/f_s = 9$)

$\lvert k \rvert$	1	3	5	7	9	11	13	15	17	19
$\hat{\varphi}_k/(U_d/2)$	0.5	$<10^{-5}$	0.001	0.09	1.08	0.09	0.002	0.04	0.36	0.36
$\hat{U}_{L,k}/\hat{U}_{L,k=1}$	1	0	0.002	0.18	0	0.18	0.004	0	0.72	0.72

*Spectrum of terminal potential φ_L shows big amplitude of fundamental, of switching harmonic (k = 9) and at **about twice switching frequency $f_p = 2 f_{sch}$** (k = 17 and 19).*

$$k = \left| \frac{f_p}{f_s} \pm 1 \right| \quad \Rightarrow \quad k = |18 \pm 1| = 17, 19$$

*Voltage harmonics with ordinal numbers, divisible by 3, do **not** occur in line-to-line voltage ! At high switching frequency f_{sch} the amplitudes of all low frequency harmonics are small.*

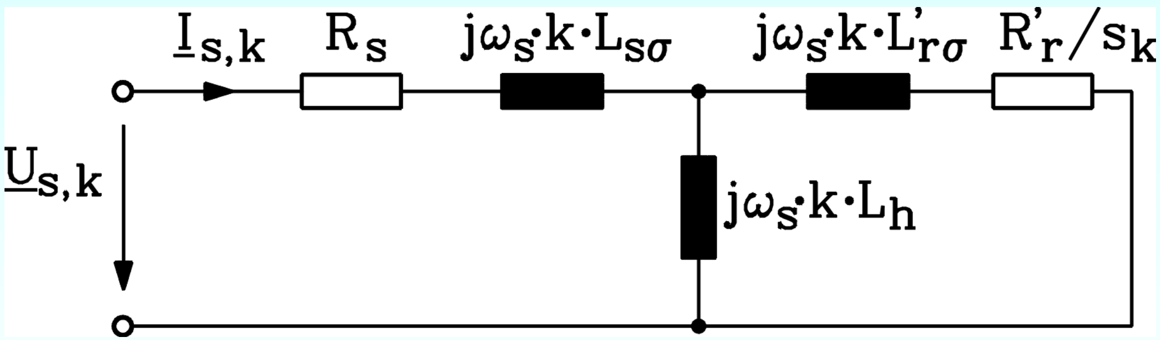
Voltage harmonics cause current harmonics

- The <u>*voltage harmonics per phase*</u> $U_{s,k}$ (frequency k-times fundamental frequency kf_s) cause current harmonics per phase $I_{s,k}$ in stator winding. These 3-phase harmonic current systems excite in air gap "high-speed" magnetic field wave (with pole count $2p$ due to winding):

***k*th synchronous velocity ("high speed"):** $n_{syn,k} = k \cdot f_s / p$

- **Rotor slip with *k*th high-speed field s_k :** $$s_k = \frac{n_{syn,k} - n}{n_{syn,k}} = \frac{kn_{syn} - n}{kn_{syn}} = 1 - \frac{1}{k} \cdot \frac{n}{n_{syn}} = 1 - \frac{1}{k} \cdot (1-s) \approx 1$$

As harmonic slip s_k is nearly unity, independent of base slip s, harmonic currents amplitude $I_{s,k}$ is nearly independent from load. Current harmonics are already present at no-load to full extent at s = 0 .



High speed fields induce rotor, causing rotor current harmonics with high frequency: $f_{rk} = s_k f_{s,k} \approx f_{s,k}$; causing big eddy currents in rotor bars and **big additional rotor losses !**

$$s_k \approx 1 \quad \Rightarrow \quad I_{s,k} \approx \frac{U_{s,k}}{\sqrt{(R_s + R'_r)^2 + (k\omega_s)^2 \cdot (L_{s\sigma} + L'_{r\sigma})^2}} \approx \frac{U_{s,k}}{|k|\omega_s (L_{s\sigma} + L'_{r\sigma})}$$

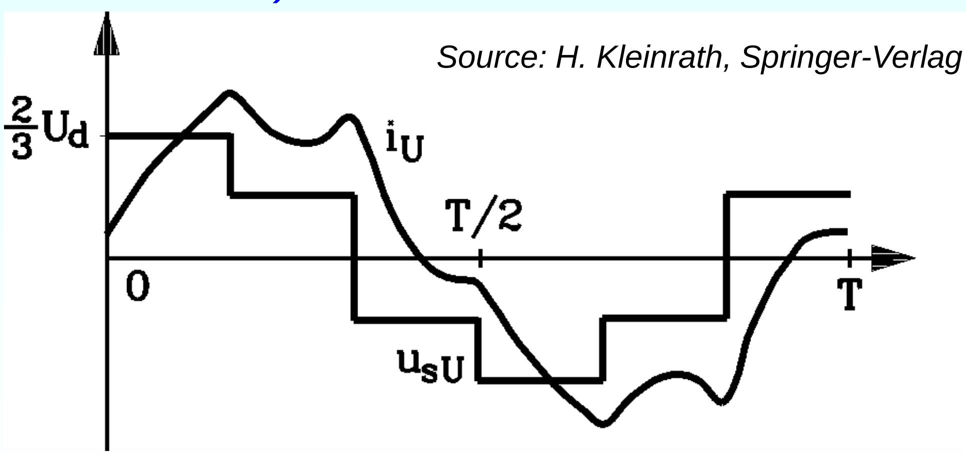
Example: Current harmonics at six-step modulation

- Amplitudes of current harmonics at six step operation:

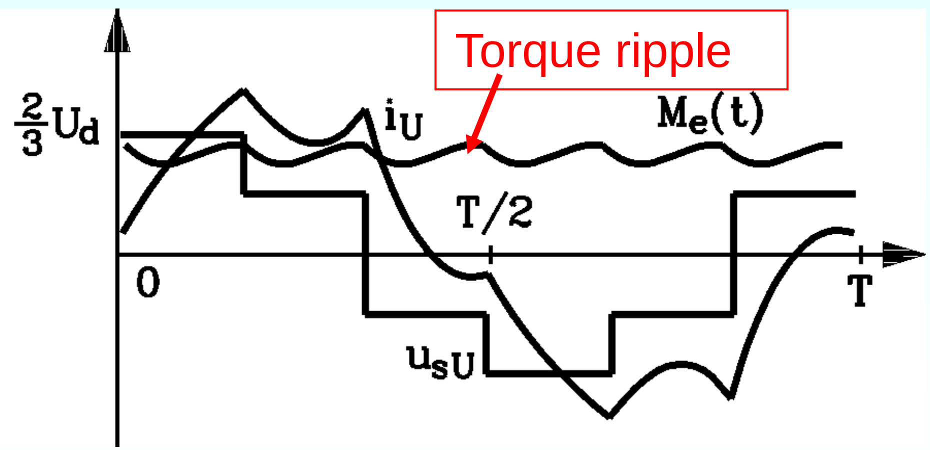
$$I_{s,k} \approx \frac{U_{s,k}}{|k|\omega_s(L_{s\sigma}+L'_{r\sigma})} \sim \frac{1}{|k|^2}$$

k	1	-5	7	-11	13
$\left\|\hat{U}_{Lk}/\hat{U}_{L1}\right\|$	**1**	**0.2**	**0.14**	**0.1**	**0.08**
$I_{s,k}/I_{s,k=1}$	**1**	**0.04**	**0.02**	**0.008**	**0.006**

- *Amplitudes of current harmonics decrease with inverse of square of ordinal number k, because leakage inductance **smoothes** the shape of current (= reduces the current harmonics !)*



FOURIER sum of 25 current harmonics

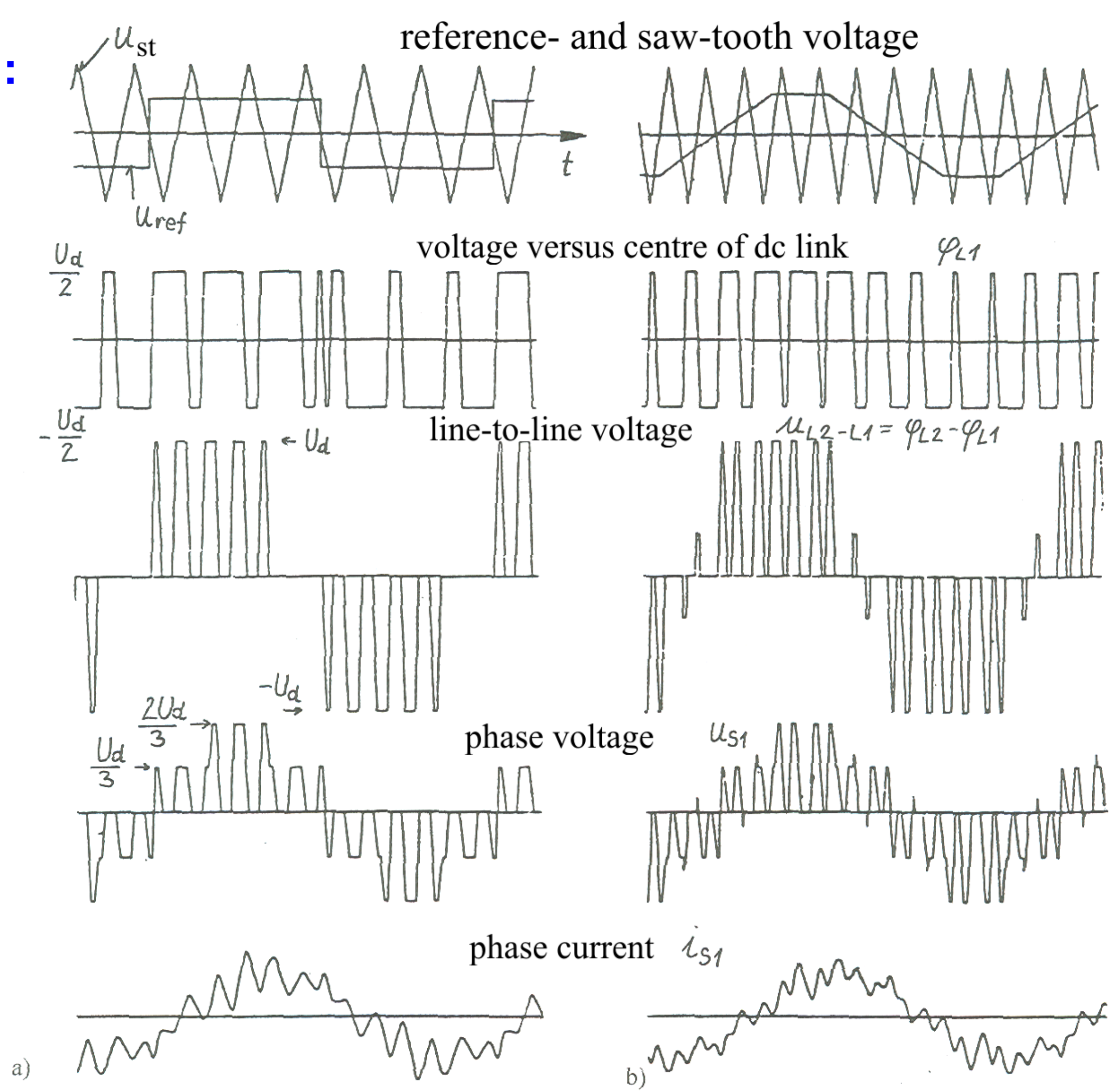


Exact solution of dynamic machine equation

Example: Current harmonics at PWM

Reference signal: rectangular

Reference signal: trapezoidal



Switching ratio:

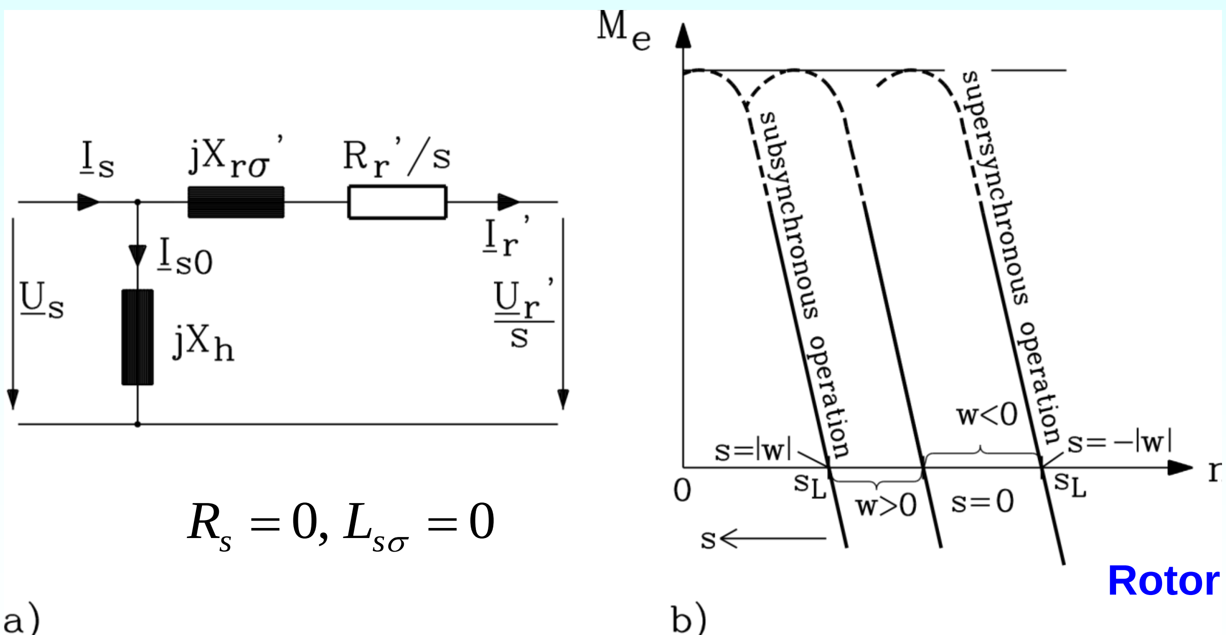
$f_{sch}/f_s = 6$

Switching ratio:

$f_{sch}/f_s = 9$

Doubly fed induction machine

- *Aim*: Speed variable operation with small inverter:
  inverter rating less than motor rating $S_{Umr} < S_{Mot}$
- *Solution:* Line-fed slip-ring induction machine, fed by small inverter in the rotor via slip rings
- **but:** Speed range $n_{min} \le n_{syn} \le n_{max}$ small. If we want $n_{min} = 0$, we get $S_{Umr} = S_{Mot}$.
- **Inverter feeds with rotor frequency an additional rotor voltage $\underline{U}'_r$ into rotor winding.**
  - Via variable **amplitude** of $\underline{U}'_r$ the speed is changed,
  - Via **phase shift** of $\underline{U}'_r$ the reactive component of stator current $\underline{I}_s$ is changed



Explanation with simplified T-equivalent circuit per phase:

$$\underline{U}_s = jX_h(\underline{I}_s - \underline{I}'_r) = jX_h \underline{I}_{s0}$$

$$\underline{U}'_r = -(R'_r + jsX'_r)\underline{I}'_r + jsX_h\underline{I}_s$$

Rotor current: $$\underline{I}'_r = \frac{\underline{U}_s - \dfrac{\underline{U}'_r}{s}}{\dfrac{R'_r}{s} + jX'_{r\sigma}}$$

Rotor additional voltage: $\underline{U}'_r = \underline{U}_s \cdot (w - jb)$

Simplified torque-speed curve of doubly fed machine

- **Electromagnetic torque M_e** : Approximation for small slip s << 1:

$$\underline{I}'_r = \frac{s\underline{U}_s - \underline{U}'_r}{R'_r + jsX'_{r\sigma}} \approx \frac{s\underline{U}_s - \underline{U}'_r}{R'_r} = \frac{\underline{U}_s}{R'_r}(s - w + jb) \qquad s << 1$$

$$P_{in} = P_\delta = m_s \operatorname{Re}\{\underline{U}_s \cdot \underline{I}'^*_r\} = m_s \frac{U_s^2}{R'_r}(s-w) \Rightarrow \boxed{M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{m_s U_s^2}{\Omega_{syn} R'_r}(s-w)}$$

By real part of additional rotor voltage w the ***M_e-n-curves are shifted in parallel !***

- Torque is ZERO at **no-load slip $s_L = w$**.
  - If no-load slip s_L is positive (**SUB-synchronous no-load points**) ⇔ Active component of additional rotor voltage IN PHASE with stator voltage
  - If s_L is negative (**SUPER-synchronous no-load points**) ⇔ Active component of additional rotor voltage is in PHASE OPPOSITION with stator voltage

$$M_e = 0 \quad \Rightarrow \quad s - w = 0 \quad \Rightarrow \quad \boxed{s_L = w = \frac{U'_{r,active}}{U_s}}$$

- **Inverter rating:** $S_{Inv} = 3U_r I_r$
- At n_{min} (⇔ $s_{L,max}$) both U_r and S_{Umr} are at maximum, thus defining inverter rating.

Components of variable speed wind converter systems

Wind rotor:

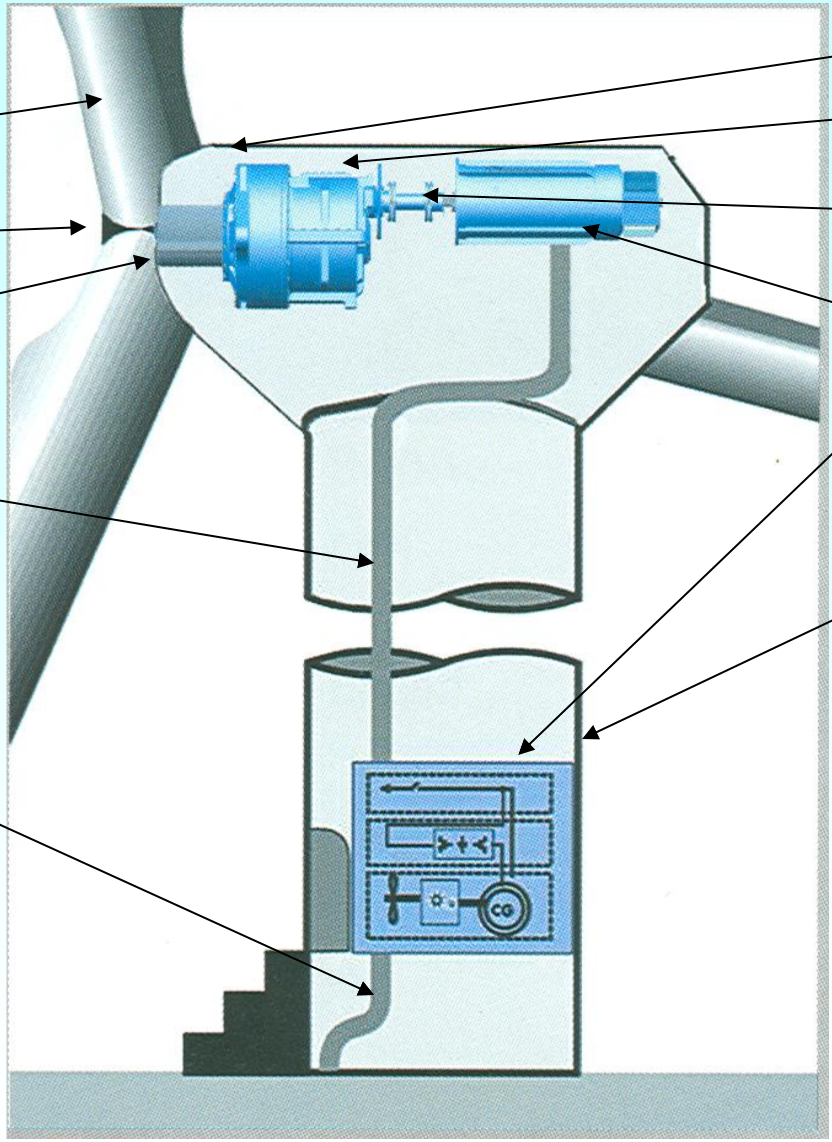
Blade

Spider

Turbine shaft

Generator three-phase cable

Transformer low-voltage three phase cable



Nacelle:

Three-stage gear

Generator shaft + coupling

Induction generator

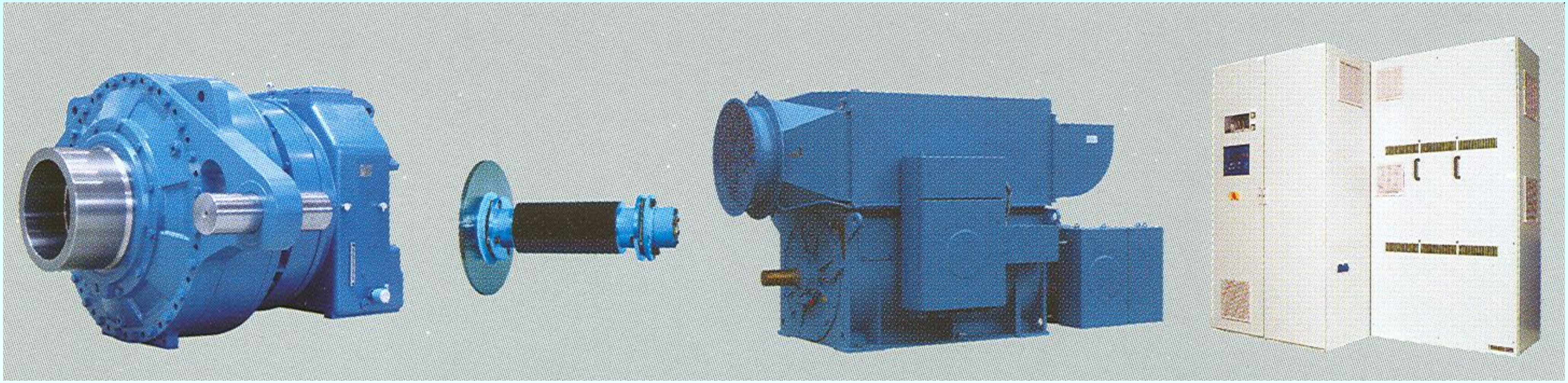
Rotor side inverter

Centre pole

Source:

Winergy, Germany

Components of doubly-fed induction generator system 2 MW

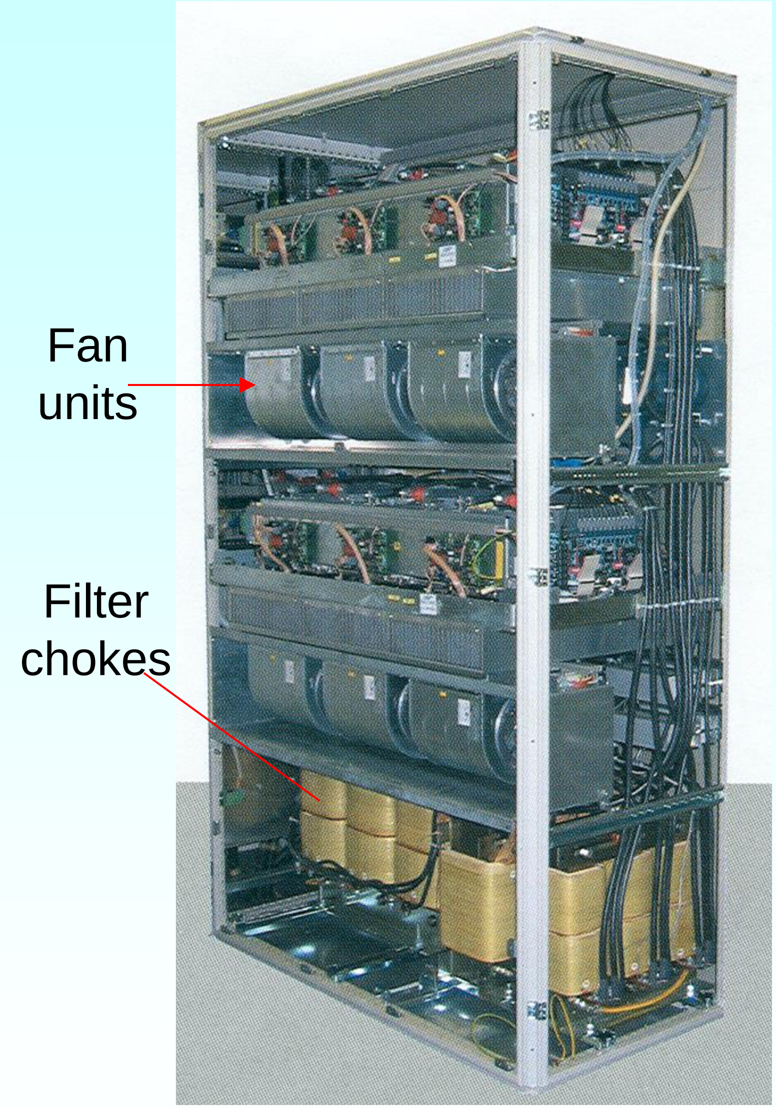


Three-stage planetary gear | generator coupling | slip-ring induction generator | rotor side inverter

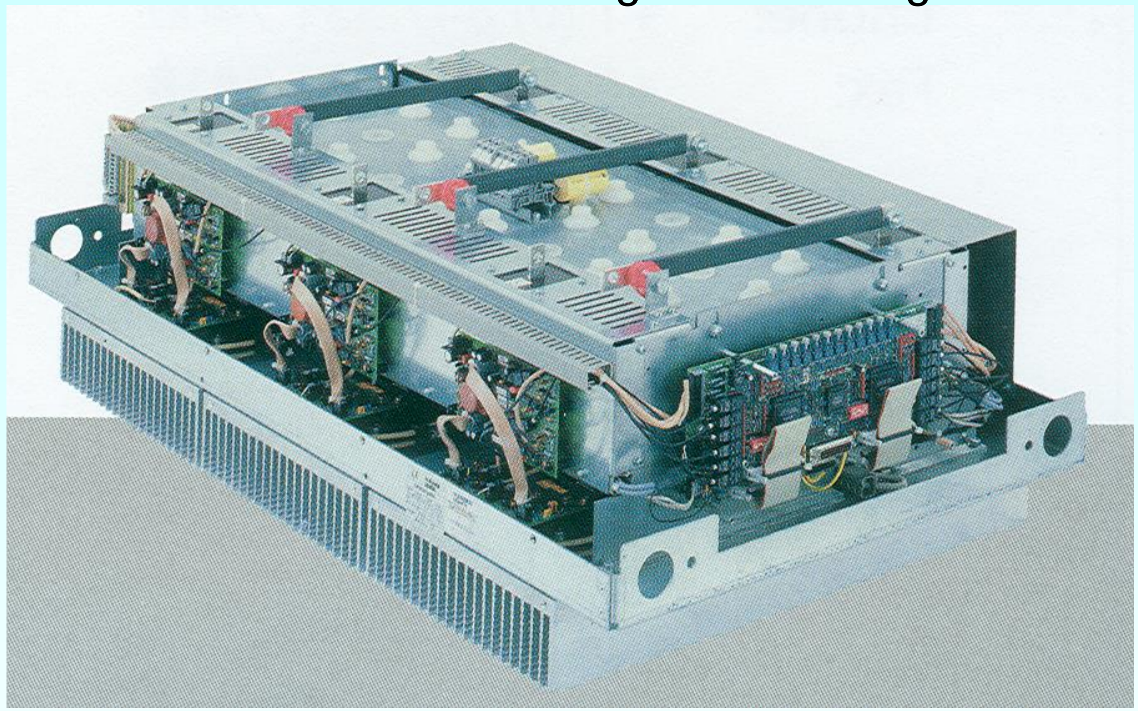
Source:

Winergy, Germany

Rotor side PWM voltage source inverters



Air cooled IGBT-inverter bridge with cooling fins



Air-cooled power electronic circuit for a 1.5 MW-wind converter has a rating of about 450 kVA = 30% of P_N

Grid side: 690 V

Rotor side: Rated rotor current

Source:

Winergy, Germany

Doubly-fed wind generator

- Wind turbine **with variable speed** allows to extract **maximum possible wind power** at each wind velocity ***v*** .
- $$P_{Wind} \sim v^3 \Rightarrow \boxed{P_{Turbine} \sim n^3}$$
- Doubly fed induction machine used as **variable speed generator, operating at grid with constant grid frequency !**
- Additional rotor voltage with rotor frequency generated by 4-quadrant PWM inverter via slip ring fed into rotor winding.

- *Example:* Wind velocity varies between $0.15P_{max}$ and P_{max}:
- Generator and gear to turbine are designed hence for speed range $n_{syn} \pm 30\%$ ($s = \pm 0.3$):

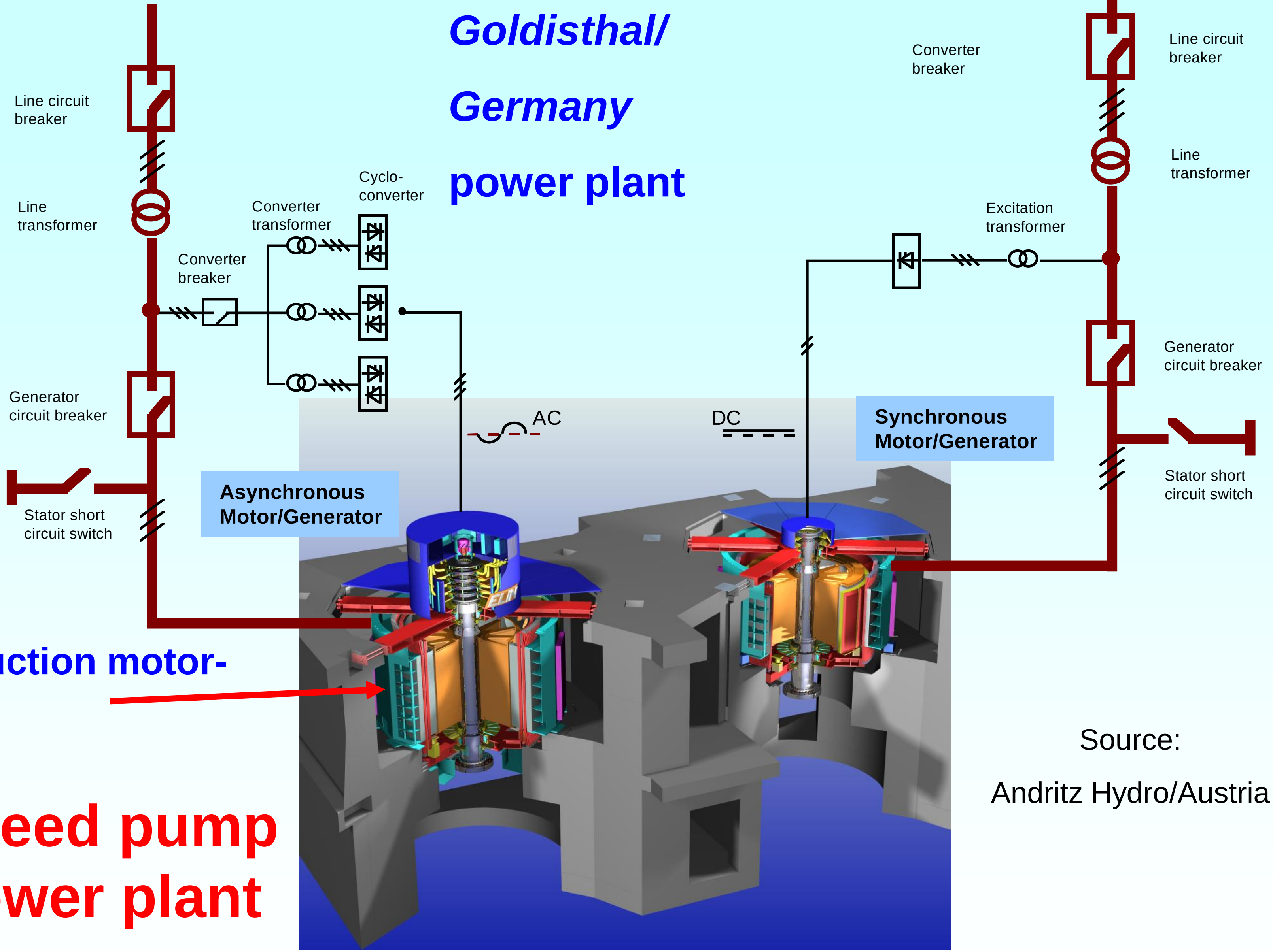
Wind speed	**Generator speed**	**Slip**	**Add. voltage**	**Power**
v_{max}	$n = 1.3n_{syn} = n_{max}$	$s = -0.3$	$w = -0.3$	$P = 100\%$
$v_{min} = 0.54v_{max}$	$n = 0.7n_{syn} = 0.54n_{max}$	$s = +0.3$	$w = +0.3$	$P = 15\%$

- Rated power of inverter at steady state operation and rated torque:

$$P_{Inverter} = sP_{\delta} \approx sP_N = 0.3P_N$$

Here inverter rating is only 30% of generator rating, thus it is a very cheap solution, which is used nowadays widely at big wind turbines 1.5 ... 5 MW.

Goldisthal/ Germany power plant



Doubly fed induction motor-generator

Source:

Andritz Hydro/Austria

Variable speed pump storage power plant

Variable speed pump storage power plant

Pump storage power plant *Goldisthal/Thuringia, Germany*:
a) Grid operated synchronous Motor/Generator:
Data: 331 MVA, 333.3/min, 18 poles, 50 Hz

b) Doubly fed induction motor-generator:
Data: 340 MVA, 300 ... 346/min, 18 poles, 50 Hz
Rotor side converter: Cyclo-converter for low frequency
Rotor slip: +10% ... -5% slip = max. frequency in rotor 5 Hz

Fixed-speed pumping: Pump operates at rated power against the constant pressure of the head of the upper storage basin. Hence only with rated power energy can be stored.
Variable-speed pumping: Pump operates at 90 … 105% rated speed. Hence it can be stored energy with variable power 73% … 115% of P_N.