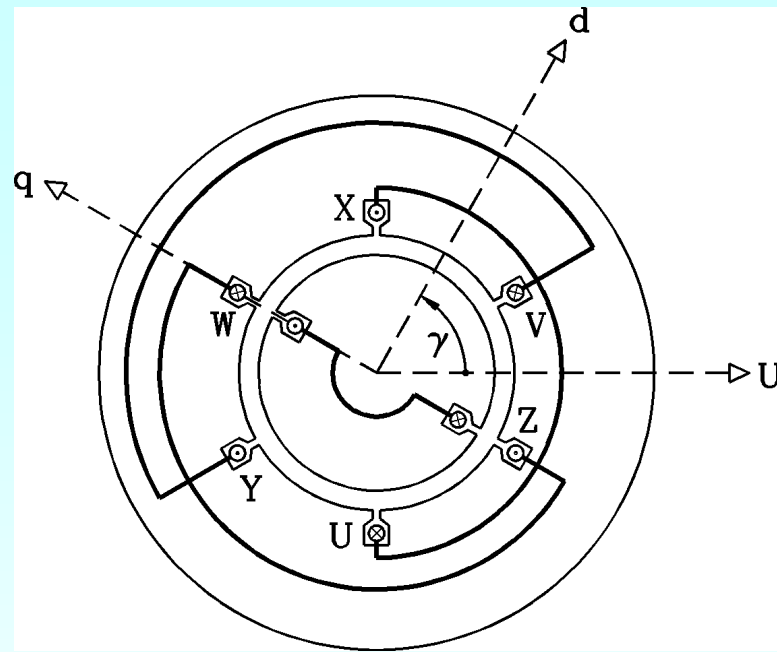


8. The Synchronous Machine

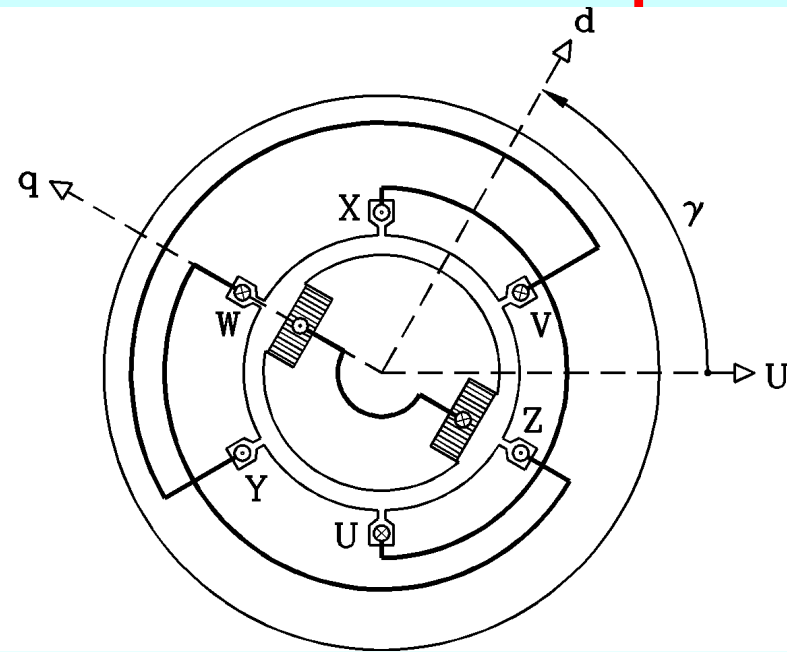


Source: Siemens AG

Synchronous machine with round rotor and salient pole rotor



ROUND ROTOR: Field winding distributed in rotor slots; constant air gap



SALIENT POLE ROTOR: concentrated field winding on rotor poles; air gap is minimum at pole centre

- **Synchronous machine:** Rotor field winding excites static magnetic rotor field with DC field current I_f .
- **MOTOR**-operation: Stator 3-phase ac current system I_s excites **stator rotating air gap field**. This field rotates with $n = f_s/p$ and attracts rotor magnetic field, which has same number of poles. So rotor will rotate **synchronously** with stator field.
- **GENERATOR**-mode: Rotor is driven mechanically, and induces with rotor field in the stator winding a 3-phase voltage system with frequency $f_s = n \cdot p$. Stator current due to this voltage excites stator field, which rotates **synchronously with rotor**.

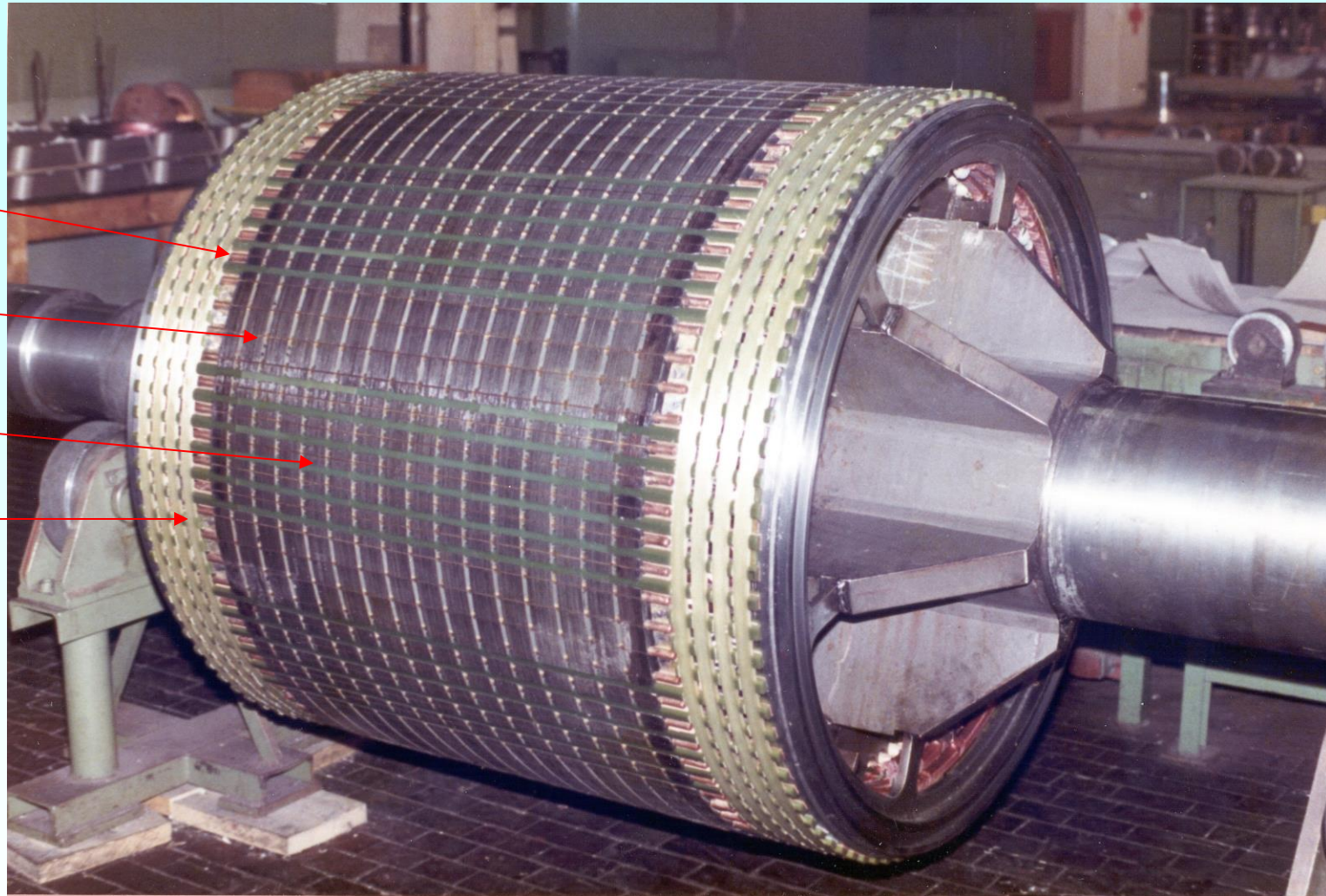
Round rotor synchronous machine, 8 poles

Three field coils
per pole: $q_r = 3$

Damper cage with
9 bars per pole

Radial ventilation
ducts

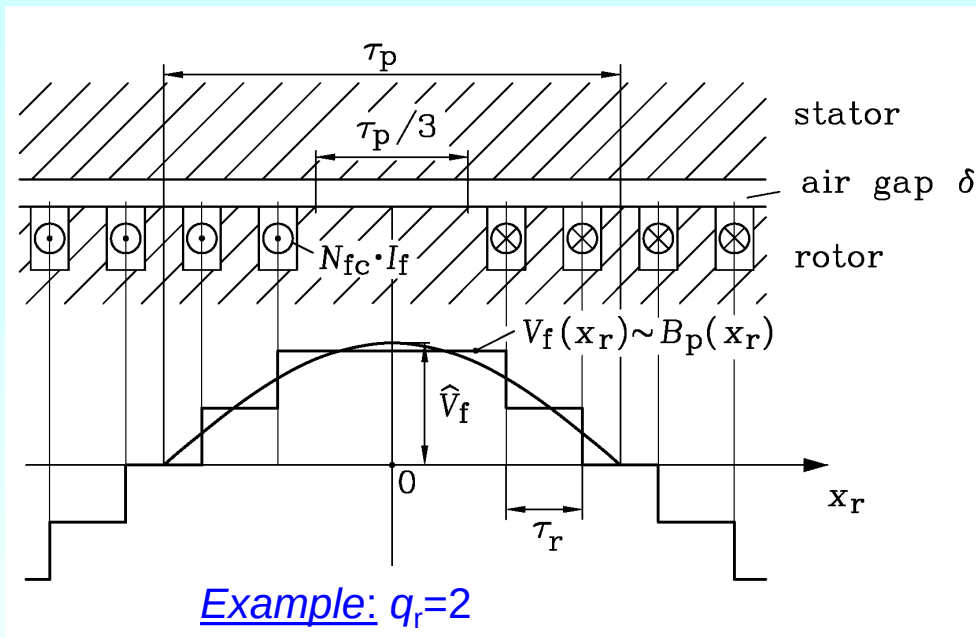
Glass fibre
bandage for fixing
rotor coil overhang



Source:

Andritz Hydro,
Bhopal, India

Rotor air gap field and stator back EMF of round rotor synchronous machine



- Rotor m.m.f. and air gap field distribution have steps due to slots and contain fundamental ($\mu = 1$):

$$\hat{V}_f = \frac{2}{\pi} \cdot \frac{N_f}{p} \cdot (k_{p,f} k_{d,f}) \cdot I_f$$

$$\hat{B}_p = \mu_0 \frac{\hat{V}_f}{\delta}, \quad N_f = 2p \cdot q_r \cdot N_{fc}$$

$$k_{p,f} = \sin\left(\frac{W}{\tau_p} \cdot \frac{\pi}{2}\right) = \sin(\pi/3) = \frac{\sqrt{3}}{2}$$

$$k_{d,f} = \frac{\sin(\pi/6)}{q_r \sin(\pi/(6q_r))}, \quad k_{wf} = k_{pf} k_{df}$$

- Back EMF U_p (synchronously induced stator voltage): Rotor field fundamental B_p induces in 3-phase stator winding at speed n a 3-phase voltage system U_p :

$$U_p = \omega \cdot \Psi_p / \sqrt{2} = \omega \cdot N_s k_{w,s} \cdot \Phi_p / \sqrt{2} = \sqrt{2} \pi f \cdot N_s k_{w,s} \cdot \frac{2}{\pi} l \tau_p \hat{B}_p$$

with frequency $f_s = n \cdot p \Rightarrow$ Current I_s will flow in stator winding.

Round rotor synchronous machine: Equivalent circuit

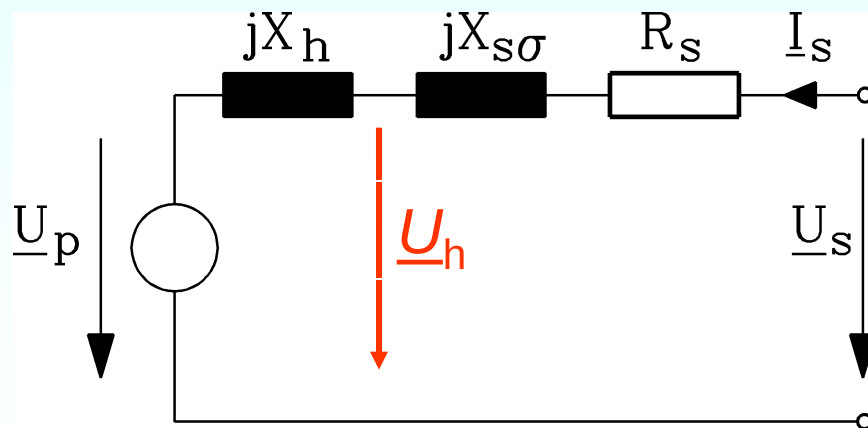
- **Stator winding:** Three phase AC winding like in induction machines with self-induced voltage due to stator rotating magnetic field, described by stator air gap field **main reactance** X_h and **stator leakage flux reactance** $X_{s\sigma}$. With **stator phase resistance** R_s we get stator voltage equation per phase:

$$\underline{U}_s = \underline{U}_p + jX_h \underline{I}_s + jX_{s\sigma} \underline{I}_s + R_s \underline{I}_s$$

$$\underline{U}_s = \underline{U}_p + jX_d \underline{I}_s + R_s \underline{I}_s$$

"synchronous reactance": $X_d = X_{s\sigma} + X_h$ contains effect of total stator magnetic field !

- **Equivalent circuit per stator phase:** for stator voltage equation (ac voltage and current). In rotor winding only DC voltage and current: $U_f = R_f \cdot I_f$



• Rotor electric circuit:

U_f : Rotor dc field voltage: (exciter voltage): U_f impresses via 2 slip rings and carbon brushes a rotor DC current (**field current I_f**) into rotor field winding. Field winding resistance is R_f .

Transfer ratio for rotor field current

- **Stator self-induced voltage:** $\underline{U}_{s,s} = jX_h \underline{I}_s$ by stator air-gap field
- **Back EMF $\underline{U}_p$:** Induced by rotor air gap field. It may be changed by field current I_f arbitrarily DURING OPERATION = **"synchronous machine is controlled voltage source"**.

a) **Amplitude of $\underline{U}_p$** is determined via I_f .

b) **Phase shift** of $\underline{U}_p$ with respect to stator voltage $\underline{U}_s$ is determined by relative position of rotor north pole axis with respect to stator north pole axis. Rotor pole position is described by **load angle ϑ** .

- **Amplitude and phase shift of $\underline{U}_p$:** may be described in equivalent circuit by **fictional AC stator current $\underline{I}'_f$** : $\underline{U}_p = jX_h \underline{I}'_f$

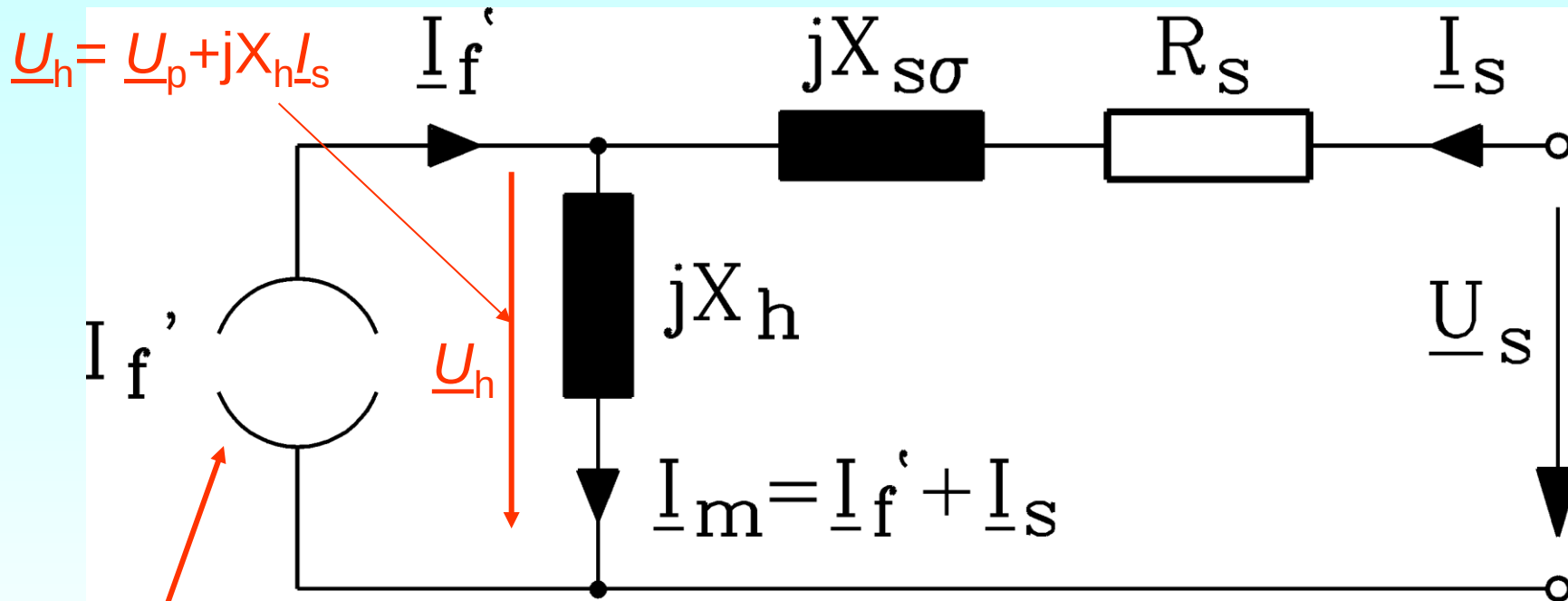
- **This defines transfer ratio of field current $\ddot{u}_{If}$:** $I'_f = \frac{1}{\ddot{u}_{If}} I_f$

$$I'_f = \frac{U_p}{U_{s,s}} I_s = \frac{B_p}{B_{s,\delta}} I_s = \hat{V}_f \cdot \frac{I_s}{\hat{V}_s} : \text{ shall be } \frac{1}{\ddot{u}_{If}} I_f$$

$$\text{With } \hat{V}_f = \frac{2}{\pi} \cdot \frac{N_f}{p} \cdot k_{wf} \cdot I_f, \quad \hat{V}_s = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s N_s}{p} \cdot k_{ws} \cdot I_s \quad \text{we get:}$$

$$\ddot{u}_{If} = \frac{m_s N_s k_{ws} \sqrt{2}}{2 N_f k_{wf}}$$

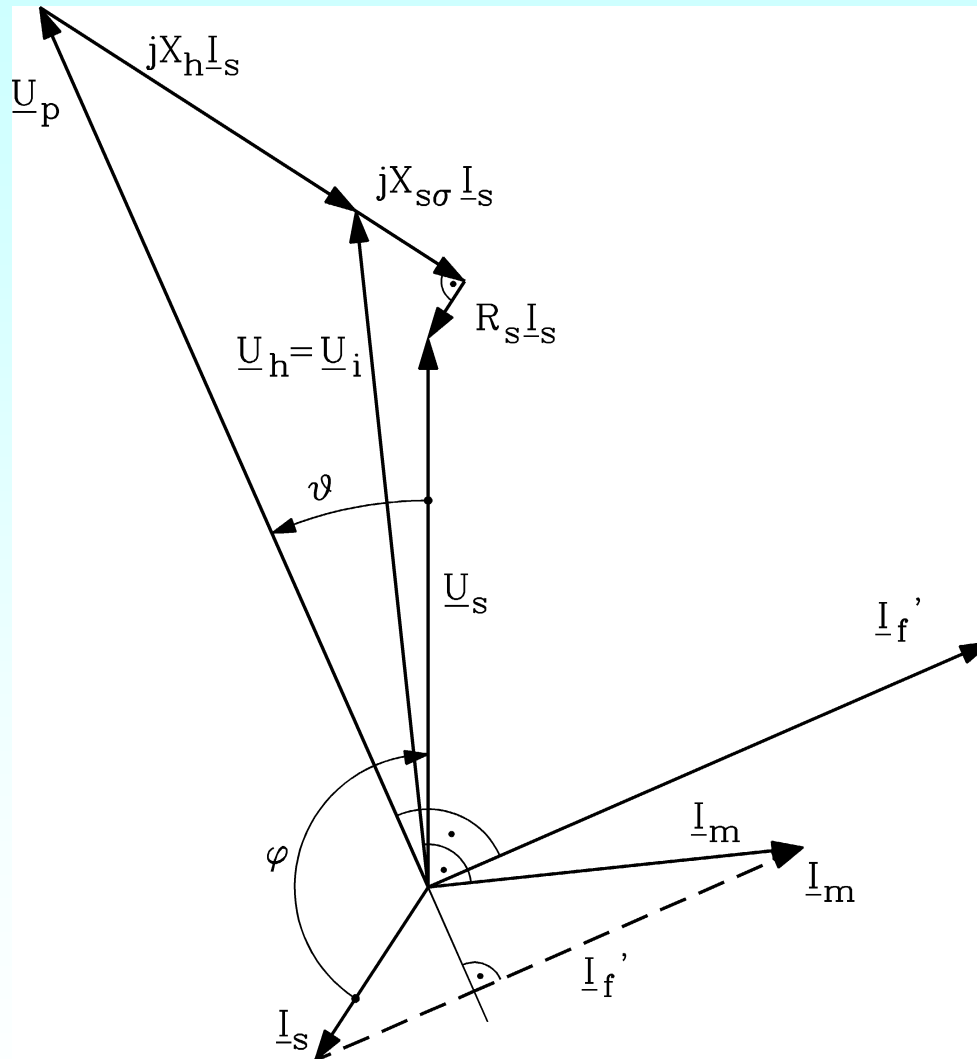
Alternative equivalent circuit: Current source for equivalent field current $\underline{I}'_f$



Fictitious AC current source $\underline{I}'_f$ generates synchronous back EMF $\underline{U}_p$ as voltage drop at magnetizing reactance

$$\underline{U}_p = jX_h \underline{I}'_f$$

Phasor diagram of round rotor synchronous machine



- **Example:** Generator, over-excited:

- **a) electrical active power:**

$$P_e = m_s U_s I_s \cos \varphi$$

Phase angle φ between -90° and -180° :
Hence $\cos \varphi$ negative: P_e is negative = power delivered to the grid

(GENERATOR).

$P_e < 0$: **Generator**,

$P_e > 0$: **Motor**.

- **b) electrical reactive power:**

$$Q = m_s U_s I_s \sin \varphi$$

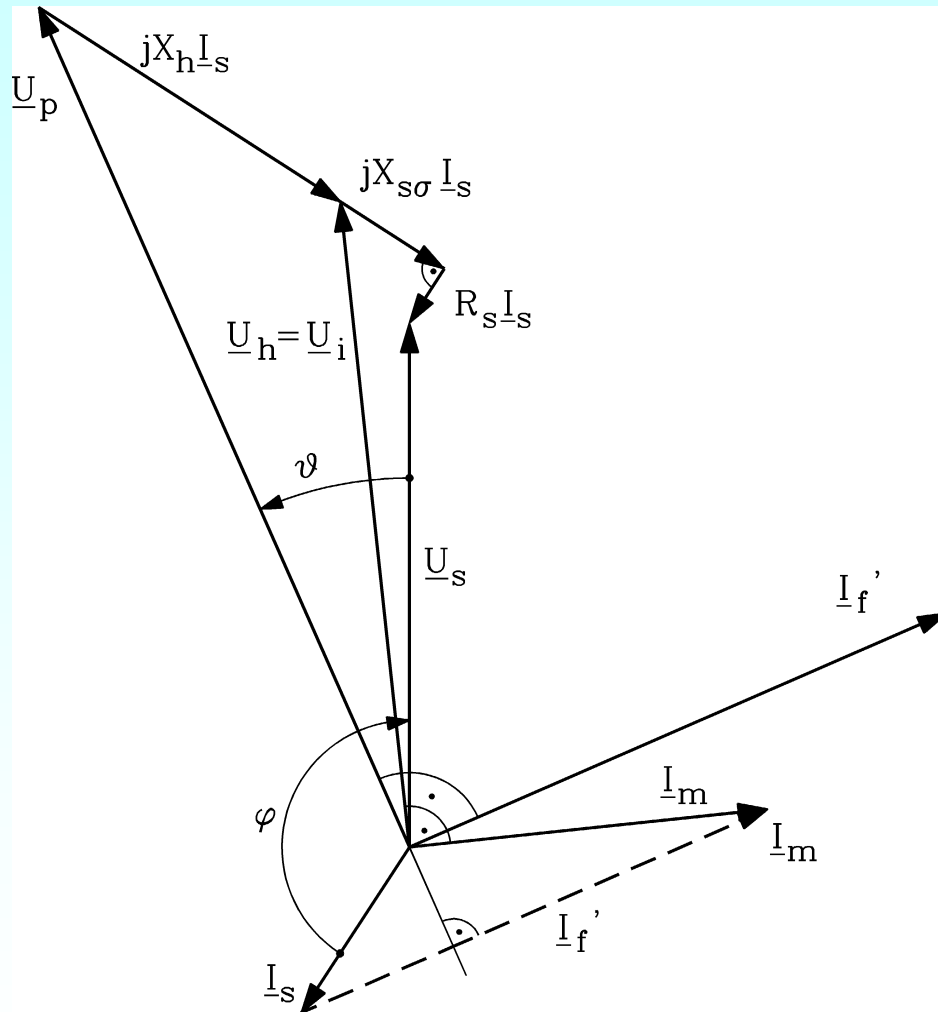
Phase angle φ negative = stator current LEADS ahead stator voltage:

$\sin \varphi$ negative: Q is negative = capacitive reactive power: Machine is **capacitive consumer**.

$Q < 0$: **over-excited**, capacitive consumer.

$Q > 0$: **under-excited**, inductive consumer.

Load angle ϑ , internal voltage $\underline{U}_h$, magnetising current $\underline{I}_m$



- **Load angle ϑ** between stator phase voltage $\underline{U}_s$ and back EMF phasor $\underline{U}_p$. Counted in mathematical positive sense (counter-clockwise).

- **Internal voltage $\underline{U}_h$** is induced in stator winding by resulting air gap field (rotor and stator field):

$$\underline{U}_h = \underline{U}_p + jX_h \underline{I}_s$$

- **Magnetising current $\underline{I}_m$:**

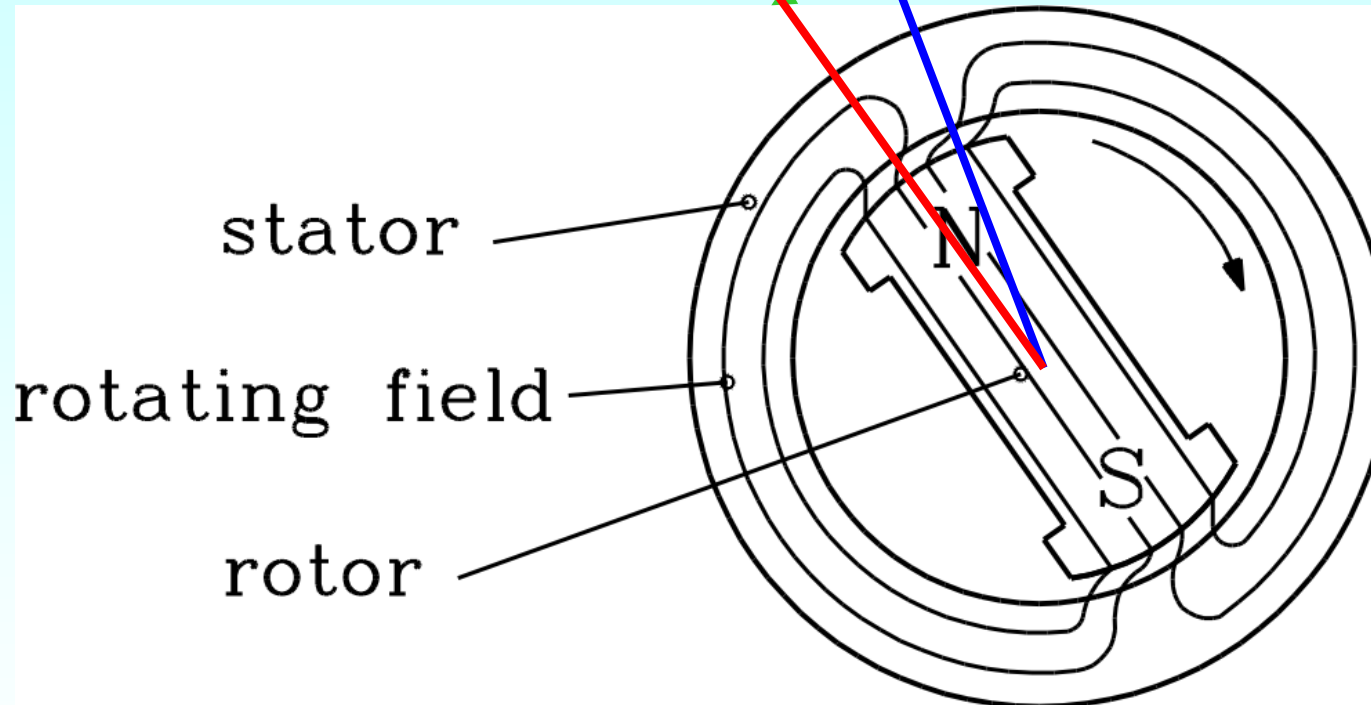
Fictitious stator current to excite resulting air gap field (rotor and stator field):

$$\underline{I}_m = \underline{I}'_f + \underline{I}_s$$

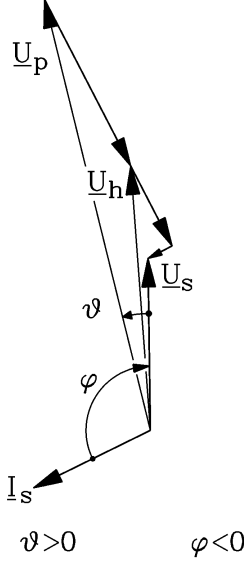
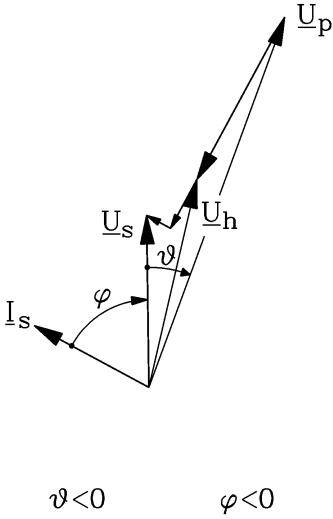
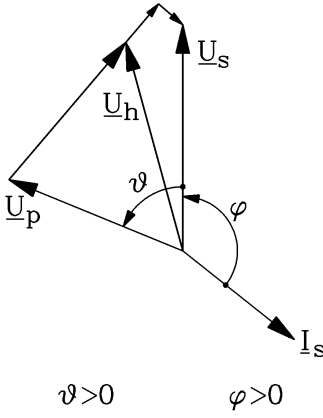
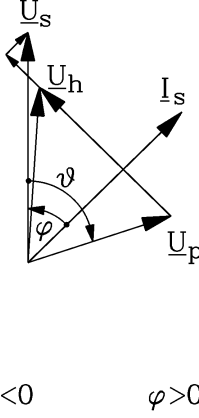
- Voltage triangle $\underline{U}_p, jX_h \underline{I}_s, \underline{U}_h$ and current triangle $\underline{I}'_f, \underline{I}_s, \underline{I}_m$ are of the same shape, but shifted by 90°.

Load angle ϑ - internal voltage U_h

$\underline{U}_h$ Load angle ϑ (exactly at $R_s, X_{s\sigma} = 0$)
 $\underline{U}_p$

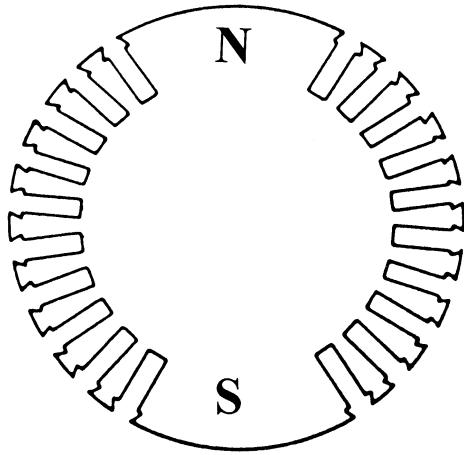


Over-/under-excitation, generator/motor-mode

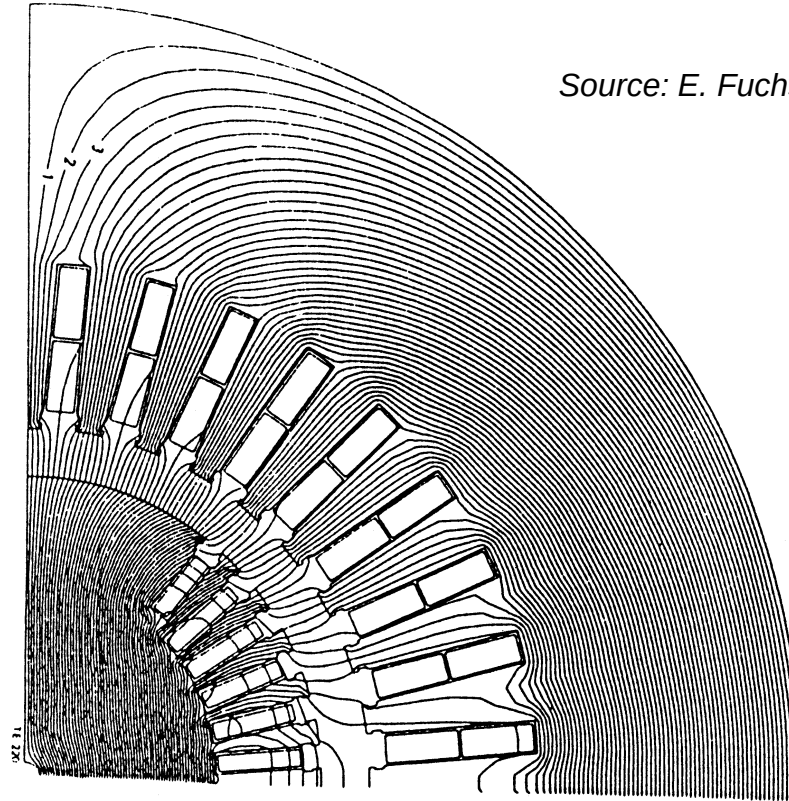
GENERATOR: rotor leading	MOTOR: rotor lagging	
 $\psi > 0$ $\varphi < 0$	 $\psi < 0$ $\varphi < 0$	overexcited
 $\psi > 0$ $\varphi > 0$	 $\psi < 0$ $\varphi > 0$	underexcited

- **Generator mode: $\vartheta > 0$:** Rotor *LEADS* ahead of resulting rotating magnetic field = Phasor $\underline{U}_p$ *LEADS* ahead of $\underline{U}_h$.
- **Motor mode: $\vartheta < 0$:** Rotor *LAGS* behind resulting magnetic field = Phasor $\underline{U}_p$ *LAGS* behind $\underline{U}_h$.
- **Over-excitation: Machine is capacitive consumer:** Phasor $\underline{U}_p$ is longer than phasor $\underline{U}_h$; big field current I_f is needed.
- **Under-excitation: Machine is inductive consumer:** Phasor $\underline{U}_p$ is shorter than phasor $\underline{U}_h$; small field current I_f is needed.
- **Facit:**
Stator- and rotor field rotate always synchronously. Generator- and motor mode are only defined by sign of load angle ϑ .

Round rotor synchronous machine: Magnetic field at no-load



Polachse = Feldachse



Source: E. Fuchs, IEEE-PAS

Rotor cross section without field winding:

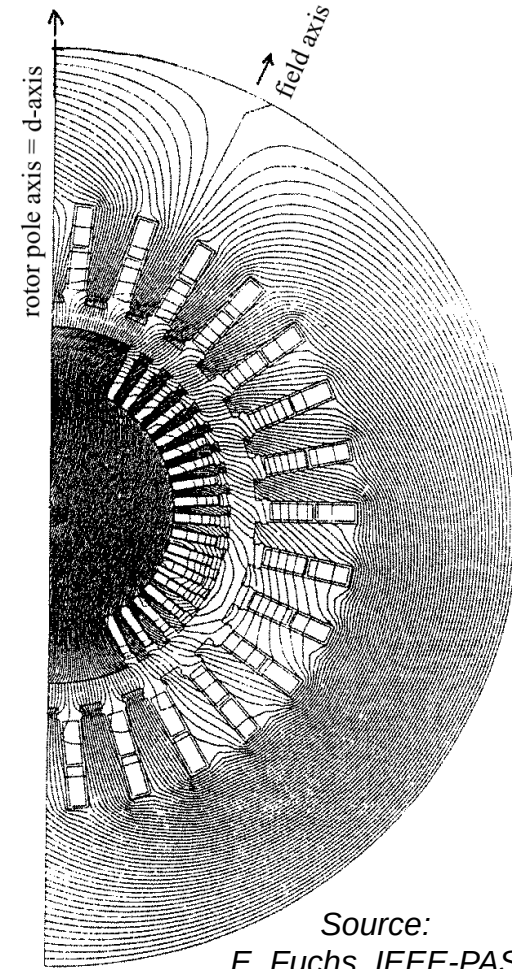
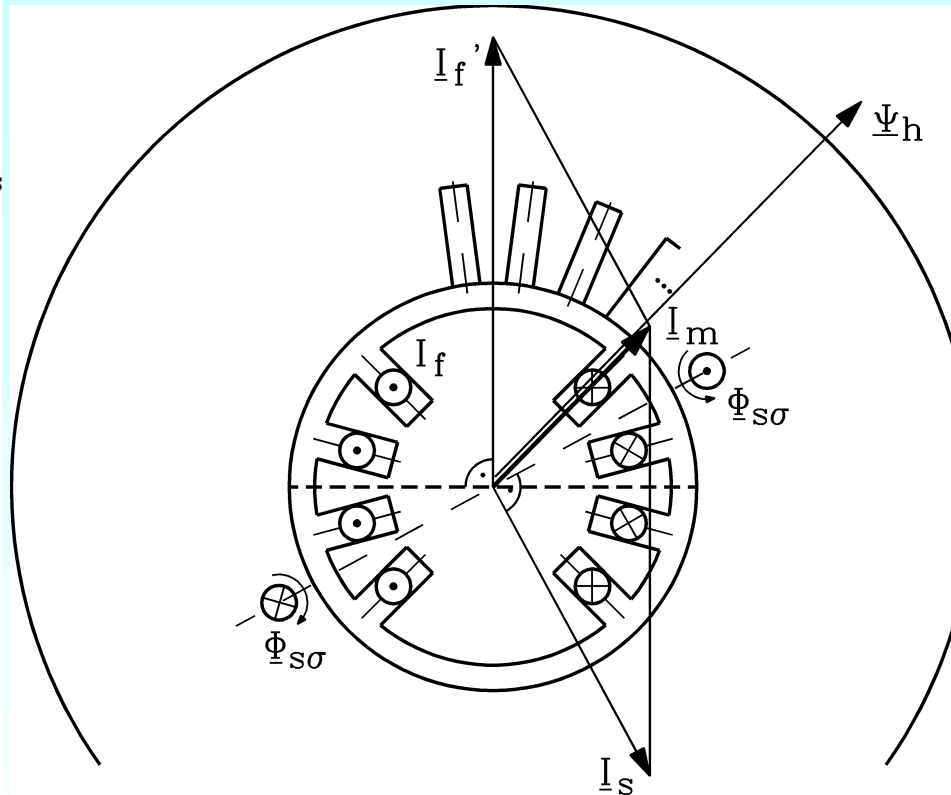
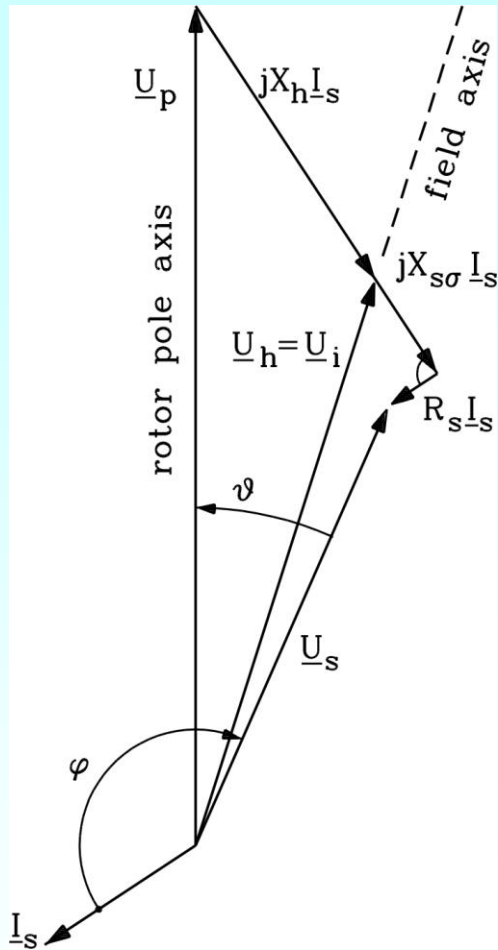
- Slots per pole $2q_r = 10$, 2-pole rotor
- Rotor may be constructed of massive iron, as rotor contains only **static magnetic field** !

Magnetic field at no-load ($I_s = 0$, $I_f > 0$):

- Field winding excited by I_f
- Stator winding without current (**no-load**)
- Field lines in air gap in radial direction = no tangential magnetic pull = **torque is zero** !

(Example: $2p = 2$, $q_s = 6$, $q_r = 6$)

Round rotor synchronous machine: Magnetic field at load



Source:
E. Fuchs, IEEE-PAS

- Magnetic field at load ($I_s > 0$, $I_f > 0$): Rotor pole axis = Direction of $\underline{U}_p$, resulting field axis = Direction of $\underline{U}_h$
- Field lines in air gap have also tangential component = tangential magnetic pull = torque !

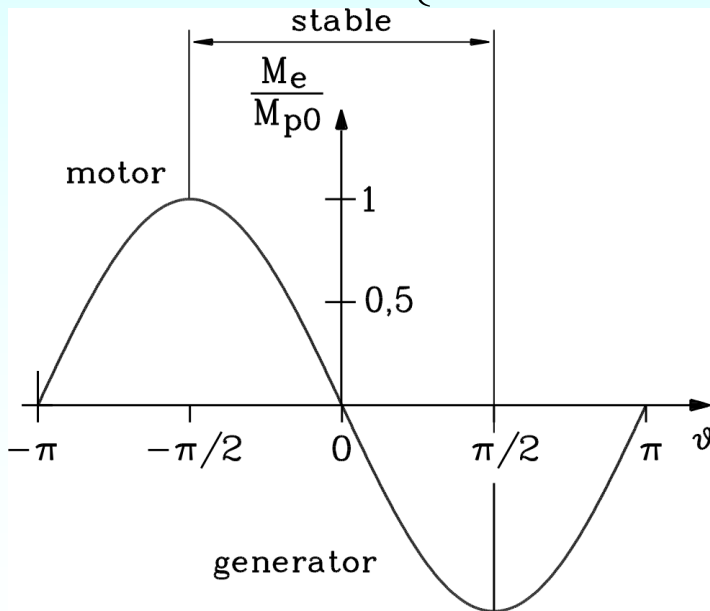
Torque of round rotor synchronous machine at $U_s = \text{const.} \ \& \ R_s = 0$

- Machine operates at **RIGID** grid: $\underline{U}_s = \text{constant} = U_s$ (= phasor put in real axis of complex plane):

$$\underline{U}_p = U_p (\cos \vartheta + j \cdot \sin \vartheta) \text{ and } \underline{I}_s = (U_s - \underline{U}_p) / (jX_d) \Rightarrow \underline{I}_s^* = (U_s - \underline{U}_p^*) / (-jX_d)$$

- Active power P_e** : $P_e = m_s U_s I_s \cos \varphi = m_s \cdot \text{Re} \{ \underline{U}_s \underline{I}_s^* \}$ (*: conjugate complex)

$$P_e = m_s \cdot \text{Re} \left\{ U_s \cdot \frac{U_s - U_p (\cos \vartheta - j \cdot \sin \vartheta)}{-jX_d} \right\} = -m_s \frac{U_s U_p}{X_d} \sin \vartheta$$



- Electromagnetic torque:**

$$M_e = \frac{P_m}{\Omega_{syn}} = \frac{P_e}{\Omega_{syn}} = -\frac{m_s}{\Omega_{syn}} \cdot \frac{U_s U_p}{X_d} \sin \vartheta = -M_{p0} \sin \vartheta$$

Note:

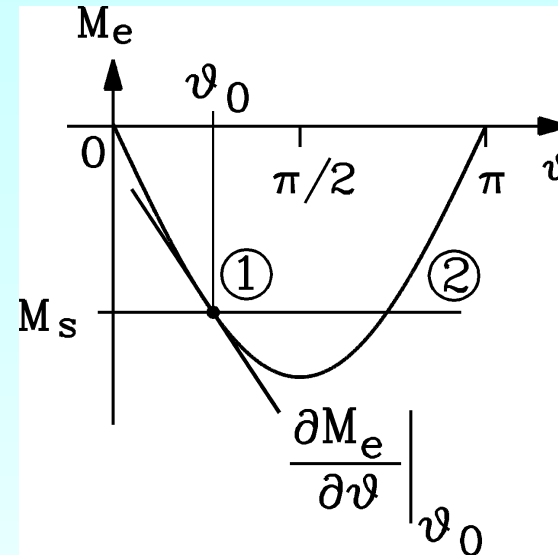
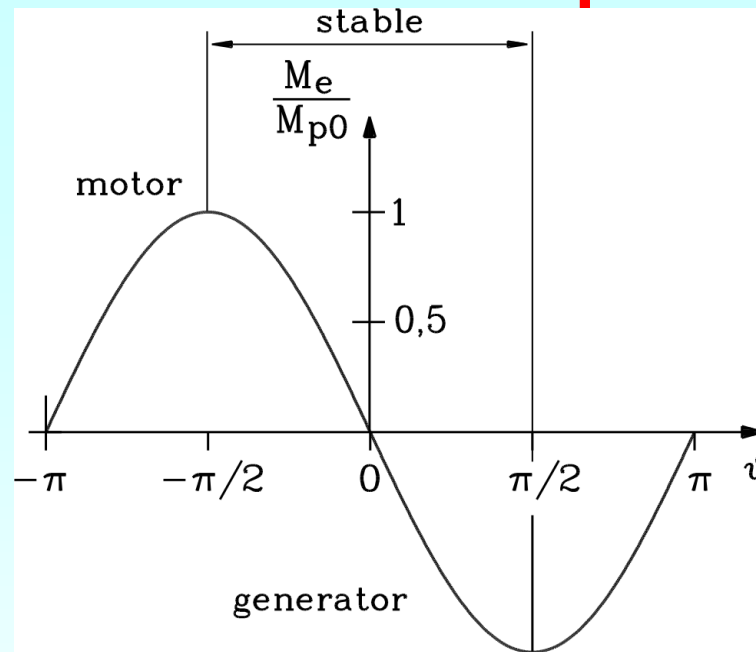
All losses neglected (“unity” efficiency).

Negative torque: Generator: M_e is braking

Positive torque: Motor: M_e is driving

Machine speed is always synchronous speed !

Stable points of operation



- **Example:** Torque-load angle curve $M(\vartheta)$: in generator mode the mechanical driving shaft torque M_s is determining operation points 1 and 2.
- Operation point 1 is **stable**, operation point 2 is **unstable**. The **stability limit** is at load angle $\pi/2$ (generator limit) and $-\pi/2$ (motor limit).

Facit: Synchronous motor and generator **pull-out torque** $\pm M_{p0}$ occurs at **pull-out load angle** $\pm \pi/2$. Rotor is "**pulled out**" of synchronism, if load torque exceeds pull-out torque. Result: Pulled-out rotor does not run synchronously with stator magnetic field, which is determined by the grid voltage. The rotor slips ! No active power is converted any longer.

Stability analysis of operation points

- Torque-load angle curve** $M_e(\vartheta)$ linearized in operation point ϑ_0 : **Tangent** as linearization: $M_e(\vartheta) \cong M_e(\vartheta_0) + \partial M_e / \partial \vartheta \cdot \Delta \vartheta$ with $\Delta \vartheta = \vartheta - \vartheta_0$

$$c_g(\vartheta_0) = \partial M_e / \partial \vartheta \big|_{\vartheta_0} : \text{Equivalent spring constant} \Leftrightarrow \Delta M_e = c_g \cdot \Delta \vartheta$$

- Change of** load angle with time causes **change of speed** $\Delta \Omega_m$:

$$d\Delta \vartheta / dt = p \cdot \Delta \Omega_m \Rightarrow \Omega_m(t) = \Omega_{syn} + \Delta \Omega_m(t)$$

- NEWTON's law of motion:** $J \frac{d\Delta \Omega_m}{dt} = M_e(\vartheta) - M_s(\vartheta) = c_g \cdot \Delta \vartheta = J \frac{d\Delta \Omega_m}{dt}$ leads to

$$J \frac{d^2 \Delta \vartheta}{dt^2} - p \cdot c_g \cdot \Delta \vartheta = 0 \quad \text{a) } |\vartheta| < \pi/2 : c_g = -|c_g| < 0, \quad \text{b) } |\vartheta| > \pi/2 : c_g = |c_g| > 0$$

$$\text{a) } |\vartheta| < \pi/2 : \Delta \ddot{\vartheta} + (p \cdot |c_g| / J) \cdot \Delta \vartheta = 0 \Rightarrow \Delta \ddot{\vartheta} + \omega_e^2 \Delta \vartheta = 0 \Rightarrow \Delta \vartheta(t) \sim \sin(\omega_e t)$$

Deviation of load angle from steady state point of operation remains limited: **STABLE** operation

$$\text{b) } |\vartheta| > \pi/2 : \Delta \ddot{\vartheta} - (p \cdot |c_g| / J) \cdot \Delta \vartheta = 0 \Rightarrow \Delta \ddot{\vartheta} - \omega_e^2 \Delta \vartheta = 0$$

$\Rightarrow \Delta \vartheta(t) \sim \sinh(\omega_e t)$ Deviation of load angle from operation point increases: **UNSTABLE**

Torsional oscillations of synchronous machine

- Deviation of load angle in stable point of operation due to disturbance:

$$|\vartheta| < \pi/2: \quad \Delta \ddot{\vartheta} + (p \cdot |c_{\vartheta}| / J) \cdot \Delta \vartheta = 0 \Rightarrow \boxed{\Delta \ddot{\vartheta} + \omega_e^2 \Delta \vartheta = 0} \Rightarrow \Delta \vartheta(t) \sim \sin(\omega_e t)$$

leads to differential equation **with oscillation as solution**. Rotor oscillates around steady state point of operation ϑ_0 , which is defined by the stator field, that is generated by the “rigid” grid.

Natural frequency of oscillation (eigen-frequency):

$$\boxed{f_e = \frac{\omega_e}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{p|c_{\vartheta}|}{J}}}$$

Facit: The synchronous machine is performing like a (non-linear) torsional spring.

Example: Operation at no-load ($M_e = 0$, $\vartheta_0 = 0$):

$$\boxed{|c_{\vartheta}| = |-M_{p0} \cdot \cos(0)| = M_{p0}}$$

With $p\Omega_{syn} = \omega_N$ and rated acceleration time $T_J = \frac{J \cdot \Omega_{syn}}{M_N}$ we get: $f_e = \frac{1}{2\pi} \sqrt{\frac{\omega_N}{T_J} \cdot \frac{M_{p0}}{M_N}}$

Synchronous motor driving a fan blower for a wind tunnel:

$P_N = 50 \text{ MW}$, $f_N = 50 \text{ Hz}$, $T_J = 10 \text{ s}$, $M_{p0}/M_N = 1.5$,

$$f_e = \frac{1}{2\pi} \sqrt{\frac{2\pi 50}{10} \cdot 1.5} = \underline{\underline{1.09 \text{ Hz}}}$$

Round rotor synchronous machine – Stator 1

**Turbo-
Generator**

Stator
stacking



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Source:

ALSTOM



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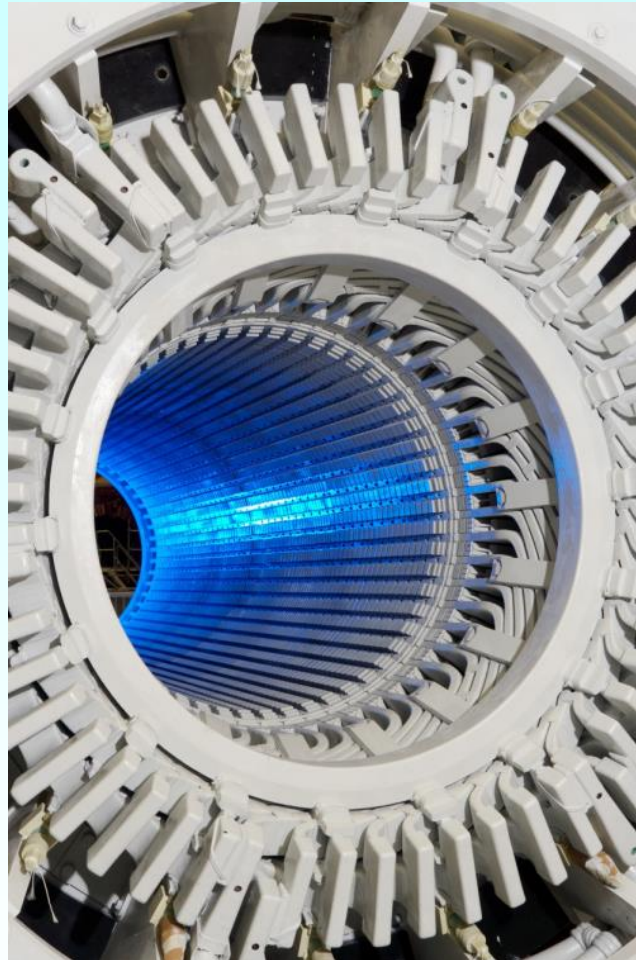
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Round rotor synchronous machine – Stator 2

Turbo-Generator

Stator complete



4-pole Turbogenerator



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Round rotor synchronous machine – Rotor 1

**Turbo-
Generator**

Rotor wound



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Round rotor synchronous machine – Rotor 2

**Turbo-
Generator**

Rotor insertion
in stator



Source:

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4-pole Turbogenerator



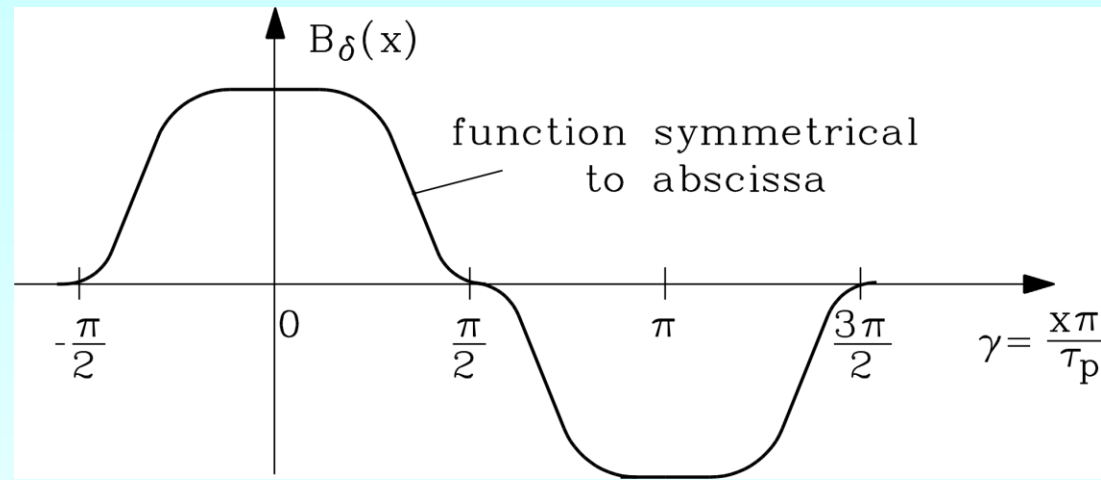
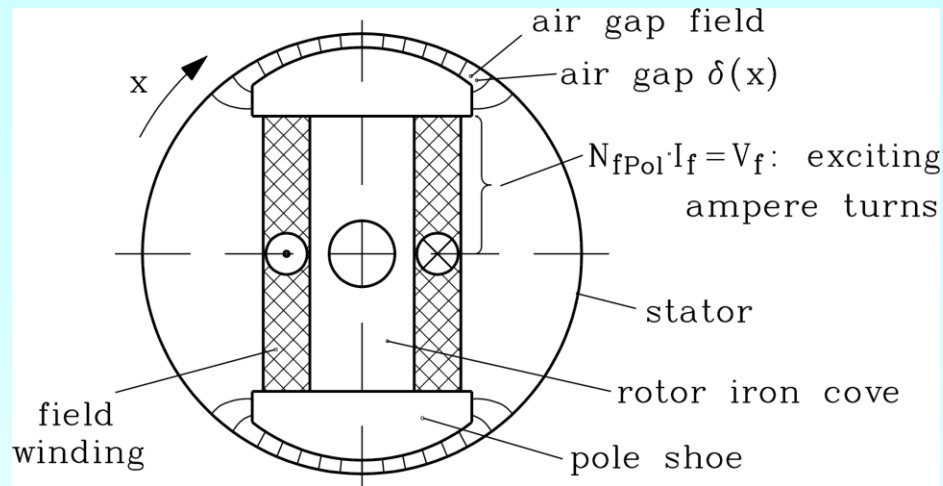
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Rotor field and back EMF of salient pole synchronous machine



- **Bell shaped rotor air gap field curve $B_\delta(x)$:** A constant m.m.f. V_f excites with a variable air gap $\delta(x)$ a bell shaped field curve. Fundamental of this “bell-shape” ($\mu = 1$):

$$B_\delta(x) = \mu_0 \frac{V_f}{\delta(x)} \rightarrow \text{FOURIER-fundamental wave: Amplitude } \hat{B}_p \text{ proportional to } I_f$$

- **Back EMF U_p :** Sinusoidal rotor field fundamental wave B_p induces **in three-phase stator winding** at speed n a three-phase voltage system U_p

$$U_p = \omega \cdot \Psi_p / \sqrt{2} = \omega \cdot N_s k_{w,s} \cdot \Phi_p / \sqrt{2} = \sqrt{2} \pi f \cdot N_s k_{w,s} \cdot \frac{2}{\pi} l \tau_p \hat{B}_p$$

with **frequency** $f = n \cdot p \Rightarrow$ Stator current I_s is flowing in stator winding.

Rotor salient pole manufacturing

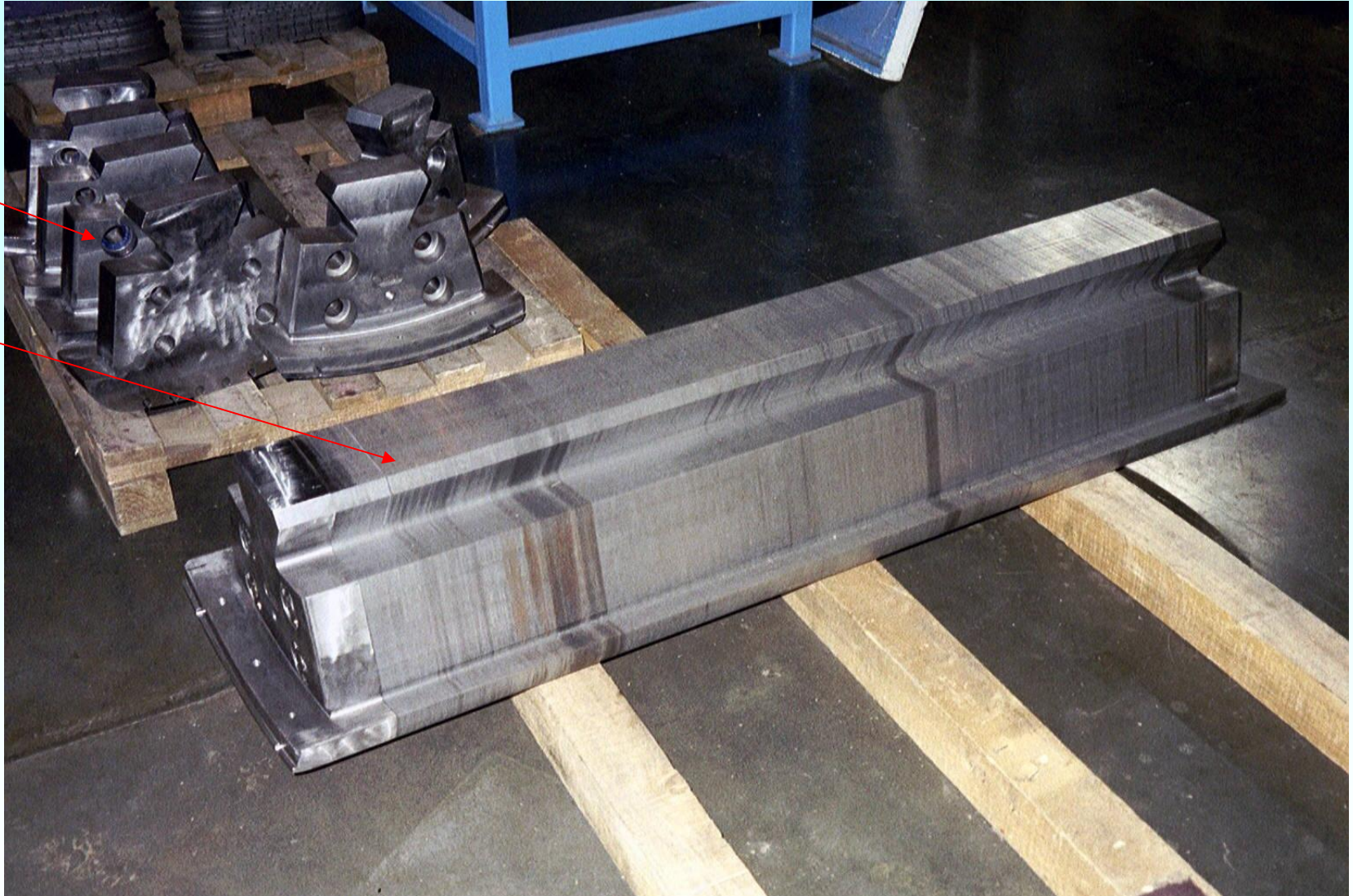
Massive
pole
pressing
plates

Dove tail
rotor pole

Lamination
pressed by
pressing
plates

Source:

Andritz Hydro,
Bhopal, India



Completed salient pole synchronous rotor, 8 poles

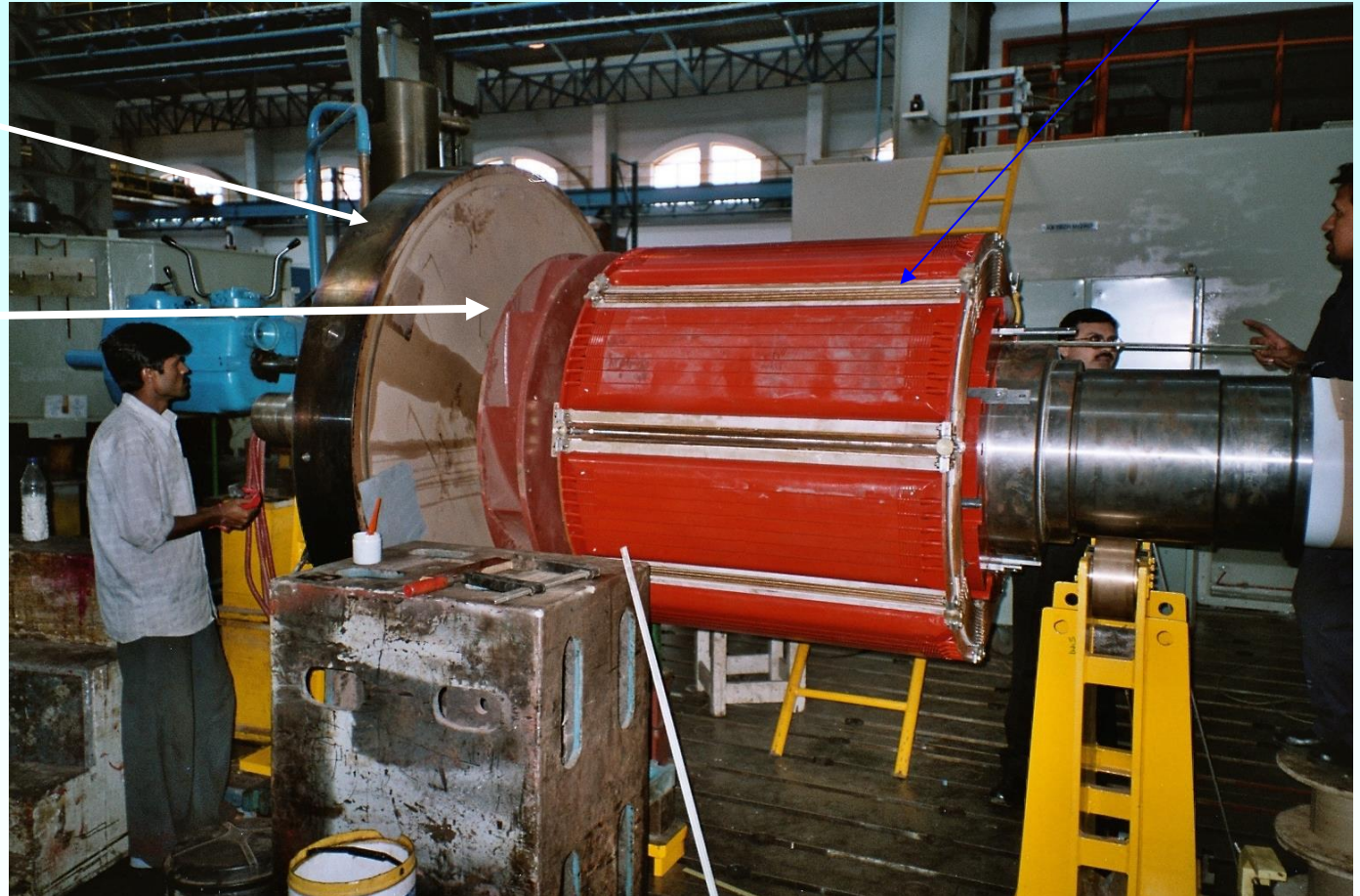
Fly wheel to increase rotor inertia to limit acceleration in case of load drop

Shaft mounted fan with backward bent radial blades for rotation in clockwise direction at fixed speed

Source:

Andritz Hydro,
Bhopal, India

Kauli power plant



Four-pole salient pole rotor with massive pole shoes for asynchronous line start up as motor

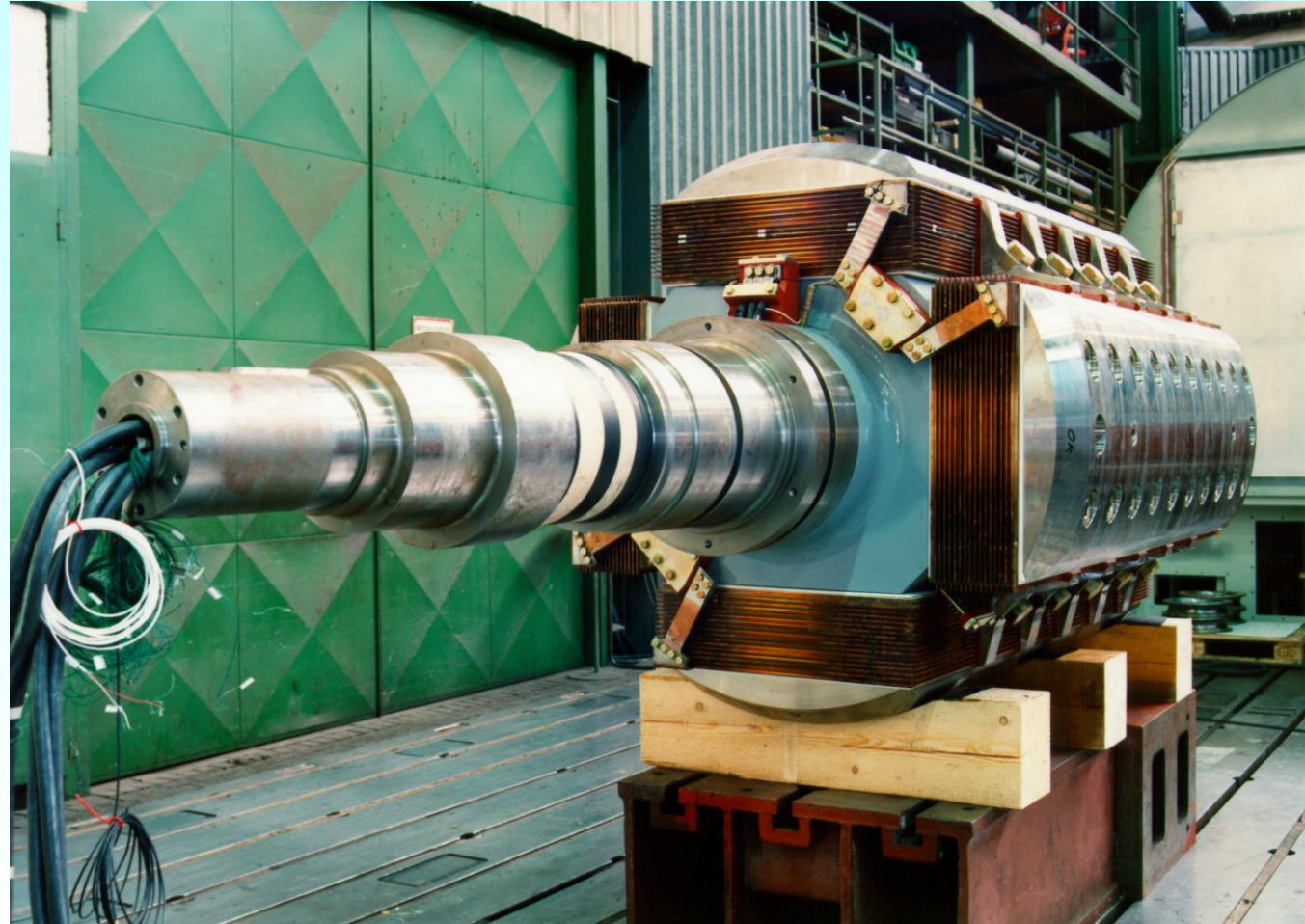
- At asynchronous line start up the stator field induces eddy currents in the rotor massive pole shoes.

- These eddy currents develop with the stator field the needed starting torque.

50 Hz, $2p = 4$, $n = 1500/\text{min}$

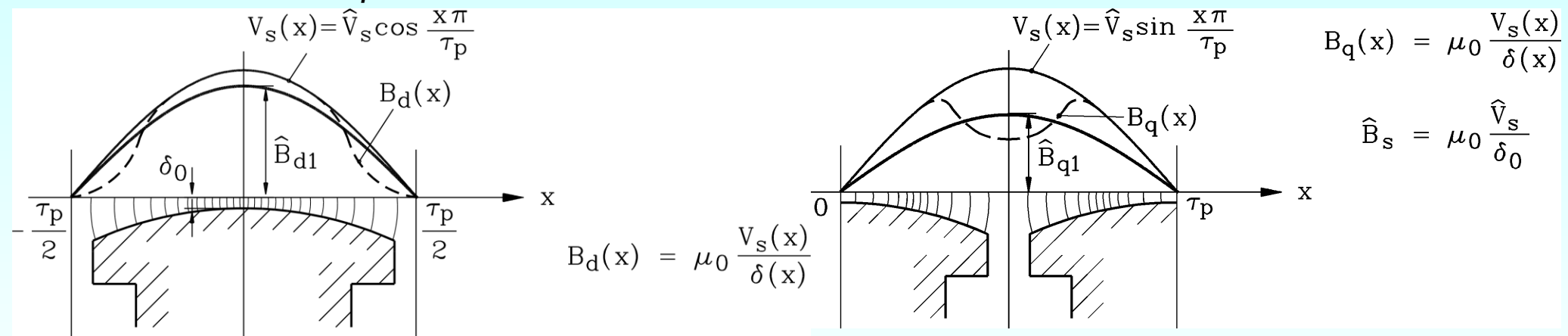
Source:

Andritz Hydro,
Austria



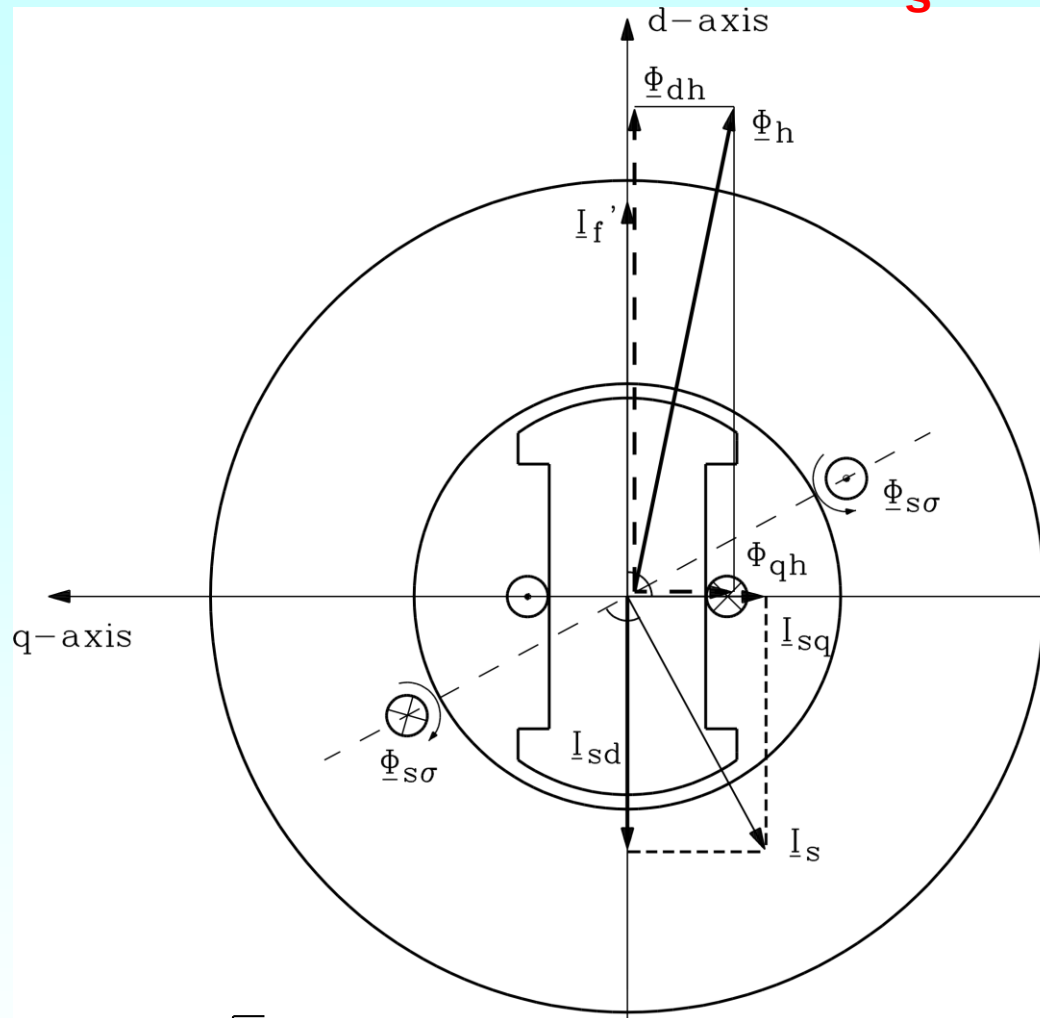
Salient pole synchronous machine: Magnetizing inductance L_h

- **Stator winding** is three-phase winding like in induction machines, BUT the air gap is **LARGER in neutral zone (inter-pole gap of q -axis)** than in **pole axis (d -axis)**. Hence for equal m.m.f. V_s (sinus fundamental $\nu = 1$) the corresponding air gap field is **SMALLER** in q -axis than in d -axis and NOT SINUSOIDAL.



- **Stator field in d -axis (direct axis):** Fundamental of field a little bit smaller than for constant air gap δ_0 : $c_d = \hat{B}_{d1} / \hat{B}_s < 1$ **ca. 0.95, thus:** $L_{dh} = c_d \cdot L_h$
- **Stator field in q -axis (quadrature axis):** Fundamental of field significantly smaller than at constant air gap δ_0 : $c_q = \hat{B}_{q1} / \hat{B}_s \ll 1$ **ca. 0.4 ... 0.5, thus** $L_{qh} = c_q \cdot L_h$

Stator current I_s : d - and q -component



$$\Psi_{dh} / \sqrt{2} = L_{dh} \cdot (I_f' + I_{sd}) \rightarrow \Phi_{dh} = \Psi_{dh} / (k_{ws} N_s)$$

$$\Psi_{qh} / \sqrt{2} = L_{qh} \cdot I_{sq} \rightarrow \Phi_{qh} = \Psi_{qh} / (k_{ws} N_s)$$

- Stator current phasor I_s decomposed into d - and q -component:

$$\underline{I}_s = \underline{I}_{sd} + \underline{I}_{sq}$$

I_{sd} is in phase or opposite phase with fictitious current I_f' . So it excites a stator air gap field in d -axis (in rotor pole axis), which together with rotor field gives d -axis air gap flux Φ_{dh} .

I_{sq} is phase-shifted by 90° to I_{sd} and excites therefore a stator air gap field in q -axis (inter-pole gap). The corresponding air gap flux is Φ_{qh} .

- Stator self-induced voltage consists of two, by 90° phase shifted components:

$$j\omega_s L_{dh} \underline{I}_{sd}$$

$$j\omega_s L_{qh} \underline{I}_{sq}$$

and of self-induced voltage of stator leakage flux:

$$j\omega_s L_{s\sigma} \underline{I}_s$$

Stator voltage equation of salient pole synchronous machine

- **Stator voltage equation per phase:** Considering self-induction of main and leakage flux L_{dh} , L_{qh} , $L_{s\sigma}$ and of rotor phase resistance R_s we get :

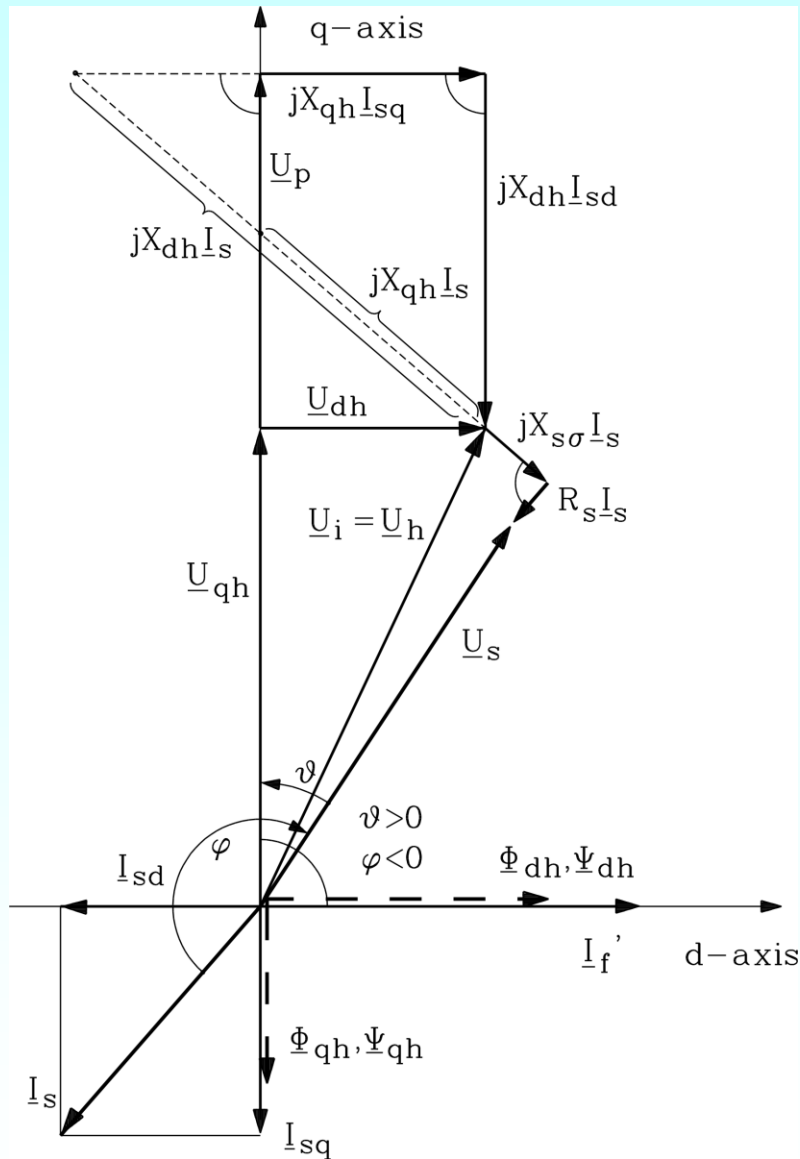
$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_{qh} \underline{I}_{sq} + j\omega_s L_{dh} \underline{I}_{sd} + \underline{U}_p \quad \underline{U}_p = j\omega_s L_{dh} \underline{I}'_f$$

$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} (\underline{I}_{sd} + \underline{I}_{sq}) + j\omega_s (L_{qh} \underline{I}_{sq} + L_{dh} \underline{I}_{sd}) + \underline{U}_p$$

- X_d : **"synchronous d-axis reactance"**: $X_d = X_{s\sigma} + X_{dh} = \omega_s L_{s\sigma} + \omega_s L_{dh}$
- X_q : **"synchronous q-axis reactance"**: $X_q = X_{s\sigma} + X_{qh} = \omega_s L_{s\sigma} + \omega_s L_{qh}$
- **Typical values:** Due to inter-pole gap it is $X_d > X_q$ (typically: $X_q = (0.5 \dots 0.6) \cdot X_d$)
e.g. salient pole hydro-generators, diesel engine generators, reluctance machines, ...
- **Note:** **Round rotor synchronous machine** may be regarded as "special case" of salient pole machine for $X_d = X_q$.

The slot openings of rotor field winding in round rotor machines may also be regarded as non-constant air gap, yielding also $X_d > X_q$ (typically: $X_q = (0.8 \dots 0.9) \cdot X_d$)

Phasor diagram of salient pole machine



- Example: Generator, over-excited:
 - $\underline{I}_f$ and $\underline{I}_{sd}$ lie in d-axis, $\underline{I}_{sq}$ lies in q-axis
 - $\underline{U}_p \sim j\underline{I}_f$ and $jX_{dh}\underline{I}_{sd}$ lie in q-axis, $jX_{qh}\underline{I}_{sq}$ lies in d-axis (!)

- Induced internal voltage $\underline{U}_h$:

$$\underline{U}_h = j\omega_s \underline{\Psi}_h = \underline{U}_{qh} + \underline{U}_{dh}$$

is decomposed into components

$$\underline{U}_{qh} = j\omega_s L_{dh} \underline{I}_{sd} + \underline{U}_p$$

$$\underline{U}_{dh} = j\omega_s L_{qh} \underline{I}_{sq}$$

- Stator voltage and current:

- Load angle ϑ ,
 - Phase angle φ
- are defined in the same way as with round rotor synchronous machines !

Torque of salient pole machine at $U_s = \text{const.}$ & $R_s = 0$

- OPERATION at "rigid" grid:** $\underline{U}_s = \text{constant}$

We choose: d -axis = Re-axis, q -axis = Im-axis of complex plane:

$$\underline{U}_s = U_{sd} + jU_{sq} \quad \underline{I}_s = I_{sd} + jI_{sq} \quad \underline{U}_p = jU_p$$

$$R_s = 0: \quad \underline{U}_s = jX_d I_{sd} + jX_q I_{sq} + \underline{U}_p \quad \Rightarrow \quad \underline{U}_s = jX_d I_{sd} - X_q I_{sq} + jU_p$$

- Active power P_e :** $P_e = m_s U_s I_s \cos \varphi = m_s \cdot \text{Re}\{\underline{U}_s \underline{I}_s^*\} = m_s (U_{sd} I_{sd} + U_{sq} I_{sq})$
 $P_e = m_s (-X_q I_{sq} I_{sd} + X_d I_{sd} I_{sq} + U_p I_{sq})$

- Electromagnetical torque:**

$$M_e = \frac{P_m}{\Omega_{syn}} = \frac{P_e}{\Omega_{syn}} = \frac{m_s}{\Omega_{syn}} \cdot (U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq})$$

- Two torque components:**

a) prop. U_p as with round rotor machines

b) **"Reluctance" torque** due to $X_d \neq X_q$. NO rotor excitation is necessary !

Synchronous reluctance machine: Reluctance torque = robust rotor WITHOUT ANY winding, but DEEP inter-pole gaps.

Torque as function of load angle ϑ

$$\underline{U}_s = jX_d I_{sd} - X_q I_{sq} + jU_p \Rightarrow \begin{cases} U_{sd} = -X_q I_{sq} & \Rightarrow I_{sq} = -\frac{U_{sd}}{X_q} \\ jU_{sq} = jX_d I_{sd} + jU_p & \Rightarrow I_{sd} = \frac{U_{sq} - U_p}{X_d} \end{cases}$$

$$\underline{U}_s = U_{sd} + jU_{sq} \begin{cases} U_{sd} = U_s \sin \vartheta \\ U_{sq} = U_s \cos \vartheta \end{cases}$$

$$M_e = \frac{m_s}{\Omega_{syn}} \cdot (U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq}) =$$

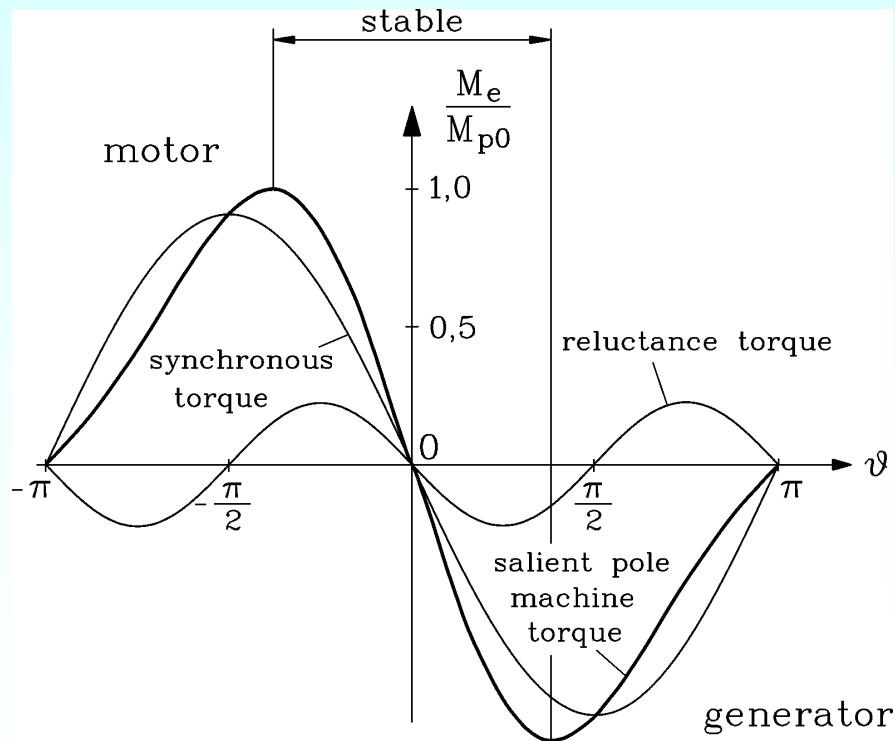
$$= \frac{m_s}{\Omega_{syn}} \cdot \left(-\frac{U_p U_s \sin \vartheta}{X_q} - \frac{X_d - X_q}{X_d X_q} \cdot U_s \sin \vartheta \cdot (U_s \cos \vartheta - U_p) \right)$$

$$M_e = -\frac{p \cdot m_s}{\omega_s} \left(\frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$$

Torque-load angle curve $M_e(\vartheta)$

- Torque is expressed by stator voltage, back EMF and load angle: I_{sd} , I_{sq} are expressed by U_s , ϑ :

$$M_e = -\frac{p \cdot m_s}{\omega_s} \left(\frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$$

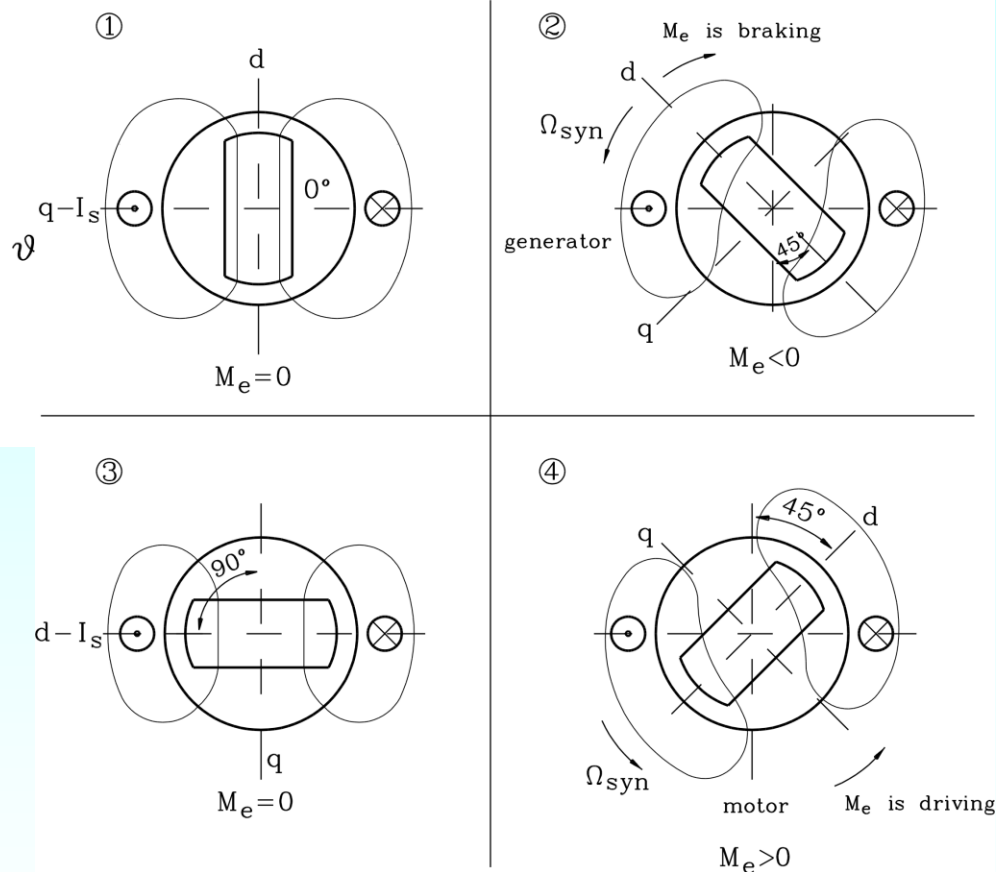
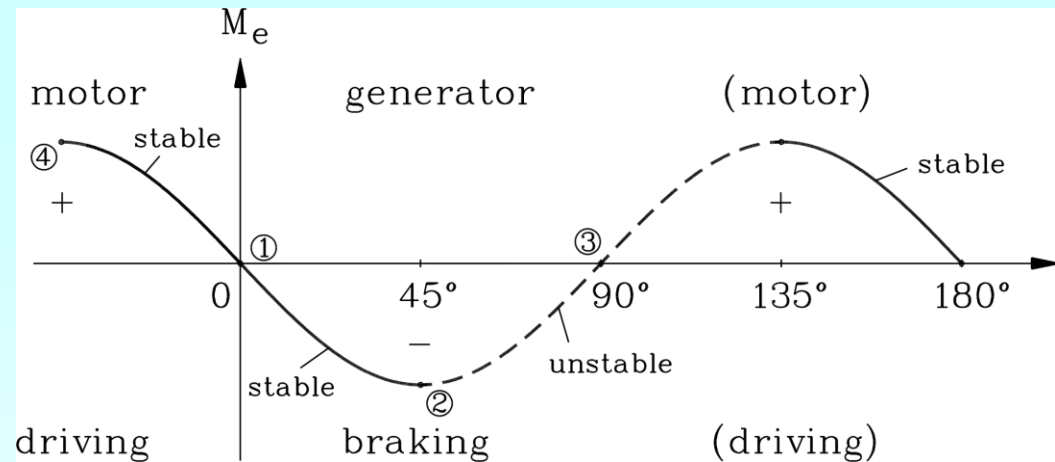


Absolute value of pull-out load angle is $< 90^\circ$,
as pull-out torque of reluctance torque occurs at load angle $\pm 45^\circ$.

Pull-out torque is increased by reluctance torque.

Equivalent spring constant c_g
bigger than in round rotor machines, as reluctance torque adds ("stiffer" $M_e(\vartheta)$ -curve).

Synchronous reluctance machine

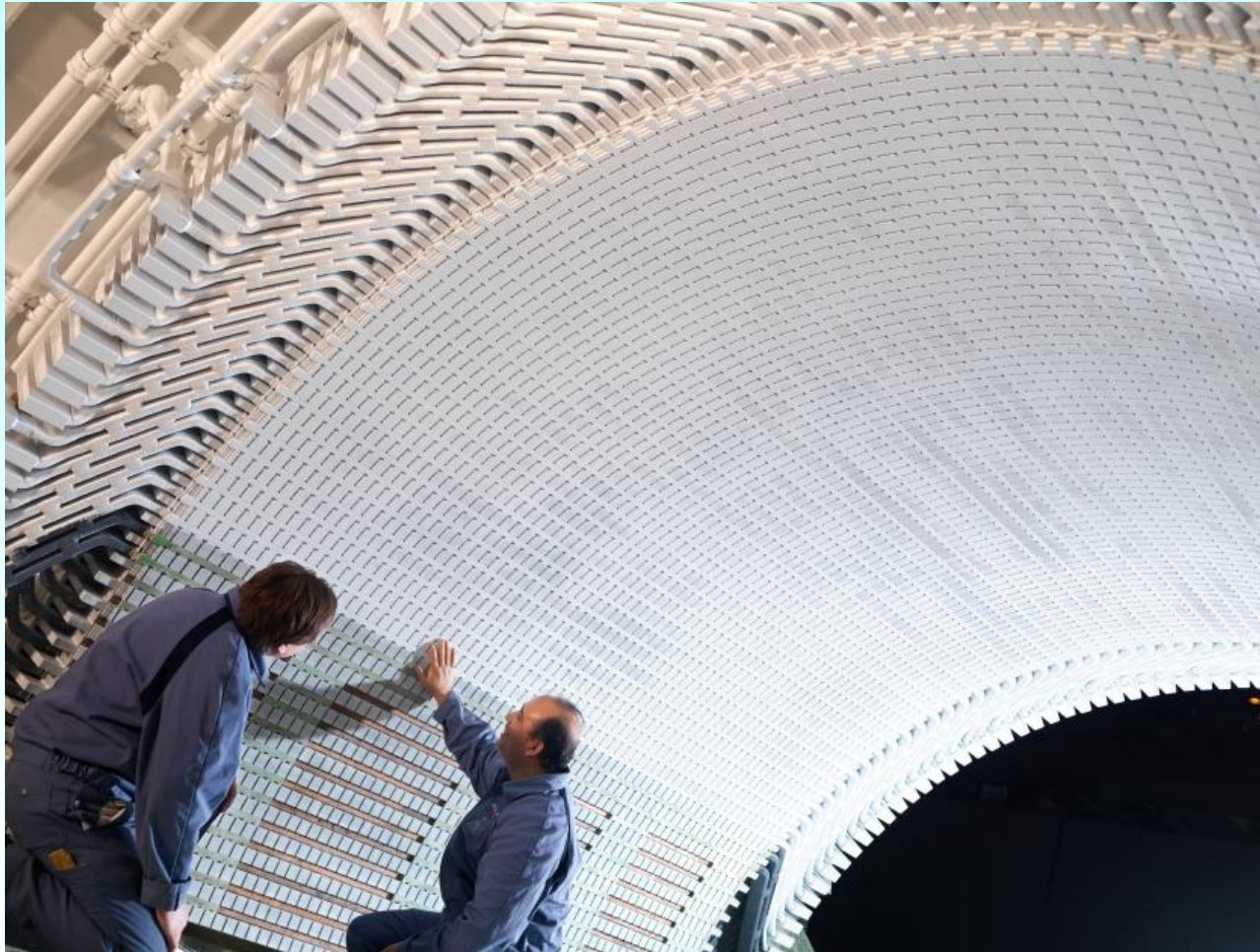


- Rotor **without any winding**, but with **deep inter-pole gaps**: $X_d > X_q$.
- Rotor d -axis wants to move into stator field axis, to allow field lines to cross air gap via the MOST SHORTEST distance possible, thus yielding the **reluctance torque**.

Salient pole synchronous machine - Stator

Hydro-
Generator

Stator



Source:

ALSTOM



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Salient pole synchronous machine - Rotor

**Hydro-
Generator**

Rotor insertion

Three Gorges
(China)
840 MVA
80 poles



Source:

ALSTOM



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Hydro Power Plant – 1

Hydro-Generator

Rotor insertion

Three Gorges
(China)
840 MVA
80 poles



Source:

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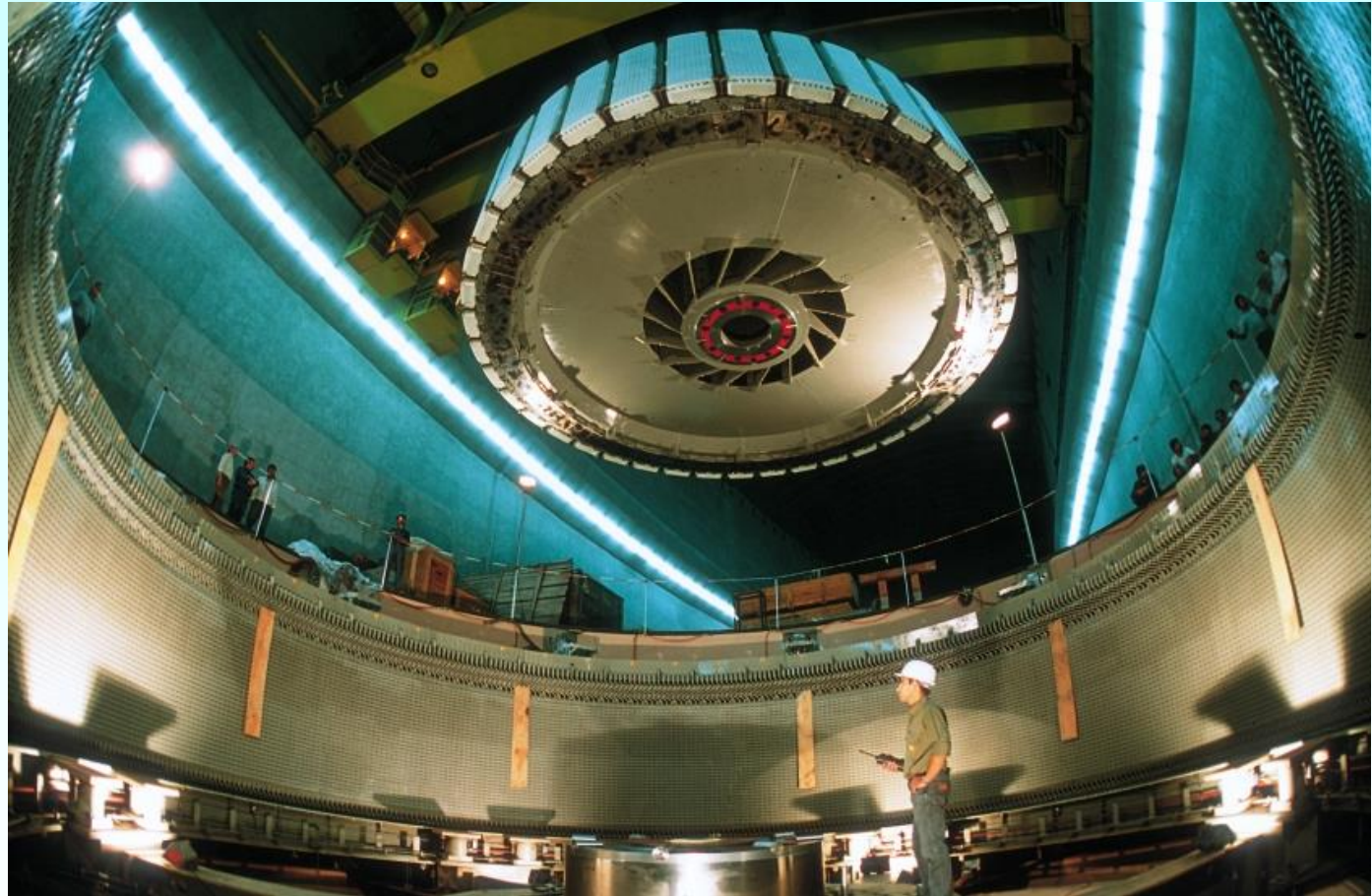


Hydro Power Plant – 2

Hydro-Generator

Rotor assembly

Karakaya
(Turkey)
315 MVA
40 poles



Source:

ALSTOM



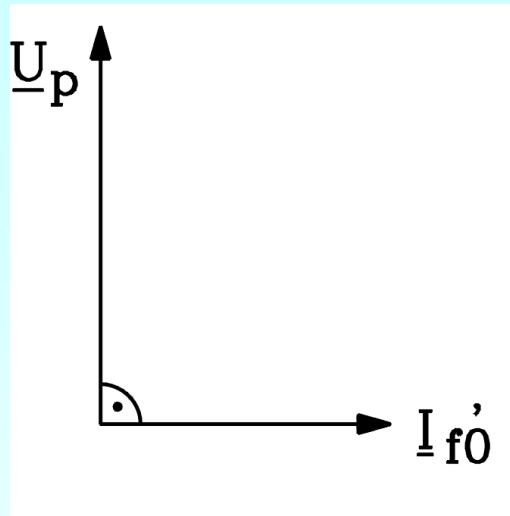
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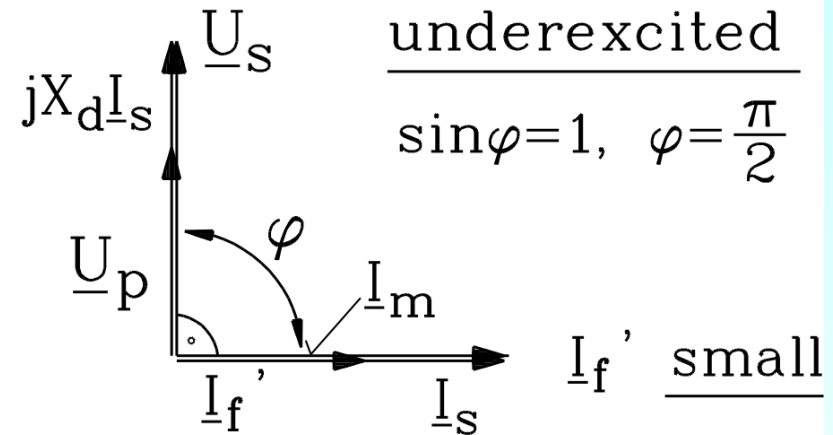
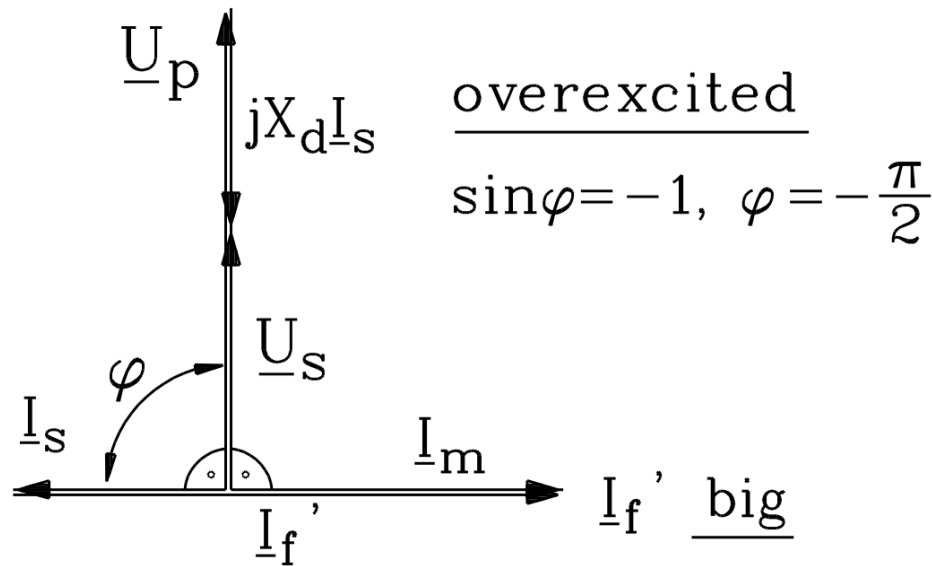
Generator no-load



Rotor with excited field winding is driven, field current is I_{f0} , stator terminals are not connected (no stator current flow): At open stator terminals U_p is measured.

$$\underline{U}_p = jX_{dh} \underline{I}_{f0}'$$

Synchronous machine as „phase shifter,, ($R_s \approx 0$)



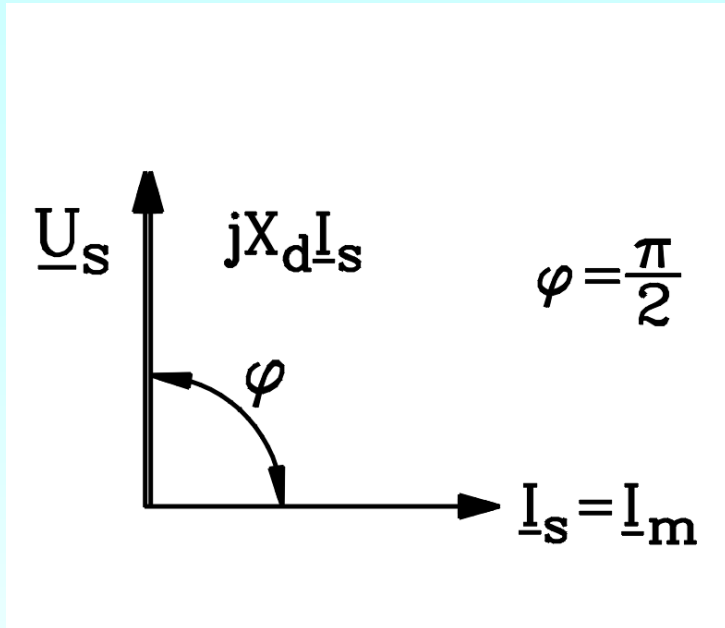
big I_f (over-excited): machine is capacitive consumer

small I_f (under-excited): machine is inductive consumer

$$R_s = 0: \underline{U}_s = \underline{U}_p + jX_d \underline{I}_s$$

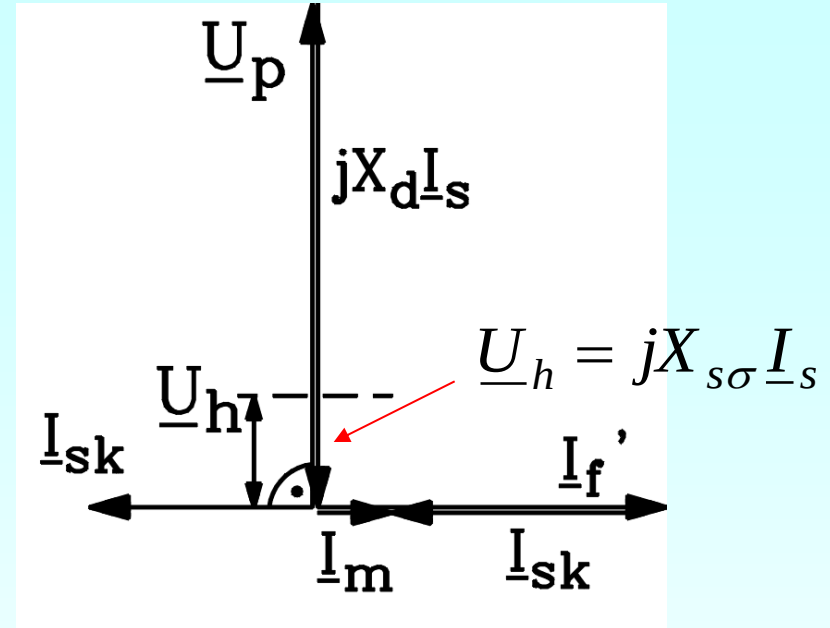
Stator connected to grid, no active power transfer, but phase angle either inductive or capacitive !

Special operating points of synchronous machines ($R_s \approx 0$)



non-excited at the grid: $I_f = 0$.
Stator winding current I_s acts as magnetizing current

$$R_s = 0: \quad \underline{U}_s = jX_d \underline{I}_s$$



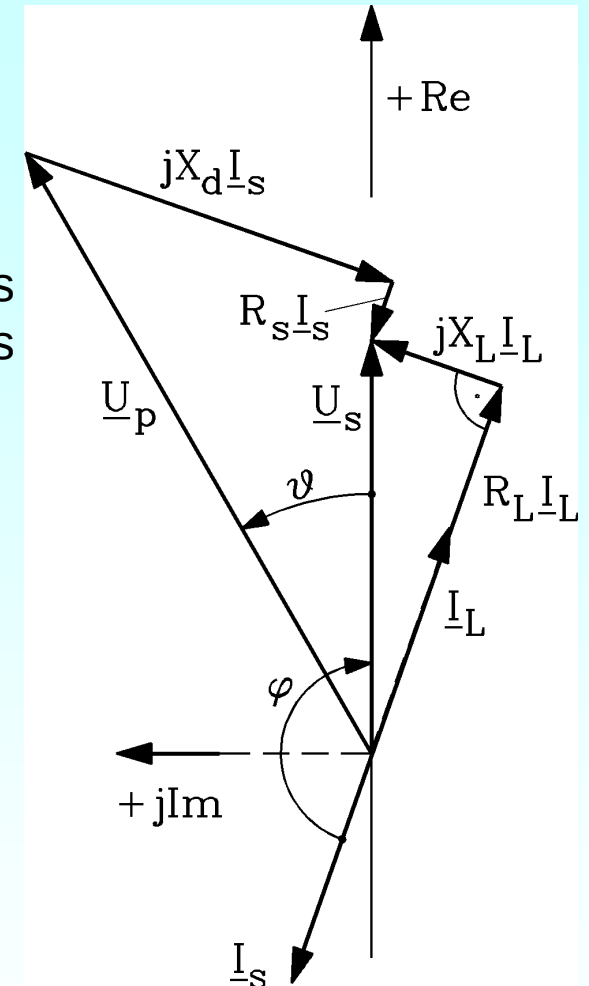
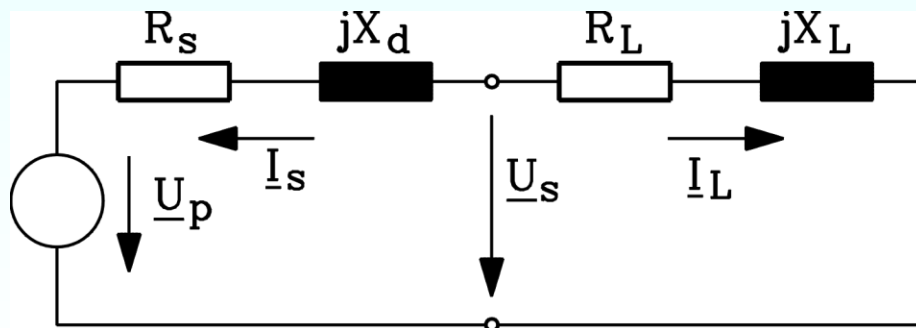
Steady state short circuit:

Stator winding short-circuited: $U_s = 0$, Rotor is driven, U_p is induced, causes **short circuit current** I_{sk} , which is limited by X_d and R_s : $I_{sk} \approx U_p / X_d$

$$R_s = 0: \quad 0 = \underline{U}_p + jX_d \underline{I}_s$$

Synchronous machine in stand-alone operation

- Examples: Automotive generator, Air plane / ship generator, generator stations on islands, off-shore platforms, oasis, mountainous regions, emergency generators in hospitals, military use (e.g. radar supply)
- **No "rigid" grid available: U_s is NOT constant:** Rotor is excited and driven, field current I_f , back EMF U_p is induced as "voltage source", U_s is depending on load. E.g.: round rotor synchronous machine:
No $M_e \sim \sin \vartheta$ - curve,
No rotor pull out at $\vartheta = \pm 90^\circ$
- Example: Mixed OHM'ic-inductive load $\underline{Z}_L$ (Load current $\underline{I}_L = -\underline{I}_s$)



Load impedance: $\underline{Z}_L$ (here: $\underline{Z}_L = R_L + jX_L$)

Stand-alone voltage-current characteristic $U_s(I_s)$ at $R_s = 0$

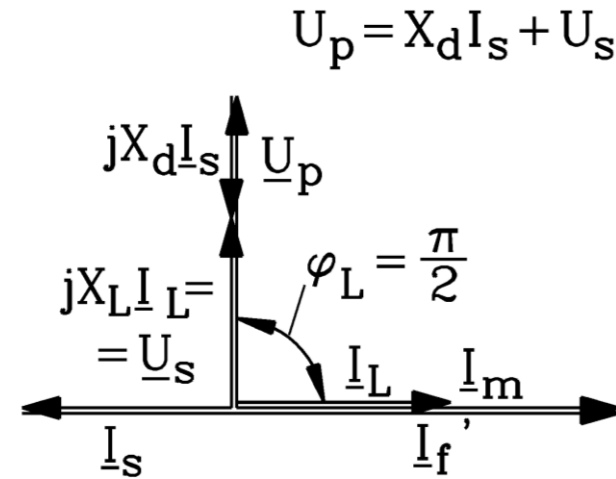
- No-load: $I_s = 0 \Rightarrow U_s = U_p = U_{s0}$;
- Short-circuit: $Z_L = 0: U_s = 0 \Rightarrow I_s = U_p/X_d = I_{sk}$

- Inductive load: $Z_L = j\omega L_L = jX_L$:

Phasor diagram: voltage drops are in line: $U_p = U_s + X_d I_s$

Voltage decreases linear with increasing load current !

$$U_s / U_p = 1 - I_s / (U_p / X_d) \quad u = 1 - i$$



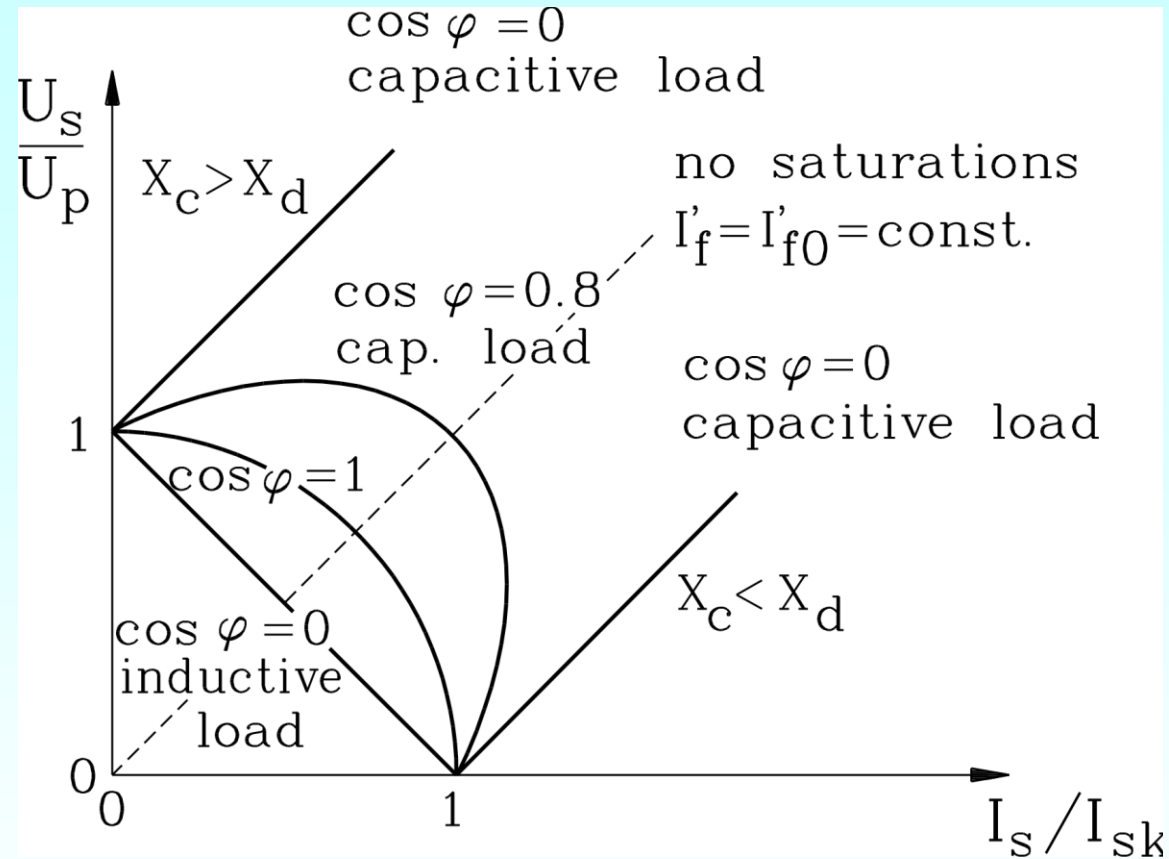
- Resistive load: $Z_L = R_L$:

Phasor diagram: right angle triangle. *Pythagoras*: $U_p^2 = U_s^2 + (X_d I_s)^2$

$$(U_s / U_p)^2 = 1 - I_s^2 / (U_p / X_d)^2 \quad u^2 = 1 - i^2$$

Voltage-current curve in per-unit of no-load voltage and short-circuit current is segment of circle !

Stand-alone operation: Voltage-current curve $U_s(I_s)$



At **mixed resistive-inductive and resistive-capacitive load** the per unit voltage-current curves are sections of ellipses.

Stand-alone capacitive load curve $U_s(I_s)$ for $R_s = 0$

- Capacitive load: $Z_L = 1/(j\omega C_L) = -jX_C$:

- From phasor diagram we see, that voltage drops are in line !
- Two cases are to be considered:

a) $\underline{U}_p$ in phase opposition to $\underline{U}_s$: $X_C < X_d$ COUNTER-EXCITATION

$$-U_p = U_s - X_d I_s$$

$$U_s / U_p = I_s / (U_p / X_d) - 1$$

$$u = -1 + i$$

b) $\underline{U}_p$ in phase with $\underline{U}_s$: $X_C > X_d$

$$U_p = U_s - X_d I_s$$

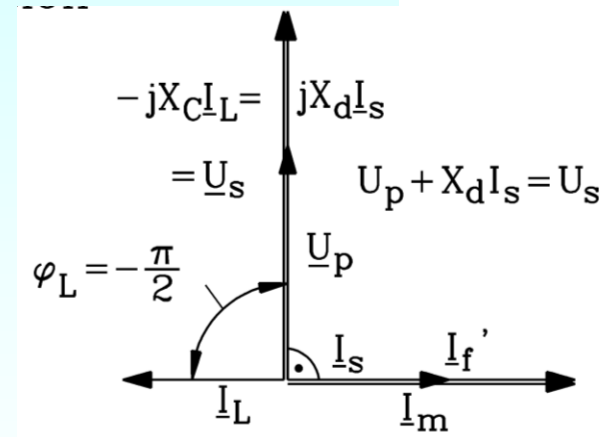
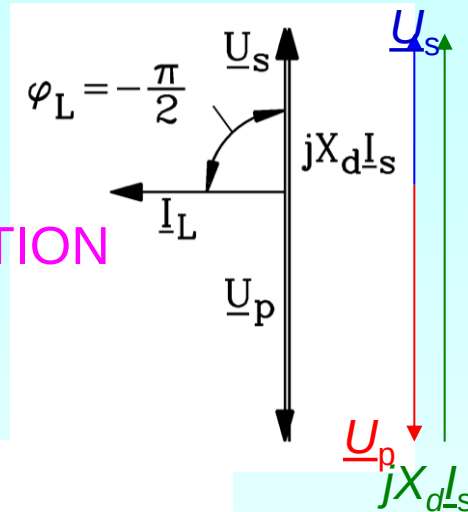
$$U_s / U_p = 1 + I_s / (U_p / X_d)$$

$$u = 1 + i$$

Usually case b) occurs !

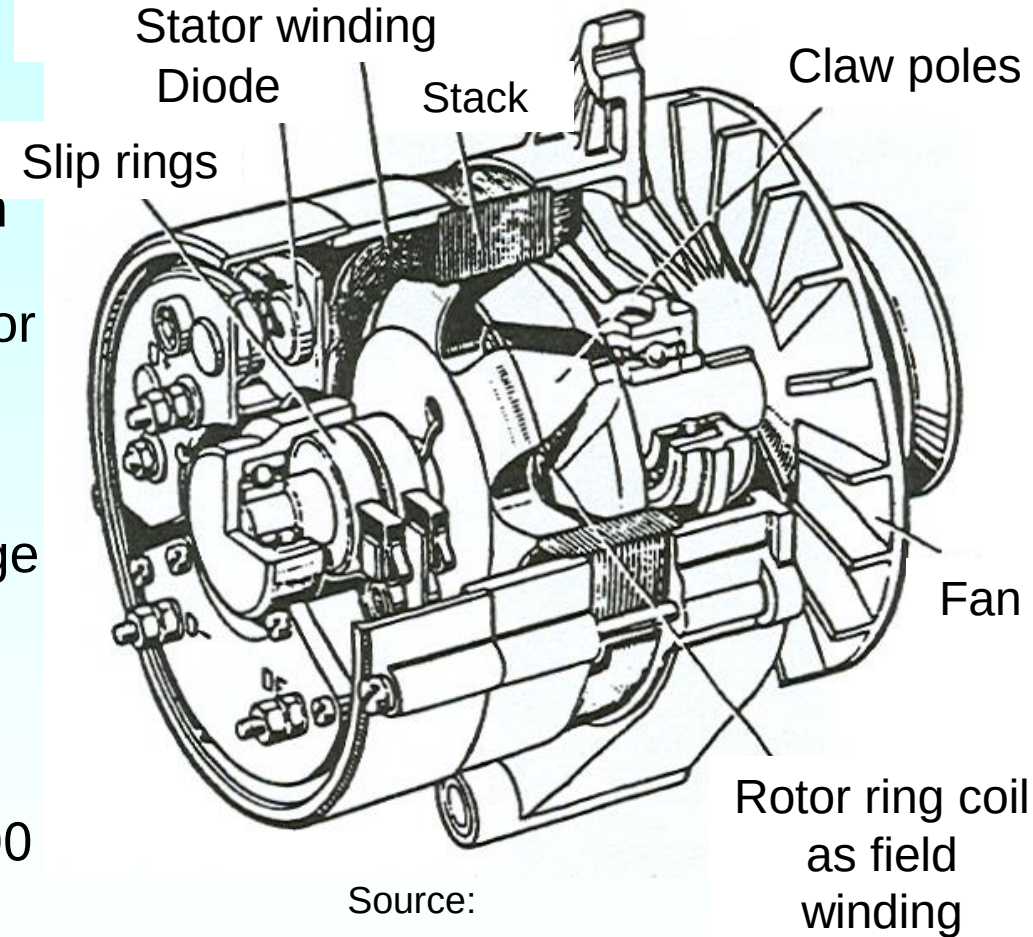
May also start from un-excited generator,
where remanence of rotor induces a (small) back EMF
in stator winding (“self-excitation of synchronous generator”).

Voltage RISES with increasing load: FERRANTI-phenomenon



Example – stand alone operation: Automotive synchronous generator

- Automotive synchronous generator:
three phases, $q = 1$, single layer wave wound winding, 12 poles
- Claw pole rotor, electrically excited
- Driven via belt with variable speed from internal combustion engine
- Diode rectifier for stator power at 12 V or 24 V DC voltage
- Diode rectifier for rotor field winding excitation
- Transistor controller keeps stator voltage constant - independently from varying speed n and stator load current I_s – via variable exciter current I_f
- **Data:** e. g.: 12...14 V, 90 A, 1 kW, 3000 ... 6500/min



Source:
Bosch, Germany