Energy Converters – CAD and System Dynamics

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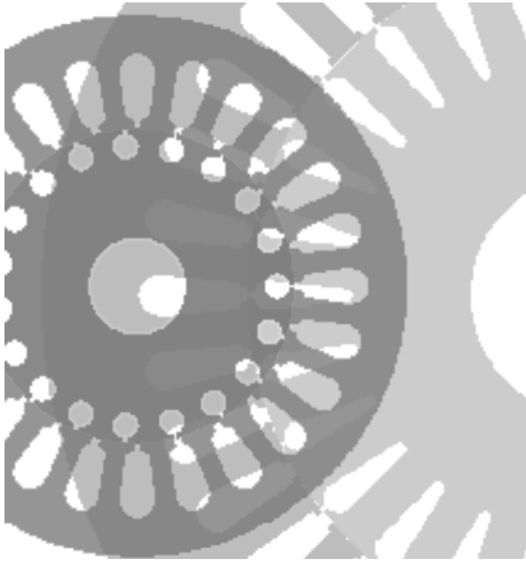
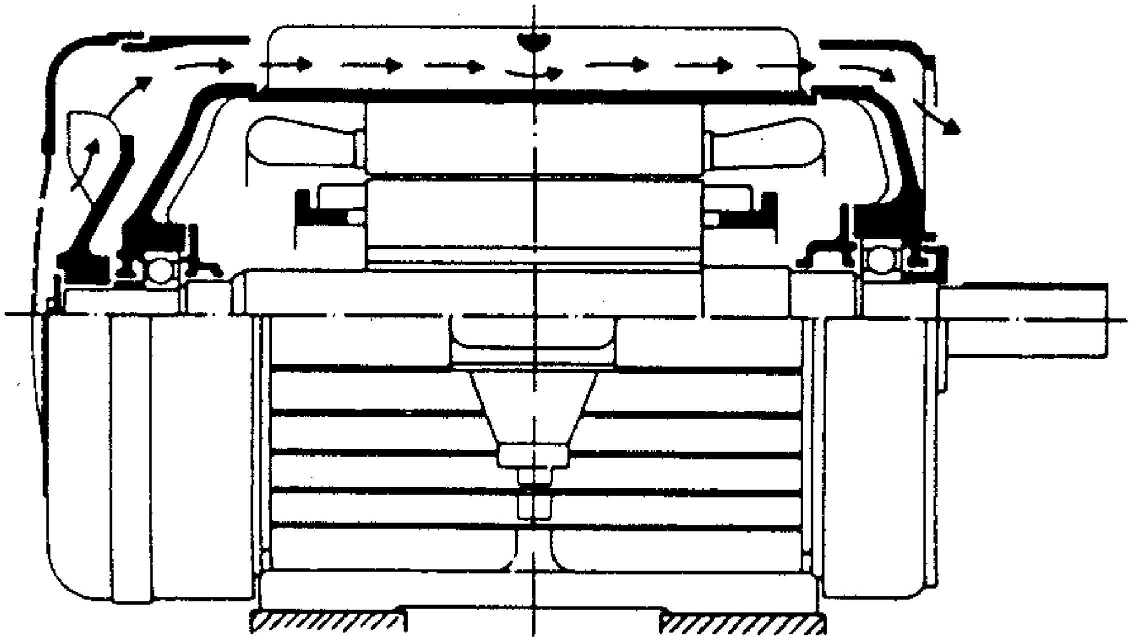
1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source: SPEED program

3. Heat transfer and cooling of electrical machines



Source: ABB, Switzerland

3. Heat transfer and cooling of electric machines

3.1 Thermal classes, cooling systems, duty types

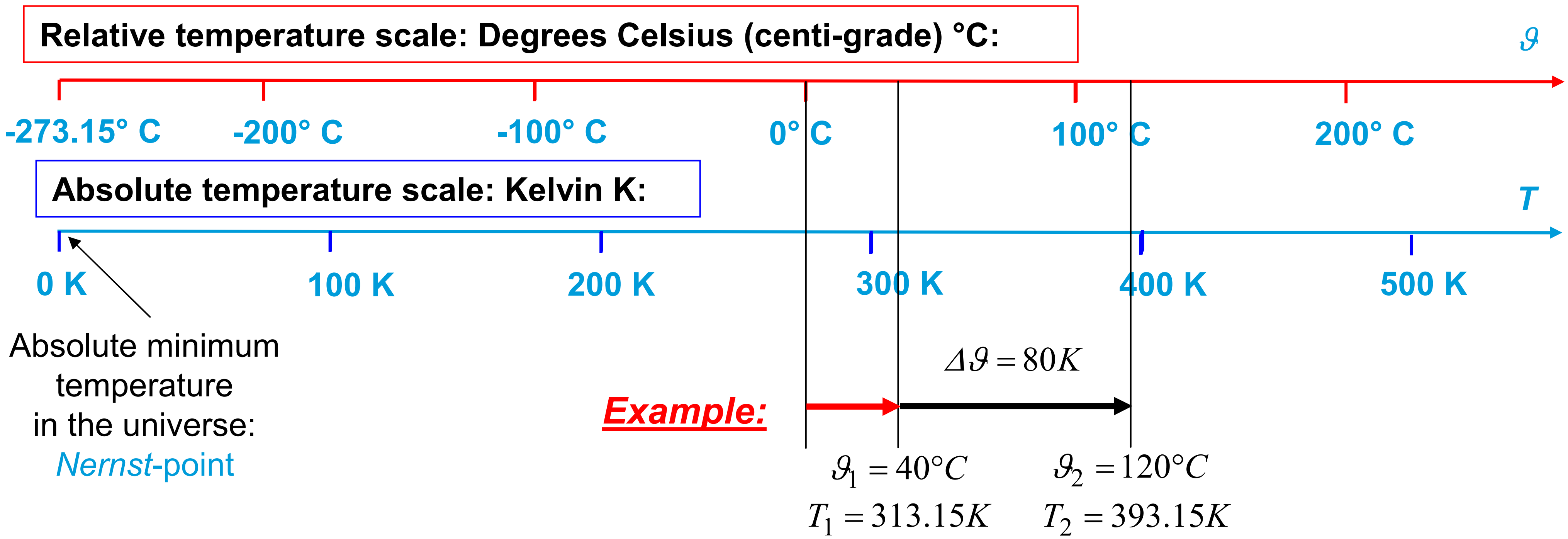
3.2 Elements for calculation of temperature rise

3.3 Heat-source plot

3.4 Thermal utilization

3.5 Simplified calculation of temperature rise

Temperature scales – Temperature rise



Temperature rise $\Delta\vartheta$ = temperature difference: $\boxed{\Delta\vartheta = \vartheta_2 - \vartheta_1 = T_2 - T_1}$

(It is measured ALSO in K!)

3. Heat transfer and cooling

Arrhenius´ law

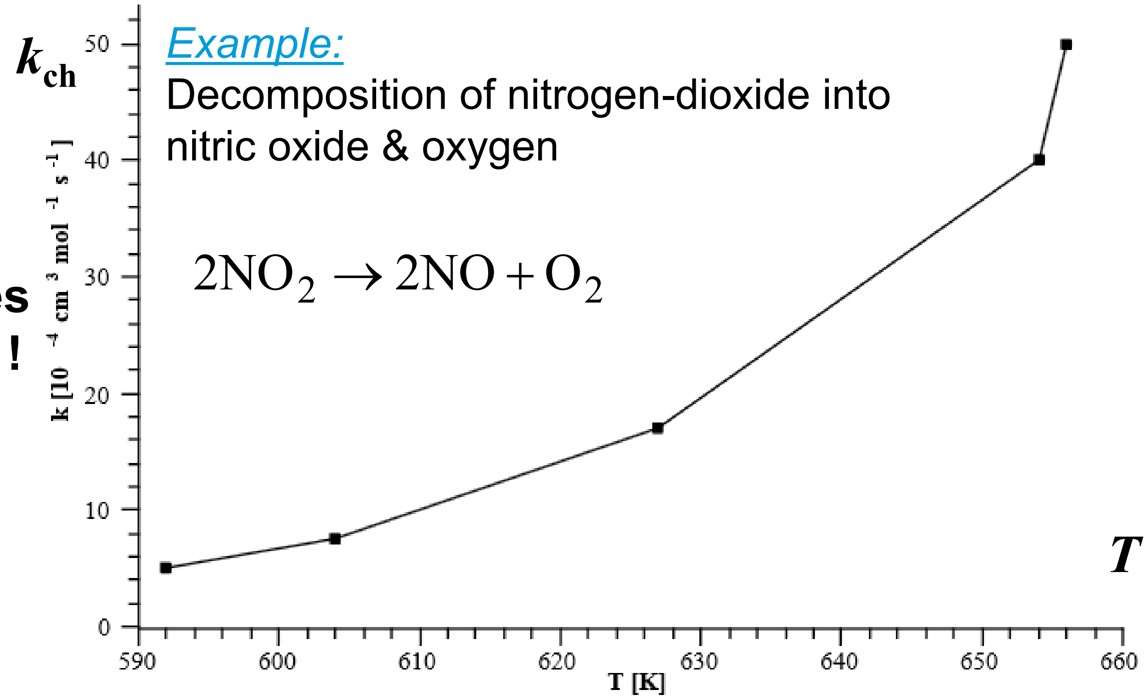
Arrhenius´ law describes the „speed“ (rate constant) k_{ch} of a chemical reaction in dependence of absolute temperature T

$$\boxed{k_{ch} \sim e^{-W_a/(k\cdot T)}}$$

W_a: Activation energy to start a chemical reaction (J)
$k = 1.38 \cdot 10^{-23}$ J/K: *Boltzmann´s* constant

Application:

Chemical degradation process of insulation materials increases exponentially with temperature !



Example:
Decomposition of nitrogen-dioxide into nitric oxide & oxygen

$$2NO_2 \rightarrow 2NO + O_2$$

Source: Wikipedia.en

3. Heat transfer and cooling

Insulation life time *L*

Experimental determination of insulation material life time *L*:

Insulation material under electrical voltage stress U is tested
e.g. with 30 specimen per temperature level T_i $(i = 1, 2, 3, \ldots)$,
until 10% of specimen (here: 3) fail due to voltage flash over
$\Rightarrow$ Elapsed time t_L for that case is **10%-life time $L_{10}(T_i)$.**

Result:

Due to *Arrhenius* law the life time L_{10} decreases exponentially
with increasing temperature T.
For a large number of tested specimen this is described by *Weibull*-distribution
as probability function.

***Montsinger´s* rule** for transformer oil and solid insulation materials:

Insulation life time L decreases by 50% (taken as average of a large number of tested specimen) with increase of temperature ϑ (or T) by $\Delta\vartheta$ = 10 K.

$$\boxed{L(\vartheta + 10K) = 0.5 \cdot L(\vartheta)}$$

3. Heat transfer and cooling

Montsinger´s rule (is based on *Arrhenius´* law)

$$\boxed{L(\vartheta) = L(\vartheta_0 + \Delta\vartheta) = L(\vartheta_0)\cdot e^{-\frac{\Delta\vartheta}{10}\cdot\ln 2}}$$

$$L(\vartheta + 10K) = 0.5\cdot L(\vartheta)$$

$$\Delta\vartheta = 10\ \mathrm{K}: \quad L(\vartheta_0 + 10\ \mathrm{K}) = L(\vartheta_0)\cdot e^{-\frac{10}{10}\cdot\ln 2} = L(\vartheta_0)\cdot e^{-\ln 2} = L(\vartheta_0)/2$$

Example:

Insulation material for Thermal Class F: $L(\vartheta = 155°C) = 100000\ \text{hours}$

$$L(\vartheta = 165°C) = 50000\ \text{hours}$$

3. Heat transfer and cooling

Thermal Classes (Insulation classes)

Electrical insulation systems for wires (used e.g. in electric machines, transformers, …) are divided into different classes by temperature and temperature rise (IEC 60085).

Thermal Class	Typical materials
130 B	Inorganic materials: e.g. mica, glass fibers, asbestos, with high-temperature binders (e.g. epoxy-resin) for 130°C
155 F	Class 130 materials with binders, stable at the higher temperature 155°C
180 H	Silicone elastomers, and Class 130 inorganic materials with high-temperature binders for 180°C
200 N	As for Class B, and including Teflon, for 200°C
220 R	Polyimide enamel (Pyre-ML) or Polyimide films (e.g. Kapton), usable at 220°C

Thermal Classes in electrical machinery

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Selected **Thermal Classes** of insulation systems according to IEC 60034-1:

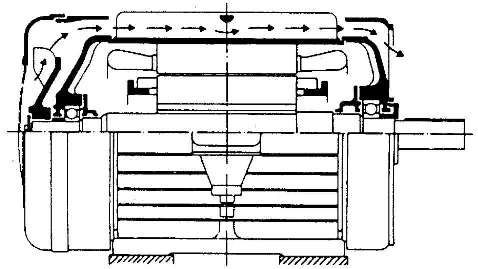
Thermal Class	B	F	H	250
Temperature limit ϑ (°C)	130	155	180	255
Maximum value of average temperature rise $\Delta\vartheta$ (K) (above ambient 40°C)	80	105 ($P_N \leq 5$ MW) 100 ($P_N > 5$ MW)	125	200

An ambient air temperature of 40°C must be assumed, which is also to be assumed the coolant inlet temperature in air-cooled components.

At elevated level above sea-level (N.N.) above 1000 m the admissible temperature rise $\Delta\vartheta$ must be reduced due the reduced mass density of air, which causes lower cooling capability.

3. Heat transfer and cooling

Thermal classes



Source: ABB, Switzerland

Example:

Thermal Class B: 40°C + 80 K + 10 K = 130°C

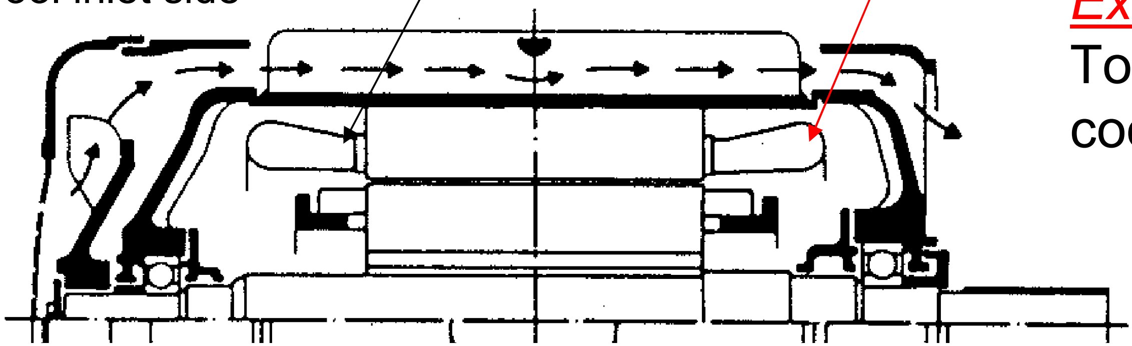
Thermal Class F: 40°C + 105 K + 10 K = 155°C (rated power ≤ 5 MW)

Thermal Class H: 40°C + 125 K + 15 K = 180°C

Ambient temperature average temperature rise add. hot spot temperature rise temperature limit

Cool inlet side

Hot outlet side



Example:

Totally enclosed fan cooled motor (TEFC)

3. Heat transfer and cooling

Different principles of cooling

Open ventilation	*Totally enclosed machines – surface cooling*	*Totally enclosed machines with heat exchanger*	*Hollow conductor cooling (H_2, de-ionized water)*
Coolant air	Coolant air or water jacket	Coolant air, Heat exchanger: Air-air or air-water	Coolant hydrogen gas, oil or de-ionized water
End shields of machine are open for coolant flow	Increase of machine surface by fins or tubes for air; Water jacket cooling	Coolant flow is directed through machine and heat exchanger in closed loop	Pump presses coolant through hollow conductors
Usually up to 500 kW, at higher power acoustic noise is too big	**Usually up to 2000 kW**	**Up to 400 MW ("Top air" turbo generators: hollow conductors)**	**Up to biggest machine power (2000 MW)**
Often shaft mounted fan	Often shaft mounted fan	Shaft mounted fans, external fans	External pump

Air cooled machines - coolant is air flow

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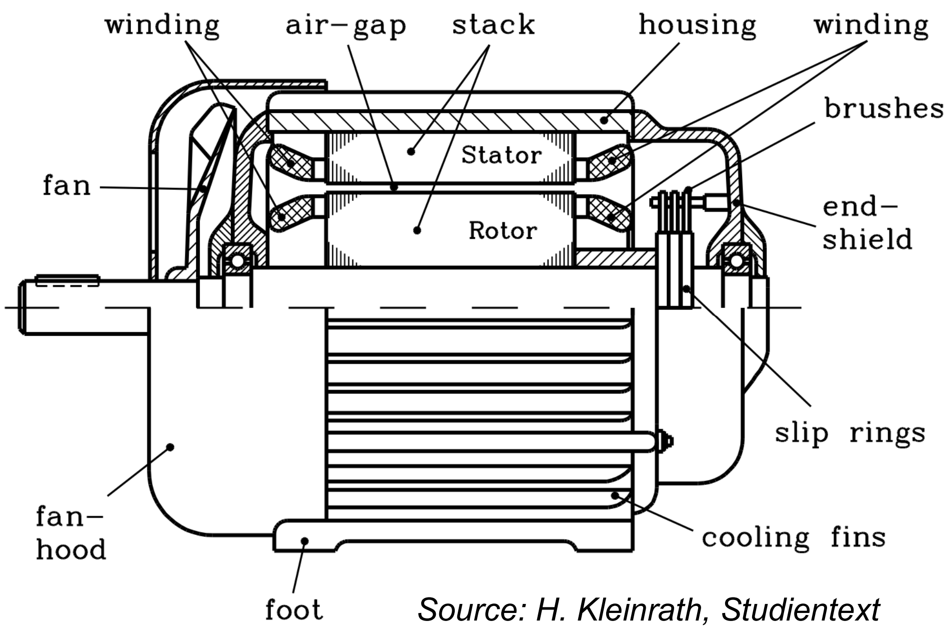
No fan	*Shaft mounted fan*	*Externally driven fan*
Cooling only due to natural convection and heat radiation	Speed dependent air flow for cooling	Air flow independent of motor speed
Used for small machines (< 1 kW), e.g. permanent magnet machines due to their lower losses	Used for constant speed drives Big machine power possible	Used for variable speed drives Big machine power possible



Source: H. Kleinrath, Studientext

IC 41: Shaft mounted fan, fan hood for guiding air flow with air inlet opening,

totally enclosed slip-ring induction machine, cooling fins on cooling surface

3. Heat transfer and cooling

International Cooling IC

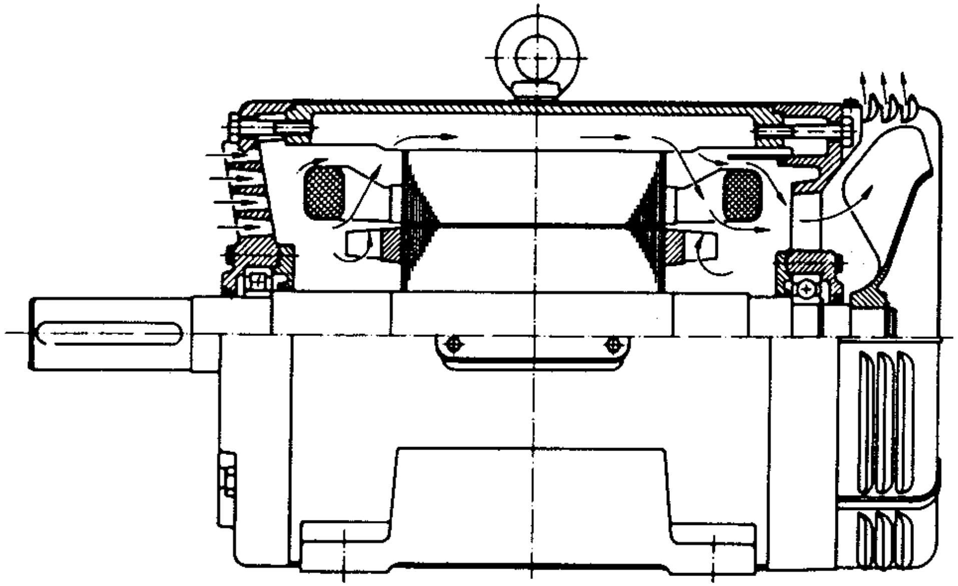
- Type of cooling of machines is abbreviated by code IC (International Cooling) according to IEC 60034-6.
- First number: Kind of coolant flow,
  Second number: How coolant is propelled.

- *Example:*

IC 41:	"4": surface cooling,	"1": Shaft mounted fan
IC 05:	"0": Open ventilation,	"5": Externally driven fan, built within the machine
IC 06:	"0": Open ventilation,	"6": Externally driven fan, mounted on the machine
IC 86:	„8": Heat exchanger on motor,	„6": As above

- Strong connection between type of cooling (IC) and mechanical degree of protection (IP) due to design of construction:
  Typical combinations are: IC06 – IP23,
  IC41 – IP44

3. Heat transfer and cooling
IC 01: Cage induction machine

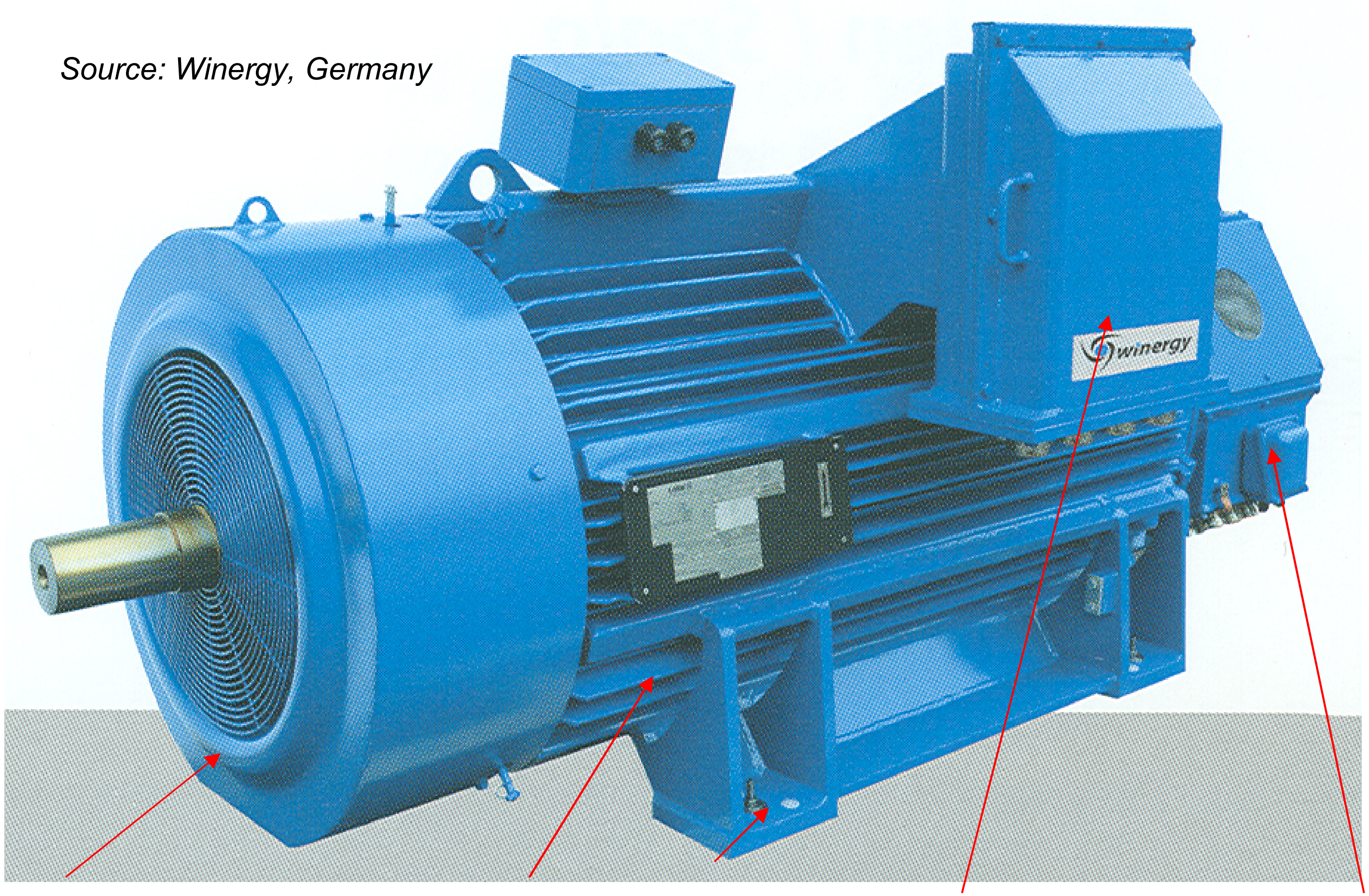


Source:
H.-O. Seinsch, Teubner-Verlag

- Shaft mounted fan, open ventilation,
- End shields with openings for coolant flow,
- Fan hood for guiding coolant flow with openings for air outlet,
- Additional small fan blades on cage rings for rotor cooling air flow

3. Heat transfer and cooling

Totally enclosed doubly-fed induction wind generator



Fan hood
Shaft mounted fan inside

Cooling fins

Feet

Power terminal box

Slip ring
terminal box

Air-cooled with iron-cast cooling fin housing

IC41

600 kW at 1155/min

3. Heat transfer and cooling

Doubly-fed induction wind generator

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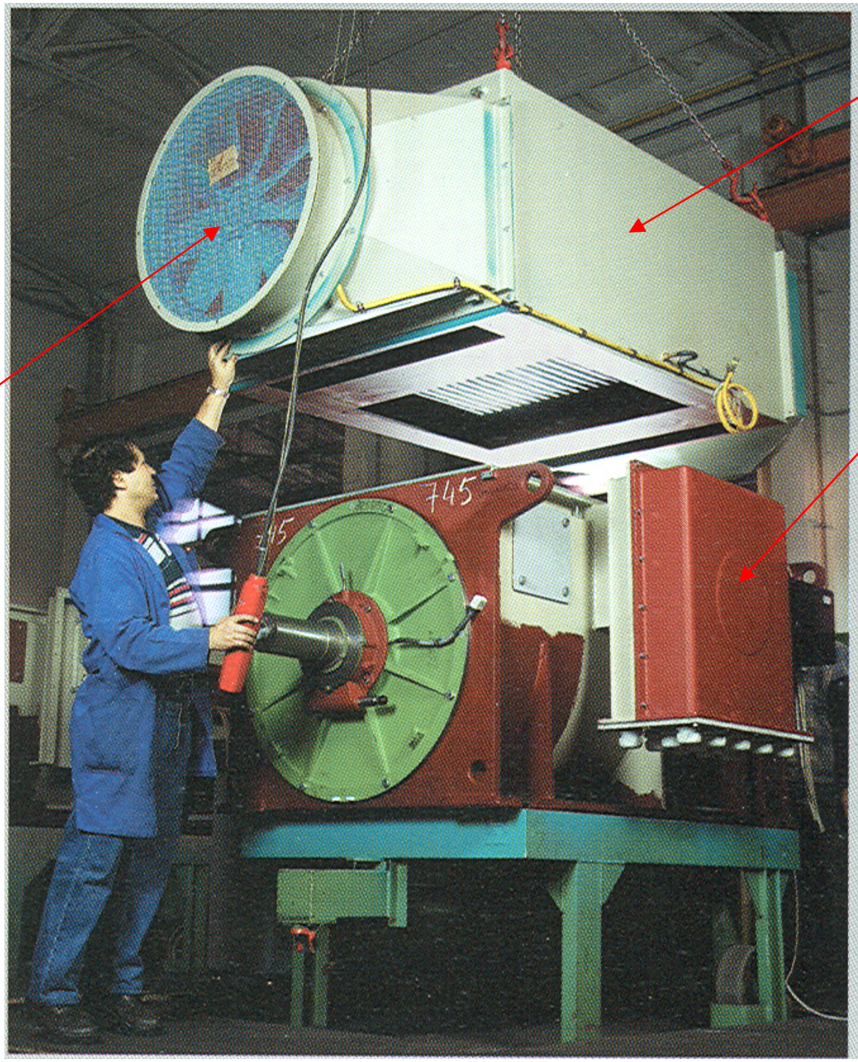
Mounting of air-air heat exchanger on slip ring induction wind generator



Air-air heat exchanger

Generator terminal box

Externally driven air inlet fan

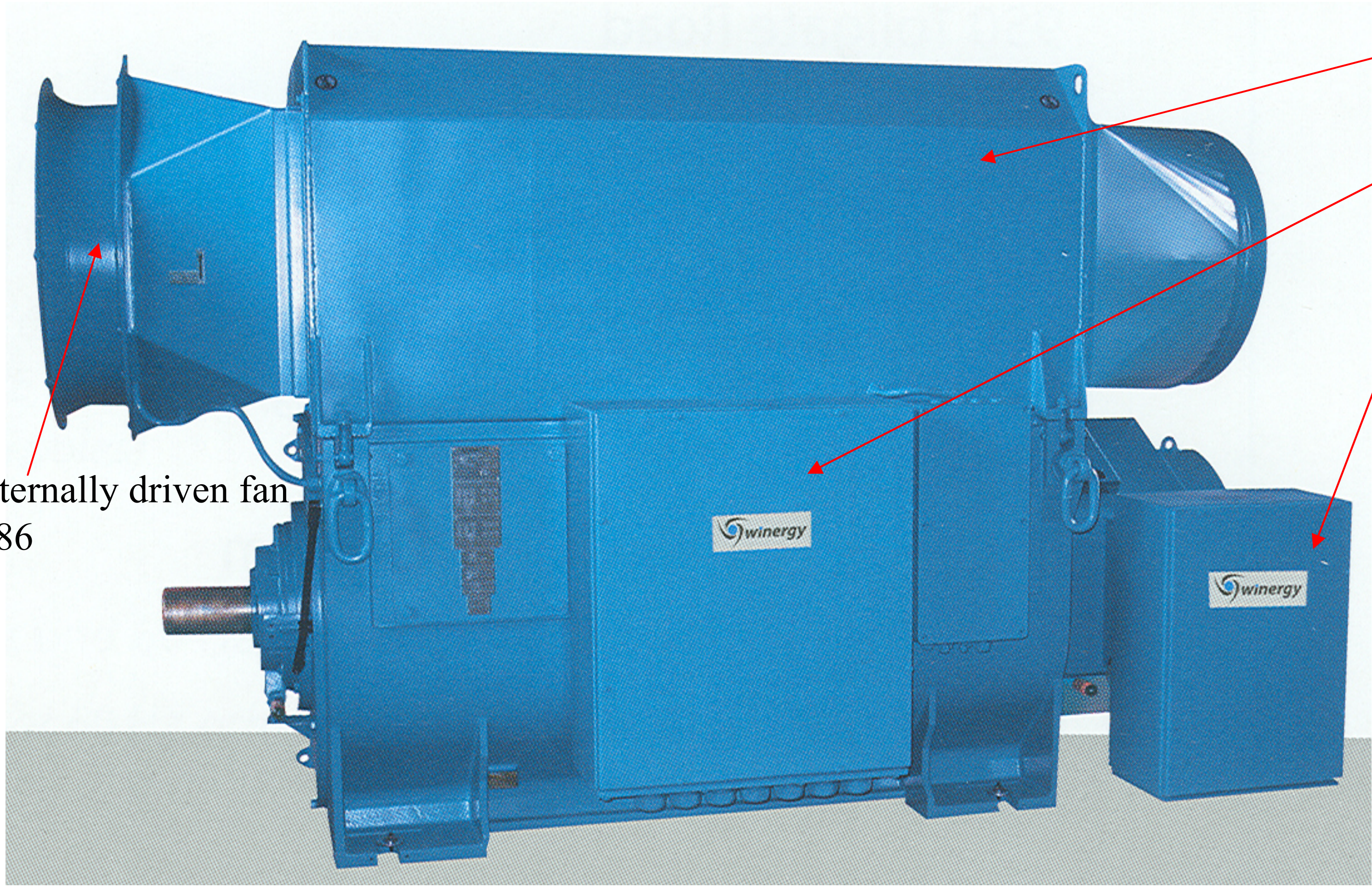
IC86

Rated electrical output power and rated speed: 1500 kW at 1800/min

Source: Winergy Germany

3. Heat transfer and cooling

Doubly-fed induction generator for wind power generation



Air-air heat exchanger

Stator winding terminal box

Rotor winding terminal box

Externally driven fan IC86

Generator data:

4 poles

2000 kW, 1800/min, 50 Hz,

slip range: -/+ 20%

Source: Winergy, Germany

3. Heat transfer and cooling

4- DC motor with commutation poles (without compensation winding)

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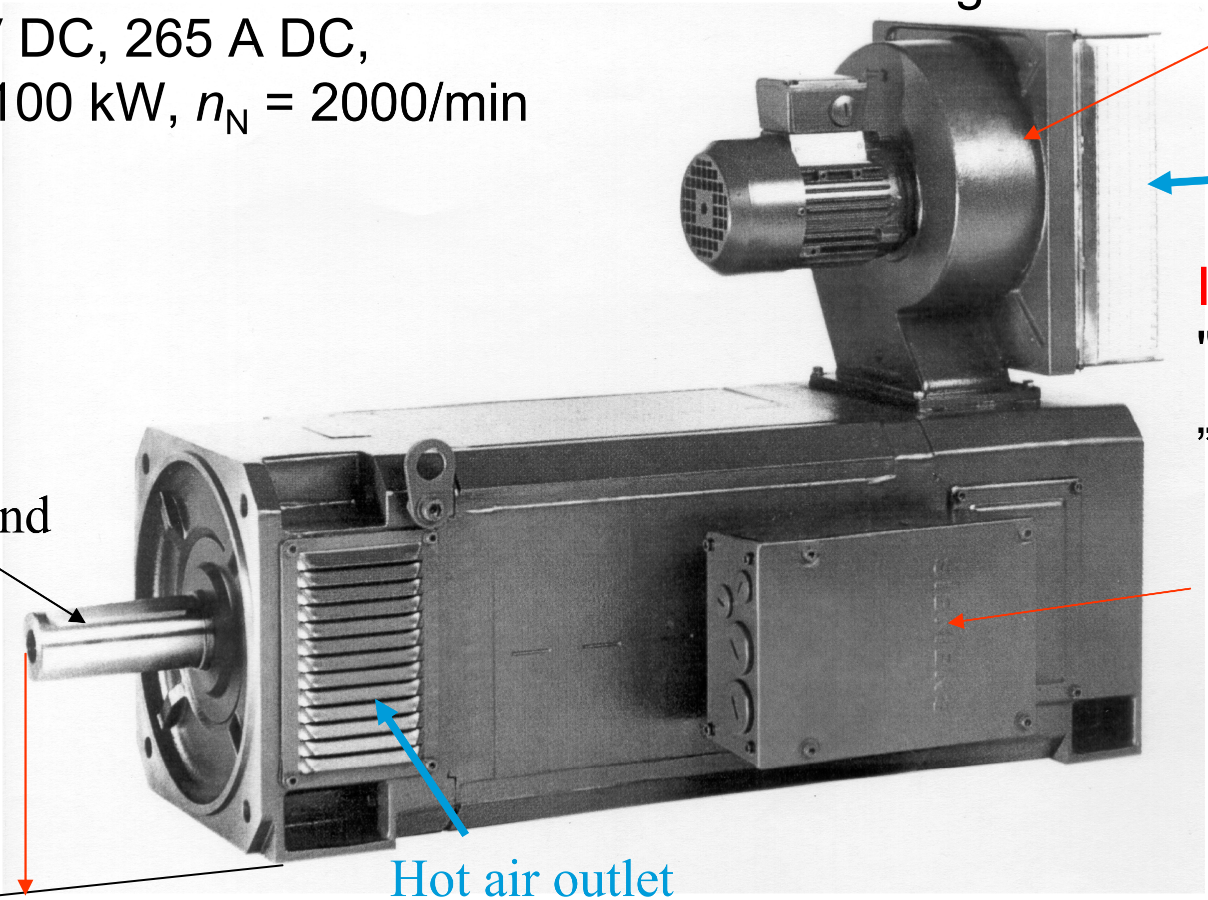
Main motor data:

400 V DC, 265 A DC,

P_N = 100 kW, n_N = 2000/min

Radial fan, driven by extra 2-pole cage induction motor (IC41)



Frame size ("shaft height") 160 mm

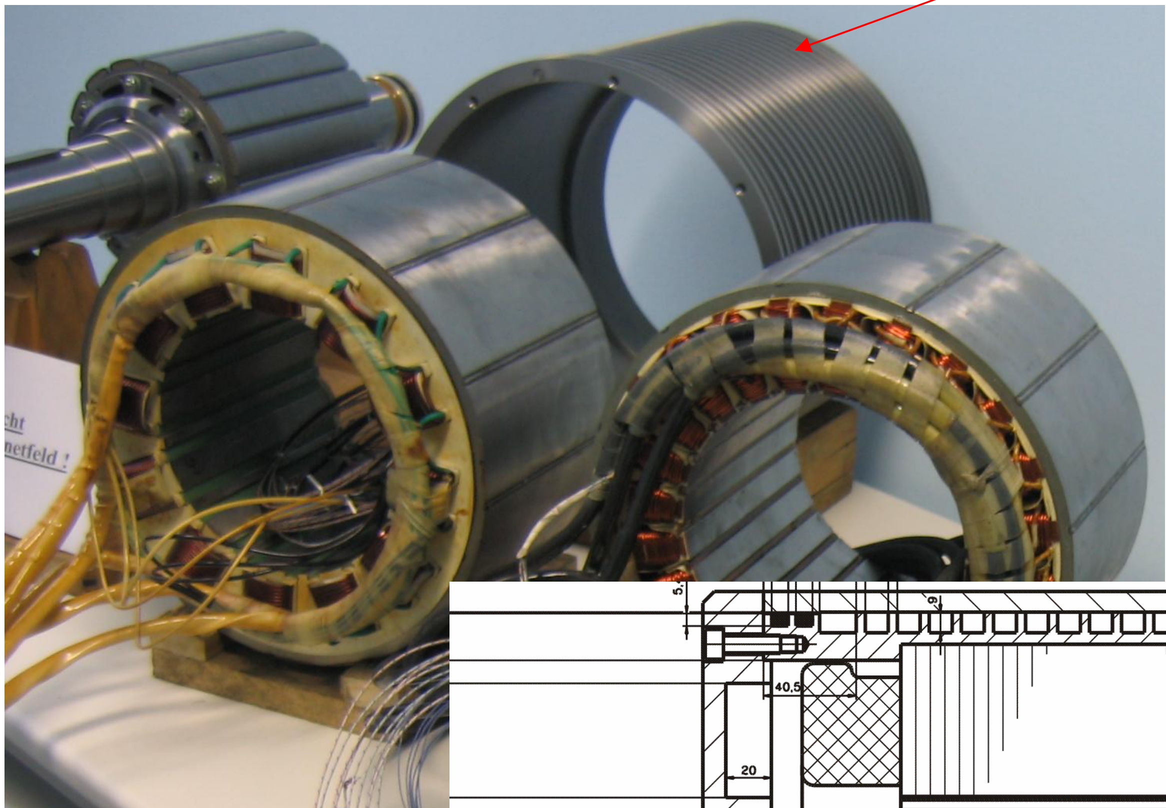
IC 06:
"0": Open ventilation,
„6": external fan, mounted on the machine

Source: Siemens AG

3. Heat transfer and cooling

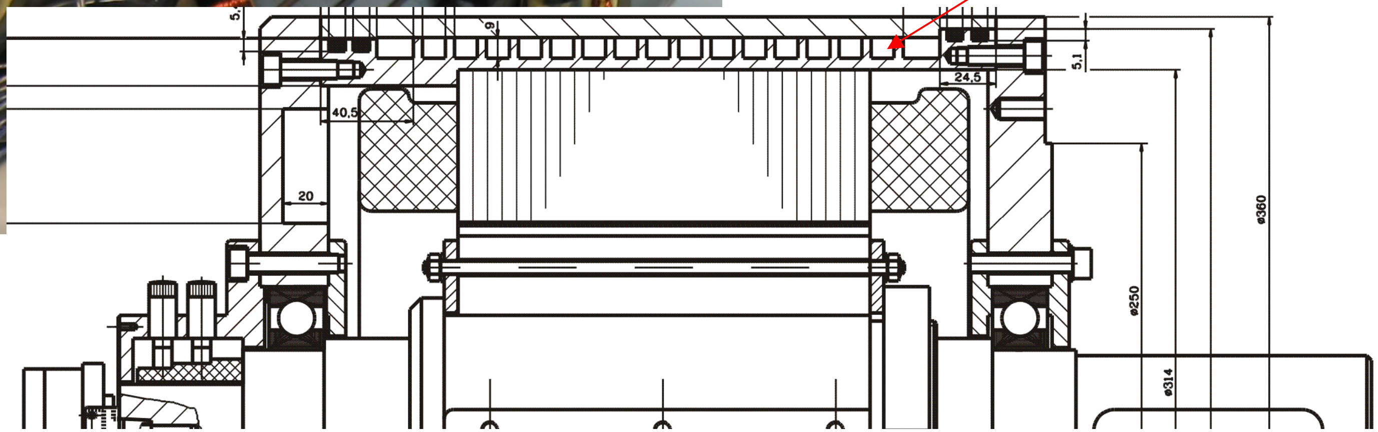
Water jacket cooling

spiral water channel



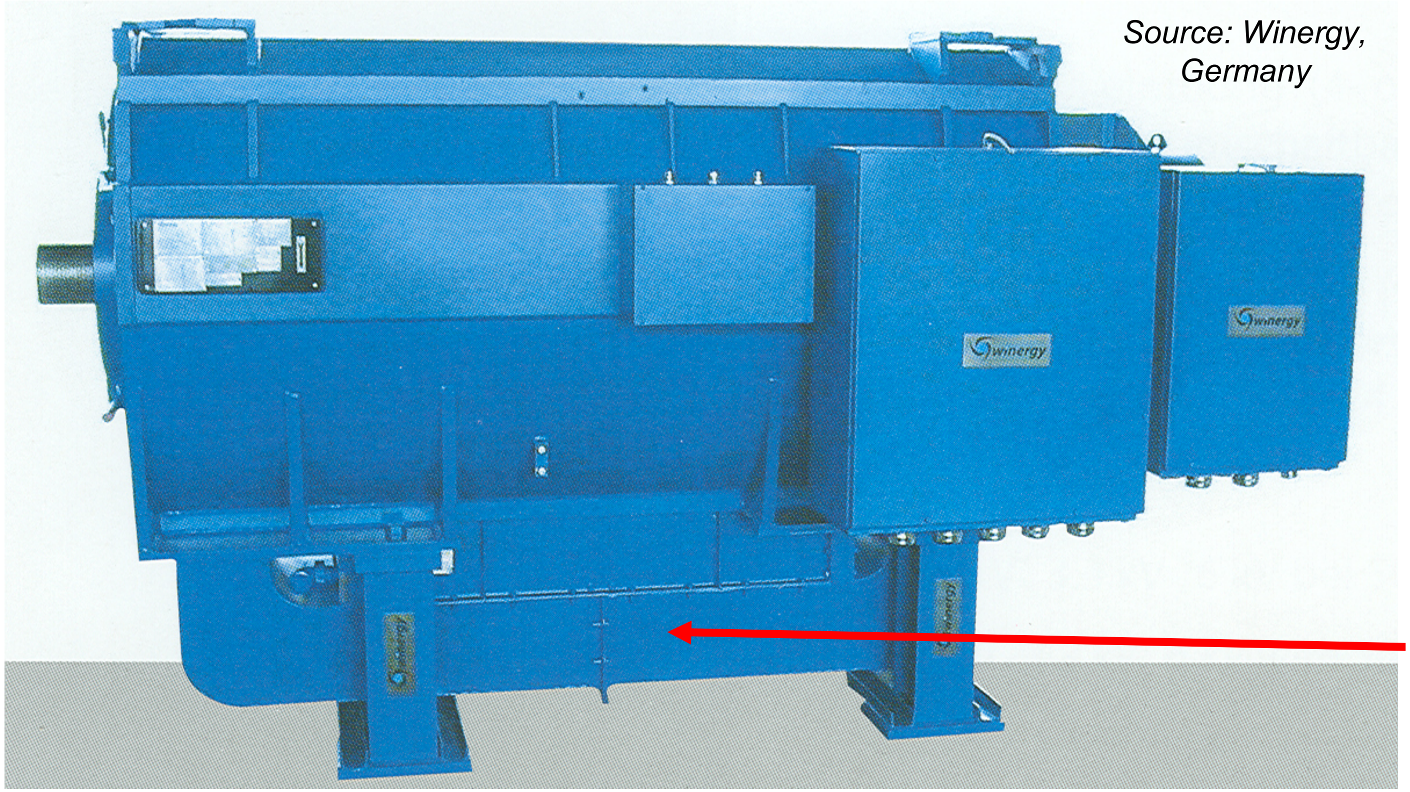
- Spiral water channel in the stator housing
- Closed motor surface
- Good for heat transfer of stator losses, hence good for PM synchronous machines due to small rotor losses

spiral water channel



3. Heat transfer and cooling

Doubly-fed induction generator for wind power generation



Main generator data:

2750 kW at 1100/min

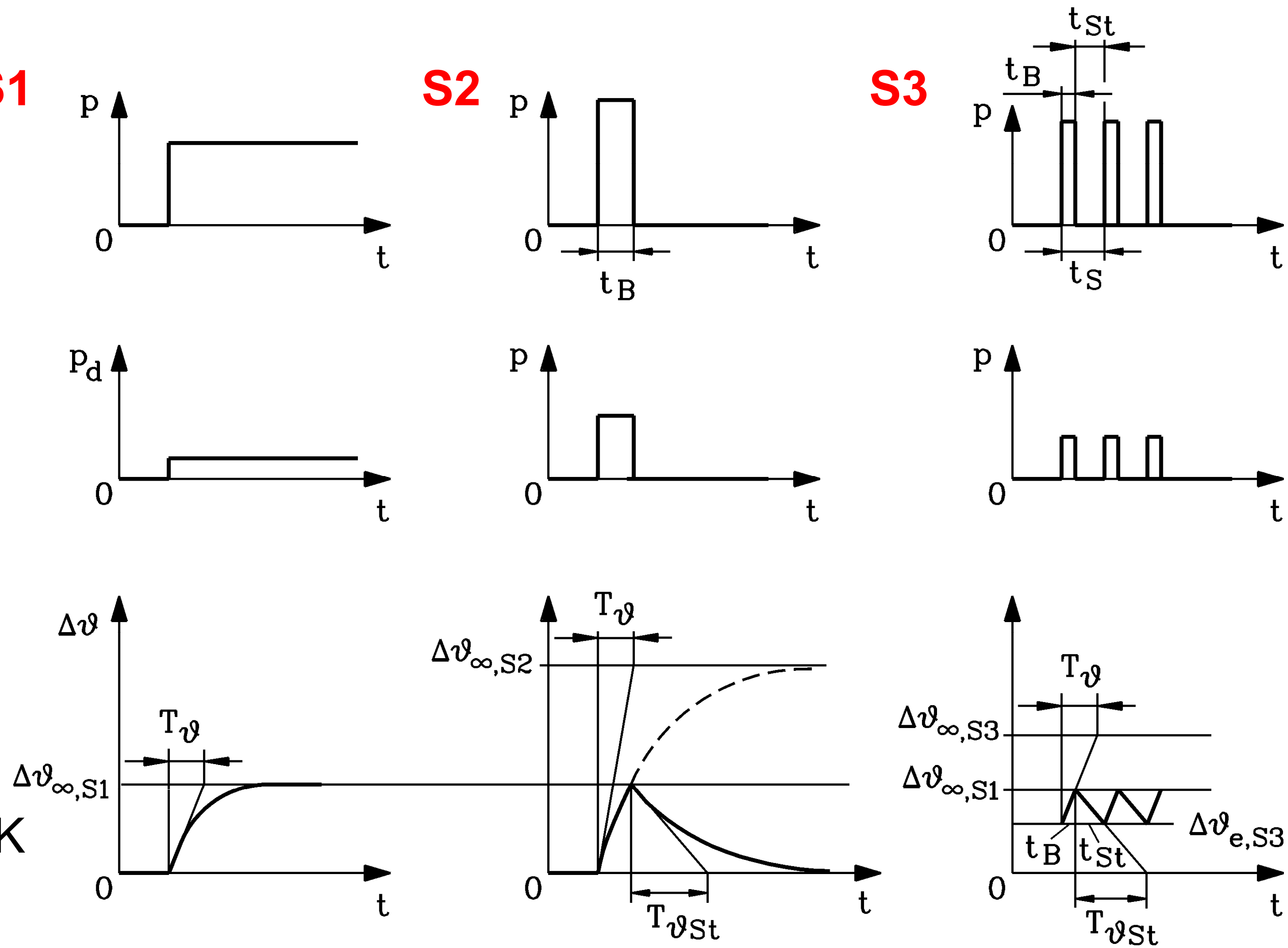
Stator winding:
Water jacket cooling

Rotor winding:
Internal air circuit:
Air-water heat exchanger beneath necessary!

- For induction machines a stator surface cooling is only sufficient up to ca. 100 kW.
- At bigger machines the increased rotor losses are not any longer cooled sufficiently.
- For bigger machines an inner circulating air flow from rotor to stator is needed.

3. Heat transfer and cooling

Duty types S1, S2, S3, ..., S10 (IEC 60034-1)



Example:

Th. Cl. F:

$\Delta\vartheta_{\infty,S1} = 105\text{K}$

Summary:
Thermal classes, cooling systems, duty types

- Standardized Thermal Classes limit the maximum insulation temperature
- MONTSINGER`s rule: Critical life-time reduction at too high temperatures
- Mostly air-cooling with standardized Cooling Classes
- Only large machines with direct air, hydrogen or water cooling
- Ten standardized Duty Classes, determined by the heating of the winding

3. Heat transfer and cooling of electric machines

3.1 Thermal classes, cooling systems, duty types

3.2 Elements for calculation of temperature rise

3.3 Heat-source plot

3.4 Thermal utilization

3.5 Simplified calculation of temperature rise

3. Heat transfer and cooling

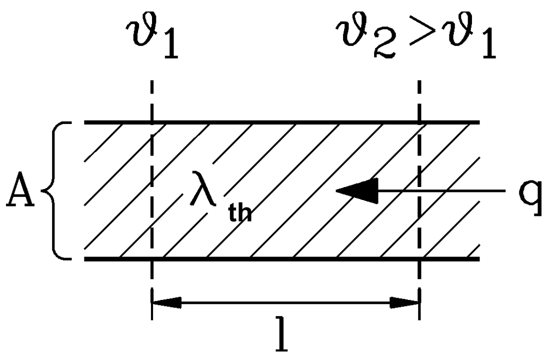
Conduction of heat

Heat resistance: $u = R \cdot i \quad \Rightarrow \quad \Delta\vartheta = R_{th} \cdot P_{th}$

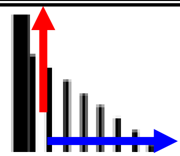
Electric current density *J* corresponds with **heat flow density** $q = P_{th}/A \;\; [\mathrm{W/m^2}]$

Conduction of heat: ***Fourier´s law***

$$\boxed{\frac{P_{th}}{A} = \lambda_{th} \cdot (\vartheta_2 - \vartheta_1)/l}$$



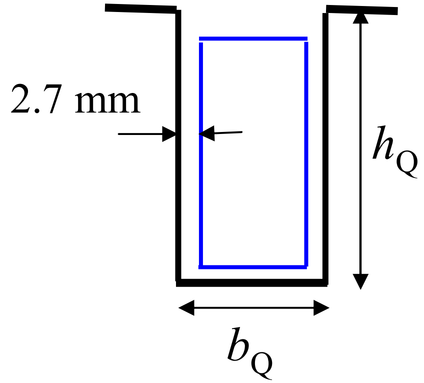
$$\boxed{R_{th} = \frac{l}{\lambda_{th} A}}$$

Material	*Thermal conductivity* λ_{th} W/(m·K)
Air at 20° / 50° / 100°C, 1 bar	0.024 / 0.028 / 0.031
Copper	380
Iron	80
Iron stack (laminated): In direction of lamination Perpendicular to lamination	20 ... 60 0.5 ... 1.2 
Insulation material	0.2
Epoxy resin	1

Heat conduction via slot insulation

Example:

550 kW-cage induction machine,
6.6 kV, 60 open stator slots,
slot height $h_Q = 69$ mm,
slot width $b_Q = 12.5$ mm,
insulation thickness $d = 2.7$ mm, stack length: $l_{Fe} = 380$ mm



Slot surface:

$$A = (2 \cdot h_Q + b_Q) \cdot l_{Fe} = (2 \cdot 69 + 12.5) \cdot 380 = 57190\,\text{mm}^2$$

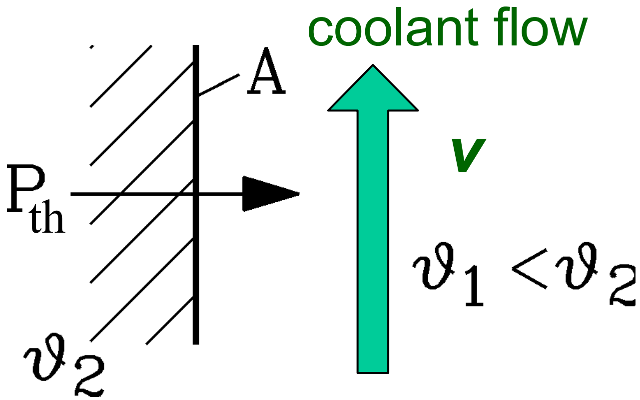
Thermal conductivity resistance from copper to iron:

$$R_{th} = \frac{d}{\lambda_{th} A} = \frac{0.0027}{0.2 \cdot 0.05719} = \underline{\underline{0.236}}\,\text{K/W}$$

With 50 W losses in the slot conductor we get a temperature rise at the insulation of $\Delta\vartheta$ = 0.236·50 = 11.8 K

Convection of heat

Heat transfer coefficient α describes the cooling effect of flowing ("convection") coolant, passing by a cooling surface A with the velocity v



$$\boxed{\frac{P_{th}}{A} = \alpha \cdot \Delta\vartheta}$$

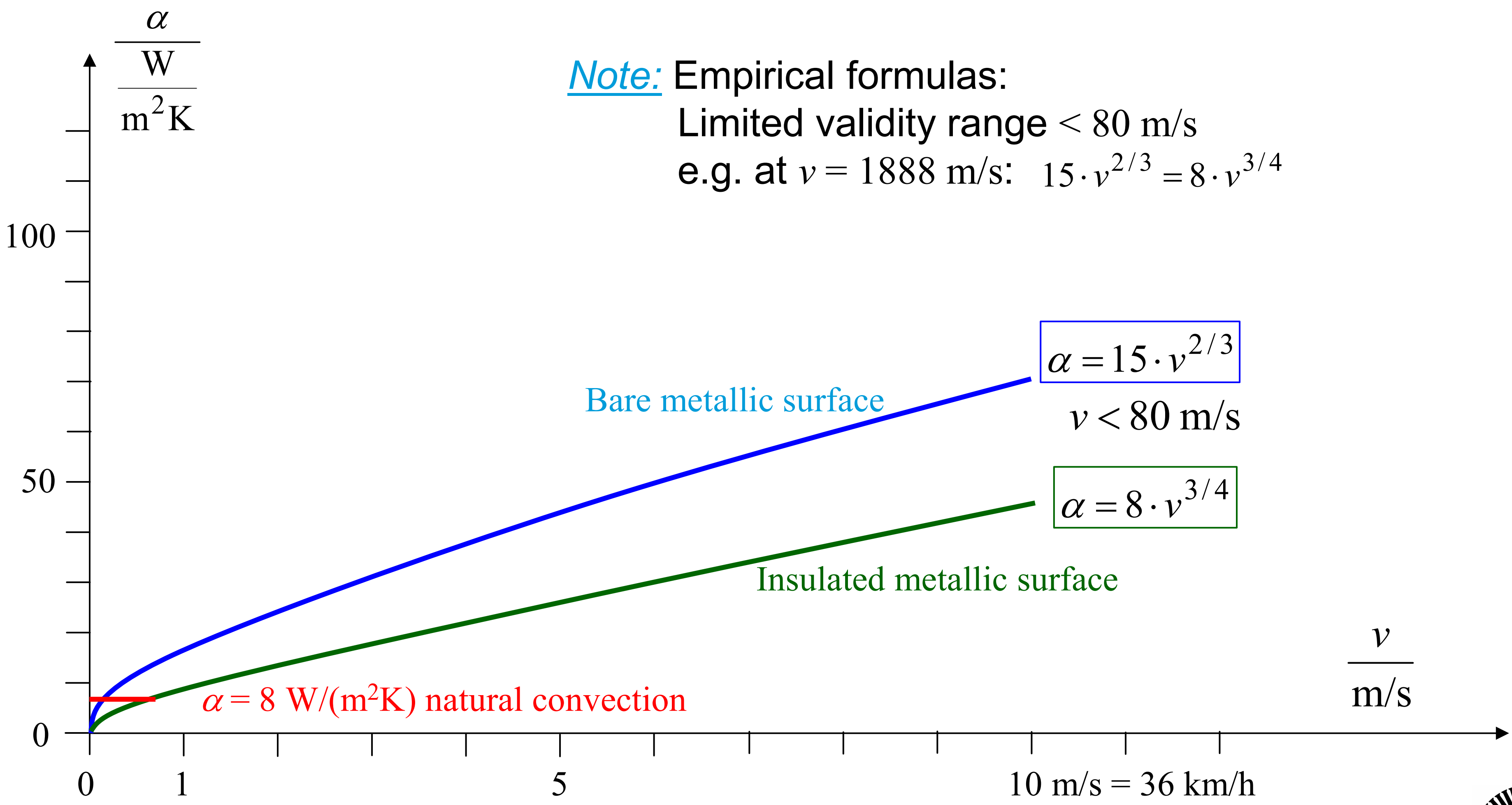
Newton´s law

$$\boxed{R_{th} = \frac{1}{\alpha \cdot A}}$$

Coolant "air" vs. Surface	α **in W/(m²K),** ***v*** **in m/s**
Nearly not moving air (v = 0 ... 0.5 m/s)	8
Moved air, bare metallic hot surface	$\alpha = 15 \cdot v^{2/3}$
Moved air, insulated winding	$\alpha = 8 \cdot v^{3/4}$

$v < 80$ m/s

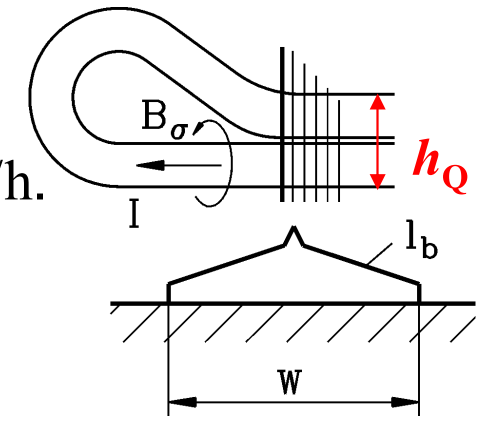
Heat transfer coefficient α



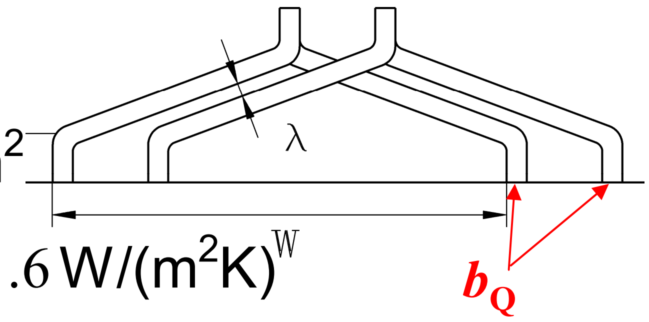
Note: Empirical formulas:
Limited validity range < 80 m/s
e.g. at $v = 1888$ m/s: $15 \cdot v^{2/3} = 8 \cdot v^{3/4}$

3. Heat transfer and cooling

Example: Heat convection at winding overhangs



550 kW-cage induction machine, 6.6 kV,
open ventilated machine, double-layer winding,
velocity of air flow in winding overhang $v = 12\ \mathrm{m/s} = 43\ \mathrm{km/h}$.
Coil height = half slot height $h_Q/2 = 69/2 = 34.5\ \mathrm{mm}$,
coil breadth = slot width $b_Q = 12.5\ \mathrm{mm}$,
length of winding overhang $l_b = 614.8\ \mathrm{mm}$



Surface of insulated stator coil in winding overhang:

$$A = 2 \cdot (h_Q/2 + b_Q) \cdot l_b = (69 + 2 \cdot 12.5) \cdot 614.8 = 57791\,\mathrm{mm}^2$$

Moved air, insulated winding: $\alpha = 8v^{3/4} = 8 \cdot 12^{3/4} = 51.6\,\mathrm{W/(m^2K)}$

$$R_{th} = \frac{1}{\alpha A} = \frac{1}{51.6 \cdot 0.057791} = \underline{\underline{0.335}}\,\mathrm{K/W}$$

With 85 W losses in one layer of winding overhang we get a temperature rise of $\Delta\vartheta$ = 0.335·85 = 28.5 K

3. Heat transfer and cooling

Radiation of heat

Heat radiation does not need any medium to transport heat:

- Transferred heat P_{th} from hot (T_2) to cold ($T_1 < T_2$) surface A
- T_1 , T_2 are absolute temperatures, measured in K
- **Heat radiation law** of *Stefan* and *Boltzmann*:

$$\boxed{\frac{P_{th}}{A} = c_s \cdot (T_2^4 - T_1^4)}$$

Example:

- Radiated losses: "black body": $c_s = 5.7 \cdot 10^{-8}$ W/(m²K⁴), "grey body": $c_s = 5 \cdot 10^{-8}$ W/(m²K⁴)
- Temperature difference: $\Delta\vartheta = 80$ K,
- Ambient temperature 20 °C, $T_1 = 20 + 273.15 = 293.15$ K

  $T_2 = T_1 + \Delta\vartheta = 293.15 + 80 = 373.15$ K
- Heat flow density:

$$q = \frac{P_{th}}{A} = c_s \cdot (T_2^4 - T_1^4) = 5 \cdot 10^{-8} \cdot (373.15^4 - 293.15^4) = \underline{\underline{600.1}}\,\mathrm{W/m^2}$$

- How big is an **equivalent heat transfer coefficient** α_e for convective heat transfer ?

$$\alpha_e = \frac{P_{th}}{A \cdot \Delta\vartheta} = \frac{q}{\Delta\vartheta} = \frac{600.1}{80} = \underline{\underline{7.5}}\,\mathrm{W/(m^2K)} \Rightarrow \text{low value, similar to natural convection!}$$

3. Heat transfer and cooling

Significance or radiation for cooling electric machines

- As the surface temperature in electric machines must be low (< 60°C to avoid skin damage at touching), the contribution of radiation to total cooling is small

- Only in machines with bad cooling (= natural convection as cooling), radiation may help significantly, especially with black painted surfaces:

$$\alpha_{Natural\ convection} + \alpha_{Radiation} = 8 + 7.5 = 15.5 \frac{\mathrm{W}}{\mathrm{m^2 K}}$$

Application: Inverter-fed PM synchronous machines as servo motors for tooling machines or robot drives. No fan = no fault can occur to the cooling system = robust cooling system, BUT: low heat transfer coefficient, so motor must be over-sized.

Example:
Two small PM servo drives, painted in black (also "infrared black" for good radiation effect)

Source: LTi-Drives, Lahnau, Germany

3. Heat transfer and cooling

Storage of heat energy = “heating up”

Storage of heat energy: $\boxed{mc \cdot \frac{d\Delta\vartheta}{dt} = P_{th} \quad \Leftrightarrow \quad C \cdot \frac{du}{dt} = i}$ *Equivalent electric circuit: Capacitor!*

Material	***Specific heat capacity c*** **Ws/(kg·K)**	***Mass density*** γ **kg/m³**
Air (at constant pressure)	**1009**	**1.226 (at 25 °C)**
Copper	**388.5**	**8900**
Iron	**502**	**7850**
Epoxy resin	**1320 ... 1450**	**1500**

Example:

Stored heat in volume V = 1 dm³ of a) air, b) copper, c) iron, heated up from 20°C to 100°C: $W_{th} = \gamma \cdot V \cdot c \cdot \Delta\vartheta$ $\quad \Delta\vartheta = 100 - 20 = 80\,\text{K}$

a) Air: $W_{th} = 1.226 \cdot 10^{-3} \cdot 1009 \cdot 80 = \underline{\underline{99}}\,\text{J}$

b) Copper: $W_{th} = 8900 \cdot 10^{-3} \cdot 388.5 \cdot 80 = \underline{\underline{276.6}}\,\text{kJ}$ (2766-times of air!)

c) Iron: $W_{th} = 7850 \cdot 10^{-3} \cdot 502 \cdot 80 = \underline{\underline{315.3}}\,\text{kJ}$ (3153-times of air!)

Summary:
Elements for calculation of temperature rise

- Three ways to dissipate heat: conduction, convection, radiation
- Cooling systems operate mostly with convection
- Internal heat flow governed by heat conduction
- Radiation of heat small; only for self-cooled machines of importance
- Heat storage for thermal transients decisive
  due to rather long thermal time constants T_ϑ

3. Heat transfer and cooling of electric machines

3.1 Thermal classes, cooling systems, duty types

3.2 Elements for calculation of temperature rise

3.3 Heat-source plot

3.4 Thermal utilization

3.5 Simplified calculation of temperature rise

3. Heat transfer and cooling

Methods for calculating of temperature rise

a) Numerical simulation: e. g. Finite Elements:
Solution of the partial differential equations of heat generation and conduction

b) Heat source plot: Lumped elements of heat flow – simplified modeling of geometry

Analogy: Electrical network:

current i	$\Rightarrow$	heat power P_{th}
potential difference u	$\Rightarrow$	temperature difference $\Delta\vartheta$
resistance R	$\Rightarrow$	thermal resistance R_{th}
capacitance C	$\Rightarrow$	thermal capacitance $m \cdot c$
$u = R \cdot i$	$\Rightarrow$	$\Delta\vartheta = R_{th} \cdot P_{th}$
$i = C \cdot (du/dt)$	$\Rightarrow$	$P_{th} = m \cdot c \cdot (d\Delta\vartheta / dt)$

3. Heat transfer and cooling

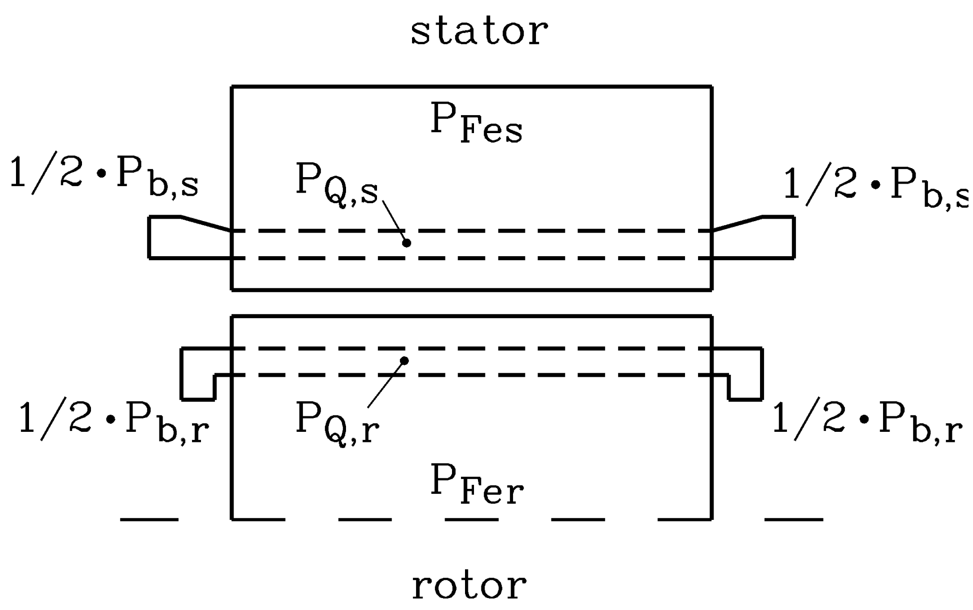
“Strategy” of heat source plot

1. Determination of losses (heat sources)
2. Feeding of sources into the heat source plot (heat storage, thermal resistances)
3. Determination of temperature differences (temperature rise) at the „nodes“

Transient heating:
Solution of coupled differential equations due to heat storage effect (heating up, cooling down)

Stationary temperature rise:
Solution of algebraic equation system, no storage effects (steady state operation)

Heat-source distribution in an AC machine



s: stator

r: rotor

Cross-section of induction machine with

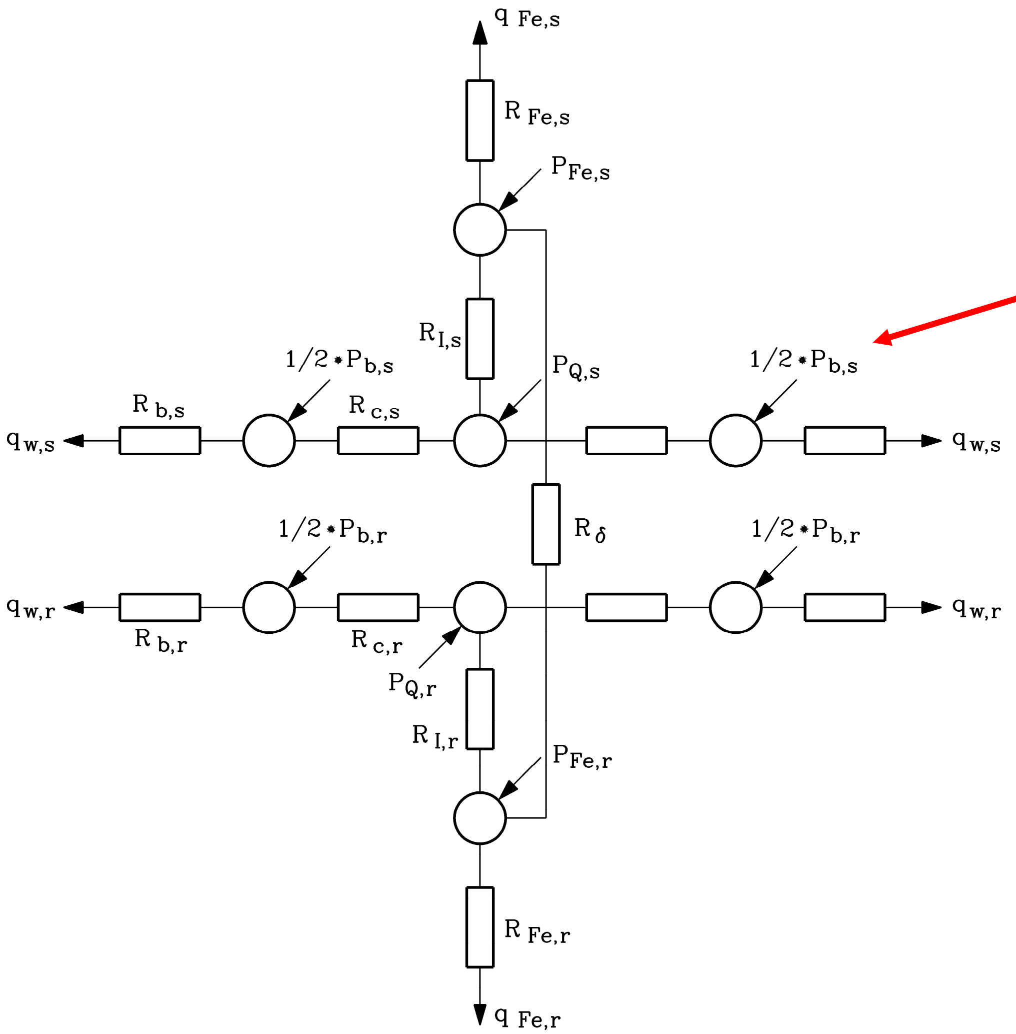
- copper losses in slot conductors of stator and rotor $P_{Q,s}$, $P_{Q,r}$,

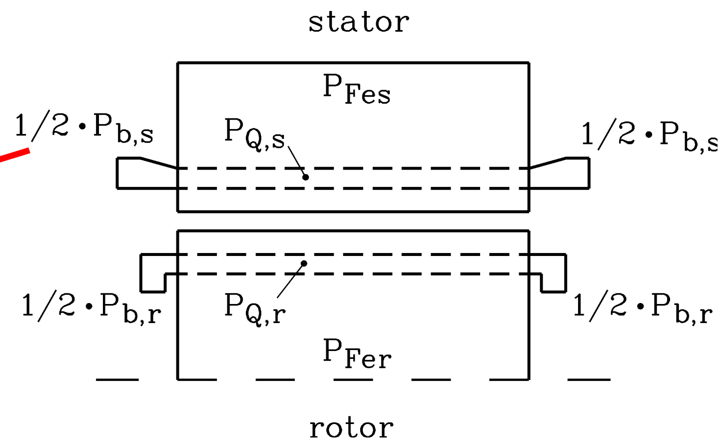
- copper losses in winding overhangs $P_{b,s}$, $P_{b,r}$

- iron losses in stator and rotor iron stack $P_{Fe,s}$, $P_{Fe,r}$

3. Heat transfer and cooling

Heat source plot of the cross-section of induction machine





Heat conductance:

in winding R_c from slot to overhang,

from copper to iron via slot insulation R_I

Heat convection:

from overhang to air R_b

from iron to air R_{Fe}

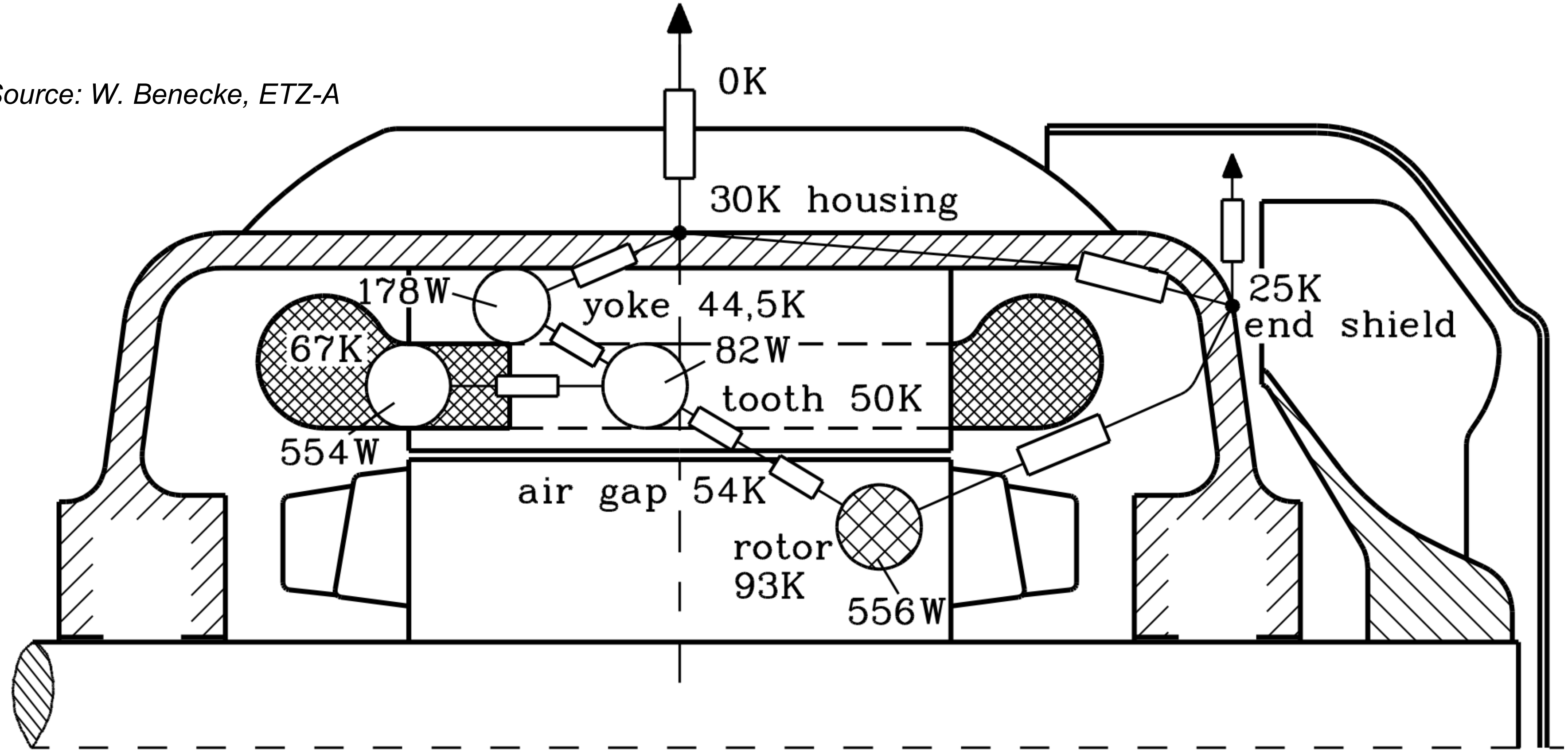
Heat conductance & convection:

from rotor to stator iron via air gap R_δ

3. Heat transfer and cooling

Heat source plot for 11 kW, 4-pole cage induction motor (1)

Source: W. Benecke, ETZ-A

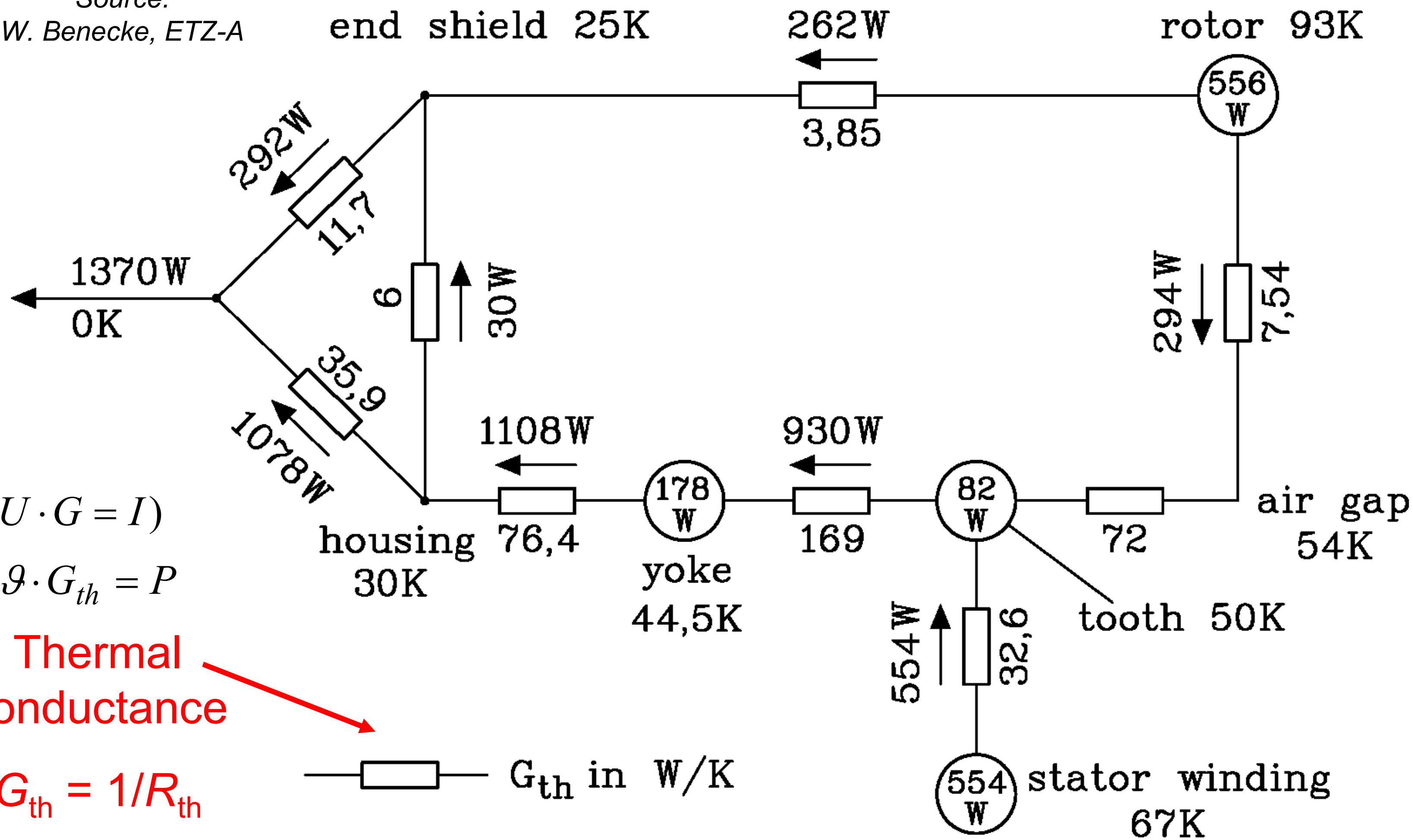


<u>*Example:*</u> Totally enclosed cage induction machine with shaft mounted fan, 11 kW, 4-pole, 50 Hz, 1450/min rated speed, Thermal Class B

3. Heat transfer and cooling

Heat source plot for 11 kW, 4-pole cage induction motor (2)

Source:
W. Benecke, ETZ-A



$(U \cdot G = I)$

$\Delta\vartheta \cdot G_{th} = P$

Thermal conductance

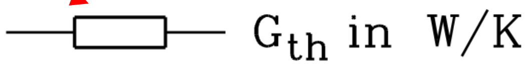
$G_{th} = 1/R_{th}$

3. Heat transfer and cooling

Heat resistance R_{th} and heat conductance $G_{th} = 1/R_{th}$

Thermal conductance

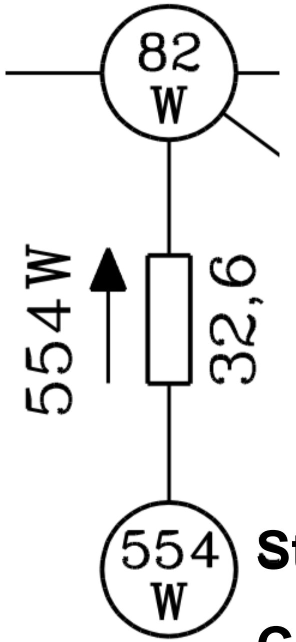
$G_{th} = 1/R_{th}$



Example:

$\Delta\vartheta$ = 67 K – 50 K = 17 K

$$\Delta\vartheta = \frac{P}{G_{th}} = \frac{554\,W}{32.6\,W/K} = 17\,K$$



Tooth 50 K

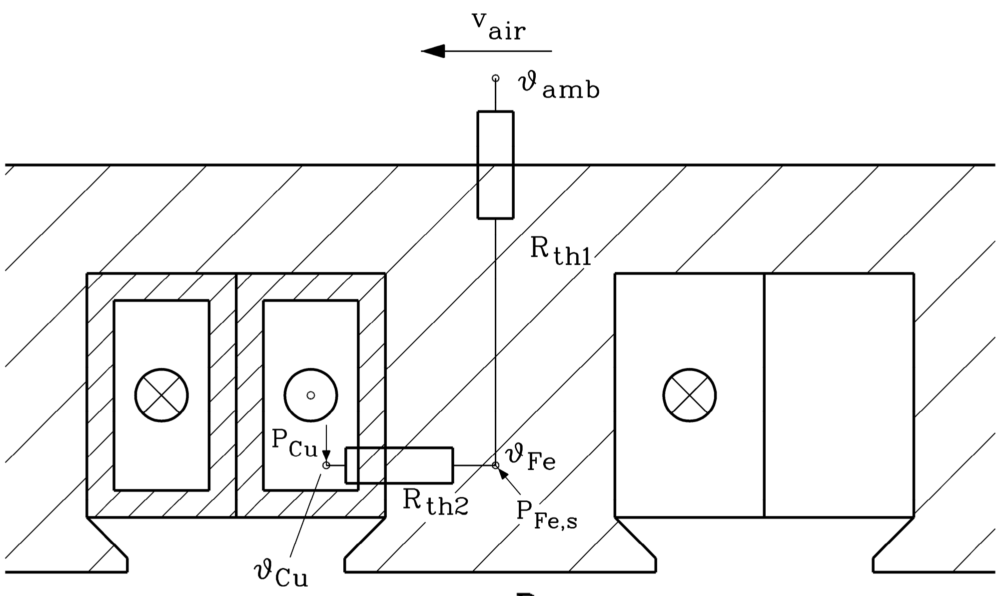
Iron losses 82 W

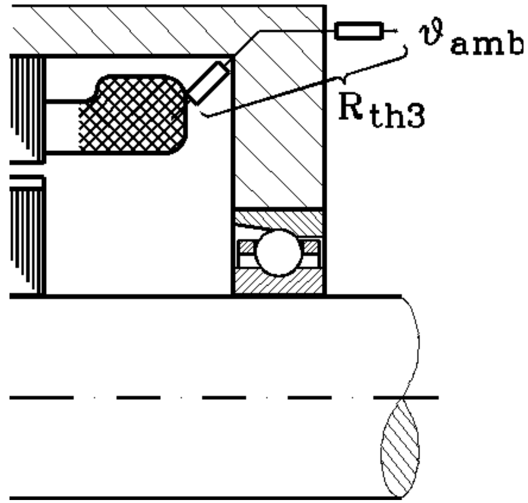
Stator winding 67 K

Copper losses 554 W

3. Heat transfer and cooling

Simplified stator thermal network

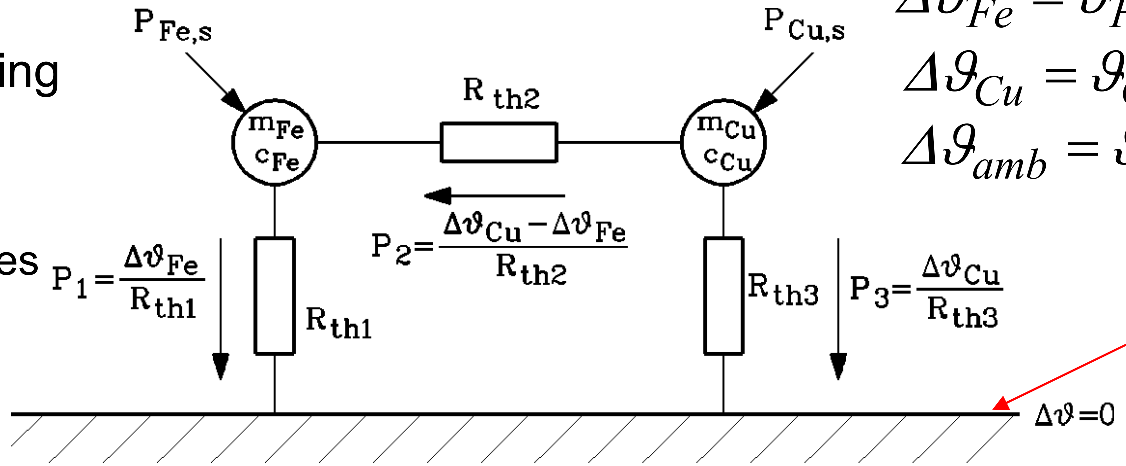




$$\Delta\vartheta_{Fe} = \vartheta_{Fe} - \vartheta_{amb}$$
$$\Delta\vartheta_{Cu} = \vartheta_{Cu} - \vartheta_{amb}$$
$$\Delta\vartheta_{amb} = \vartheta_{amb} - \vartheta_{amb} = 0$$

<u>Here:</u> Only stator heating considered

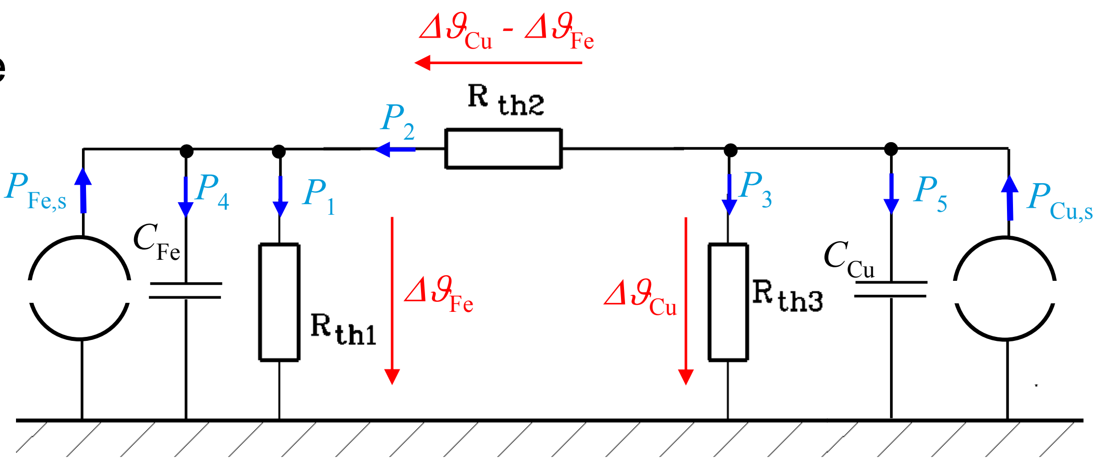
e.g.
PM synchronous machines with tooth-coils
(see: Lecture: Motor development)



Electric equivalent circuit of transient „two-body“ problem

- Heat source (power) = current source
- Heat conduction & convection =
  = resistance
- Heat storage = capacitance
- Temperature difference = voltage
- Power flow = current

$$C_{Cu} = m_{Cu} \cdot c_{Cu}, \quad C_{Fe} = m_{Fe} \cdot c_{Fe}$$



$$P_1 = \frac{\Delta\vartheta_{Fe}}{R_{th1}} \qquad P_2 = \frac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} \qquad P_4 = C_{Fe} \cdot \frac{d\Delta\vartheta_{Fe}}{dt}$$

$$P_3 = \frac{\Delta\vartheta_{Cu}}{R_{th3}} \qquad P_5 = C_{Cu} \cdot \frac{d\Delta\vartheta_{Cu}}{dt}$$

Kirchhoff´s nodal law:

$$\begin{cases} P_{Cu,s} = P_5 + P_3 + P_2 = m_{Cu} \cdot c_{Cu} \cdot \dfrac{d\Delta\vartheta_{Cu}}{dt} + \dfrac{\Delta\vartheta_{Cu}}{R_{th3}} + \dfrac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} \\ P_{Fe,s} = P_4 + P_1 - P_2 = m_{Fe} \cdot c_{Fe} \cdot \dfrac{d\Delta\vartheta_{Fe}}{dt} + \dfrac{\Delta\vartheta_{Fe}}{R_{th1}} - \dfrac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} \end{cases}$$

3. Heat transfer and cooling

„Two-body“ problem: Copper winding and iron core

- The two unknown temperature differences of copper $\Delta\vartheta_{Cu}$ and iron $\Delta\vartheta_{Fe}$

$$m_{Cu} \cdot c_{Cu} \cdot \frac{d\Delta\vartheta_{Cu}}{dt} + \frac{\Delta\vartheta_{Cu}}{R_{th3}} + \frac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} = P_{Cu,s}$$

$$m_{Fe} \cdot c_{Fe} \cdot \frac{d\Delta\vartheta_{Fe}}{dt} + \frac{\Delta\vartheta_{Fe}}{R_{th1}} - \frac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} = P_{Fe,s}$$

- Two bodies: Copper and iron = Two 1st order linear differential equations = One 2nd order linear differential equation = **Two thermal time constants $T_{\vartheta 1}$, $T_{\vartheta 2}$!**
  Usually: Copper mass much smaller than iron mass: $m_{Fe} >> m_{Cu}$

- Therefore:
  LONG time constant $T_{\vartheta 1} > T_{\vartheta 2}$ related mainly to iron mass
  SHORT time constant $T_{\vartheta 2}$ related mainly to copper mass

- *Example:* 550 kW 4-pole cage induction machine: Stator: m_{Fe} = 631 kg > m_{Cu} = 142 kg
  Inactive iron mass $m_{Fe,Housing}$ has to be considered in addition!

3. Heat transfer and cooling

Steady state thermal condition

- **Steady state temperature rise** corresponds to $\underline{d/dt = 0}$:

(1) $$\frac{\Delta\vartheta_{Cu}}{R_{th3}} + \frac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} = P_{Cu,s}$$

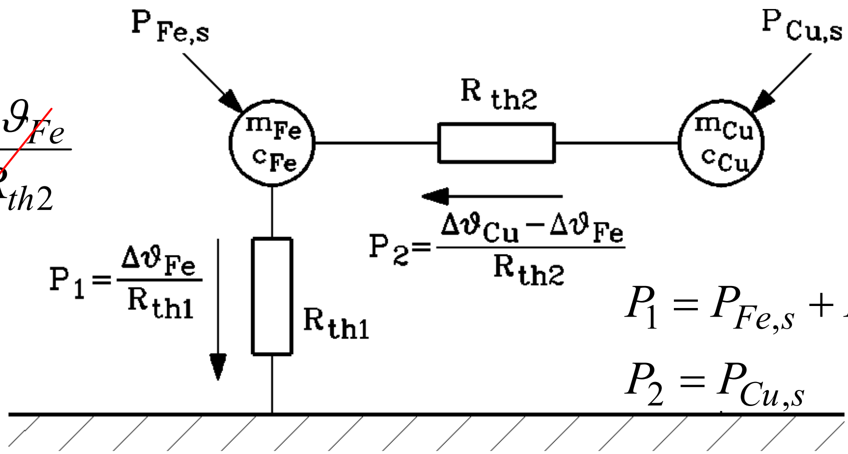
(2) $$\frac{\Delta\vartheta_{Fe}}{R_{th1}} - \frac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} = P_{Fe,s}$$

- **Simplified: $R_{th3} >> R_{th1}$, R_{th2}:** **(1)** $\dfrac{\Delta\vartheta_{Cu}}{R_{th3}} \approx 0 \Rightarrow \dfrac{\Delta\vartheta_{Cu} - \Delta\vartheta_{Fe}}{R_{th2}} = P_{Cu,s}$

(1) $$\boxed{\Delta\vartheta_{Cu} = P_{Cu,s} \cdot R_{th2} + \Delta\vartheta_{Fe}}$$

$$\frac{\Delta\vartheta_{Fe}}{R_{th1}} + \cancel{\frac{\Delta\vartheta_{Fe}}{R_{th2}}} = P_{Fe,s} + \frac{\Delta\vartheta_{Cu}}{R_{th2}} = P_{Fe,s} + P_{Cu,s} + \cancel{\frac{\Delta\vartheta_{Fe}}{R_{th2}}}$$

(2) $$\boxed{\Delta\vartheta_{Fe} = (P_{Fe,s} + P_{Cu,s}) \cdot R_{th1}}$$



$$P_1 = P_{Fe,s} + P_{Cu,s}$$

$$P_2 = P_{Cu,s}$$

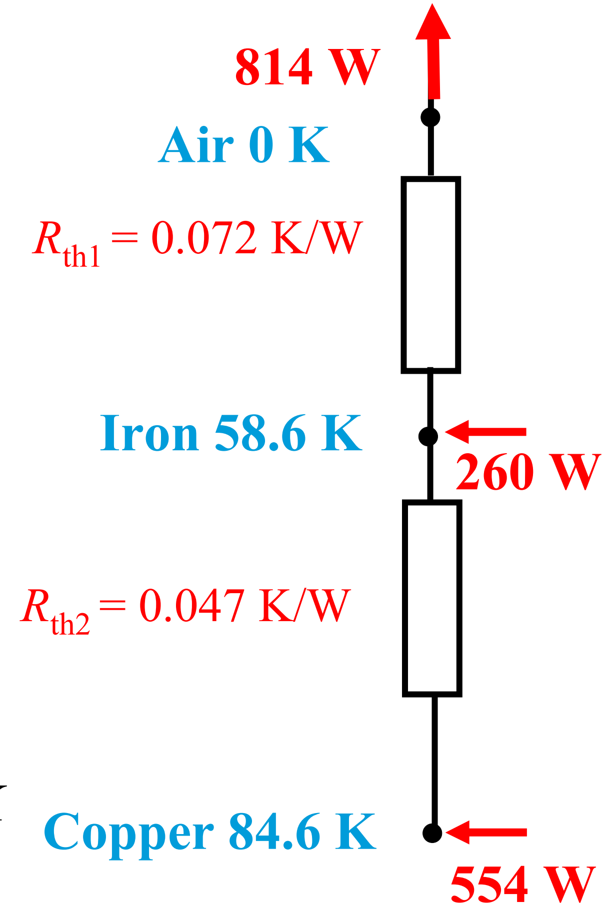
3. Heat transfer and cooling

Example: Stator heat flow from conductors to surface

11 kW cage induction motor, Thermal Class F
frame size 160 mm, totally enclosed, 50 Hz, four poles
shaft mounted fan, motor mass 76 kg:

$$P_{Cu,s} = 554W,\ P_{Fe,s} = 260W,$$

$$R_{th2} = 0.047K/W,\ R_{th1} = 0.072K/W$$



Steady state temperature rise:

In stator iron:

$$\Delta\vartheta_{Fe,s} = (P_{Fe,s} + P_{Cu,s}) \cdot R_{th1} = (260 + 554) \cdot 0.072 = \underline{\underline{58.6K}}$$

In stator winding:

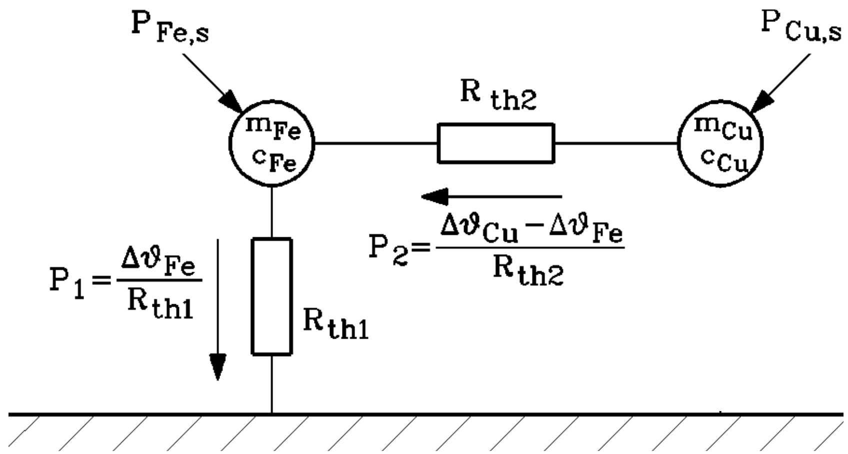
$$\Delta\vartheta_{Cu,s} = P_{Cu,s} \cdot R_{th2} + \Delta\vartheta_{Fe,s} = 554 \cdot 0.047 + 58.6 = \underline{\underline{84.6K}} < 105K$$

Result:

The winding temperature rise does not exceed the Thermal Class F limit.

3. Heat transfer and cooling

Simplified transient „two-body“ problem: $R_{th3} \to \infty$



$$\frac{d^2\Delta\vartheta_{Cu}}{dt^2} + \frac{d\Delta\vartheta_{Cu}}{dt} \cdot \left(\frac{1}{\tau} + \frac{1}{\tau_{Cu}}\right) + \frac{\Delta\vartheta_{Cu}}{\tau_{Cu} \cdot \tau_{Fe}} = \frac{1}{m_{Cu}c_{Cu} \cdot m_{Fe}c_{Fe}} \cdot \left(\frac{P_{Cu,s}}{R_{th1} \| R_{th2}} + \frac{P_{Fe,s}}{R_{th2}}\right)$$

$$\frac{d^2\Delta\vartheta_{Fe}}{dt^2} + \frac{d\Delta\vartheta_{Fe}}{dt} \cdot \left(\frac{1}{\tau} + \frac{1}{\tau_{Cu}}\right) + \frac{\Delta\vartheta_{Fe}}{\tau_{Cu} \cdot \tau_{Fe}} = \frac{1}{m_{Cu}c_{Cu} \cdot m_{Fe}c_{Fe}} \cdot \frac{P_{Cu,s} + P_{Fe,s}}{R_{th2}}$$

$$\tau = m_{Fe}c_{Fe} \cdot (R_{th1} \| R_{th2}) = m_{Fe}c_{Fe} \cdot \frac{R_{th1} \cdot R_{th2}}{R_{th1} + R_{th2}}$$

$$\tau_{Fe} = m_{Fe}c_{Fe} \cdot R_{th1}$$

$$\tau_{Cu} = m_{Cu}c_{Cu} \cdot R_{th2}$$

3. Heat transfer and cooling

Solution for simplified transient „two-body“ problem

Initial conditions:

$$\Delta\vartheta_{Cu}(0)=0,\ \Delta\vartheta_{Fe}(0)=0 \rightarrow \Delta\dot{\vartheta}_{Cu}(0)=P_{Cu,s}/(m_{Cu}c_{Cu}),\ \Delta\dot{\vartheta}_{Fe}(0)=P_{Fe,s}/(m_{Fe}c_{Fe})$$

Solution: $t \geq 0$:

$$\Delta\vartheta_{Cu}(t)=\frac{1}{\sqrt{\left(\frac{1}{\tau}+\frac{1}{\tau_{Cu}}\right)^2-\frac{4}{\tau_{Cu}\tau_{Fe}}}}\cdot\left[\left(\frac{P_{Cu,s}}{m_{Cu}c_{Cu}}-\frac{\Delta\vartheta_{Cu\infty}}{T_{\vartheta 2}}\right)\cdot e^{-\frac{t}{T_{\vartheta 1}}}-\left(\frac{P_{Cu,s}}{m_{Cu}c_{Cu}}-\frac{\Delta\vartheta_{Cu\infty}}{T_{\vartheta 1}}\right)\cdot e^{-\frac{t}{T_{\vartheta 2}}}\right]+\Delta\vartheta_{Cu\infty}$$

$$\Delta\vartheta_{Fe}(t)=\frac{1}{\sqrt{\left(\frac{1}{\tau}+\frac{1}{\tau_{Cu}}\right)^2-\frac{4}{\tau_{Cu}\tau_{Fe}}}}\cdot\left[\left(\frac{P_{Fe,s}}{m_{Fe}c_{Fe}}-\frac{\Delta\vartheta_{Fe\infty}}{T_{\vartheta 2}}\right)\cdot e^{-\frac{t}{T_{\vartheta 1}}}-\left(\frac{P_{Fe,s}}{m_{Fe}c_{Fe}}-\frac{\Delta\vartheta_{Fe\infty}}{T_{\vartheta 1}}\right)\cdot e^{-\frac{t}{T_{\vartheta 2}}}\right]+\Delta\vartheta_{Fe\infty}$$

Steady-state temperature rise: $\Delta\vartheta_{Cu\infty}=P_{Cu,s}\cdot R_{th2}+\Delta\vartheta_{Fe\infty}$ $\qquad$ $\Delta\vartheta_{Fe\infty}=(P_{Fe,s}+P_{Cu,s})\cdot R_{th1}$

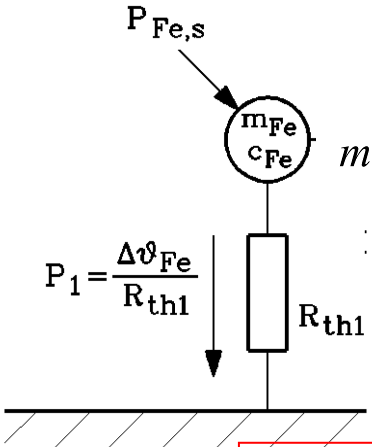
Two time constants: $T_{\vartheta 1} >> T_{\vartheta 2}$

$$T_{\vartheta 1}=\frac{2}{\frac{1}{\tau}+\frac{1}{\tau_{Cu}}-\sqrt{\left(\frac{1}{\tau}+\frac{1}{\tau_{Cu}}\right)^2-\frac{4}{\tau_{Cu}\tau_{Fe}}}} \qquad T_{\vartheta 2}=\frac{2}{\frac{1}{\tau}+\frac{1}{\tau_{Cu}}+\sqrt{\left(\frac{1}{\tau}+\frac{1}{\tau_{Cu}}\right)^2-\frac{4}{\tau_{Cu}\tau_{Fe}}}}$$

3. Heat transfer and cooling

Consideration only of iron or copper: „Single-body“ problem

Iron “alone”:

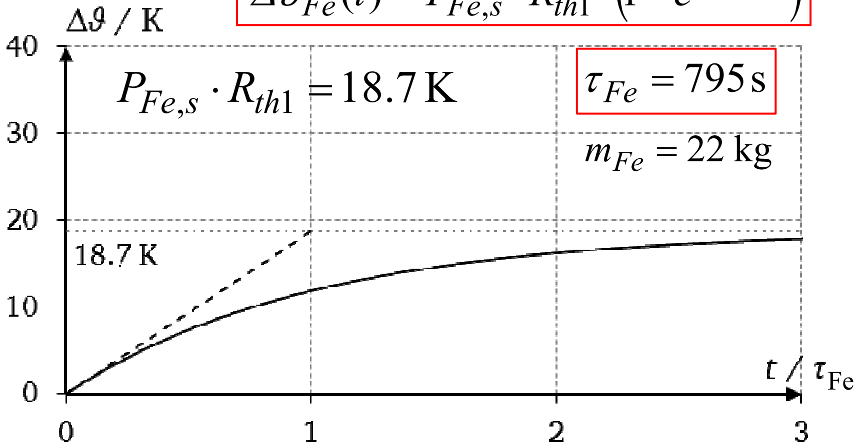


$$\boxed{\tau_{Fe} = m_{Fe} \cdot c_{Fe} \cdot R_{th1}}$$

$$m_{Fe} \cdot c_{Fe} \cdot \frac{d\Delta\vartheta_{Fe}}{dt} + \frac{\Delta\vartheta_{Fe}}{R_{th1}} = P_{Fe,s}$$

$$\Delta\vartheta_{Fe}(0) = 0$$

$$\boxed{t \ge 0: \quad \Delta\vartheta_{Fe}(t) = P_{Fe,s} \cdot R_{th1} \cdot \left(1 - e^{-t/\tau_{Fe}}\right)}$$

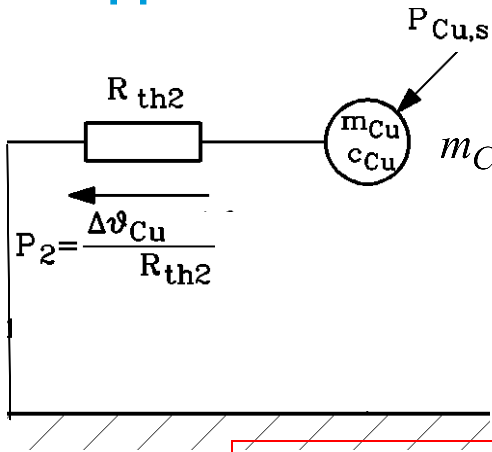


$P_{Fe,s} \cdot R_{th1} = 18.7\,\mathrm{K}$

$\boxed{\tau_{Fe} = 795\,\mathrm{s}}$

$m_{Fe} = 22\,\mathrm{kg}$

Copper “alone”:

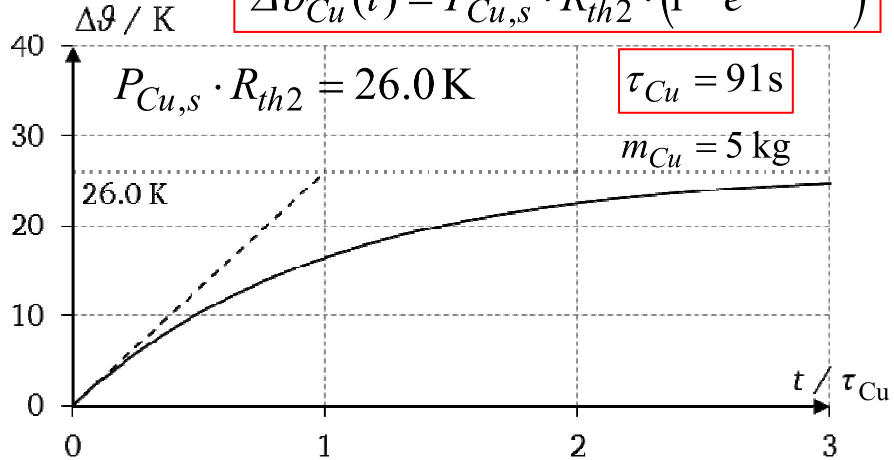


$$\boxed{\tau_{Cu} = m_{Cu} \cdot c_{Cu} \cdot R_{th2}}$$

$$m_{Cu} \cdot c_{Cu} \cdot \frac{d\Delta\vartheta_{Cu}}{dt} + \frac{\Delta\vartheta_{Cu}}{R_{th2}} = P_{Cu,s}$$

$$\Delta\vartheta_{Cu}(0) = 0$$

$$\boxed{t \ge 0: \quad \Delta\vartheta_{Cu}(t) = P_{Cu,s} \cdot R_{th2} \cdot \left(1 - e^{-t/\tau_{Cu}}\right)}$$



$P_{Cu,s} \cdot R_{th2} = 26.0\,\mathrm{K}$

$\boxed{\tau_{Cu} = 91\,\mathrm{s}}$

$m_{Cu} = 5\,\mathrm{kg}$

3. Heat transfer and cooling

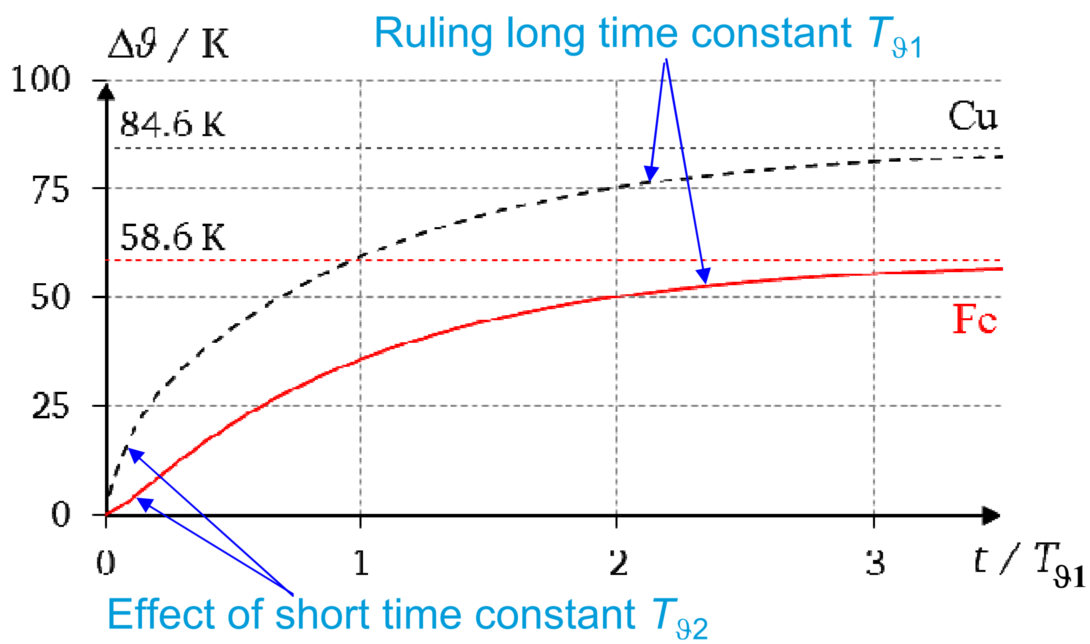
Example: Transient „Two-body" problem

11 kW cage induction motor, Th. Cl. F, frame size 160 mm, totally enclosed, 50 Hz, four poles, shaft mounted fan, total motor mass 76 kg, stator: copper mass: 5 kg, active iron mass: 22 kg

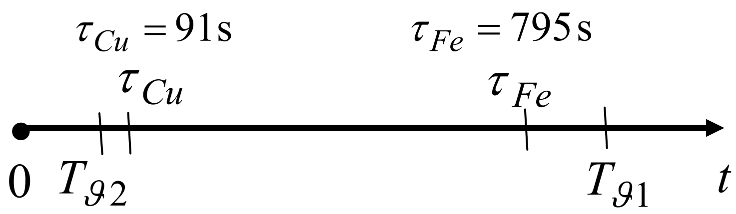
$$P_{Cu,s} = 554\,\mathrm{W},\ P_{Fe,s} = 260\,\mathrm{W},\ R_{th2} = 0.047\,\mathrm{K/W},\ R_{th1} = 0.072\,\mathrm{K/W},\ R_{th3} \to \infty,$$

$$m_{Fe}c_{Fe} = 11044\ \mathrm{J/K},\ m_{Cu}c_{Cu} = 1943\ \mathrm{J/K}$$

$$\tau = 314\,\mathrm{s},\ \tau_{Fe} = 795\,\mathrm{s},\ \tau_{Cu} = 91\,\mathrm{s}$$



$$\boxed{\Delta\vartheta_{Fe\infty} = 58.6\,\mathrm{K},\ \Delta\vartheta_{Cu\infty} = 84.6\,\mathrm{K}}$$



$$\boxed{T_{\vartheta 2} = 76\,\mathrm{s}} \qquad \boxed{T_{\vartheta 1} = 950\,\mathrm{s}}$$

Long iron and short copper time constant

Example: 550 kW 4-pole cage induction machine: Stator: m_{Fe} = 631 kg > m_{Cu} = 142 kg
m_{Cu}/m_{Fe} = 0.23 << 1. With $R_{th1} \approx R_{th2}$, $c_{Cu} \approx c_{Fe}$ we get:

$$\tau_{Fe} = m_{Fe} c_{Fe} \cdot R_{th1} >> \tau_{Cu} = m_{Cu} c_{Cu} \cdot R_{th2} \qquad \tau = \tau_{Fe} \cdot R_{th2} / (R_{th1} + R_{th2}) = \tau_{Fe} \cdot r \approx 0.5 \cdot \tau_{Fe} >> \tau_{Cu}$$

$$\sqrt{\left(\frac{1}{\tau} + \frac{1}{\tau_{Cu}}\right)^2 - \frac{4}{\tau_{Cu}\tau_{Fe}}} = \frac{1}{\tau_{Cu}} \cdot \sqrt{\left(1 + \frac{\tau_{Cu}/r}{\tau_{Fe}}\right)^2 - \frac{4\tau_{Cu}}{\tau_{Fe}}} \approx \frac{1}{\tau_{Cu}} \cdot \sqrt{1 - \frac{2\tau_{Cu} \cdot (2 - 1/r)}{\tau_{Fe}}} \approx$$

$$\approx \frac{1}{\tau_{Cu}} \cdot \left(1 - \frac{\tau_{Cu} \cdot (2 - 1/r)}{\tau_{Fe}}\right) = \frac{1}{\tau_{Cu}} - \frac{2 - 1/r}{\tau_{Fe}}$$

$$\boxed{x = \frac{\tau_{Cu}}{\tau_{Fe}} << 1 : \left(\frac{\tau_{Cu}}{\tau_{Fe}}\right)^2 << 1}$$

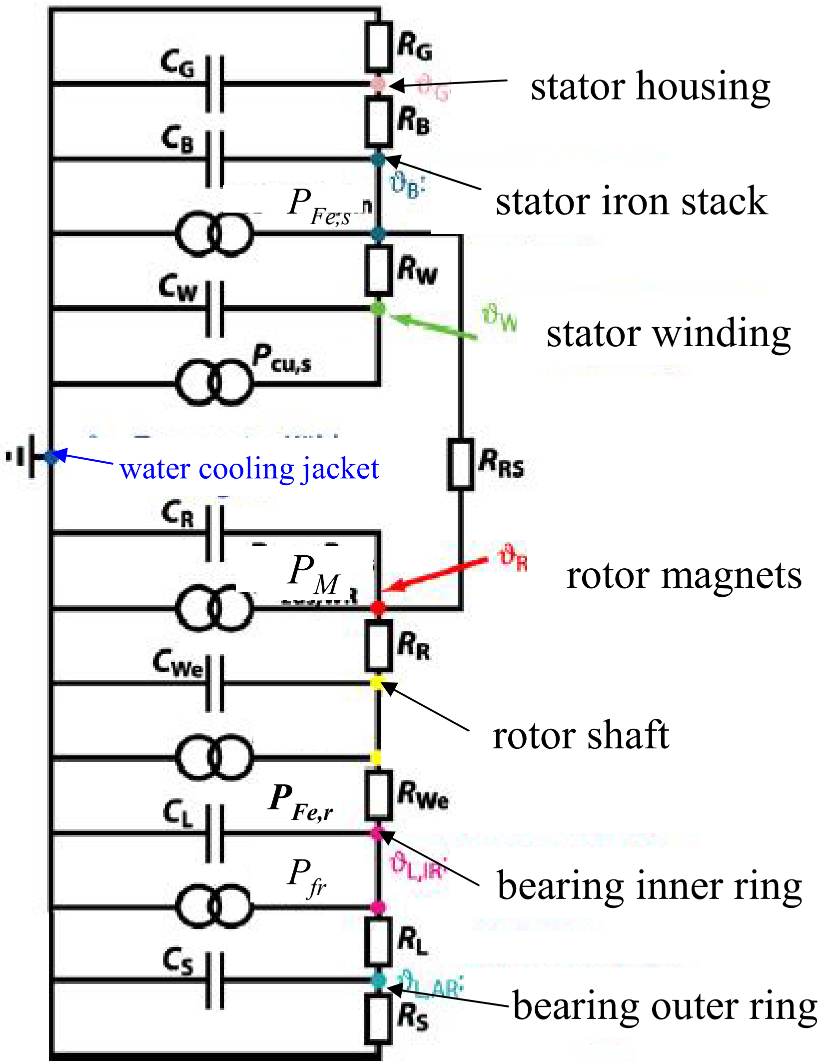
$$\boxed{x = \frac{\tau_{Cu}}{\tau_{Fe}} << 1 : \sqrt{1 - x} \approx 1 - x/2}$$

$$T_{\vartheta 1} = \frac{2}{\frac{1}{\tau} + \frac{1}{\tau_{Cu}} - \sqrt{\left(\frac{1}{\tau} + \frac{1}{\tau_{Cu}}\right)^2 - \frac{4}{\tau_{Cu}\tau_{Fe}}}} \approx \frac{2}{\frac{1/r}{\tau_{Fe}} + \frac{1}{\tau_{Cu}} - \frac{1}{\tau_{Cu}} + \frac{2 - 1/r}{\tau_{Fe}}} = \tau_{Fe}$$

$$T_{\vartheta 2} = \frac{2}{\frac{1}{\tau} + \frac{1}{\tau_{Cu}} + \sqrt{\left(\frac{1}{\tau} + \frac{1}{\tau_{Cu}}\right)^2 - \frac{4}{\tau_{Cu}\tau_{Fe}}}} \approx \frac{2}{\frac{1/r}{\tau_{Fe}} + \frac{1}{\tau_{Cu}} + \frac{1}{\tau_{Cu}} - \frac{2 - 1/r}{\tau_{Fe}}} = \frac{\tau_{Cu}}{1 + \frac{\tau_{Cu}}{\tau_{Fe}} \cdot \left(\frac{1}{r} - 1\right)} \approx \tau_{Cu}$$

$$\boxed{T_{\vartheta 1} \approx \tau_{Fe} \quad T_{\vartheta 2} \approx \tau_{Cu}}$$

Heat source plot of a PM synchronous machine



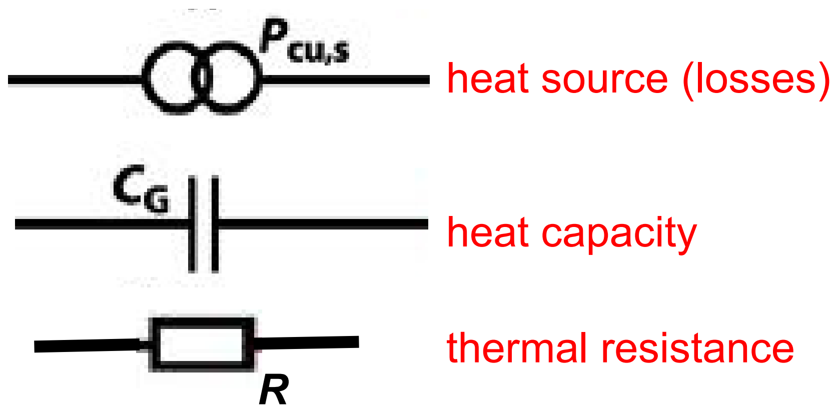
Example:

Heat source plot of a PM synchronous machine
with stator water jacket cooling – "7-body-problem":

7 unknown temperatures = 7th order differential equation

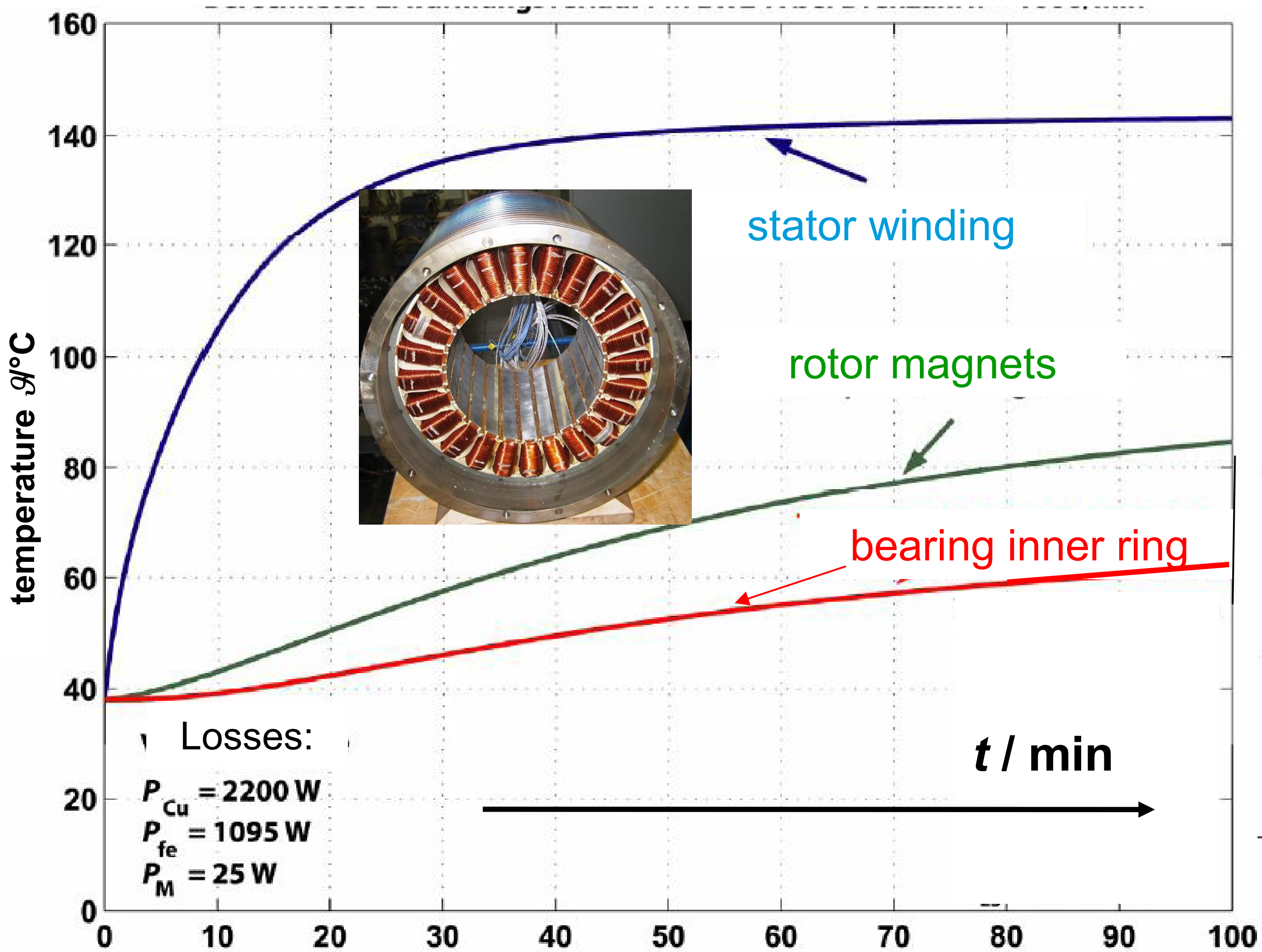
5 heat sources:
Stator & rotor iron losses,
stator copper losses,
rotor eddy current losses in magnets,
friction losses in bearings

7 nodes, where the 7 temperatures are calculated



3. Heat transfer and cooling

Example: Calculated temperature rise at three „nodes"



16-pole PM synchronous motor, surface mounted magnets, tooth-coil winding, $q = ½$,

45 kW, 1000/min

Thermal resistances:

R_G = 0.001 K/W
R_B = 0.0233 K/W
R_W = 0.0128 K/W
R_{RS} = 0.26 K/W
R_R = 0.031 K/W
R_L = 0.238 K/W
R_{We} = 0.219 K/W
R_{LS} = 0,0299 K/W

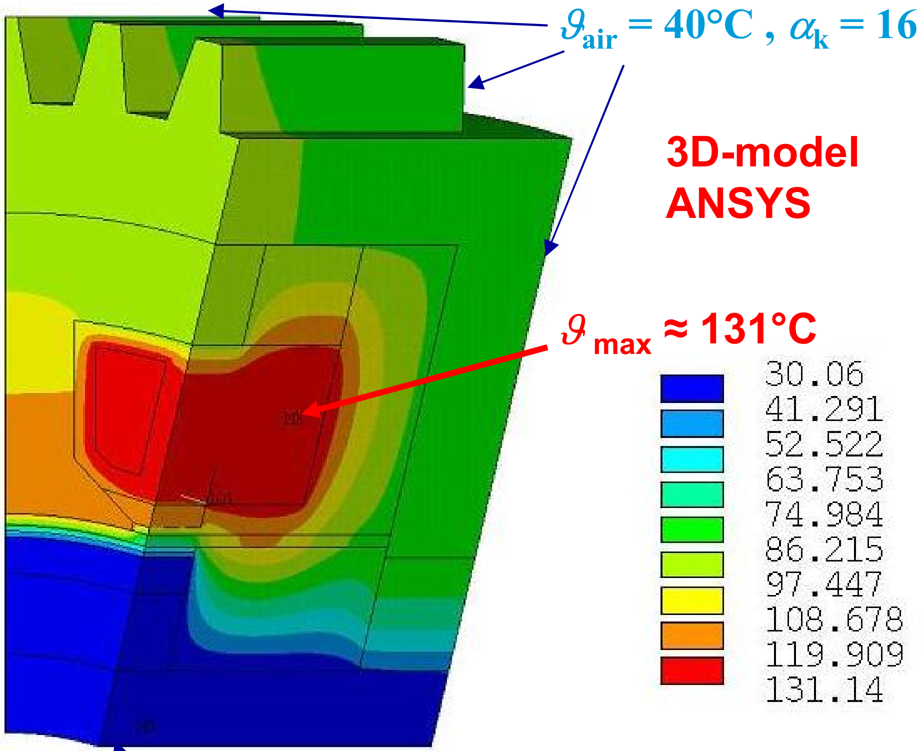
3. Heat transfer and cooling

Example: 3D heat flow at axially short motors

Numerical 3D Finite-Element thermal steady-state calculation of an axially short PM synchronous machine

"Thermally equivalent" slot insulation = mix of slot insulation paper, epoxy resin and air voids at a slot fill factor 55% $\Rightarrow$ reduction of number of finite elements in the slot region

$\vartheta_{air} = 40°C$, $\alpha_k = 16\ W/(m^2K)$



$\vartheta_{max} \approx 131°C$

$\vartheta_{water} = 30°C$, $\alpha_k = 9950\ W/(m^2K)$

3D-model ANSYS

PM synchronous machine:
No fan, only cooling fins.
Hollow shaft rotates in water as a pump drive

Rated data:

$n_N = 5000\ min^{-1}$
$I_N = 7.74\ A$
$M_N = 9\ Nm$

Motor losses	
Motor efficiency	
P_{Cu} [W]	61.5
P_{Fe} [W]	279.9
P_{fr} [W]	232.6
η_{mot} [%]	89.15

Result:
Maximum temperature 131°C in the winding overhang = 24 K below the admissible temperature limit Th. Cl. F 155°C

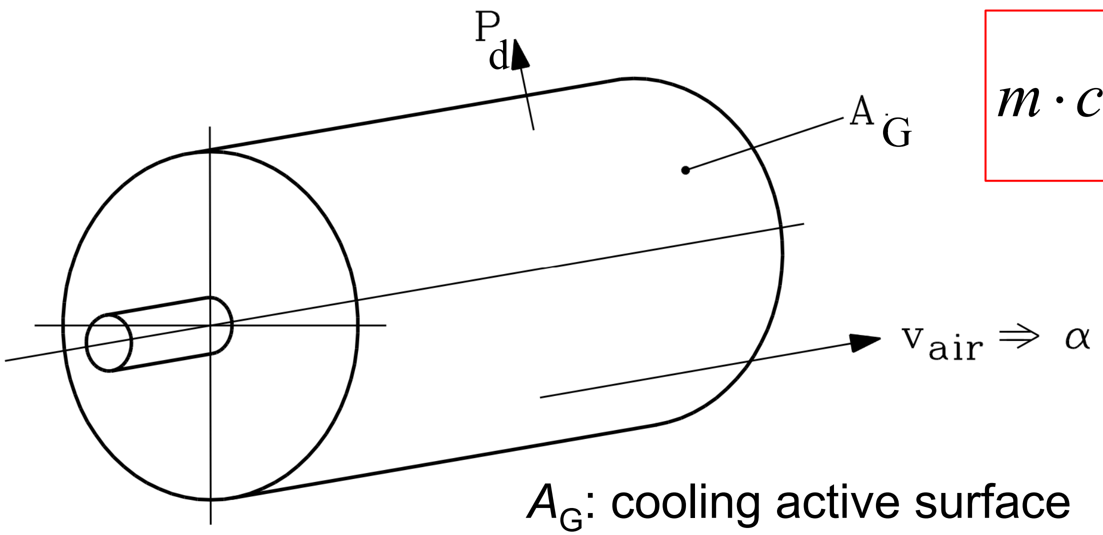
3. Heat transfer and cooling

„Single-body“ problem: Total losses in machine

Simplified calculation for winding temperature rise:

Machine is **„Single-body = homogenous body replica“:**

- Total losses P_d within machine = heat source
- Convective heat transfer from machine surface A_G to coolant flow: $R_{th} = \dfrac{1}{\alpha A_G}$
- Total motor mass is taken for heat storage.



A_G: cooling active surface

$$\boxed{m \cdot c \cdot \frac{d\Delta\vartheta_{Cu}}{dt} + \alpha \cdot A_G \cdot \Delta\vartheta_{Cu} = P_d}$$

Steady-state solution: $d/dt = 0$:

$$\alpha \cdot A_G \cdot \Delta\vartheta_{Cu} = P_d$$

$$\boxed{\Delta\vartheta_{Cu} = P_d / (\alpha \cdot A_G)}$$

3. Heat transfer and cooling

Temperature rise of a homogeneous body

$$\boxed{m \cdot c \cdot \frac{d\Delta\vartheta_{Cu}}{dt} + \alpha \cdot A_G \cdot \Delta\vartheta_{Cu} = P_d}$$

Solution of 1st order differential linear equation: Superposition of homogenous and particular solution:

$$\Delta\vartheta_{Cu}(t) = \Delta\vartheta_{Cu,h}(t) + \Delta\vartheta_{Cu,p}(t)$$

- Homogenous differential equation: Solution is exponential function

$$m \cdot c \cdot \frac{d\Delta\vartheta_{Cu,h}}{dt} + \alpha \cdot A_G \cdot \Delta\vartheta_{Cu,h} = 0 \qquad \frac{d\Delta\vartheta_{Cu,h}}{dt} + \frac{\alpha \cdot A_G}{m \cdot c} \cdot \Delta\vartheta_{Cu,h} = 0 \qquad \frac{d\Delta\vartheta_{Cu,h}}{dt} + \frac{\Delta\vartheta_{Cu,h}}{T_\vartheta} = 0$$

$$T_\vartheta = \frac{m \cdot c}{\alpha \cdot A_G} \qquad \Delta\vartheta_{Cu,h}(t) = C \cdot e^{-t/T_\vartheta}$$

- Particular solution: As right hand side is constant, it must also be constant: $\Delta\vartheta_{Cu,p}(t) = K$

$$m \cdot c \cdot \frac{dK}{dt} + \alpha \cdot A_G \cdot K = P_d \Rightarrow K = P_d / (\alpha \cdot A_G) = \Delta\vartheta(t \to \infty) = \Delta\vartheta_\infty$$

- Resulting solution must satisfy initial condition via *C*: $\Delta\vartheta_{Cu}(t = 0) = \Delta\vartheta_0$

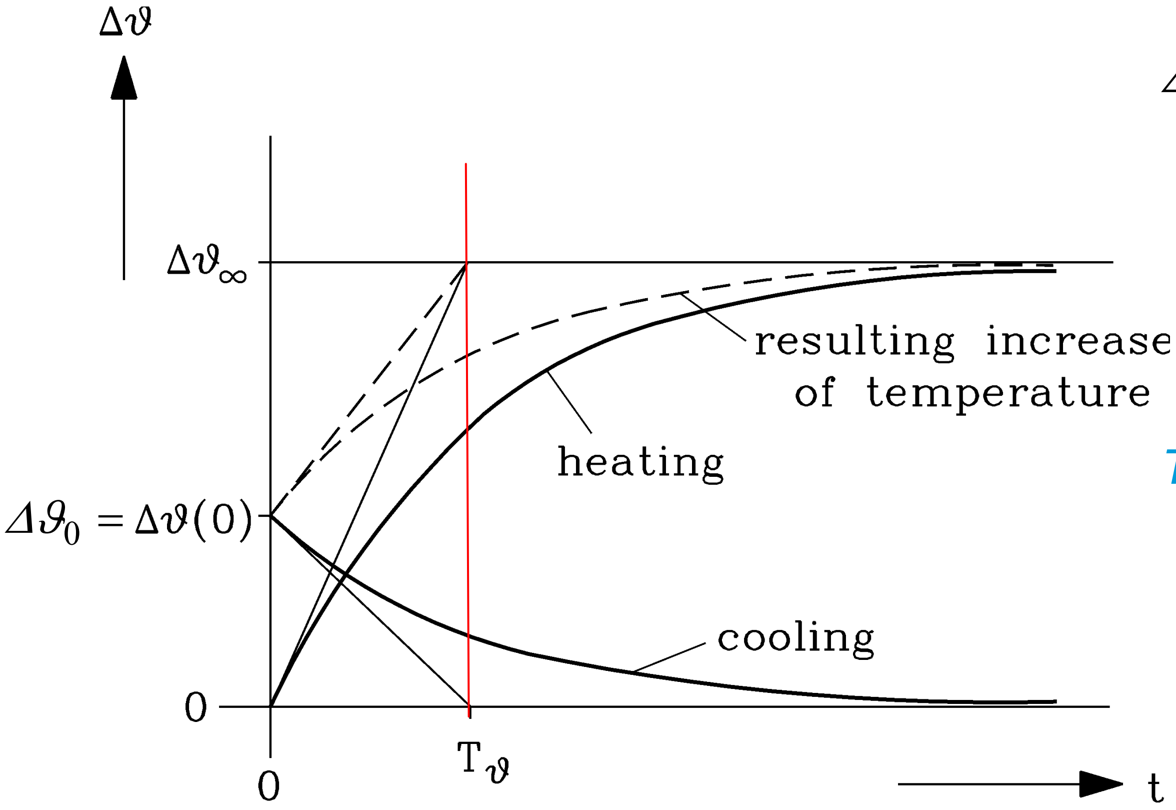
$$\Delta\vartheta_{Cu}(t) = \Delta\vartheta_{Cu,h}(t) + \Delta\vartheta_{Cu,p}(t) = C \cdot e^{-t/T_\vartheta} + \Delta\vartheta_\infty \qquad \Delta\vartheta_0 = C \cdot e^{0/T_\vartheta} + \Delta\vartheta_\infty \Rightarrow C = \Delta\vartheta_0 - \Delta\vartheta_\infty$$

$$\boxed{\Delta\vartheta_{Cu}(t) = \left(\Delta\vartheta_0 - \Delta\vartheta_\infty\right) \cdot e^{-t/T_\vartheta} + \Delta\vartheta_\infty}$$

3. Heat transfer and cooling

Transient solution of „homogenous-body“ replica at S1 duty

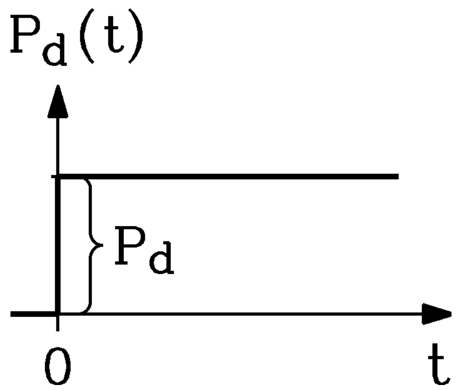
$$\boxed{\Delta\vartheta_{Cu} = \Delta\vartheta_{\infty} \cdot (1 - e^{-t/T_{\vartheta}}) + \Delta\vartheta_0 \cdot e^{-t/T_{\vartheta}}}$$



$$\Delta\vartheta_{\infty} = \frac{P_d}{\alpha \cdot A_G}$$ **Stationary temperature rise**

$$T_{\vartheta} = \frac{m \cdot c}{\alpha \cdot A_G}$$ **Thermal time constant**

Time function of losses at S1-duty:



Thermal time constant at operation and stand still

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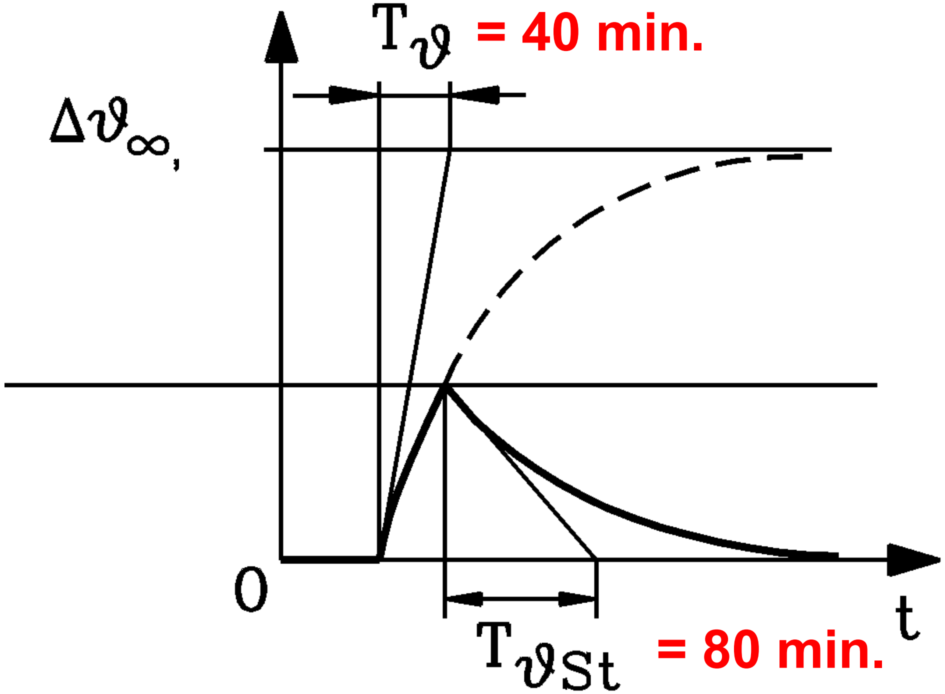
Example:



Machines with shaft mounted fan have

a) a **shorter** thermal time constant T_ϑ, when rotating, as the fan blows, and

b) a **longer** thermal time constant $T_{\vartheta,St}$ at **stand still**, as the air is not moved.

In case b) the heat transfer coefficient α is smaller!

$$T_\vartheta = \frac{m \cdot c}{\alpha \cdot A_G}$$

$$\boxed{T_{\vartheta,St} = (1.5 \ldots 2.0) \cdot T_\vartheta}$$

3. Heat transfer and cooling

Summary: Homogenous-body replica for temperature rise

- Simplified calculation for winding temperature rise: **„Homogenous-body replica“:**

a) Heat source: Total losses P_d within machine (or “equivalent losses” P_{de})

b) Convective heat transfer from machine surface A_G to coolant flow: $R_{th} = \dfrac{1}{\alpha \cdot A_G}$

c) Total motor mass with equivalent specific thermal capacity c_e is taken for heat storage.

$$m \cdot c_e \cdot \frac{d\Delta\vartheta_{Cu}}{dt} + \alpha \cdot A_G \cdot \Delta\vartheta_{Cu} = P_{de} \qquad \Delta\vartheta_\infty = \frac{P_{de}}{\alpha \cdot A_G}$$

- **Initial condition** is winding temperature rise at $t = 0$: $\Delta\vartheta_{Cu}(0) = \Delta\vartheta_0$.
- Homogenous-body **thermal time constant**: $T_\vartheta = \dfrac{m \cdot c_e}{\alpha \cdot A_G}$ $\quad m \sim l^3,\ A_G \sim l^2$
- *Thermal time constants scales with: $T_\vartheta \sim l$ = Thermal time constant rises with motor size.*
- Small machines: $T_\vartheta \approx 10\ min.$ (several 100 W), big machines: $T_\vartheta \approx 3\ h$ (several MW).

Summary:
Heat-source plot

- Heat sources as power input
- Thermal resistances for steady state temperature calculation
- Heat capacities for transient temperature calculation
- Each heat capacity element gives an additional order of differential equation
- Stator copper and iron losses give a thermal „two-body“ problem
- Equivalent total losses P_{de} give the „single-body“ problem:
  e.g. for stator winding temperature: $P_{de} = P_{Cu,s} + 0.5 \cdot P_{Fe} + 0.3 \cdot P_{Cu,r}$
- „Single-body“ problem gives one thermal time constant
  via equivalent specific heat capacity c_e
  e.g. for stator winding temperature: $c_e = (c_{Cu} \cdot m_{Cu} + c_{Fe} \cdot m_{Fe})/(m_{Fe} + m_{Cu})$

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3. Heat transfer and cooling of electric machines

- 3.1 Thermal classes, cooling systems, duty types
- 3.2 Elements for calculation of temperature rise
- 3.3 Heat-source plot
- 3.4 Thermal utilization
- 3.5 Simplified calculation of temperature rise

3. Heat transfer and cooling
Thermal utilization (see Chapter 1)

Steady-state copper temperature rise: $\Delta\vartheta_{Cu} = P_{Cu} / (\alpha \cdot A_G)$

A_G: cooling active surface

Copper losses: $P_{Cu} = m \cdot \frac{1}{\kappa} \cdot \frac{N \cdot 2 \cdot (l_{Fe} + l_b)}{a_a \cdot A_{Cu}} \cdot I^2$

With **current loading** $A = \frac{2mNI}{2p\tau_p}$ and **current density** $J = \frac{I}{a_a A_{Cu}}$ we get:

$$\Delta\vartheta_{Cu} = A \cdot J \cdot \frac{1}{\alpha \cdot \kappa} \cdot \frac{2p\tau_p \cdot (l_{Fe} + l_b)}{A_G} \quad \Rightarrow \quad \boxed{\underline{\Delta\vartheta_{Cu} \sim A \cdot J}}$$

3. Heat transfer and cooling

Example: Thermal utilization for Th. Cl. F

- Thermal Class F: (IEC60034-1): $\Delta\vartheta_{Cu}$ = 105 K at ϑ_{amb} = 40°C → ϑ_{Cu} = 145°C

  Standard induction machine (totally enclosed, shaft-mounted fan cooled: TEFC):

  $\alpha \approx 50$ W/(m²K), κ_{Cu}(145°C) = 38 MS/m

  $$\frac{2p\tau_p \cdot (l_{Fe} + l_b)}{A_G} \approx 1$$

- **Typical values: *A* = 250 A/cm, *J* = 7 A/mm²: *A·J* = 1750 A/cm·A/mm²**

  $$\Delta\vartheta_{Cu} = A \cdot J \cdot \frac{1}{\alpha \cdot \kappa} \cdot \frac{2p\tau_p \cdot (l_{Fe} + l_b)}{A_G} = \frac{25000 \cdot 7 \cdot 10^6}{50 \cdot 38 \cdot 10^6} \cdot 1 = 92\mathrm{K} < 105\mathrm{K}$$

- ***Result:***

  With $A \cdot J \leq 1800$ A/cm·A/mm² TEFC motors are roughly within Th. Cl. F temperature rise (Note: Other loss components neglected!) .

Summary:
Thermal utilization

- Thermal utilization is related to steady-state „single-body“ problem
- Heat source is only given by copper losses
- Thermal utilization coefficient $A \cdot J$ only valid, if copper losses dominate
- Thermal utilization often used as a „first guess“ for temperature
- For detailed machine design the heat-source plot is needed

3. Heat transfer and cooling of electric machines

3.1 Thermal classes, cooling systems, duty types

3.2 Elements for calculation of temperature rise

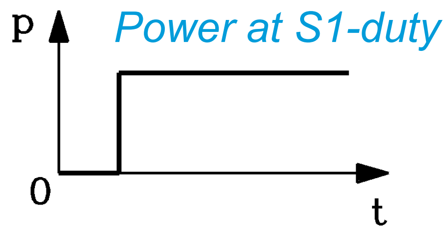
3.3 Heat-source plot

3.4 Thermal utilization

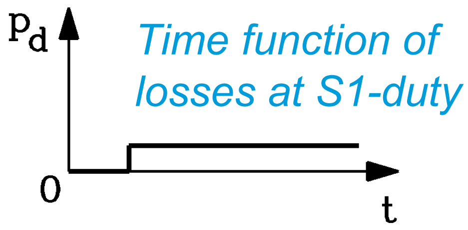
3.5 Simplified calculation of temperature rise

3. Heat transfer and cooling

Transient temperature rise at S1 duty

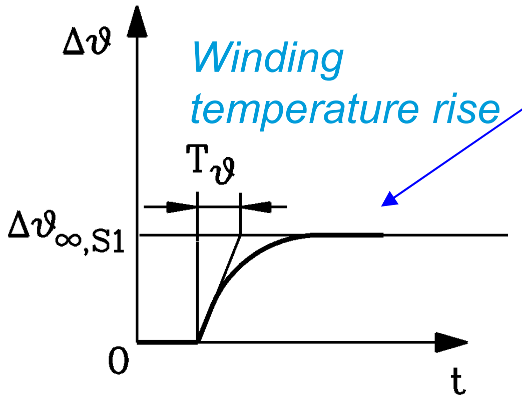


$$\Delta\vartheta_{Cu} = \Delta\vartheta_\infty \cdot (1 - e^{-t/T_\vartheta}) + \Delta\vartheta_0 \cdot e^{-t/T_\vartheta}$$



Example: $\Delta\vartheta_0 = 0$

$$\boxed{\Delta\vartheta_{Cu}(t) = \Delta\vartheta_\infty \cdot (1 - e^{-t/T_\vartheta})}$$



Steady state temperature $\Delta\vartheta_\infty = P_{Cu}/(\alpha A_G)$ is reached after about <u>three time constants</u>:

$$\Delta\vartheta_{Cu}(3T_\vartheta) = \Delta\vartheta_\infty \cdot (1 - e^{-3}) = 0.95 \cdot \Delta\vartheta_\infty \, .$$

3. Heat transfer and cooling

Increased power P_{S2} at short time duty S2

- Due to short-time operation t_B output power may be increased up to P_{S2}:

$$\Delta\vartheta(t_B) = \Delta\vartheta_{\infty,S2} \cdot \left(1 - e^{-t_B/T_\vartheta}\right) \le \Delta\vartheta_{\infty,S1} \qquad \Delta\vartheta_\infty \sim P_d$$

Power is estimated as: $P_{S2} = 3 \cdot U_s I_s \cdot \cos\varphi_s \cdot \eta \sim I_s = I_{s,S2}$

Losses are estimated as: $P_d \approx P_{Cu,s+r} = 3 \cdot (R_s I_s^2 + R'_r I'^2_r) \sim I_s^2 \Rightarrow P_{d,S2} \sim I_{s,S2}^2$

$$\underline{I}_s \approx -\underline{I}'_r$$

- **Increased power P_{S2} estimated:**

$$\boxed{\frac{P_{S2}}{P_{S1}} = \frac{I_{s,S2}}{I_{s,S1}} = \sqrt{\frac{P_{d,S2}}{P_{d,S1}}} = \sqrt{\frac{\Delta\vartheta_{\infty,S2}}{\Delta\vartheta_{\infty,S1}}} = \frac{1}{\sqrt{1 - e^{-t_B/T_\vartheta}}}}$$

- *Example:*

500 kW cage induction motor, thermal time constant T_ϑ = 40 min.
Motor shall be operated in S2 duty with operation time t_B = 30 min.

$$\frac{P_{S2}}{P_{S1}} = \frac{1}{\sqrt{1 - e^{-t_B/T_\vartheta}}} = \frac{1}{\sqrt{1 - e^{-30/40}}} = \sqrt{1.9} = \underline{\underline{1.38}}$$

Motor power may be increased for S2-operation by 38% up to 690 kW.

3. Heat transfer and cooling

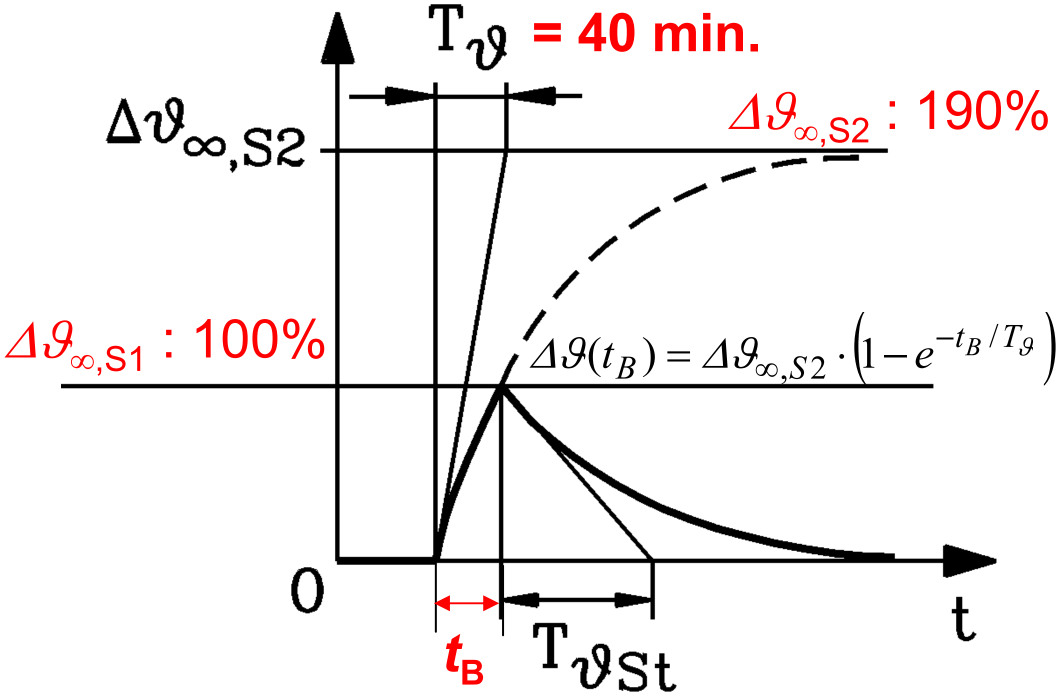
Short time duty: S2 duty

Power 138%

= 30 min.

Losses 1.38^2: 190%

$\Delta\vartheta_{\infty,S1}$



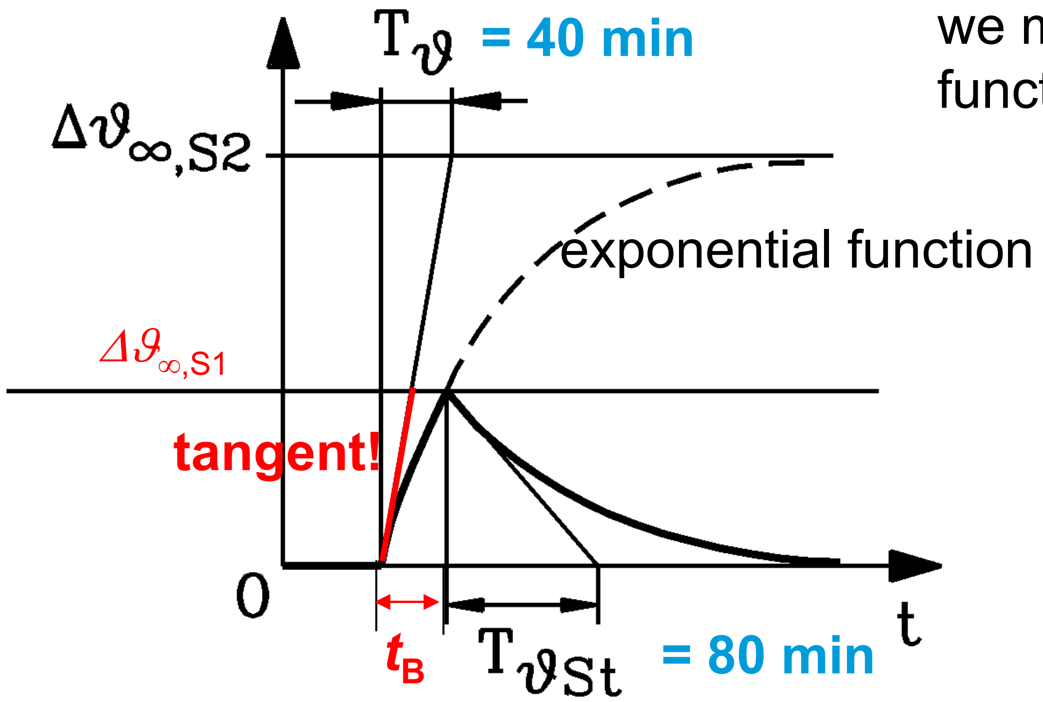
schematic sketch!

3. Heat transfer and cooling

Linear approximation of exponential temperature function

If a considered time span t_B is much shorter than the thermal time constant T_ϑ, we may approximate the exponential function via the tangent!

Example:



$$t_B << T_\vartheta :$$

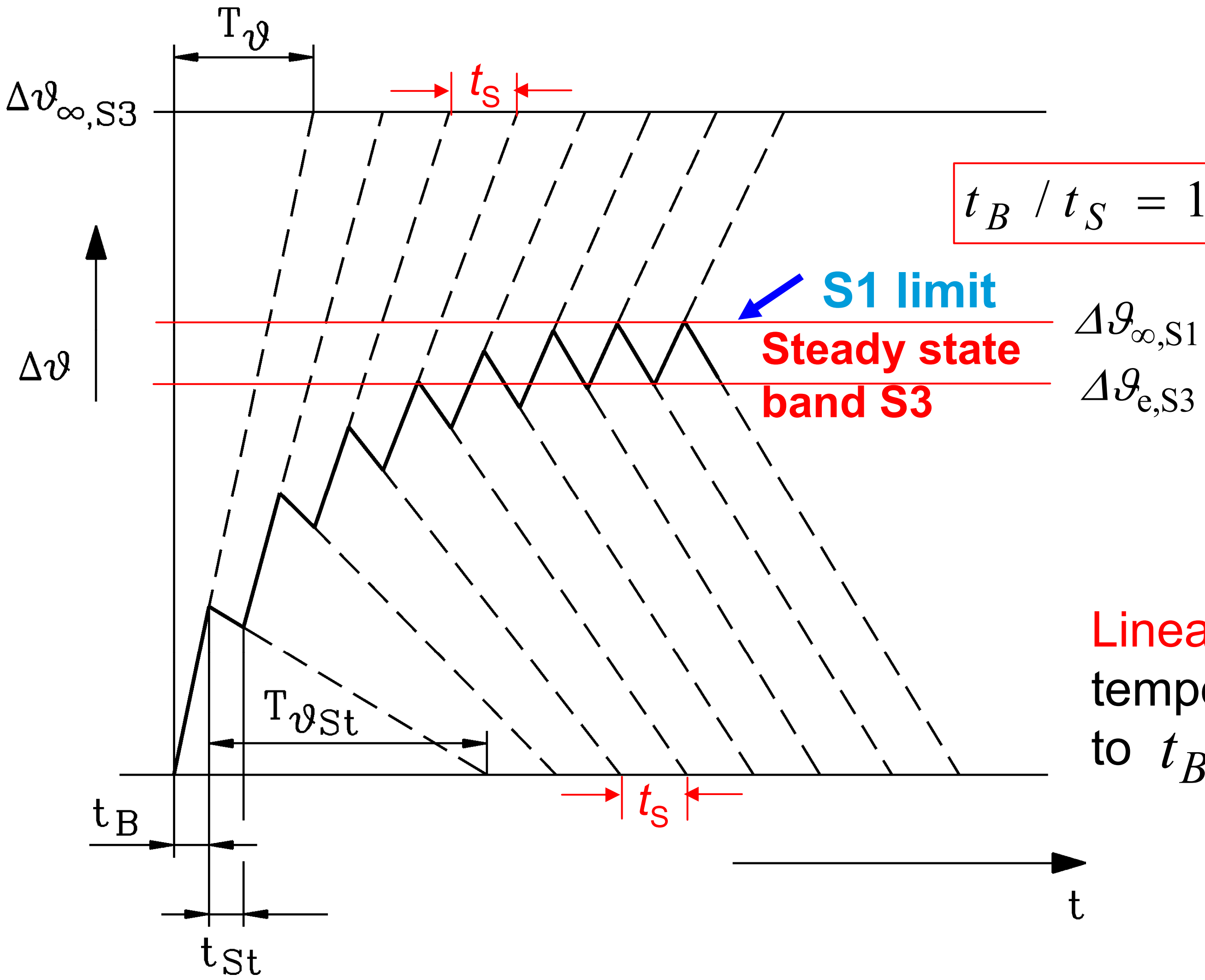
$$0 \le t \le t_B : 1 - e^{-t/T_\vartheta} \approx t / T_\vartheta$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \approx 1 + x, \qquad x << 1$$

$$1 - e^{-t/T_\vartheta} \approx 1 - (1 - t/T_\vartheta) = t / T_\vartheta$$

3. Heat transfer and cooling

Temperature rise at intermittent periodic duty S3



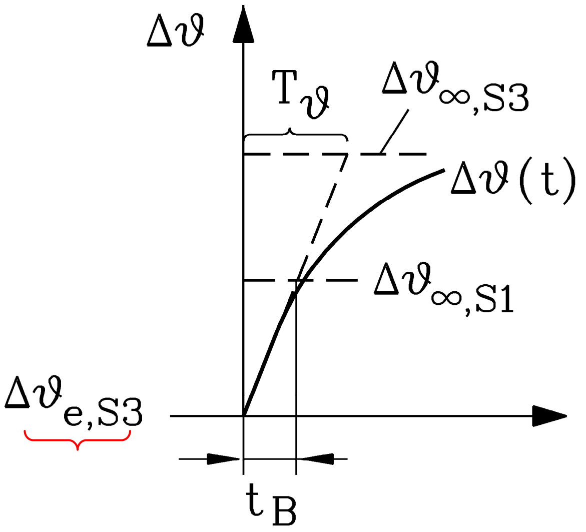
$$t_B \,/\, t_S = 15\ \%,\ \ 25\ \%,\ \ 40\ \%,\ \ 60\ \%$$

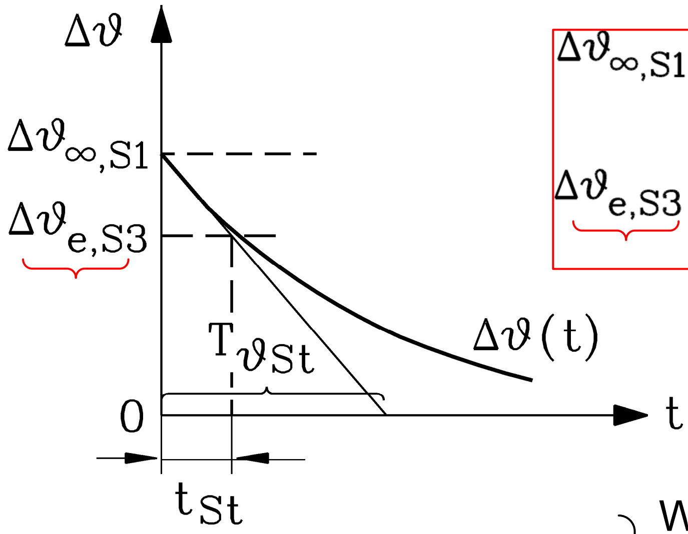
$$t_S = t_B + t_{St}$$

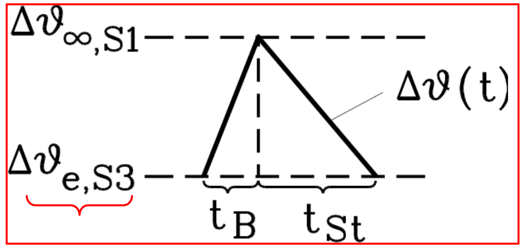
Linear instead of exponential temperature rise function due to $t_B \ll T_\vartheta,\ t_{St} \ll T_{\vartheta,St}$

3. Heat transfer and cooling

Increased power P_{S3} possible at intermittent periodic duty S3







$$\Delta\vartheta_{\infty,S1} - \underbrace{\Delta\vartheta_{e,S3}} = (\Delta\vartheta_{\infty,S3} - \underbrace{\Delta\vartheta_{e,S3}})\cdot\frac{t_B}{T_\vartheta}$$

$$\Delta\vartheta_{\infty,S1} - \underbrace{\Delta\vartheta_{e,S3}} = \Delta\vartheta_{\infty,S1}\cdot\frac{t_{St}}{T_{\vartheta,St}}$$

We eliminate $\underbrace{\Delta\vartheta_{e,S3}}$ and get:

$$\frac{\Delta\vartheta_{\infty,S3}}{\Delta\vartheta_{\infty,S1}} = 1 + \frac{T_\vartheta\cdot t_{St}}{T_{\vartheta,St}\cdot t_B} - \frac{t_{St}}{T_{\vartheta,St}}$$

- **By taking linear instead of exponential temperature rise and fall we obtain:**

$$\boxed{\frac{P_{S3}}{P_{S1}} = \frac{I_{s,S3}}{I_{s,S1}} = \sqrt{\frac{P_{d,S3}}{P_{d,S1}}} = \sqrt{\frac{\Delta\vartheta_{\infty,S3}}{\Delta\vartheta_{\infty,S1}}} = \sqrt{1 + \frac{T_\vartheta\cdot t_{St}}{T_{\vartheta,St}\cdot t_B} - \frac{t_{St}}{T_{\vartheta,St}}}}$$

Example:
500 kW cage induction motor, shaft mounted fan:

Thermal time constants T_ϑ = 40 min, $T_{\vartheta,St}$ = 80 min.

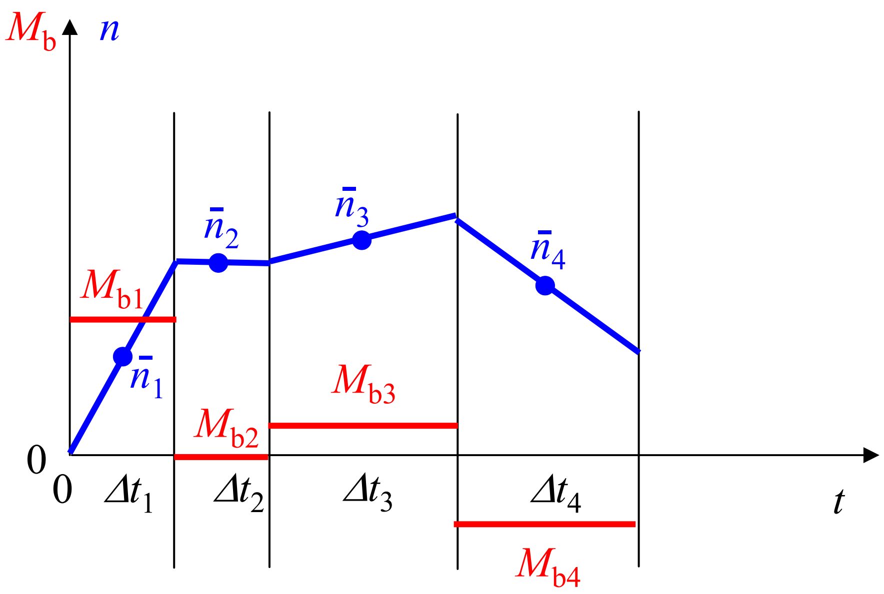
Motor shall be operated in S3 duty with
- operation time t_B = 2 min, stand still time t_{St} = 3 min, both << T_ϑ, $T_{\vartheta,St}$.

- $t_B / t_S = 2/(2+3) = 2/5 = \underline{\underline{40\%}}$
- $\frac{P_{S3}}{P_{S1}} = \sqrt{1 + \frac{T_\vartheta \cdot t_{St}}{T_{\vartheta,St} \cdot t_B} - \frac{t_{St}}{T_{\vartheta,St}}} = \sqrt{1 + \frac{40 \cdot 3}{80 \cdot 2} - \frac{3}{80}} = \sqrt{1.71} = \underline{\underline{1.31}}$

Motor power may be increased for S3-operation by 31% up to 655 kW.

Equivalent thermal torque M_{eff} (1)

- **Arbitrary dynamic duty cycle:**
  K different n_i-M_i-load points with short load durations $\Delta t_i \ll T_\vartheta$ each, $i = 1, \dots, K$:
- Δt_i "short" compared to the thermal time constant T_ϑ: $\Delta t_i / T_\vartheta \ll 1$
- Estimate of winding temperature via the method of „equivalent thermal torque" M_{eff}



- **Torque M_e** ~ current x magnetic flux = $I \cdot \Phi$
- **Losses P_d:** mainly $I^2 \cdot R \sim I^2$
- **Temperature rise** prop. to losses: $\Delta\vartheta \sim P_d$
- Load cycle with K time sections Δt_i, $i = 1, \dots, K$:
- **Duration of load cycle:** $T = \Delta t_1 + \Delta t_2 + \dots + \Delta t_K$
- **Averaged speed** during load cycle:

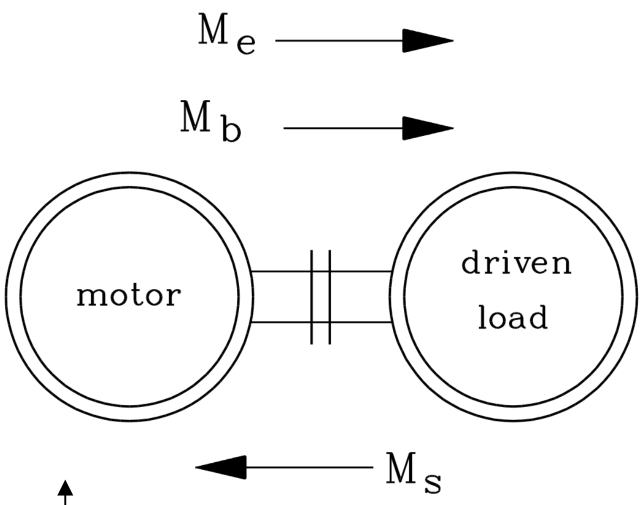
$$n_{av} = \frac{1}{T} \cdot \left(|\bar{n}_1| \cdot \Delta t_1 + |\bar{n}_2| \cdot \Delta t_2 + \dots + |\bar{n}_K| \cdot \Delta t_K \right)$$

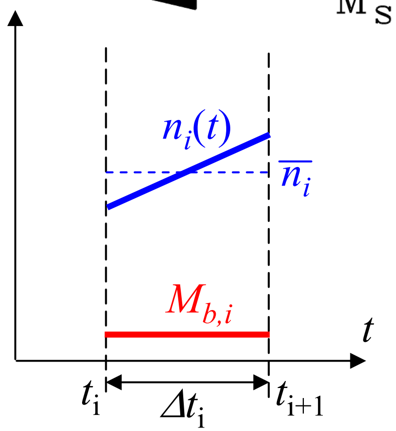
$$\boxed{M_{eff} = \sqrt{\frac{1}{T} \cdot \left(M_{e1}^2 \cdot \Delta t_1 + M_{e2}^2 \cdot \Delta t_2 + \dots + M_{eK}^2 \cdot \Delta t_K \right)}}$$

3. Heat transfer and cooling

Accelerating/braking torque calculation M_b

$$\boxed{J \cdot 2\pi \cdot \frac{dn}{dt} = M_b}$$





- M_e: Electromagnetic torque
- $M_s > 0$ at $n > 0$ and $M_s < 0$ at $n < 0$: Shaft torque of load
- M_b: Accelerating torque
- $M_d > 0$ at $n > 0$ and $M_d < 0$ at $n < 0$:
  Braking torque due to machine losses (friction, …)

$$M_b = M_e - M_d - M_s$$

$M_b > 0$: Acceleration: $dn/dt > 0$

$M_b = 0$: Constant speed: $dn/dt = 0$

$M_b < 0$: Deceleration (braking): $dn/dt < 0$

$M_s = 0$: No-load operation

$$t_i \le t \le t_i + \Delta t_i : M_{b,i} = \text{const.} : n_i(t) = n(t_i) + (M_{b,i} / (J \cdot 2\pi)) \cdot t$$

$$M_{b,i} = M_{e,i} - M_{d,i} - M_{s,i}$$

3. Heat transfer and cooling

Equivalent thermal torque M_{eff} (2)

- **r.m.s. current during duty cycle:** $I_{rms} = \sqrt{\frac{1}{T} \cdot \left(I_1^2 \cdot \Delta t_1 + I_2^2 \cdot \Delta t_2 + \ldots + I_K^2 \cdot \Delta t_K \right)}$
- **Losses during duty cycle:** $P_d = 3 \cdot R \cdot I_{rms}^2 \sim \Delta \vartheta_{Cu}$
- **Equivalent thermal torque:** $M_{eff} \sim I_{rms} \cdot \Phi$ (flux Φ assumed to be constant, e.g. PM flux)

$$\boxed{M_{eff} = \sqrt{\frac{1}{T} \cdot \left(M_{e1}^2 \cdot \Delta t_1 + M_{e2}^2 \cdot \Delta t_2 + \ldots + M_{eK}^2 \cdot \Delta t_K \right)}}$$

- *Result:*
  If M_{eff} and n_{av} are below the rated values M_{N} & n_{N}, then we can expect, that $\vartheta_{\mathrm{Cu}} < \vartheta_{\mathrm{Cu,lim}}$!

$$\boxed{M_{\mathrm{eff}} < M_{\mathrm{N}} , n_{\mathrm{av}} < n_{\mathrm{N}} \Rightarrow \vartheta_{\mathrm{Cu}} < \vartheta_{\mathrm{Cu,lim}}}$$

- For more detailed calculation the transient heat source plot is needed!

3. Heat transfer and cooling

Example: Equivalent torque of PM synchronous motor (1)

Data:

Rated shaft torque $M_N = 17.5$ Nm, maximum shaft torque $M_{e,max} = 42$ Nm

Loss torque (e.g. friction losses), assumed as independent from speed: $M_d = 2.73$ Nm

Rated speed $n_N = 1500$/min, maximum speed $n_{max} = 3000$/min

Electromagnetic torque M_e:

Rated load torque: $M_s = 17.5$ Nm

At braking operation:

Loss torque reduces necessary braking torque: $M_{b,e} = M_e - M_d < 0$

No-load motor operation: $M_e = M_d$

Electromagnetic torque M_e ~ current · magnetic flux = $I \cdot \Phi$

The electromagnetic torque M_e has to be used for the determination of M_{eff} !

3. Heat transfer and cooling

Example: Equivalent torque of PM synchronous motor (2)

i		Δt_i / s	n/min^{-1}	$\lvert\bar{n}_i\rvert$ min^{-1}	$M_{s,i}$/Nm	$M_{e,i}$/Nm
1	Speed up with $M_{e,max}$ from $n = 0$ to n_{max}	0.12	0 ... 3000	1500	0	42 + 2.73 = 44.73
2	Rotate with M_d at n_{max} for 0.6 s	0.6	3000	3000	0	2.73
3	Braking with $-M_{e,max}$ to working speed n	0.08*)	3000 ... 1000	2000	0	-42 + 2.73 = -39.27
4	Load torque M_N at working speed for 3 s	3	1000	1000	17.5	17.5 + 2.73 = 20.23
5	Braking with $-M_{e,max}$ to stand still $n = 0$	0.04**)	1000 ... 0	500	0	-39.27
6	Motor stop for 0.5 s	0.5	0	0	0	0
7	Speed up with $-M_{e,max}$ from $n = 0$ to $-n_{max}$	0.12	0 ... -3000	1500	0	-42 - 2.73 = -44.73
8	Rotate with $-M_d$ at $-n_{max}$ for 1.5 s	1.5	-3000	3000	0	-2.73
9	Braking with $M_{e,max}$ to stand still $n = 0$	0.12	-3000 ... 0	1500	0	42 - 2.73 = 39.27
10	Motor stop for 3 s	3	0	0	0	0
	Duration of duty cycle $T = 9.08$ s	**9.08**		**1100**		**$M_{eff} = 15.2$ Nm**

$$*)\ (2/3)\cdot 0.12 = 0.08\,\text{s} \qquad **)\ (1/3)\cdot 0.12 = 0.04\,\text{s}$$

$M_{eff} = 15.2\ \text{Nm} < M_N = 17.5\ \text{Nm}$

$n_{av} = 1100/\text{min} < n_N = 1500/\text{min}$

The motor winding temperature should be well below the admissible limit!

Summary:
Simplified calculation of temperature rise

- Duty types are ruled by the thermal time constant T_ϑ of „single-body" problem
- Only duty types S1, S2, S3 treated here
- Temperature rise for steady-state, short-time & intermittent operation derived
- The method of equivalent thermal torque M_{eff} may be used for dynamic load cycles, if $\Delta t_i << T_\vartheta$