Energy Converters – CAD and System Dynamics

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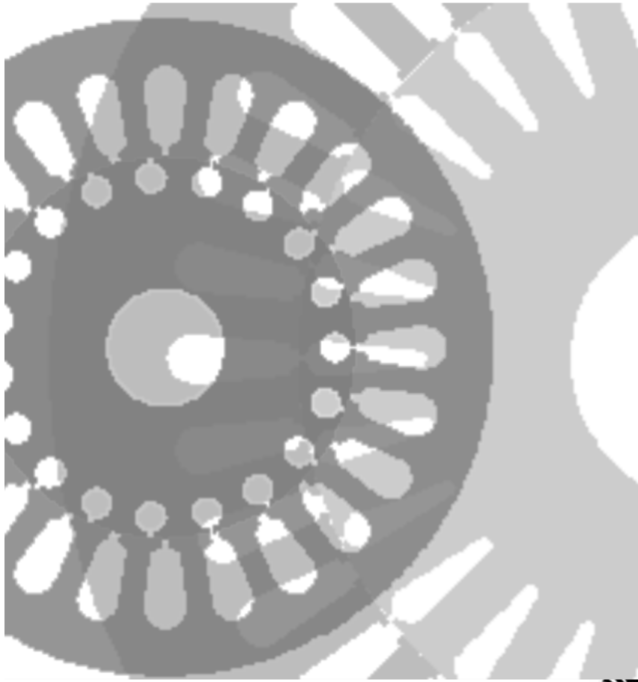
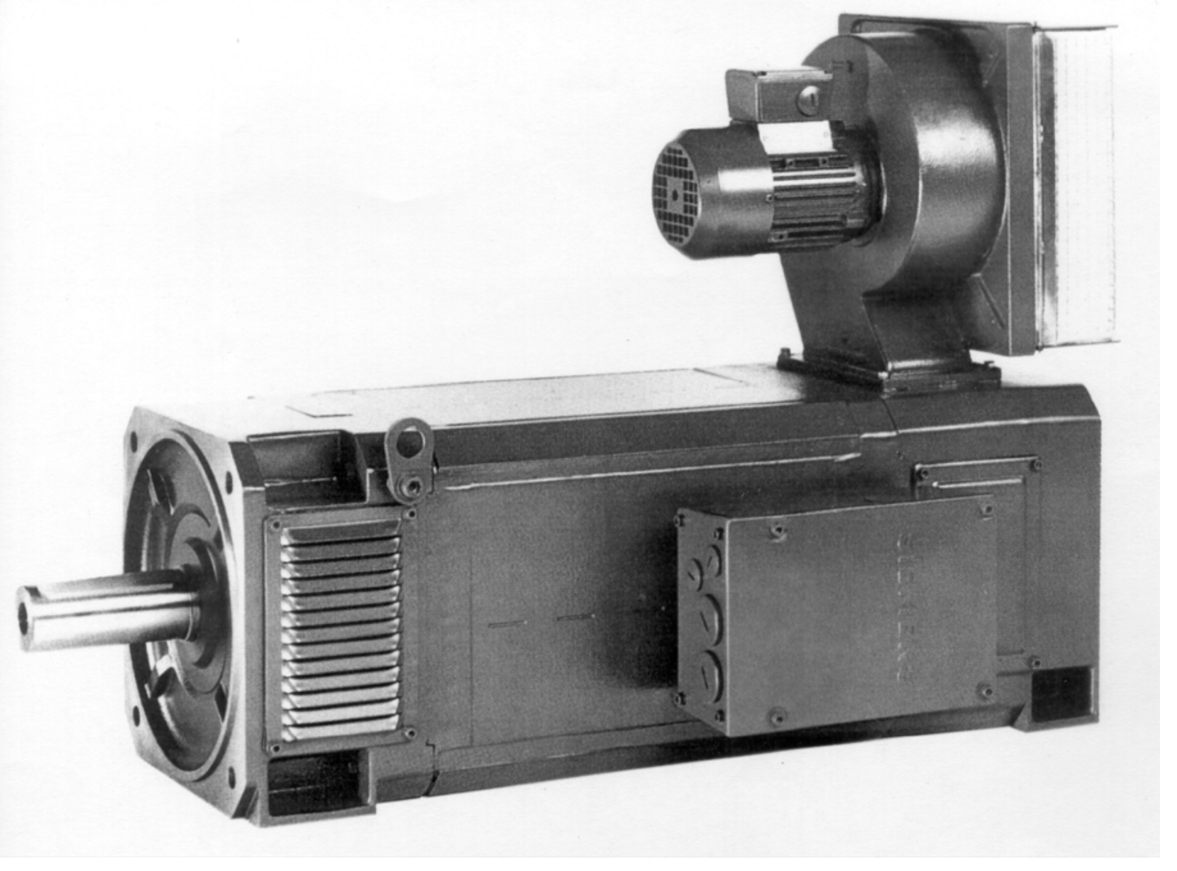
1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source: SPEED program

5. Dynamics of DC machines



Source: Siemens AG

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5. Dynamics of DC machines

5. Dynamics of DC machines

Dynamic system equations of separately excited DC machines

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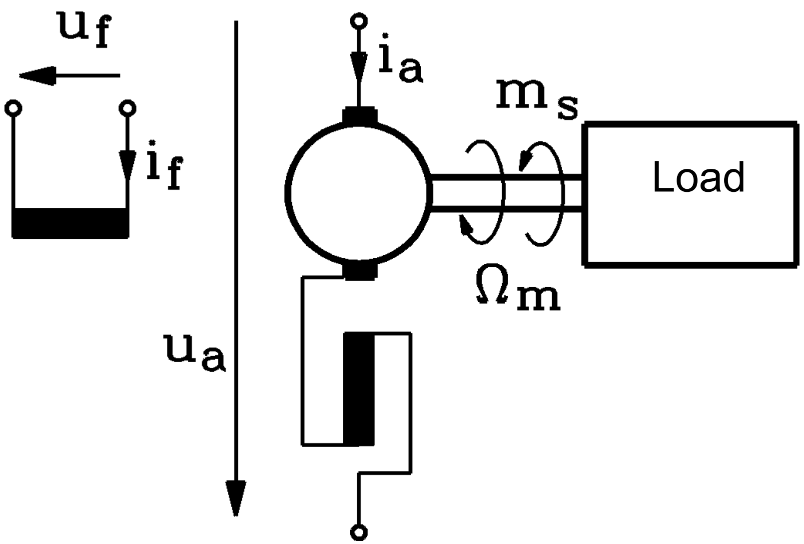


Machine constant: $k_2 = \frac{1}{2\pi} \cdot \frac{z \cdot 2p}{2a}$

Induced voltage of motion: $u_i(t) = k_2 \cdot \Omega_m(t) \cdot \Phi(i_f)$

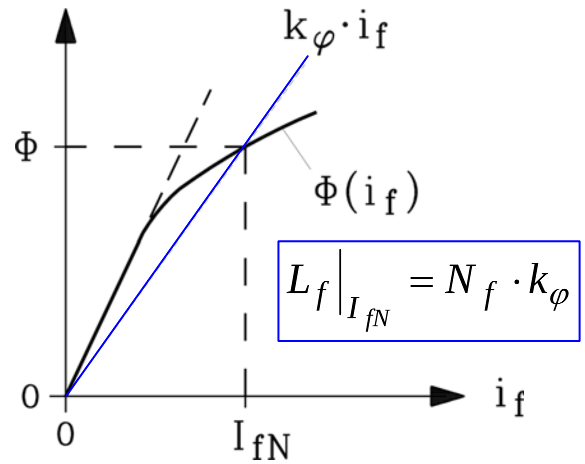
Machine torque: $m_e(t) = k_2 \cdot i_a(t) \cdot \Phi(i_f)$

Main flux per pole: $\Phi(i_f)$

$\Omega_m(t) = 2\pi \cdot n(t)$

Armature circuit: $u_a(t) = R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + u_i(t)$

Mechanical acceleration: $J \cdot \frac{d\Omega_m}{dt} = m_e(t) - m_s(t)$



Field circuit: $u_f(t) = R_f \cdot i_f(t) + L_f \cdot \frac{di_f(t)}{dt}$

Non-linear differential equations, even if L_f = const.

5. Dynamics of DC machines

Saturation-dependent field inductance $L_f(i_f)$

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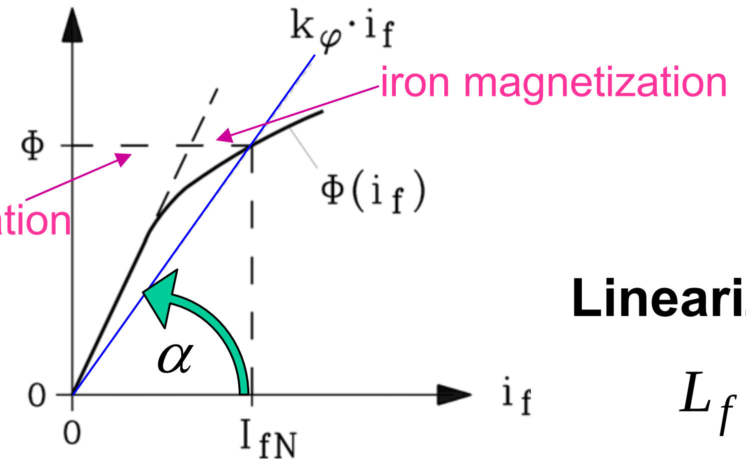
Main flux per pole: $\Phi(i_f)$

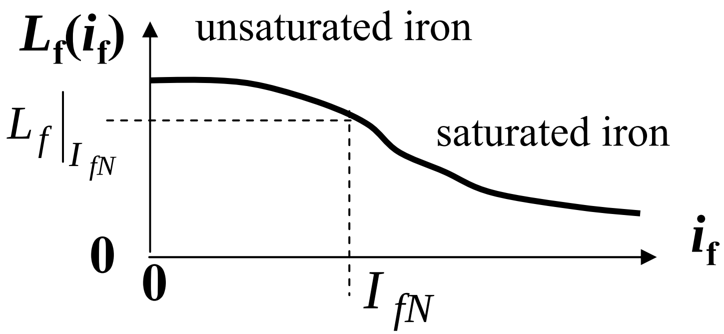


Linearized field inductance:

$$L_f\big|_{I_{fN}} = N_f \cdot k_\varphi \cdot i_f / i_f$$

$$L_f\big|_{I_{fN}} = N_f \cdot k_\varphi$$

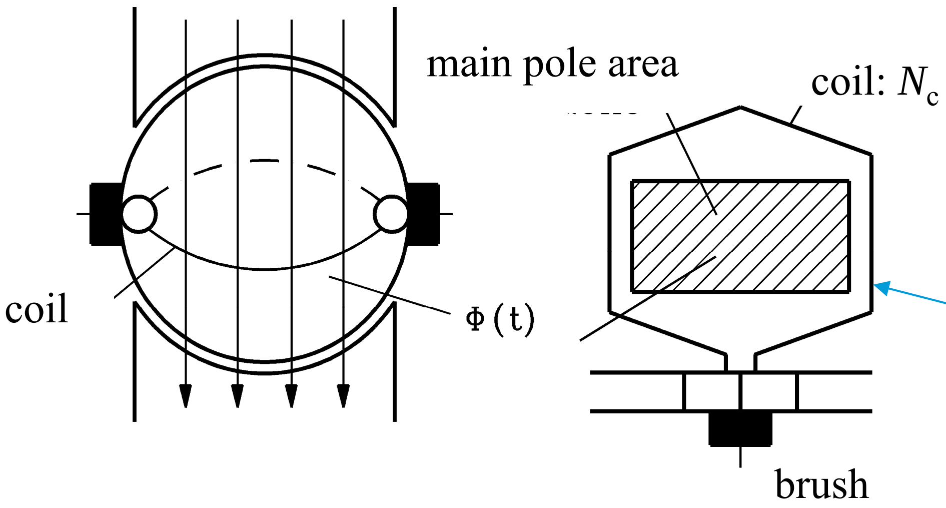
Field inductance: $L_f(i_f) = N_f \cdot \Phi(i_f)/i_f \sim \tan\alpha$



5. Dynamics of DC machines
No magnetic coupling between armature and field circuit

- Armature field: **Armature self-inductance L_a**
  Main field: **Field self-inductance L_f** .
- **Mutual inductance M_{af}** only between

a) commutating armature coils (= short-circuited by brushes) and field coil: $M_{af,com}$,
b) otherwise **zero: M_{af} = 0.**



$$u_a(t) = i_a(t)\cdot R_a + L_a \cdot di_a(t)/dt + u_i(t)$$
$$u_i(t) = k_2 \cdot \Omega_m(t)\cdot \Phi(t),\ \ \Phi(t) = \Phi(i_f(t))$$
$$u_f(t) = i_f(t)\cdot R_f + d(L_f(i_f)\cdot i_f)/dt$$

$M_{af,com}$ neglected ⇒ M_{af} = 0, only L_a, L_f needed!

Armature and field circuit decoupled!

L_a = const.: Unsaturated small armature field!

Operation at constant field current I_f

$$u_f(t) = U_f = R_f \cdot I_f$$

$$\boxed{\begin{aligned} u_a(t) &= R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + k_2 \cdot \Omega_m(t) \cdot \Phi \\ J \cdot \frac{d\Omega_m}{dt} &= k_2 \cdot i_a(t) \cdot \Phi - m_s(t) \end{aligned}}$$

If flux is kept constant, then the set of **differential equations is linear.**

- One linear differential equation of second order with constant coefficients:

$$\frac{d^2 i_a}{dt^2} + \frac{1}{T_a} \cdot \frac{di_a}{dt} + \frac{1}{T_a \cdot T_m} \cdot i_a = \frac{1}{T_a \cdot R_a} \cdot \frac{du_a}{dt} + \frac{1}{T_a \cdot T_m} \cdot \frac{1}{k_2 \Phi} \cdot m_s$$

$$\frac{d^2 \Omega_m}{dt^2} + \frac{1}{T_a} \cdot \frac{d\Omega_m}{dt} + \frac{1}{T_a \cdot T_m} \cdot \Omega_m = \frac{1}{T_a \cdot T_m} \cdot \frac{1}{k_2 \Phi} u_a - \frac{1}{T_a} \cdot \frac{1}{J} \cdot m_s - \frac{1}{J} \cdot \frac{dm_s}{dt}$$

- Electrical time constant of armature: $\boxed{T_a = L_a / R_a}$
- Mechanical time constant of machine and load: $\boxed{T_m = \frac{J \cdot R_a}{(k_2 \Phi)^2}}$

5. Dynamics of DC machines

Features of separately excited DC machine at constant main flux

- Changing of armature current and rotor speed is ruled by armature voltage u_a and is disturbed by load torque m_s, which both are contained in "right side" of system differential equation.

- DC machine at constant main flux Φ = const.: LINEAR system: DC machine may be controlled in an easy way.

- Mechanical time constant $T_m \sim 1/\Phi^2$ decreases via the square of increasing flux.

- Machine gets "weaker" ($T_m \uparrow$) at:

a) Flux weakening $\Phi\downarrow$

b) Increased stator resistance R_a, which causes voltage drop, thus reducing internal voltage of motion (= induced voltage) U_i,

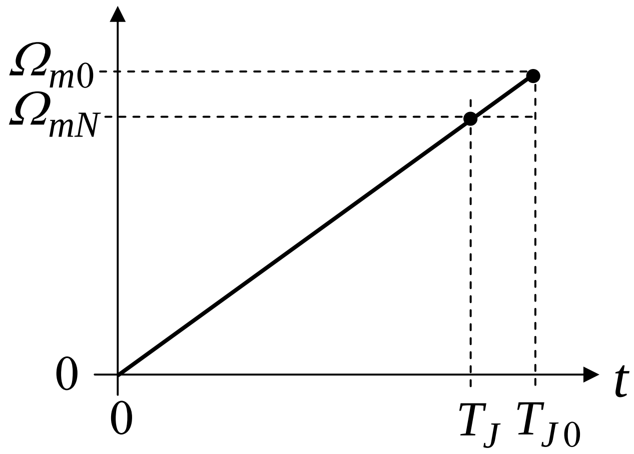
$\Rightarrow$ Mechanical time constant = response to load step: T_m increases !

5. Dynamics of DC machines

Starting time constant T_J of electric machines

$$\boxed{T_J = \frac{J_M \cdot \Omega_{mN}}{M_N} = \frac{J_M \cdot 2\pi \cdot n_N}{M_N}}$$

$$J_M \cdot \frac{d\Omega_m}{dt} = M_N \quad \rightarrow \quad \Omega_m(t) = \frac{M_N}{J_M} \int_{t=0}^{t} dt = \frac{M_N}{J_M} \cdot t$$



- Relationship between starting time constant T_J and mechanical time constant T_m is given with per unit resistance r_a and flux ϕ:

$$r_a = \frac{R_a}{U_N / I_N}, \phi = \Phi / \Phi_N : \quad T_{J0} = J_M \cdot \frac{\Omega_{m0}}{M_N} = J_M \cdot \frac{U_N /(k_2 \Phi_N)}{k_2 \Phi_N I_N} = T_m \cdot \frac{1}{r_a} \cdot \left(\frac{\Phi}{\Phi_N} \right)^2$$

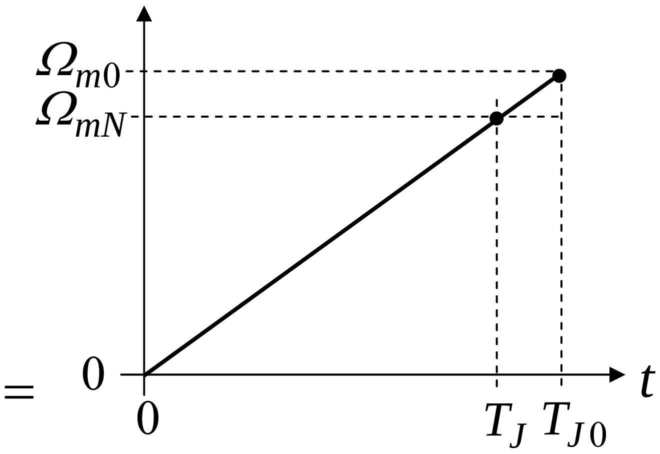
- *Example:*

$$r_a = 0.05,\ \Phi = \Phi_N :\ T_{J0} = 20 \cdot T_m, \quad T_{J0} = 10\ \text{s},\ T_m = 0.5\ \text{s}$$

5. Dynamics of DC machines

Starting time constant T_J vs. mechanical time constant T_m

$$r_a = \frac{R_a}{U_N / I_N} = \frac{R_a}{Z_N}, \quad \phi = \Phi / \Phi_N :$$



$$T_{J0} = J_M \cdot \frac{\Omega_{m0}}{M_N} = J_M \cdot \frac{U_N /(k_2 \Phi_N)}{k_2 \Phi_N I_N} = J_M \cdot \frac{U_N}{I_N} \cdot \frac{1}{(k_2 \Phi_N)^2} =$$

$$= \underbrace{J_M \cdot \frac{R_a}{(k_2 \Phi)^2}}_{T_m} \cdot \frac{1}{R_a} \cdot \frac{U_N}{I_N} \cdot \frac{(k_2 \Phi)^2}{(k_2 \Phi_N)^2} = T_m \cdot \frac{1}{r_a} \cdot \left(\frac{\Phi}{\Phi_N} \right)^2 = T_m \cdot \frac{1}{r_a} \cdot \phi^2$$

Starting time to no-load speed n_0: $\quad T_{J0} = J_M \cdot \frac{\Omega_{m0}}{M_N}$

Starting time to rated speed n_N: $\quad T_J = J_M \cdot \frac{\Omega_{mN}}{M_N} < T_{J0}$

Summary:
Dynamic system equations of separately excited DC machine

- Separately excited DC machine treated in this lecture
- Second order differential electro-mechanical equation for armature circuit
- At constant main flux Φ = const.: Linear differential equation
- Long mechanical and short electrical time constant $T_m >> T_a$
- Do not mix mechanical time constant T_m and starting time constant T_J!

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5. Dynamics of DC machines

5.1 Dynamic system equations of separately excited DC machine

5.2 Dynamic response of electrical and mechanical system of separately excited DC machine

5.3 Dynamics of coupled electric-mechanical system of separately excited DC machine

5.4 Linearized model of separately excited DC machine for variable flux

5.5 Transfer function of separately excited DC machine

5.6 Dynamic simulation of separately excited DC machine

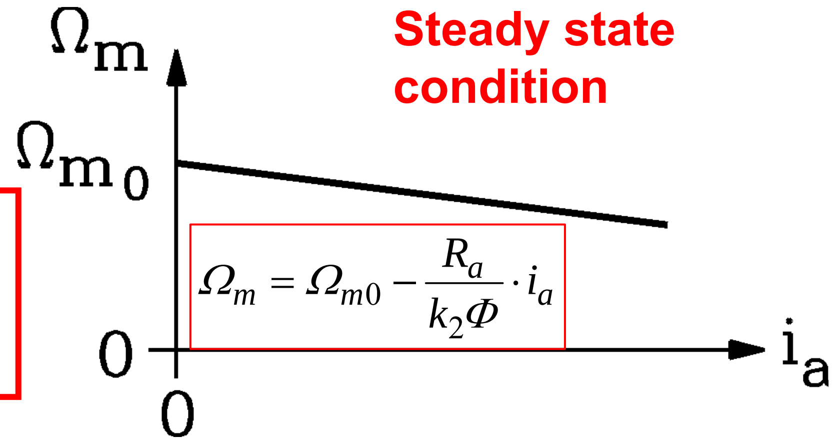
5.7 Converter operated separately excited DC machine

Dynamics of mechanical system of DC machine (1)

Taking $T_a << T_m$ leads to $T_a \to 0$:

$$\boxed{\frac{d\Omega_m}{dt} + \frac{1}{T_m} \cdot \Omega_m = \frac{1}{T_m} \cdot \frac{1}{k_2 \Phi} u_a - \frac{1}{J} \cdot m_s}$$

Steady state condition



a) Steady state condition: d./dt = 0 :

$$\Omega_m = \frac{1}{k_2 \Phi} u_a - \frac{T_m}{J} \cdot m_s = \Omega_{m0} - \frac{T_m}{J} \cdot m_s$$

$$\Omega_{m0} = \frac{1}{k_2 \Phi} u_a$$ No-load speed

$$\boxed{\Omega_m = \Omega_{m0} - \frac{R_a}{k_2 \Phi} \cdot i_a}$$

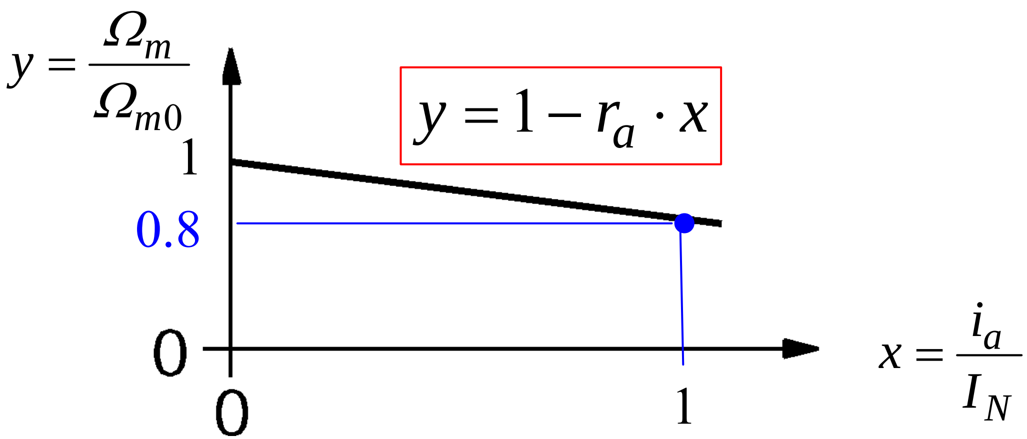
$$\left.\begin{aligned} m_s &= m_e = k_2 \Phi \cdot i_a \\ T_m / J &= R_a / (k_2 \Phi)^2 \end{aligned}\right\} \frac{T_m}{J} m_s = \frac{R_a}{k_2 \Phi} i_a$$

5. Dynamics of DC machines
Steady-state characteristic in p.u.

$$\Omega_m = \Omega_{m0} - \frac{R_a}{k_2\Phi}\cdot i_a \quad \Rightarrow \quad \frac{\Omega_m}{\Omega_{m0}} = 1 - \frac{R_a}{k_2\Phi\cdot\Omega_{m0}}\cdot i_a$$

$$\Phi = \Phi_N,\ \Omega_m = \Omega_{m0} \Leftrightarrow i_a = 0 : u_i = k_2\Phi_N\cdot\Omega_{m0} = u_a = U_N$$

$$\frac{\Omega_m}{\Omega_{m0}} = 1 - \frac{R_a}{U_N / I_N}\cdot\frac{i_a}{I_N} = 1 - r_a\cdot\frac{i_a}{I_N}$$



Example:

Small two-pole PM-excited DC motor:
$r_a = 0.2$

Speed at rated current:

$$\frac{\Omega_m}{\Omega_{m0}} = 1 - r_a\cdot\frac{i_a}{I_N} = 1 - 0.2\cdot 1 = 0.8$$

5. Dynamics of DC machines
Dynamics of mechanical system of DC machine (2)

$$\boxed{\frac{d\Omega_m}{dt} + \frac{1}{T_m} \cdot \Omega_m = \frac{1}{T_m} \cdot \frac{1}{k_2 \Phi} u_a - \frac{1}{J} \cdot m_s}$$

b) Dynamic operation:

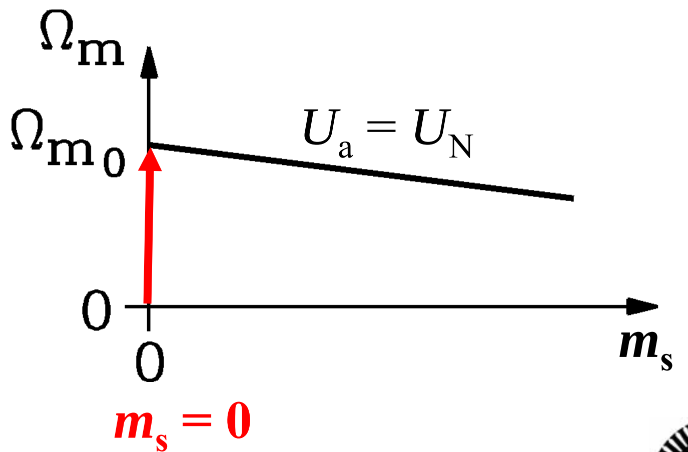
Example:

Switching on of DC machine at a) rated flux and b) no-load $m_s = 0$:

Initial condition: Zero speed $\Omega_m(0) = 0$. Armature voltage is switched from zero to rated value $U_a = U_N$ for $t > 0$:

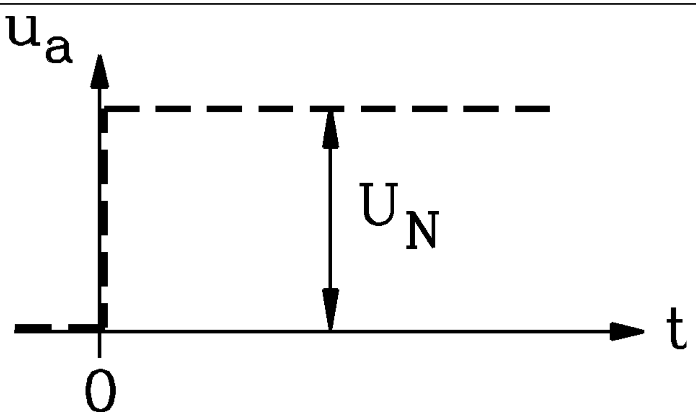
$$\frac{d\Omega_m}{dt} + \frac{1}{T_m} \cdot \Omega_m = \frac{1}{T_m} \cdot \frac{1}{k_2 \Phi} u_a \qquad \Omega_{m0} = \frac{1}{k_2 \Phi} u_a$$

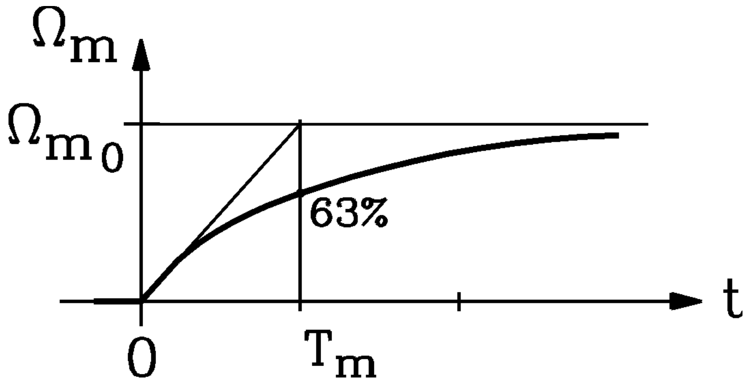
Solution: $\boxed{\Omega_m(t) = \Omega_{m0} \cdot \left(1 - \exp(-t / T_m)\right)}$



5. Dynamics of DC machines

Dynamics of mechanical system of DC machine (3)





$$\boxed{\Omega_m(t) = \Omega_{m0} \cdot \left(1 - \exp(-t / T_m)\right)} \qquad t \geq 0$$

Dynamic speed response of separately excited DC machine to switching with armature voltage step, leading to exponential increase of speed !

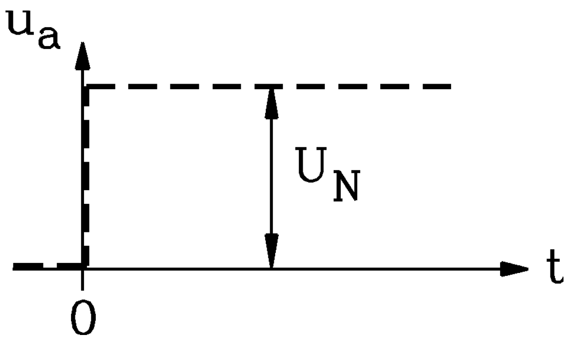
$$m_e = J \cdot \frac{d\Omega_m}{dt} = J \cdot \frac{\Omega_{m0}}{T_m} \cdot \exp(-t / T_m)$$ **yields armature current** $$i_a = \frac{U_N}{R_a} \cdot \exp(-t / T_m)$$

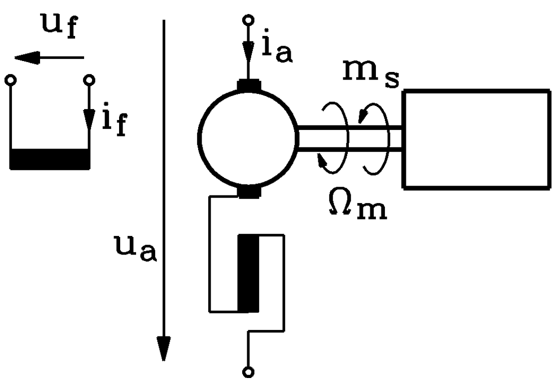
$(m_s = 0)$ $\qquad t \geq 0$

$$\frac{m_e}{k_2\Phi} = i_a \Rightarrow \frac{J\Omega_{m0}}{T_m \cdot k_2\Phi} = J \frac{u_a}{k_2\Phi} \cdot \frac{1}{k_2\Phi} \cdot \frac{(k_2\Phi)^2}{JR_a} = \frac{u_a}{R_a}$$

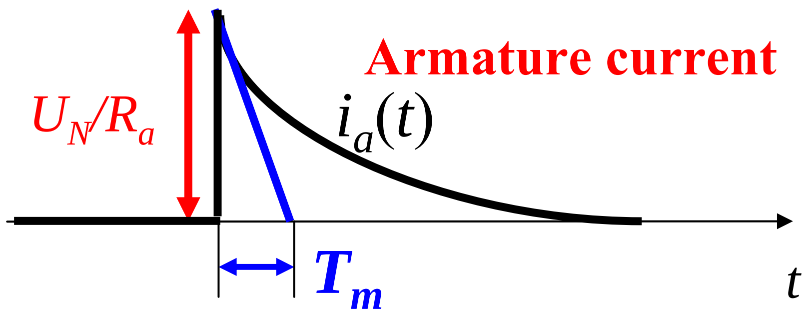
5. Dynamics of DC machines

Dynamics of mechanical system of DC machine (4)





- "Jumping" armature current (due to T_a = 0) and decreasing armature current due to speeding up of the rotor to no-load speed.
- At (ideal) no-load the armature current also in motor operation is zero!



$$\boxed{i_a(t) = \frac{U_N}{R_a} \cdot e^{-t/T_m}}$$

Attention: Starting resistor is necessary:

$$\frac{R_a}{U_N / I_N} = 0.025 \quad \rightarrow \quad \hat{i}_a = \frac{U_N}{R_a} = \frac{I_N}{0.025} = 40 \cdot I_N$$

Too big!

5. Dynamics of DC machines

Dynamics of electrical system of DC machine (1)

Taking $T_a << T_m$ leads to $T_m \to \infty$, if electrical system only is investigated.

$$\boxed{\frac{di_a}{dt} + \frac{1}{T_a} \cdot i_a = \frac{u_a}{T_a \cdot R_a} - \frac{k_2 \Phi \cdot \Omega_m}{T_a \cdot R_a} = \frac{u_a - u_i}{L_a}}$$

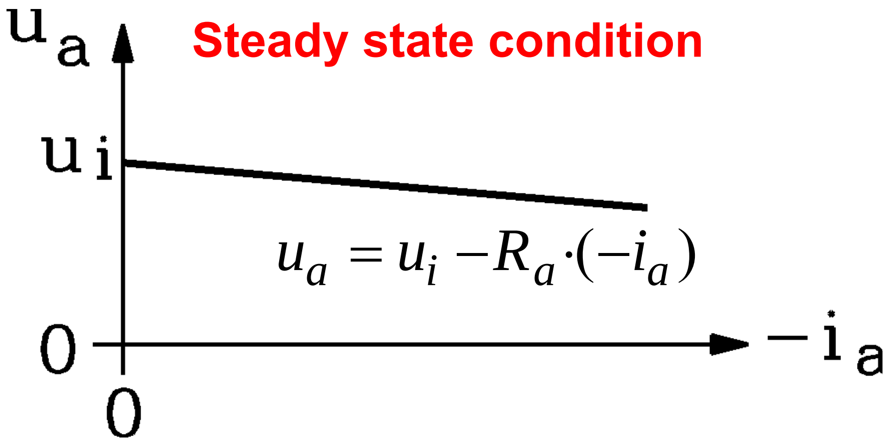
$J \to \infty$:

$d\Omega_m / dt = 0 \to \Omega_m = const.$

a) *Steady state condition:* *d./dt* = 0:

$$i_a = I_a = \frac{u_a - u_i}{R_a} = \frac{U_a - U_i}{R_a}$$

$$u_a = u_i + i_a \cdot R_a = u_i - (-i_a) \cdot R_a$$



Consumer reference system: $-i_a > 0$: Generator operation

5. Dynamics of DC machines

Dynamics of electrical system of DC machine (2)

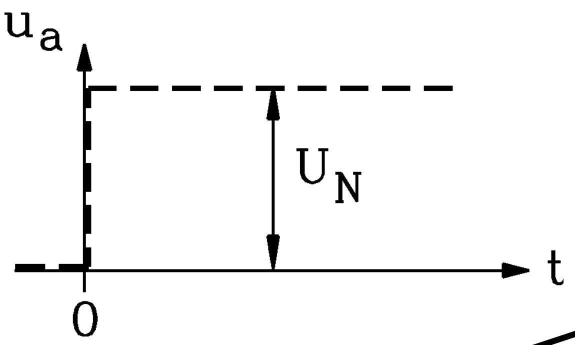
b) Dynamic operation:

Example:

Switching on of DC machine at rated flux, no-load, zero speed: $\Omega_m = 0$

Initial condition: Zero current $i_a(0) = 0$

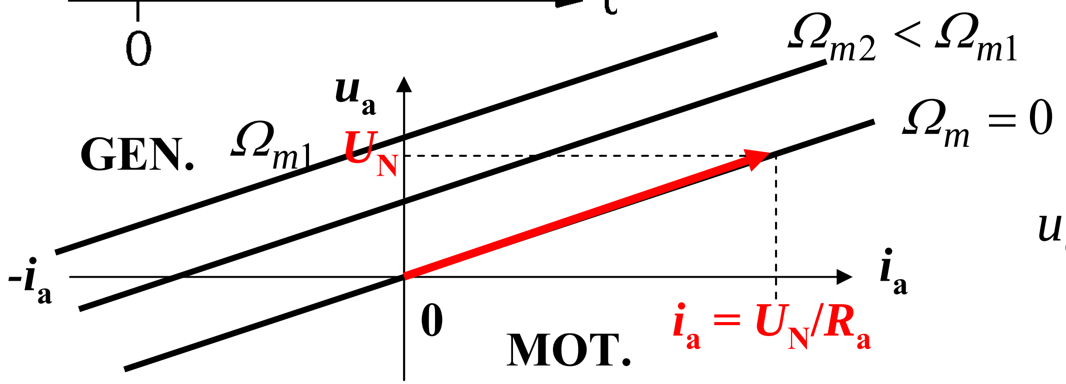
Armature voltage is switched from zero to rated value $u_a = U_N$ for $t > 0$.



$$\frac{di_a}{dt} + \frac{1}{T_a} \cdot i_a = \frac{U_N}{T_a \cdot R_a}$$

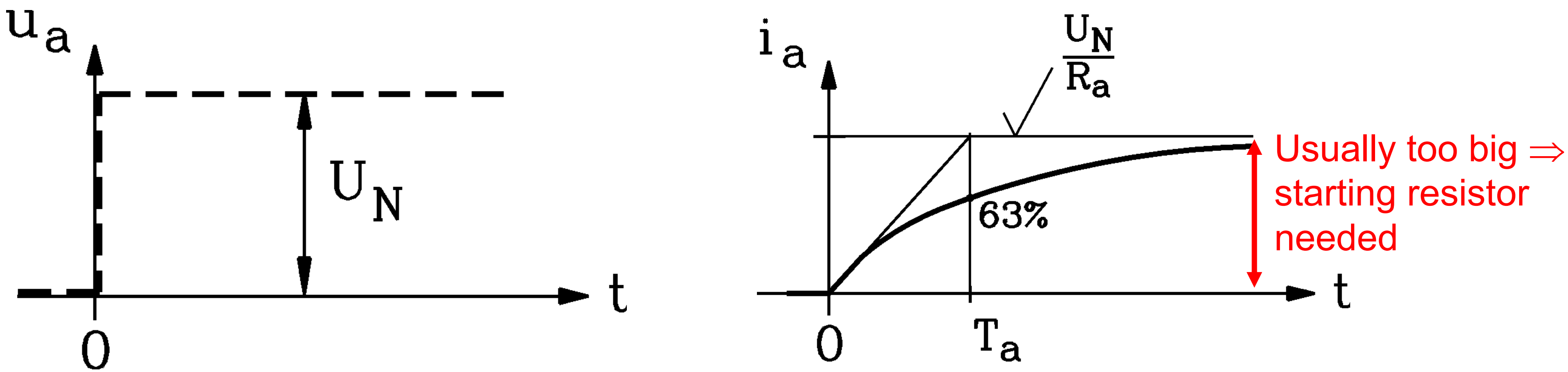
Solution: $t \geq 0$

$$\boxed{i_a(t) = \frac{U_N}{R_a} \cdot \left(1 - e^{-t/T_a}\right)}$$



$$u_a = u_i + i_a \cdot R_a = k_2 \cdot \Omega_m \cdot \Phi_N + i_a \cdot R_a$$

Dynamics of electrical system of DC machine (3)



$$\boxed{i_a(t) = \frac{U_N}{R_a} \cdot \left(1 - e^{-t/T_a}\right)} \qquad t \geq 0$$

- Dynamic current response of separately excited DC machine at stand still to switching with armature voltage step of rated voltage, leading to exponential increase of armature current.

Armature current and so electromagnetic torque react to armature voltage steps with the short armature time constant T_a.

- So separately excited DC machines are dynamic drives.

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Summary:
Dynamic response of electrical and mechanical system of separately excited DC machine

- Separate solving of electrical and mechanical machine behaviour
- Steady-state solution and step response
- Consumer reference system used
- Steady-state solutions already known from bachelor´s course

5. Dynamics of DC machines

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5. Dynamics of DC machines
Dynamics of coupled electric-mechanical system (1)

$$\boxed{\frac{d^2\Omega_m}{dt^2}+\frac{1}{T_a}\cdot\frac{d\Omega_m}{dt}+\frac{1}{T_a\cdot T_m}\cdot\Omega_m=\frac{1}{T_a\cdot T_m}\cdot\frac{1}{k_2\Phi}u_a(t)-\frac{1}{T_a}\cdot\frac{1}{J}\cdot m_s(t)-\frac{1}{J}\cdot\frac{dm_s(t)}{dt}}$$

Homogeneous differential equation: $\frac{d^2\Omega_m}{dt^2}+\frac{1}{T_a}\cdot\frac{d\Omega_m}{dt}+\frac{1}{T_a\cdot T_m}\cdot\Omega_m=0$

$$\Omega_{mh}(t)=C_1\cdot e^{\lambda_1 t}+C_2\cdot e^{\lambda_2 t}$$

Characteristic equation for λ:

$$\lambda^2+\frac{1}{T_a}\cdot\lambda+\frac{1}{T_a\cdot T_m}=0 \quad\rightarrow\quad \lambda_{1,2}=-\frac{1}{2T_a}\pm\frac{1}{2T_a}\cdot\sqrt{1-\frac{4T_a}{T_m}}$$

If $T_m<4T_a$, then $\sqrt{1-\frac{4T_a}{T_m}}=j\cdot\sqrt{\frac{4T_a}{T_m}-1}$

5. Dynamics of DC machines
Dynamics of coupled electric-mechanical system (2)

a)	$T_m > 4T_a$	λ_1, λ_2 are real values, so transient speed response contains **TWO time constants** $T_1 = -1/\lambda_1, T_2 = -1/\lambda_2$, a short and a long one.
b)	$T_m = 4T_a$	$\lambda_1 = \lambda_2 = \lambda$ is real value, so transient speed response contains **ONE time constant** $T = -1/\lambda$ **("aperiodic limit").**
c)	$T_m < 4T_a$	$\underline{\lambda}_1, \underline{\lambda}_2$ are complex values $\underline{\lambda}_1 = -\delta + j\cdot\omega_d$, $\underline{\lambda}_2 = -\delta - j\cdot\omega_d$, so transient speed response **oscillates** with **frequency** $f_d = \omega_d/(2\pi)$, which is damped by **damping coefficient** δ.

a) $\Omega_{mh}(t) = C_1 \cdot e^{-t/T_1} + C_2 \cdot e^{-t/T_2}$

long time constant: $T_1 = \dfrac{2T_a}{1 - \sqrt{1 - \dfrac{4T_a}{T_m}}}$, **short** time constant $T_2 = \dfrac{2T_a}{1 + \sqrt{1 - \dfrac{4T_a}{T_m}}}$

$\boxed{T_1 \le T_m}$ $\boxed{T_2 \ge T_a}$

5. Dynamics of DC machines

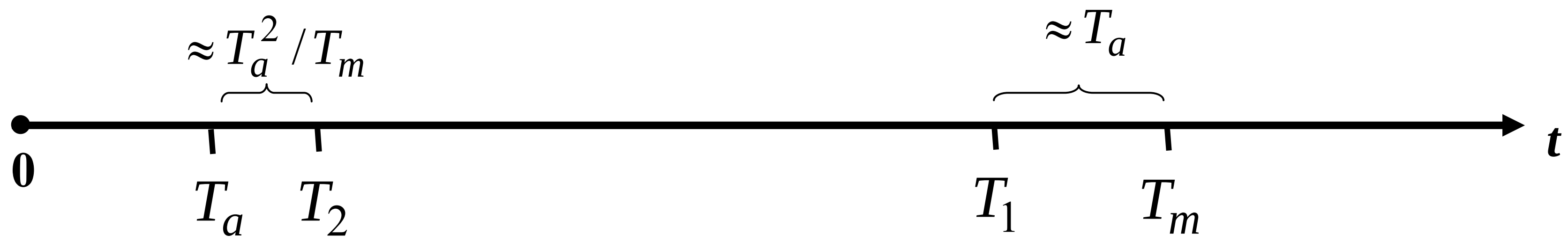
Long and short time constant T_1 and T_2

Long time constant: $\boxed{T_1 \le T_m}$

$$T_1\Big|_{T_a<<T_m} = \frac{2T_a}{1-\sqrt{1-\dfrac{4T_a}{T_m}}} \approx \frac{2T_a}{1-\left(1-\dfrac{2T_a}{T_m}-2\left(\dfrac{T_a}{T_m}\right)^2\right)} = \frac{T_m}{1+\dfrac{T_a}{T_m}} \approx T_m - T_a$$

Short time constant: $\boxed{T_2 \ge T_a}$

$$T_2\Big|_{T_a<<T_m} = \frac{2T_a}{1+\sqrt{1-\dfrac{4T_a}{T_m}}} \approx \frac{2T_a}{1+\left(1-\dfrac{2T_a}{T_m}\right)} = \frac{T_a}{1-\dfrac{T_a}{T_m}} \approx T_a \cdot \left(1+\frac{T_a}{T_m}\right)$$



Dynamics of coupled electric-mechanical system (3)

b) $\Omega_{mh}(t) = C_1 \cdot e^{-t/T} + C_2 \cdot t \cdot e^{-t/T}$, $T = 2T_a$

c) $\Omega_{mh}(t) = A \cdot e^{-\delta \cdot t} \cdot \cos(\omega_d \cdot t) + B \cdot e^{-\delta \cdot t} \cdot \sin(\omega_d \cdot t)$ *A, B*: integration constants

- Damping coefficient: $\delta = \dfrac{1}{2T_a}$
- Eigen-frequency: $f_d = \dfrac{1}{2\pi \cdot T_a} \cdot \sqrt{\dfrac{T_a}{T_m} - \dfrac{1}{4}} = \dfrac{1}{2\pi \cdot T_a} \cdot \sqrt{\dfrac{T_a}{T_J} \cdot \dfrac{\phi^2}{r_a} - \dfrac{1}{4}}$, Period: $T_d = \dfrac{1}{f_d}$
- Number N_H of half periods, until oscillations are damped to 5%:

$$e^{-t^* \cdot \delta} = 0.05 \quad \rightarrow \quad t^* = 3/\delta \quad \rightarrow \quad N_H = \frac{t^*}{T_d/2} = \frac{3/\delta}{\pi/\omega_d} \approx \frac{\omega_d}{\delta}$$

Dynamics of coupled electric-mechanical system (4)

Example:

Separately excited DC machine:

Inertia T_J = 1 s, electric parameters: T_a = 50 ms, r_a = 0.05, $\Phi = \Phi_N$.

- **With $T_J \cong T_{J0}$: Mechanical time constant:** $T_m = T_J \cdot \frac{r_a}{\phi^2} = 1 \cdot 0.05 / 1^2 = \underline{\underline{50}}$ **ms**
- **As** $T_m = 50ms < 4T_a = 4 \cdot 50 = 200ms$, **DC machine oscillates:**
- **frequency** $f_d = \frac{1}{2\pi \cdot 0.05} \cdot \sqrt{\frac{0.05}{0.05} - \frac{1}{4}} = \underline{\underline{2.76}}$ **Hz**
- **angular frequency** $\omega_d = 2\pi f_d = \underline{\underline{17.3}} /$ **s,**
- **oscillation period of** $T_d = 1/f_d = 1/2.76 = \underline{\underline{362}}$ **ms.**
- **Damping coefficient:** $\delta = \frac{1}{2T_a} = \frac{1}{2 \cdot 0.05} = \underline{\underline{10}} /$ **s**
- **After** $N_H \approx \frac{\omega_d}{\delta} = \frac{17.3}{10} = \underline{\underline{1.73}}$ **half-periods the oscillation is damped down to 5% of initial value.**

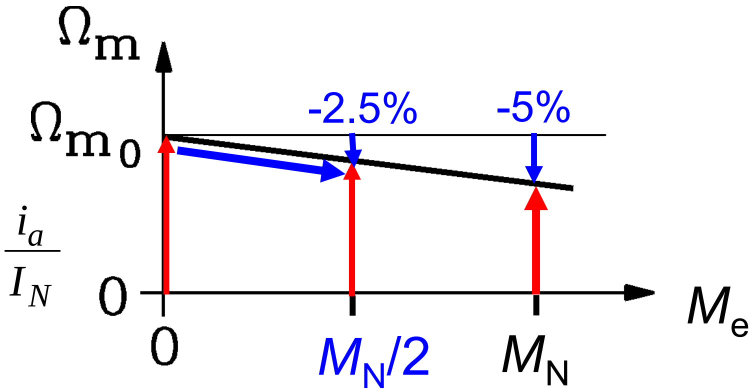
5. Dynamics of DC machines
Speed response to mechanical load torque step

Data: $T_J = 1s, T_a = 50ms, r_a = 0.05, \phi = 1$, step in shaft torque of $\Delta M_s / M_N = 0.5$

Result: $\omega_d = 17.3/s, T_d = 363ms, T_m = 50ms$

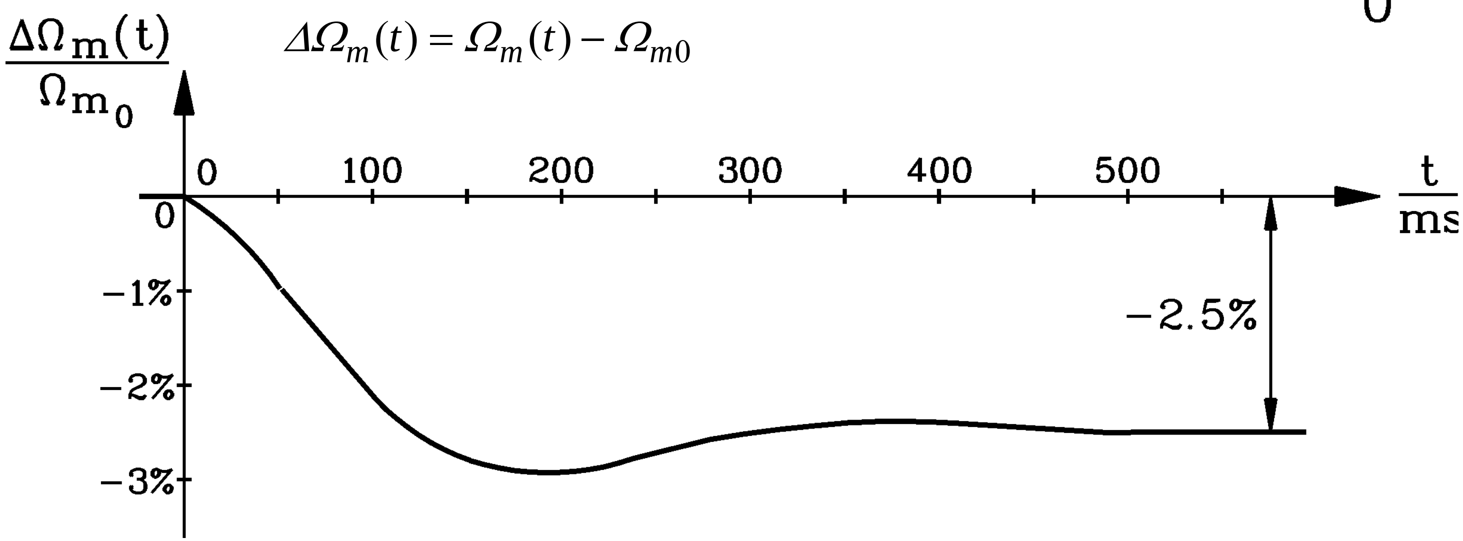
$$\Omega_m = \Omega_{m0} - \frac{R_a}{k_2\Phi} \cdot I_a$$

$$\frac{\Omega_m}{\Omega_{m0}} = 1 - \frac{R_a}{U_a} \cdot I_a \bigg|_{U_N} = 1 - r_a \cdot \frac{i_a}{I_N}$$



Transient speed response:

$$\Delta\Omega_m(t) = \Omega_m(t) - \Omega_{m0}$$

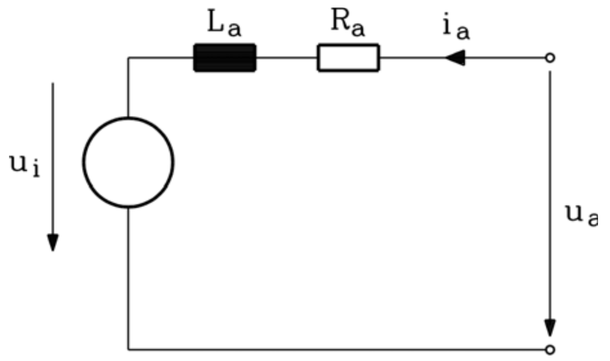


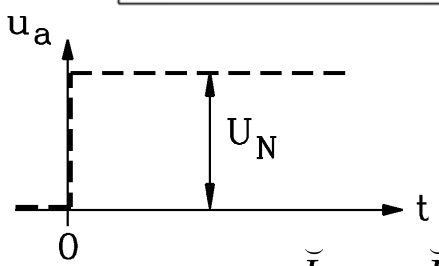
5. Dynamics of DC machines

Example: Start-up of unloaded, separately excited DC motor

($m_s = 0$, Φ = const., $4 \cdot T_a < T_m$) (1)

Tutorial





$$\frac{d^2 i_a}{dt^2} + \frac{1}{T_a} \cdot \frac{di_a}{dt} + \frac{1}{T_a \cdot T_m} \cdot i_a = \frac{1}{T_a \cdot R_a} \cdot \frac{du_a}{dt} + \frac{1}{T_a \cdot T_m} \cdot \frac{1}{k_2 \Phi} \cdot m_s$$

$$\frac{d^2 i_a}{dt^2} + \frac{1}{T_a} \cdot \frac{di_a}{dt} + \frac{1}{T_a \cdot T_m} \cdot i_a = \frac{1}{T_a \cdot R_a} \cdot \frac{du_a}{dt}$$

$$\Omega_m(0) = 0 \Rightarrow u_i(0) = 0, \quad i_a(0) = 0 \quad L_a \cdot i_a'(0) = u_a(0) - u_i(0) - R_a i_a(0) = u_a(0)$$

$$L\{i_a'\} = s \cdot \breve{I}_a - i_a(0) = s \cdot \breve{I}_a \qquad L\{i_a''\} = s^2 \cdot \breve{I}_a - s \cdot i_a(0) - i_a'(0) = s^2 \cdot \breve{I}_a - u_a(0)/L_a$$

$$s^2 \breve{I}_a + \frac{s \cdot \breve{I}_a}{T_a} + \frac{\breve{I}_a}{T_a \cdot T_m} - \cancel{\frac{u_a(0)}{L_a}} = \frac{s \cdot \breve{U}_a - \cancel{u_a(0)}}{T_a R_a} \qquad \breve{U}_a = U_a / s$$

$$s^2 \breve{I}_a + \frac{s \cdot \breve{I}_a}{T_a} + \frac{\breve{I}_a}{T_a \cdot T_m} = \frac{U_a}{T_a R_a} \Rightarrow \breve{I}_a = \frac{U_a}{T_a R_a} \cdot \frac{1}{s^2 + s/T_a + 1/(T_a T_m)} = \frac{U_a}{T_a R_a} \cdot \frac{1}{\lambda_1 - \lambda_2} \cdot \left(\frac{1}{s - \lambda_1} - \frac{1}{s - \lambda_2} \right)$$

$$i_a(t) = \frac{U_a}{2 T_a R_a} \cdot \frac{2 T_a}{\sqrt{1 - 4 T_a / T_m}} \cdot \left(e^{-t/T_1} - e^{-t/T_2} \right)$$

$4 \cdot T_a < T_m$: λ_1, λ_2 are real numbers!

$$s^2 + s/T_a + 1/(T_a \cdot T_m) = (s - \lambda_1) \cdot (s - \lambda_2)$$

$$\boxed{i_a(t) = \frac{U_a / R_a}{\sqrt{1 - 4 T_a / T_m}} \cdot \left(e^{-t/T_1} - e^{-t/T_2} \right)}$$

e.g.: $T_1 \approx T_m$, $T_2 \approx T_a$

$$\lambda_1 - \lambda_2 = \frac{1}{T_a} \cdot \sqrt{1 - 4 T_a / T_m}$$

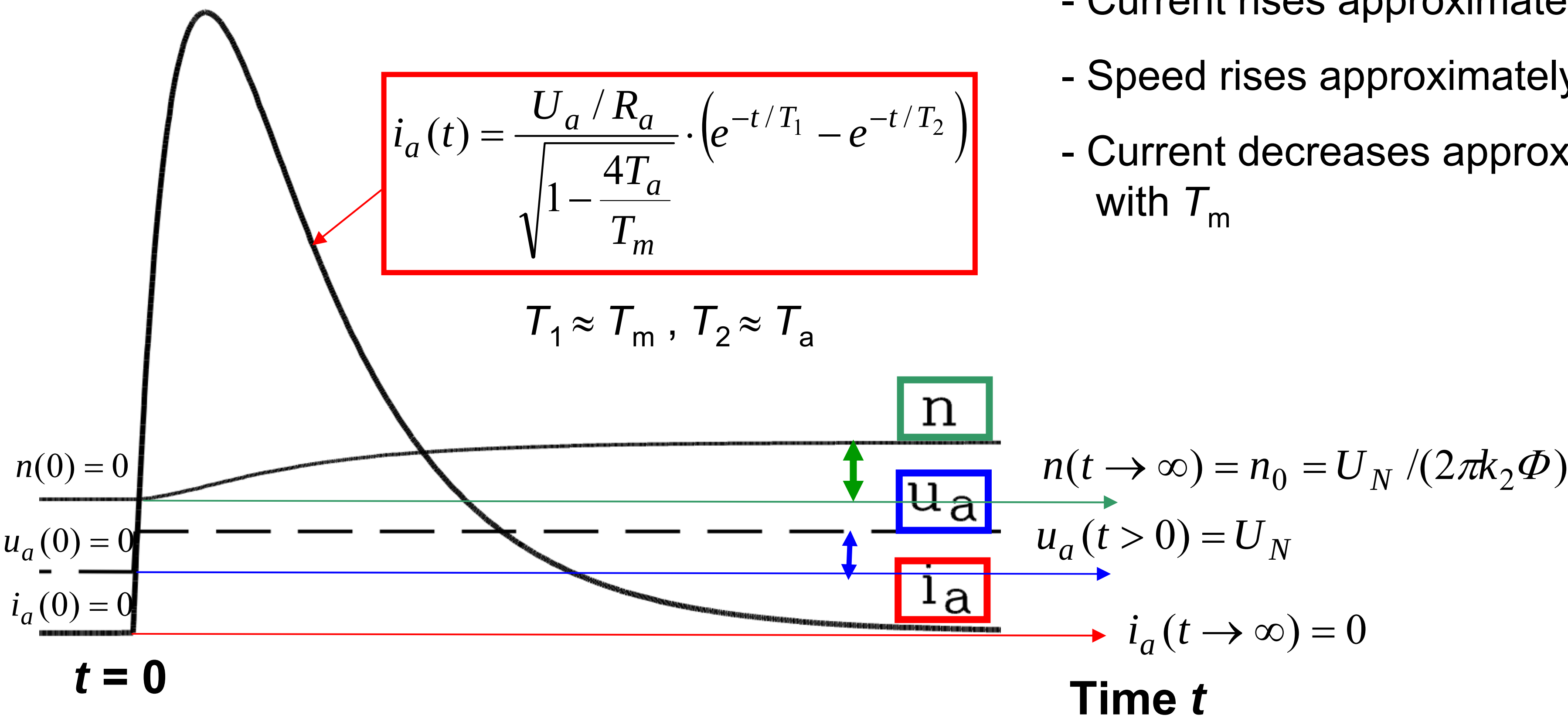
Example: Start-up of unloaded, separately excited DC motor ($m_s = 0$, Φ = const., $4 \cdot T_a < T_m$) (2)

Qualitative solution:

- Current rises approximately with T_a
- Speed rises approximately with T_m
- Current decreases approximately with T_m

$$i_a(t) = \frac{U_a / R_a}{\sqrt{1 - \frac{4T_a}{T_m}}} \cdot \left(e^{-t/T_1} - e^{-t/T_2}\right)$$

$T_1 \approx T_m$, $T_2 \approx T_a$



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Summary:
Dynamics of coupled electric-mechanical system of separately excited DC machine

- Mechanical and electrical transients are
  a) change of speed and b) change of armature current
- DC machine may oscillate, if mechanical time constant is “short”: $< 4T_a$
- In most cases the big load inertia does not allow any oscillation
- Dynamic current overshoot calculated during motor start-up

5. Dynamics of DC machines
Linearized model for variable flux

$$u_a(t) = i_a(t)\cdot R_a + L_a \cdot di_a(t)/dt + \boxed{k_2 \cdot \Omega_m(t)\cdot \Phi(t)} \qquad \Phi(t) = \Phi(i_f(t))$$

$$J \cdot d\Omega_m(t)/dt = \boxed{k_2 \cdot \Phi(t)\cdot i_a(t)} - m_s(t)$$

Non-linear expressions

$$u_f(t) = i_f(t)\cdot R_f + \boxed{d(L_f(i_f)\cdot i_f)/dt} \qquad L_f = N_f \cdot \Phi(i_f)/i_f$$

- For investigations of small disturbances the equations are linearized !
- Small transient deviations $\Delta u_a(t)$, $\Delta i_a(t)$, $\Delta \Omega_m(t)$, $\Delta m_s(t)$, $\Delta \Phi(t)$, $\Delta i_f(t)$, $\Delta u_f(t)$ from the steady state operation U_a, I_a, Ω_m, M_s, Φ, I_f, U_f !

5. Dynamics of DC machines
Linearized model variables

- For investigations of small disturbances the equations are linearized !
- Small transient deviations:

$$u_a(t) = U_a + \Delta u_a(t)$$

$$i_a(t) = I_a + \Delta i_a(t)$$

$$\Omega_m(t) = \Omega_m + \Delta\Omega_m(t)$$

$$m_s(t) = M_s + \Delta m_s(t)$$

$$\Phi(t) = \Phi + \Delta\Phi(t)$$

$$u_f(t) = U_f + \Delta u_f(t)$$

$$i_f(t) = I_f + \Delta i_f(t)$$

- **Small transient deviations** from steady state operation:
  e.g.: $\Delta u_a(t)$ from U_a

- "Small": Per unit deviation < 10%: $\underbrace{|\Delta u_a(t)/U_a|}_{<0.1} << 1$

5. Dynamics of DC machines

Linearized differential equations at variable flux

- Neglect product of small deviations in voltage equation:

$$u_i(t) = k_2 \cdot (\Omega_m + \Delta\Omega_m(t)) \cdot (\Phi + \Delta\Phi(t)) = k_2 \cdot \Omega_m \cdot \Phi \cdot (1 + \frac{\Delta\Omega_m(t)}{\Omega_m}) \cdot (1 + \frac{\Delta\Phi(t)}{\Phi})$$

$$(1 + \frac{\Delta\Omega_m(t)}{\Omega_m}) \cdot (1 + \frac{\Delta\Phi(t)}{\Phi}) = 1 + \frac{\Delta\Omega_m(t)}{\Omega_m} + \frac{\Delta\Phi(t)}{\Phi} + \frac{\Delta\Omega_m(t)}{\Omega_m} \cdot \frac{\Delta\Phi(t)}{\Phi} \approx 1 + \frac{\Delta\Omega_m(t)}{\Omega_m} + \frac{\Delta\Phi(t)}{\Phi}$$

$$1.21 = (1 + 0.1) \cdot (1 + 0.1) = 1 + 0.1 + 0.1 + 0.1 \cdot 0.1 = 1 + 0.1 + 0.1 + 0.01 \approx 1 + 0.1 + 0.1 = 1.20$$

- Result for induced voltage $u_i(t)$ and torque $m_e(t)$:

$$\boxed{u_i(t) \cong k_2 \cdot \Omega_m \cdot \Phi + k_2 \cdot \Delta\Omega_m(t) \cdot \Phi + k_2 \cdot \Omega_m \cdot \Delta\Phi(t) = U_i + \Delta u_{i,\Omega_m}(t) + \Delta u_{i,\Phi}(t)}$$

$$\boxed{m_e(t) = k_2 \cdot (I_a + \Delta i_a(t)) \cdot (\Phi + \Delta\Phi(t)) \approx \underbrace{k_2 \cdot I_a \cdot \Phi}_{M_e = k_2 \cdot I_a \cdot \Phi} + k_2 \cdot \Delta i_a(t) \cdot \Phi + k_2 \cdot I_a \cdot \Delta\Phi(t)}$$

5. Dynamics of DC machines
Linearized dynamic equations (1)

A) LINEAR **differential equations, neglecting products of Δ :**

$$U_a + \Delta u_a \approx R_a \cdot (I_a + \Delta i_a) + L_a \cdot \frac{d(I_a + \Delta i_a)}{dt} + U_i + k_2 \cdot \Delta\Omega_m \cdot \Phi + k_2 \cdot \Omega_m \cdot \Delta\Phi$$

$$J \cdot \frac{d(\Omega_m + \Delta\Omega_m)}{dt} \approx M_e + k_2 \cdot \Delta i_a \cdot \Phi + k_2 \cdot I_a \cdot \Delta\Phi - M_s - \Delta m_s$$

$$\boxed{\frac{dI_a}{dt} = 0, \quad \frac{d\Omega_m}{dt} = 0}$$

$$\cancel{U_a} + \Delta u_a \approx R_a \cdot (\cancel{I_a} + \Delta i_a) + L_a \cdot \frac{d\Delta i_a}{dt} + \cancel{U_i} + k_2 \cdot \Delta\Omega_m \cdot \Phi + k_2 \cdot \Omega_m \cdot \Delta\Phi$$

$$J \cdot \frac{d\Delta\Omega_m}{dt} \approx \cancel{M_e} + k_2 \cdot \Delta i_a \cdot \Phi + k_2 \cdot I_a \cdot \Delta\Phi - \cancel{M_s} - \Delta m_s$$

Stationary equilibrium cancels !

$$\boxed{\begin{aligned} \Delta u_a(t) &\approx R_a \cdot \Delta i_a(t) + L_a \cdot \frac{d\Delta i_a(t)}{dt} + k_2 \cdot \Delta\Omega_m(t) \cdot \Phi + k_2 \cdot \Omega_m \cdot \Delta\Phi(t) \\ J \cdot \frac{d\Delta\Omega_m(t)}{dt} &\approx k_2 \cdot \Delta i_a(t) \cdot \Phi + k_2 \cdot I_a \cdot \Delta\Phi(t) - \Delta m_s(t) \end{aligned}}$$

5. Dynamics of DC machines
Linearized dynamic equations (2)

NON-LINEAR **field circuit:** $\Phi(t) = \Phi(i_f(t))$

$$u_f(t) = R_f \cdot i_f(t) + N_f \cdot \frac{d\Phi(t)}{dt}$$

$$u_f(t) = U_f + \Delta u_f = R_f \cdot I_f + R_f \cdot \Delta i_f + N_f \cdot \underbrace{\frac{d(\Phi(I_f) + \Delta\Phi(t))}{dt}}_{\frac{d\Delta\Phi(t)}{dt}}$$

$$\cancel{U_f} + \Delta u_f = R_f \cdot \cancel{I_f} + R_f \cdot \Delta i_f + N_f \cdot \frac{d\Delta\Phi(t)}{dt}$$

$$\boxed{\Delta u_f = R_f \cdot \Delta i_f + N_f \cdot \frac{d\Delta\Phi(t)}{dt}}$$

5. Dynamics of DC machines
Linearized dynamic equations (3)

B) LINEAR **differential equations of deviations :**

$$\Delta u_a(t) \approx R_a \cdot \Delta i_a(t) + L_a \cdot \frac{d\Delta i_a(t)}{dt} + k_2 \cdot \Delta\Omega_m(t) \cdot \Phi + k_2 \cdot \Omega_m \cdot \Delta\Phi(t)$$

$$J \cdot \frac{d\Delta\Omega_m(t)}{dt} \approx k_2 \cdot \Delta i_a(t) \cdot \Phi + k_2 \cdot I_a \cdot \Delta\Phi(t) - \Delta m_s(t)$$

$$\Delta u_f \approx R_f \cdot \Delta i_f + N_f \cdot \frac{d\Delta\Phi(t)}{dt}$$

"Small signal theory":

Linear differential equation system is only valid within the limits of deviation from steady state operation of about +/- 10 ... 20 %.

Constant parameters of differential equations depend on steady state values.

5. Dynamics of DC machines

Linearized dynamic equations (4)

C) In case of linear systems NO linearization is necessary:
"Large signal theory"!

Example:

Separately excited DC machines with constant flux operation !

$$\Delta\Phi(t) = 0 \Rightarrow \Phi = const. \Rightarrow i_f = I_f = const. \Leftrightarrow \Delta i_f = 0$$

$$\Delta u_a(t) = R_a \cdot \Delta i_a(t) + L_a \cdot \frac{d\Delta i_a(t)}{dt} + k_2 \cdot \Delta\Omega_m(t) \cdot \Phi$$

$$J \cdot \frac{d\Delta\Omega_m(t)}{dt} = k_2 \cdot \Delta i_a(t) \cdot \Phi - \Delta m_s(t) \qquad \Delta u_f = R_f \cdot \Delta i_f = 0$$

$$u_a(t) = R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + k_2 \cdot \Omega_m(t) \cdot \Phi$$

$$J \cdot \frac{d\Omega_m(t)}{dt} = k_2 \cdot i_a(t) \cdot \Phi - m_s(t) \qquad u_f = U_f = R_f \cdot i_f = R_f \cdot I_f$$

Summary:
Linearized model of separately excited DC machine for variable flux

- Method of linearization of non-linear equations shown
- Linearization only possible for small disturbances („perturbation method“)
- Perturbation method leads to small signal theory
- Separately excited DC machine at constant main flux linear also for large signals

5. Dynamics of DC machines

Separately excited DC machine: Transfer function (1)

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- Here: Constant flux operation (Φ = const.) = linear system !

- Initial conditions set to zero:

$$\Delta u_a(0)=0,\ \Delta i_a(0)=0,\ \Delta\Omega_m(0)=0,\ \Delta\Phi(0)=0,\ \Delta m_s(0)=0,\ \Delta i_f(0)=0$$

- *Laplace* transform: $\boxed{L\left\{\frac{d\Delta i_a(t)}{dt}\right\}=s\cdot\Delta\breve{i}_a(s)-\Delta i_a(0)=s\cdot\Delta\breve{i}_a(s)}$ **and so on ....**

Φ = const.:

$$\boxed{\begin{aligned}&\Delta\breve{u}_a(s)=R_a\cdot\Delta\breve{i}_a(s)+s\cdot L_a\cdot\Delta\breve{i}_a(s)+k_2\cdot\Delta\breve{\Omega}_m(s)\cdot\Phi\\&J\cdot s\cdot\Delta\breve{\Omega}_m(s)=k_2\cdot\Delta\breve{i}_a(s)\cdot\Phi-\Delta\breve{m}_s(s)\end{aligned}}$$

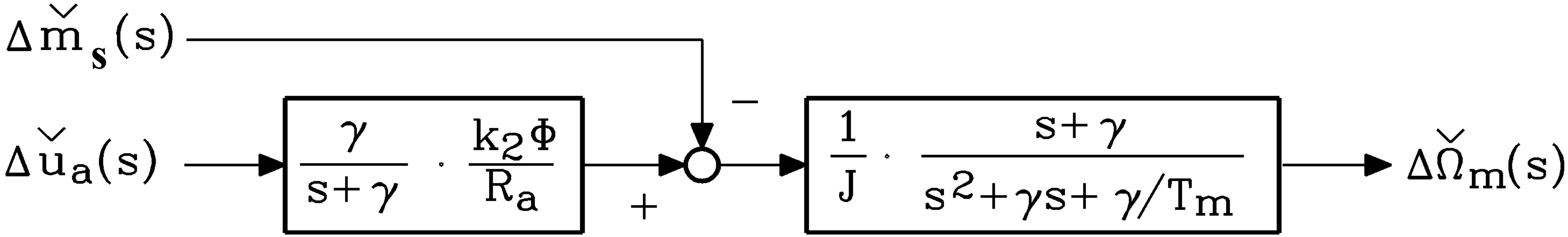
$$\boxed{\Delta\breve{\Omega}_m(s)=\frac{s+\gamma}{s^2+\gamma\cdot s+\frac{\gamma}{T_m}}\cdot\frac{1}{J}\cdot\left[\frac{\gamma}{s+\gamma}\cdot\frac{k_2\Phi}{R_a}\cdot\Delta\breve{u}_a(s)-\Delta\breve{m}_s(s)\right]}\qquad\gamma=1/T_a$$

5. Dynamics of DC machines

Separately excited DC machine: Transfer function (2)

$$\boxed{\Delta\breve{\Omega}_m(s) = \frac{s+\gamma}{s^2+\gamma\cdot s+\dfrac{\gamma}{T_m}}\cdot\frac{1}{J}\cdot\left[\frac{\gamma}{s+\gamma}\cdot\frac{k_2\Phi}{R_a}\cdot\Delta\breve{u}_a(s)-\Delta\breve{m}_s(s)\right]}$$

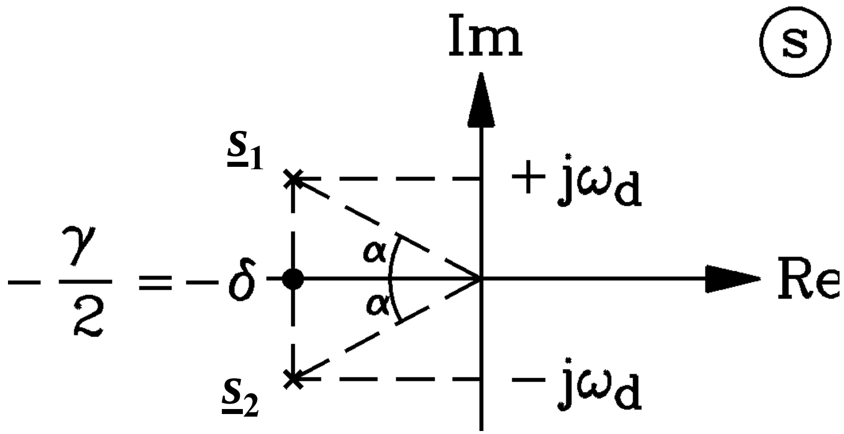
$$\gamma = 1/T_a = 2\delta$$



5. Dynamics of DC machines

Characteristic polynomial of 2nd order differential equation

$$\boxed{P(s) = 0 = s^2 + \gamma \cdot s + \frac{\gamma}{T_m} = (s - \underline{s}_1) \cdot (s - \underline{s}_2)}$$

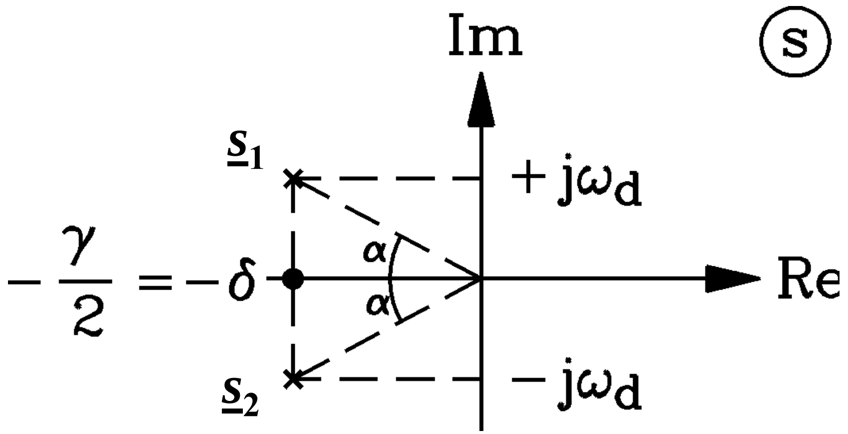


„Roots“: If $T_m < 4T_a$: Complex:

$$\underline{s}_{1,2} = \underline{\lambda}_{1,2} = -\delta \pm j \cdot \omega_d$$

$$\boxed{\tan\alpha = \frac{\omega_d}{\delta} = N_H}$$

Roots characteristic polynomial of 2nd order in *s*-plane



$$\tan\alpha = \frac{\omega_d}{\delta} = N_H$$

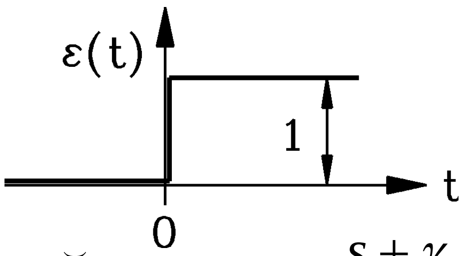
- Real part of roots in the **left half plane** $(\mathrm{Re}(\underline{s}_1), \mathrm{Re}(\underline{s}_2) < 0)$: **Stable** operation.
- **If imaginary part** of roots is zero $(\mathrm{Im}(\underline{s}_1) = \mathrm{Im}(\underline{s}_2) = 0)$: **No oscillations** occur.
- Real part of roots $\mathrm{Re}(\underline{s}_1), \mathrm{Re}(\underline{s}_2)$ *small*: Time constants are **very long**.
- **Pairs of conjugate complex roots** $\underline{s}_1, \underline{s}_2$: Sine & cosine function as transients.
- Imaginary part of root is far off origin $(\mathrm{Im}(\underline{s}_1) >> 1)$: Oscillation frequency **is high**.
- **Tangent of angle α** = number N_H of half periods of transient oscillation, till damped to 5% of initial value.

5. Dynamics of DC machines

Example: Speed response to mechanical load torque step (1)

- **Constant armature voltage, load step after no-load operation:**

$u_a(t) = U_a = const.;\ \Delta u_a(t) = 0$ Load torque step: $\boxed{\Delta m_s(t) = \Delta M_s \cdot \varepsilon(t)}$



$$\varepsilon(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\boxed{L\{\Delta m_s(t)\} = \frac{\Delta M_s}{s}}$$

$$\Delta\breve{\Omega}_m(s) = \frac{s+\gamma}{s^2 + \gamma \cdot s + \frac{\gamma}{T_m}} \cdot \frac{1}{J} \cdot \left[\frac{\gamma}{s+\gamma} \cdot \frac{k_2\Phi}{R_a} \cdot 0 - \frac{\Delta M_s}{s} \right] = -\frac{s+\gamma}{s^2 + \gamma \cdot s + \frac{\gamma}{T_m}} \cdot \frac{\Delta M_s}{J \cdot s}$$

- **Decomposition in single terms for inverse *Laplace* transform: for $t > 0$**

***Laplace* domain:**

$$\frac{s+2\delta}{(s+\delta)^2 + \omega_d^2} \cdot \frac{1}{s} = \frac{2\delta}{\delta^2 + \omega_d^2} \cdot \left(\frac{1}{s} - \frac{s+\delta}{(s+\delta)^2 + \omega_d^2} + \frac{\omega_d^2 - \delta^2}{2\delta \cdot \omega_d} \cdot \frac{\omega_d}{(s+\delta)^2 + \omega_d^2} \right)$$

Time domain:

$$T_m \cdot \left(1 - e^{-\delta \cdot t} \cdot \cos(\omega_d \cdot t) + \frac{\omega_d^2 - \delta^2}{2\delta \cdot \omega_d} \cdot e^{-\delta \cdot t} \cdot \sin(\omega_d \cdot t) \right)$$

5. Dynamics of DC machines

Side calculations for inverse *Laplace* transform

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$$\frac{s+\gamma}{s^2+\gamma\cdot s+\frac{\gamma}{T_m}}\cdot\frac{1}{s}=\frac{s+2\delta}{(s+\delta)^2+\omega_d^2}\cdot\frac{1}{s}=\frac{A}{s}+\frac{Bs+C}{(s+\delta)^2+\omega_d^2}$$

$$A\cdot((s+\delta)^2+\omega_d^2)+Bs^2+Cs=s+2\delta$$

$$s^2(A+B)+s(A\cdot 2\delta+C)+A(\delta_2+\omega_d^2)=s+2\delta \longrightarrow \left\{\begin{array}{l} A+B=0 \\ A\cdot 2\delta+C=1 \\ A\cdot(\delta^2+\omega_d^2)=2\delta \end{array}\right\} \begin{array}{l} A=2\delta/(\delta^2+\omega_d^2) \\ B=-2\delta/(\delta^2+\omega_d^2) \\ C=1-(2\delta)^2/(\delta^2+\omega_d^2) \end{array}$$

$$\frac{s+2\delta}{(s+\delta)^2+\omega_d^2}\cdot\frac{1}{s}=\frac{2\delta}{\delta^2+\omega_d^2}\cdot\left[\frac{1}{s}+\frac{-s+(\omega_d^2-3\delta^2)/(2\delta)}{(s+\delta)^2+\omega_d^2}\right]$$

$$\frac{s+2\delta}{(s+\delta)^2+\omega_d^2}\cdot\frac{1}{s}=\frac{2\delta}{\delta^2+\omega_d^2}\cdot\left(\frac{1}{s}-\frac{s+\delta}{(s+\delta)^2+\omega_d^2}+\frac{\omega_d^2-\delta^2}{2\delta\cdot\omega_d}\cdot\frac{\omega_d}{(s+\delta)^2+\omega_d^2}\right)$$

Inverse *Laplace* transform

$$\frac{2\delta}{\delta^2+\omega_d^2}\cdot\left(1-e^{-\delta\cdot t}\cdot\cos(\omega_d\cdot t)+\frac{\omega_d^2-\delta^2}{2\delta\cdot\omega_d}\cdot e^{-\delta\cdot t}\cdot\sin(\omega_d\cdot t)\right)$$

5. Dynamics of DC machines

Side calculations for more elegant formula description

$$\boxed{\frac{2\delta}{\delta^2+\omega_d^2}\cdot\left[1-e^{-\delta\cdot t}\cdot\left(\cos(\omega_d\cdot t)-\frac{\omega_d^2-\delta^2}{2\delta\cdot\omega_d}\cdot\sin(\omega_d\cdot t)\right)\right]}$$

$$\delta=\frac{1}{2T_a},\ \omega_d=\delta\cdot\sqrt{\frac{4T_a}{T_m}-1}:\ \boxed{\frac{2\delta}{\delta^2+\omega_d^2}}=\frac{2\delta}{\delta^2+\delta^2\cdot\left(\frac{4T_a}{T_m}-1\right)}=\frac{2}{\delta}\cdot\frac{1}{1+\frac{4T_a}{T_m}-1}=\frac{4T_a}{\frac{4T_a}{T_m}}=\boxed{T_m}$$

$$\cos\omega_d t-\frac{\omega_d^2-\delta^2}{2\delta\cdot\omega_d}\cdot\sin\omega_d t=K\cdot\cos\omega_d t-L\cdot\sin\omega_d t=M\cdot\cos(\omega_d t-\psi)$$

$$M\cdot\cos(\omega_d t-\psi)=M\cos\omega_d t\cos\psi+M\sin\omega_d t\sin\psi\rightarrow K=M\cos\psi=1,\ L=-M\sin\psi=\frac{\omega_d^2-\delta^2}{2\delta\cdot\omega_d}$$

$$M^2(\cos^2\psi+\sin^2\psi)=M^2=K^2+L^2\rightarrow M=\sqrt{K^2+L^2}=\sqrt{1+\left(\frac{\omega_d^2-\delta^2}{2\delta\omega_d}\right)^2}$$

$$M=\frac{1}{2\delta\omega_d}\cdot\sqrt{(2\delta\omega_d)^2+(\omega_d^2-\delta^2)^2}=\frac{1}{2\delta\omega_d}\cdot\sqrt{(\omega_d^2+\delta^2)^2}=\frac{\omega_d^2+\delta^2}{2\delta\omega_d}=\frac{\delta^2(4T_a/T_m-1)+\delta^2}{2\delta\omega_d}$$

$$\boxed{M=\frac{\delta\cdot 4T_a/T_m}{2\omega_d}=\frac{1}{\omega_d T_m}}\qquad \boxed{\cos\psi=1/M=\omega_d T_m\rightarrow\psi=\arccos(\omega_d T_m)}$$

5. Dynamics of DC machines

Example: Speed response to mechanical load torque step (2)

Result: $$\Delta\Omega_m(t) = -\frac{\Delta M_s}{J}\cdot T_m\cdot\left[1-\frac{1}{\omega_d T_m}\cdot e^{-\frac{t}{2T_a}}\cdot\cos(\omega_d t-\psi)\right] \qquad \psi = \arccos(\omega_d T_m)$$

$$\frac{\Delta\Omega_m(t)}{\Omega_{m0}} = -\frac{\Delta M_s}{M_N}\cdot\frac{r_a}{\phi^2}\cdot\left[1-\frac{1}{\omega_d T_m}\cdot e^{-\frac{t}{2T_a}}\cdot\cos(\omega_d t-\psi)\right] \qquad \frac{T_m}{J\Omega_{m0}} = \frac{r_a}{\phi^2}\frac{J\Omega_{m0}}{M_N}\frac{1}{J\Omega_{m0}} = \frac{r_a}{\phi^2 M_N}$$

a) Data:

$T_J = 1s, T_a = 50ms, r_a = 0.05, \phi = 1$, step in shaft torque of $\Delta M_s / M_N = 0.5$

b) Result with data values:

$\omega_d = 17.3/s, T_d = 363ms, T_m = 50ms$, $\psi = \arccos(17.3\cdot 0.05) = 0.526$.

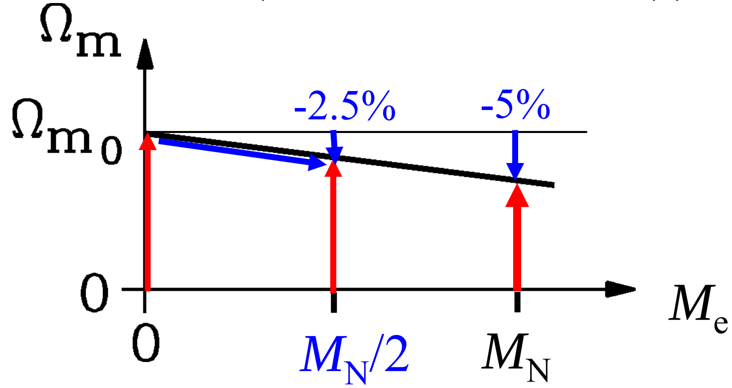
Transient speed response: (*t* in seconds)

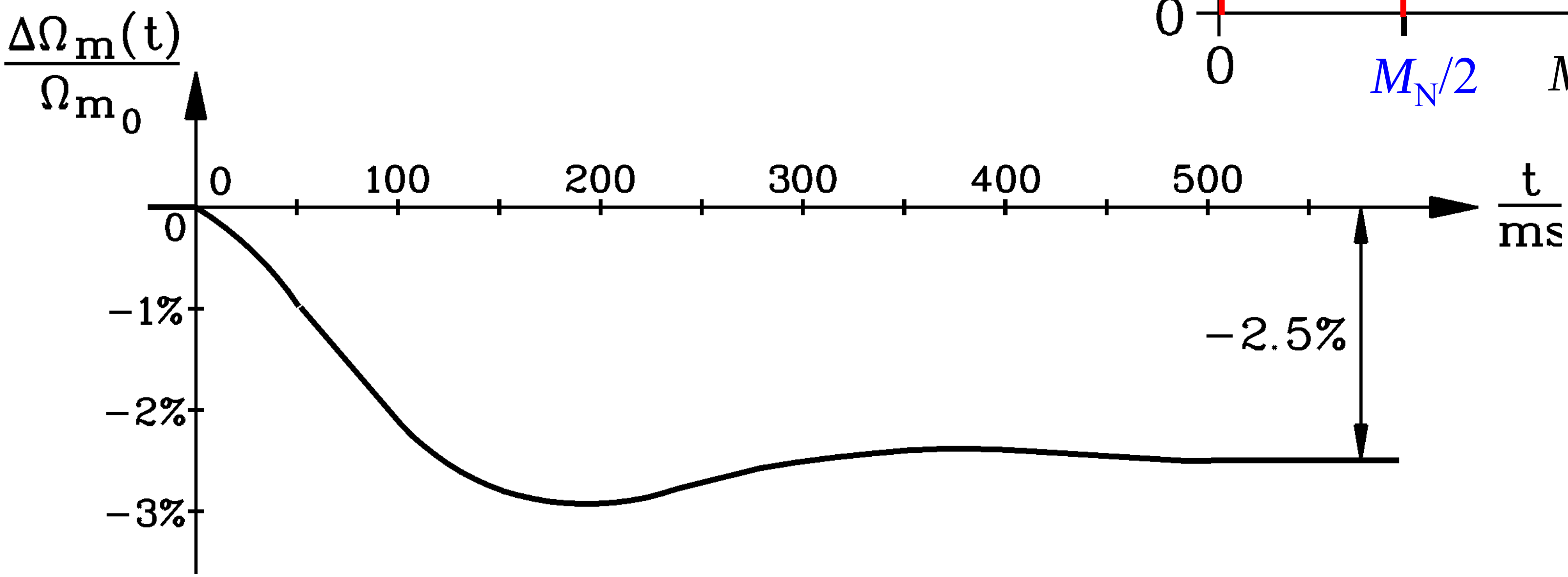
$$\boxed{\frac{\Delta\Omega_m(t)}{\Omega_{m0}} = -0.5\cdot 0.05\cdot\left(1-\frac{1}{0.865}\cdot e^{-\frac{t}{2\cdot 0.05}}\cdot\cos(17.3\cdot t-0.526)\right)}$$

5. Dynamics of DC machines

Example: Speed response to mechanical load torque step (3)

$$\frac{\Delta\Omega_m(t)}{\Omega_{m0}} = -0.5\cdot 0.05\cdot\left(1-\frac{1}{0.865}\cdot e^{-\frac{t}{2\cdot 0.05}}\cdot\cos(17.3\cdot t-0.526)\right)$$



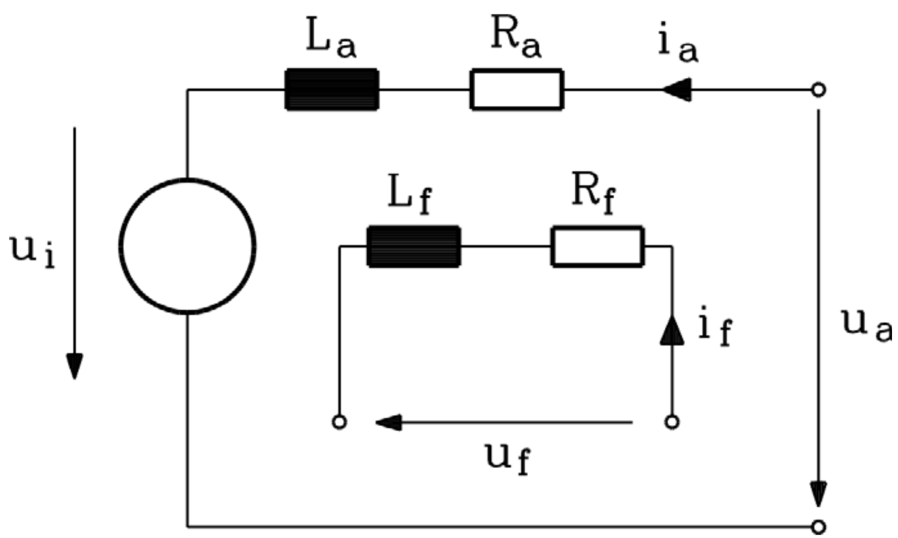


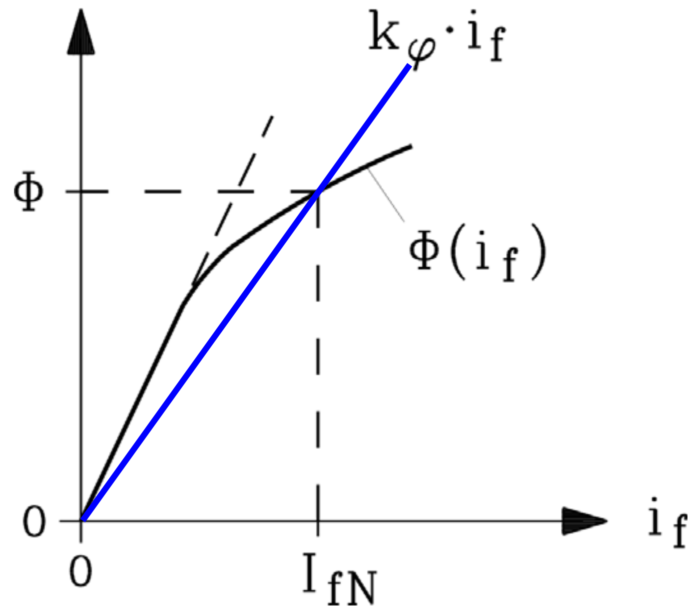
Summary:
Transfer function of separately excited DC machine

- (Linearized) differential equation is „transfer function“ in LAPLACE domain
- Step response calculated via transfer function
- Denominator of transfer function is characteristic polynomial of diff. equation
- Zeros (= “roots”) of characteristic polynomial are „poles“ of the system in the *s*-plane
- Negative inverse of „poles“ = time constants or natural frequencies of the system
- Positive time constants for stable operation needed
- Hence poles must lie in the negative *s*-half-plane

5. Dynamics of DC machines

5. Dynamics of DC machines
Input for dynamic simulations





- General model calculation possible via numerical integration
- Here: Only: a) Linearized non-linear magnetization characteristic

„flux vs. field current" $\Phi = k_\varphi \cdot i_f \Rightarrow \psi_f = N_f \cdot \Phi = L_f \cdot i_f \Rightarrow L_f = N_f \cdot k_\varphi$

b) Separately excited machine

c) Constant flux operation

5. Dynamics of DC machines

First order differential equations for *RUNGE-KUTTA* method, here for Φ = const. (1)

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$$u_a(t) = R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + u_i(t) \quad \Rightarrow \quad \frac{di_a(t)}{dt} = \frac{u_a(t)}{L_a} - \frac{R_a \cdot i_a(t)}{L_a} - \frac{k_2\Phi}{L_a} \cdot \Omega_m(t)$$

$$J \cdot \frac{d\Omega_m}{dt} = m_e(t) - m_s(t) \quad \Rightarrow \quad \frac{d\Omega_m}{dt} = \frac{k_2\Phi}{J} \cdot i_a(t) - \frac{m_s(t)}{J}$$

- Initial conditions: $i_a(0), \quad \Omega_m(0)$
- The functions $u_a(t)$, $m_s(t)$ must be given for $t \geq 0$!
- **In case of varying flux Φ add the third differential equation:** $\Phi(i_f(t))$

$$u_f(t) = R_f \cdot i_f(t) + N_f \cdot \frac{d\Phi(t)}{dt} \quad \Rightarrow \quad \frac{d\Phi(t)}{dt} = \frac{u_f(t)}{N_f} - \frac{R_f \cdot i_f(t)}{N_f} \qquad \Phi \rightarrow i_f$$

- The function $u_f(t)$ must be given for $t \geq 0$! Initial condition $i_f(0), \Phi(i_f(0))$

First order differential equations for *RUNGE-KUTTA* method, here for Φ = const. (2)

Example:

Separately excited DC machine: **Motor** data,
Motor fed from ideal DC voltage ("stiff battery" = zero internal resistance):

$$U_N = 460\ \mathrm{V},\ P_N = 142\ \mathrm{kW},\ n_N = 625/\mathrm{min}$$

$$I_N = 320\ \mathrm{A},\ I_{fN} = 6.5\ \mathrm{A},\ J_M = 7\ \mathrm{kg}\cdot\mathrm{m}^2$$

$$R_a = 0.05\ \Omega,\ L_a = 1.5\ \mathrm{mH},\ R_f = 25\ \Omega,\ L_f = 64\ \mathrm{H}$$

Load inertia:	**(i): 8 kgm²**	**Total inertia: J = 15 kgm² = J_N**
	(ii): 143 kgm²	**Total inertia: J = 150 kgm²**

Two cases investigated:

a) Load step with rated torque at no-load speed, rated armature voltage and rated flux

b) Armature voltage step of 20% rated voltage at rated motor operation

5. Dynamics of DC machines

Tutorial

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Steady state characteristics of the DC motor at U_N

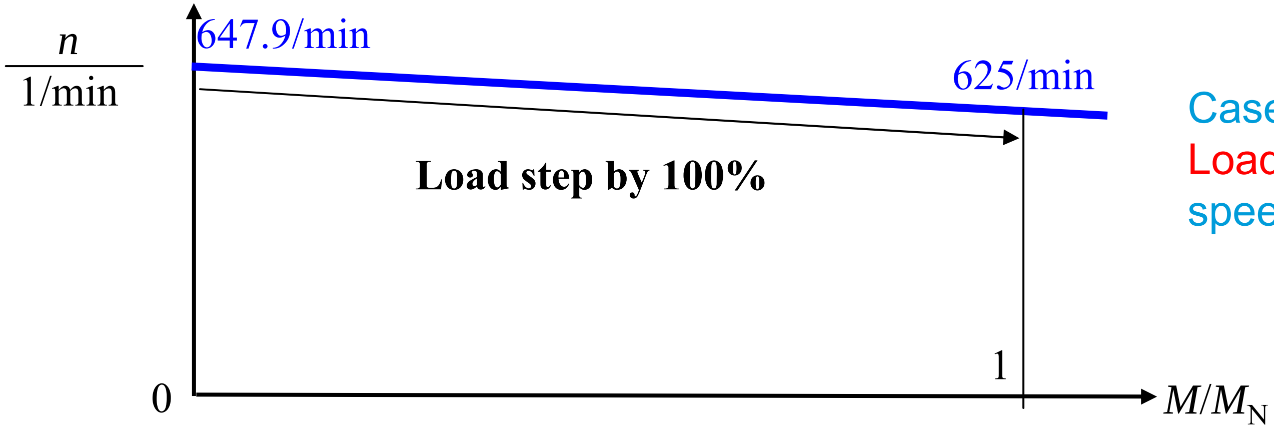
Rated torque: $M_N = \frac{P_N}{2\pi \cdot n_N} = \frac{142000}{2\pi \cdot (625/60)} = 2169.6 Nm$

Induced voltage at rated speed and torque: $U_i = U_N - I_N \cdot R_a = 460 - 320 \cdot 0.05 = 444V$

Motor constant and flux per pole: $k_2 \Phi_N = \frac{U_i}{\Omega_{mN}} = \frac{444}{2\pi \cdot (625/60)} = 6.78 Vs$

Motor efficiency: $\eta = \frac{P_N}{U_N \cdot I_N + R_f I_{fN}^2} = \frac{142000}{460 \cdot 320 + 25 \cdot 6.5^2} = 95.78\%$

No-load speed at rated armature voltage and main flux: $n_0 = \frac{U_N}{2\pi \cdot k_2 \Phi_N} = \frac{460 \cdot 60}{2\pi \cdot 6.78} = 647.9/\mathrm{min}$



Case a)
Load step with rated torque at no-load speed, rated armature voltage and flux

5. Dynamics of DC machines

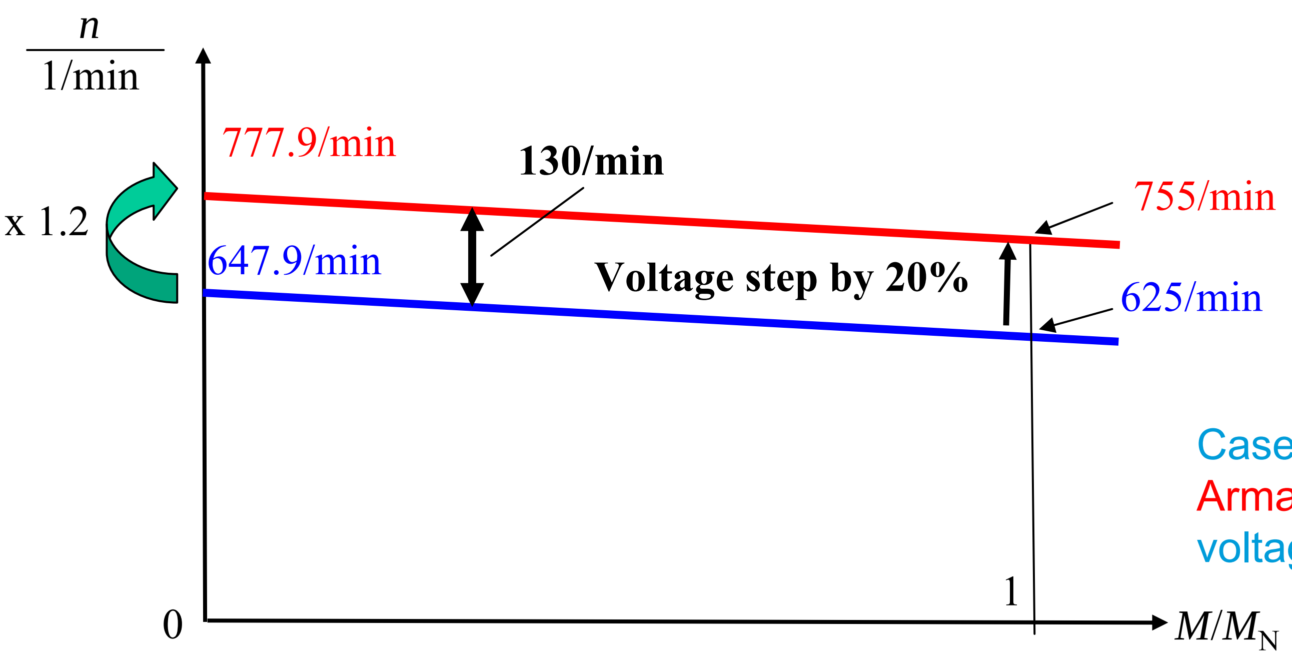
Tutorial

Steady state characteristics of the DC motor at $1.2U_N$

Induced voltage at rated torque: $U_i = 1.2 \cdot U_N - I_N \cdot R_a = 552 - 320 \cdot 0.05 = 536V$

Motor speed at constant flux/pole: $\Omega_m = \frac{U_i}{k_2 \Phi_N} \rightarrow n = \frac{536}{6.78} \cdot \frac{60}{2\pi} = 755/\min$

No-load speed at 120% armature voltage & main flux: $n_0 = \frac{1.2U_N}{2\pi \cdot k_2 \Phi_N} = \frac{552 \cdot 60}{2\pi \cdot 6.78} = 777.9/\min$



Case b)
Armature voltage step of 20% rated voltage at rated motor operation

5. Dynamics of DC machines

Calculation of time constants

Tutorial

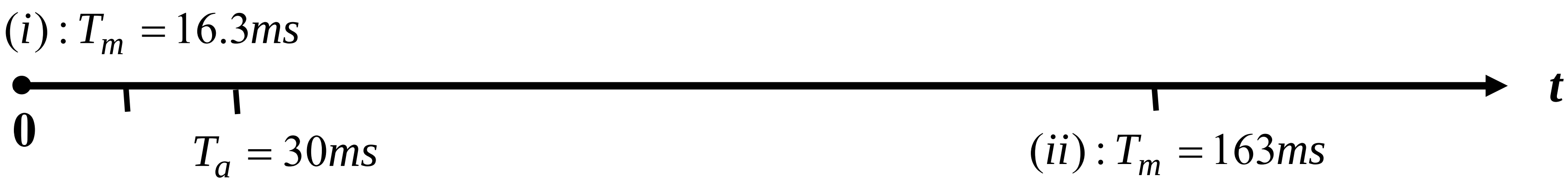
$$T_a = \frac{L_a}{R_a} = \frac{0.0015}{0.05} = 30ms, \qquad T_f = \frac{L_f}{R_f} = \frac{64}{25} = 2.56s,$$

$$T_m = \frac{R_a \cdot J_N}{(k_2 \Phi_N)^2} = \frac{0.05 \cdot 15}{6.78^2} = 0.0163s$$

(i) at J_N = 15 kg·m^2

$$T_m = \frac{R_a \cdot J}{(k_2 \Phi_N)^2} = \frac{0.05 \cdot 150}{6.78^2} = 0.163s$$

(ii) at J = 150 kg·m^2



5. Dynamics of DC machines

Variation of total inertia: (i) J = 15 kgm², (ii) J = 150 kgm²

(i) Total inertia 15 kgm²:

$T_m = 16.3 ms < 4T_a = 120 ms$: damped oscillations occur:

Damping coefficient: $\delta = \frac{1}{2T_a} = \frac{1}{2 \cdot 0.03} = 16.67 / s$.

Natural frequency: $f_d = \frac{1}{2\pi \cdot T_a}\sqrt{\frac{T_a}{T_m} - \frac{1}{4}} = \frac{1}{2\pi \cdot 0.03}\sqrt{\frac{0.03}{0.0163} - \frac{1}{4}} = 6.69 Hz$

Period of oscillation is $T_d = \frac{1}{f_d} = \frac{1}{6.69} = 149.5 ms$

After $N_H = \frac{\omega_d}{\delta} = \frac{2\pi \cdot 6.69}{16.67} = 2.5$ half periods the oscillation is reduced down to 5%.

5. Dynamics of DC machines

Tutorial

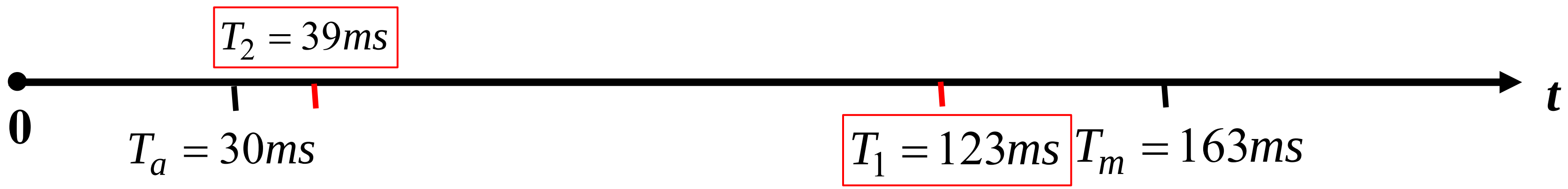
Variation of total inertia: (i) 15 kgm², (ii) 150 kgm²

(ii) Total inertia increased by factor 10: 150 kgm²

$T_m = 163\,ms > 4T_a = 120\,ms$: no oscillations occur:

long time constant: $T_1 = \dfrac{2T_a}{1-\sqrt{1-\dfrac{4T_a}{T_m}}} = \dfrac{2\cdot 30}{1-\sqrt{1-\dfrac{4\cdot 30}{163}}} = 123.3ms$,

short time constant $T_2 = \dfrac{2T_a}{1+\sqrt{1-\dfrac{4T_a}{T_m}}} = \dfrac{2\cdot 30}{1+\sqrt{1-\dfrac{4\cdot 30}{163}}} = 39.6ms$



Tutorial

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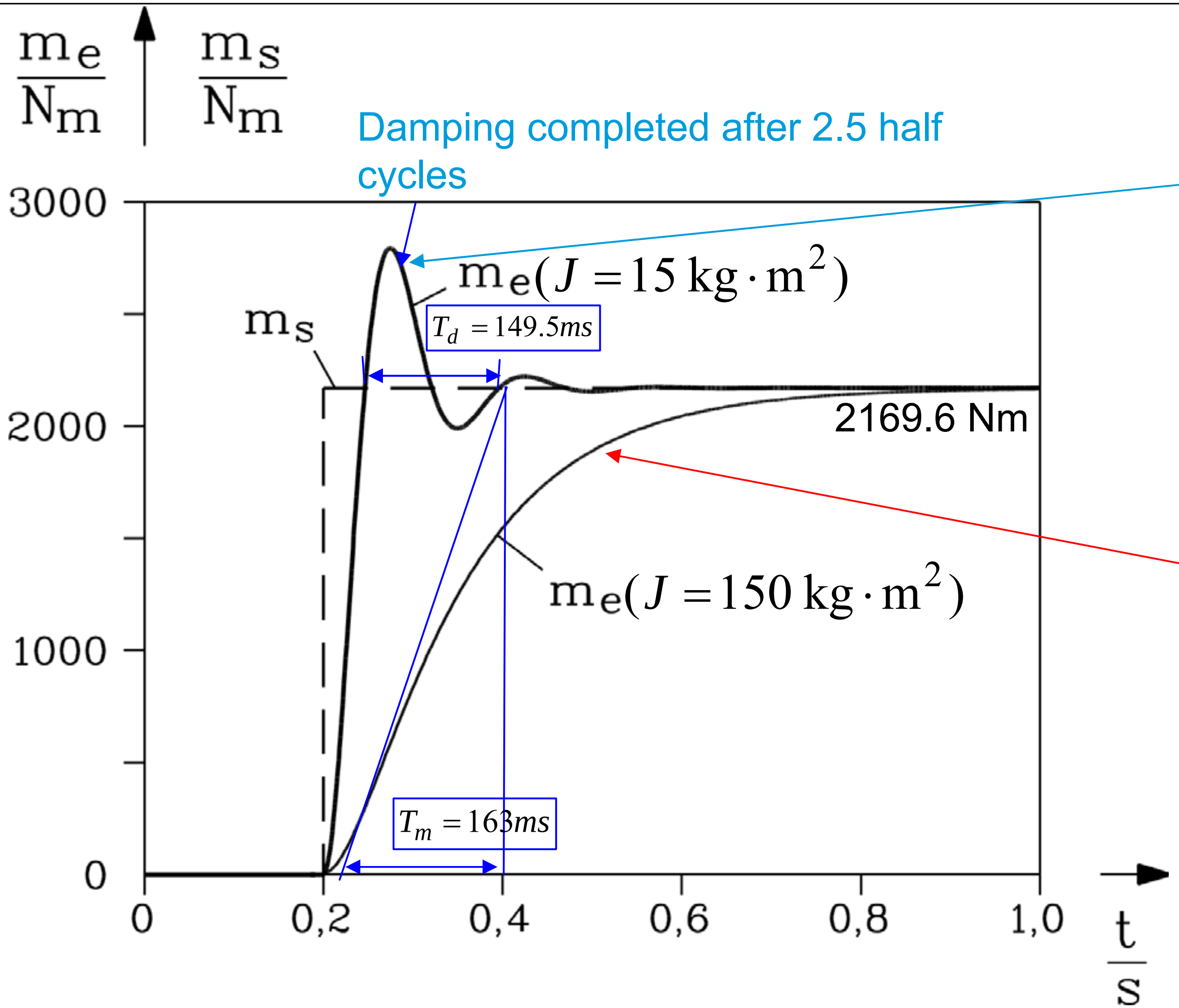
a) Load step with rated torque at no-load speed, rated armature voltage and flux



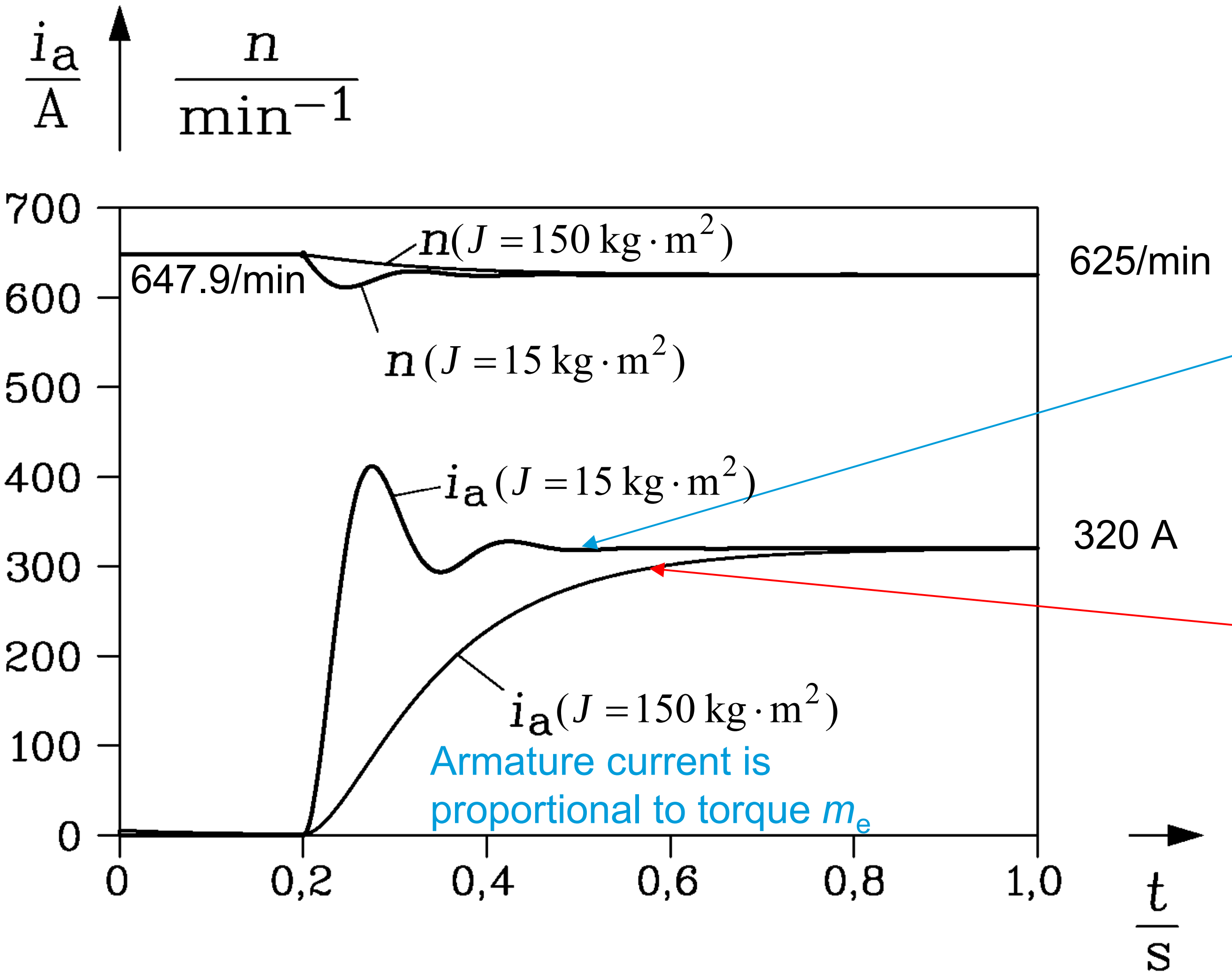
At **low** total inertia DC machine is oscillating

At **big total inertia** no oscillation occurs

5. Dynamics of DC machines

a) Load step with rated torque at no-load speed, rated armature voltage and flux

Tutorial



At low total inertia DC machine is oscillating

At big total inertia no oscillation occurs

5. Dynamics of DC machines

b) Armature voltage step of 20% rated voltage at rated motor operation

Tutorial

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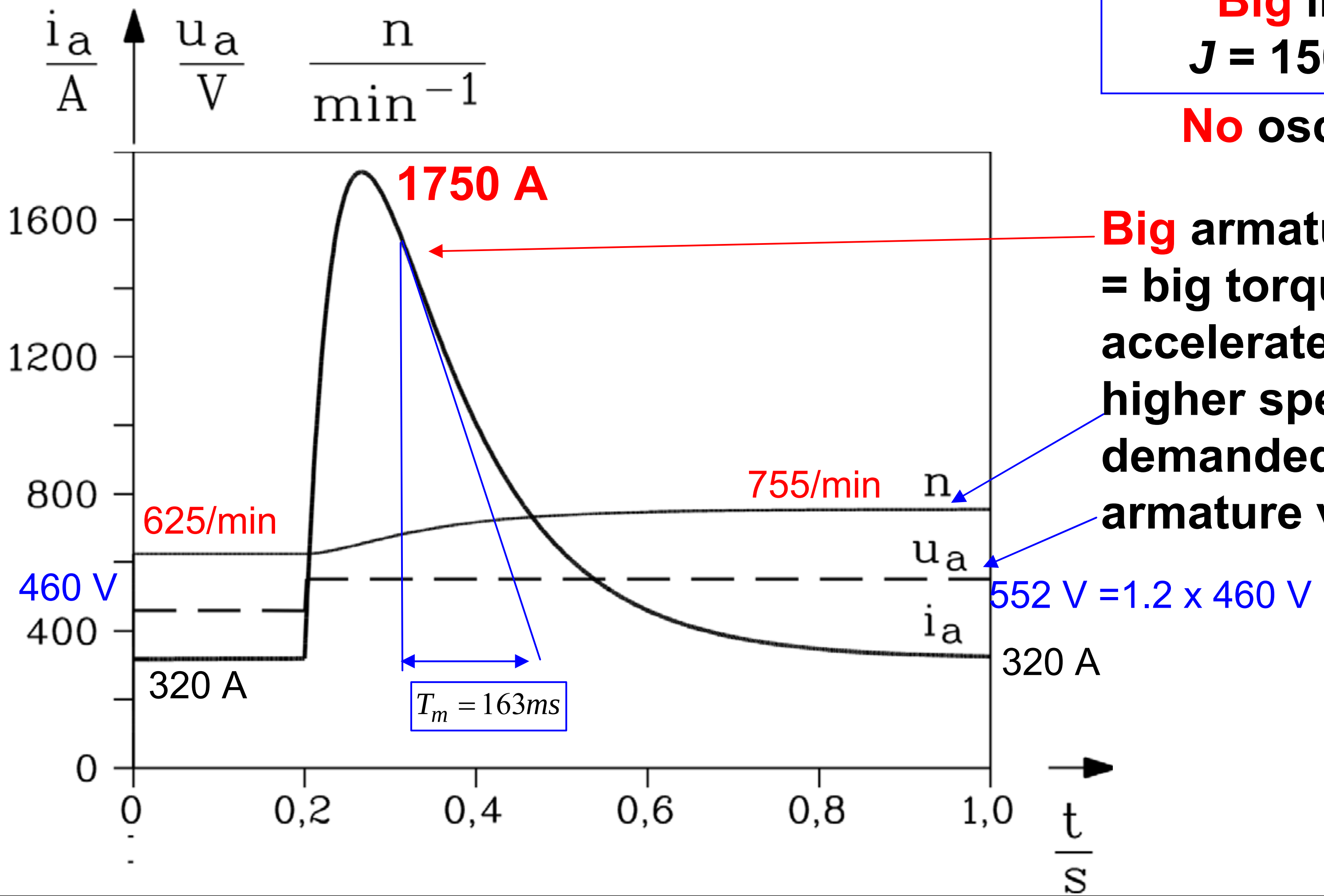


Big inertia
J = 150 kgm²

No oscillations

Big armature current = big torque: to accelerate inertia to higher speed, demanded by higher armature voltage

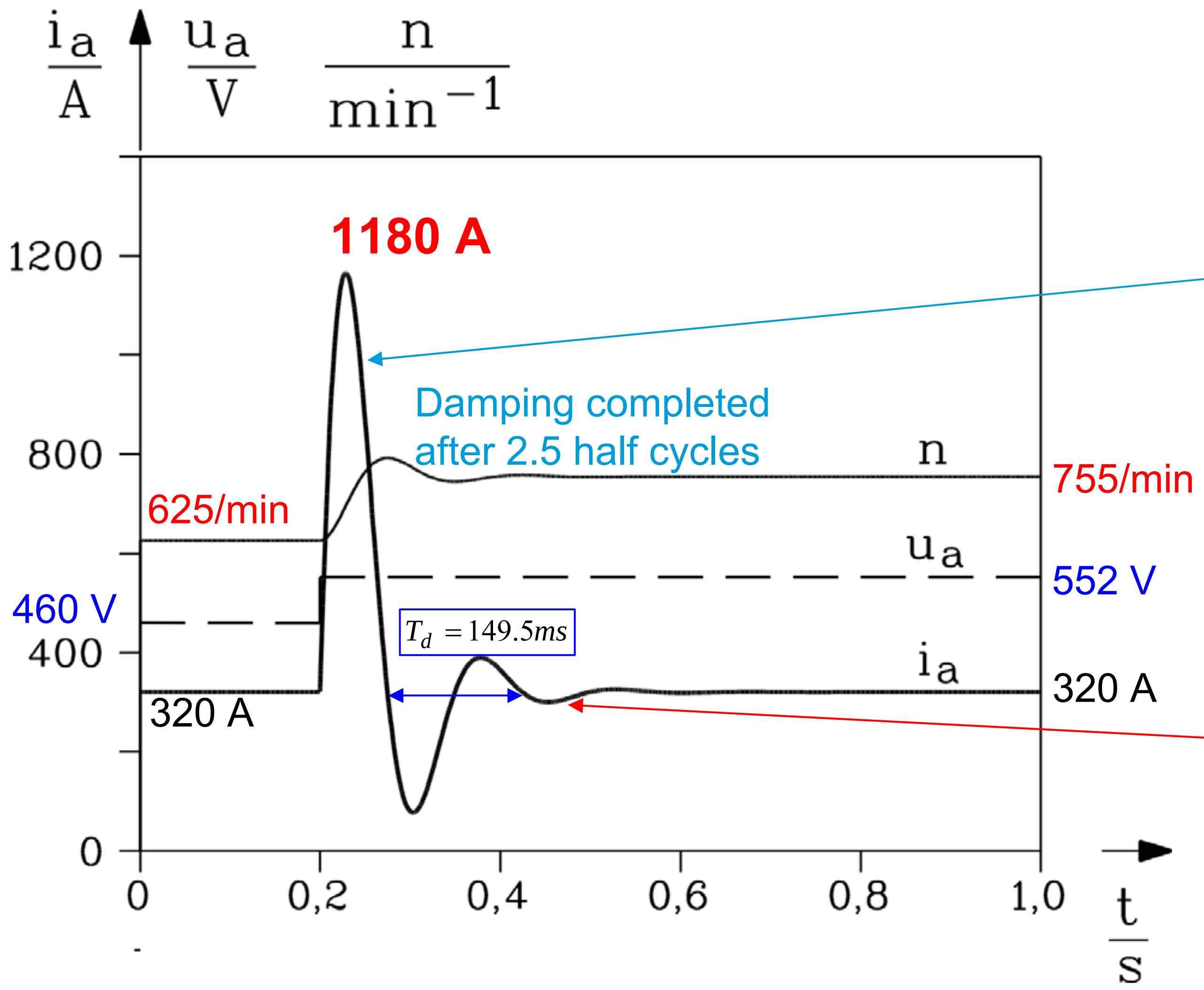
5. Dynamics of DC machines

b) Armature voltage step of 20% rated voltage at rated motor operation

Tutorial

Small inertia J = 15 kgm²

Oscillation occurs



Smaller peak armature current (1180 A < 1750 A) = smaller peak torque: to accelerate the lower inertia to higher speed

Oscillation occurs

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Summary:
Dynamic simulation of separately excited DC machine

- Linear differential equation is solved numerically via RUNGE-KUTTA
- Parameter variation: Big vs. small inertia = without / with oscillations
  Usually: Load inertia big, so no oscillations occur!
- Step response to torque as disturbing signal
- Step response to armature voltage as commanding signal
- Further example in the text book with variable series resistor

5. Dynamics of DC machines

5.1 Dynamic system equations of separately excited DC machine

5.2 Dynamic response of electrical and mechanical system of separately excited DC machine

5.3 Dynamics of coupled electric-mechanical system of separately excited DC machine

5.4 Linearized model of separately excited DC machine for variable flux

5.5 Transfer function of separately excited DC machine

5.6 Dynamic simulation of separately excited DC machine

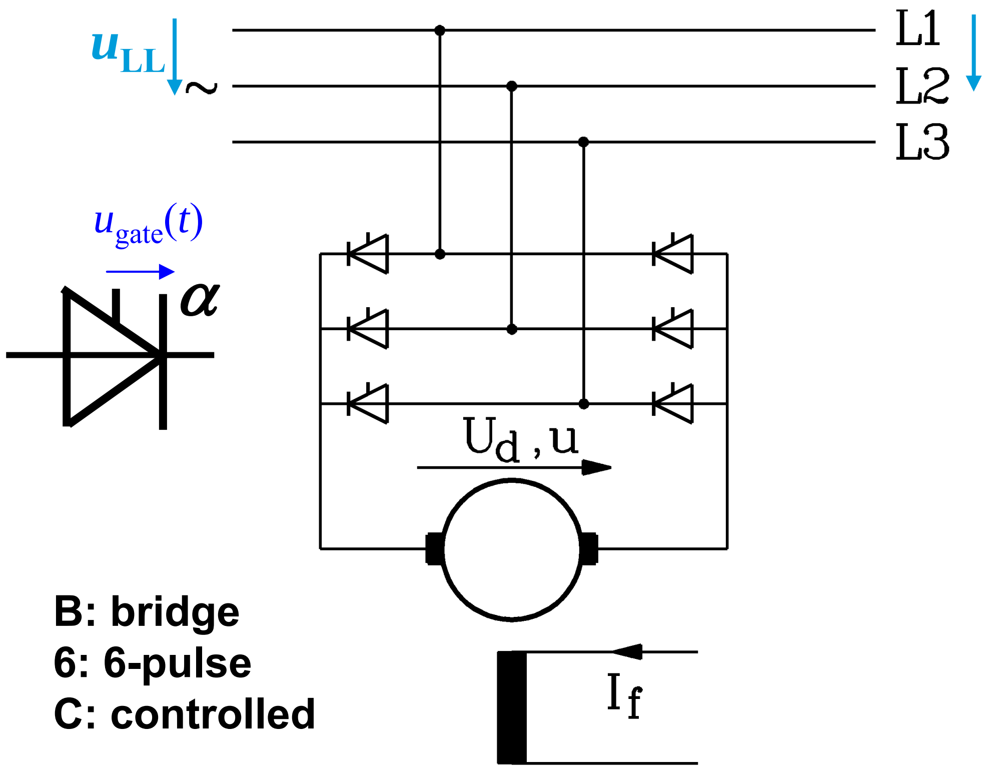
5.7 Converter operated separately excited DC machine

Converter-operated separately excited DC machine

Rectified voltage from the grid: $U_d(\alpha) = U_{d0} \cdot \cos\alpha$

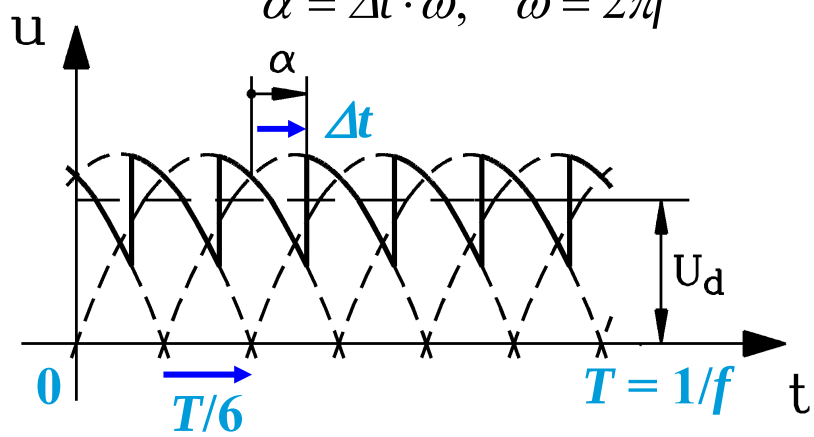
$$U_{d0} = \frac{3}{\pi} \cdot U_{LL} \cdot \sqrt{2}$$

$$U_d(\alpha = 0) = U_{d0}$$



B: bridge
6: 6-pulse
C: controlled

$$\alpha = \Delta t \cdot \omega, \quad \omega = 2\pi f$$



Six-pulse ripple

Thyristor control: $U_d > 0$, $U_d < 0$ possible

B6C bridge

rectified voltage

5. Dynamics of DC machines

Armature current with six-pulse ripple due to B6C converter

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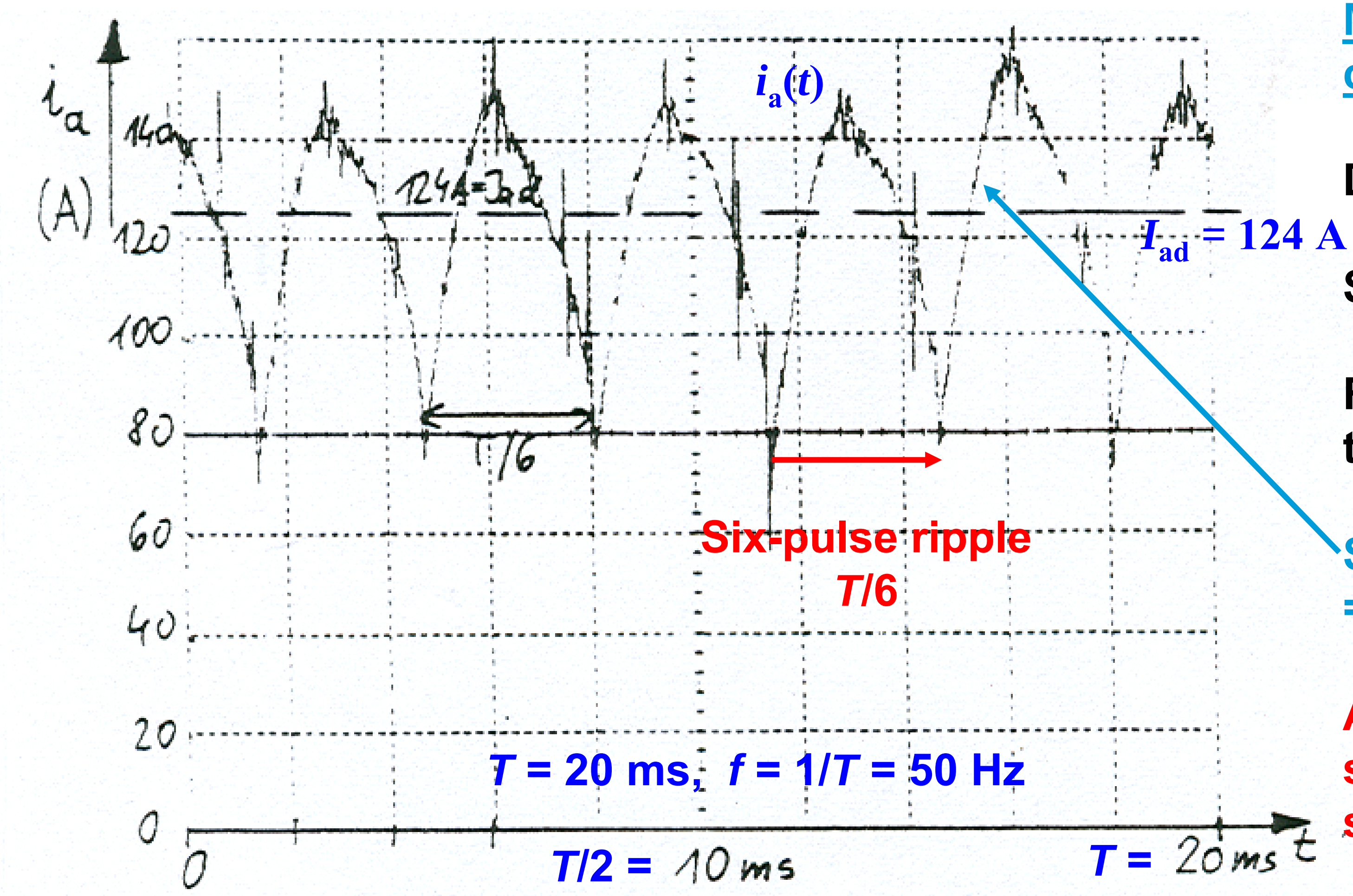
Calculation example in "Collection of Exercises"



Measured armature current

DC motor 40 kW

Separately excited

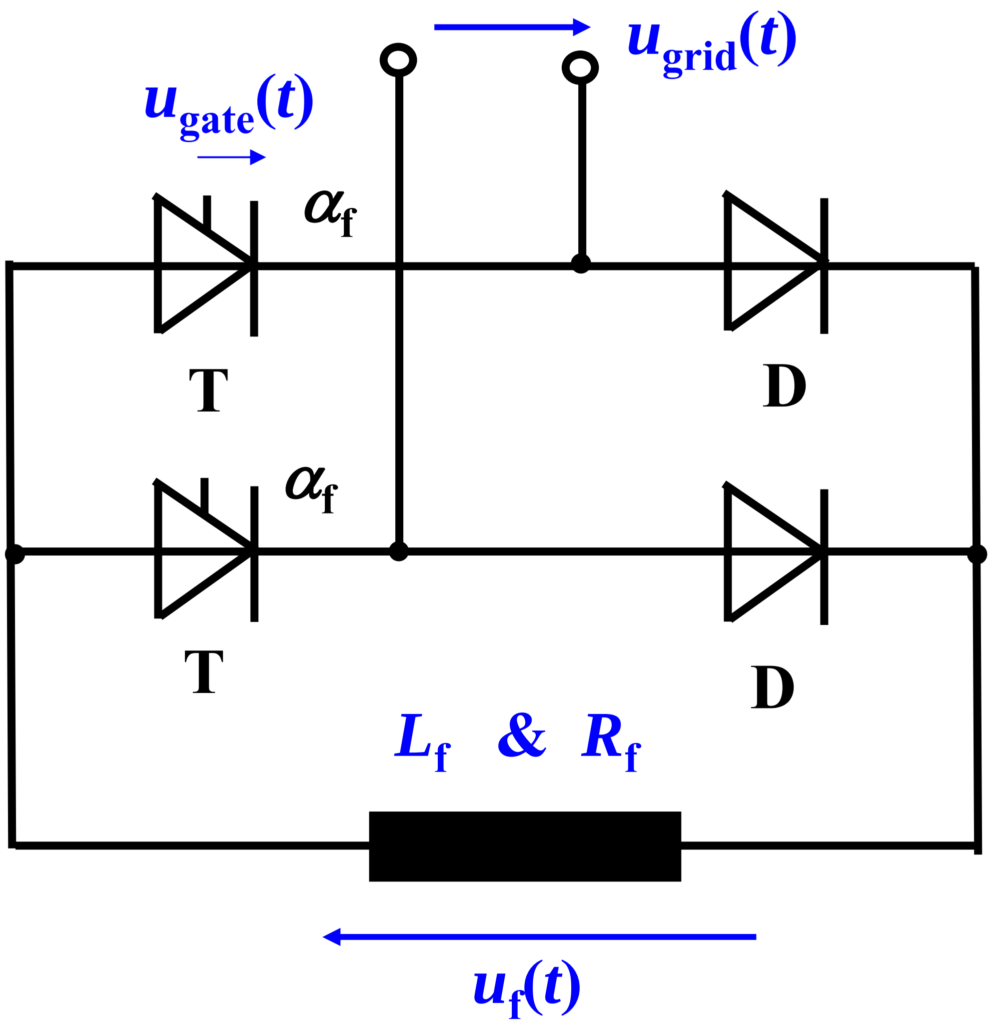
Fed by B6C armature thyristor bridge

Six-pulse ripple = 6x50 = 300 Hz

Armature inductance is small: $L_a \sim N_a^2 \sim z^2$, so i_a-ripple is big!

5. Dynamics of DC machines

B2H-converter (cheap solution)



D: diode

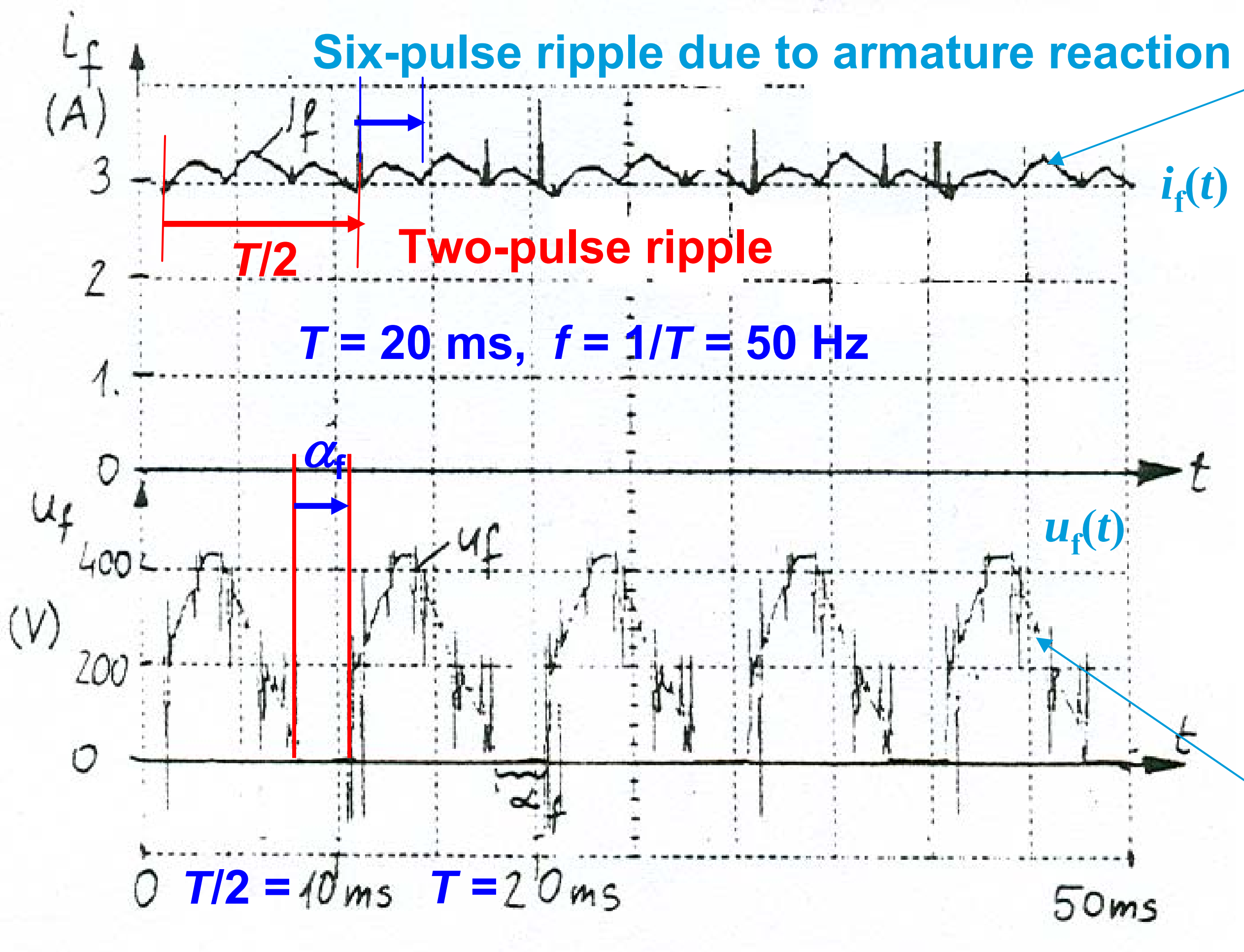
T: thyristor

Due to diodes:
only $U_f > 0$ possible

B: bridge
2: 2-pulse
H: half-controlled

5. Dynamics of DC machines

Field current: Two-pulse ripple due to B2H-converter



Measured field current

DC motor 40 kW

Separately excited

Field current fed by B2H thyristor bridge

Two-pulse ripple = 2x50 = 100 Hz

Field inductance is big: $L_f \sim N_f^2$, so i_f-ripple is small!

Measured field voltage

5. Dynamics of DC machines

Cross-coupling of stator and armature & compole field via iron saturation

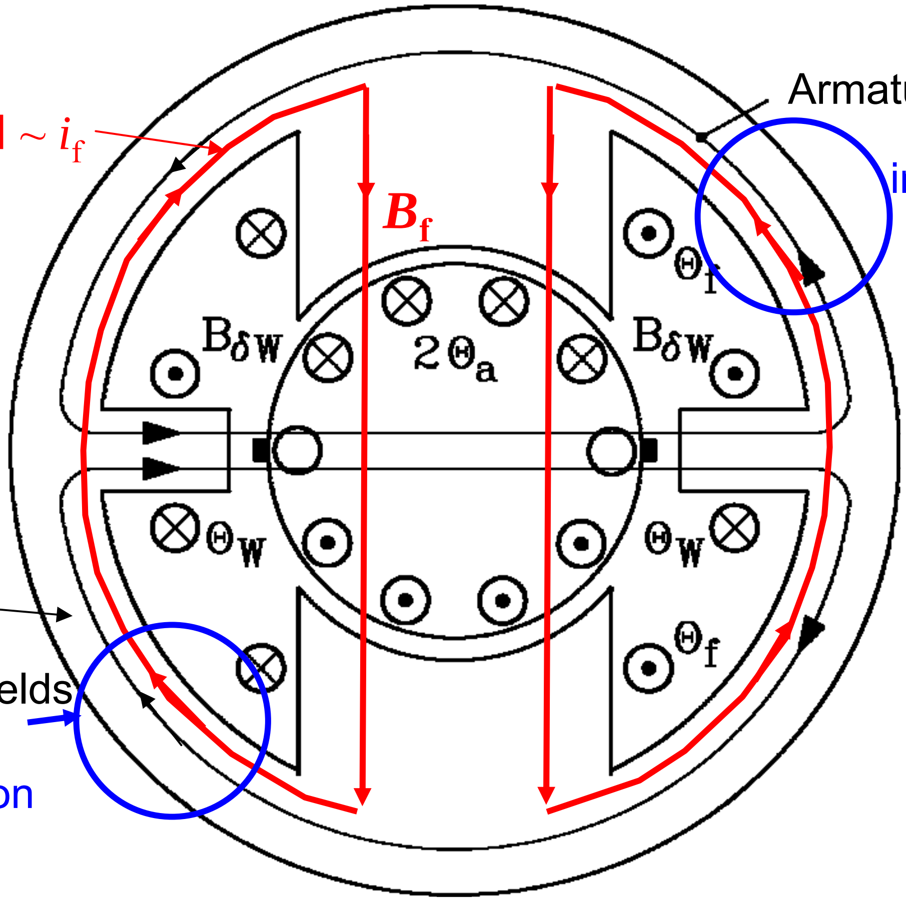
$$\Theta_f = N_{f,pole} \cdot i_f \rightarrow B_f$$

$$\left.\begin{array}{l}\Theta_a \\ \Theta_W\end{array}\right\} \sim i_a \rightarrow B_{\delta W}$$

Stator field $\sim i_f$

Armature & compole field ~ i_a

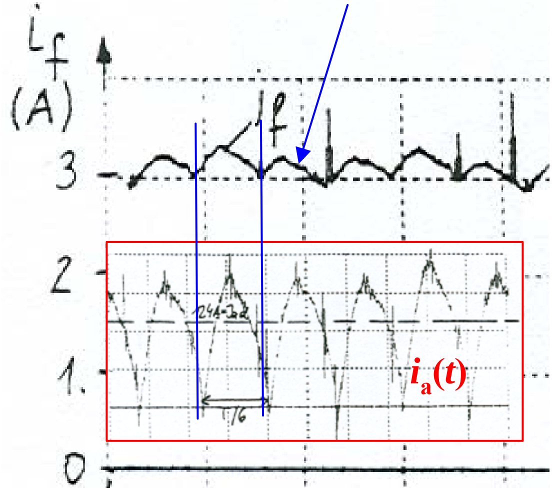
increased saturation



yoke

i_a- and i_f-excited magnetic fields **add** = increased yoke flux density = increased saturation

(small) six-pulse field current ripple



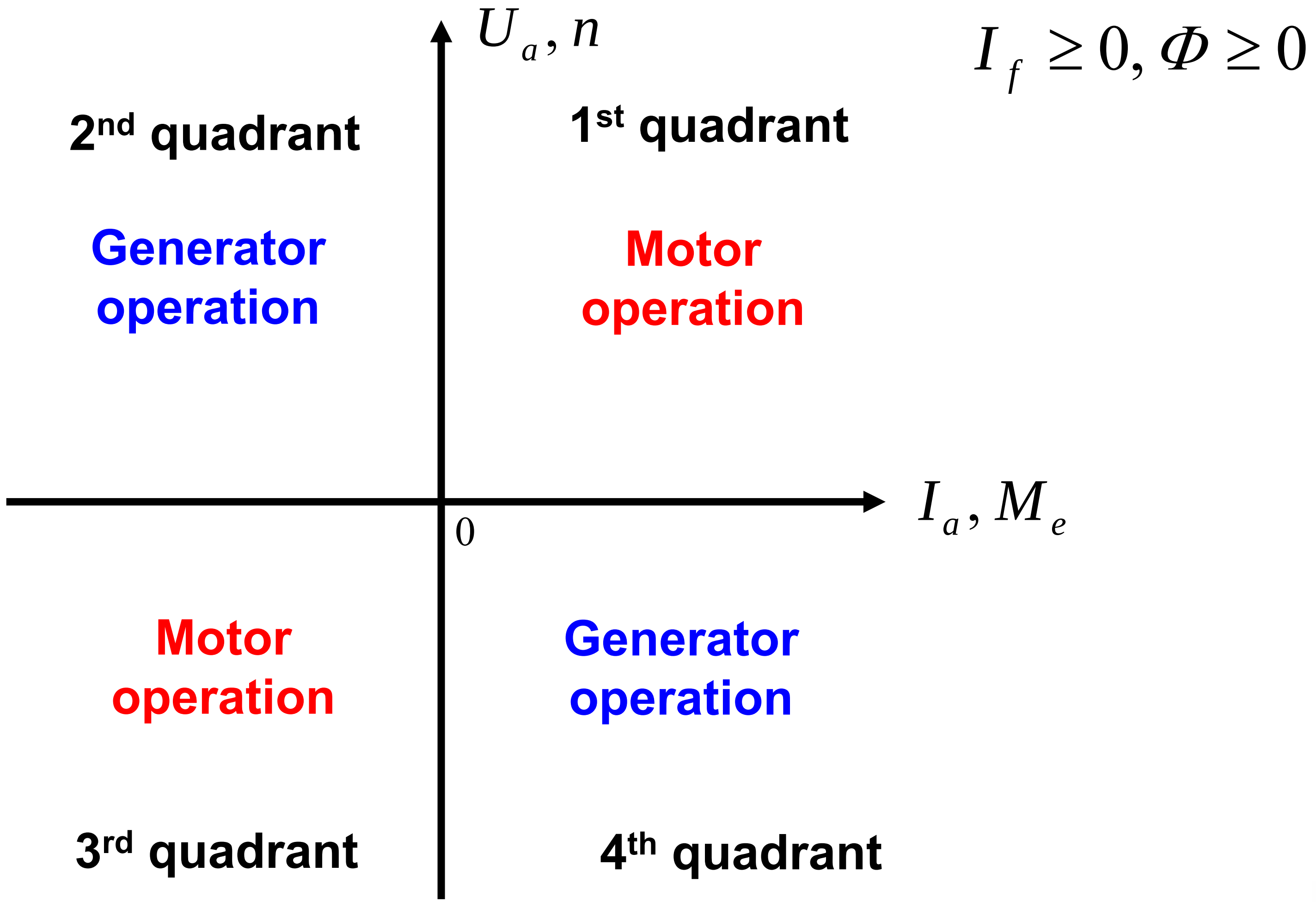
Due to the magnetic common path in the yoke a change of the armature current i_a causes a change of yoke iron permeability μ_{Fe}. Hence also the stator field B_f changes and induces a voltages in the field coil, which causes a (small) six-pulse ripple of field current i_f.

5. Dynamics of DC machines

Different configurations of controlled rectifier bridges for armature and field circuit of separately excited DC machines

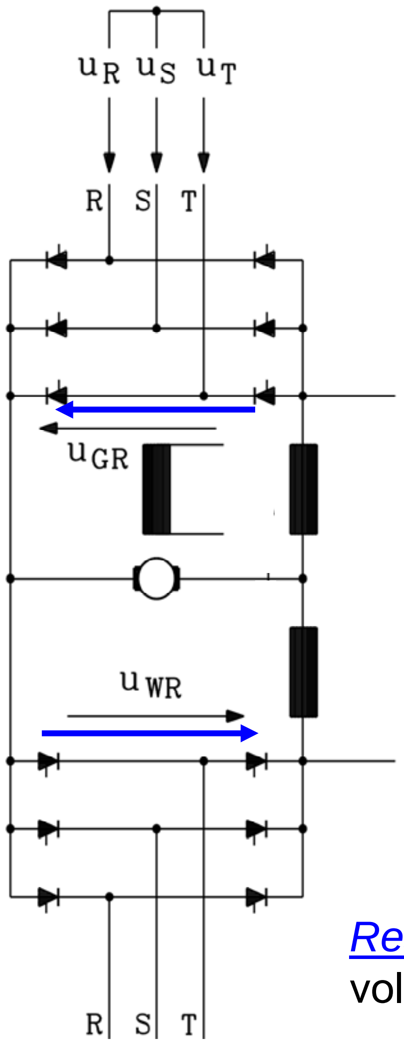
Armature $U_a <> 0, I_a > 0$	**B6C:** One six-pulse, voltage controlled thyristor bridge, 6 thyristors. Voltage and current ripple: 6*f* = 300 Hz. $n \leq\geq 0, M \geq 0$, but often only: **One quadrant operation:** $n \geq 0, M \geq 0$
Armature $U_a <> 0, I_a <> 0$	**(B6C)A(B6C):** Two anti-parallel six-pulse, voltage controlled thyristor bridges, 12 thyristors Voltage and current ripple: 6*f* = 300 Hz **Four quadrants operation**: $n <> 0, M <> 0$
Field $U_f \geq 0, I_f \geq 0, \Phi \geq 0$	**B2H:** One two-pulse, voltage controlled thyristor-diode bridge, 2 thyristors, 2 diodes Voltage and current ripple: 2*f* = 100 Hz
Field $U_f <> 0, I_f \geq 0, \Phi \geq 0$	**B2C:** One two-pulse, voltage controlled thyristor bridge, 4 thyristors Voltage and current ripple: 2*f* = 100 Hz
Field $U_f <> 0, I_f \geq 0, \Phi \geq 0$	**B6C:** One six-pulse, voltage controlled thyristor bridge, 6 thyristors Voltage and current ripple: 6*f* = 300 Hz

Four-quadrant operation

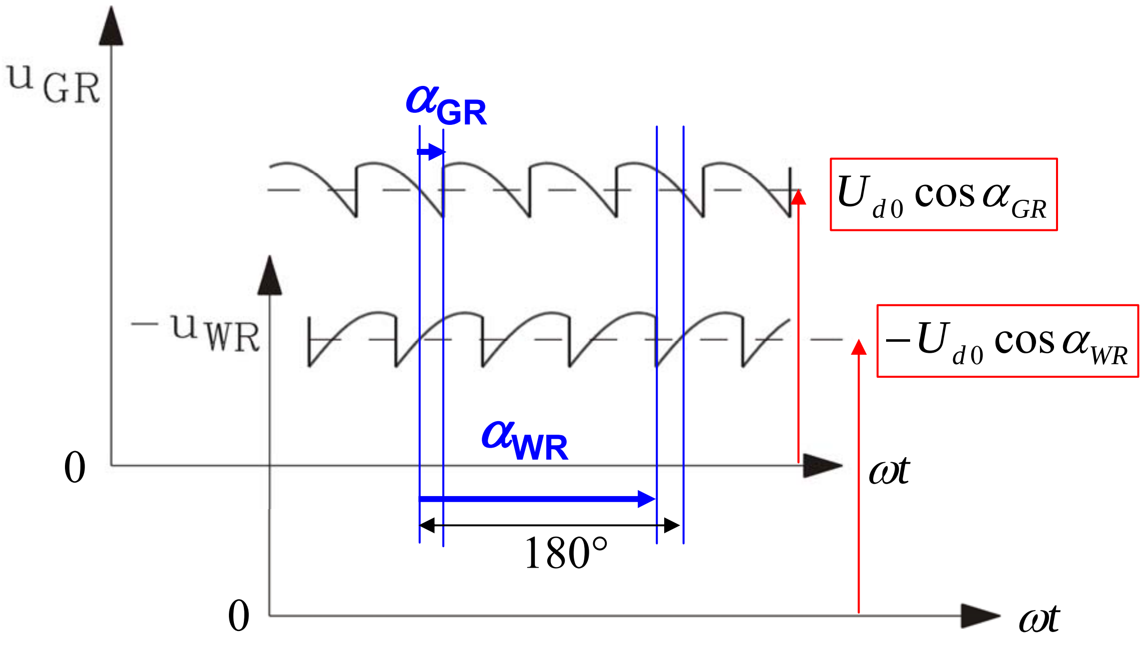


5. Dynamics of DC machines

Parallel B6C bridge operation



Two anti-parallel bridges operate with the same average output voltage in parallel.

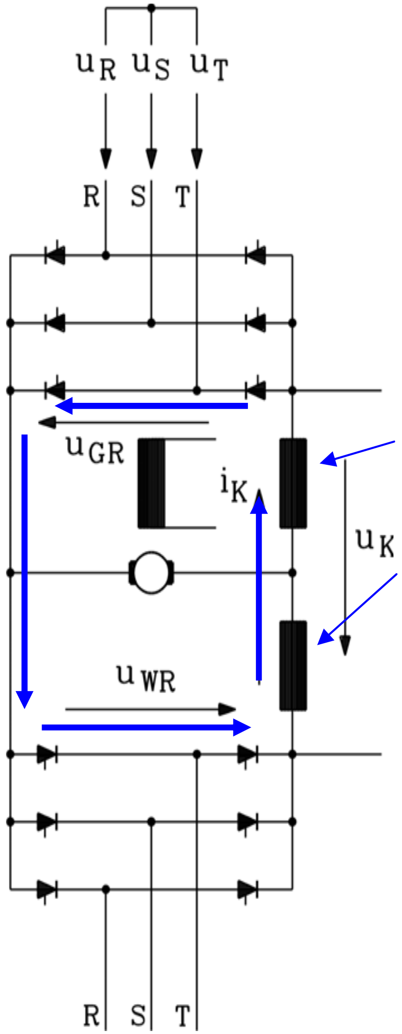


$$\boxed{U_{d0}\cos\alpha_{GR} = -U_{d0}\cos\alpha_{WR}}$$

$$\cos\alpha_{GR} = -\cos\alpha_{WR} \Rightarrow \alpha_{WR} = 180° - \alpha_{GR}$$

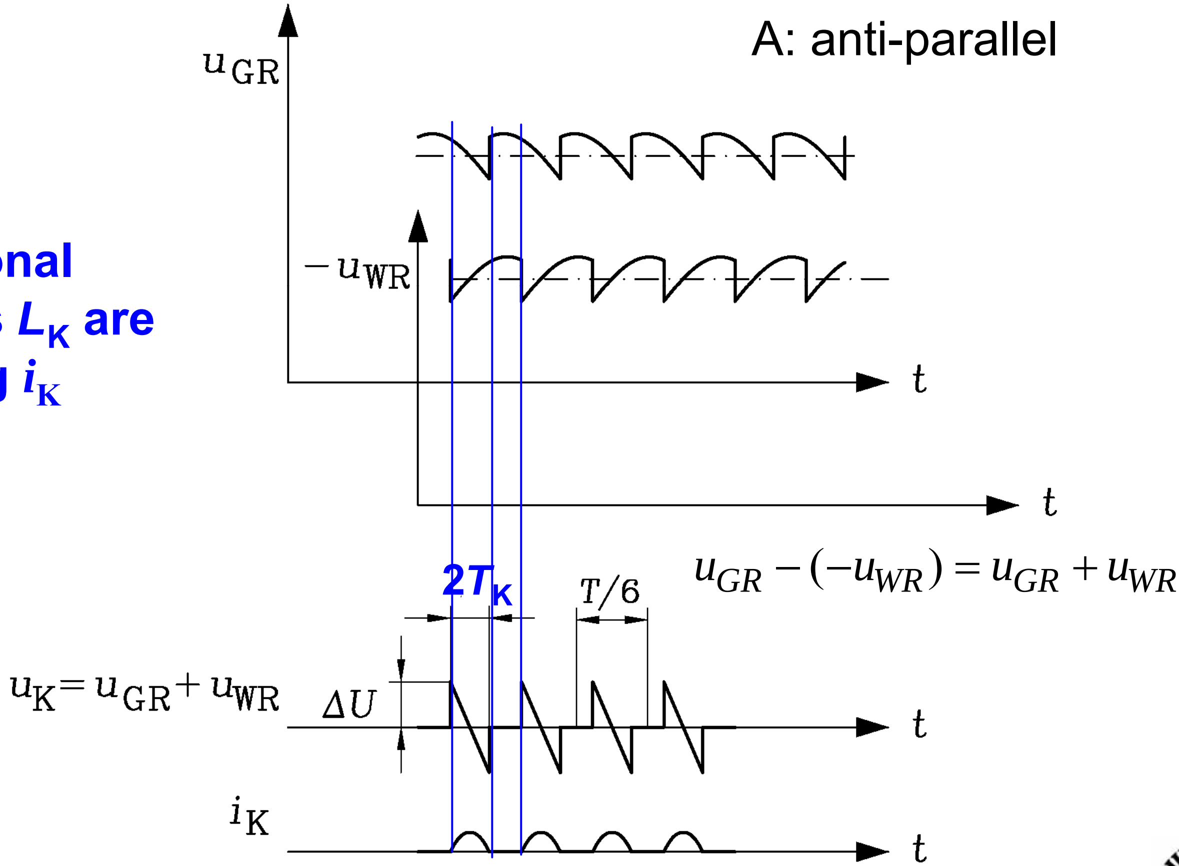
Result: With the condition $\alpha_{WR} = 180° - \alpha_{GR}$ both bridges deliver the same average voltage and can therefore operate in parallel on the DC machine.

5. Dynamics of DC machines
(B6C)A(B6C)-bridge operation



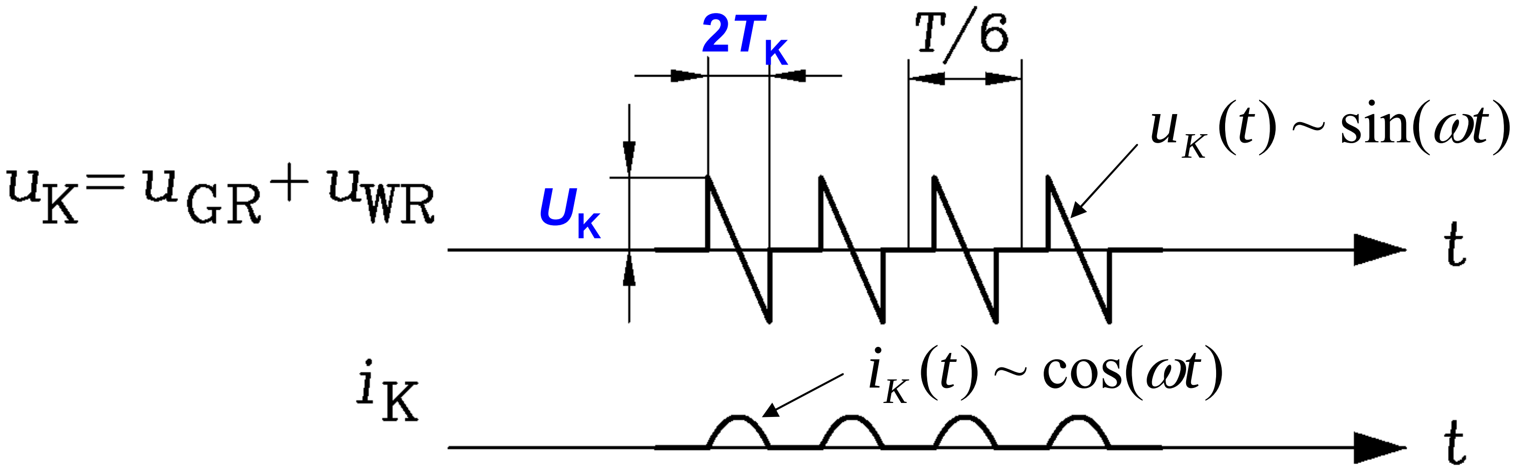
If both anti-parallel bridges are on at the same time,
a **parasitic circular current i_K** flows via both bridges.

Additional chokes L_K are limiting i_K



5. Dynamics of DC machines

Exact calculation of parasitic circular current i_K



$$\boxed{2T_K \le T/6}$$

$$u_K(t) = L_K \cdot di_K/dt = -U_K \cdot \sin(\omega t)/\sin(\omega T_K) \qquad -T_K \le t \le T_K$$

$$i_K(t) = \int u_K(t) \cdot dt / L_K = \frac{U_K}{\omega L_K \cdot \sin(\omega T_K)} \cdot \left(\cos(\omega t) - C\right) \qquad i_K(t = -T_K) = 0 \quad C = \cos(\omega T_K)$$

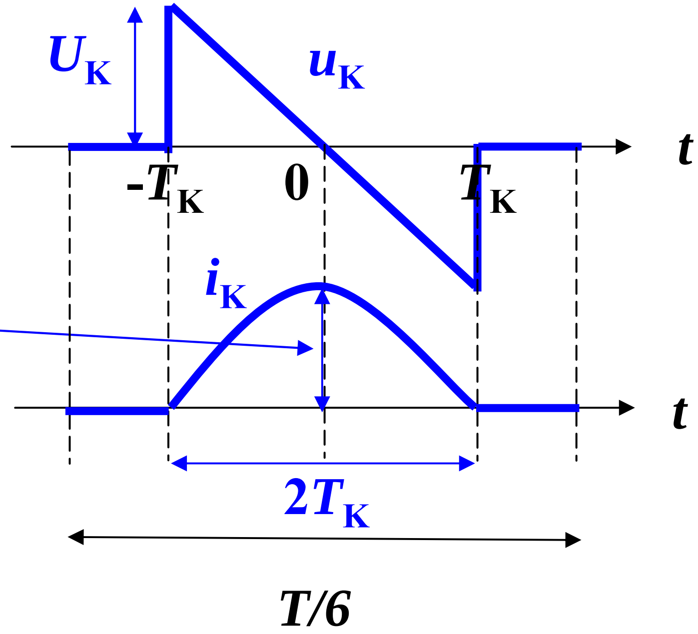
$$\boxed{i_K(t) = \frac{U_K}{\omega L_K} \cdot \frac{\cos(\omega t) - \cos(\omega T_K)}{\sin(\omega T_K)}} \qquad \boxed{\hat{i}_K = i_K(t=0) = \frac{U_K}{\omega L_K} \cdot \frac{1 - \cos(\omega T_K)}{\sin(\omega T_K)}}$$

For small values T_K the **parabolic approximation of i_K** is derived!

5. Dynamics of DC machines

Parabolic approximation of small parasitic circular current i_K

$$\hat{i}_K = \frac{U_K}{\omega L_K} \cdot \frac{1-\cos(\omega T_K)}{\sin(\omega T_K)}$$



For small values of α : $2T_K << T/6 \Rightarrow \omega T_K << 1$

$$\hat{i}_K = \frac{U_K}{\omega L_K} \cdot \frac{1-\cos(\omega T_K)}{\sin(\omega T_K)} \approx \frac{U_K}{\omega L_K} \cdot \frac{1-(1-(\omega T_K)^2/2)}{\omega T_K} = \frac{U_K T_K}{2L_K}$$

$$\boxed{\hat{i}_K = \frac{U_K T_K}{2L_K}}$$

Voltage limits of converter-operated DC machines

Three-phase AC 400 V grid, B6C-bridge:

maximum voltage: $U_d(\alpha=0)=U_{d0}\cdot\cos 0=U_{d0}=\frac{3}{\pi}\cdot U_{LL}\cdot\sqrt{2}=\frac{3}{\pi}\cdot 400\cdot\sqrt{2}=540\ \mathrm{V}$

rated voltage: $U_d(\alpha=30°)=U_{d0}\cdot\cos 30°=540\cdot\frac{\sqrt{3}}{2}=460\ \mathrm{V}$

voltage margin for voltage control: $\Delta U_d=540\ \mathrm{V}-460\ \mathrm{V}=80\ \mathrm{V}$

So thyristor bridge is operated between $30°<\alpha<150°$.

Example:

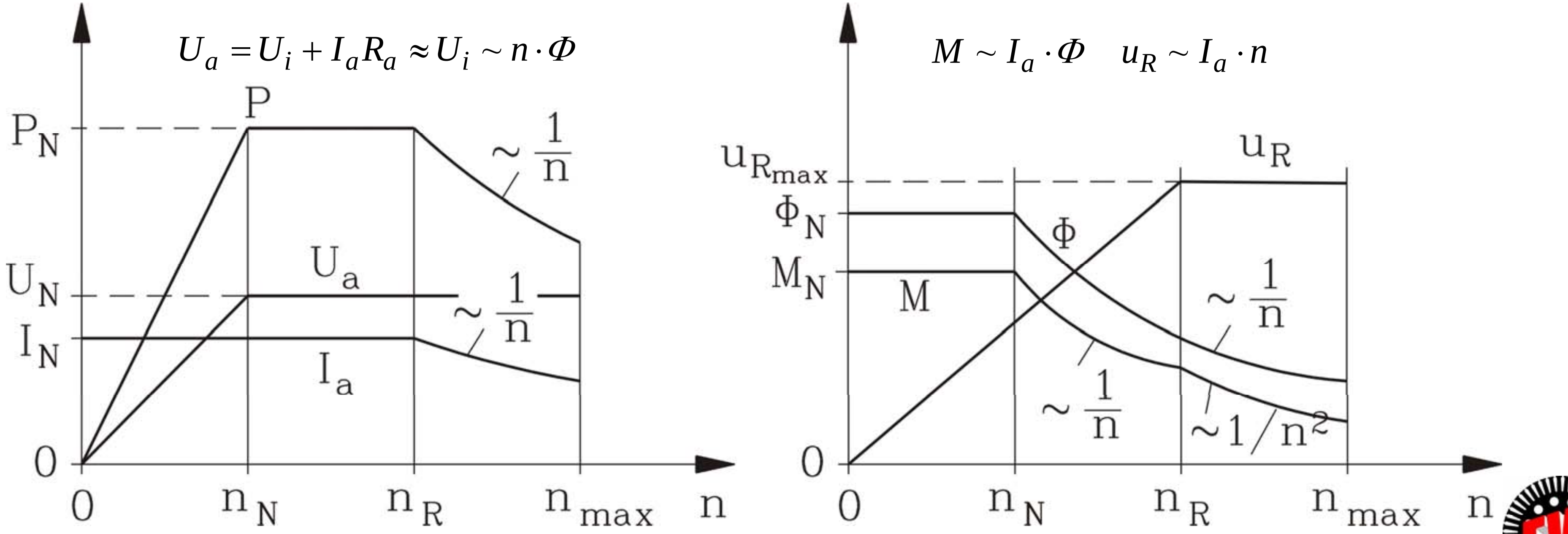
Three-phase AC 400 V grid, usual rated and maximum voltages:

	Maximum voltage	Rated voltage	
B6C	540 V	460 V	1 quadrant operation
(B6C)A(B6C)	540 V	400 V	4 quadrant operation

5. Dynamics of DC machines

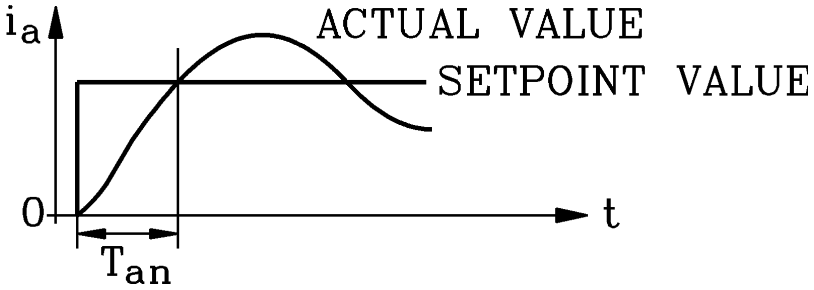
Limits of converter-fed DC machine operation

$0 \le n \le n_N$	**Voltage controlled** DC machine: Limits: Maximum armature current i_a, maximum main flux Φ
$n_N \le n \le n_R$	Flux controlled DC machine (**Field weakening**): $\Phi\downarrow$ Limits: Maximum armature current i_a and voltage u_a
$n_R \le n \le n_{max}$	Flux controlled DC machine (Field weakening): $\Phi\downarrow$ Limits: Maximum armature voltage u_a and **maximum reactance voltage of commutation $u_{R,max} = 12$ V**



5. Dynamics of DC machines

Typical response times of converter-fed DC machines



$$T_f = L_f / R_f >> T_a = L_a / R_a$$

Fast DC machine reaction via change of i_a, NOT via change of i_f!

Time for reversal of armature current.

a) (B6C)A(B6C)	b) (B6C)A(B6C)	c) Mechanical switch
Both bridges always active	Only one bridge active	Polarity changer
< 0.5 ms	5 ... 10 ms	50 ... 1500 ms

The larger numbers correspond with larger drives of several hundreds of kW up to MW range.

Time for reversal of field current.

a) (B6C)A(B6C)	b) Mechanical switch
0.5 ... 2 s	1 ... 2.5 s

The larger numbers correspond with larger drives of several hundreds of kW up to MW range.

Summary:
Converter operated separately excited DC machine

- Thyristor converter operation of DC machine is a dynamic operation
- Armature current time signal may be calculated analytically (see: Collection of Exercises)
- Anti-parallel thyristor rectifier for reversed torque
- Circulating parasitic current between the two thyristor bridges possible
- Limiting operation curves for variable speed DC machine
- Typical dynamic response times of controlled DC machines rise with increased machine size
- Fast control via change of armature current due to $T_a << T_f$