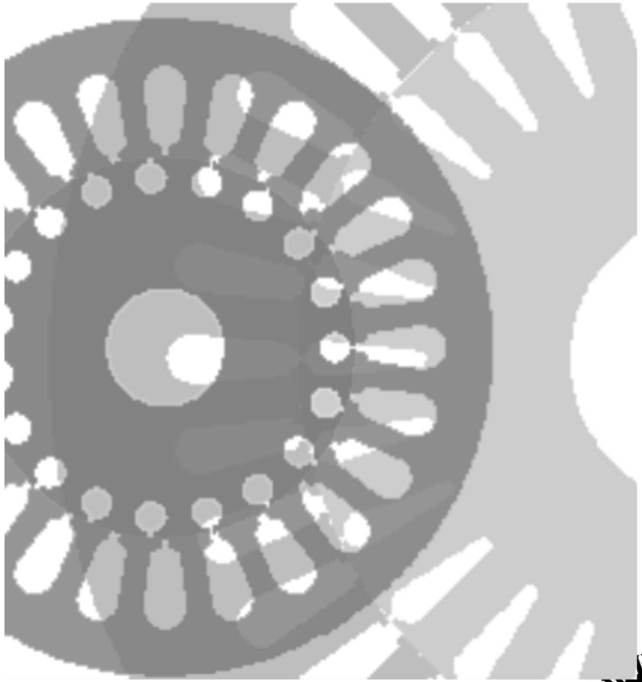
Energy Converters – CAD and System Dynamics

TECHNISCHE UNIVERSITÄT DARMSTADT

1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



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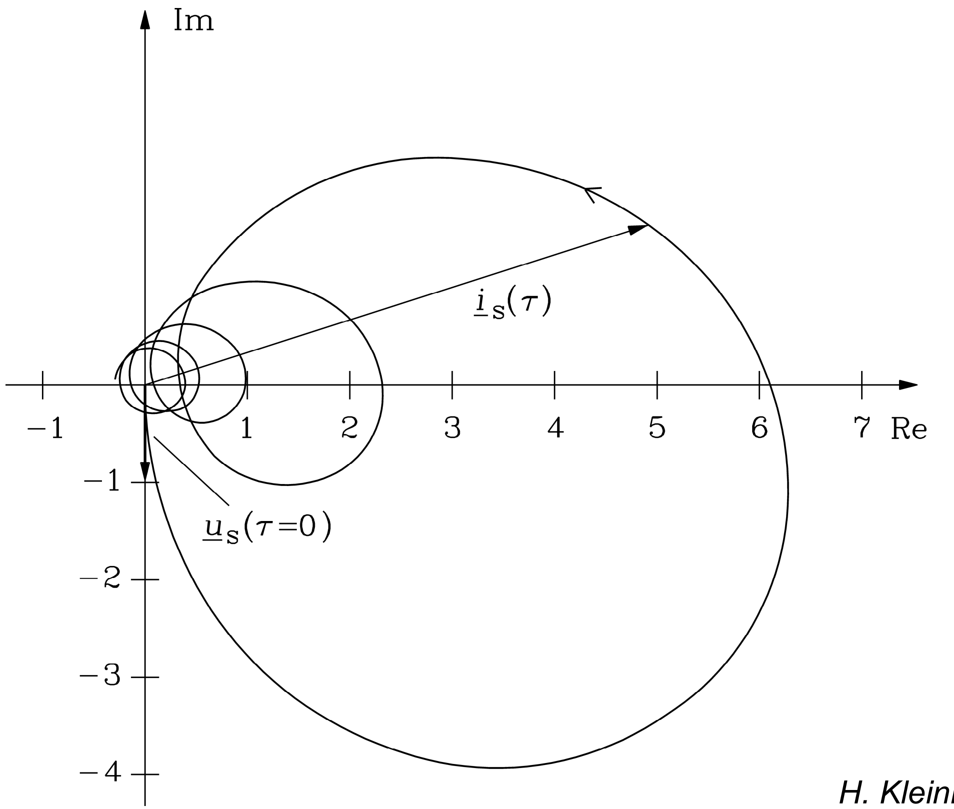
SPEED program

6. Space vector theory



Source:
H. Kleinrath, Springer-Verlag

6. Space vector theory

6.1 M.M.F. space vector definition

6.2 M.M.F. space vector and phase currents

6.3 Current, flux linkage and voltage space vectors

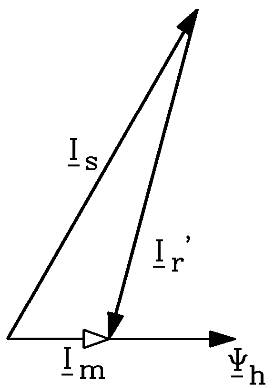
6.4 Space vector transformation

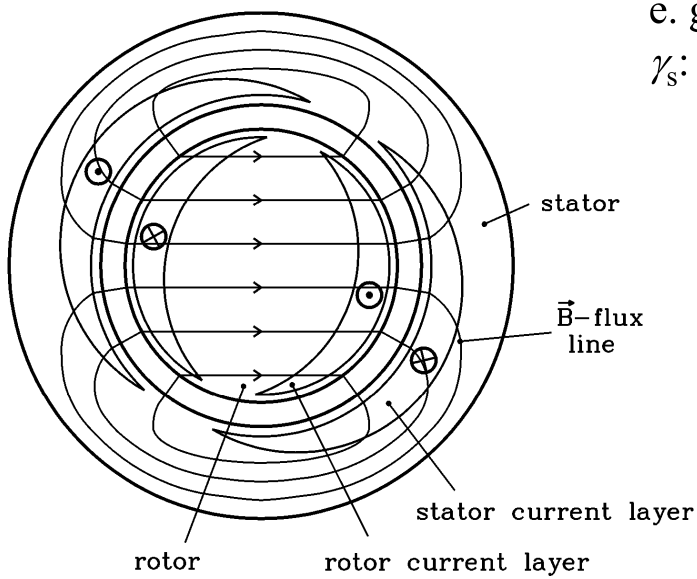
6.5 Influence of zero sequence current system

6.6 Magnetic energy

6. Space vector theory

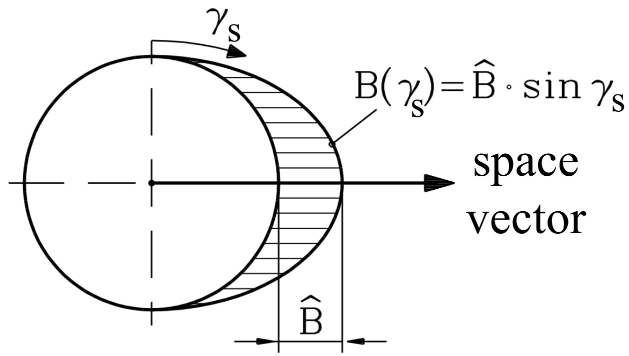
Concept of space vector ("Raumzeiger")





e. g. stator reference frame:

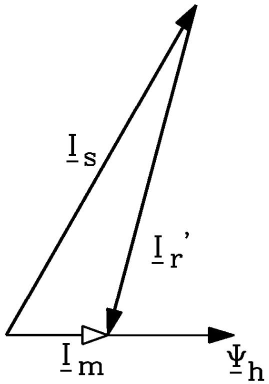
γ_s: stator circumference angle: $\gamma_s = x_s \pi / \tau_p$

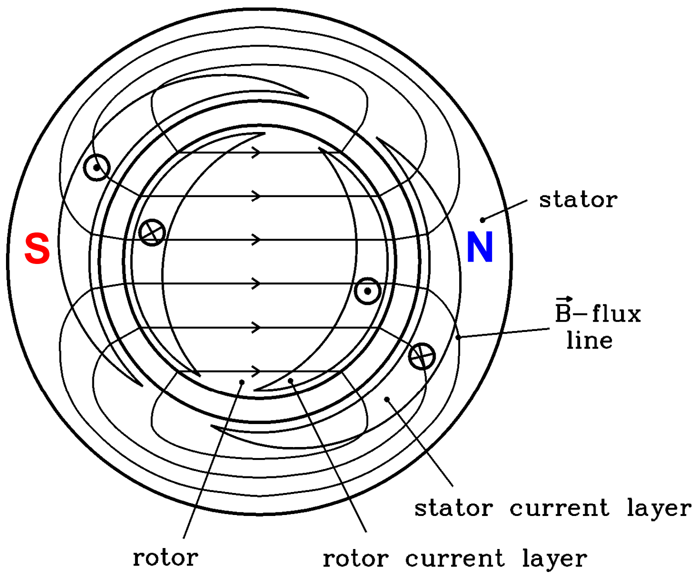


- Stator and rotor <u>fundamental</u> air gap field are excited by **sinusoidal distributed stator and rotor current load (= "current layer" $A_s(\gamma_s)$, $A_r(\gamma_s)$)**
- Superposition of both fundamental fields yields the **resulting magnetizing fundamental air gap field wave $B(\gamma_s)$**
- Each sinusoidal distributed air gap field wave is described by a **space vector** in the machine´s axial cross section plane

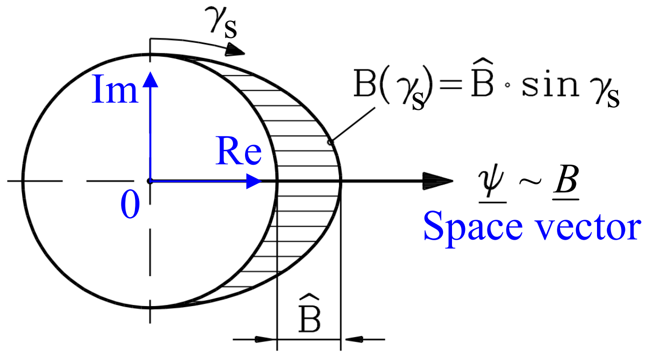
6. Space vector theory

Definition of space vector ("Raumzeiger")





e. g. stator reference frame:
γ_s: stator circumference angle



- Space vector length = field wave amplitude $\hat{B}$
- Space vector orientation = position γ_s of field wave maxima
- Space vector direction = position of north pole **N**.

- **Use of complex coordinate frame for machine cross section ⇒ complex space vector!**
- **Alternatively the space vectors $\underline{B}$ or $\underline{\Psi}$ or $\underline{I}$ are used for the magnetic fundamental air gap field wave ⇒ e.g.: $\underline{B}_\delta$ or $\underline{\Psi}_h$ or $\underline{I}_m$**

$$\hat{\underline{I}}_m = \underline{\Psi}_h / L_h = (N_s k_{ws} \cdot \frac{2}{\pi} \tau_p l_e / L_h) \cdot \underline{B}_\delta$$

M.M.F. space vector definition

- **Dynamic situation:** The three phase currents are no longer of sine wave time function, but $I_U(t)$, $I_V(t)$, $I_W(t)$ **vary arbitrarily**.

Steady state:

fixed frequency Ω, amplitude $\hat{I}$ & phase shift

$$I_U(t) = \hat{I} \cdot \cos(\Omega \cdot t)$$

$$I_V(t) = \hat{I} \cdot \cos(\Omega \cdot t - 2\pi / 3)$$

$$I_W(t) = \hat{I} \cdot \cos(\Omega \cdot t - 4\pi / 3)$$

Dynamic situation:

currents change arbitrarily

$$I_U(t)$$

$$I_V(t)$$

$$I_W(t)$$

- In many cases the three-phase winding is **star-connected**:

$$I_U(t) + I_V(t) + I_W(t) = 0$$

- a) **Delta-connected** winding or
  b) star-connected winding with **connection of neutral point**:

$$I_U(t) + I_V(t) + I_W(t) \neq 0 \quad \text{„neutral point clamped“}$$

6. Space vector theory

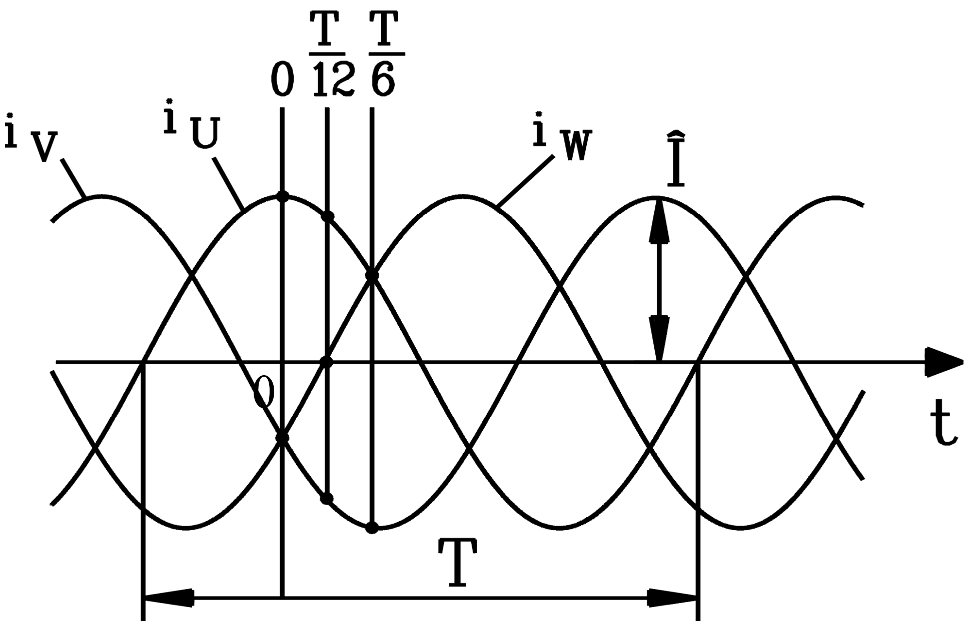
Three-phase system:
Sinusoidal symmetrical vs. arbitrary currents

Three-phase sinusoidal AC current system

Fixed frequency $f = 1/T$

Fixed amplitude $\hat{I}$

Fixed phase shift $0, 2\pi/3, 4\pi/3$

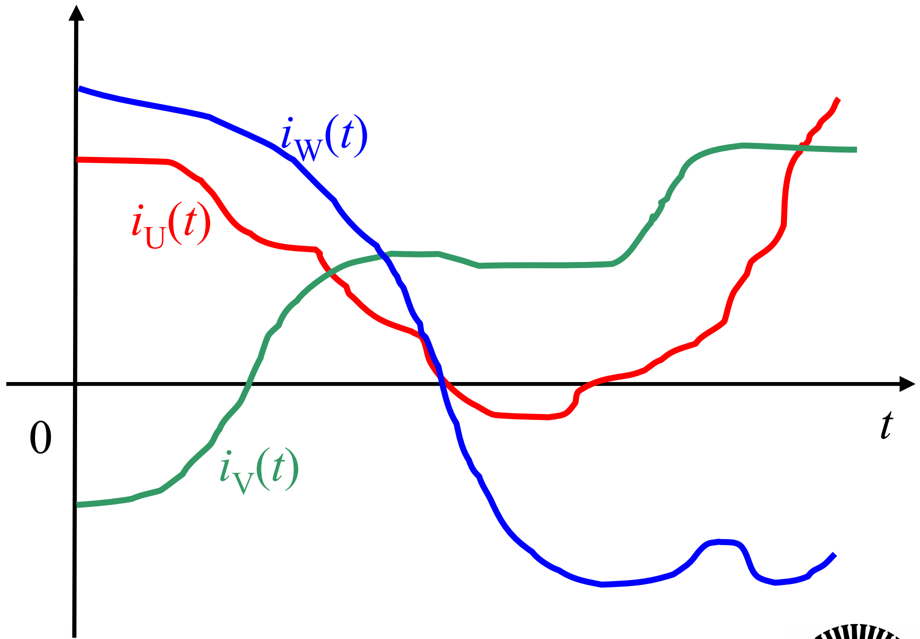


Three-phase arbitrary current system

No frequency detectable

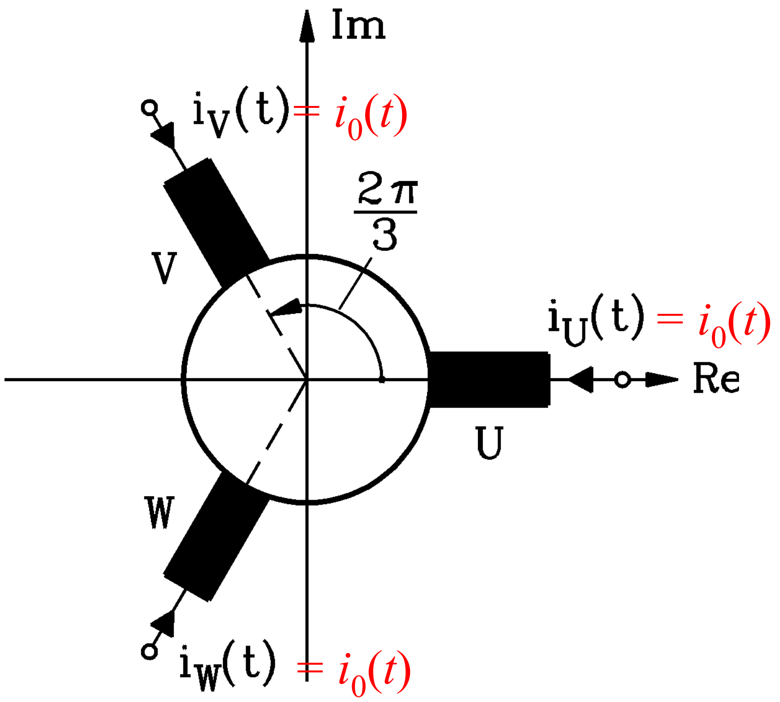
No defined amplitude

No phase-shift defined

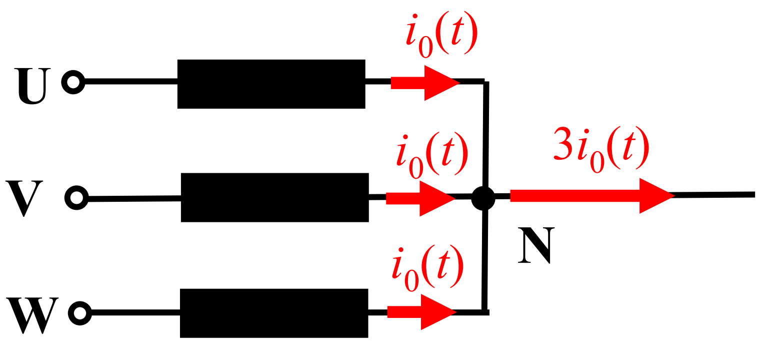


6. Space vector theory

Common mode current $i_0(t)$ in all three phases identical!



Example: Neutral point N clamped



$$\left.\begin{aligned} i_{US}(t) &= i_U(t) - i_0(t) \\ i_{VS}(t) &= i_V(t) - i_0(t) \\ i_{WS}(t) &= i_W(t) - i_0(t) \end{aligned}\right\} +$$

$$0 = i_U(t) + i_V(t) + i_W(t) - 3i_0(t)$$

„Common-mode free“ current system:

$$i_{US}(t) + i_{VS}(t) + i_{WS}(t) = 0$$

Original current system:

$$i_U(t) + i_V(t) + i_W(t) \neq 0$$

$$\boxed{i_0(t) = \left[i_U(t) + i_V(t) + i_W(t)\right]/3}$$

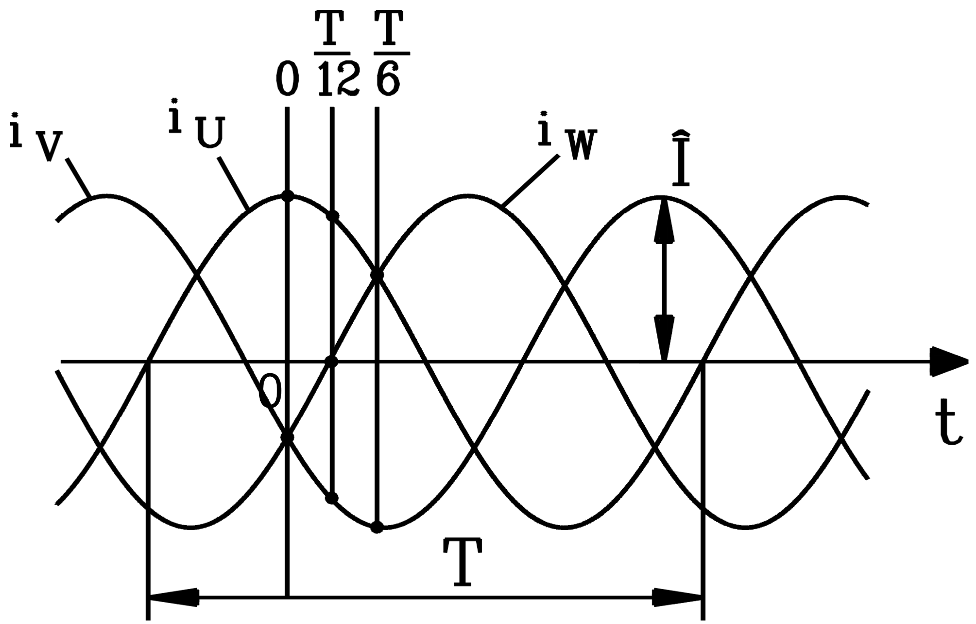
6. Space vector theory

Common mode current $i_0(t)$

$$\boxed{i_0(t) = \left[i_U(t) + i_V(t) + i_W(t) \right] / 3}$$

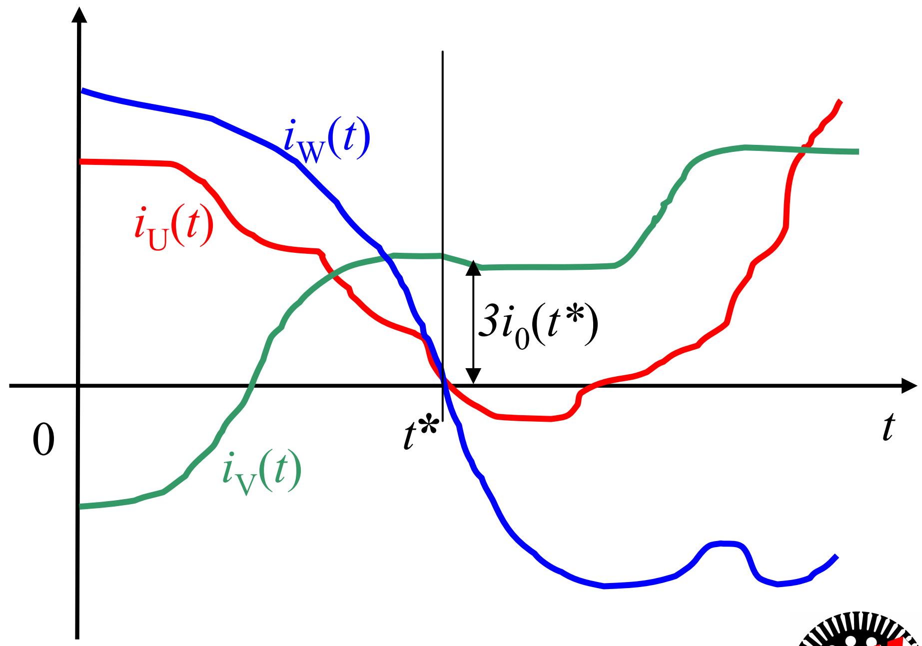
Three-phase sinusoidal AC current system

$$i_0(t) = 0$$



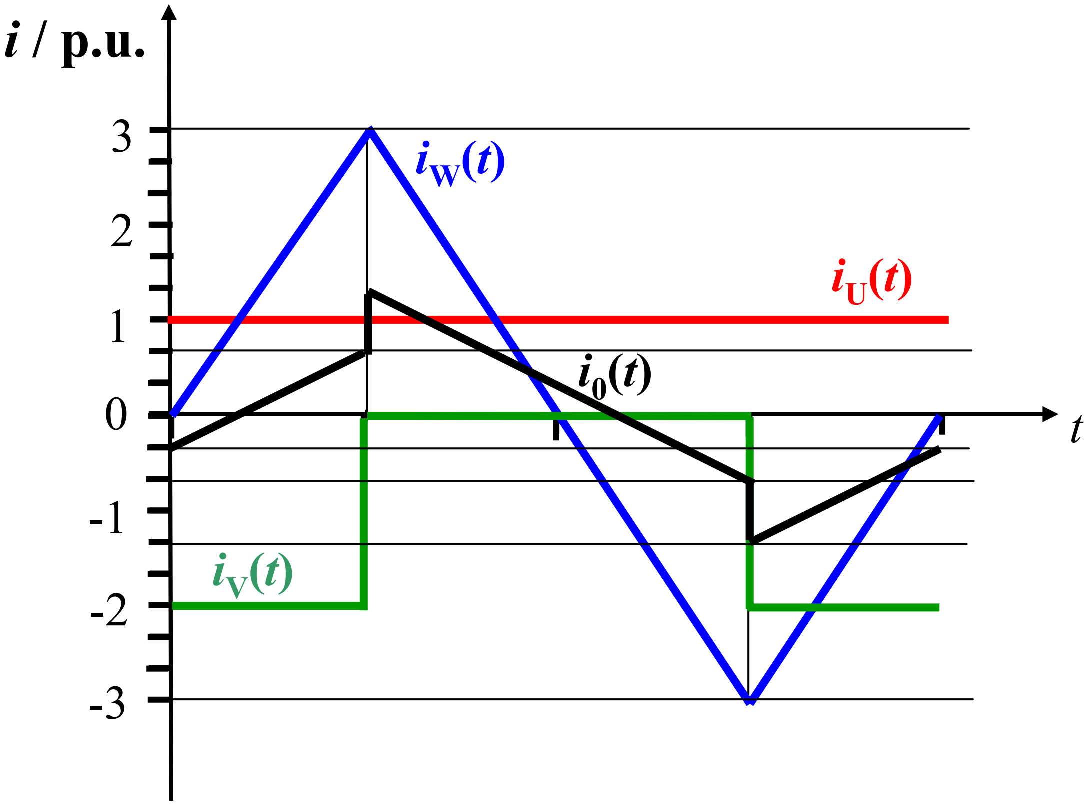
Three-phase arbitrary current system

$$i_0(t) = 0 \quad \text{or} \quad i_0(t) \neq 0$$

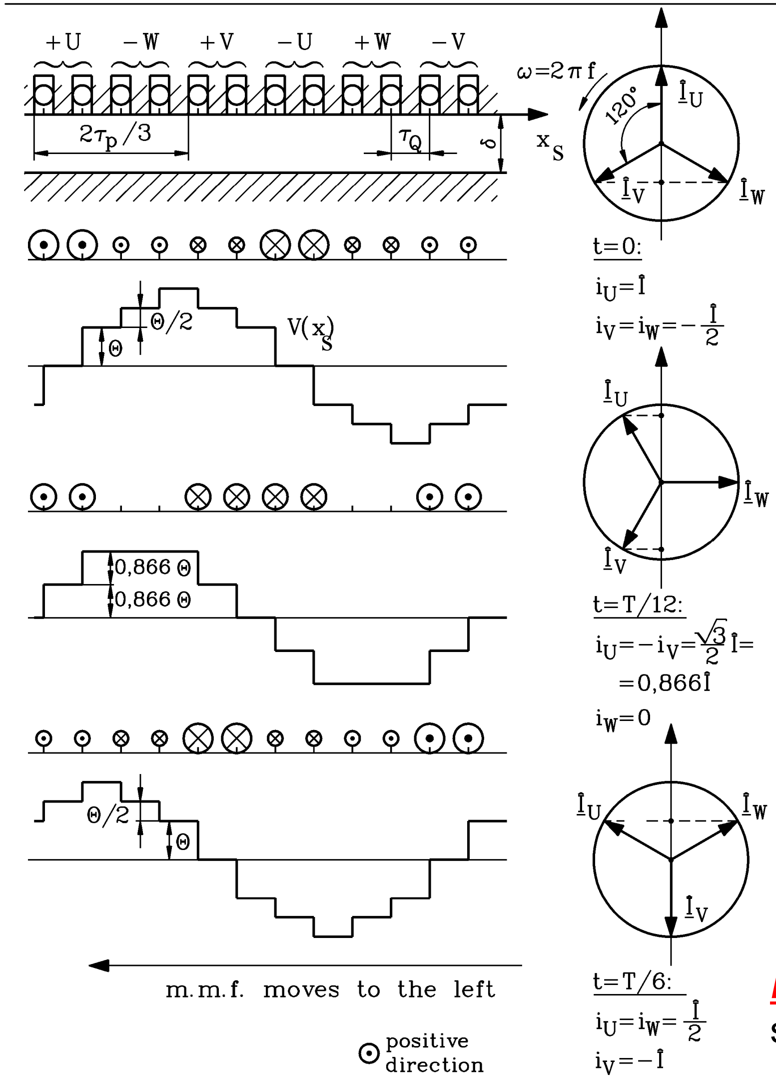


Example: Common mode current $i_0(t)$

$$\boxed{i_0(t) = \left[i_U(t) + i_V(t) + i_W(t)\right]/3}$$



6. Space vector theory
Three phase sinusoidal currents



- "Steady state" operation
- **Fundamental m.m.f. wave $V_{\nu=1}(x_s)$ moves with constant velocity**

$$\boxed{v_{syn}=\frac{2\tau_p}{T}=2f\tau_p}$$

Synchronous velocity !

Synchronous rotational speed n_{syn}
in case of rotating field arrangement:

$$\omega_{syn}=2\pi n_{syn}=\frac{v_{syn}}{d_{si}/2}=\frac{v_{syn}}{p\tau_p/\pi}=\frac{2\pi f}{p}$$

$$\boxed{n_{syn}=\frac{f}{p}}$$

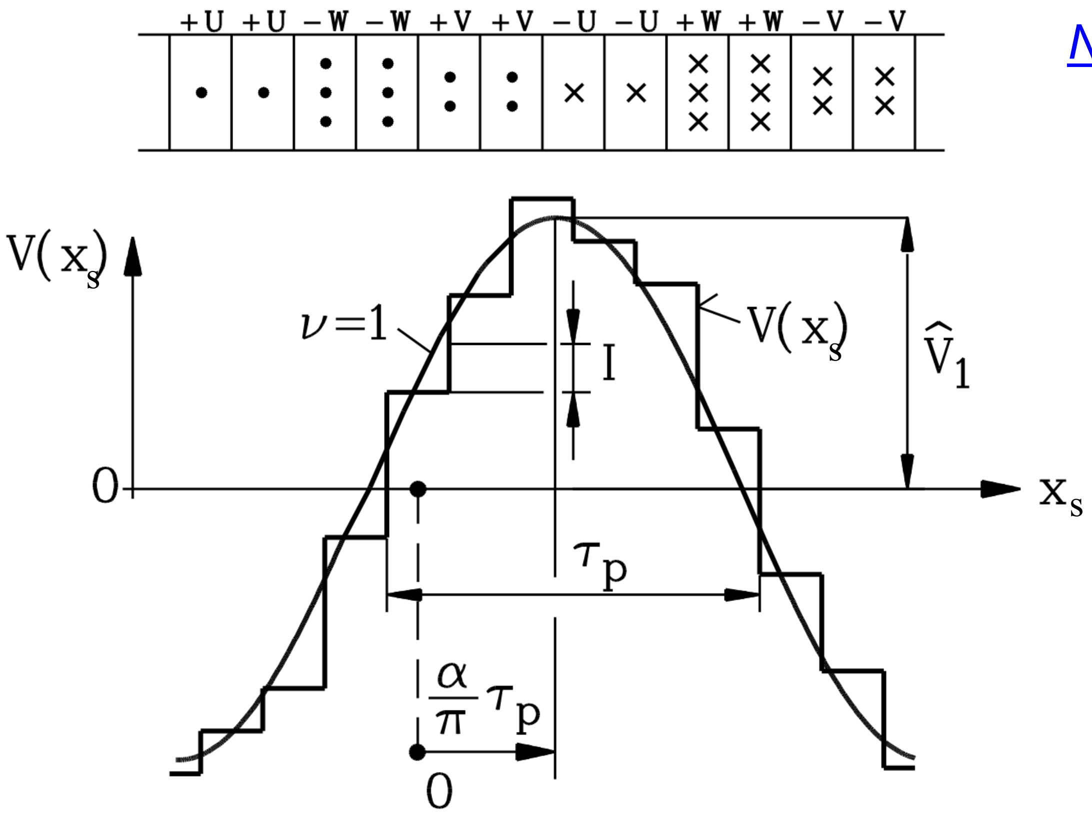
Example: $q = 2$,
single-layer winding

6. Space vector theory

M.M.F. $V(x_s)$ of three phase winding at arbitrary currents

Three-phase AC star-connected winding with arbitrary phase currents excites a MMF distribution $V(x)$ with a **dominant sine wave fundamental** $V_1(x_s)$

Example: $q = 2$, $N_c = 1$, single layer winding, $I_U(t) = I,\ I_V(t) = 2I,\ I_W(t) = -3I$



Note: Star connection: Common mode current I_0 is ZERO!

$$3 \cdot I_0(t) = I_U(t) + I_V(t) + I_W(t) =$$
$$= I + 2I - 3I = 0$$

Fourier **m.m.f. fundamental:**

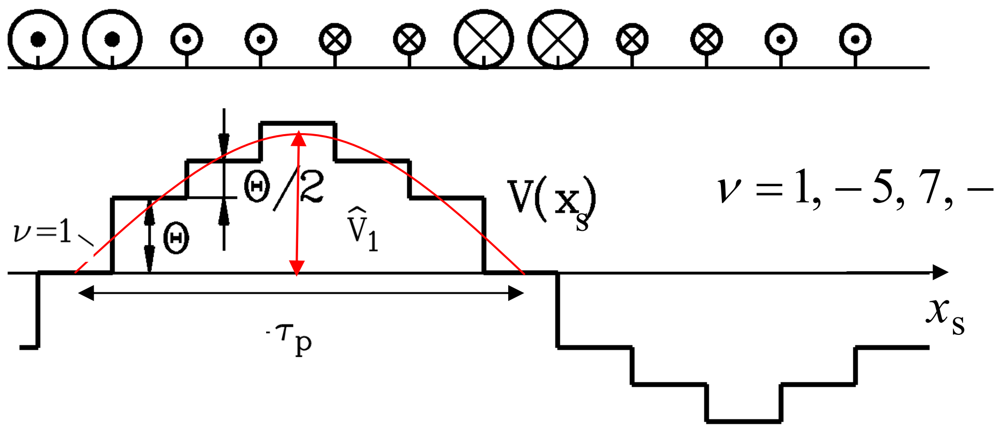
$$\boxed{V_1(x_s) = \hat{V}_1 \cdot \cos\left(\frac{x_s \pi}{\tau_p} - \alpha\right)}$$

$\alpha = 0$: in U-axis

6. Space vector theory

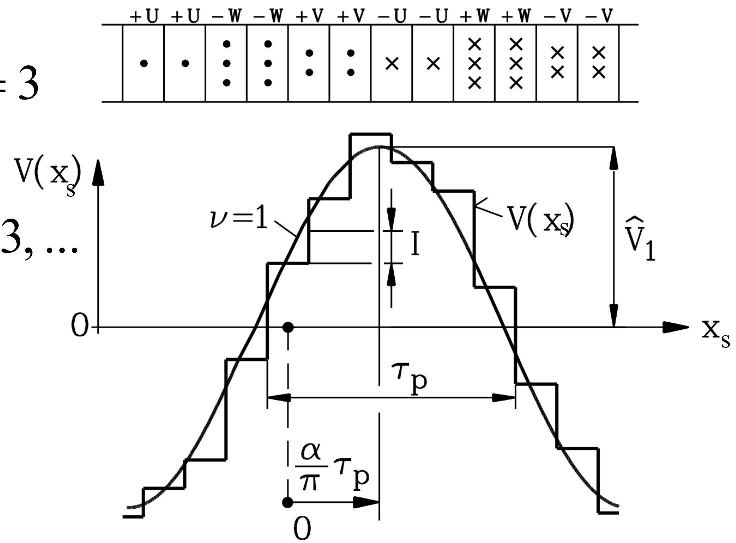
M.M.F. of three phase winding at “common mode current = zero”

a) Three phase sinusoidal currents | **b) Three phase arbitrary currents**



$m_s = 3$

$\nu = 1, -5, 7, -11, 13, \ldots$



- In both cases a step like m.m.f. distribution with a dominant fundamental $\nu = 1$ is excited:
  - Double pole pitch $2\tau_p$ = wave length
  - Amplitude is derived from FOURIER-analysis: $\hat{V}_1$

Fundamental moves with a) constant speed n_{syn} b) arbitrary speed $n(t) = \dot{\alpha}(t)/(2\pi \cdot p)$

Fundamental amplitude is a) constant b) of arbitrary value $\hat{V}_1(t)$

- Here: ONLY THE FUNDAMENTAL ν = 1 IS FURTHER CONSIDERED !

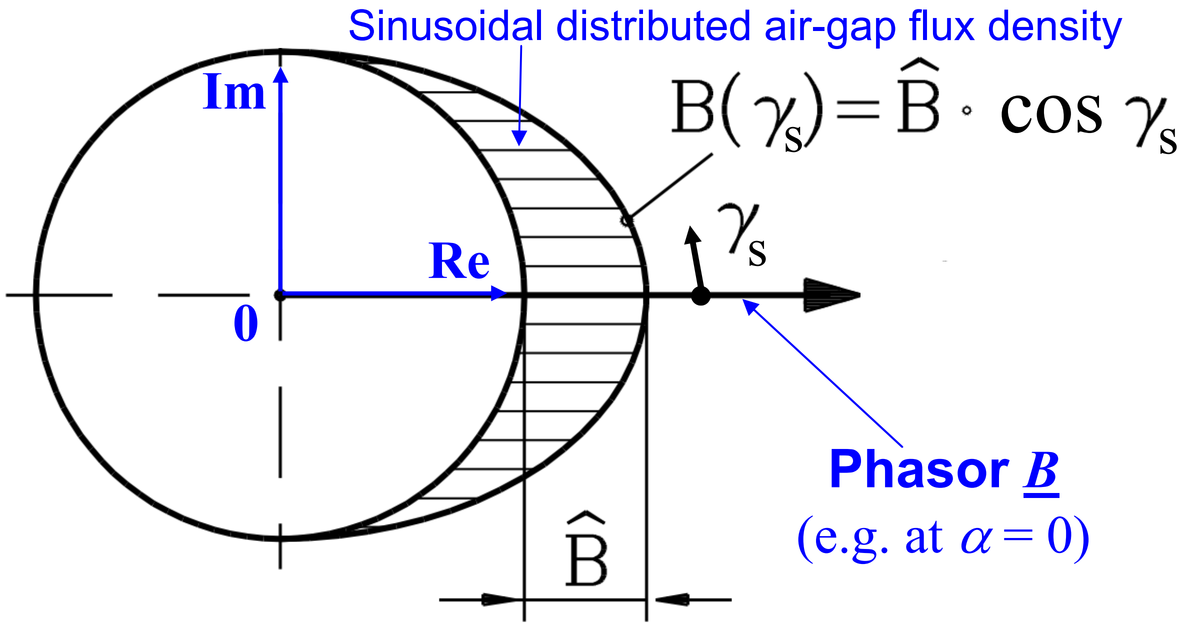
6. Space vector theory

"Arrow" for <u>fundamental</u> air gap magnetic flux density *B*

- Magnetic air gap field *B*: No slotting, no (or constant) iron saturation:

An arrow („phasor") may represent the sinusoidal *B*- or *V*-distribution !

1. **Arrow length = $\hat{B}$**
2. **Arrow orientation = at maximum field**
3. **Arrow points in NORTH pole direction**

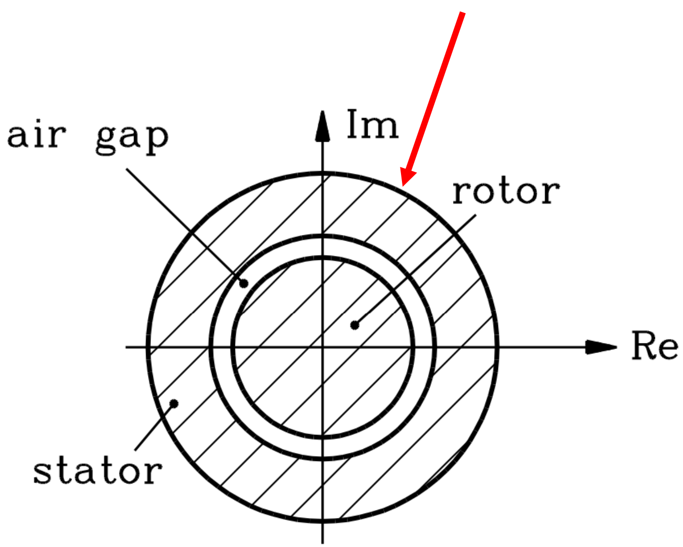


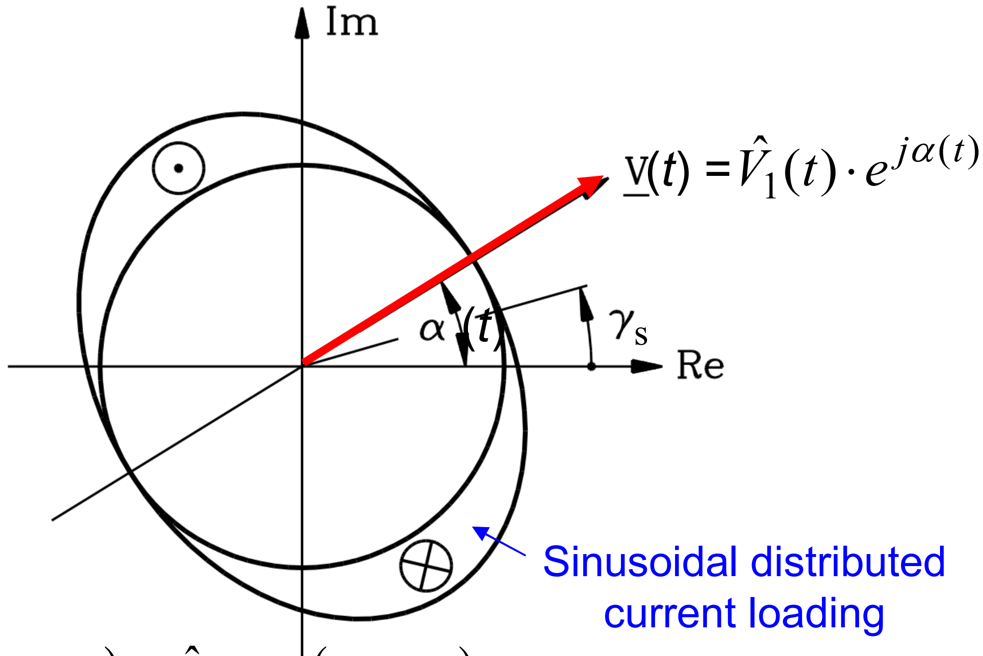
$$B_1(x_s,t) = \mu_0 \cdot \frac{\hat{V}_1(t)}{\delta} \cdot \cos\left(\frac{x_s\pi}{\tau_p} - \alpha(t)\right) = \hat{B}(t) \cdot \cos\left(\frac{x_s\pi}{\tau_p} - \alpha(t)\right) = \hat{B}(t) \cdot \cos(\gamma_s - \alpha(t))$$

6. Space vector theory

Complex space vector = arrow of MMF and *B*-fundamental

- Prof. *Kovacs* (*Budapest*): **Complex co-ordinate system** in machine cross section
- MMF fundamental may be represented as **complex space phasor ("space vector"),** lying in *machine cross section plane*.





$$V_1(x,t) = \hat{V}_1(t)\cdot\cos\left(\frac{x_s\pi}{\tau_p} - \alpha(t)\right) = \hat{V}_1\cdot\cos(\gamma_s - \alpha) = \hat{V}_1\cdot\cos(\alpha - \gamma_s)$$

$$\hat{V}_1(t)\cdot\cos(\alpha(t) - \gamma_s) = \mathrm{Re}\left\{\hat{V}_1(t)\cdot e^{j\alpha(t)}\cdot e^{-j\gamma_s}\right\} = \mathrm{Re}\left\{\underline{V}(t)\cdot e^{-j\gamma_s}\right\}$$

Summary:
M.M.F. space vector definition

- Three-phase system with arbitrary phase currents
- 2*p*-pole distributed winding leads to 2*p*-pole count also with arbitrary currents
- Movement of 2*p*-field follows the arbitrary time function of currents
- 2*p*-fundamental field described by space vector in magnitude and position
- Zero-sequence current system is here omitted,
  but – if existing - does not contribute to 2*p*-fundamental (see later in this chapter)

6. Space vector theory

6.1 M.M.F. space vector definition

6.2 M.M.F. space vector and phase currents

6.3 Current, flux linkage and voltage space vectors

6.4 Space vector transformation

6.5 Influence of zero sequence current system

6.6 Magnetic energy

6. Space vector theory

1st step: Single phase excitation: Magnetic alternating field

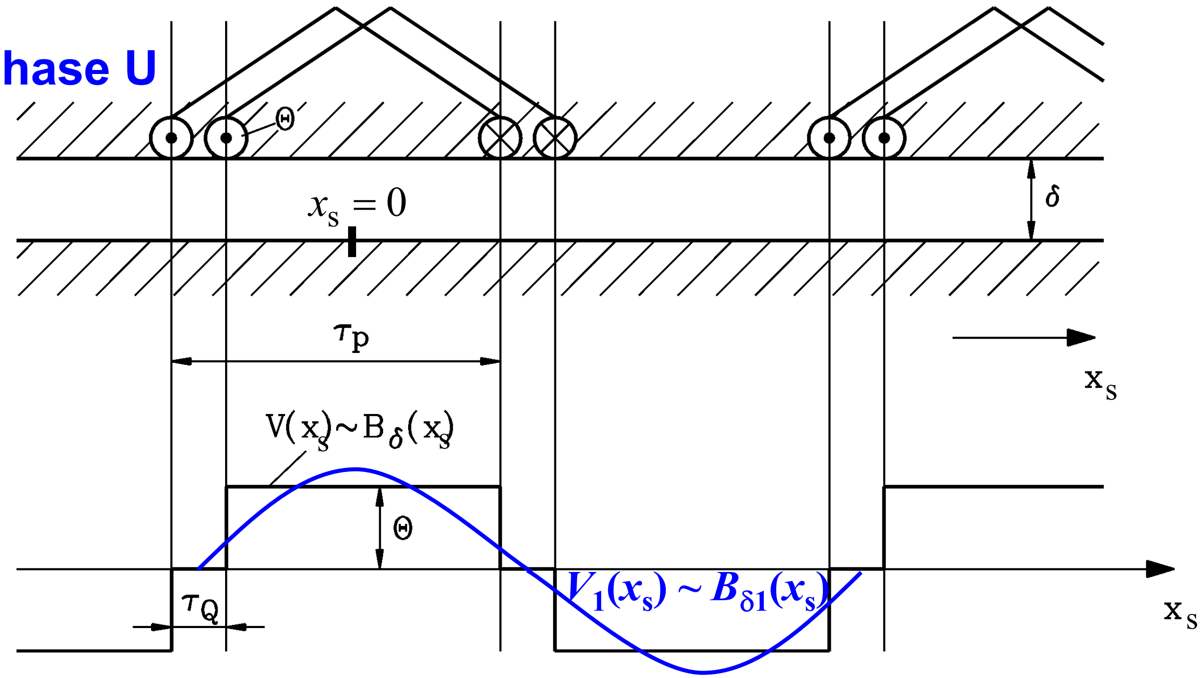
- Feeding the coil groups with sinusoidal alternating current i_c:
  Amplitude $\hat{I}_c$, frequency f, angular frequency $\omega = 2\pi f$, $T = 1/f$: period of oscillation

$$i_c(t) = \hat{I}_c \cdot \cos\omega t \quad \Rightarrow \quad B_\delta(x_s, t) = B_\delta(x_s) \cdot \cos\omega t$$

- Air gap field oscillates also sinusoidal with time, BUT maintains **its spatial distribution (its shape =** its distribution along x_s) ! The amplitude of (radial) field component $B_\delta(x_s, t)$ at locus x_s changes with time between positive and negative maximum value.

Example: q = 2, single-layer winding

x_s = 0: at winding axis of phase U



6. Space vector theory

FOURIER-series of field of one phase with pitched coil groups

- *FOURIER*-series of m.m.f. of one phase $V_{ph}(\gamma_s, t)$: **$q \geq 1$, $W/\tau_p \leq 1$**

$$\hat{V}_{\nu,ph}(t) = \frac{N}{2p} \cdot I(t) \cdot \frac{4}{\nu\pi} \cdot k_{p,\nu} \cdot k_{d,\nu}\ , \qquad \nu = 1, 3, 5, \ldots \quad N: \text{turns per phase}$$

- Phase current: **Arbitrary current $I(t)$, not:** $I(t) = I_{rms} \cdot \sqrt{2} \cdot \cos(\omega t)$
- The MMF distribution $V_{ph}(\gamma_s, t)$ (and hence the air gap field $B_\delta(\gamma_s, t)$) is a sum of pulsating, standing waves **(„Pulsating field“ with ~ $I(t)$)**

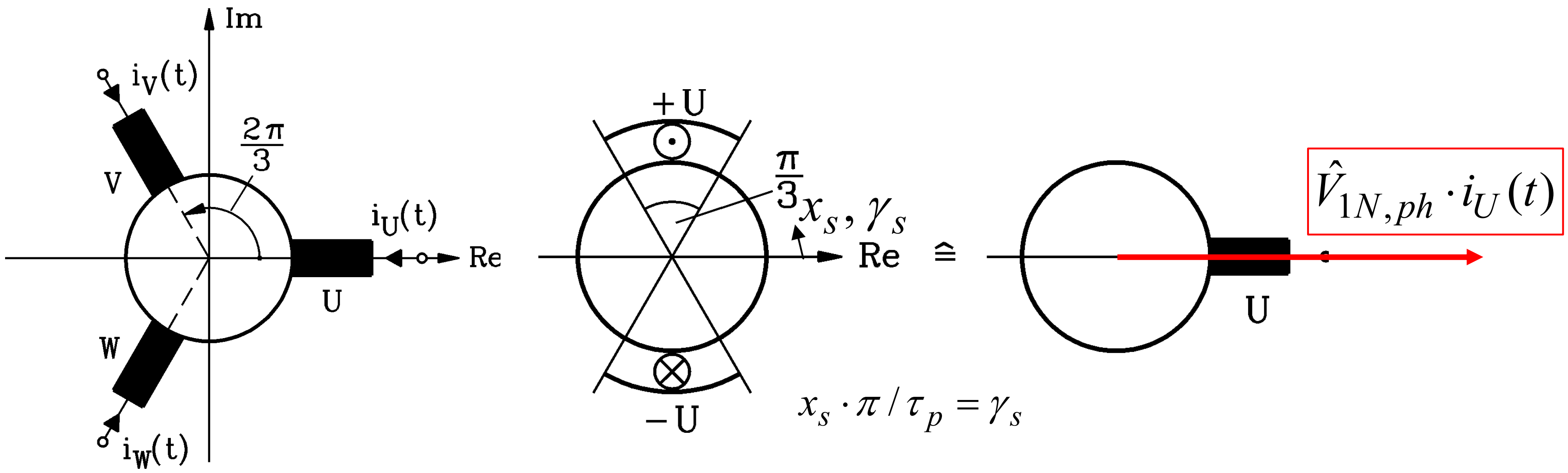
$$V_{ph}(\gamma_s, t) = \sum_{\nu=1,3,5,\ldots}^{\infty} \hat{V}_{\nu,ph}(t) \cdot \cos(\nu \cdot \gamma_s)$$

- **"Winding coefficient“:** $k_{w,\nu} = k_{p,\nu} \cdot k_{d,\nu}$

- Only fundamental $\nu = 1$ considered, at arbitrary current $I(t)$: $\hat{V}_{1,ph}(t) = \frac{N}{2p} \cdot I(t) \cdot \frac{4}{\pi} \cdot k_{w,1}$

- **At rated current I_N:** $\boxed{\hat{V}_{1N,ph} = \frac{N}{2p} \cdot \hat{I}_N \cdot \frac{4}{\pi} \cdot k_{w,1}}$

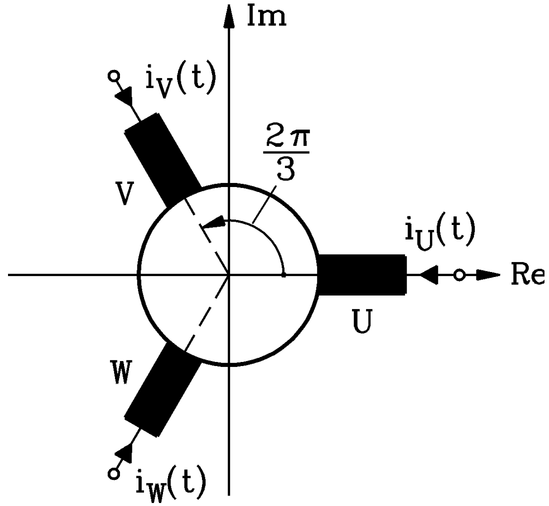
Calculation of m.m.f. space vector $\underline{V}(t)$ from arbitrary currents for a three-phase system



2nd step: Fundamental of phase MMF distribution of phase U is directly proportional to the phase current value $I_U(t)$: $\hat{V}_{1U,ph}(t) = \hat{V}_{1N,ph} \cdot i_U(t)$

$$V_{1U}(x_s,t) = \hat{V}_{1U,ph}(t) \cdot \cos\left(\frac{x_s \pi}{\tau_p}\right) = \hat{V}_{1N,ph} \cdot \frac{I_U(t)}{\hat{I}_N} \cdot \cos\left(\frac{x_s \pi}{\tau_p}\right) = \hat{V}_{1N,ph} \cdot i_U(t) \cdot \cos(\gamma_s)$$

6. Space vector theory
Calculation of m.m.f. space vectors for each phase



3rd step:

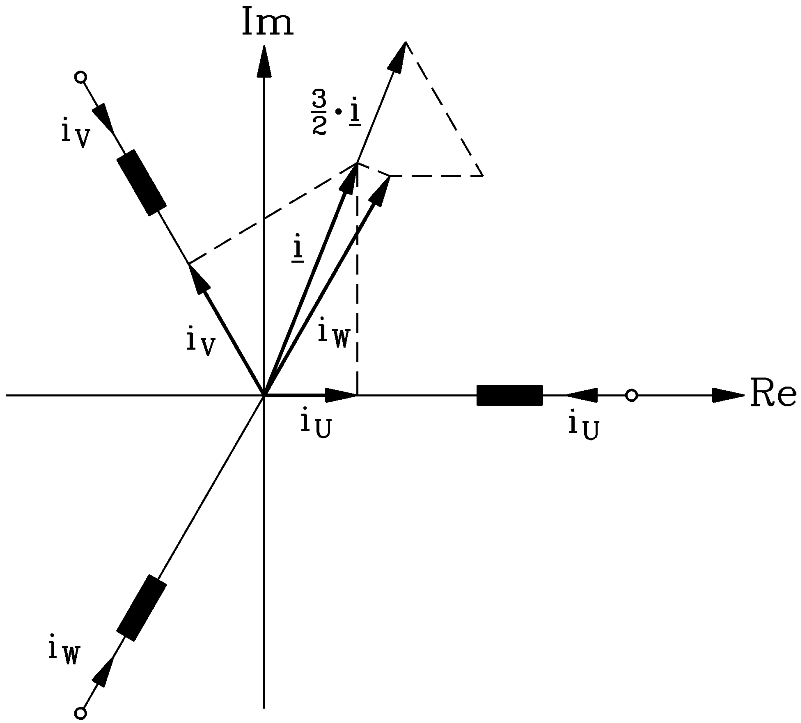
MMF fundamentals of phases V, W, excited by the arbitrary currents $I_v(t)$, $I_W(t)$, are spatially shifted by 120°, 240° (in el. degrees)!

$$V_{1V}(x_s,t) = \hat{V}_{1N,ph} \cdot \frac{I_V(t)}{\hat{I}_N} \cdot \cos\left(\frac{x_s \pi}{\tau_p} - 2\pi/3\right) = \hat{V}_{1N,ph} \cdot i_V(t) \cdot \cos(\gamma_s - 2\pi/3) = V_{1V}(\gamma_s,t)$$

$$V_{1W}(x_s,t) = \hat{V}_{1N,ph} \cdot \frac{I_W(t)}{\hat{I}_N} \cdot \cos\left(\frac{x_s \pi}{\tau_p} - 4\pi/3\right) = \hat{V}_{1N,ph} \cdot i_W(t) \cdot \cos(\gamma_s - 4\pi/3) = V_{1W}(\gamma_s,t)$$

6. Space vector theory

Calculation of m.m.f. space vectors for each phase



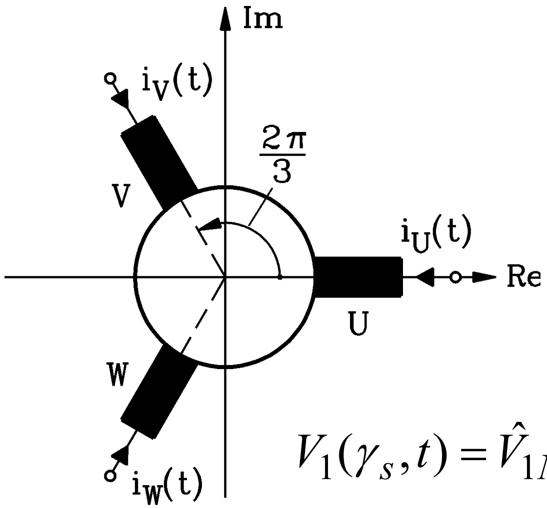
4th step:

Superposition of MMF fundamentals of all three phases U, V, W ,
and divide by 3/2 to get a per-unit value.

Calculation of MMF space vector $\underline{V}(t)$ from actual phase currents

$$\underline{a} = e^{j\cdot(2\pi/3)}$$

$$\underline{a}^2 = e^{j\cdot(4\pi/3)}$$



$$\left.\begin{aligned} V_{1U}(\gamma_s,t) &= \hat{V}_{1N,ph}\cdot i_U(t)\cdot\cos(-\gamma_s) \\ V_{1V}(\gamma_s,t) &= \hat{V}_{1N,ph}\cdot i_V(t)\cdot\cos(-\gamma_s+2\pi/3) \\ V_{1W}(\gamma_s,t) &= \hat{V}_{1N,ph}\cdot i_W(t)\cdot\cos(-\gamma_s+4\pi/3) \end{aligned}\right\}+$$

$$V_1(\gamma_s,t) = V_{1U}(\gamma_s,t)+V_{1V}(\gamma_s,t)+V_{1W}(\gamma_s,t)$$

$$V_1(\gamma_s,t) = \hat{V}_{1N,ph}\cdot\mathrm{Re}\left\{i_U(t)\cdot e^{-j\gamma_s}+i_V(t)\cdot e^{-j\gamma_s}\cdot\underline{a}+i_W(t)\cdot e^{-j\gamma_s}\cdot\underline{a}^2\right\}$$

- Positions of the phasors of the MMF fundamentals of the three phases U, V, W :
are spatially shifted by 120°, 240° (in el. degrees) ⇒ **Multiplication with $\underline{a}$, $\underline{a}^2$**

$$\cos(-\gamma_s) = \mathrm{Re}\left\{e^{-j\gamma_s}\right\} \qquad \cos(-\gamma_s+2\pi/3) = \mathrm{Re}\left\{e^{-j\gamma_s}\cdot\underline{a}\right\} \qquad \cos(-\gamma_s+4\pi/3) = \mathrm{Re}\left\{e^{-j\gamma_s}\cdot\underline{a}^2\right\}$$

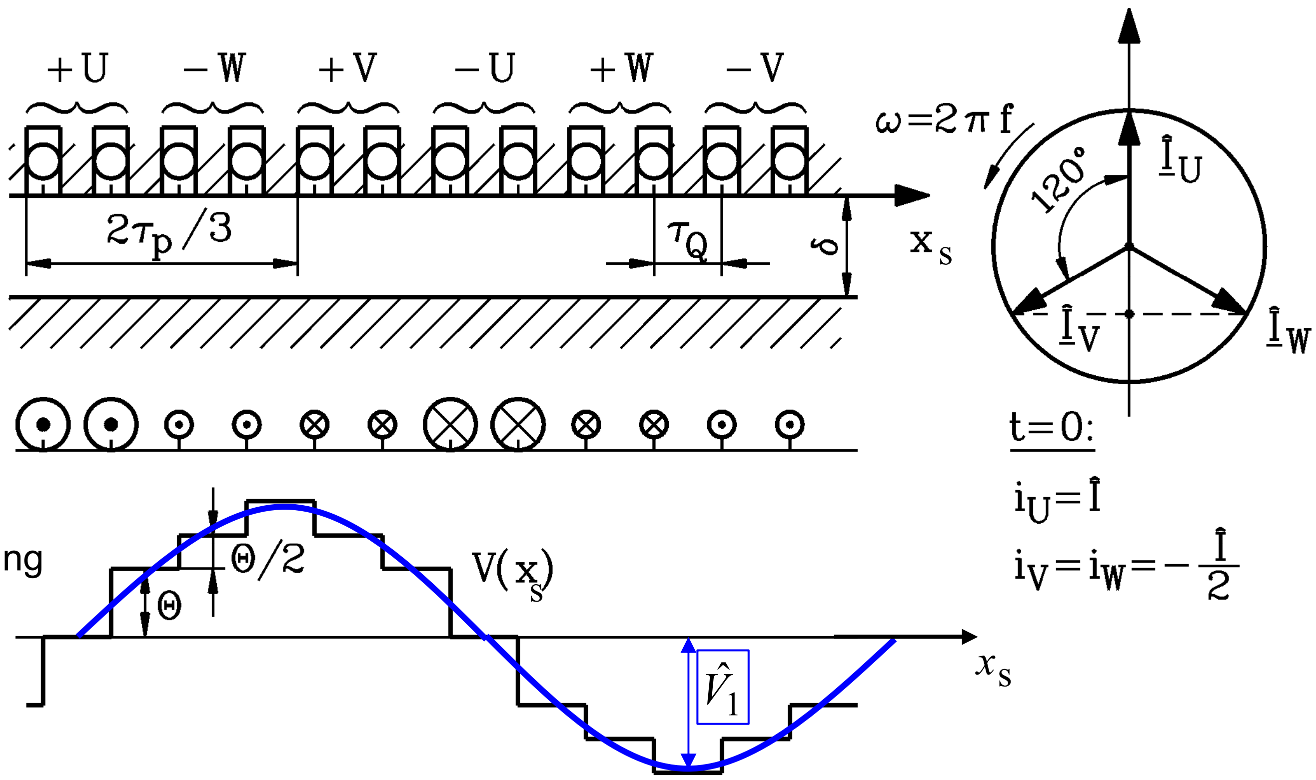
$$V_1(\gamma_s,t) = \hat{V}_{1N,ph}\cdot\mathrm{Re}\left\{\left[i_U(t)+i_V(t)\cdot\underline{a}+i_W(t)\cdot\underline{a}^2\right]\cdot e^{-j\gamma_s}\right\} = \mathrm{Re}\left\{\underline{V}(t)\cdot e^{-j\gamma_s}\right\}$$

$$\boxed{\underline{V}(t) = \hat{V}_{1N,ph}\cdot\left[i_U(t)+\underline{a}\cdot i_V(t)+\underline{a}^2\cdot i_W(t)\right]}$$

6. Space vector theory

Space fundamental MMF of a symmetrical 3-phase winding, fed by a 3-phase sinusoidal current system

Example: $q = 2$,
single-layer winding



$$\hat{V}_1 = \frac{3}{2}\cdot\hat{V}_{1,ph} = \frac{\sqrt{2}}{\pi}\cdot\frac{3}{p}\cdot N\cdot k_{w,1}\cdot I \qquad \hat{I}_U = \hat{I}_V = \hat{I}_W = \hat{I} = \sqrt{2}\cdot I$$

6. Space vector theory

Per unit space vectors

- Fundamental wave m.m.f. amplitude at rated sinusoidal current system:

$$\hat{V}_{1,N} = \frac{3}{2} \cdot \hat{V}_{1N,ph} = \frac{\sqrt{2}}{\pi} \cdot \frac{3}{p} \cdot N \cdot k_{w,1} \cdot I_N$$

$$I_U(t) = I_N \cdot \sqrt{2} \cdot \cos(\omega t),\ I_V(t) = I_N \cdot \sqrt{2} \cdot \cos(\omega t - 2\pi/3),\ \ I_W(t) = I_N \cdot \sqrt{2} \cdot \cos(\omega t - 4\pi/3)$$

- **M.M.F. space vector** as ***per unit value*** of rated amplitude $\hat{V}_{1,N}$:

Arbitrary currents !

$$\boxed{\underline{v}(t) = \frac{\underline{V}(t)}{\hat{V}_{1N}} = \frac{2}{3} \cdot \left[i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t) \right]}$$

- **Current space vector**: $\underline{I}(t) = \frac{2}{3} \cdot \left[I_U(t) + \underline{a} \cdot I_V(t) + \underline{a}^2 \cdot I_W(t) \right]$ **(Definition !)**

- The per unit current space vector **is identical** with per unit m.m.f. space vector:

$$\boxed{\underline{i}(t) = \frac{\underline{I}(t)}{\hat{I}_N} = \frac{2}{3} \cdot \left[i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t) \right] = \underline{v}(t)}$$

Summary:
M.M.F. space vector and phase currents

- Cross section plane of machine scaled with complex coordinate frame
- Space vector is a complex phasor in the cross-section coordinate frame
- Space vector formulation of 2*p*-fundamental field acc. to Prof. KOVACS
- Per-unit MMF space vector $\underline{v}(t)$ identical with per-unit current space vector $\underline{i}(t)$

6. Space vector theory

6.1 M.M.F. space vector definition

6.2 M.M.F. space vector and phase currents

6.3 Current, flux linkage and voltage space vectors

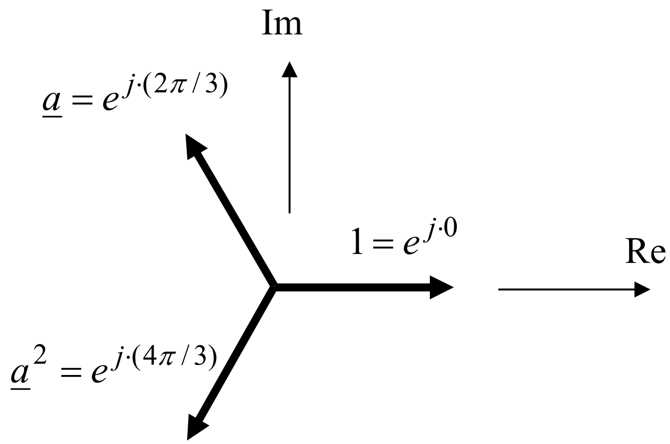
6.4 Space vector transformation

6.5 Influence of zero sequence current system

6.6 Magnetic energy

6. Space vector theory

Phase shifter $\underline{a}$: Calculation methods



$$\underline{a}^2 = \left(e^{j\cdot(2\pi/3)}\right)^2 = e^{j\cdot(4\pi/3)}$$

$$\underline{a}^3 = \left(e^{j\cdot(2\pi/3)}\right)^3 = e^{j\cdot(6\pi/3)} = e^{j\cdot 2\pi} = 1$$

$$1/\underline{a} = \left(e^{j\cdot(2\pi/3)}\right)^{-1} = e^{j\cdot(-2\pi/3)} = e^{j\cdot(4\pi/3)} = \underline{a}^2$$

$$1/\underline{a}^2 = \underline{a} \Leftrightarrow \underline{a}^3 = 1$$

$$1 + \underline{a} + \underline{a}^2 = 0$$

Example:

$$1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}} = 1 + \underline{a}^2 + \underline{a}^4 = 1 + \underline{a}^2 + \underline{a}^3 \cdot \underline{a} = 1 + \underline{a}^2 + \underline{a} = 0$$

6. Space vector theory

Current space vector for three phase AC sine wave system

- Three phase sine wave current system:

a) $I_U(t) = \hat{I}\cdot\cos(\Omega\cdot t),\ I_V(t) = \hat{I}\cdot\cos(\Omega\cdot t - 2\pi/3),\ I_W(t) = \hat{I}\cdot\cos(\Omega\cdot t - 4\pi/3)$.

b) $I_U(t) = \hat{I}\cdot\dfrac{e^{j\Omega\cdot t} + e^{-j\Omega\cdot t}}{2},\ I_V(t) = \hat{I}\cdot\dfrac{e^{j(\Omega\cdot t - 2\pi/3)} + e^{-j(\Omega\cdot t - 2\pi/3)}}{2}$

$$I_W(t) = \hat{I}\cdot\frac{e^{j(\Omega\cdot t - 4\pi/3)} + e^{-j(\Omega\cdot t - 4\pi/3)}}{2}$$

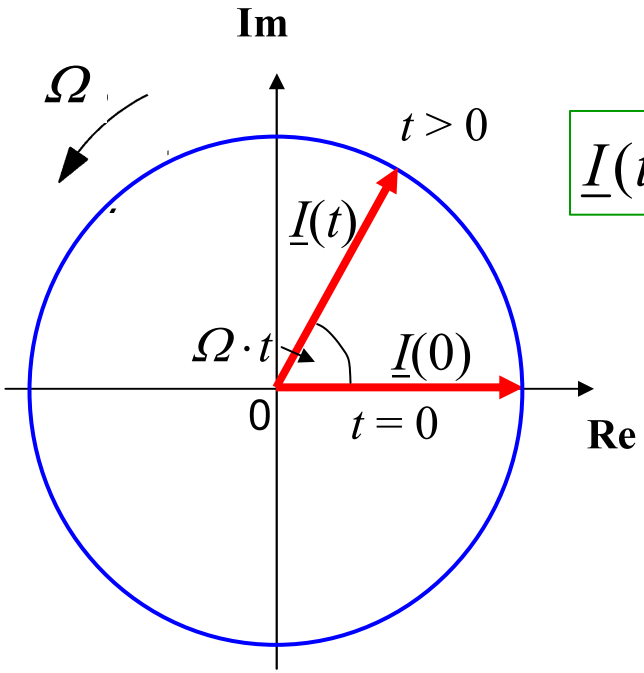
- Current space vector:

$$\underline{I}(t) = \frac{2}{3}\cdot\left[\hat{I}\cdot\frac{e^{j\Omega\cdot t} + e^{-j\Omega\cdot t}}{2} + e^{j\frac{2\pi}{3}}\cdot\hat{I}\cdot\frac{e^{j(\Omega\cdot t - 2\pi/3)} + e^{-j(\Omega\cdot t - 2\pi/3)}}{2} + \right.$$

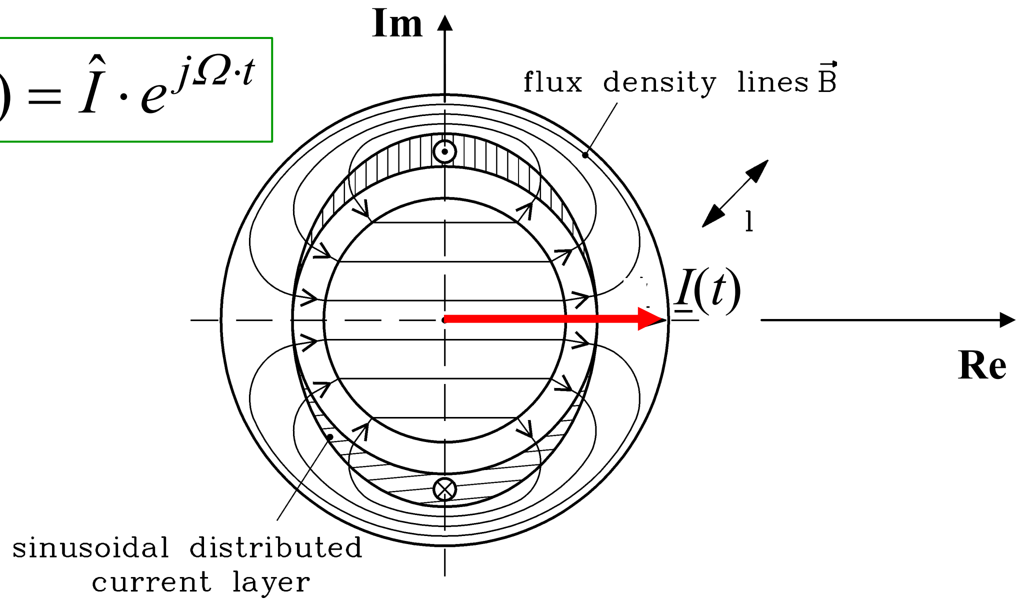
$$\left. + e^{j\frac{4\pi}{3}}\cdot\hat{I}\cdot\frac{e^{j(\Omega\cdot t - 4\pi/3)} + e^{-j(\Omega\cdot t - 4\pi/3)}}{2}\right] = \frac{2}{3}\cdot\left[\frac{3\hat{I}e^{j\Omega\cdot t}}{2} + \frac{\hat{I}e^{-j\Omega\cdot t}}{2}\cdot\left(\underbrace{1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}}}_{0}\right)\right] = \underline{\underline{\hat{I}\cdot e^{j\Omega\cdot t}}}$$

- The current space vector of a three phase AC sine wave **system is a rotating vector of constant amplitude,** which is equal to the phase current amplitude. Rotation frequency is the electric frequency of the phase currents.

Current space vector at stationary operation



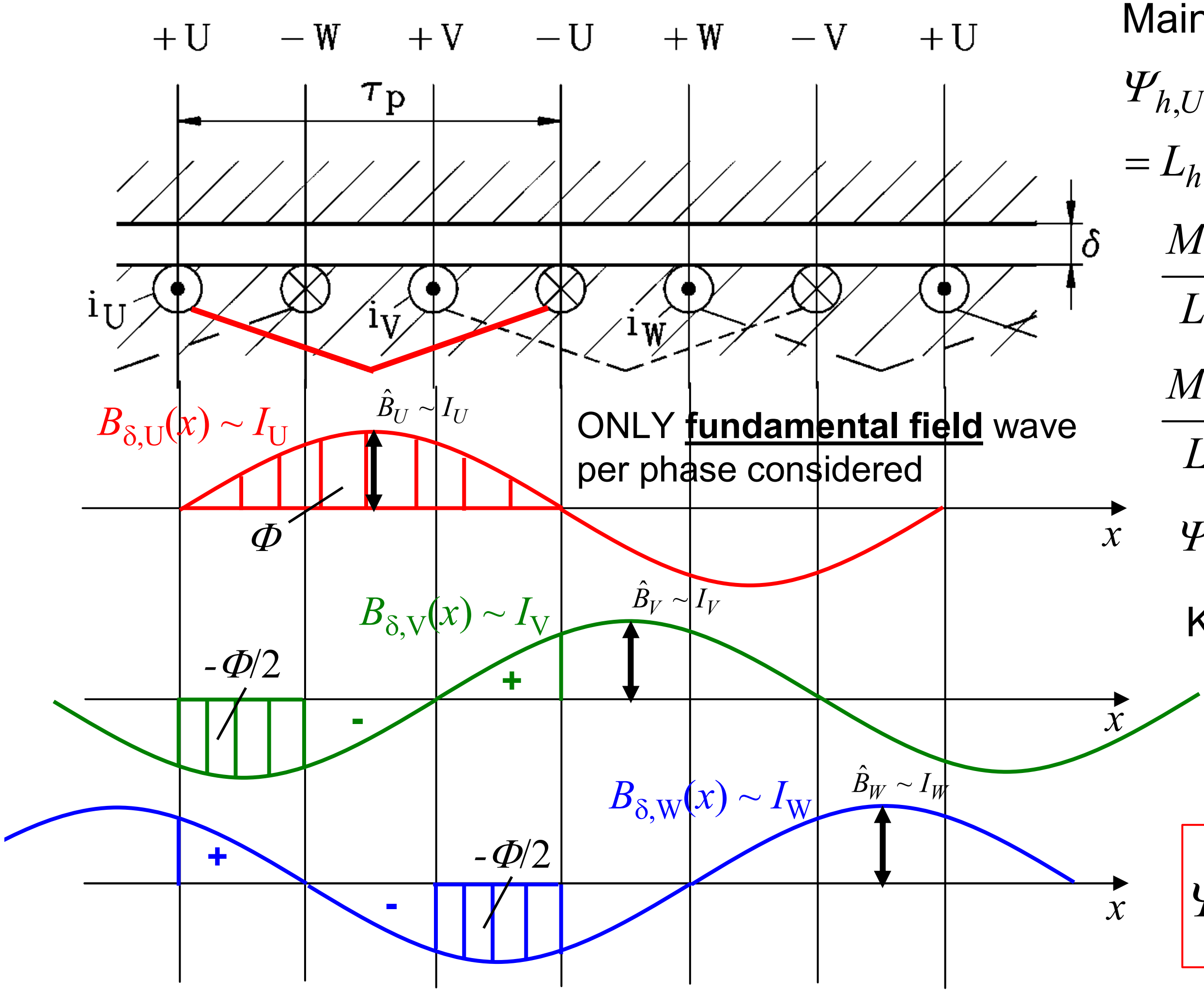
$$\boxed{\underline{I}(t) = \hat{I} \cdot e^{j\Omega \cdot t}}$$



- Per unit: $\underline{i}(t) = \dfrac{\underline{I}(t)}{\hat{I}_N} = \dfrac{\hat{I}}{\hat{I}_N} \cdot e^{j\Omega \cdot t} = i \cdot e^{j\Omega \cdot t} \quad e.g.: i = 1 : \underline{i}(t) = 1 \cdot e^{j\Omega \cdot t}$
- Real fundamental field wave rotates in 2p-pole machine at $n_{syn} = f_s/p$
- Space vector rotates in 2p = 2-pole machine model with $n_{syn} = f_s$

6. Space vector theory

Main flux linkage $\Psi_h(t)$ per phase (m = 3: U, V, W)



Main flux linkage per phase U:

$$\Psi_{h,U}(t) = \Psi_{h,UU}(t) + \Psi_{h,UV}(t) + \Psi_{h,UW}(t) =$$

$$= L_{h,ph} \cdot I_U(t) + M_{h,UV} \cdot I_V(t) + M_{h,UW} \cdot I_W(t)$$

$$\frac{M_{h,UV}}{L_{h,ph}} = \frac{M_{h,UV} \cdot i}{L_{h,ph} \cdot i} = \frac{-\Phi/2}{\Phi} = -\frac{1}{2}$$

$$\frac{M_{h,UW}}{L_{h,ph}} = -\frac{1}{2}$$

$$\Psi_{h,U}(t) = L_{h,ph} \cdot I_U - \frac{L_{h,ph}}{2} \cdot I_V - \frac{L_{h,ph}}{2} \cdot I_W$$

KIRCHHOFF´s law: $I_U(t) + I_V(t) + I_W(t) = 0$

$$I_U = -I_V - I_W$$

$$\Psi_{h,U} = L_{h,ph} \cdot I_U + \frac{L_{h,ph}}{2} \cdot (-I_V - I_W)$$

$$\boxed{\Psi_{h,U}(t) = \frac{3L_{h,ph}}{2} \cdot I_U(t) = L_h \cdot I_U(t)}$$

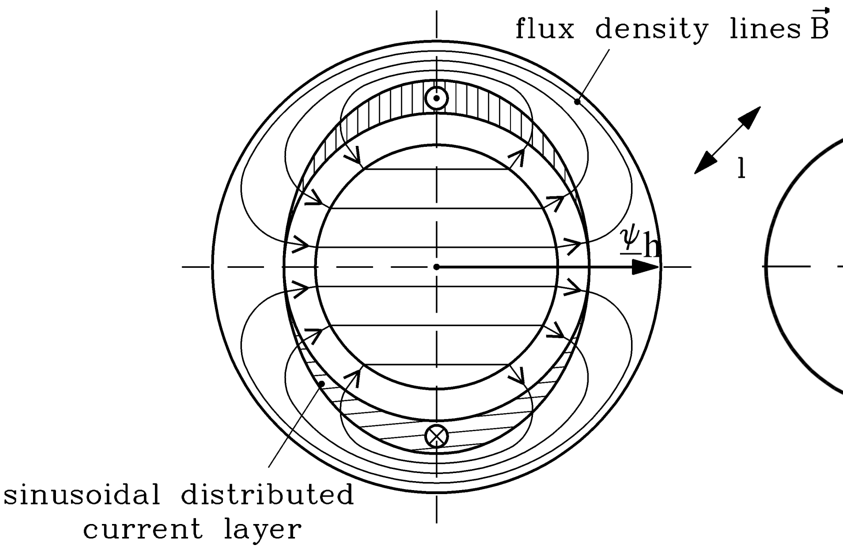
6. Space vector theory

Main flux linkage space vector $\underline{\Psi}_h(t)$

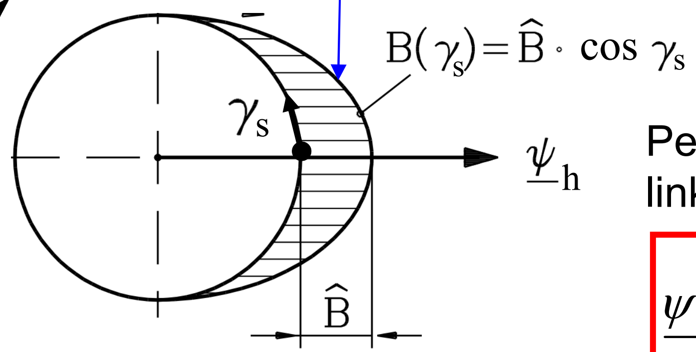
$$L_h\big|_{\mu_{Fe}\to\infty} = \mu_0 \cdot (N \cdot k_{w,1})^2 \cdot \frac{6 \cdot \tau_p l_e}{\pi^2 \cdot p \cdot \delta_e}$$

$$\underbrace{\Psi_{h,U}(t) = \frac{3L_{h,ph}}{2} \cdot I_U(t) = L_h \cdot I_U(t) \qquad \Psi_{h,V}(t) = L_h \cdot I_V(t) \qquad \Psi_{h,W}(t) = L_h \cdot I_W(t)}$$

$$\underline{\Psi}_h(t) = \frac{2}{3} \cdot \left[\Psi_{hU}(t) + \underline{a} \cdot \Psi_{hV}(t) + \underline{a}^2 \cdot \Psi_{hW}(t)\right] \longrightarrow \boxed{\underline{\Psi}_h(t) = L_h \cdot \underline{I}(t)}$$



Resulting sine wave air-gap flux density

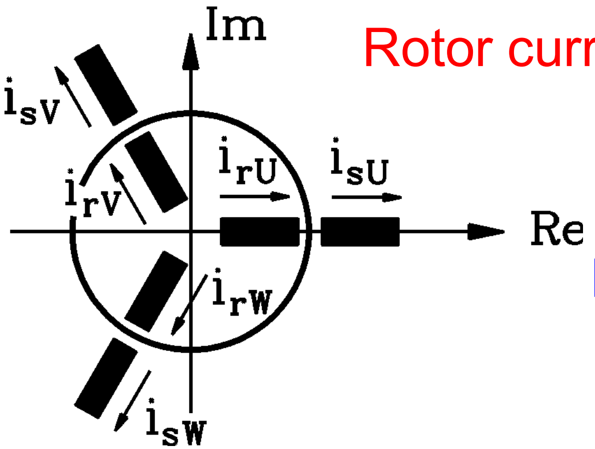


Per-unit main flux linkage space vector

$$\boxed{\underline{\psi}_h(t) = \frac{\underline{\Psi}_h(t)}{\Psi_{hN}}}$$

6. Space vector theory
Rotor current space vector

- **Cage induction machine:** Rotor contains Q_r rotor bars
- **Slip-ring induction machine:** Rotor contains the 3 rotor phase winding



Rotor current transfer ratio: Q_r rotor phases ⇒ 3 equivalent phases

$$I'_r(t) = \frac{I_r(t)}{\ddot{u}_I} \qquad \ddot{u}_I = \frac{m_s \cdot N_s \cdot k_{w,1,s}}{m_r \cdot N_r \cdot k_{w,1,r}}$$

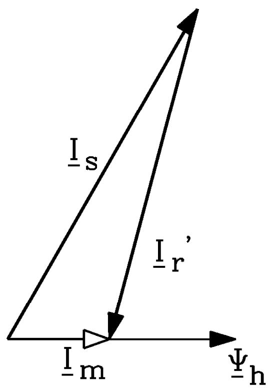
Rotor current space vector: ("3 rotor phases")

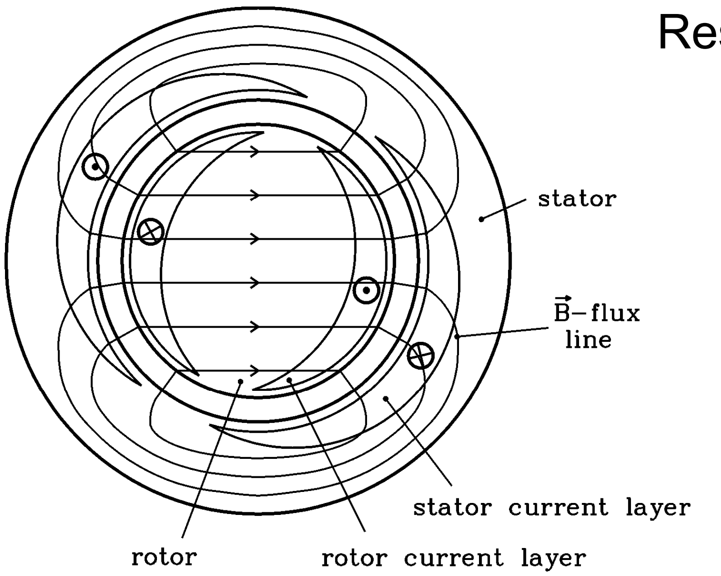
$$\boxed{\underline{I}'_r(t) = \frac{2}{3} \cdot \left[I'_{rU}(t) + \underline{a} \cdot I'_{rV}(t) + \underline{a}^2 \cdot I'_{rW}(t) \right]}$$

	stator AC winding	rotor cage winding	Wound rotor
phase count	$m_s = 3$	Q_r	$m_r = 3$
turns per phase	N_s	1/2	N_r
winding factor ($\nu = 1$)	$k_{w,1,s}$	1	$k_{w,1,r}$

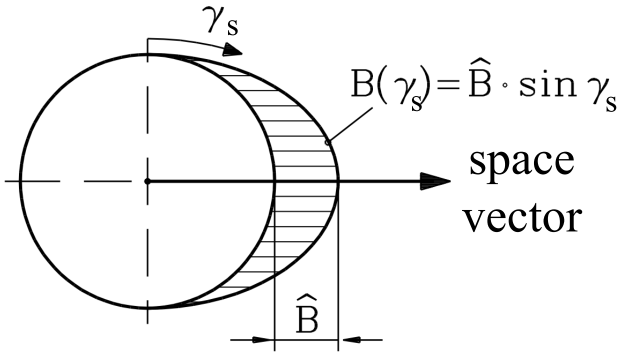
6. Space vector theory

Magnetizing current space vector $\underline{i}_m$





Resulting air-gap field ~ $\underline{i}_m$ ~ $\underline{\psi}_h$



- Stator current space vector $\underline{i}_s$ is directly proportional to fundamental stator field wave
- Cage rotor is described by equivalent three-phase system
- Rotor current space vector $\underline{i}'_r$ represents rotor fundamental field wave
- Addition of stator and rotor space vector $\underline{i}_s$, $\underline{i}'_r$ in the same coordinate system $\Rightarrow$ leads to magnetizing current space vector $\underline{i}_m$ $\Rightarrow$ Gives resulting air-gap field wave, which is ~ $\underline{i}_m$ ~ $\underline{\psi}_h$

6. Space vector theory
Voltage space vector (1)

- Adding stator leakage flux linkage per phase yields stator flux linkage space vector: $\underline{\Psi}_s(t)$

$$\text{e.g.}: \Psi_{sU}(t) = (L_h + L_\sigma)\cdot I_{sU}(t) \quad \text{per phase U, V, W}$$

$$\underline{\Psi}_s(t) = (L_h + L_\sigma)\cdot \underline{I}_s(t)$$

- At $R_s = 0$ the stator terminal voltage per phase is the resulting induced voltage per phase:

$$U_{sU}(t) = d\Psi_{sU}(t)/dt,\, U_{sV}(t) = d\Psi_{sV}(t)/dt,\, U_{sW}(t) = d\Psi_{sW}(t)/dt$$

$$\frac{d}{dt}\underline{\Psi}_s(t) = \frac{2}{3}\cdot\frac{d}{dt}\left[\Psi_{sU}(t) + \underline{a}\cdot\Psi_{sV}(t) + \underline{a}^2\cdot\Psi_{sW}(t)\right] = \underbrace{\frac{2}{3}\cdot\left[U_{sU}(t) + \underline{a}\cdot U_{sV}(t) + \underline{a}^2\cdot U_{sW}(t)\right]}_{\underline{U}_s(t)}$$

- Definition of (stator) voltage space vector: $\boxed{\underline{U}(t) = \frac{2}{3}\cdot\left[U_U(t) + \underline{a}\cdot U_V(t) + \underline{a}^2\cdot U_W(t)\right]}$

- Voltage space vector is an „artificial“ quantity, which does not represent the real rotating field wave in the machine cross section. It is defined as analogous quantity!

6. Space vector theory

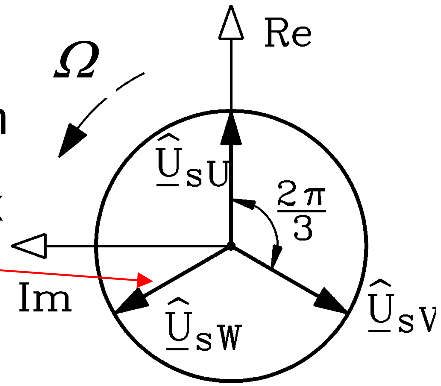
Voltage space vector (2)

$$\boxed{\underline{U}(t) = \frac{2}{3} \cdot \left[U_U(t) + \underline{a} \cdot U_V(t) + \underline{a}^2 \cdot U_W(t) \right]}$$

Example:

Three phase sine wave voltage system

Three voltage time phasors in complex time plane



3-phase sine wave system:

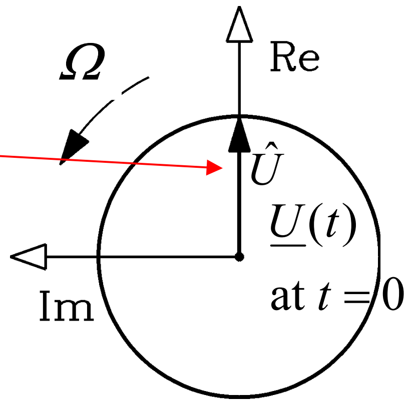
$$\hat{\underline{U}}_{sU} = \hat{U}, U_U(t) = \hat{U} \cdot \cos(\Omega \cdot t)$$

$$\hat{\underline{U}}_{sV} = \hat{U} \cdot e^{-j\frac{2\pi}{3}}$$

$$\hat{\underline{U}}_{sW} = \hat{U} \cdot e^{-j\frac{4\pi}{3}}$$

One voltage space vector

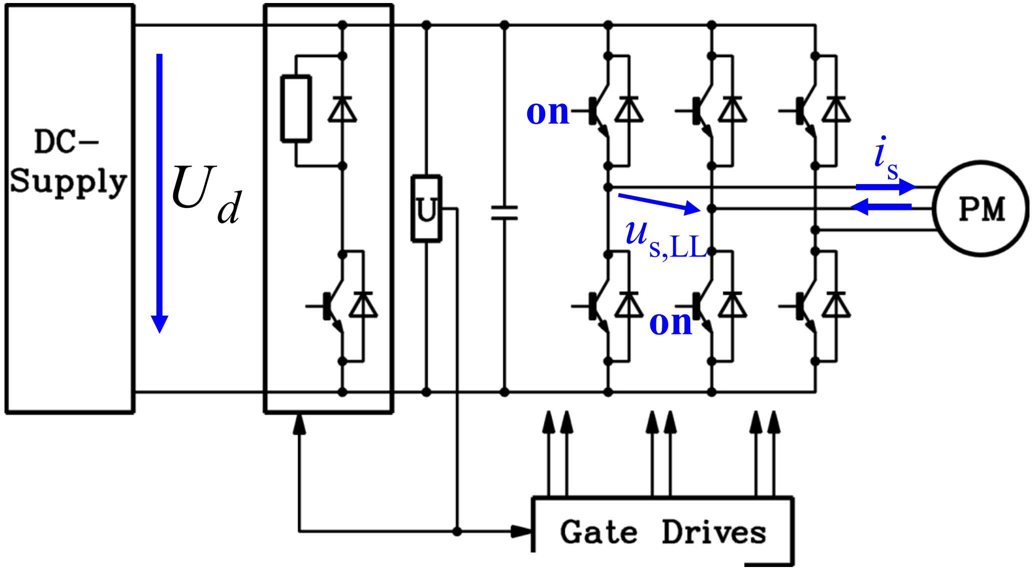
Rotating with electric frequency in complex space plane = machine cross section



Voltage space vector:

$$\boxed{\underline{U}(t) = \hat{U} \cdot e^{j\Omega \cdot t}}$$

Example: Pulse width modulated inverter operation

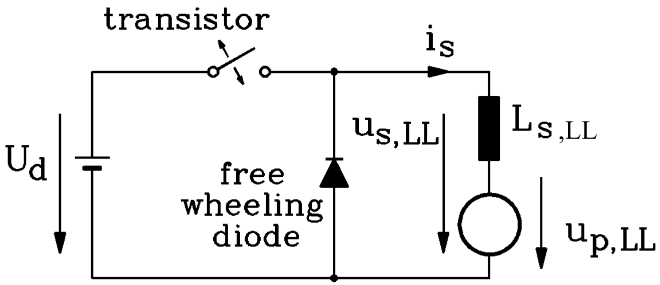


DC link voltage source inverter with switching transistors and free-wheeling diodes

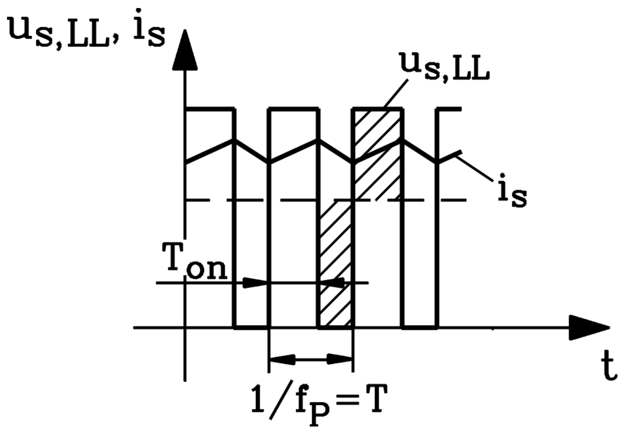
e.g.: PM synchronous machine, R_s neglected:

$$U_d = u_{s,LL}(t) = d\psi_{s,LL}/dt$$

$$U_d - u_{p,LL} \approx L_{s,LL} \cdot di_s/dt$$



Equivalent switching scheme of DC link voltage source inverter, connected to the two phases with switching transistor and free-wheeling diode

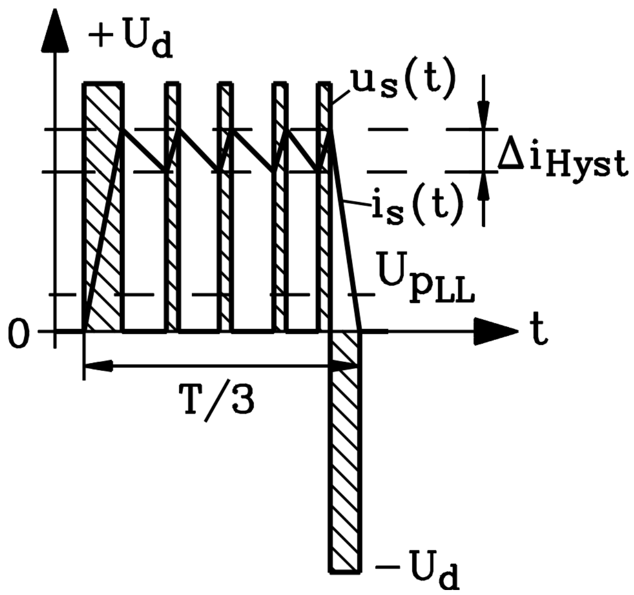


Current ripple and chopped inverter voltage

6. Space vector theory

Example: Block current operation

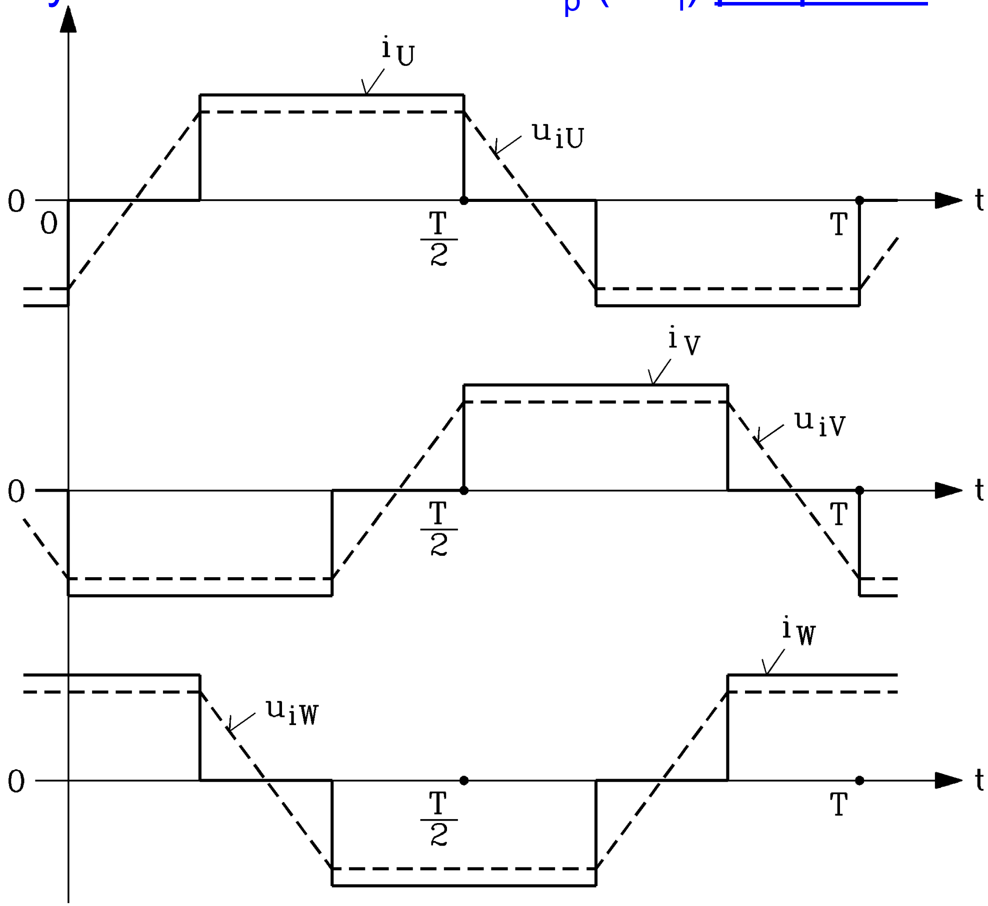
Block current commutation



Shaping of block current with hysteresis band control

For details see: Lecture "Motor development for electrical drive systems"

Idealized shape of stator block current & synchronous back EMF u_p (= u_i) per phase



Flux linkage space vector at inverter operation

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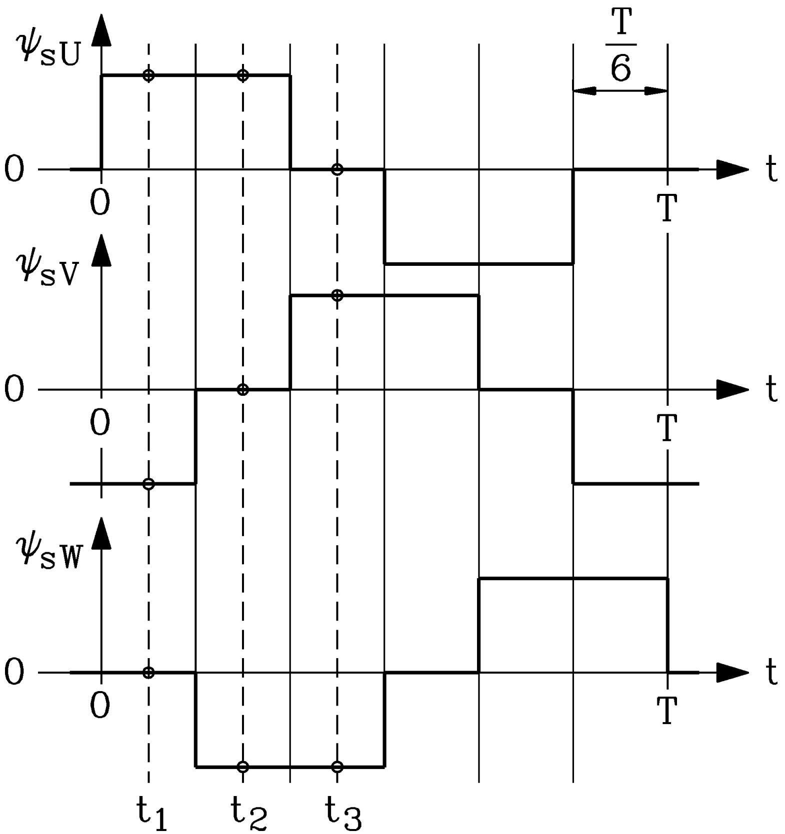
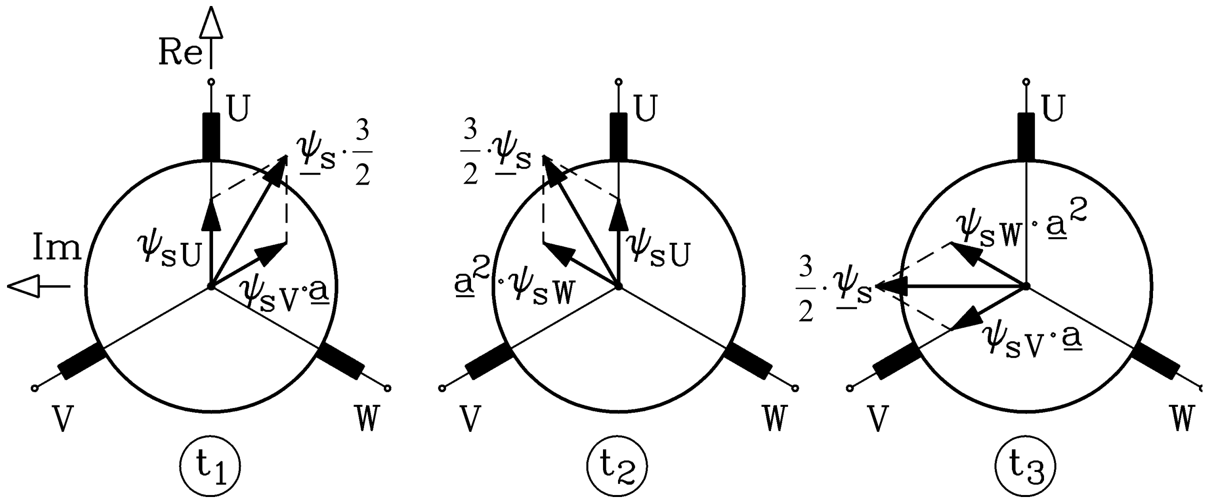
Example: Inverter-fed AC PM synchronous machine with block-commutated stator currents.
Give stator flux linkage space vector WITHOUT rotor flux linkage space vector (= no rotor magnets)!



$$\underline{\psi}_s(t) = (L_h + L_\sigma)\cdot \underline{i}(t)$$

$$\underline{\psi}_s(t) = \frac{2}{3}\cdot\left[\psi_{sU}(t) + \underline{a}\cdot\psi_{sV}(t) + \underline{a}^2\cdot\psi_{sW}(t)\right]$$



The flux linkage space vector is at rest for 1/6 of the period T, then jumps by 60° into next position.

Summary:
Current, voltage and flux linkage space vector

- Current space vector $\underline{i}$ directly proportional to fundamental field amplitude $\underline{B}$
- Analogue definitions for voltage and flux linkage space vectors $\underline{u}$, $\underline{\psi}$
- Cage rotor described by equivalent three-phase system
- Rotor current space vector $\underline{i}'_r$ represents rotor field
- Addition of stator and rotor space vector $\underline{i}_s$, $\underline{i}'_r$ in the same coordinate system $\Rightarrow$ leads to $\underline{i}_m$ $\Rightarrow$ Gives resulting air-gap field $\underline{i}_m \sim \underline{\psi}_h$
- Orbit of space vector in cross-section plane is determined by phase current shape
- Sine wave current system: Space vector orbits on a circle

6. Space vector theory

6.1 M.M.F. space vector definition

6.2 M.M.F. space vector and phase currents

6.3 Current, flux linkage and voltage space vectors

6.4 Space vector transformation

6.5 Influence of zero sequence current system

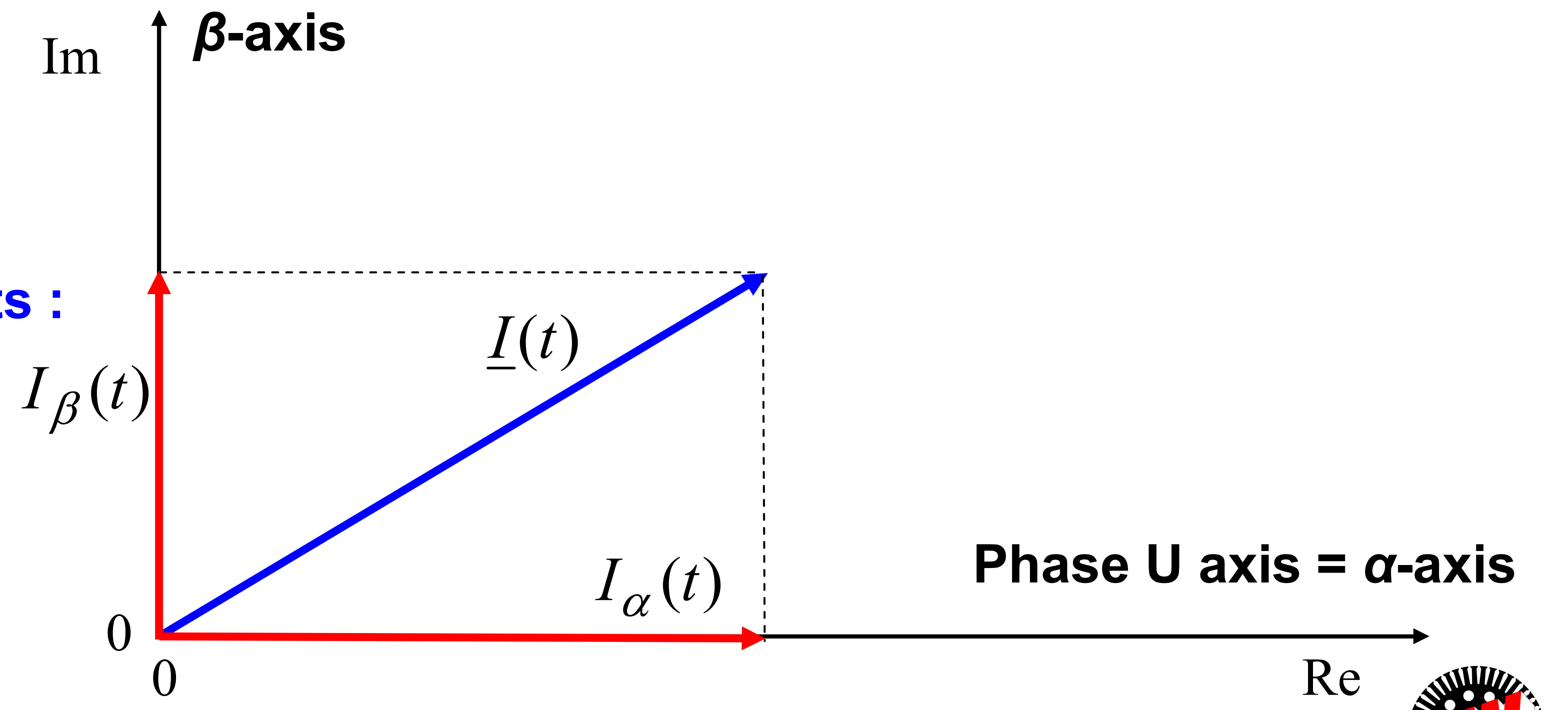
6.6 Magnetic energy

6. Space vector theory
Space vector transformation

- **Phase currents (voltages, flux linkages) U, V, W into α, β - components of space vector:**

$$\underline{I}(t) = \frac{2}{3} \cdot \left[I_U(t) + \underline{a} \cdot I_V(t) + \underline{a}^2 \cdot I_W(t) \right] = I_\alpha(t) + j \cdot I_\beta(t)$$

α, β -components :



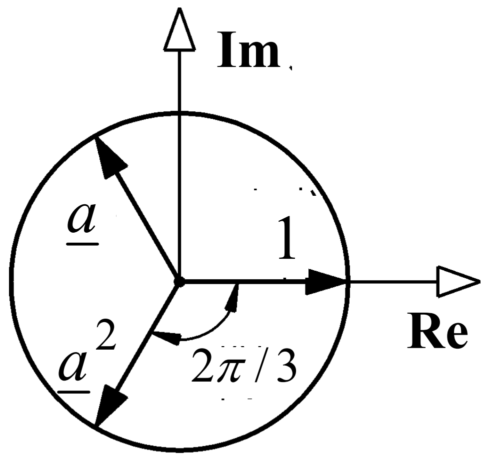
6. Space vector theory

Space vector transformation (*Edith CLARKE*)

- **Phase currents (voltages, flux linkages) of U, V, W into α, β - components of space vector: CLARKE´s transformation**

$$\underline{I}(t) = \frac{2}{3} \cdot \left[I_U(t) + \underline{a} \cdot I_V(t) + \underline{a}^2 \cdot I_W(t) \right] = I_\alpha(t) + j \cdot I_\beta(t)$$

$$\underline{a} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j\frac{2\pi}{3}} \qquad \underline{a}^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j\frac{4\pi}{3}}$$



- **α, β -components:**

$$\boxed{I_\alpha(t) = \frac{2}{3} I_U(t) - \frac{1}{3}\big(I_V(t) + I_W(t)\big) = I_U(t),^{*)} \; I_\beta(t) = (I_V(t) - I_W(t))/\sqrt{3}}$$

***): if $I_U + I_V + I_W = 0$**

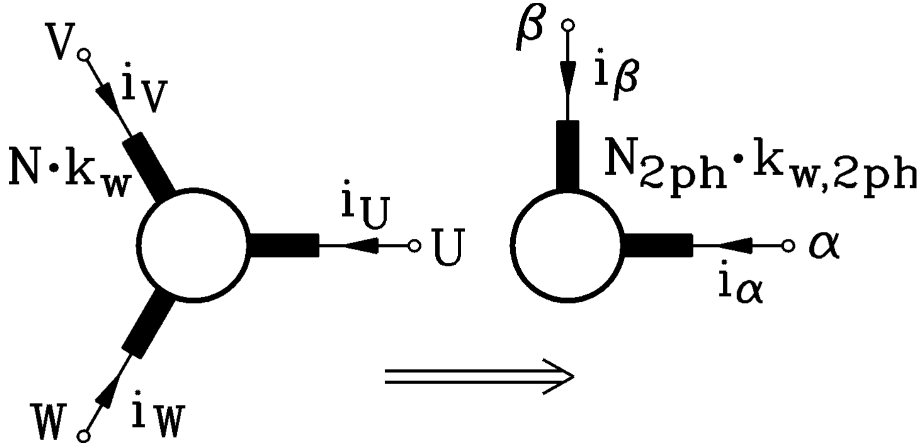
*- Three phase currents U, V, W a transformed into one space vector with two space vector components α, β with perpendicular directions ! = **"Two-axes theory".***

6. Space vector theory

Two-axis theory corresponds to two-phase windings

Fundamental air-gap field amplitude: $$B_{\delta,1} = \frac{\mu_0}{\delta} \cdot \frac{1}{\pi} \cdot \frac{m_s}{p} \cdot N_s k_{w,s} \cdot \hat{I}_s$$

Real three-phase machine



Equivalent two-phase machine

$$B_{\delta,1} = \frac{\mu_0}{\delta} \cdot \frac{1}{\pi} \cdot \frac{3}{p} \cdot N_{3ph} k_{w,3ph} \cdot \hat{I}_{3ph}$$

$$B_{\delta,1} = \frac{\mu_0}{\delta} \cdot \frac{1}{\pi} \cdot \frac{2}{p} \cdot N_{2ph} k_{w,2ph} \cdot \hat{I}_{2ph}$$

If $i_{3ph} = i_{2ph}$, we need for the same flux density: $$\boxed{N_{2ph} k_{w,2ph} = \frac{3}{2} \cdot N_{3ph} k_{w,3ph}}$$

Space vector transformation for $I_0(t) = 0$ (1)

Clarke´s matrix transformation:

U, V, W $\Rightarrow \alpha, \beta$:

$$\begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix} \cdot \begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix} = (A) \cdot \begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix}$$

$\alpha, \beta \Rightarrow$ U, V, W :

$(I_U + I_V + I_W = 0)$

$$\begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \cdot \begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \end{pmatrix} = (A)^{-1} \cdot \begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \end{pmatrix}$$

$$\text{e.g.}: I_V = -\frac{I_\alpha}{2} + \frac{\sqrt{3} \cdot I_\beta}{2}$$

6. Space vector theory

Space vector transformation for $I_0(t) = 0$ (2)

$$\begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}}_{(A)^{-1}} \cdot \underbrace{\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}}_{(A)} \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{3} \cdot \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{I_U + I_V + I_W = 0} \right] \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix}$$

$$\begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix}$$

$$\begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}}_{(A)} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}}_{(A)^{-1}} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix}$$

6. Space vector theory

Inverse space vector transformation: $\alpha, \beta \Rightarrow$ U, V, W

$$\boxed{\begin{aligned} I_U(t) &= \mathrm{Re}\{\underline{I}(t)\} \\ I_V(t) &= \mathrm{Re}\{\underline{a}^2 \cdot \underline{I}(t)\} \\ I_W(t) &= \mathrm{Re}\{\underline{a} \cdot \underline{I}(t)\} \end{aligned}}$$

Valid for: $I_U(t) + I_V(t) + I_W(t) = 0$

Proof:

$$I_U(t) = \mathrm{Re}\{\underline{I}(t)\} = \frac{2}{3} \cdot \left(I_U(t) + \mathrm{Re}\{\underline{a}\} \cdot I_V(t) + \mathrm{Re}\{\underline{a}^2\} \cdot I_W(t)\right) = \frac{2}{3} \cdot \left(I_U(t) - \frac{I_V(t)}{2} - \frac{I_W(t)}{2}\right) =$$

$$= \frac{2}{3} \cdot \left(I_U(t) + \frac{I_U(t)}{2}\right) = I_U(t)$$

$$I_V = \mathrm{Re}\{\underline{a}^2 \underline{I}\} = \frac{2}{3} \cdot \left(\mathrm{Re}\{\underline{a}^2\} \cdot I_U + \mathrm{Re}\{\underline{a}^3\} \cdot I_V + \mathrm{Re}\{\underline{a}^4\} \cdot I_W\right) = \frac{2}{3} \cdot \left(\mathrm{Re}\{\underline{a}^2\} \cdot I_U + \mathrm{Re}\{1\} \cdot I_V + \mathrm{Re}\{\underline{a}\} \cdot I_W\right) =$$

$$= \frac{2}{3} \cdot \left(-\frac{I_U + I_W}{2} + I_V\right) = \frac{2}{3} \cdot \left(\frac{I_V}{2} + I_V\right) = I_V$$

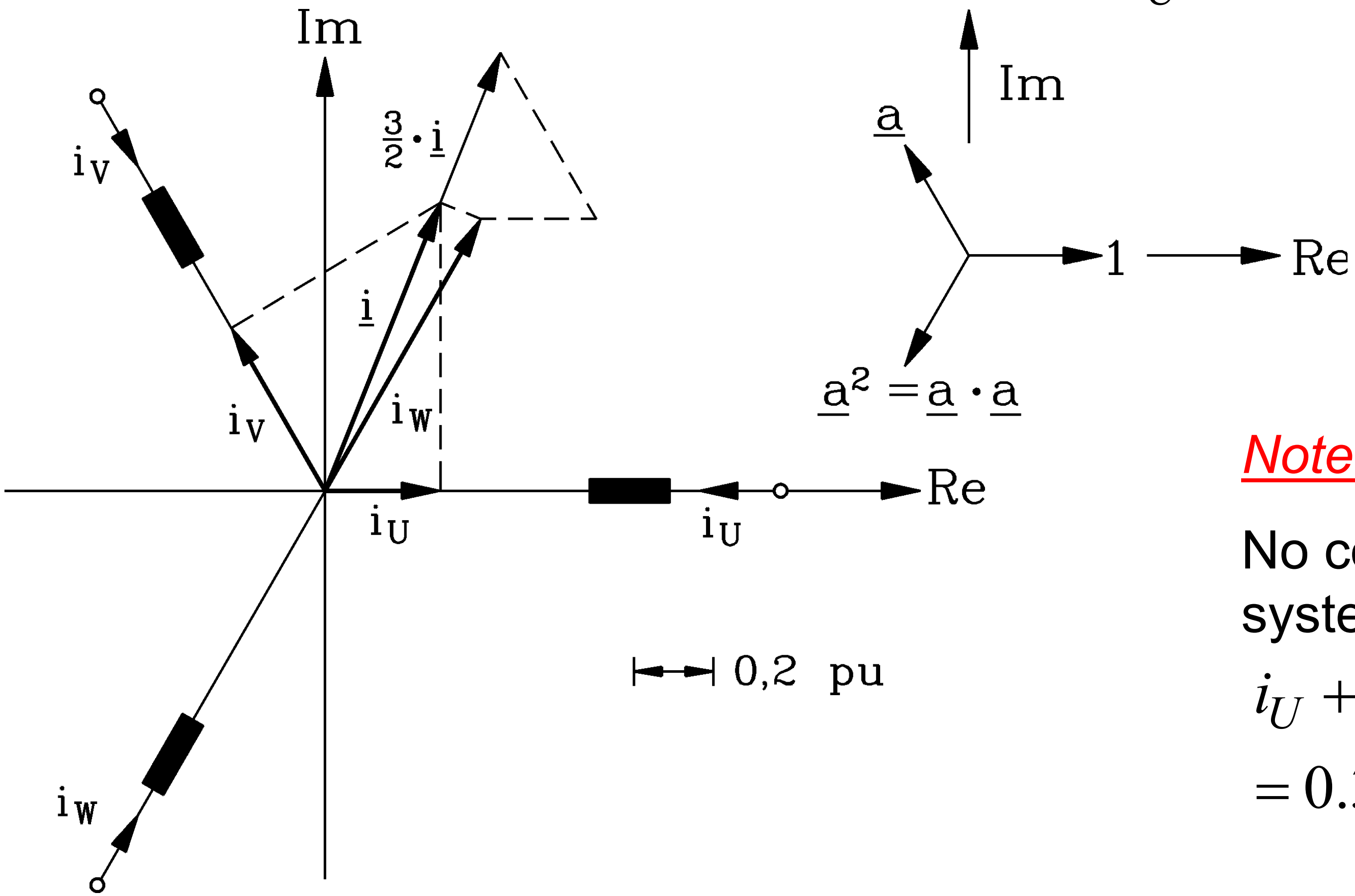
In the same way one proofs the last relationship $I_W(t) = \mathrm{Re}\{\underline{a}\underline{I}(t)\}$.

Note: e.g.: $I_V = \mathrm{Re}\{\underline{a}^2 \cdot \underline{I}\} = \mathrm{Re}\left\{\left(-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}\right) \cdot \left(I_\alpha + j \cdot I_\beta\right)\right\} = -\frac{I_\alpha}{2} + \frac{\sqrt{3} \cdot I_\beta}{2}$

Space vector transformation graphically

Example:

Star connected winding, per unit current values: $i_U = 0.3, i_V = 0.5, i_W = -0.8$



Note:

No common mode system:

$$i_U + i_V + i_W =$$

$$= 0.3 + 0.5 - 0.8 = 0$$

Summary:
Space vector transformation

- Coordinate system $(\mathrm{Re}, \mathrm{Im})$ of machine cross-section alternatively as α-β-system
- „Two axis"-theory in α-β-components
- Transformation from U, V, W into α-β-components = space vector transformation
- *Edith CLARKE´s* transformation: Originally introduced for power systems, not for electrical machines

6. Space vector theory

6.1 M.M.F. space vector definition

6.2 M.M.F. space vector and phase currents

6.3 Current, flux linkage and voltage space vectors

6.4 Space vector transformation

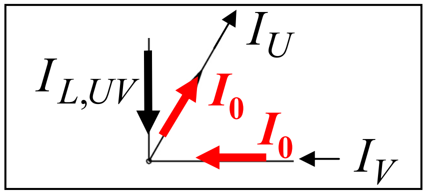
6.5 Influence of zero sequence current system

6.6 Magnetic energy

6. Space vector theory

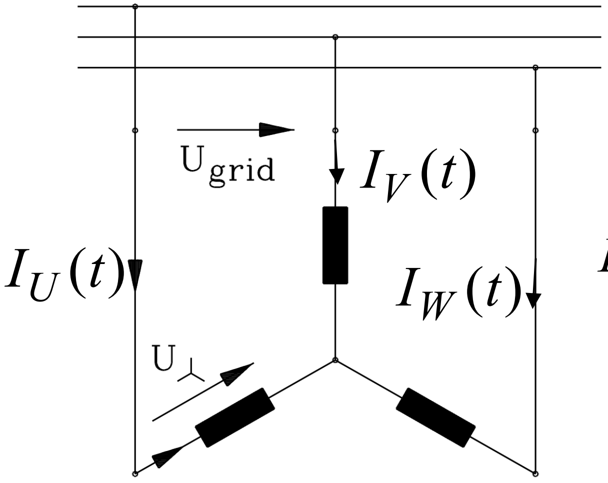
Influence of zero sequence current system

- In all three phases U, V, W the *same, identical* zero-sequence current $I_0(t)$ flows.
- Dynamic operation: Arbitrary time function: $I_0(t)$ is called COMMON MODE current !

Star connected winding	*Delta connected winding*	*Star connected winding, neutral point connected*
No common mode current	Common mode current flows ***circulating*** in delta connection, but is not visible in grid connections	Common mode current flows in each line and phase and with ***3-times in neutral point*** connection
$I_U(t)+I_V(t)+I_W(t)=$ $=3\cdot I_0(t)=0$ $I_0(t)=0$	Phase currents: $I_U(t)=I_V(t)=I_W(t)=I_0(t)$ Line currents: $I_{L,UV}(t)=I_U(t)-I_V(t)=0$ 	Phase currents: $I_U(t)=I_V(t)=I_W(t)=I_0(t)$ Line currents: $I_{L,U}(t)=I_U(t)=I_0(t)$ Neutral current: $I_n(t)=I_U(t)+I_V(t)+I_W(t)=$ $=3\cdot I_0(t)$

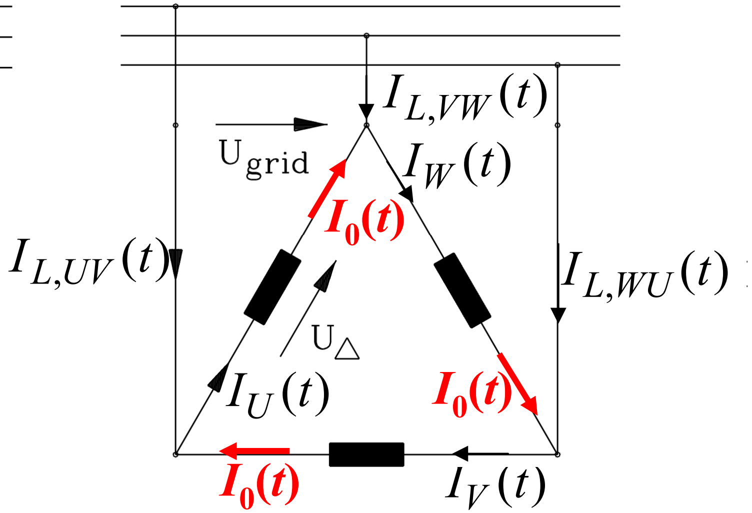
6. Space vector theory

Zero sequence (= common mode) current $I_0(t)$



$$I_U(t)+I_V(t)+I_W(t)=$$
$$=3\cdot I_0(t)=0$$

No **common mode current**



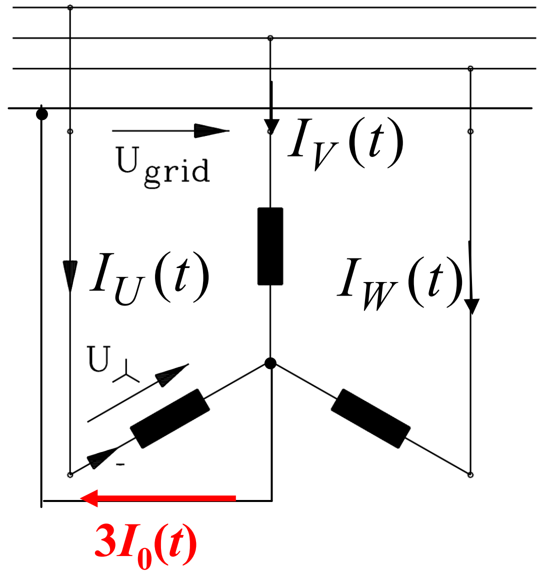
$$I_{L,UV}(t)+I_{L,VW}(t)+I_{L,WU}(t)=0$$

$$\left.\begin{array}{l} I_{L,UV}(t)=I_U(t)-I_V(t) \\ I_{L,VW}(t)=I_V(t)-I_W(t) \\ I_{L,WU}(t)=I_W(t)-I_U(t) \end{array}\right\} +$$

$$\left.\begin{array}{l} I_{US}(t)=I_U(t)-I_0(t) \\ I_{VS}(t)=I_V(t)-I_0(t) \\ I_{WS}(t)=I_W(t)-I_0(t) \end{array}\right\} +$$

$$0=I_U(t)+I_V(t)+I_W(t)-3I_0(t)$$

Circulating **common mode current**



$$I_n(t)=I_U(t)+I_V(t)+I_W(t)=$$
$$=3\cdot I_0(t)$$

3-times **common mode current** ***in neutral point connection***

6. Space vector theory

Calculation of zero sequence current

Example:

- Star connected 3-phase winding, neutral point connected:
- At time *t*: Measured per unit currents: $i_U = 0.3, i_V = 0.5, i_W = -0.2$
- Per unit currents: $i(t) = I(t)/\hat{I}_N$

- *Question:* How big is zero sequence current a) in phases, b) in neutral clamp ?

a) $i_0(t) = \frac{1}{3}\cdot(i_U + i_V + i_W) = \frac{1}{3}\cdot(0.3 + 0.5 - 0.2) = \underline{\underline{0.2}}$

b) $i_n(t) = 3i_0 = 3\cdot 0.2 = \underline{\underline{0.6}}$

Result: In neutral **clamp flows 60% of rated current !**

The "common-mode free" current values are:

$$i_{US} = i_U - i_0 = 0.3 - 0.2 = 0.1$$

$$i_{VS} = i_V - i_0 = 0.5 - 0.2 = 0.3$$

$$i_{WS} = i_W - i_0 = -0.2 - 0.2 = -0.4$$

Result: One can always decompose a three-phase system into a zero sequence system and a „I_0"-free three phase system.

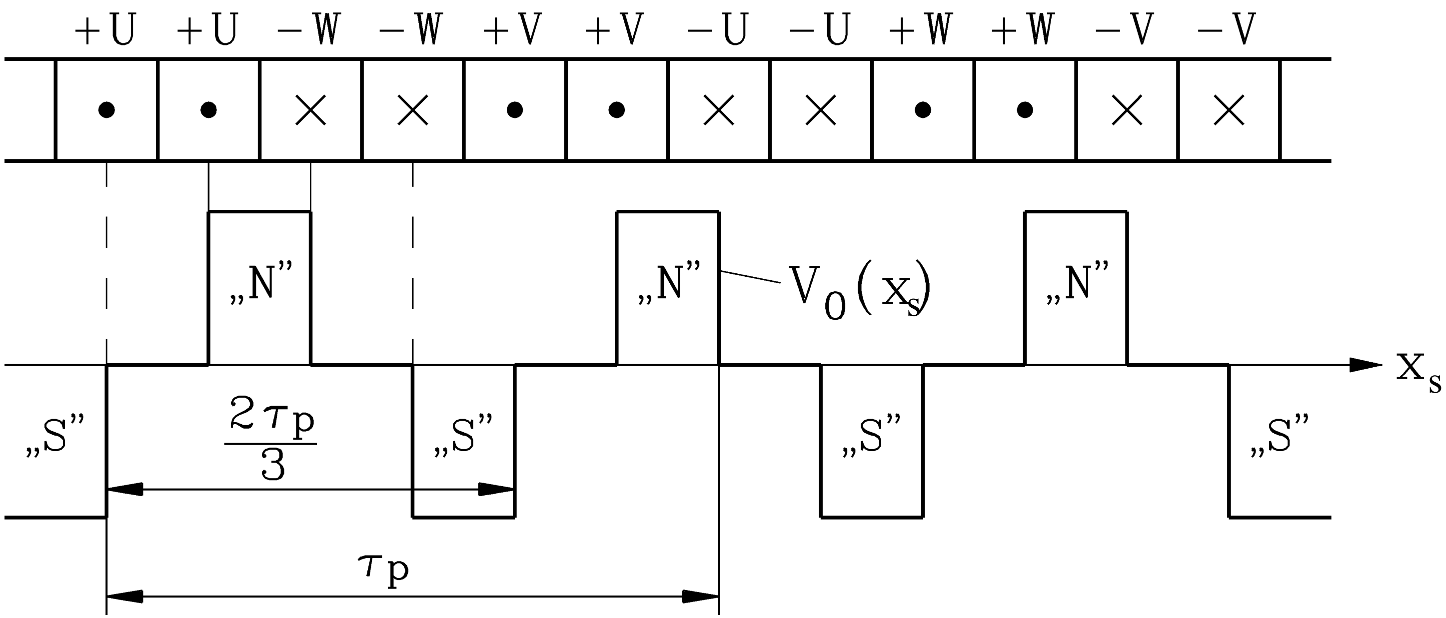
6. Space vector theory

Air gap magnetic field due to zero sequence current

- **Example:** $i_U = 0.3, i_V = 0.5, i_W = -0.2 \quad \Rightarrow \quad i_0(t) = \frac{1}{3} \cdot (0.3 + 0.5 - 0.2) = \underline{\underline{0.2}}$

Magnetic air gap field, excited by zero sequence current (= flowing in all three phases).

Here: Single layer winding with *q* = 2 slots per pole and phase:



- *Ampere´s* law yields a zero-sequence M.M.F. distribution $V_0(x_s)$ with three pole pairs instead of one along double pole pitch $2\tau_p$.

6. Space vector theory

Effect of zero sequence air gap field

- Zero sequence field is not travelling, but STANDING, but time varying acc. to $i_0(t)$.
- 3 pole pairs of stator with one pole pair of rotor (e.g. PM machine) do not generate torque.
- Zero sequence current excites flux waves, which do not contribute to torque generation of fundamental wave.
- **Space vector of zero sequence current is therefore zero.**
- ***Proof:*** $\underline{I}_0(t) = \frac{2}{3} \cdot \left(I_0(t) + \underline{a} \cdot I_0(t) + \underline{a}^2 \cdot I_0(t) \right) = \frac{2}{3} \cdot I_0(t) \cdot \left(1 + \underline{a} + \underline{a}^2 \right) =$

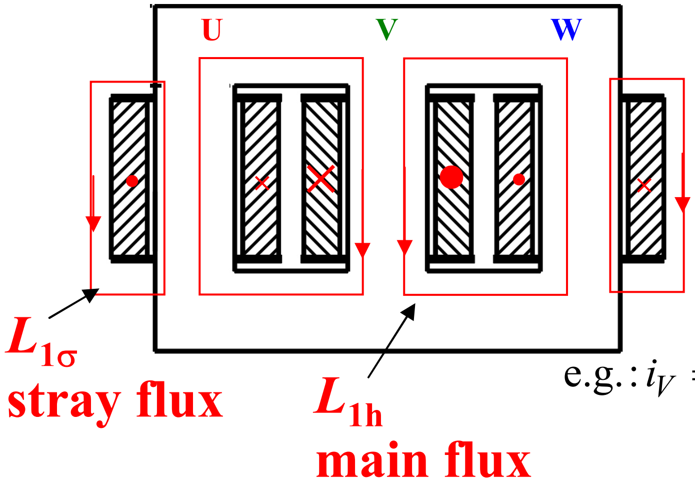
$$= \frac{2}{3} \cdot I_0(t) \cdot \left(1 + e^{j\frac{2\pi}{3}} + e^{j\frac{4\pi}{3}} \right) = 0$$

- ***Facit:*** **Additional losses, pulsating radial forces (leading to noise or vibration), in cage induction machines also parasitic braking torque may occur, so AVOID common mode current !**

6. Space vector theory

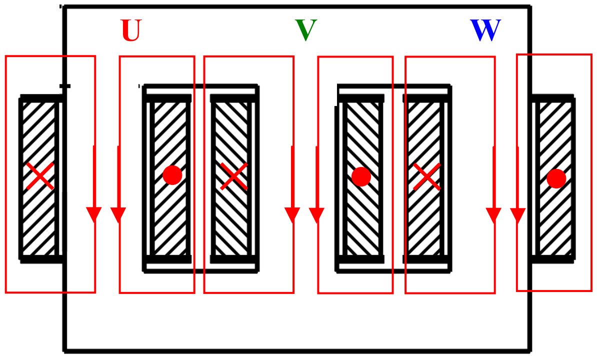
Effect of zero sequence current in 3-leg transformers

Symmetrical current system e.g. Yy0



e.g.: $i_V = -2i_U = -2i_W$

Common-mode current system possible e.g. at Dy



$i_U = i_V = i_W = i_0$

$$L_{1,0} << L_{1h}$$

Per phase at primary side at no-load ($I_2 = 0$):

$$\Psi_{1,ph}(t) = (L_{1\sigma} + L_{1h}) \cdot I_{1,ph}(t)$$

Big magnetizing inductance $L_{1h} \sim \mu_{Fe}$!
(No air-gap, moderate iron saturation)

$$\Psi_{1,ph,0}(t) = L_{1,0} \cdot I_{1,ph,0}(t)$$

Small zero-sequence inductance $L_{1,0} \sim \mu_0$!
(Flux lines pass via air = similar to stray flux!)

Effect of zero sequence current in 5-leg transformers

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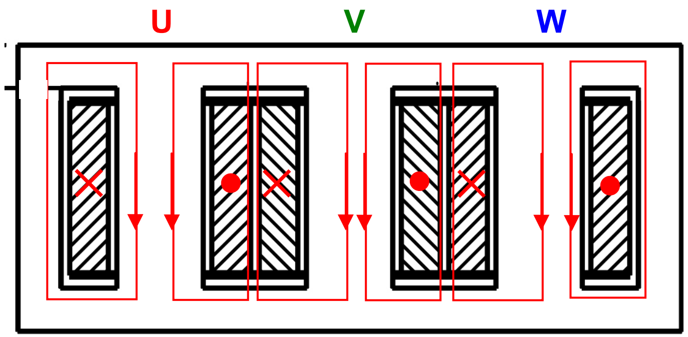
Common-mode current system



$$i_U = i_V = i_W = i_0$$

$$L_{1,0} < L_{1h}$$

Per phase at primary side at no-load ($I_2 = 0$):

$$\Psi_{1,ph,0}(t) = L_{1,0} \cdot I_{1,ph,0}(t)$$

Big zero-sequence inductance $L_{1,0} \sim \mu_{\mathrm{Fe}}$!
Flux lines pass partially via the outer legs and not via air $\Rightarrow$ zero-sequence flux bigger than stray flux!

(i) In 5-leg transformers the zero-sequence inductance $L_{1,0} < L_{1h}$ is much bigger than in 3-leg transformers

(ii) and decreases with increasing current $I_0(t)$ due to iron saturation of the outer legs

6. Space vector theory

Space vector *CLARKE* transformation including $I_0(t)$

Clarke´s matrix transformation from 3-phase system U, V, W to two-axes system α, β and zero-sequence system 0:

U, V, W ⇒ α, β, 0 :

$$\begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \\ I_0(t) \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix} = (A) \cdot \begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix}$$

α, β, 0 ⇒ U, V, W :

$$\begin{pmatrix} I_U(t) \\ I_V(t) \\ I_W(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \\ I_0(t) \end{pmatrix} = (A)^{-1} \cdot \begin{pmatrix} I_\alpha(t) \\ I_\beta(t) \\ I_0(t) \end{pmatrix}$$

Note: Transformation from *m*-phase system to two-axes system α, β and zero-sequence system 0 yields a (3 x *m*)-matrix!

6. Space vector theory
CLARKE transformation identities

$$\begin{pmatrix} I_\alpha \\ I_\beta \\ I_0 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}}_{(A)} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}}_{(A)^{-1}} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \\ I_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \\ I_0 \end{pmatrix}$$

$$\begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}}_{(A)^{-1}} \cdot \underbrace{\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}}_{(A)} \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} I_U \\ I_V \\ I_W \end{pmatrix}$$

Summary: Influence of zero sequence current system

- Zero-sequence current system excites a 6*p*-air gap field
- Hence zero sequence system does not contribute to space vectors, as these are related to 2*p*-air gap field waves
- Insulated star-point: No zero sequence current system possible
- (3x3)-*CLARKE´*s transformation includes zero sequence system
- Zero sequence systems should be avoided: otherwise additional losses, pulsating torque, vibration forces, noise, …

Energy Converters – CAD and System Dynamics

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6. Space vector theory

6.1 M.M.F. space vector definition

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6.3 Current, flux linkage and voltage space vectors

6.4 Space vector transformation

6.5 Influence of zero sequence current system

(6.6 Magnetic energy: leads via power balance to torque equation)