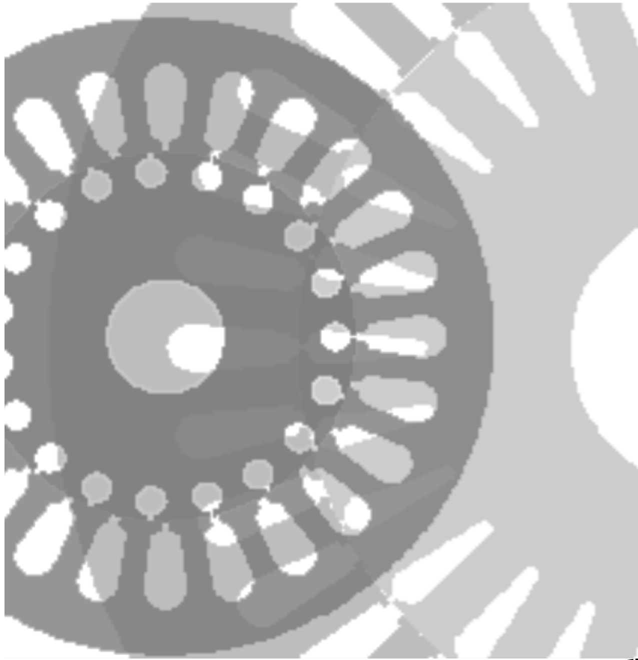
Energy Converters – CAD and System Dynamics

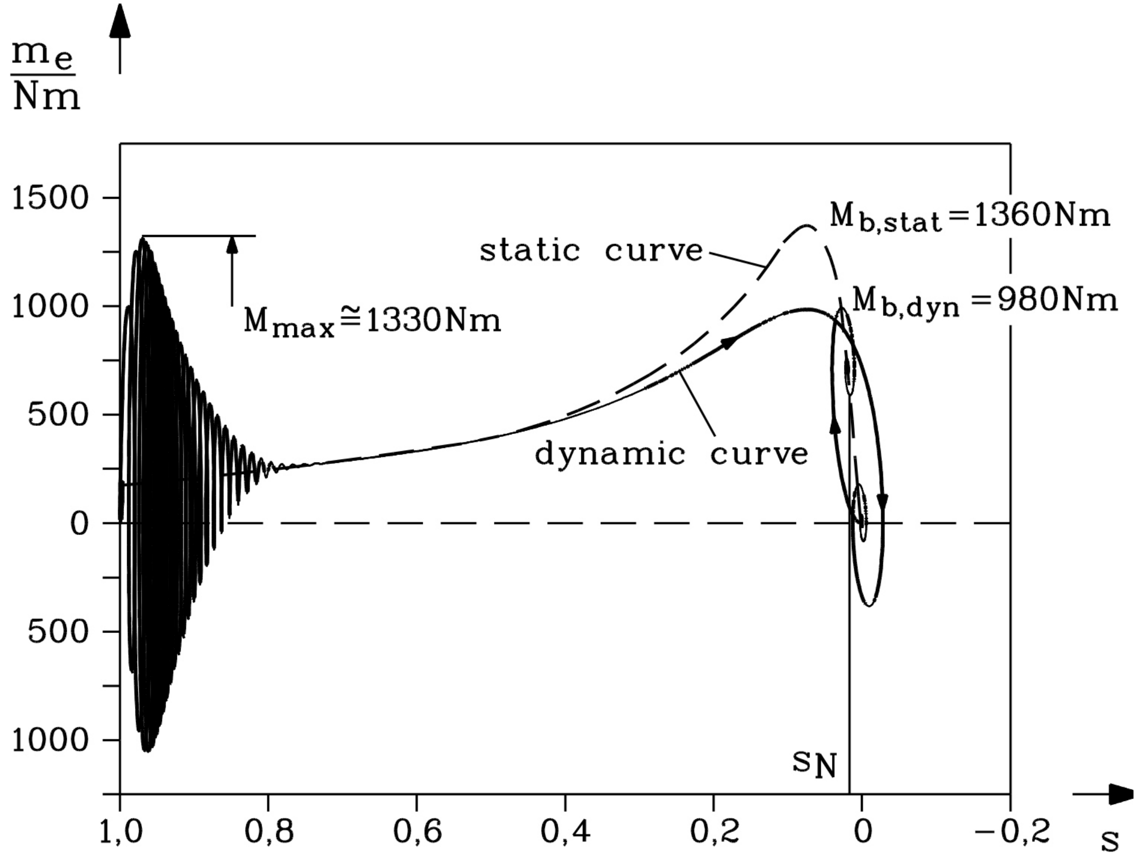
TECHNISCHE UNIVERSITÄT DARMSTADT

1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source: SPEED program

7. Dynamics of induction machines



7. Dynamics of induction machines

7. Dynamics of induction machines
Per unit calculation (p.u.)

Example: *Ohm´s law:*

- $U = 10\,\mathrm{V}, R = 2\,\Omega$: How big is current I?
- Rated voltage and current: $U_N = 5\,\mathrm{V}, I_N = 5\,\mathrm{A}$

(i) Calculated with physical numbers: $I = U/R = 10\,\mathrm{V}/2\,\Omega = \underline{\underline{5\,\mathrm{A}}}$.

- Check of physical units: $\mathrm{V}/\Omega = \mathrm{V}/((\mathrm{V}/\mathrm{A}) = \mathrm{A}$

(ii) Calculated with per unit numbers:

$u = U/U_N = 10/5 = 2, Z_N = U_N/I_N = 5/5 = 1\,\Omega$.

$r = R/Z_N = 2/1 = 2 \qquad \Rightarrow i = u/r = 2/2 = \underline{\underline{1}}$

Note: $i = \underline{\underline{1\,p.u.}}$ is equal to $i = I/I_N = \underline{\underline{1}} \quad \Rightarrow \quad I = i \cdot I_N = 1 \cdot 5\,\mathrm{A} = 5\,\mathrm{A}$

Drawback: p.u. have the physical units 1, so checking of results of analytical calculations by physical units check is no longer possible.

Benefits: The calculation result gives directly an impression of the degree of loading of the electric device.

Basic rules for per unit calculation

- Values for per unit calculation are taken from machine data plate
- In three phase systems the **rated impedance Z_N** has to be calculated with **phase values = ** ratio of rated **phase** voltage versus **rated** phase current**.**
- Data plate voltage & current values are ALWAYS line values !
- Electric machine models are based on phase values **in order to be independent from the kind of winding connection (Y or D)**. For per unit voltage, current and impedance calculation: rated **phase values are taken.**
- Symbols for per unit values are **small letters** ($u(t)$, $i(t)$, ...).
- For time varying voltage, current etc. in physical units **capital letters** are used here ($U(t)$, $I(t)$, ...).

7. Dynamics of induction machines
Typical data plate of electric machine

- ***Example:*** Six-pole cage AC induction machine:

Type MKG-222 M06 F3B-9	...... Motor Company/2003
AC-Motor	Nr. 691 502
400 V Y	84 A
45 kW	1490/3000 /min S1
75 Hz	$\cos\varphi = 0.88$
Th.Cl. F	IP 44

- We calculate from the data plate the rated u & i phase values and the rated impedance:

$$U_{N,ph} = U_N / \sqrt{3} = 400/\sqrt{3} = 231\ \text{V} \approx 230\ \text{V},\ I_{N,ph} = I_N$$

$$Z_N = U_{N,ph} / I_{N,ph} = 230/84 = \underline{\underline{2.74}}\ \Omega$$

7. Dynamics of induction machines

Summary of per unit values (1)

- **Per unit time:** $\tau = \omega_N \cdot t$
- **Per unit electric angular frequency:** $\omega_s = \Omega_s / \omega_N \qquad \omega_r = \Omega_r / \omega_N$
- **Per unit mechanical angular frequency:** $\omega_m = \Omega_m \cdot p / \omega_N$
- **Per unit electric resistance:** $r_s = R_s / Z_N \qquad r_r' = R_r' / Z_N \qquad Z_N = U_{N,ph} / I_{N,ph}$

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e.g.: $f_\mathrm{N} = 50\mathrm{Hz},\ t = 1\mathrm{s}:\ \boxed{\tau = \omega_N \cdot t = (2\pi \cdot 50) \cdot 1 = 314.16}$ 50 cycles: $50 \cdot 2\pi = 50 \cdot 6.28 = 314.16$

e.g.: $f_\mathrm{N} = 50\mathrm{Hz},\ f_s = 150\mathrm{Hz}:\quad \Omega_s = 2\pi f_s = 2\pi \cdot 150/\mathrm{s} = 942.5/\mathrm{s}$

$\omega_\mathrm{N} = 2\pi \cdot 50 = 314.16/\mathrm{s} \quad \boxed{\omega_\mathrm{s} = \Omega_s / \omega_\mathrm{N} = 942.5/314.16 = 3.0}$

e.g.: $f_\mathrm{N} = 50\mathrm{Hz},\ \omega_\mathrm{N} = 314.16/\mathrm{s},\ n = 1000/\mathrm{min},\ 2p = 8:$

$\Omega_m = 2\pi n = 2\pi \cdot (1000/60) = 104.7/\mathrm{s}$

$(\Omega_\mathrm{syn} = 2\pi \cdot f_\mathrm{N} / p = 2\pi \cdot 50/4 = 78.54/\mathrm{s}) \quad \boxed{\omega_m = \Omega_m / \Omega_\mathrm{syn} = 104.7/78.54 = 1.33}$

$\boxed{\omega_m = \Omega_m \cdot p / \omega_\mathrm{N} = 104.7 \cdot 4 / 314.16 = 1.33}$

7. Dynamics of induction machines
Summary of per unit values (2)

- **Per unit inductance:** $x_s = \omega_N \cdot L_s / Z_N$ $\quad x_h = \omega_N \cdot L_h / Z_N$ $\quad x'_r = \omega_N \cdot L'_r / Z_N$
- **Per unit electric voltage:** $u_s = U_s / (\sqrt{2} U_{N,ph})$ $\quad u'_r = U'_r / (\sqrt{2} U_{N,ph})$
- **Per unit electric current:** $i_s = I_s / (\sqrt{2} I_{N,ph})$ $\quad i'_r = I'_r / (\sqrt{2} I_{N,ph})$
- **Per unit magnetic flux linkage:** $\psi = \Psi / \Psi_N$ $\quad \Psi_N = \dfrac{\sqrt{2} \cdot U_{N,ph}}{\omega_N}$

$$\text{e.g.}: L_s = 10mH,\ Z_N = 2\Omega,\ f_N = 50Hz: \quad \boxed{x_s = \omega_N \cdot L_s / Z_N = (2\pi \cdot 50) \cdot 0.01/2 = 1.57}$$

$$(X_s = \omega_N \cdot L_s = (2\pi \cdot 50) \cdot 0.01 = 3.14\Omega) \quad x_s = X_s / Z_N = 3.14/2 = 1.57$$

$$\text{e.g.}: \Psi = 3\text{Vs},\ U_{N,ph} = 231V,\ f_N = 50Hz: \quad \Psi_N = \frac{\sqrt{2} \cdot U_{N,ph}}{\omega_N} = \frac{\sqrt{2} \cdot 231}{2\pi \cdot 50} = 1.04Vs$$

$$\boxed{\psi = \Psi / \Psi_N = 3/1.04 = 2.885}$$

Result: High flux linkage = high iron saturation must be expected!

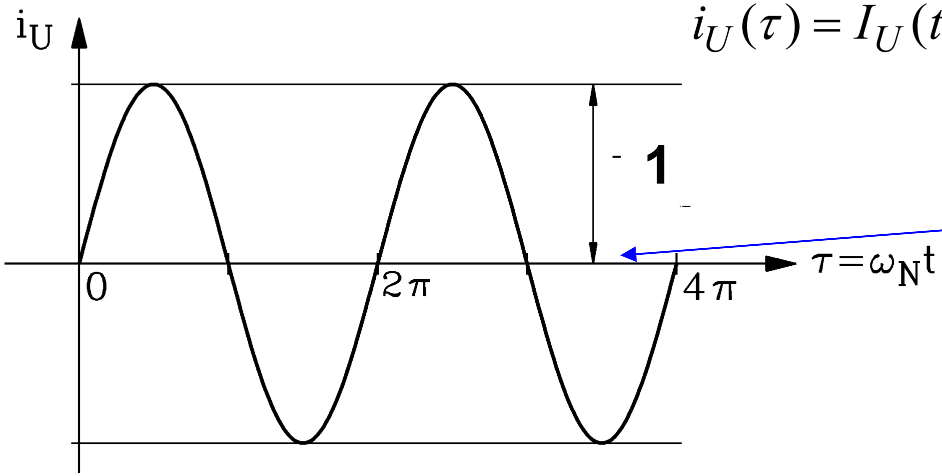
7. Dynamics of induction machines
Per unit electric phase voltage and current

- In dynamic calculations instantaneous values $U(t)$, $I(t)$ are derived as results.
- Therefore in AC machinery the per unit calculation is done with the momentary peak values (amplitudes) of the stationary sinusoidal rated operational values.

$$u_s(\tau) = U_s(t)/(\sqrt{2}\cdot U_{N,ph}) \qquad i_s(\tau) = I_s(t)/(\sqrt{2}\cdot I_{N,ph})$$

- ***Example:*** Sinusoidal rated operation: $I_U(t) = \sqrt{2}\cdot I_{N,ph}\cdot \sin(2\pi f_N t)$

$$i_U(\tau) = I_U(t)/(\sqrt{2}\cdot I_{N,ph}) = 1\cdot \sin(\tau)$$



One RATED “per-unit” period is ALWAYS 2π, independent from rated frequency f_N

$$\text{e.g.}: t = \frac{1}{f_N} : \tau = \omega_N \cdot t = 2\pi\cdot\frac{f_N}{f_N} = 2\pi$$

Summary of per unit values (3)

a) Per unit torque:

- Reference: Rated apparent torque M_B = rated APPARENT power S_N vs. synchronous speed !

- This M_B includes power factor $\cos\varphi_N$ and machine efficiency η_N !

$$m = M / M_B \qquad M_B = \frac{S_N}{\omega_N / p}$$

Note:

Induction motor operation: Rated apparent torque M_B is bigger than rated torque M_N :

$$M_B = \frac{S_N}{\omega_N / p} = \frac{P_N / (\cos\varphi_N \cdot \eta_N)}{\frac{\Omega_{mN}}{1 - s_N}} = \frac{P_N}{\Omega_{mN}} \cdot \frac{1 - s_N}{\cos\varphi_N \cdot \eta_N} = M_N \cdot \frac{1 - s_N}{\cos\varphi_N \cdot \eta_N}$$

$$M_B = M_N \cdot \frac{1 - s_N}{\cos\varphi_N \cdot \eta_N}$$

b) Per unit moment of inertia:

- Rotor inertia J is calculated from T_J as **per unit starting time constant τ_J.**

$$\tau_J = \omega_N \cdot T_J \qquad T_J = J \cdot \frac{\omega_N / p}{M_B}$$

7. Dynamics of induction machines
Per unit calculation from data plate values

Example:
4-pole cage induction motor

400 V Y	34.5 A	
18.5 kW	1465 /min	S1
50 Hz	$\cos\varphi = 0.84$	$J = 0.054\ \mathrm{kgm}^2$
Th.Cl. F	IP 54	

$$Z_N = U_{N,ph} / I_{N,ph} = 231/34.5 = \underline{\underline{6.67\ \Omega}}$$

$$S_N = 3 \cdot U_{N,ph} \cdot I_{N,ph} = 3 \cdot 231 \cdot 34.5 = \underline{\underline{23.9\ \mathrm{kVA}}}$$

$$M_B = \frac{S_N}{\Omega_{syn,N}} = \frac{23909}{2\pi \cdot 50/2} = \underline{\underline{152.2\ \mathrm{Nm}}}$$

$$M_N = \frac{P_N}{\Omega_{mN}} = \frac{18500}{2\pi \cdot (1465/60)} = \underline{\underline{120.6\ \mathrm{Nm}}}$$

$$s_N = \frac{1500 - 1465}{1500} = 0.0233$$

$$\eta_N = \frac{P_N}{S_N \cos\varphi_N} = \frac{18.5}{23.9 \cdot 0.84} = 0.9215$$

$$\boxed{\frac{M_B}{M_N} = \frac{152.2}{120.6} = 1.262 = \frac{1 - s_N}{\cos\varphi_N \cdot \eta_N} = \frac{1 - 0.0233}{0.84 \cdot 0.9215}}$$

$$\hat{\Psi}_N = \frac{\sqrt{2} \cdot 231}{314} = \underline{\underline{1.036\ \mathrm{Vs}}}$$

$$T_J = 0.054 \cdot \frac{314}{2} \cdot \frac{1}{152.23} = \underline{\underline{0.056\ \mathrm{s}}}$$

$$\tau_J = \omega_N T_J = 314 \cdot 0.056 = \underline{\underline{17.58}}$$

7. Dynamics of induction machines
Dynamic p.u. equations

- Voltage equation per phase

  a) in physical units: $\boxed{U(t) = R \cdot I(t) + \frac{d\Psi(t)}{dt}}$

  b) in per unit system:

$$u(\omega_N t) = \frac{U(t)}{\sqrt{2} \cdot U_{N,ph}} = \frac{R \cdot I(t)}{\sqrt{2} \cdot U_{N,ph}} \cdot \frac{\sqrt{2} \cdot I_{N,ph}}{\sqrt{2} \cdot I_{N,ph}} + \frac{d\Psi(t)}{\frac{\sqrt{2} \cdot U_{N,ph}}{\omega_N} \cdot d(\omega_N t)} \rightarrow \boxed{\underline{\underline{u(\tau) = r \cdot i(\tau) + \frac{d\psi(\tau)}{d\tau}}}}$$

- Mechanical equation

  a) in physical units: $\boxed{J \cdot \frac{d\Omega_m(t)}{dt} = M_e(t) - M_s(t)}$

  b) in per unit system:

$$\omega_N \cdot J \cdot \frac{\omega_N / p}{M_B} \cdot \frac{d\Omega_m(t)}{\frac{\omega_N}{p} d(\omega_N t)} = \frac{M_e(t) - M_s(t)}{M_B} \rightarrow \boxed{\underline{\underline{\tau_J \cdot \frac{d\omega_m(\tau)}{d\tau} = m_e(\tau) - m_s(\tau)}}}$$

Summary:
Per unit calculation

- Name-plate data used for per-unit calculation
- Usually phase quantities used for p.u.
- Not the rated real torque M_N, but the rated apparent torque M_B used for p.u.
- Fast estimate of percentage of loading of a device possible by p.u. values

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7. Dynamics of induction machines
Three phase dynamic voltage equation

$$
\left.\begin{aligned}
u_{s,U}(\tau) &= r_s \cdot i_{s,U}(\tau) + \frac{d\psi_{s,U}(\tau)}{d\tau} \\
u_{s,V}(\tau) &= r_s \cdot i_{s,V}(\tau) + \frac{d\psi_{s,V}(\tau)}{d\tau} \\
u_{s,W}(\tau) &= r_s \cdot i_{s,W}(\tau) + \frac{d\psi_{s,W}(\tau)}{d\tau}
\end{aligned}\right|
\left.\begin{aligned}
&\cdot\frac{2}{3} \\
&\cdot\frac{2}{3}\cdot\underline{a} \\
&\cdot\frac{2}{3}\cdot\underline{a}^2
\end{aligned}\right\} +
\left.\begin{aligned}
&\cdot\frac{1}{3} \\
&\cdot\frac{1}{3} \\
&\cdot\frac{1}{3}
\end{aligned}\right\} +
$$

From 3 phase voltages

$\Downarrow$

$$\underline{u}_s(\tau) = r_s \cdot \underline{i}_s(\tau) + \frac{d\underline{\psi}_s(\tau)}{d\tau}$$

ONE space vector equation

$$u_{s0}(\tau) = r_s \cdot i_{s0}(\tau) + \frac{d\psi_{s0}(\tau)}{d\tau}$$

ONE zero sequence equation

Voltage space vector:

$$\underline{u}_s(\tau) = \frac{2}{3}\cdot\left(u_{s,U}(\tau) + \underline{a}\cdot u_{s,V}(\tau) + \underline{a}^2\cdot u_{s,W}(\tau)\right)$$

Zero sequence voltage:

$$u_{s,0}(\tau) = \frac{1}{3}\cdot\left(u_{s,U}(\tau) + u_{s,V}(\tau) + u_{s,W}(\tau)\right)$$

7. Dynamics of induction machines
Stator and rotor reference frame

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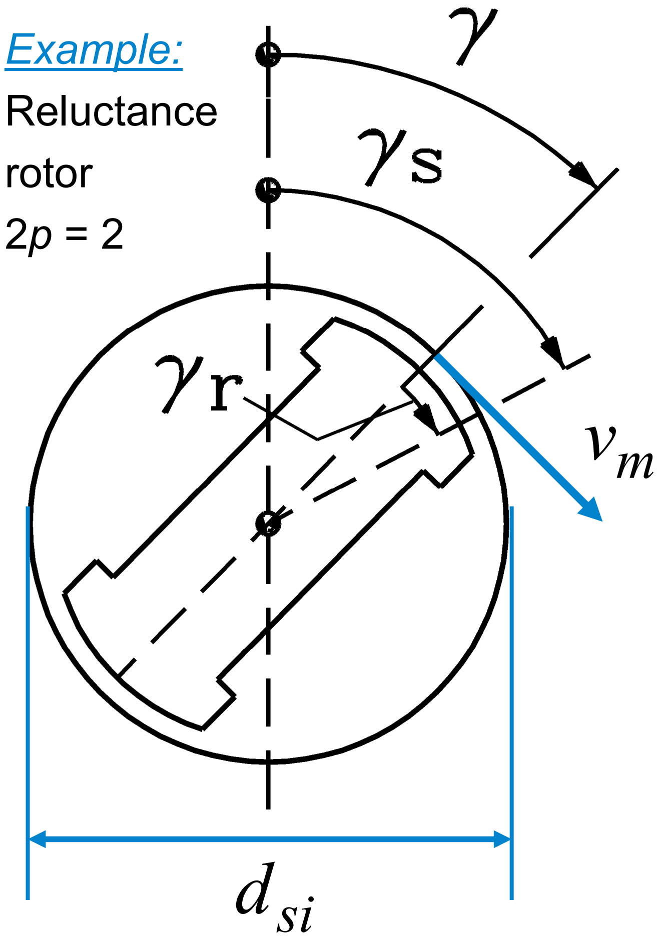
Example:
Reluctance rotor
$2p = 2$



$\gamma_s(t)$: Circumference angle in stator reference frame

$\gamma_r(t)$: Circumference angle in rotor reference frame

„elecrical degrees“: $2p\tau_p = p \cdot 2\tau_p \Leftrightarrow p \cdot 2\pi$

Relationship between circumference angles in stator and rotor reference frame:
(„***Galilei*-transformation**“):

$$\boxed{\gamma_s(t) = \gamma_r + \gamma(t) = \gamma_r + p \cdot \Omega_m \cdot t}$$

$$\Omega_m = 2\pi \cdot n = \frac{v_m}{d_{si}/2} \qquad p\Omega_m = 2\pi \cdot n \cdot p$$

7. Dynamics of induction machines

Reference frames = Co-ordinate systems

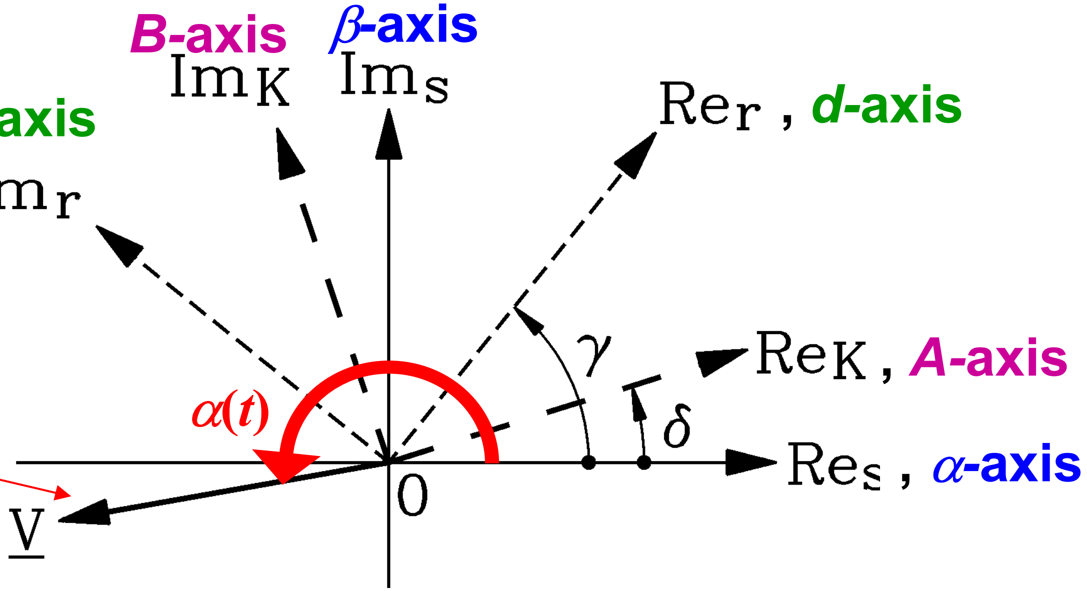
- **Stator reference frame (s):** α-axis is Re_s-axis, β-axis is Im_s-axis

- **Rotor reference frame (r):** *d*-axis is Re_r-axis, *q*-axis is Im_r-axis

- **Arbitrary reference frame (K):** *A*-axis is Re_K-axis, *B*-axis is Im_K-axis

$$\boxed{\underline{V}(t)_{(s)} = V(t) \cdot e^{j\alpha(t)}}$$

Space vector $\underline{V}$



7. Dynamics of induction machines

Space vector V in different reference frames

Rotor reference frame is shifted by **rotation angle** $\gamma(t)$, measured in "electric degrees”:

$$\gamma(t) = p \cdot \int_0^t \Omega_m(t) \cdot dt + \gamma_0 = \int_0^t \frac{\Omega_m(t)}{\omega_N / p} \cdot \omega_N \cdot dt + \gamma_0 = \int_0^\tau \omega_m(\tau) \cdot d\tau + \gamma_0 = \gamma(\tau)$$

in stator reference frame *s*	in rotor reference frame *r*	in reference frame *K*
$\underline{V}_{(s)} = V \cdot e^{j\alpha}$	$\underline{V}_{(r)} = V \cdot e^{j\alpha} \cdot e^{-j\gamma}$	$\underline{V}_{(K)} = V \cdot e^{j\alpha} \cdot e^{-j\delta}$
	$\underline{V}_{(r)} = \underline{V}_{(s)} \cdot e^{-j\gamma}$	$\underline{V}_{(K)} = \underline{V}_{(s)} \cdot e^{-j\delta}$

Space vector coordinate transformation:

- from stator reference frame to reference frame *K*: multiplication by $\cdot e^{-j\delta(\tau)}$
- from rotor reference frame to frame *K* : by multiplication with $\cdot e^{-j(\delta(\tau)-\gamma(\tau))}$.

$$\underline{V}_{(K)} = V \cdot e^{j\alpha} \cdot e^{-j\delta} = V \cdot e^{j\alpha} \cdot e^{-j\gamma} \cdot e^{j\gamma} \cdot e^{-j\delta} = \underline{V}_{(r)} \cdot e^{-j(\delta-\gamma)}$$

7. Dynamics of induction machines
Three phase rotor dynamic voltage equation

- **Space vector rotor voltage equation:**
  in rotor reference frame

$$\underline{u}'_r(\tau) = r'_r \cdot \underline{i}'_r(\tau) + \left.\frac{d\underline{\psi}'_r(\tau)}{d\tau}\right|_{(r)}$$

$$u'_{r0}(\tau) = r'_r \cdot i'_{r0}(\tau) + \left.\frac{d\psi'_{r0}(\tau)}{d\tau}\right|_{(r)}$$

Subscript (r) means:
"in rotor reference frame"

- **Note: Cage induction machine:**
  Q_r phases: May be treated as a 3-phase machine,
  transformed into space vector formulation !

7. Dynamics of induction machines

Transformation of voltage equation between reference frames

- **Rule for differentiation of product of two functions**:

$$\frac{d\left(\underline{\psi}_s(\tau)\cdot e^{-j\delta(\tau)}\right)}{d\tau}=e^{-j\delta(\tau)}\cdot\frac{d\underline{\psi}_s(\tau)}{d\tau}+\underline{\psi}_s(\tau)\cdot\frac{d\left(e^{-j\delta(\tau)}\right)}{d\tau}=e^{-j\delta(\tau)}\cdot\frac{d\underline{\psi}_s(\tau)}{d\tau}-j\cdot\underline{\psi}_s(\tau)\cdot e^{-j\delta(\tau)}\cdot\frac{d(\delta(\tau))}{d\tau}$$

- **Transformation of voltage equations into arbitrary co-ordinate system K:**

$$\underline{u}_{s(K)}=\underline{u}_s\cdot e^{-j\delta}=r_s\cdot\underline{i}_s\cdot e^{-j\delta}+\frac{d\underline{\psi}_s}{d\tau}\cdot e^{-j\delta}=r_s\cdot\underline{i}_{s(K)}+\frac{d\underline{\psi}_{s(K)}}{d\tau}+j\cdot\frac{d\delta}{d\tau}\cdot\underline{\psi}_{s(K)}$$

$$\underline{u}'_{r(K)}=\underline{u}'_r\cdot e^{-j(\delta-\gamma)}=r'_r\cdot\underline{i}'_r\cdot e^{-j(\delta-\gamma)}+\frac{d\underline{\psi}'_r}{d\tau}\cdot e^{-j(\delta-\gamma)}=r'_r\cdot\underline{i}'_{r(K)}+\frac{d\underline{\psi}'_{r(K)}}{d\tau}+j\cdot\frac{d(\delta-\gamma)}{d\tau}\cdot\underline{\psi}'_{r(K)}$$

$$\boxed{\begin{aligned}\underline{u}_{s(K)}&=r_s\cdot\underline{i}_{s(K)}+\frac{d\underline{\psi}_{s(K)}}{d\tau}+j\cdot\frac{d\delta}{d\tau}\cdot\underline{\psi}_{s(K)}\\ \underline{u}'_{r(K)}&=r'_r\cdot\underline{i}'_{r(K)}+\frac{d\underline{\psi}'_{r(K)}}{d\tau}+j\cdot\frac{d(\delta-\gamma)}{d\tau}\cdot\underline{\psi}'_{r(K)}\end{aligned}}$$

7. Dynamics of induction machines

Transformer and rotary induction

Transformer induction

Rotary induction

$$\boxed{\begin{aligned} \underline{u}_{s(K)} &= r_s \cdot \underline{i}_{s(K)} + \frac{d\underline{\psi}_{s(K)}}{d\tau} + j \cdot \frac{d\delta}{d\tau} \cdot \underline{\psi}_{s(K)} \\ \underline{u}'_{r(K)} &= r'_r \cdot \underline{i}'_{r(K)} + \frac{d\underline{\psi}'_{r(K)}}{d\tau} + j \cdot \frac{d(\delta - \gamma)}{d\tau} \cdot \underline{\psi}'_{r(K)} \end{aligned}}$$

Mainly used reference frames (1)

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Reference frame	*Angular rotation of reference frame*
stator reference frame (α, β)	$\delta(\tau)=0:\omega_K=\dfrac{d\delta}{d\tau}=0$
rotor reference frame (d, q)	$\delta=\gamma:\omega_K(\tau)=\dfrac{d\gamma}{d\tau}=\omega_m$
synchronous reference frame (a, b)	$\dfrac{d\delta(\tau)}{d\tau}=\omega_K(\tau)=\dfrac{\Omega_{syn}(\tau)}{\omega_N/p}=\dfrac{\Omega_s(\tau)}{\omega_N}=\omega_s$

Stator and rotor equation in stator reference frame (α-β-system):

$$\boxed{\begin{aligned} \underline{u}_s &= r_s\cdot\underline{i}_s+\frac{d\underline{\psi}_s}{d\tau} \\ 0 &= r_r'\cdot\underline{i}'_r+\frac{d\underline{\psi}'_r}{d\tau}-j\cdot\omega_m\cdot\underline{\psi}'_r \end{aligned}}$$

Induction machine:

$\underline{u}'_r=0+j\cdot 0$

7. Dynamics of induction machines

Mainly used reference frames (2)

Stator reference frame (α, β) $\delta(\tau)=0:\omega_K=\dfrac{d\delta}{d\tau}=0$

$$\underline{u}_{s(s)}=r_s\cdot\underline{i}_{s(s)}+\frac{d\underline{\psi}_{s(s)}}{d\tau}\qquad 0=r_r'\cdot\underline{i}'_{r(s)}+\frac{d\underline{\psi}'_{r(s)}}{d\tau}-j\cdot\omega_m\cdot\underline{\psi}'_{r(s)}$$

Rotor reference frame (d, q) $\delta=\gamma:\omega_K(\tau)=\dfrac{d\gamma}{d\tau}=\omega_m$

$$\underline{u}_{s(r)}=r_s\cdot\underline{i}_{s(r)}+\frac{d\underline{\psi}_{s(r)}}{d\tau}+j\cdot\omega_m\cdot\underline{\psi}'_{s(r)}\qquad 0=r_r'\cdot\underline{i}'_{r(r)}+\frac{d\underline{\psi}'_{r(r)}}{d\tau}$$

Synchronous reference frame (a, b) $\omega_K(\tau)=\dfrac{\Omega_{syn}(\tau)}{\omega_N/p}=\dfrac{\Omega_s(\tau)}{\omega_N}=\omega_s$

$$\underline{u}_{s(syn)}=r_s\cdot\underline{i}_{s(syn)}+\frac{d\underline{\psi}_{s(syn)}}{d\tau}+j\cdot\omega_s\cdot\underline{\psi}_{s(syn)}$$

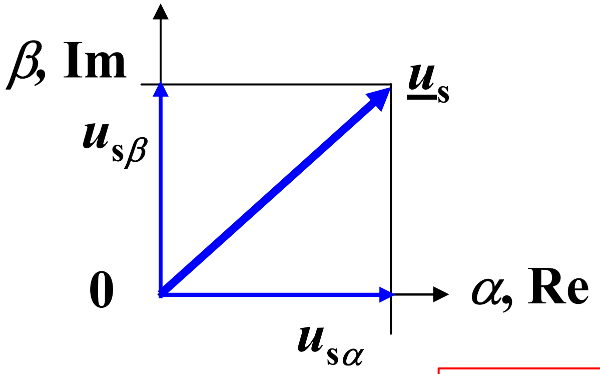
$$\underline{u}'_{r(syn)}=r_r'\cdot\underline{i}'_{r(syn)}+\frac{d\underline{\psi}'_{r(syn)}}{d\tau}+j\cdot(\omega_s-\omega_m)\cdot\underline{\psi}'_{r(syn)}$$

7. Dynamics of induction machines

Example: Voltage equations in stator reference frame

- **In stator reference frame: α - β -system: components:**

$$\underline{u}_s = u_{s\alpha} + j \cdot u_{s\beta}$$



Induction machine:

$$\underline{u}'_r = 0$$

$$\underline{u}_s = r_s \cdot \underline{i}_s + \frac{d\underline{\psi}_s}{d\tau}$$

$$0 = r'_r \cdot \underline{i}'_r + \frac{d\underline{\psi}'_r}{d\tau} - j \cdot \omega_m \cdot \underline{\psi}'_r$$

$$u_{s\alpha} = r_s \cdot i_{s\alpha} + d\psi_{s\alpha} / d\tau$$

$$u_{s\beta} = r_s \cdot i_{s\beta} + d\psi_{s\beta} / d\tau$$

$$0 = r'_r \cdot i'_{r\alpha} + d\psi'_{r\alpha} / d\tau + \omega_m \cdot \psi'_{r\beta}$$

$$0 = r'_r \cdot i'_{r\beta} + d\psi'_{r\beta} / d\tau - \omega_m \cdot \psi'_{r\alpha}$$

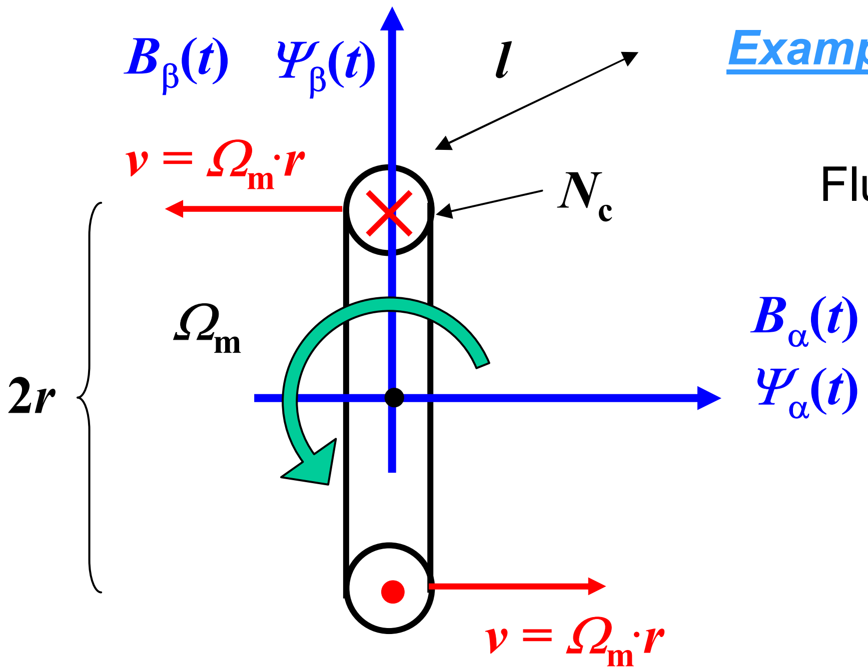
$$\underline{i}_s = i_{s\alpha} + j \cdot i_{s\beta} \qquad \underline{i}'_r = i'_{r\alpha} + j \cdot i'_{r\beta}$$

$$\underline{\psi}_s = \psi_{s\alpha} + j \cdot \psi_{s\beta} \qquad \underline{\psi}'_r = \psi'_{r\alpha} + j \cdot \psi'_{r\beta}$$

"Transformer" part "Rotary" part

of voltage induction

Transformer and rotary part of induction



Example: Rotating rotor coil (N_c turns) in stator B-field

Flux linkages
$$\begin{cases} 2r \cdot l \cdot B_\alpha = \Phi_\alpha & \Psi_\alpha = N_c \cdot \Phi_\alpha \\ 2r \cdot l \cdot B_\beta = \Phi_\beta & \Psi_\beta = N_c \cdot \Phi_\beta \end{cases}$$

$$U(t) + U_i(t) = I(t) \cdot R \Rightarrow U(t) = -U_i(t) + I(t) \cdot R$$

$$U_i(t) = -\partial \Psi / \partial t + N_c \cdot \int_l (\vec{v} \times \vec{B}) \cdot d\vec{s}$$

$$U_{i,\alpha}(t) = \underbrace{-(2r \cdot l) \cdot N_c \cdot \partial B_\alpha / \partial t}_{\text{“Transformer”}} - \underbrace{2v \cdot N_c \cdot B_\beta \cdot l}_{\text{“Rotary”}}$$

$$2 \cdot v \cdot N_c \cdot B_\beta \cdot l = 2(\Omega_m \cdot r) \cdot N_c \cdot B_\beta \cdot l = \Omega_m \cdot N_c \cdot 2r \cdot l \cdot B_\beta = \Omega_m \cdot N_c \cdot \Phi_\beta = \Omega_m \cdot \Psi_\beta$$

$$p = 1: \; -U_{i,\alpha}(t) = -(U_{i,tr,\alpha} + U_{i,rot,\alpha}) = d\Psi_\alpha / dt + \Omega_m \cdot \Psi_\beta$$

$$\boxed{U_\alpha(t) = R \cdot I_\alpha(t) + d\Psi_\alpha / dt + \Omega_m \cdot \Psi_\beta}$$

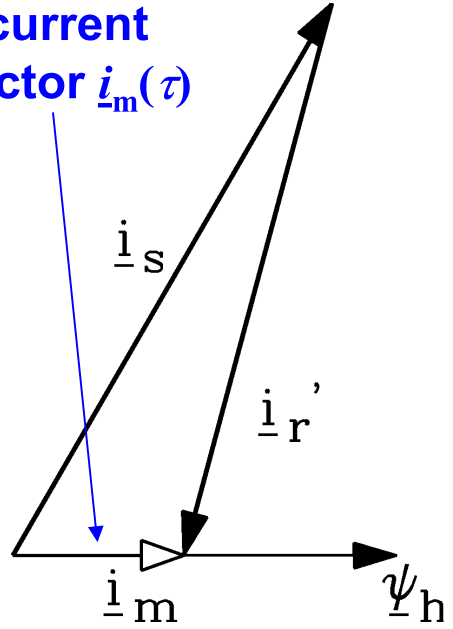
Summary:
Dynamic voltage equations and reference frames of induction machine

- Space vector formulation allows <u>one</u> voltage equation instead of <u>three</u> U, V, W
- One stator and one rotor voltage equation
- Different reference frames may be used: stator, rotor, arbitrary
- Voltage induction separated into „transformer“ and „rotary“ part

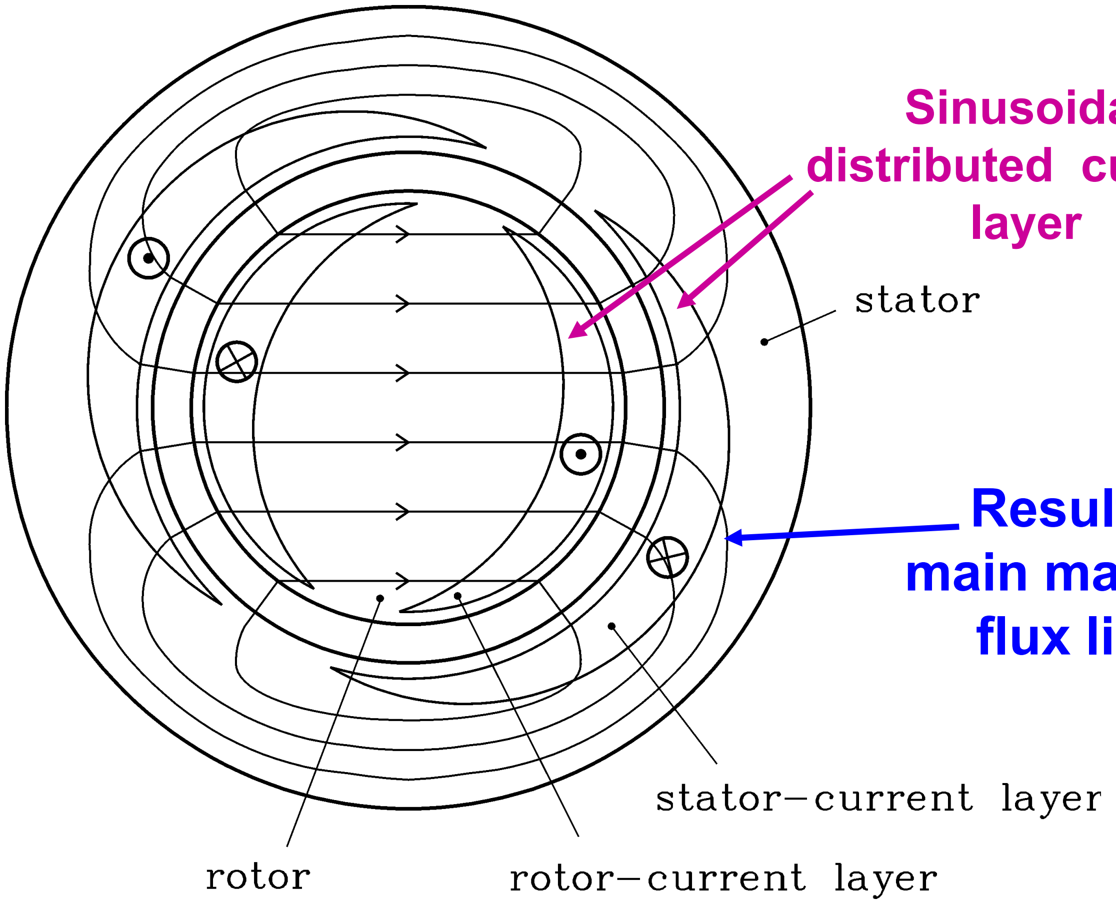
7. Dynamics of induction machines
Main flux linkage

Main flux linkage (fundamental wave, excited by *sinusoidal* distributed current layer of stator and rotor = magnetizing current layer):

Magnetizing space current vector $\underline{i}_m(\tau)$



$$\boxed{\underline{\psi}_h(\tau) = x_h \cdot \underline{i}_m(\tau)}$$



7. Dynamics of induction machines
Dynamic stator and rotor flux linkages

- Stator and rotor space current vector excite **ALSO leakage flux linkage**:

$$\underline{\psi}_{s\sigma}(\tau) = x_{s\sigma} \cdot \underline{i}_s(\tau) \qquad \underline{\psi}'_{r\sigma}(\tau) = x'_{r\sigma} \cdot \underline{i}'_r(\tau)$$

- Per unit **stray inductance**: $x_{s\sigma} = \dfrac{\omega_N \cdot L_{s\sigma}}{Z_N}$, $x'_{r\sigma} = \dfrac{\omega_N \cdot L'_{r\sigma}}{Z_N}$

- Resulting flux linkage in stator and rotor:

$$\boxed{\begin{aligned} \underline{\psi}_s &= (x_h + x_{s\sigma}) \cdot \underline{i}_s + x_h \cdot \underline{i}'_r = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r = x_{s\sigma} \cdot \underline{i}_s + x_h \cdot \underline{i}_m = \underline{\psi}_{s\sigma} + \underline{\psi}_h \\ \underline{\psi}'_r &= x_h \cdot \underline{i}_s + (x_h + x'_{r\sigma}) \cdot \underline{i}'_r = x_h \cdot \underline{i}_s + x'_r \cdot \underline{i}'_r = x_h \cdot \underline{i}_m + x'_{r\sigma} \cdot \underline{i}'_r = \underline{\psi}_h + \underline{\psi}'_{r\sigma} \end{aligned}}$$

- Total leakage flux space vector is described by *Blondel´*s total leakage coefficient:

$$\boxed{\sigma = 1 - \frac{x_h^2}{x_s \cdot x'_r}}$$

7. Dynamics of induction machines

Flux linkage equations independent of reference frame

- Flux linkage equation <u>independent</u> of reference frame!

  Example: (s) $\rightarrow$ (K)

$$\underline{\psi}_{s(K)} = \underline{\psi}_{s(s)} \cdot e^{-j\delta}, \underline{i}_{s(K)} = \underline{i}_{s(s)} \cdot e^{-j\delta}, \underline{i}'_{r(K)} = \underline{i}'_{r(s)} \cdot e^{-j\delta}$$

$$\underline{\psi}_{s(s)} = x_s \cdot \underline{i}_{s(s)} + x_h \cdot \underline{i}'_{r(s)}$$

$$\underline{\psi}_{s(K)} = x_s \cdot \underline{i}_{s(K)} + x_h \cdot \underline{i}'_{r(K)} = x_s \cdot \underline{i}_{s(s)} \cdot e^{-j\cdot\delta} + x_h \cdot \underline{i}'_{r(s)} \cdot e^{-j\cdot\delta}$$

$$\underline{\psi}_{s(K)} = \underbrace{(x_s \cdot \underline{i}_{s(s)} + x_h \cdot \underline{i}'_{r(s)})}_{\underline{\psi}_{s(s)}} \cdot e^{-j\cdot\delta} = \underline{\psi}_{s(s)} \cdot e^{-j\cdot\delta} = \underline{\psi}_{s(K)}$$

Calculation of p.u. stator and rotor flux linkages (1)

Example 1:

Induction machine operated at three-phase symmetrical sinus voltage system (u_s = 1) with rated frequency $\omega_s = 1$.

- Stator resistance neglected $r_s = 0$, calculation in stator reference frame:
- Inductance data: $x_h = 2.5, \quad x_s = 2.6, \quad x'_r = 2.58$:

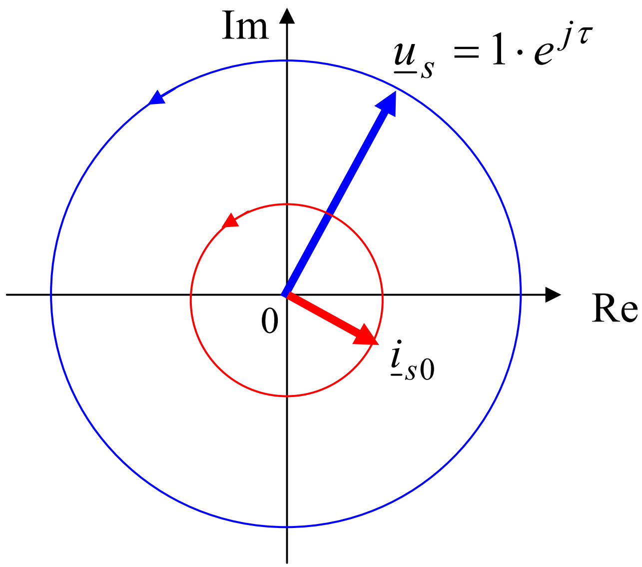
No-load current $\underline{i}_{s0}$: $\underline{i}'_{r0} = 0$

voltage space vector: $\underline{u}_s = 1 \cdot e^{j\tau}$

flux linkage $\underline{\psi}_s = x_s \cdot \underline{i}_{s0}$

$$\underline{u}_s = r_s \cdot \underline{i}_s + \frac{d\underline{\psi}_s}{d\tau} \approx \frac{d\underline{\psi}_s}{d\tau} = x_s \cdot \frac{d\underline{i}_{s0}}{d\tau} = e^{j\tau} \quad \rightarrow$$

$$\rightarrow \quad \underline{\underline{\underline{i}_{s0} = -j \cdot \frac{1}{x_s} \cdot e^{j\tau}}} \qquad i_{s0} = \frac{1}{2.6} = \underline{\underline{0.38}}$$



Rotor flux linkage ψ'_r at very high rotor slip is ZERO

- Induction machine: Rotor winding is short-circuited: $\underline{u}'_r(\tau) = 0 = 0 + j \cdot 0$
- At very high rotor slip the rotor flux linkage changes very fast: $r'_r \cdot \underline{i}'_r(\tau) << d\underline{\psi}'_r(\tau)/d\tau$

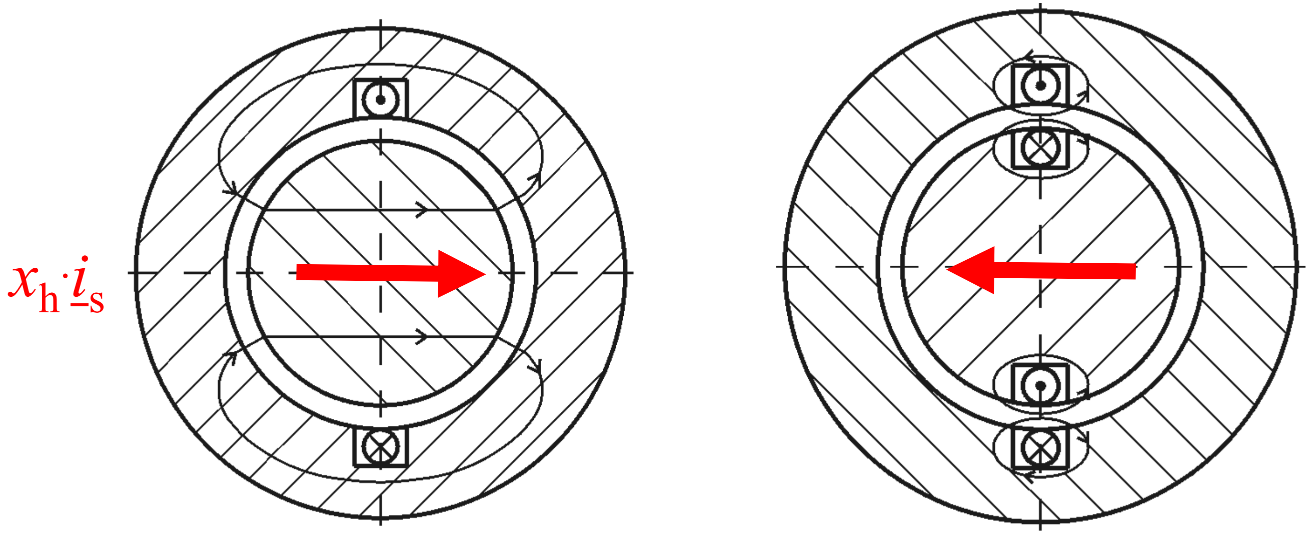
- Rotor voltage equation in rotor ref. frame, very high rotor slip:

$$0 = \underline{u}'_r(\tau) = r'_r \cdot \underline{i}'_r(\tau) + d\underline{\psi}'_r(\tau)/d\tau \approx d\underline{\psi}'_r(\tau)/d\tau \qquad \boxed{\Rightarrow \underline{\psi}'_r(\tau) = \text{const.} = 0}$$

Result:
- Rotor flux linkage at very high slip is **zero**! (No DC rotor flux: "const. = 0")
- Stator & rotor main flux $\underline{\psi}_h$ and rotor stray flux $\underline{\psi}'_{r\sigma}$ cancel, so total rotor flux is **zero**!

$$\boxed{\underline{\psi}'_r = 0 = \underline{\psi}_h + \underline{\psi}'_{r\sigma}}$$

$$0 = x_h \cdot (\underline{i}_s + \underline{i}'_r) + x'_{r\sigma} \cdot \underline{i}'_r$$



7. Dynamics of induction machines

Stator flux linkage ψ_s at high rotor slip is total leakage

Very big slip |*Slip*| >> 1:

$$\left.\begin{aligned}\underline{\psi}_s &= x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r \\ \underline{\psi}'_r &= 0 = x_h \cdot \underline{i}_s + x'_r \cdot \underline{i}'_r\end{aligned}\right\} \quad \underline{i}'_r = -(x_h / x'_r) \cdot \underline{i}_s$$

Stator flux linkage: $\underline{\psi}_s = x_s \cdot \underline{i}_s - (x_h^2 / x'_r) \cdot \underline{i}_s = x_s \underline{i}_s \cdot (1 - (x_h^2 / (x'_r x_s))) = \sigma \cdot x_s \cdot \underline{i}_s$

or: $\underline{\psi}_s = x_{s\sigma} \cdot \underline{i}_s + x_h \cdot \underline{i}_s + x_h \cdot \underline{i}'_r = x_{s\sigma} \cdot \underline{i}_s - x_h \cdot (x'_r / x_h) \cdot \underline{i}'_r + x_h \cdot \underline{i}'_r =$

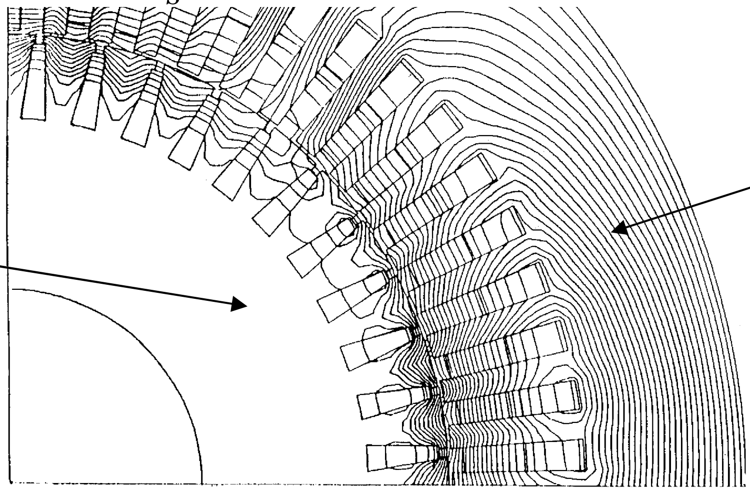
$= x_{s\sigma} \cdot \underline{i}_s - x'_{r\sigma} \cdot \underline{i}'_r$ $\quad\boxed{\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s = x_{s\sigma} \cdot \underline{i}_s - x'_{r\sigma} \cdot \underline{i}'_r}$

With $\underline{i}'_r = -(x_h / x'_r) \cdot \underline{i}_s \approx -\underline{i}_s$: $\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s = x_{s\sigma} \cdot \underline{i}_s - x'_{r\sigma} \cdot \underline{i}'_r \approx (x_{s\sigma} + x'_{r\sigma}) \cdot \underline{i}_s$

$$\boxed{\sigma \cdot x_s \approx x_{s\sigma} + x'_{r\sigma}}$$

Rotor flux linkage is zero!

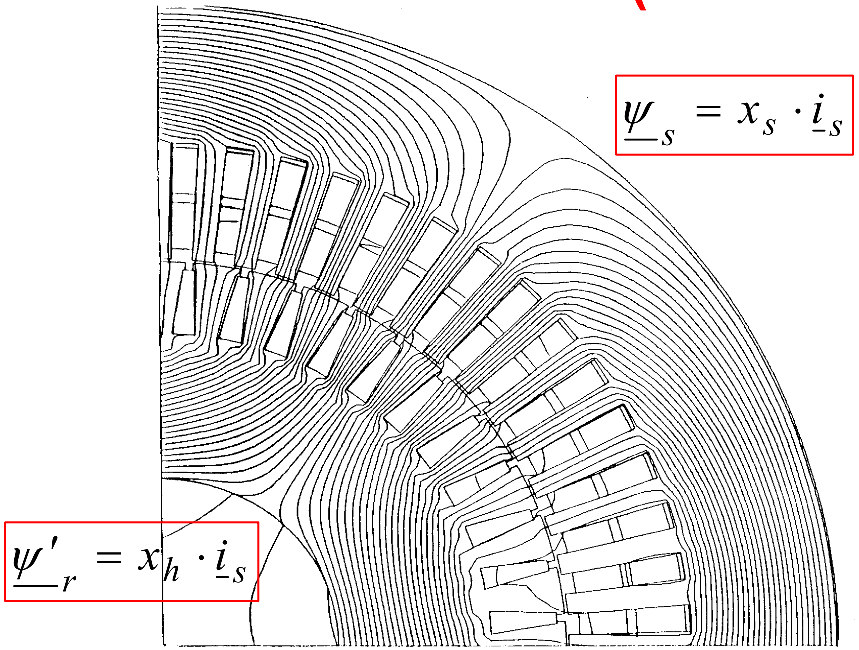
Example: *Slip* = 1



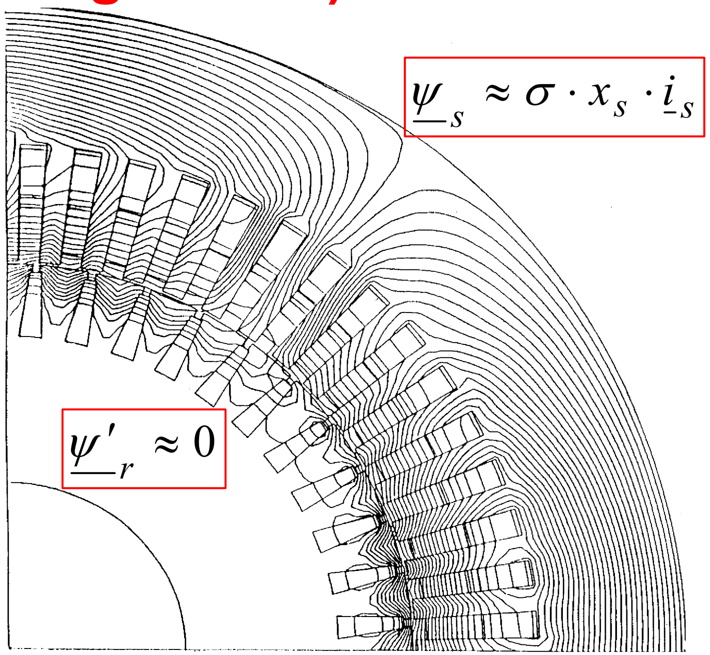
Stator flux linkage is at high slip nearly equal to the total leakage flux!

2. Design of Induction Machines
Rotor shielding effect at big slip

At big slip (already *Slip* = 1!) the rotor inner part is nearly free of flux ("rotor shielding effect")



At no-load (*Slip* = 0, rotor current zero)



At stand still (locked rotor) *Slip* = 1

Numerically calculated two-dimensional magnetic flux density *B* of a three-phase, 4-pole high voltage cage induction machine with wedge rotor slots (Q_s / Q_r = 60/44) at rated voltage

Calculation of per unit stator and rotor flux linkages (2)

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Example 2:

Induction machine operated at three-phase symmetrical sinus voltage system (u_s = 1) with rated frequency $\omega_s = 1$ at high slip (e.g.: *Slip* ≥ 1):

- Calculation in stator reference frame:
- Inductance data: $x_h = 2.5, \quad x_s = 2.6, \quad x'_r = 2.58$:
- Current data at big slip:

$$|s| >> 1: \; |i'_r| = |-(x_h / x'_r) \cdot i_s| = (x_h / x'_r) \cdot |i_s| = (2.5/2.58) \cdot |i_s| = 0.97 \cdot |i_s| \quad \Rightarrow \quad \underline{i}_s \approx -\underline{i}'_r \, .$$

Leakage inductances:

$$x_{s\sigma} = x_s - x_h = 2.6 - 2.5 = \underline{\underline{0.1}}, \quad x'_{r\sigma} = x'_r - x_h = 2.58 - 2.5 = \underline{\underline{0.08}}$$

- Total leakage coefficient: $\sigma = 1 - \dfrac{x_h^2}{x_s \cdot x'_r} = 1 - \dfrac{2.5^2}{2.6 \cdot 2.58} = \underline{\underline{0.068}}$

- Stator flux linkage at *s* ≥ 1: $|\underline{\psi}_s| = \sigma \cdot x_s \cdot |\underline{i}_s| = 0.068 \cdot 2.6 \cdot |\underline{i}_s| = \underline{\underline{0.177 \cdot |\underline{i}_s|}}$ with total

leakage flux linkage $|\underline{\psi}_s| = |\underline{\psi}_\sigma| = |x_{s\sigma} \cdot \underline{i}_s - x'_{r\sigma} \cdot \underline{i}'_r| = |0.1 - 0.08 \cdot (-0.97)| \cdot |\underline{i}_s| = \underline{\underline{0.177 \cdot |\underline{i}_s|}}$

7. Dynamics of induction machines

Example: Rotor flux linkage is ZERO at very big rotor slip

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Very big slip $|Slip| \gg 1$: $\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r$ $\qquad$ $\underline{\psi}'_r = 0 = x_h \cdot \underline{i}_s + x'_r \cdot \underline{i}'_r$

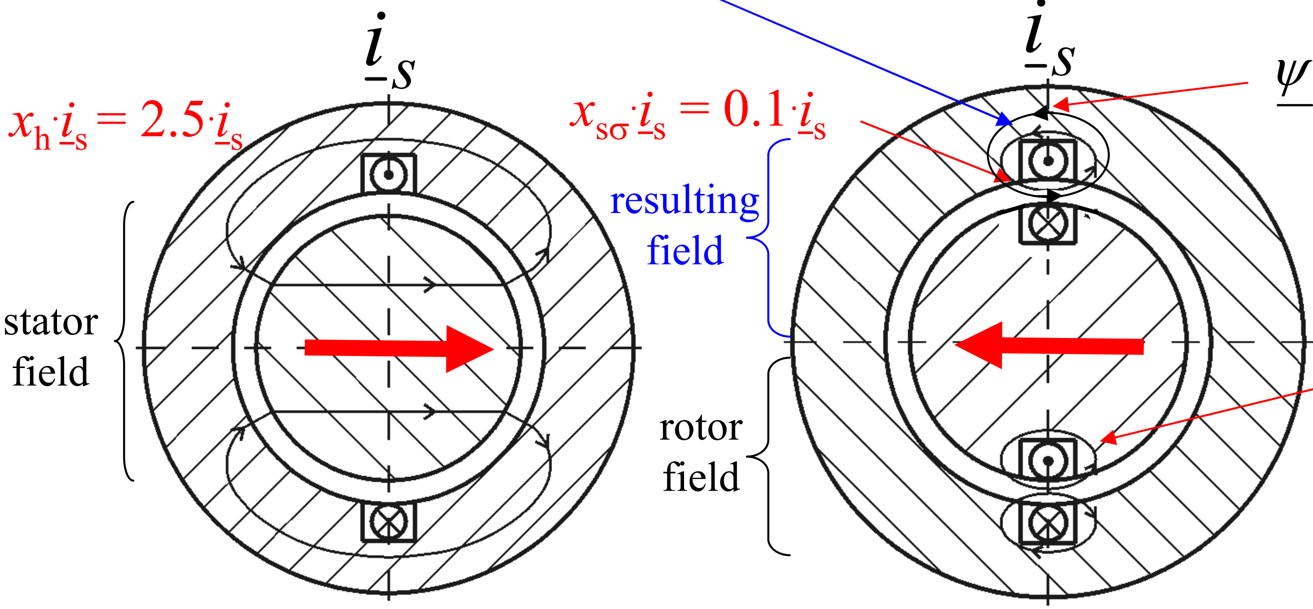
$$\underline{i}'_r = -(x_h / x'_r) \cdot \underline{i}_s$$

$x_h = 2.5,\ x_{s\sigma} = 0.1,\ x'_{r\sigma} = 0.08 : \underline{i}'_r = -0.969 \cdot \underline{i}_s$

$$0 = x_h \cdot \underline{i}_s + x_h \cdot \underline{i}'_r + x'_{r\sigma} \cdot \underline{i}'_r$$

$$0 = 2.5 \cdot \underline{i}_s - 2.423 \cdot \underline{i}_s - 0.077 \cdot \underline{i}_s$$

Also the rotor leakage flux linkage is due to the rotor shielding effect linked with the stator winding!



$$\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s = 0.177 \cdot \underline{i}_s \approx$$

$$\approx (x_{s\sigma} + x'_{r\sigma}) \cdot \underline{i}_s = 0.18 \cdot \underline{i}_s$$

$$x_h \cdot \underline{i}'_r + x'_{r\sigma} \cdot \underline{i}'_r = $$

$$= (2.423 + 0.077) \cdot \underline{i}_s = 2.5 \cdot \underline{i}_s$$

Summary:
Dynamic flux linkage equations

- Stator and rotor current space vectors $\underline{i}_s$, $\underline{i}'_r$ excite resulting air gap flux linkage vector $\underline{\psi}_h$
- Flux linkage equations independent of reference frame
- Separation of main and stray flux $\underline{\psi}_h$, $\underline{\psi}_\sigma$ possible
- Physical separation of stator and rotor stray flux linkage $\underline{\psi}_{s\sigma}$, $\underline{\psi}'_{r\sigma}$ by measurement not possible
- Rotor total flux linkage $\underline{\psi}'_{r\sigma}$ decreases with increasing slip due to rotor short-circuit
- Saturation may be introduced by current-depending inductances $x_h(i_m)$

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Torque equation

- **Introduction of flux linkage and current:**

Amplitude of flux linkage

$$\hat{\Psi}_h = N_s \cdot k_{w1} \cdot \frac{2}{\pi} \tau_p l \cdot B_\delta$$

Amplitude of current loading:

$$\hat{A}_s = \sqrt{2} \cdot k_{w1} \cdot A_s \quad \left(A_s = \frac{2 \cdot m_s \cdot N_s \cdot I_s}{2p \cdot \tau_p} \right)$$

$$M_e = \frac{(p\tau_p)^2}{\pi} \cdot l \cdot \hat{A}_s \cdot B_\delta \cdot \cos\varphi_\delta \quad \Rightarrow \overset{m_s = 3}{\Rightarrow} \Rightarrow \quad \boxed{M_e = \frac{3}{2} \cdot p \cdot \hat{I}_s \cdot \hat{\Psi}_h \cdot \cos\varphi_\delta}$$

- **Per unit torque equation:**

$$m_e(\tau) = \frac{M_e(t)}{M_B} = \frac{\omega_N / p}{3 \cdot U_{N,ph} \cdot I_{N,ph}} \cdot \frac{3}{\sqrt{2} \cdot \sqrt{2}} \cdot p \cdot \hat{I}_s(t) \cdot \hat{\Psi}_h(t) \cdot \cos\varphi_\delta(t) = i_s(\tau) \cdot \psi_h(\tau) \cdot \cos\varphi_\delta(\tau)$$

$$\boxed{m_e(\tau) = \frac{M_e(t)}{M_B} = i_s \cdot \psi_h \cdot \cos\varphi_\delta = i_{s\perp}(\tau) \cdot \psi_h(\tau)}$$

Torque is product of main flux linkage vector and *orthogonal* component of current space vector !

Per unit torque equation

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Example:

Stator current loading and resulting stator & rotor field

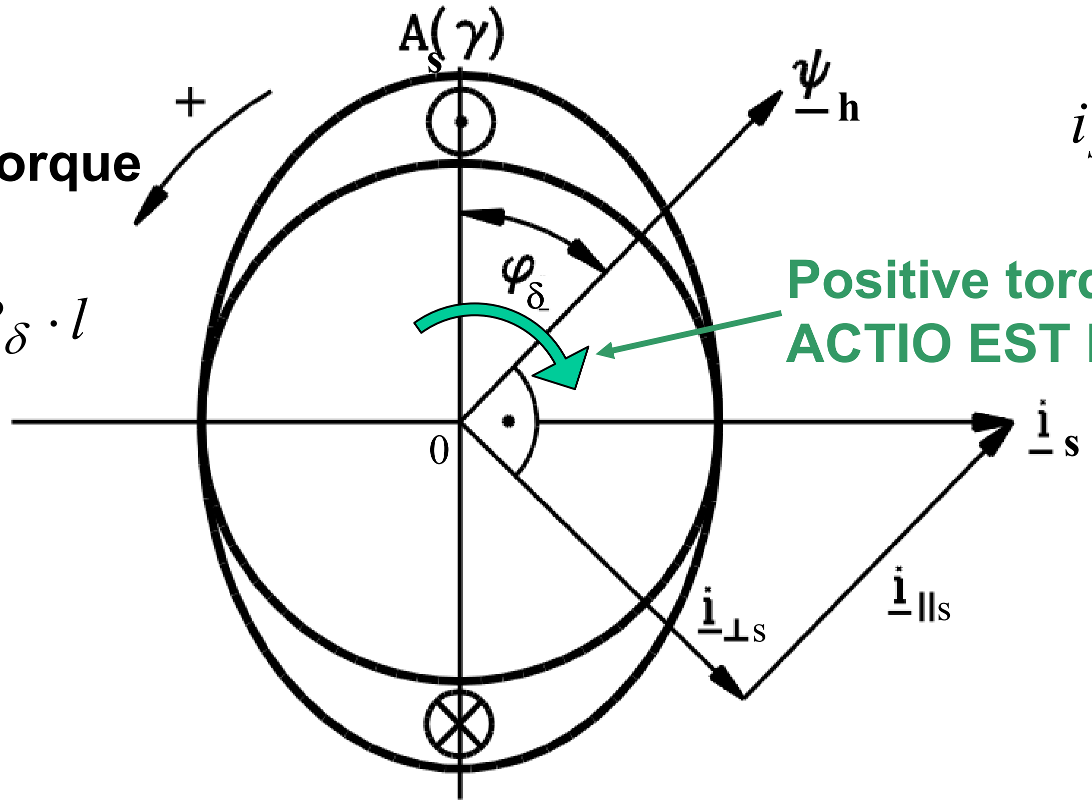
$$\boxed{m_e = \frac{M_e}{M_B} = i_s \cdot \psi_h \cdot \cos\varphi_\delta = i_{s\perp} \cdot \psi_h}$$

Positive torque on stator

$$F_s = I_s \cdot B_\delta \cdot l$$

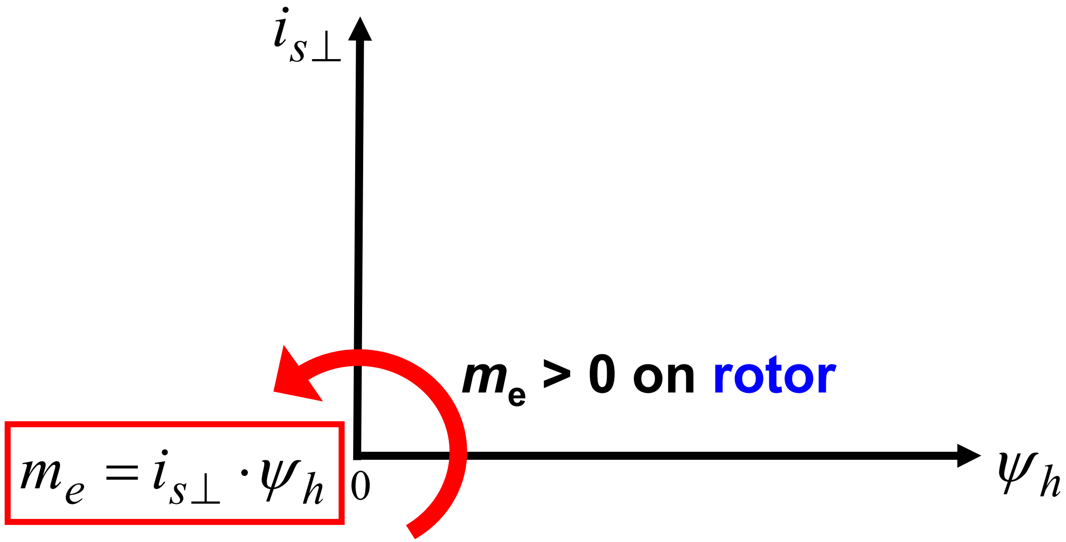
$$i_{s\perp} = i_s \cdot \cos\varphi_\delta$$

Positive torque on rotor = ACTIO EST REACTIO



7. Dynamics of induction machines

Per Rotor torque direction with stator current



- Positive counting of rotor torque in counter-clockwise sense!

  (Mathematical positive counting sense!)

- Flux (linkage) space vector turns into direction of current space vector

7. Dynamics of induction machines
Orthogonal vector components define torque

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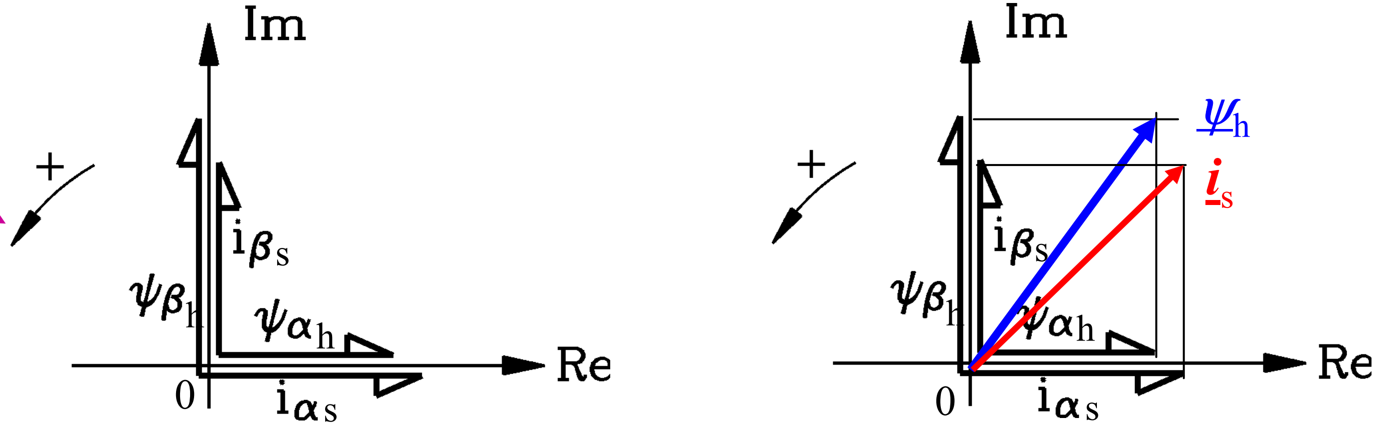
- **Introduction of vector product:** $m_e = i_{s\perp} \cdot \psi_h = i_{s\beta} \cdot \psi_{h\alpha} - i_{s\alpha} \cdot \psi_{h\beta} = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\}$

- If **stator** current space vector component is 90° **leading** to **flux linkage** space vector $\Rightarrow$ **torque on rotor** is **positive**.

Positive direction of torque on ROTOR



$$m_e = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\} = \mathrm{Im}\left\{(i_{s\alpha} + j \cdot i_{s\beta}) \cdot (\psi_{h\alpha} + j \cdot \psi_{h\beta})^*\right\} = \mathrm{Im}\left\{(i_{s\alpha} + j \cdot i_{s\beta}) \cdot (\psi_{h\alpha} - j \cdot \psi_{h\beta})\right\} =$$

$$= \mathrm{Im}\left\{i_{s\alpha} \cdot \psi_{h\alpha} - j \cdot i_{s\alpha} \cdot \psi_{h\beta} + j \cdot i_{s\beta} \cdot \psi_{h\alpha} + i_{s\beta} \cdot \psi_{h\beta}\right\} = i_{s\beta} \cdot \psi_{h\alpha} - i_{s\alpha} \cdot \psi_{h\beta}$$

7. Dynamics of induction machines
Different formulations for the torque (1)

$$m_e = i_{s\perp} \cdot \psi_h = i_{s\beta} \cdot \psi_{h\alpha} - i_{s\alpha} \cdot \psi_{h\beta} = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\}$$

- Stator stray flux does not generate torque !

$$m_e = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_s^*\right\}$$

Proof: $m_e = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot \left(x_h \underline{i}_s^* + x_h \underline{i}_r'^*\right)\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot \left(x_{s\sigma} \underline{i}_s^* + x_h \underline{i}_s^* + x_h \underline{i}_r'^*\right)\right\} =$

$$= \mathrm{Im}\left\{\underline{i}_s \cdot \left(\underline{\psi}_{s\sigma} + \underline{\psi}_h\right)^*\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_s^*\right\}$$

- Stator flux does not generate torque with stator current !

$$m_e = \mathrm{Im}\left\{\underline{i}_s \cdot \underline{\psi}_h^*\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot \left(x_h \underline{i}_s^* + x_h \underline{i}_r'^*\right)\right\} = \mathrm{Im}\left\{\underline{i}_s \cdot x_h \underline{i}_r'^*\right\}$$

- Note: $\underline{z} = a + j \cdot b \Rightarrow \underline{z}^* = a - j \cdot b \Rightarrow \mathrm{Im}\{\underline{z}\} = b = -\mathrm{Im}\{\underline{z}^*\}$

$$m_e = \mathrm{Im}\left\{x_h \cdot \underline{i}_s \cdot \underline{i}_r'^*\right\} = -\mathrm{Im}\left\{x_h \cdot \underline{i}_s^* \cdot \underline{i}_r'\right\}$$

7. Dynamics of induction machines
Different formulations for the torque (2)

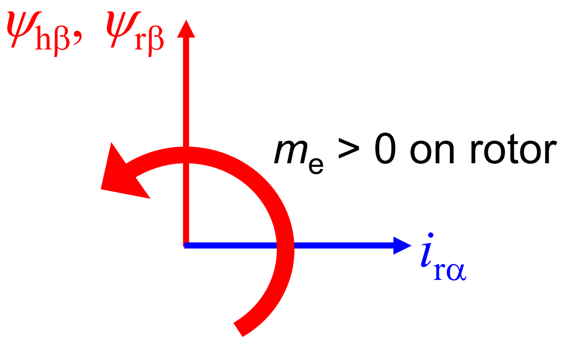
$$m_e = i_{s\perp} \cdot \psi_h = i_{s\beta} \cdot \psi_{h\alpha} - i_{s\alpha} \cdot \psi_{h\beta} = \mathrm{Im}\left\{ \underline{i}_s \cdot \underline{\psi}_h^* \right\}$$

$$m_e = i_{s\beta} \cdot \psi_{s\alpha} - i_{s\alpha} \cdot \psi_{s\beta} = \mathrm{Im}\left\{ \underline{i}_s \cdot \underline{\psi}_s^* \right\}$$

- Other formulation for torque with rotor flux linkage !

$$m_e = -\mathrm{Im}\left\{ x_h \cdot \underline{i}_s^* \cdot \underline{i}'_r \right\} = -\mathrm{Im}\left\{ x_h \cdot (\underline{i}_s^* + \underline{i}'^*_r) \cdot \underline{i}'_r \right\} = -\mathrm{Im}\left\{ \left(x_h \cdot (\underline{i}_s^* + \underline{i}'^*_r) + x'_{r\sigma} \cdot \underline{i}'^*_r \right) \cdot \underline{i}'_r \right\}$$

$$m_e = -\mathrm{Im}\left\{ \underline{i}'_r \cdot \underline{\psi}_h^* \right\} = -\mathrm{Im}\left\{ \underline{i}'_r \cdot \underline{\psi}'^*_r \right\} \Rightarrow \boxed{m_e = \psi'_{r\beta} \cdot i'_{r\alpha} - \psi'_{r\alpha} \cdot i'_{r\beta}}$$



- If **rotor** current space vector component is 90° **lagging** to **flux linkage** space vector $\Rightarrow$ **torque on rotor** is **positive**.

7. Dynamics of induction machines
Different formulations for the torque (3)

- **Torque formulation with rotor flux linkage and stator current (& vice versa)!**

$$m_e = \mathrm{Im}\left\{\underline{i}_s \cdot x_h \underline{i}'^*_r\right\} = \mathrm{Im}\left\{\frac{\underline{\psi}_s - x_h \underline{i}'_r}{x_s} \cdot x_h \underline{i}'^*_r\right\} = \mathrm{Im}\left\{\frac{x_h}{x_s} \cdot \underline{\psi}_s \cdot \underline{i}'^*_r\right\} = -\mathrm{Im}\left\{\frac{x_h}{x_s} \cdot \underline{\psi}^*_s \cdot \underline{i}'_r\right\}$$

$$m_e = \mathrm{Im}\left\{x_h \underline{i}_s \cdot \underline{i}'^*_r\right\} = \mathrm{Im}\left\{x_h \underline{i}_s \cdot \left(\frac{\underline{\psi}'_r - x_h \underline{i}_s}{x'_r}\right)^*\right\} = \mathrm{Im}\left\{\frac{x_h}{x'_r} \cdot \underline{\psi}'^*_r \cdot \underline{i}_s\right\} = -\mathrm{Im}\left\{\frac{x_h}{x'_r} \cdot \underline{\psi}'_r \cdot \underline{i}^*_s\right\}$$

- **Torque equation <u>independent</u> of reference frame!**

<u>Example:</u> **(s) $\rightarrow$ (K)**

$$\underline{\psi}_{s(K)} = \underline{\psi}_{s(s)} \cdot e^{-j\delta}, \quad \underline{i}'_{r(K)} = \underline{i}'_{r(s)} \cdot e^{-j\delta}, \quad \underline{i}'^*_{r(K)} = \underline{i}'^*_{r(s)} \cdot e^{j\delta}$$

$$m_e = \mathrm{Im}\left\{\frac{x_h}{x_s} \cdot \underline{\psi}_{s(s)} \cdot \underline{i}'^*_{r(s)}\right\} = \mathrm{Im}\left\{\frac{x_h}{x_s} \cdot \underline{\psi}_{s(K)} \cdot e^{j\cdot\delta} \cdot \underline{i}'^*_{r(K)} \cdot e^{-j\cdot\delta}\right\} = \mathrm{Im}\left\{\frac{x_h}{x_s} \cdot \underline{\psi}_{s(K)} \cdot \underline{i}'^*_{r(K)}\right\}$$

Summary:
Torque equation

- Orthogonal components of flux and current space vector yield torque
- Different formulations of torque with stator or rotor quantities
- Torque equation independent of reference frame
- Stray flux does not generate torque

7. Dynamics of induction machines

Dynamic equations in stator reference frame

- Set of dynamic equations
- Components
- In stator reference frame
- α - β -system

VOLTAGE: 4 equations

FLUX LINKAGE: 4 equations

TORQUE: 1 equation

9 equations In TOTAL !

$$u_{s\alpha} = r_s \cdot i_{s\alpha} + d\psi_{s\alpha} / d\tau$$

$$u_{s\beta} = r_s \cdot i_{s\beta} + d\psi_{s\beta} / d\tau$$

$$0 = r'_r \cdot i'_{r\alpha} + d\psi'_{r\alpha} / d\tau + \omega_m \cdot \psi'_{r\beta}$$

$$0 = r'_r \cdot i'_{r\beta} + d\psi'_{r\beta} / d\tau - \omega_m \cdot \psi'_{r\alpha}$$

$$\psi_{s\alpha} = x_s \cdot i_{s\alpha} + x_h \cdot i'_{r\alpha}$$

$$\psi_{s\beta} = x_s \cdot i_{s\beta} + x_h \cdot i'_{r\beta}$$

$$\psi'_{r\alpha} = x_h \cdot i_{s\alpha} + x'_r \cdot i'_{r\alpha}$$

$$\psi'_{r\beta} = x_h \cdot i_{s\beta} + x'_r \cdot i'_{r\beta}$$

$$\tau_J \cdot \frac{d\omega_m}{d\tau} = (\psi'_{r\beta} \cdot i'_{r\alpha} - \psi'_{r\alpha} \cdot i'_{r\beta}) - m_s$$

7. Dynamics of induction machines

A different flux linkage formulation

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$$\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r = x_s \cdot \underline{i}_s + x_h \cdot \frac{\underline{\psi}'_r - x_h \cdot \underline{i}_s}{x'_r} \longleftarrow \underline{\psi}'_r = x_h \cdot \underline{i}_s + x'_r \cdot \underline{i}'_r$$

$$\underline{\psi}_s = x_s \cdot \underline{i}_s \cdot \left(1 - \frac{x_h^2}{x_s x'_r}\right) + \frac{x_h}{x'_r} \cdot \underline{\psi}'_r = \sigma \cdot x_s \cdot \underline{i}_s + \frac{x_h}{x'_r} \cdot \underline{\psi}'_r$$

$$\boxed{\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s + \frac{x_h}{x'_r} \cdot \underline{\psi}'_r}$$

In the same way:

$$\boxed{\underline{\psi}'_r = \sigma \cdot x'_r \cdot \underline{i}'_r + \frac{x_h}{x_s} \cdot \underline{\psi}_s}$$

Flux linkage formulation for MATLAB/Simulink model

Previous different formulation for flux linkage, useful for MATLAB/Simulink model, in components in the stator reference frame:

$$\psi_{s\alpha} = x_s \cdot i_{s\alpha} + x_h \cdot i'_{r\alpha} = \sigma \cdot x_s \cdot i_{s\alpha} + \frac{x_h}{x'_r} \cdot \psi'_{r\alpha}$$

$$\psi_{s\beta} = x_s \cdot i_{s\beta} + x_h \cdot i'_{r\beta} = \sigma \cdot x_s \cdot i_{s\beta} + \frac{x_h}{x'_r} \cdot \psi'_{r\beta}$$

$$\psi'_{r\alpha} = x_h \cdot i_{s\alpha} + x'_r \cdot i'_{r\alpha} = \sigma \cdot x'_r \cdot i'_{r\alpha} + \frac{x_h}{x_s} \cdot \psi_{s\alpha}$$

$$\psi'_{r\beta} = x_h \cdot i_{s\beta} + x'_r \cdot i'_{r\beta} = \sigma \cdot x'_r \cdot i'_{r\beta} + \frac{x_h}{x_s} \cdot \psi_{s\beta}$$

Dynamic equations in stator reference frame for MATLAB/Simulink model WITHOUT mechanical equation

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$$u_{s\alpha} = r_s \cdot i_{s\alpha} + d\psi_{s\alpha} / d\tau$$

$$u_{s\beta} = r_s \cdot i_{s\beta} + d\psi_{s\beta} / d\tau$$

$$0 = r'_r \cdot i'_{r\alpha} + d\psi'_{r\alpha} / d\tau + \omega_m \cdot \psi'_{r\beta}$$

$$0 = r'_r \cdot i'_{r\beta} + d\psi'_{r\beta} / d\tau - \omega_m \cdot \psi'_{r\alpha}$$

$$\psi_{s\alpha} = \sigma \cdot x_s \cdot i_{s\alpha} + \frac{x_h}{x'_r} \cdot \psi'_{r\alpha}$$

$$\psi_{s\beta} = \sigma \cdot x_s \cdot i_{s\beta} + \frac{x_h}{x'_r} \cdot \psi'_{r\beta}$$

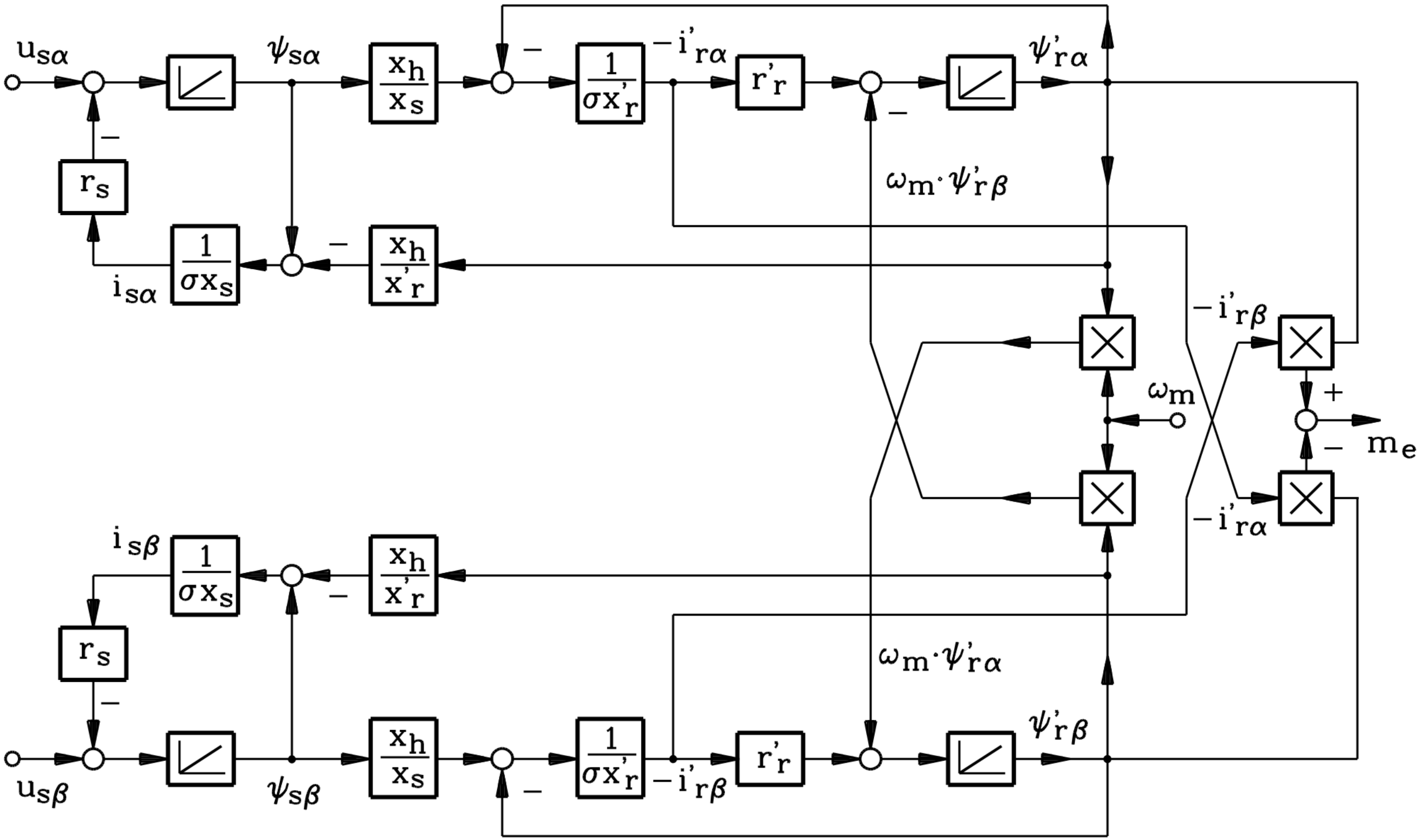
$$\psi'_{r\alpha} = \sigma \cdot x'_r \cdot i'_{r\alpha} + \frac{x_h}{x_s} \cdot \psi_{s\alpha}$$

$$\psi'_{r\beta} = \sigma \cdot x'_r \cdot i'_{r\beta} + \frac{x_h}{x_s} \cdot \psi_{s\beta}$$

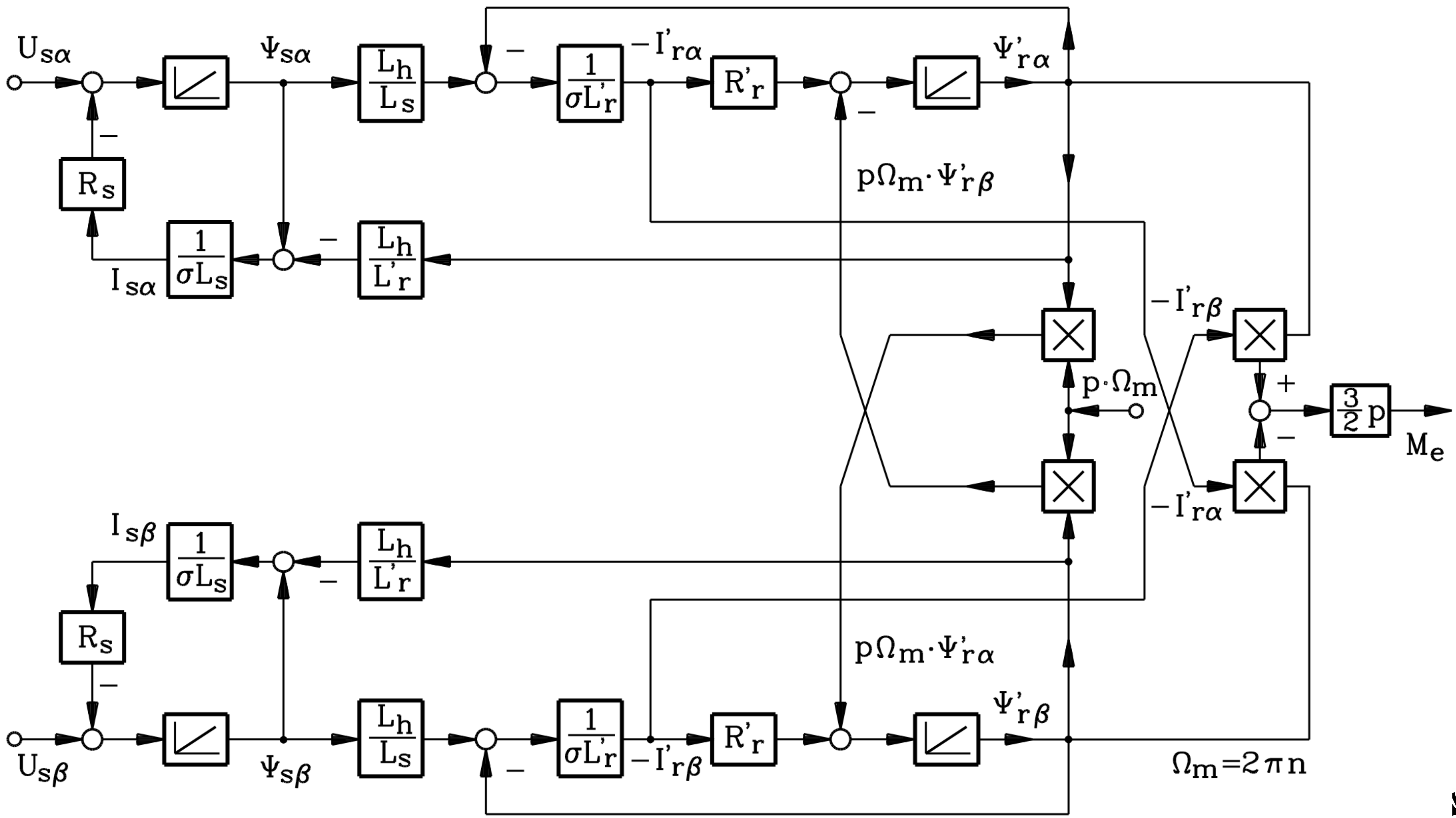
$$m_e = \psi'_{r\beta} \cdot i'_{r\alpha} - \psi'_{r\alpha} \cdot i'_{r\beta}$$

7. Dynamics of induction machines

Per unit formulation of equations in stator reference frame



7. Dynamics of induction machines
Formulation of equations in physical units



Summary:
Dynamic equations of induction machines in stator reference frame

- In the α-β-frame:
  4 voltage equations, four flux linkage equations, one mechanical equation
- Mechanical equation may be replaced by more detailed description:
  e.g. torsion oscillations (resonance frequencies)
- Time-step solution via RUNGE-KUTTA

Energy Converters – CAD and System Dynamics

7. Dynamics of induction machines

7. Dynamics of induction machines

Operation of induction machine at constant speed

- At constant speed **ONLY** voltage and flux linkage equations remain to be solved, **NO** torque equation !

- Equations are linear, so *Laplace* transformation is used to get transfer function „current from voltage“.

Example:

Switching voltage to an already running motor:

e.g. **Y-D-start-up:** Motor is running after Y-start up with no-load speed ω_{m0} = const.: then stator winding is switched in D to three-phase grid voltage system.

Grid voltage: $u_U(\tau) = u \cdot \cos(\tau),\ u_V(\tau) = u \cdot \cos(\tau - 2\pi/3),\ u_W(\tau) = u \cdot \cos(\tau - 4\pi/3)$

Space vector: $\underline{u}_s(\tau) = \frac{2}{3} \cdot \left(u_U(\tau) + \underline{a} \cdot u_V(\tau) + \underline{a}^2 \cdot u_W(\tau)\right) = u \cdot e^{j\tau}$

Laplace transform: $\breve{\underline{u}}_s = \frac{u}{s - j}$

7. Dynamics of induction machines

Laplace-transform of voltage & flux linkage equations

(in stator reference frame)

$$r_s \cdot \underline{i}_s(\tau) + \frac{d\underline{\psi}_s}{d\tau} = \underline{u}_s(\tau) \longrightarrow r_s \cdot \underline{\breve{i}}_s + s \cdot \underline{\breve{\psi}}_s = \underline{\breve{u}}_s + \underline{\psi}_{s0}$$

$$r'_r \cdot \underline{i}'_r(\tau) + \frac{d\underline{\psi}'_r}{d\tau} - j \cdot \omega_m \cdot \underline{\psi}'_r(\tau) = 0 \longrightarrow r'_r \cdot \underline{\breve{i}}'_r + (s - j \cdot \omega_m) \cdot \underline{\breve{\psi}}'_r = \underline{\psi}'_{r0}$$

$$\underline{\psi}_s(\tau) = x_s \cdot \underline{i}_s(\tau) + x_h \cdot \underline{i}'_r(\tau) \longrightarrow \underline{\breve{\psi}}_s = x_s \cdot \underline{\breve{i}}_s + x_h \cdot \underline{\breve{i}}'_r$$

$$\underline{\psi}'_r(\tau) = x_h \cdot \underline{i}_s(\tau) + x'_r \cdot \underline{i}'_r(\tau) \longrightarrow \underline{\breve{\psi}}'_r = x_h \cdot \underline{\breve{i}}_s + x'_r \cdot \underline{\breve{i}}'_r$$

Initial conditions: *Example*: Flux, current, voltage is zero ! $\underline{\psi}_{s0} = 0, \underline{\psi}'_{r0} = 0$

$$(r_s + s \cdot x_s) \cdot \underline{\breve{i}}_s + s \cdot x_h \cdot \underline{\breve{i}}'_r = \underline{\breve{u}}_s (+\underline{\psi}_{s0})$$

$$(s - j \cdot \omega_m) \cdot x_h \cdot \underline{\breve{i}}_s + (r'_r + (s - j \cdot \omega_m) \cdot x'_r) \cdot \underline{\breve{i}}'_r = 0 (+\underline{\psi}'_{r0})$$

Unknowns: Stator and rotor current space vectors $\underline{\breve{i}}_s(s), \underline{\breve{i}}'_r(s)$

7. Dynamics of induction machines

Solution of 2nd order linear algebraic equation system

$$\left(r_s + s\cdot x_s\right)\cdot \breve{\underline{i}}_s + s\cdot x_h \cdot \breve{\underline{i}}'_r = \breve{\underline{u}}_s$$

$$\left(s - j\cdot\omega_m\right)\cdot x_h \cdot \breve{\underline{i}}_s + \left(r'_r + \left(s - j\cdot\omega_m\right)\cdot x'_r\right)\cdot \breve{\underline{i}}'_r = 0$$

$$\breve{\underline{i}}_s = \frac{u}{s-j}\cdot\frac{r'_r + (s - j\omega_m)\cdot x'_r}{(r_s + s\cdot x_s)\cdot(r'_r + (s - j\omega_m)\cdot x'_r) - s\cdot x_h^2\cdot(s - j\omega_m)} =$$

$$= \frac{u}{s-j}\cdot\frac{r'_r + (s - j\omega_m)\cdot x'_r}{\sigma\cdot x_s\cdot x'_r\cdot\left(s^2 + s\cdot\left(\dfrac{r_s x'_r + x_s r'_r}{\sigma\cdot x_s\cdot x'_r} - j\omega_m\right) + \dfrac{r_s\cdot(r'_r - j\omega_m\cdot x'_r)}{\sigma\cdot x_s\cdot x'_r}\right)} =$$

$$= \frac{u}{s-j}\cdot\frac{r'_r + (s - j\omega_m)\cdot x'_r}{\sigma\cdot x_s\cdot x'_r\cdot\left(s - \underline{s}_a\right)\cdot\left(s - \underline{s}_b\right)}$$

Solutions for stator & rotor current vectors:

$$\boxed{\breve{\underline{i}}_s = \frac{u}{s-j}\cdot\frac{r'_r + (s - j\omega_m)\cdot x'_r}{\sigma\cdot x_s\cdot x'_r\cdot\left(s - \underline{s}_a\right)\cdot\left(s - \underline{s}_b\right)}}$$

$$\boxed{\breve{\underline{i}}'_r = -\frac{u}{s-j}\cdot\frac{x_h\cdot(s - j\omega_m)}{\sigma\cdot x_s\cdot x'_r\cdot\left(s - \underline{s}_a\right)\cdot\left(s - \underline{s}_b\right)}}$$

Second order characteristic polynomial

$$P_2(s) = s^2 + s \cdot \left(\frac{r_s \cdot x_r' + x_s \cdot r_r'}{\sigma \cdot x_s \cdot x_r'} - j\omega_m \right) + \frac{r_s \cdot (r_r' - j\omega_m x_r')}{\sigma x_s x_r'}$$

$$P_2(s) = s^2 + s \cdot (\alpha_s + \alpha_r - j\omega_m) + \alpha_s \cdot (\sigma \cdot \alpha_r - j\omega_m)$$

The polynomial describes the transient electrical behaviour of the induction machine ! $P_2(s) = (s - \underline{s}_a) \cdot (s - \underline{s}_b)$

We define:

- Stator and rotor short-circuit time constant: $\tau_{s\sigma} = \frac{1}{\alpha_s} = \frac{\sigma \cdot x_s}{r_s}, \quad \tau_{r\sigma} = \frac{1}{\alpha_r} = \frac{\sigma \cdot x_r'}{r_r'}$

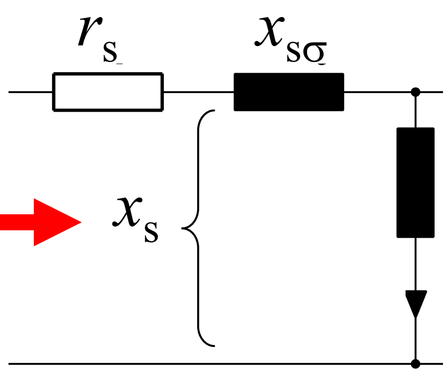
- Stator and rotor open-circuit time constant: $\tau_s = \frac{x_s}{r_s}, \quad \tau_r = \frac{x_r'}{r_r'}$

Stator open-circuit & short-circuit time constant

- Stator open-circuit time constant

- Rotor circuit interrupted

$$\tau_s = \frac{x_s}{r_s} \rightarrow x_s$$



ω_m = 1 (Slip: zero)

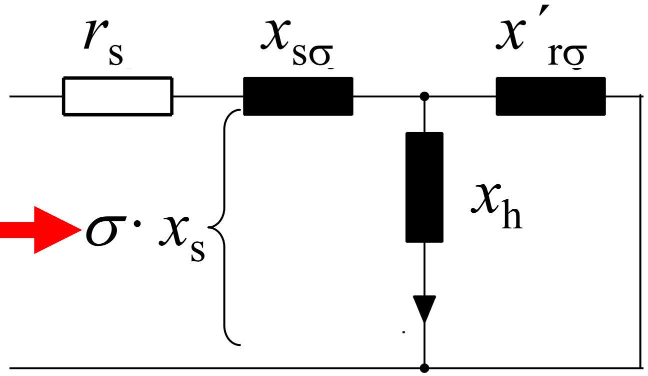
No rotor current = OPEN circuit

Change of total flux, including main flux

- Stator short-circuit time constant

- Rotor circuit has no resistance

$$\tau_{s\sigma} = \frac{\sigma \cdot x_s}{r_s} \rightarrow \sigma \cdot x_s$$



$\omega_m = \pm\infty$ (Slip: infinite)

$\omega_m = \pm\infty$, current similar to:
ω_m = 0 (Slip: Unity)

$$r_r' / Slip = 0 \approx r_r' / 1 = r_r' = 0.01 \ldots 0.05$$

"SHORT circuit" = STAND STILL

Change of stray flux

7. Dynamics of induction machines

Typical values for open-circuit & short-circuit time constant

$$\sigma \approx 0.1, \quad x_s \approx x_r' \approx 3, \quad r_s \approx r_r' \approx 0.06$$

- Stator and rotor short-circuit time constant = SHORT time constant

$$\tau_{s\sigma} = \frac{1}{\alpha_s} = \frac{\sigma \cdot x_s}{r_s} \approx \frac{0.1 \cdot 3}{0.06} = 5, \quad \tau_{r\sigma} = \frac{1}{\alpha_r} = \frac{\sigma \cdot x_r'}{r_r'} \approx \frac{0.1 \cdot 3}{0.06} = 5$$

$$\alpha_s \approx \frac{1}{5} = 0.2, \quad \alpha_r \approx \frac{1}{5} = 0.2 \qquad \Rightarrow \quad \alpha_s \approx \alpha_r = 0.2$$

- Stator and rotor open-circuit time constant = LONG time constant

$$\tau_s = \frac{1}{\sigma \cdot \alpha_s} = \frac{x_s}{r_s} \approx \frac{3}{0.06} = 50, \quad \tau_r = \frac{1}{\sigma \cdot \alpha_r} = \frac{x_r'}{r_r'} \approx \frac{3}{0.06} = 50$$

(**Note:** τ = 50 means 50/(2π) $\approx$ 8 periods at rated frequency!)

Complex linear transfer function of electrical performance

- Two poles (roots) $\underline{s}_a$, $\underline{s}_b$ in *Laplace* $\underline{s}$-plane, depending on SPEED !

$$s^2 + s \cdot \left(\frac{r_s x'_r + x_s r'_r}{\sigma x_s x'_r} - j\omega_m \right) + \frac{r_s (r'_r - j\omega_m x'_r)}{\sigma x_s x'_r} = s^2 + s \cdot (\alpha_s + \alpha_r - j\omega_m) + \alpha_s \cdot (\sigma \cdot \alpha_r - j\omega_m) = 0$$

$$s^2 + s \cdot \underline{p} + \underline{q} = 0 \quad \Rightarrow \quad \underline{s}_a = -\frac{\underline{p}}{2} - \sqrt{\left(-\frac{\underline{p}}{2}\right)^2 - \underline{q}}\,, \quad \underline{s}_b = -\frac{\underline{p}}{2} + \sqrt{\left(-\frac{\underline{p}}{2}\right)^2 - \underline{q}}$$

$$\underline{s}_{a,b} = \left(-\frac{\underline{p}}{2}\right) \cdot \left(1 \pm \sqrt{1 - \frac{4\underline{q}}{\underline{p}^2}}\right)$$

We discuss two special cases:

A) SPEED ZERO = SLIP 1: $\omega_m = 0$:

No natural oscillation frequency ($\underline{s}_a$, $\underline{s}_b$ = real numbers !)

B) SUFFICIENT HIGH SPEED: $\omega_m \neq 0$ (e.g.: synchronous rated speed: *Slip* = 0, $\omega_m = 1$)

$$Slip = 1 - \omega_m$$

$$(Slip = 1 - \frac{\Omega_m}{\omega_N / p})$$

Two short time constants and two natural oscillation frequencies !
$\underline{s}_a$, $\underline{s}_b$: complex numbers $\rightarrow$ simplified: Only $\underline{s}_a$ complex number!

Two real roots s_a, s_b of the characteristic polynomial at $\omega_m = 0$

Example:

Starting the induction machine at zero speed and blocked rotor (= switching three-phase grid voltage system to stator winding at $\omega_m = 0$)

Zero speed operation (stand still): $\boxed{\omega_m = 0}$

$$s^2 + s\cdot(\alpha_s + \alpha_r - j\omega_m) + \alpha_s\cdot(\sigma\cdot\alpha_r - j\omega_m) = s^2 + s\cdot(\alpha_s + \alpha_r) + \sigma\cdot\alpha_s\alpha_r = 0$$

$$s_{a,b} = -\frac{\alpha_s + \alpha_r}{2} \mp \sqrt{\underbrace{\left(-\frac{\alpha_s + \alpha_r}{2}\right)^2 - \sigma\alpha_s\alpha_r}_{0 \le \sigma \le 1:\ > 0}}$$

Worst case: No main flux linkage $\sigma = 1$

$$\sigma = 1:\ \left(\frac{\alpha_s + \alpha_r}{2}\right)^2 - \alpha_s\alpha_r = \left(\frac{\alpha_s - \alpha_r}{2}\right)^2 > 0$$

The two roots s_a, s_b are in any case at $\omega_m = 0$ real numbers!

7. Dynamics of induction machines

Two <u>real</u> roots s_a, s_b of the characteristic polynomial at $\omega_m = 0$

$$s^2 + s\cdot(\alpha_s + \alpha_r) + \sigma\cdot\alpha_s\alpha_r = 0$$

$$\left(\frac{4\cdot\sigma\cdot\alpha_s\cdot\alpha_r}{(\alpha_s+\alpha_r)^2} \approx \frac{4\cdot\sigma\cdot\alpha^2}{4\cdot\alpha^2} = \sigma \approx 0.1 << 1\right)$$

$$s_{a,b} = -\frac{\alpha_s+\alpha_r}{2}\cdot\left(1 \pm \sqrt{1-\frac{4\cdot\sigma\cdot\alpha_s\cdot\alpha_r}{(\alpha_s+\alpha_r)^2}}\right) \approx -\frac{\alpha_s+\alpha_r}{2}\cdot\left(1 \pm \left(1-\frac{2\cdot\sigma\cdot\alpha_s\cdot\alpha_r}{(\alpha_s+\alpha_r)^2}\right)\right)$$

As $\sigma \approx 0.1$ is small: $\sqrt{1-x} \approx 1 - x/2,\ x << 1$

$$\boxed{s_a = -(\alpha_s+\alpha_r) + \frac{\sigma\cdot\alpha_s\cdot\alpha_r}{\alpha_s+\alpha_r} \approx -(\alpha_s+\alpha_r)}$$

$$\boxed{s_b \approx -\frac{\sigma\cdot\alpha_s\cdot\alpha_r}{\alpha_s+\alpha_r} = -\frac{1}{\frac{1}{\sigma\cdot\alpha_r}+\frac{1}{\sigma\cdot\alpha_s}}}$$

$$\boxed{\tau_1 = -\frac{1}{s_a} \approx \frac{1}{1/\tau_{s\sigma}+1/\tau_{r\sigma}} = \frac{1}{\frac{r_s}{\sigma\cdot x_s}+\frac{r'_r}{\sigma\cdot x'_r}}}$$

$$\boxed{\tau_2 = -\frac{1}{s_b} \approx \tau_s + \tau_r = \frac{x_s}{r_s} + \frac{x'_r}{r'_r}}$$

short time constant
(change of stray flux)

long time constant
(change of main flux)

7. Dynamics of induction machines

Two complex roots $\underline{s}_a$, $\underline{s}_b$ of the characteristic polynomial at $\omega_m \neq 0$

At bigger speed: $\boxed{|\omega_m| > \alpha_s, \alpha_r \approx 0.2}$ **e.g.: synchronous speed ω_m = 1**

$$s^2 + s\cdot(\alpha_s + \alpha_r - j\omega_m) + \alpha_s\cdot(\sigma\cdot\alpha_r - j\omega_m) = 0$$

$$\underline{s}_{a,b} = -\frac{\alpha_s + \alpha_r - j\omega_m}{2}\cdot\left(1 \pm \sqrt{1 - \frac{4\alpha_s\cdot(\sigma\alpha_r - j\omega_m)}{(\alpha_s + \alpha_r - j\omega_m)^2}}\right) \approx -\frac{\alpha_s + \alpha_r - j\omega_m}{2}\cdot\left(1 \pm \sqrt{\left(1 - \frac{j4\alpha_s}{\omega_m}\right)}\right)$$

If $\sigma \approx 0.1$ α_s, $\alpha_r \approx 0.2$ are smaller than ω_m: $\sqrt{1-x} \approx 1 - x/2,\ x << 1$

$$\underline{s}_{a,b} \approx -\frac{\alpha_s + \alpha_r - j\omega_m}{2}\cdot\left(1 \pm (1 - \frac{j2\alpha_s}{\omega_m})\right)$$

$$\boxed{\underline{s}_a = -\alpha_r + j\cdot\left(\omega_m + \frac{\alpha_s\cdot(\alpha_s + \alpha_r)}{\omega_m}\right) \approx -\alpha_r + j\omega_m}$$

$$\boxed{\underline{s}_b = -\alpha_s - j\cdot\frac{(\alpha_s + \alpha_r)\cdot\alpha_s}{\omega_m} \approx -\alpha_s}$$

$$\boxed{\tau_1 \approx \frac{1}{\alpha_r} = \tau_{r\sigma},\ \tau_2 \approx \frac{1}{\alpha_s} = \tau_{s\sigma}}$$

Two short time constants (only stray flux changes)

$$\boxed{\omega_{d,1} \approx \omega_m,\ \omega_{d,2} \approx 0}$$

Two natural frequencies

Simplified electric time constants for $r_s = r'_r = r,\ x_s = x'_r = x$

A) At zero speed and low speed: No natural oscillation frequency (= "real numbers" !)

Short time constant: $\tau_1 \approx \dfrac{1}{\dfrac{r_s}{\sigma \cdot x_s} + \dfrac{r'_r}{\sigma \cdot x'_r}} = \dfrac{1}{\dfrac{2r}{\sigma \cdot x}}$ $\boxed{\tau_1 \approx \dfrac{\sigma \cdot x}{2r} = \tau_\sigma / 2}$

Long time constant: $\tau_2 \approx \dfrac{x_s}{r_s} + \dfrac{x'_r}{r'_r} = 2 \cdot \dfrac{x}{r}$ $\boxed{\tau_2 \approx 2 \cdot \dfrac{x}{r} = 2\tau}$

0 τ_1 $\tau_\sigma \approx$ 5 … 10 $\tau = x/r \approx$ 50 … 100 $\tau_2 \approx$ 100 … 200

B) High speed operation:

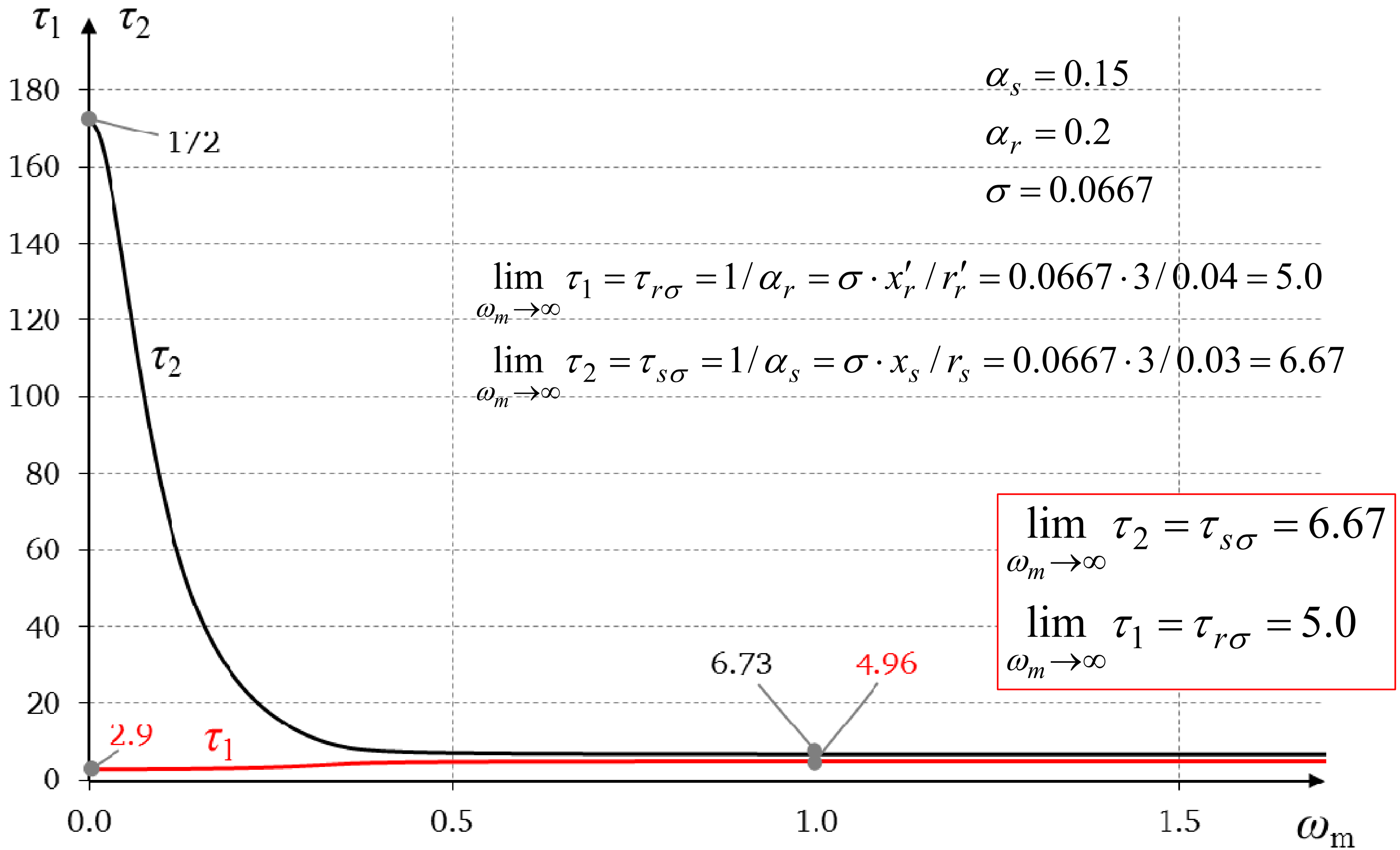
Two short time constants: $\tau_1 = \tau_{s\sigma}, \quad \tau_2 = \tau_{r\sigma}$ $\boxed{\tau_1 \approx \tau_2 \approx \tau_\sigma}$

Natural oscillation frequencies: $\omega_{d,1} \approx \omega_m \quad \omega_{d,2} \approx 0$

0 $\tau_1 = \tau_\sigma = \tau_2 =$ 5 ... 10

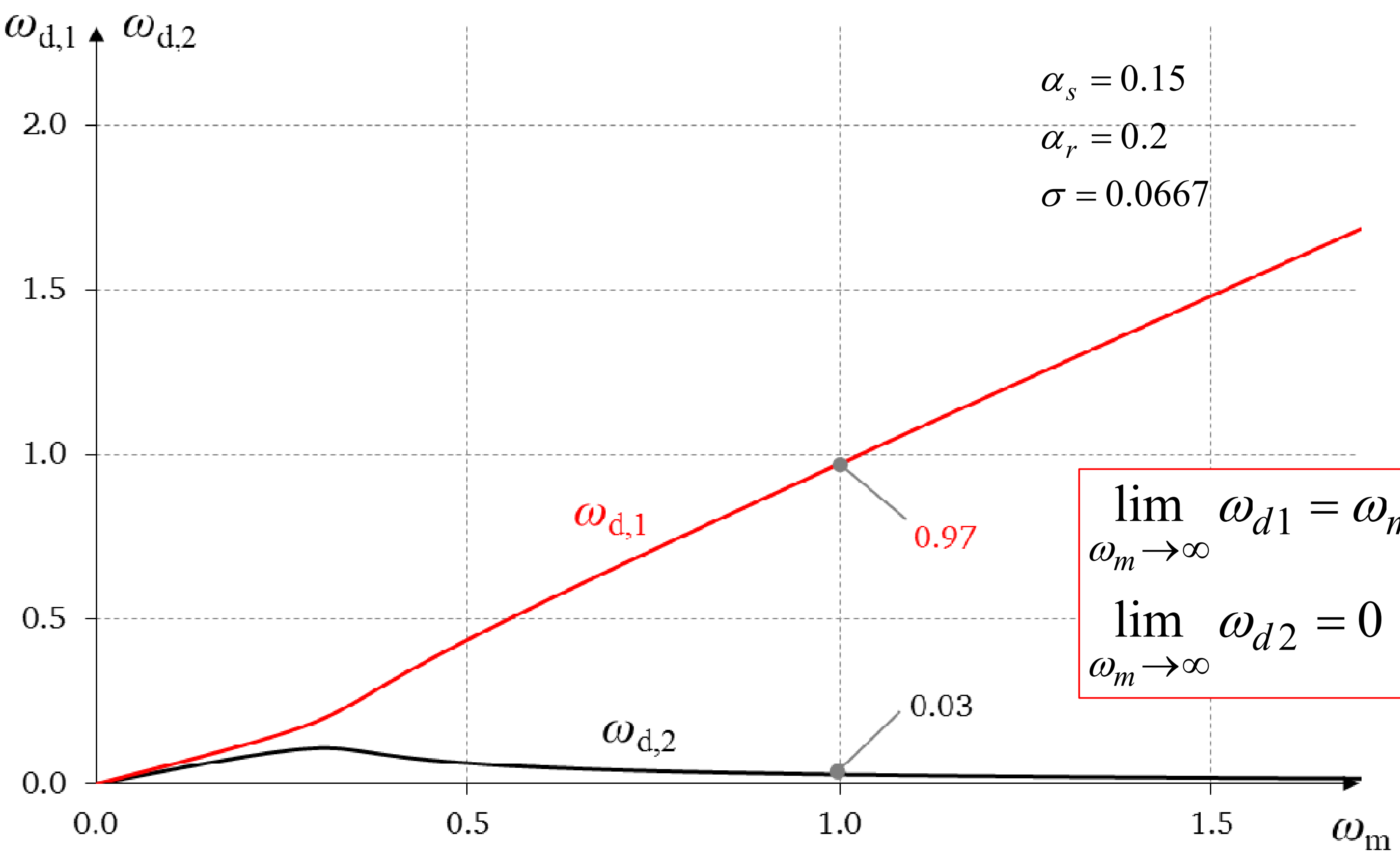
7. Dynamics of induction machines

Two IM electric time constants τ_1, τ_2 depend on speed ω_m



7. Dynamics of induction machines

Imaginary parts ω_{d1}, ω_{d2} of roots of electric transfer function



$$\lim_{\omega_m \to \infty} \omega_{d1} = \omega_m$$

$$\lim_{\omega_m \to \infty} \omega_{d2} = 0$$

Dynamic performance of induction machine at constant speed ω_m = const.

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- ***Induction machines*** *react with current to sudden change in voltage with TWO time constants τ_1, τ_2, because we have TWO coupled electric circuits (stator and rotor circuit).*
- *The time constants τ_1, τ_2 depend on speed ω_m!*
- *The phase windings U, V, W are coupled via the main flux and act as ONE winding system = ONE time constant per winding system!*
- *At not too low speed τ_1, τ_2 are nearly equal, being the short* ***stator and rotor short-circuit time constants*** *$\tau_{s\sigma}, \tau_{r\sigma}$.*

$\Rightarrow$ During rotation the induction machine´s main flux Φ_h remains also at sudden changes nearly constant; only the stray flux Φ_σ changes.

- *At low speed & stand still τ_1 is short and τ_2 is long as the sum of* ***stator and rotor open-circuit time constant*** *$\tau_2 = \tau_s + \tau_r$*

Compare: *DC machines:*

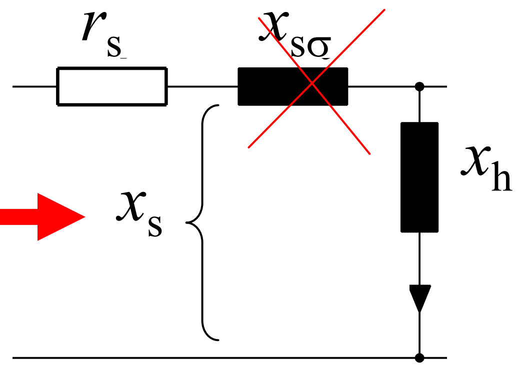
- *Only one short electric time constant due to one armature circuit: $T_a = L_a / R_a$*
- *Time constant T_a independent of speed n!*

7. Dynamics of induction machines

Dynamic Time-constants for change of main and stray flux

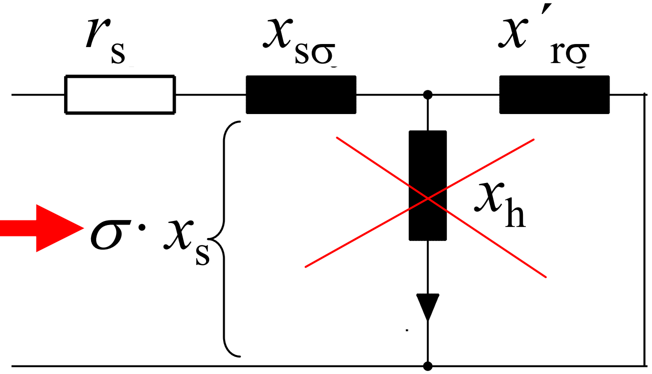
- Long **stator and rotor open-circuit time constant**: Change of main flux

$$\tau_s = \frac{x_s}{r_s} \rightarrow x_s$$



- Short **stator and rotor short-circuit time constant**:
  Change of stator and rotor stray flux

$$\tau_{s\sigma} = \frac{\sigma \cdot x_s}{r_s} \rightarrow \sigma \cdot x_s$$



7. Dynamics of induction machines

Example: Switching stator winding to grid at ω_m = const.

Grid voltage: $u_U(\tau) = u \cdot \cos(\tau),\ u_V(\tau) = u \cdot \cos(\tau - 2\pi/3),\ u_W(\tau) = u \cdot \cos(\tau - 4\pi/3)$

Space vector: $\underline{u}_s(\tau) = \frac{2}{3} \cdot \left(u_U(\tau) + \underline{a} \cdot u_V(\tau) + \underline{a}^2 \cdot u_W(\tau) \right) = u \cdot e^{j\tau} \rightarrow \breve{\underline{u}}_s = \frac{u}{s - j}$

Solution for stator current vector:

$$\boxed{\breve{\underline{i}}_s = \frac{u}{s - j} \cdot \frac{r_r' + (s - j\omega_m) \cdot x_r'}{\sigma \cdot x_s \cdot x_r' \cdot (s - \underline{s}_a) \cdot (s - \underline{s}_b)}}$$

Laplace transform **current space vector :** $\breve{\underline{i}}_s = \frac{\underline{A}}{s - j} + \frac{\underline{B}}{s - \underline{s}_a} + \frac{\underline{C}}{s - \underline{s}_b}$

Inverse transform **current space vector :** $\underline{i}_s(\tau) = \underline{A} \cdot e^{j \cdot \tau} + \underline{B} \cdot e^{\underline{s}_a \cdot \tau} + \underline{C} \cdot e^{\underline{s}_b \cdot \tau}$

Laplace transform: $\breve{\underline{u}}_s = \frac{u}{s - j}$

7. Dynamics of induction machines

Two **complex** roots s_a, s_b of characteristic polynomial at $|\omega_m| > 0.2$

$$\underline{s}_a = \mathrm{Re}\{\underline{s}_a\} + j\cdot\mathrm{Im}\{\underline{s}_a\} = -\frac{1}{\tau_a} + j\cdot\omega_{d,a} = -\frac{1}{\tau_1} + j\cdot\omega_{d,1} \approx -\alpha_r + j\cdot\omega_m = \frac{1}{\tau_{r\sigma}} + j\cdot\omega_m$$

$$\underline{s}_b = \mathrm{Re}\{\underline{s}_b\} + j\cdot\mathrm{Im}\{\underline{s}_b\} = -\frac{1}{\tau_b} + j\cdot\omega_{d,b} = -\frac{1}{\tau_2} + j\cdot\omega_{d,2} \approx -\alpha_s + j\cdot 0 = \frac{1}{\tau_{s\sigma}}$$

$$\boxed{\tau_1 \approx \frac{1}{\alpha_r} = \tau_{r\sigma},\ \tau_2 \approx \frac{1}{\alpha_s} = \tau_{s\sigma}}$$

Two short time constants (only stray flux changes)

$$\boxed{\omega_{d,1} \approx \omega_m,\ \omega_{d,2} \approx 0}$$

Two natural frequencies

$$\underline{B}\cdot e^{\underline{s}_a\cdot\tau} = \underline{B}\cdot e^{\mathrm{Re}\{\underline{s}_a\}\tau}\cdot e^{j\cdot\mathrm{Im}\{\underline{s}_a\}\tau} = \underline{B}\cdot e^{-\frac{\tau}{\tau_a}}\cdot e^{j\cdot\omega_{d,a}\cdot\tau} = \underline{B}\cdot e^{-\frac{\tau}{\tau_1}}\cdot e^{j\cdot\omega_{d,1}\cdot\tau}$$

$$\underline{C}\cdot e^{\underline{s}_b\cdot\tau} = \underline{C}\cdot e^{\mathrm{Re}\{\underline{s}_b\}\tau}\cdot e^{j\cdot\mathrm{Im}\{\underline{s}_b\}\tau} = \underline{C}\cdot e^{-\frac{\tau}{\tau_b}}\cdot e^{j\cdot\omega_{d,b}\cdot\tau} = \underline{C}\cdot e^{-\frac{\tau}{\tau_2}}\cdot e^{j\cdot\omega_{d,2}\cdot\tau}$$

$$\boxed{\underline{B}\cdot e^{\underline{s}_a\cdot\tau} \approx \underline{B}\cdot e^{-\frac{\tau}{\tau_{r\sigma}}}\cdot e^{j\cdot\omega_m\cdot\tau}} \qquad \boxed{\underline{C}\cdot e^{\underline{s}_b\cdot\tau} \approx \underline{C}\cdot e^{-\frac{\tau}{\tau_{s\sigma}}}}$$

7. Dynamics of induction machines

Inverse *Laplace* transformation

Inverse transform **current space vector :** $\underline{i}_s(\tau) = \underline{A} \cdot e^{j\cdot\tau} + \underline{B} \cdot e^{\underline{s}_a \cdot\tau} + \underline{C} \cdot e^{\underline{s}_b \cdot\tau}$

a) **Homogeneous part** of solution: $\underline{i}_{s,h}(\tau) = \underline{B} \cdot e^{\underline{s}_a \cdot\tau} + \underline{C} \cdot e^{\underline{s}_b \cdot\tau}$

b) **Particular** solution: $\underline{i}_{s,p}(\tau) = \underline{A} \cdot e^{j\cdot\tau}$

Stator and rotor current change with **two time constants**

$$\boxed{\tau_a = -\frac{1}{\mathrm{Re}(\underline{s}_a)} = \tau_1,\ \tau_b = -\frac{1}{\mathrm{Re}(\underline{s}_b)} = \tau_2}$$

having **two natural oscillation frequencies**

$$\boxed{\omega_{d,a} = \mathrm{Im}(\underline{s}_a) = \omega_{d,1},\ \omega_{d,b} = \mathrm{Im}(\underline{s}_b) = \omega_{d,2}}$$

$$\underline{i}_{s,h}(\tau) = \underline{B} \cdot e^{-\tau/\tau_1} \cdot e^{j\omega_{d,1}\tau} + \underline{C} \cdot e^{-\tau/\tau_2} \cdot e^{j\omega_{d,2}\tau}$$

$\tau_1, \tau_2, \omega_{d,1}, \omega_{d,2}$ **depend** on resistances r_s, r_r', inductances x_s, x_r', x_h

AND **on rotor speed** ω_m

Homogeneous solution = transient part

$$\underline{i}_{s,h}(\tau) = \underline{B} \cdot e^{-\tau/\tau_1} \cdot e^{j\omega_{d,1}\tau} + \underline{C} \cdot e^{-\tau/\tau_2} \cdot e^{j\omega_{d,2}\tau}$$

$$\left.\begin{array}{l} \tau_1 \approx \tau_{r\sigma}, \tau_2 \approx \tau_{s\sigma} \\ \omega_{d,1} \approx \omega_m, \omega_{d,2} \approx 0 \end{array}\right\}$$

Caused by rotor DC flux **stator DC flux**

$$\underline{i}_{s,h}(\tau) \cong \underline{B} \cdot e^{-\tau/\tau_{r\sigma}} \cdot e^{j\omega_m\tau} + \underline{C} \cdot e^{-\tau/\tau_{s\sigma}}$$

Transient AC part **transient DC part**

Particular solution = steady-state part

$$\underline{i}_{s,p}(\tau) = \underline{A} \cdot e^{j\cdot\tau}$$

Rotating stator current space vector with constant amplitude according to impressed voltage space vector $\underline{u}_s(\tau) = u \cdot e^{j\tau}$

7. Dynamics of induction machines

Determination of constants $\underline{A}$, $\underline{B}$, $\underline{C}$ by *Heaviside´s* rule

- $$\breve{\underline{i}}_s = \frac{u}{s-j} \cdot \frac{r'_r + (s - j\omega_m)\cdot x'_r}{\sigma \cdot x_s \cdot x'_r \cdot (s - \underline{s}_a)\cdot(s - \underline{s}_b)}$$

- *If condition:* Order of numerator polynomial $Z(s)$ less than of denominator polynomial $N(s)$ ⇒

⇒ *Heaviside´s* rule:

$$\boxed{L^{-1}\left\{\frac{Z(s)}{N(s)}\right\} = L^{-1}\left\{\frac{Z(s)}{(s-\underline{s}_1)\cdot(s-\underline{s}_2)\cdot\ldots\cdot(s-\underline{s}_n)}\right\} = \sum_{i=1}^{n} \frac{Z(\underline{s}_i)}{\prod\limits_{k=1\ldots n \wedge k\neq i}(\underline{s}_i - \underline{s}_k)} \cdot e^{\underline{s}_i \cdot t}}$$

- $$\breve{\underline{i}}_s = \frac{\underline{A}}{s-j} + \frac{\underline{B}}{s-\underline{s}_a} + \frac{\underline{C}}{s-\underline{s}_b}$$

- So we get:

$$i = 1 : \underline{s}_1 = j : \underline{A} = \frac{u\cdot(r'_r + (j - j\omega_m)\cdot x'_r)}{\sigma \cdot x_s \cdot x'_r \cdot (j - \underline{s}_a)\cdot(j - \underline{s}_b)}$$

$$i = 2 : \underline{s}_2 = \underline{s}_a : \underline{B} = \frac{u\cdot(r'_r + (\underline{s}_a - j\omega_m)\cdot x'_r)}{\sigma \cdot x_s \cdot x'_r \cdot (\underline{s}_a - j)\cdot(\underline{s}_a - \underline{s}_b)}$$

$$i = 3 : \underline{s}_3 = \underline{s}_b : \underline{C} = \frac{u\cdot(r'_r + (\underline{s}_b - j\omega_m)\cdot x'_r)}{\sigma \cdot x_s \cdot x'_r \cdot (\underline{s}_b - j)\cdot(\underline{s}_b - \underline{s}_a)}$$

7. Dynamics of induction machines

Determination of non-damped solution: $r_s = r'_r = 0$

- $$\breve{\underline{i}}_s = \frac{u}{s-j} \cdot \frac{r'_r + (s - j\omega_m)\cdot x'_r}{\sigma \cdot x_s \cdot x'_r \cdot \left(s^2 + s \cdot \left(\frac{r_s x'_r + x_s r'_r}{\sigma \cdot x_s \cdot x'_r} - j\omega_m \right) + \frac{r_s \cdot (r'_r - j\omega_m \cdot x'_r)}{\sigma \cdot x_s \cdot x'_r} \right)}$$

$r_s = r'_r = 0$

$$\breve{\underline{i}}_s = \frac{u}{s-j} \cdot \frac{s - j\omega_m}{\sigma \cdot x_s \cdot \left(s^2 - s \cdot j\omega_m\right)} = \frac{u}{s-j} \cdot \frac{1}{\sigma \cdot x_s \cdot s} = \frac{u}{s-j} \cdot \frac{s - j\omega_m}{\sigma \cdot x_s \cdot (s - \underline{s}_a)\cdot(s - \underline{s}_b)}$$

$r_s = r'_r = 0$

- $$\underline{s}_{a,b} = -\frac{\alpha_s + \alpha_r - j\omega_m}{2} \cdot \left(1 \pm \sqrt{1 - \frac{4\alpha_s \cdot (\sigma\alpha_r - j\omega_m)}{(\alpha_s + \alpha_r - j\omega_m)^2}} \right) = \frac{j\omega_m}{2} \cdot \left(1 \pm \sqrt{1} \right) = \begin{cases} \underline{s}_a = j\omega_m \\ \underline{s}_b = 0 \end{cases}$$

7. Dynamics of induction machines

Non-damped transient solution is independent of speed ω_m

- **Simplification: Damping is neglected:** $r_s = 0, r_r' = 0$: Determination of roots $\underline{s}_a, \underline{s}_b$:

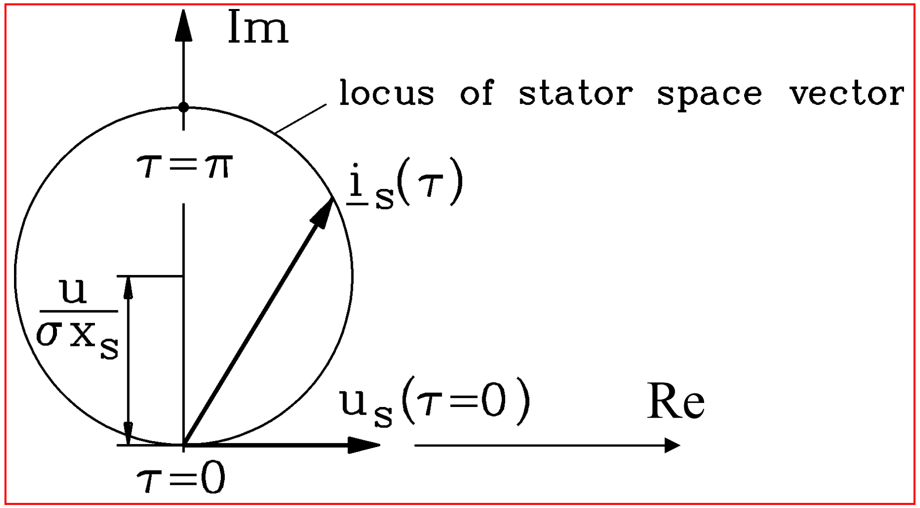
$$s^2 + s\cdot(-j\omega_m) = 0 \quad \rightarrow \quad \underline{s}_a = j\omega_m, \underline{s}_b = 0 \qquad \underline{A} = -\underline{C} = -ju/(\sigma x_s), \underline{B} = 0$$

$$\breve{\underline{i}}_s = \frac{u}{s-j}\cdot\frac{\cancel{(s-j\omega_m)}\cdot x_r'}{\sigma\cdot x_s\cdot x_r'\cdot\cancel{(s-j\omega_m)}\cdot(s-0)} = \frac{u}{\sigma\cdot x_s\cdot s(s-j)} = -\frac{\underline{A}}{s-j} + \frac{\underline{C}}{s} = -\frac{j\cdot u}{\sigma\cdot x_s}\cdot\left(\frac{1}{s-j} - \frac{1}{s}\right)$$

Inverse transformation: $\underline{\underline{\underline{i}_s(\tau) = \frac{j\cdot u}{\sigma\cdot x_s}\cdot\left(1 - e^{j\cdot\tau}\right)}}$

transient stationary

solution



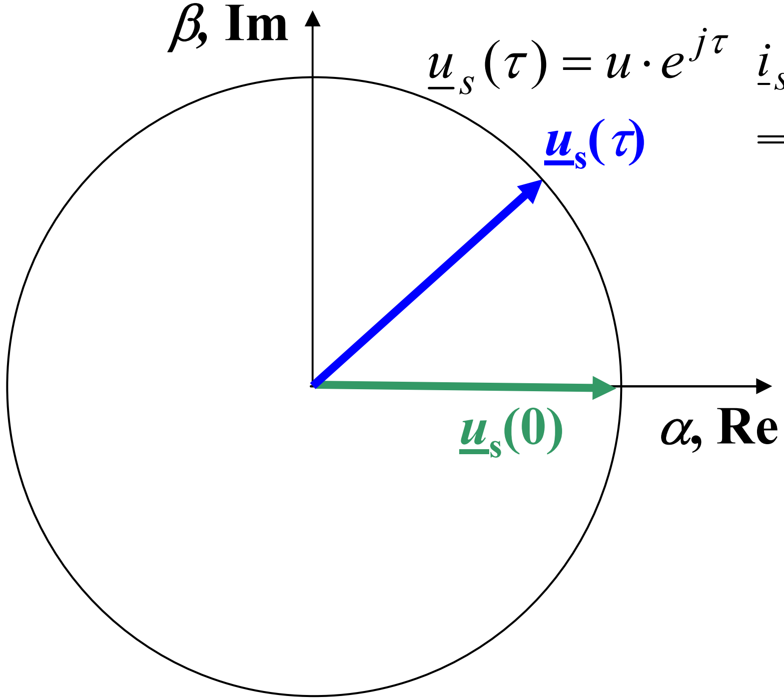
- **Time function of U phase current:**

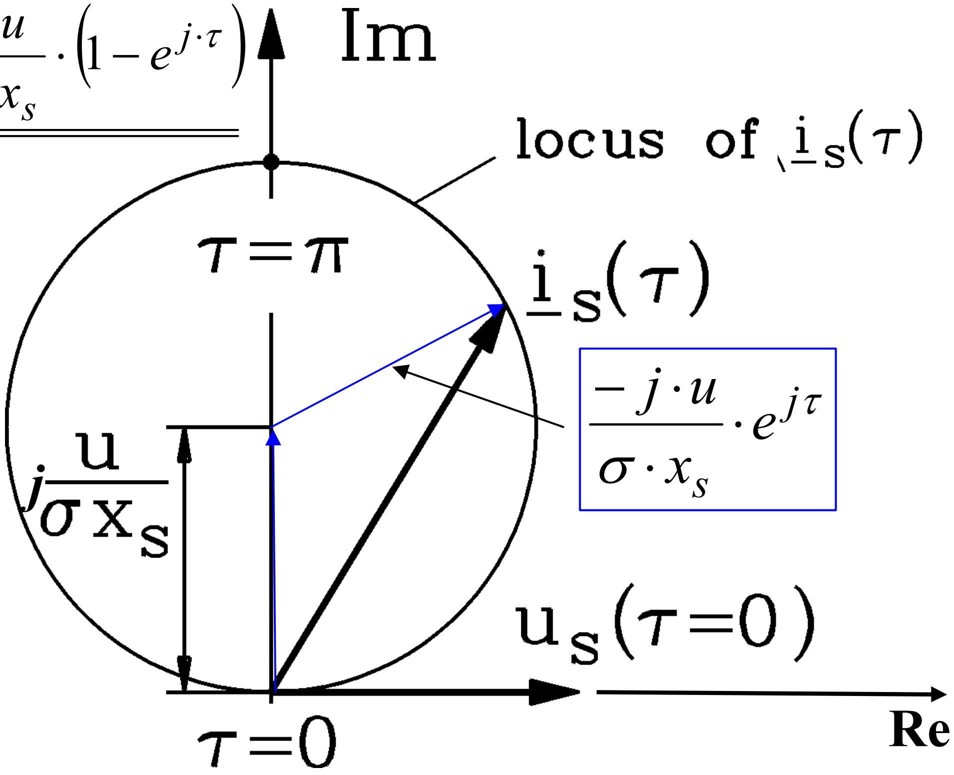
$$i_U(\tau) = \mathrm{Re}\{\underline{i}_s(\tau)\} = \mathrm{Re}\left\{\frac{j\cdot u}{\sigma\cdot x_s}\cdot\left(1-e^{j\cdot\tau}\right)\right\} = \frac{u}{\sigma\cdot x_s}\cdot\mathrm{Re}\{j - j\cdot(\cos\tau + j\sin\tau)\} = \underline{\underline{\frac{u}{\sigma\cdot x_s}\cdot\sin\tau}}$$

7. Dynamics of induction machines

Non-damped stator current: Initial condition: $\underline{u}_s(0) = u$

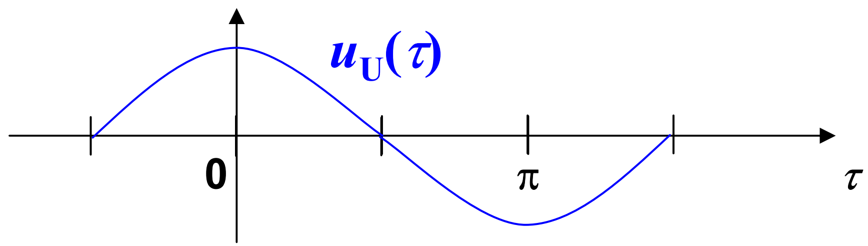
$$\underline{u}_s(\tau) = u \cdot e^{j\tau} \qquad \underline{i}_s(\tau) = \frac{j \cdot u}{\sigma \cdot x_s} \cdot \left(1 - e^{j\cdot\tau}\right)$$

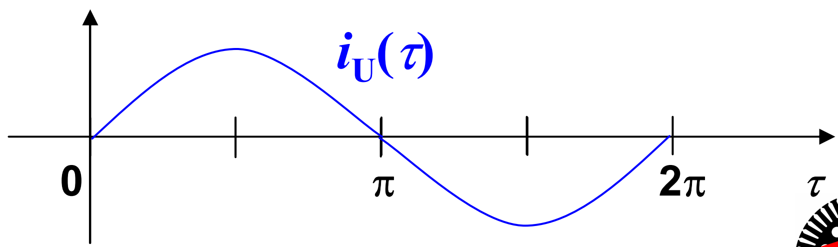




$$\frac{-j \cdot u}{\sigma \cdot x_s} \cdot e^{j\tau}$$

$$u_U(\tau) = \mathrm{Re}\{\underline{u}_s(\tau)\} = u \cdot \cos\tau$$

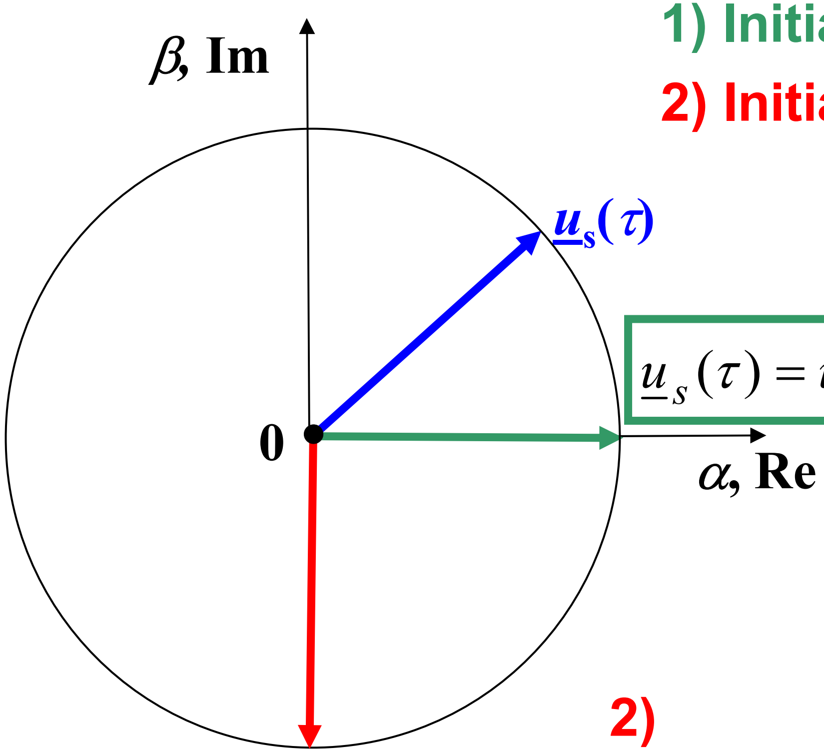




Two special initial conditions in phase U for voltage space vector

1) Initial condition „best case“: $u_s(0) = \pm u$

2) Initial condition „worst case“: $u_s(0) = 0$



$$\boxed{\underline{u}_s(\tau) = u \cdot e^{j\tau}}$$

1) Initial condition „best case“: $u_s(0) = u$

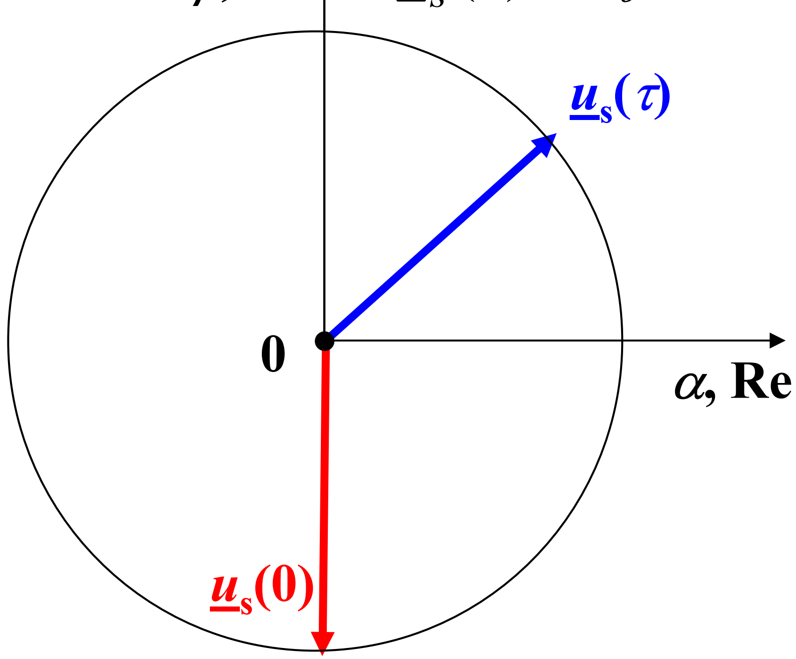
$$\boxed{\underline{u}_s(\tau) = -j \cdot u \cdot e^{j\tau}}$$

2) Initial condition „worst case“: $u_s(0) = 0$

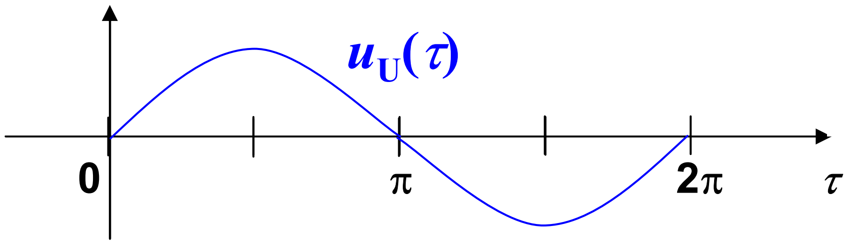
7. Dynamics of induction machines

Non-damped stator current: Initial condition: $\underline{u}_s(0) = 0$

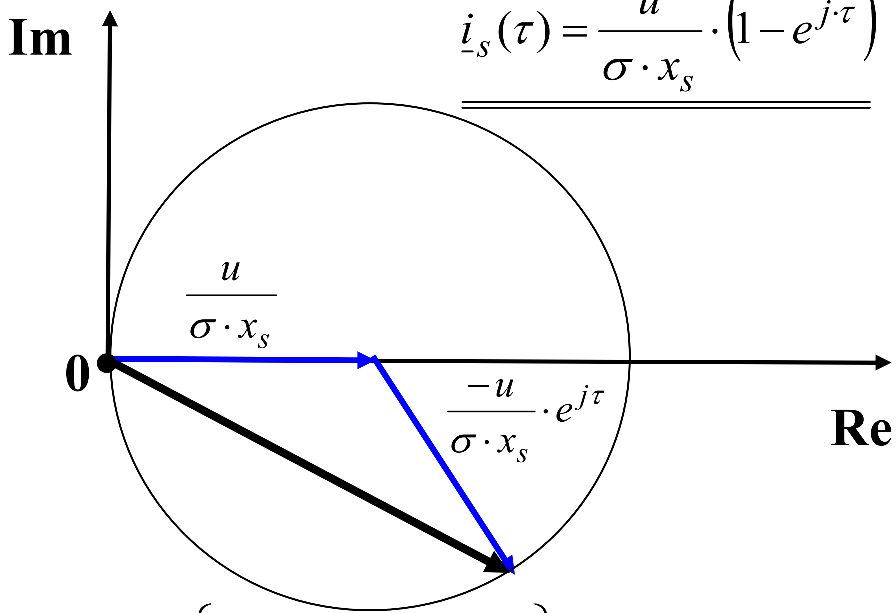
$$\underline{u}_s(\tau) = -j \cdot u \cdot e^{j\tau}$$



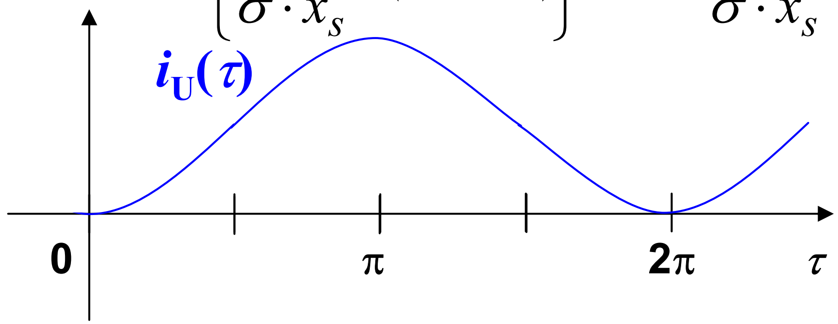
$$u_U(\tau) = \mathrm{Re}\{\underline{u}_s(\tau)\} = u \cdot \sin\tau$$



$$\underline{i}_s(\tau) = \frac{u}{\sigma \cdot x_s} \cdot \left(1 - e^{j\cdot\tau}\right)$$

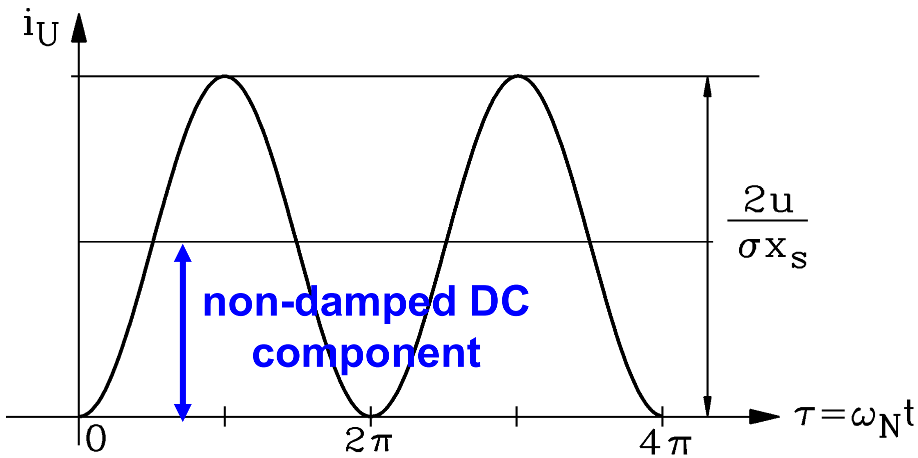


$$i_U(\tau) = \mathrm{Re}\left\{\frac{u}{\sigma \cdot x_s} \cdot \left(1 - e^{j\cdot\tau}\right)\right\} = \frac{u \cdot (1 - \cos\tau)}{\sigma \cdot x_s}$$



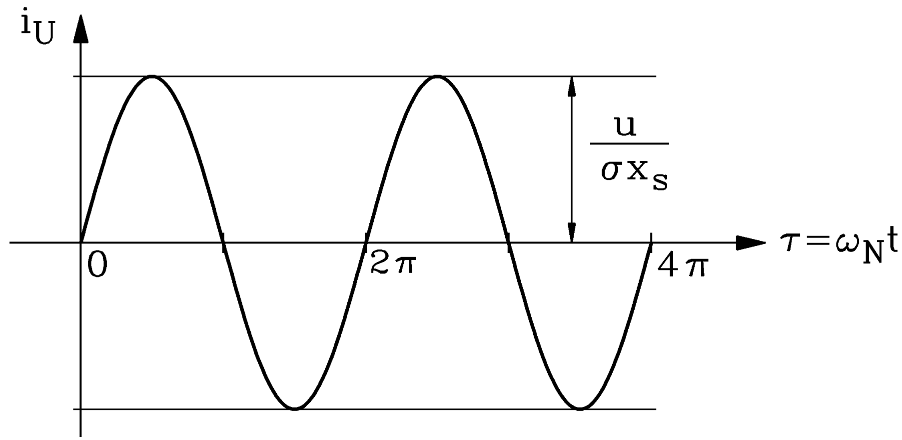
7. Dynamics of induction machines

Dynamic turn-on current - depends on switching-on instant



$$u_U(\tau) = u \cdot \cos(\tau - \pi/2) = u \cdot \sin\tau$$

$$\rightarrow \quad u_U(\tau = 0) = 0$$



$$u_U(\tau) = u \cdot \cos(\tau) \rightarrow$$

$$u_U(0) = u$$

Switching on at zero voltage	***Switching on at maximum voltage***
Current: DC component AND AC component	Current: no DC component
Peak current **200%**	Peak current **100%**
Peak occurs at half period after switching on	Peak occurs at quarter period after switching on
$i_{s,peak} = 2u_s/(\sigma \cdot x_s) = 2 \cdot 1/(0.0667 \cdot 3) = 10$	$i_{s,peak} = u_s/(\sigma \cdot x_s) = 1/(0.0667 \cdot 3) = 5$
worst case	***best case***

Complete stationary solution (1)

Steady state solution (= particular solution of differential equation):

$$\underline{i}_{s,p}(\tau) = \underline{A} \cdot e^{j\cdot\tau} \qquad \underline{A} = \frac{u \cdot (r_r' + (j - j\omega_m) \cdot x_r')}{\sigma \cdot x_s \cdot x_r' \cdot (j - \underline{s}_a) \cdot (j - \underline{s}_b)}$$

$$\boxed{1 - \omega_m = Slip}$$

$$\sigma \cdot x_s x_r' \cdot (j - \underline{s}_a) \cdot (j - \underline{s}_b) = \sigma \cdot x_s x_r' \cdot (j^2 + j \cdot (\alpha_s + \alpha_r - j\omega_m) + \alpha_s \cdot (\sigma \cdot \alpha_r - j\omega_m)) =$$

$$= (r_s + j \cdot x_s) \cdot (r_r' + (j - j\omega_m) \cdot x_r') - j \cdot x_h^2 \cdot (j - j\omega_m) =$$

$$= (r_s + j \cdot x_s) \cdot (r_r' + j \cdot Slip \cdot x_r') + x_h^2 \cdot Slip =$$

$$= r_s r_r' - x_s \cdot Slip \cdot x_r' + x_h^2 \cdot Slip + j \cdot (Slip \cdot r_s x_r' + x_s r_r') =$$

$$= r_s r_r' - Slip \cdot \sigma \cdot x_s x_r' + j \cdot (Slip \cdot r_s x_r' + x_s r_r')$$

$$\underline{i}_{s,p}(\tau) = \frac{u \cdot (r_r' + j \cdot Slip \cdot x_r')}{r_s r_r' - Slip \cdot \sigma \cdot x_s x_r' + j \cdot (Slip \cdot r_s x_r' + x_s r_r')} \cdot e^{j\cdot\tau}$$

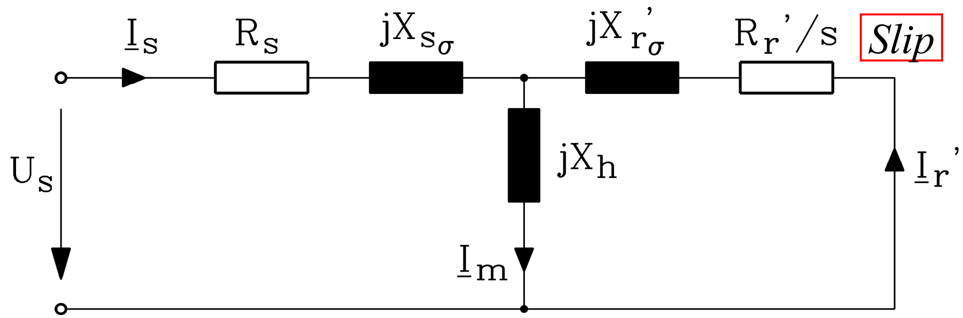
7. Dynamics of induction machines

Stationary solution equivalent circuit

- **Solution of the two linear equations** of T-equivalent circuit:
- Two unknowns $\underline{I}_s, \underline{I}'_r$

$$\underline{U}_s = R_s \underline{I}_s + jX_s \underline{I}_s + jX_h \underline{I}'_r$$

$$0 = \frac{R_r}{s} \underline{I}'_r + jX'_r \underline{I}'_r + jX_h \underline{I}_s$$

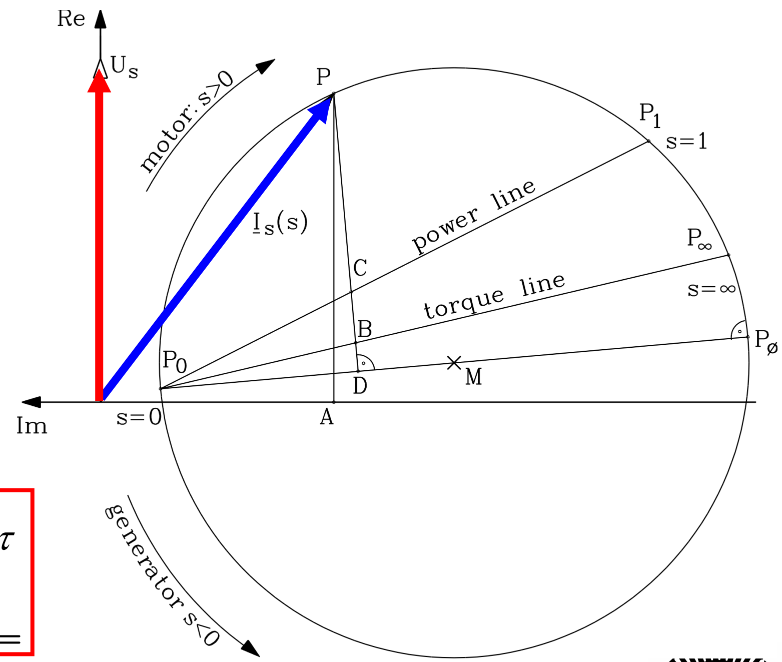


- **Rotor and stator current:** $\underline{I}'_r = -\underline{I}_s \cdot \dfrac{jX_h}{\dfrac{R'_r}{s} + jX'_r}$ $(s = Slip)$

$$\boxed{\underline{I}_s = \underline{U}_s \cdot \frac{R'_r + j \cdot sX'_r}{R_s R'_r - s \cdot \sigma \cdot X_s X'_r + j(s \cdot R_s X'_r + X_s R'_r)}}$$

- **Compare:**

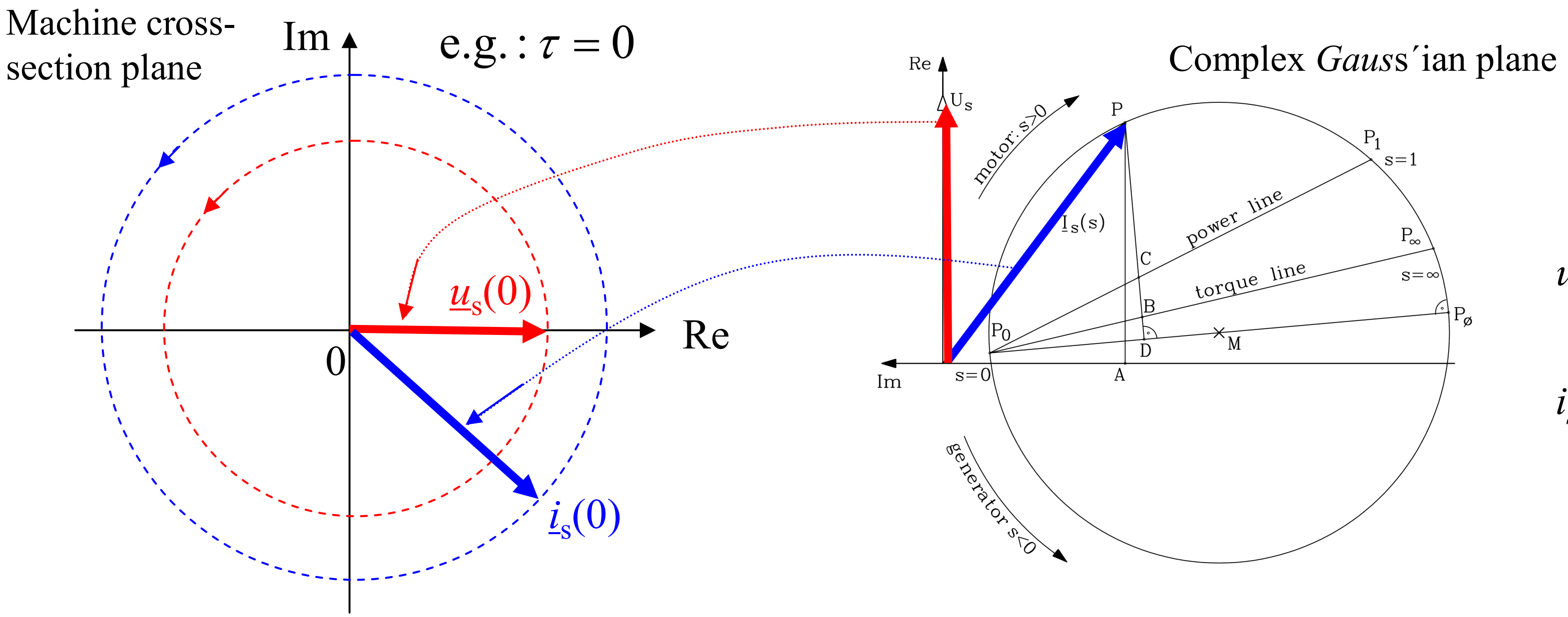
$$\boxed{\underline{i}_{s,p}(\tau) = u \cdot \frac{r'_r + j \cdot Slip \cdot x'_r}{r_s r'_r - Slip \cdot \sigma \cdot x_s x'_r + j \cdot \left(Slip \cdot r_s x'_r + x_s r'_r\right)} \cdot e^{j \cdot \tau}}$$



Complete stationary solution (2)

- $$\underline{\underline{\underline{i}_s(\tau) = \underline{i}_{s,p}(\tau) = \frac{u \cdot (r_r' + j \cdot Slip \cdot x_r')}{r_s r_r' - Slip \cdot \sigma \cdot x_s x_r' + j \cdot \left(Slip \cdot r_s x_r' + x_s r_r'\right)} \cdot e^{j \cdot \tau}}}, \quad \underline{u}_s(\tau) = u \cdot e^{j \cdot \tau}$$

- Stationary solution gives for stator current space vector the well-known *OSSANNA* "circle diagram" as locus of all solutions $\underline{i}_{s,p}$ for varying speed **(= varying *Slip s*)**



$$u_s = u = \frac{U_s \cdot \sqrt{2}}{\hat{U}_{sN}}$$

$$i_s = \frac{I_s \cdot \sqrt{2}}{\hat{I}_{sN}}$$

7. Dynamics of induction machines

Example: Complete transient solution with damping

$$s^2 + s\cdot(\alpha_s + \alpha_r - j\omega_m) + \alpha_s \cdot (\sigma \cdot \alpha_r - j\omega_m) = 0$$

$$\tau_{s\sigma} = 1/\alpha_s = \sigma \cdot x_s / r_s = 0.0667 \cdot 3/0.03 = 6.67,$$

$$\tau_{r\sigma} = 1/\alpha_r = \sigma \cdot x'_r / r'_r = 0.0667 \cdot 3/0.04 = 5.0$$

$\omega_m = 1$: **Roots for transient solution**

$$\underline{s}_a = -\frac{1}{\tau_1} + j\cdot\omega_{d,1} = \underbrace{-\frac{1}{4.96}}_{-0.202} + j\cdot 0.971, \quad \underline{s}_b = -\frac{1}{\tau_2} + j\cdot\omega_{d,2} = \underbrace{-\frac{1}{6.73}}_{-0.149} + j\cdot 0.0288$$

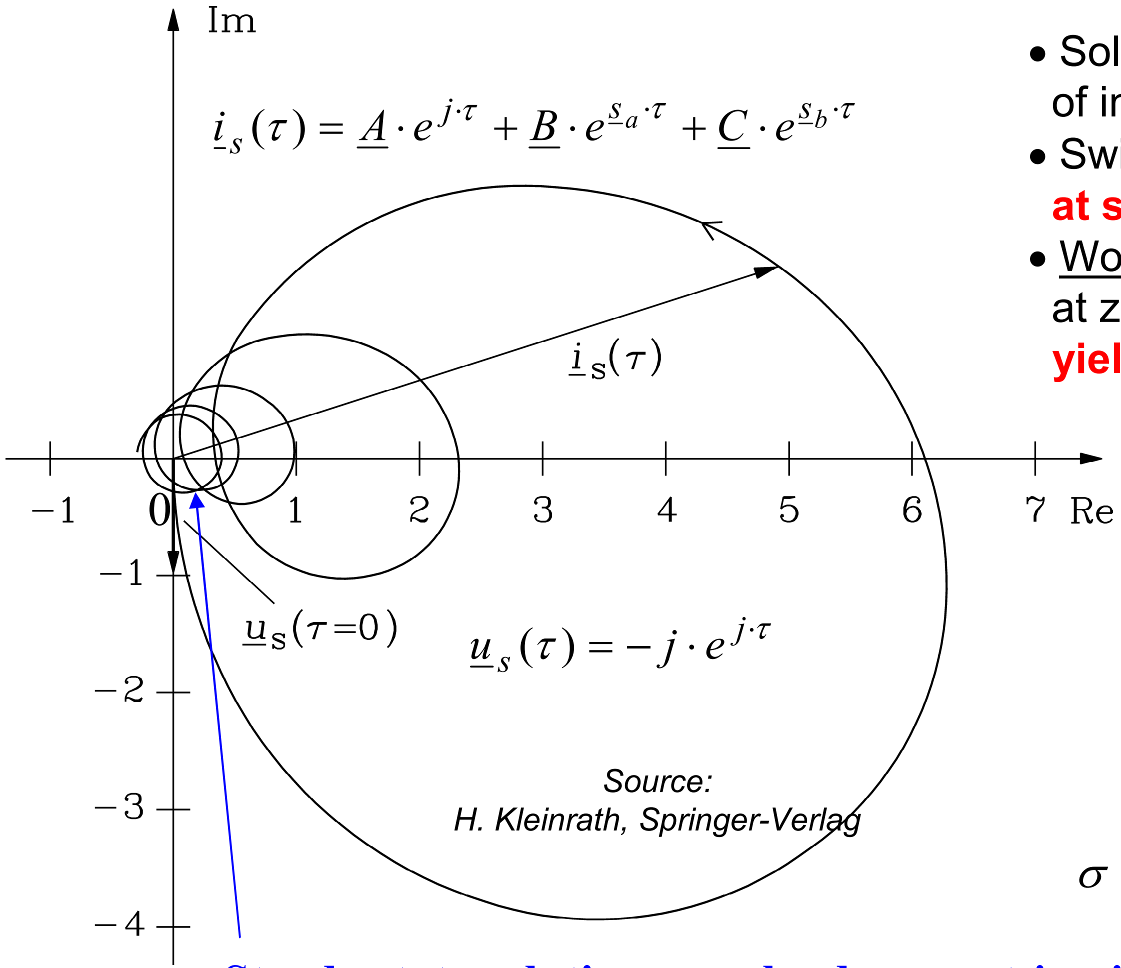
Solution for "inrush" current of induction machine, being switched to grid, when already running

(i) at synchronous speed: $i_s = 0.33$ (ii) at rated speed: $i_s = 1.0$

(i)	(ii)
$\omega_m = 1, \quad Slip = 0$	$\omega_m = 0.96, \quad Slip = 0.04$
$\underline{s}_a = -0.202 + j0.971, \underline{s}_b = -0.149 + j0.029$	$\underline{s}_a = -0.202 + j0.93, \underline{s}_b = -0.149 + j0.03$
Steady state current = no-load current = 0.33	steady state current = rated current = 1

7. Dynamics of induction machines

Stator current space vector solution with damping, *Slip* = 0



- Solution for "inrush" current space vector $\underline{i}_s$ of induction machine
- Switched to sinusoidal grid, when running **at synchronous** speed $\omega_m = 1$
- Worst-case for phase U, where switching occurs at zero voltage, **yielding maximum current peak**

$$\sigma = 0.0667,\ x_s = 3,\ x'_r = 3,\ r_s = 0.03,\ r'_r = 0.04$$

Steady state solution = no-load current $i_s = i_{s0} = 0.33$ p.u.

7. Dynamics of induction machines

Stator „inrush" current $i_{U}(\tau)$ at $\omega_m = 1$, (*Slip* = 0)

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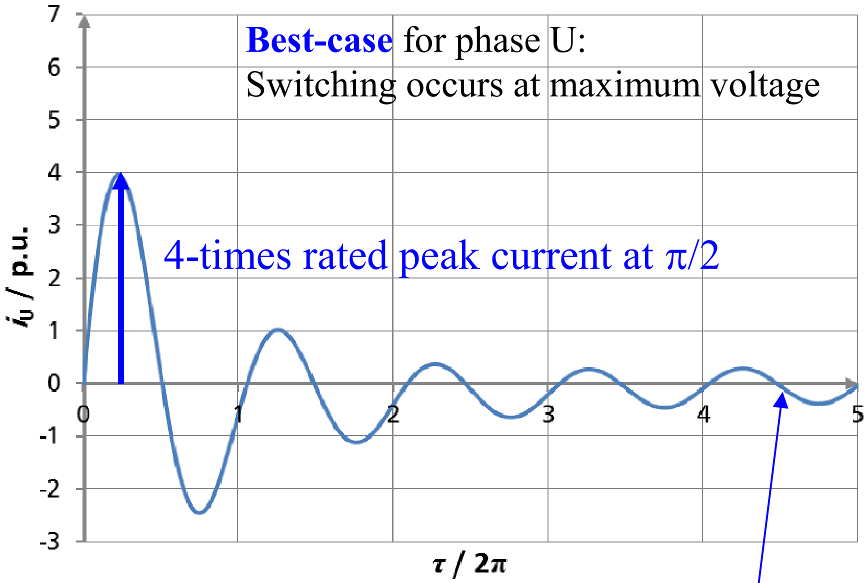
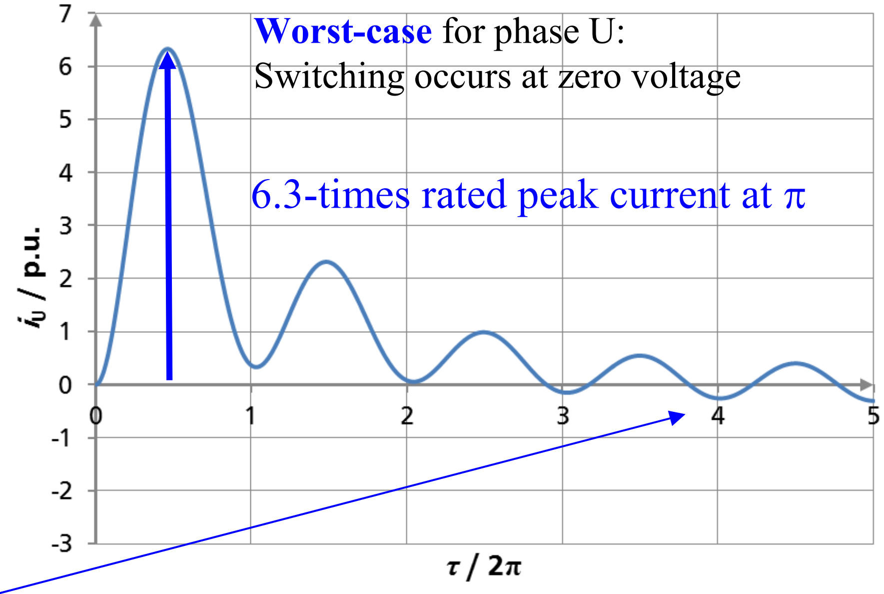
Induction machine switched to sinusoidal grid, when running **at synchronous** speed $\omega_m = 1$

$\sigma = 0.0667,\ x_s = 3,\ x_r' = 3,\ r_s = 0.03,\ r_r' = 0.04 \qquad |\underline{u}_s(\tau)| = 1$

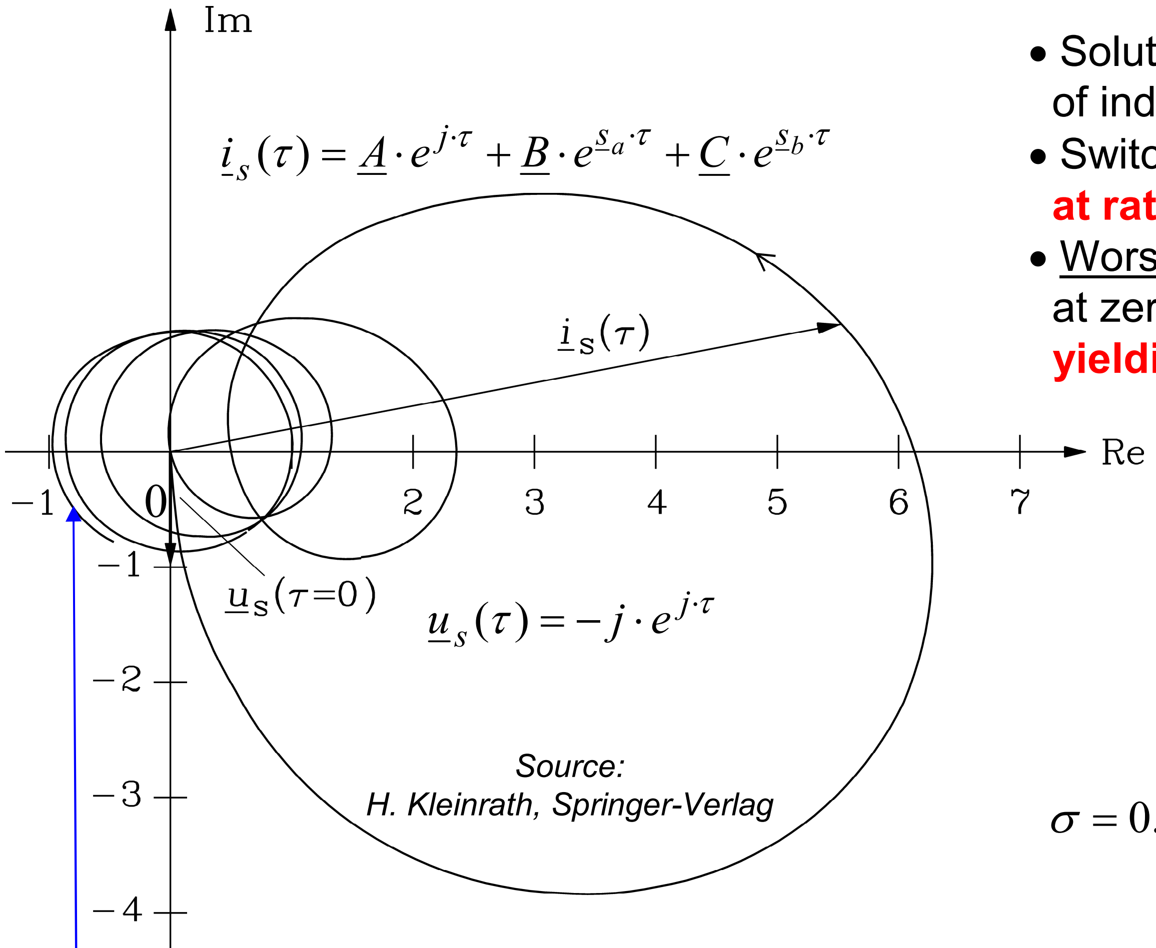




Steady state solution = no-load current $i_s = i_{s0} = 0.33$ p.u.

7. Dynamics of induction machines

Stator current space vector solution with damping, at rated *Slip* = 0.04



- Solution for "inrush" current space vector $\underline{i}_\mathrm{s}$ of induction machine
- Switched to sinusoidal grid, when running **at rated** speed $\omega_\mathrm{m} = 0.96$
- Worst-case for phase U, where switching occurs at zero voltage, **yielding maximum current peak**

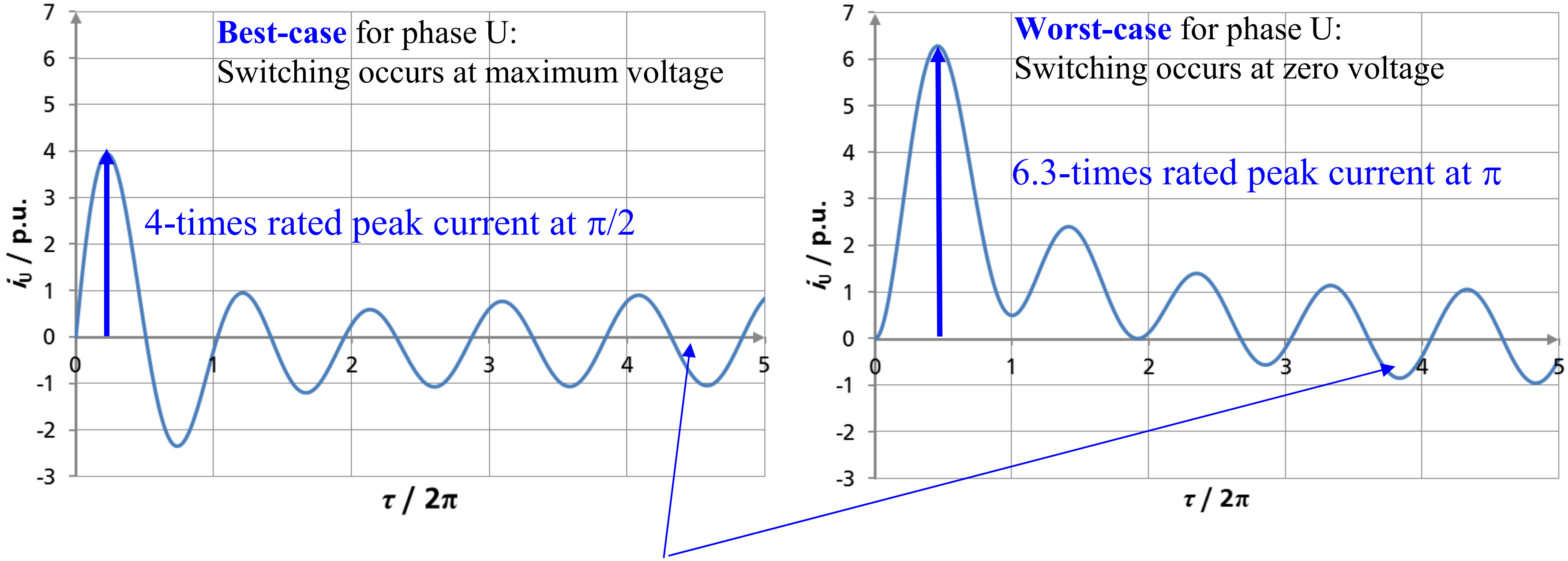
$\sigma = 0.0667,\ x_s = 3,\ x'_r = 3,\ r_s = 0.03,\ r'_r = 0.04$

Steady state solution = rated current, $i_\mathrm{s} = i_\mathrm{sN} = 1.0$ p.u.

Stator „inrush“ current $i_U(\tau)$ at ω_m = 0.96, rated *Slip* = 0.04

Induction machine switched to sinusoidal grid, when running **at rated** speed $\omega_m = 0.96$

$\sigma = 0.0667,\ x_s = 3,\ x_r' = 3,\ r_s = 0.03,\ r_r' = 0.04$ $\qquad |\underline{u}_s(\tau)| = 1$



Steady state solution = rated current $i_s = i_{sN} = 1.0$ p.u.

7. Dynamics of induction machines

Transient response of induction machine at elevated speed at $|\omega_m| > 0.2$

- **Voltage switching** such as

a) switching on motor (= in-rush current),
b) sudden short-circuit,
...

leads to a **DC current component**, which is largest,
when switching occurs at zero voltage in the considered phase.

- **DC current component limited** by rotor and stator stray inductances and resistances

- **DC current component vanishes** with two short time constants $\tau_1 \approx \tau_{r\sigma}$, $\tau_2 \approx \tau_{s\sigma}$, determined by the rotor and stator stray inductances and resistances

7. Dynamics of induction machines

Sudden short-circuit of a 4-pole no-load induction motor

- Maximum short circuit current, when short circuit occurs at voltage zero crossing
- Stator current space vector at (nearly) constant speed $\omega_m = 1$ contains transient DC and AC component:

$$\underline{i}_s \cong \underline{i}_{s0} \cdot \left(\frac{e^{-\tau/\tau_{s\sigma}}}{\sigma} + \left(1 - \frac{1}{\sigma}\right) \cdot e^{-\tau/\tau_{r\sigma}} \cdot e^{j\omega_m \tau} \right)$$

$$i_s(\tau = 0) = i_{s0} \cong u_s / x_s = 0.33 \quad \text{No-load current}$$

- Maximum short circuit phase current amplitude:

$$\hat{i}_{s,U} = \frac{u}{x_s} \cdot \left(\frac{2}{\sigma} - 1 \right) = 6.87 \text{ (at } \tau = \pi \text{, undamped)}$$

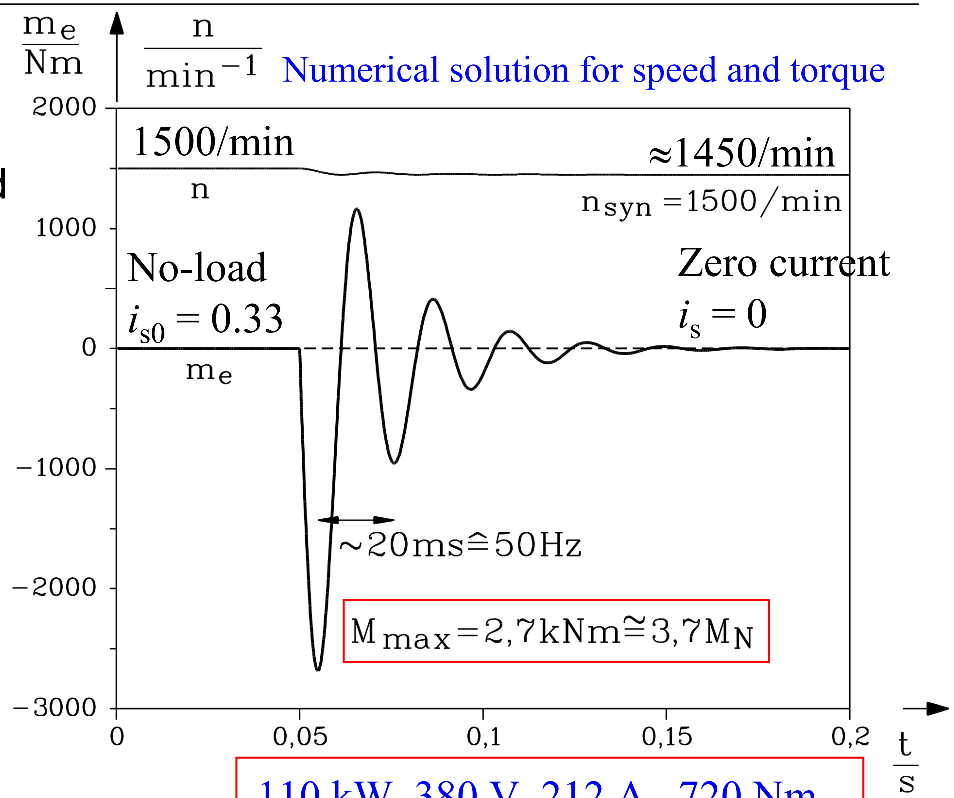
- Dynamic short circuit torque:

$$m_e(\tau) = -\mathrm{Im}\left\{ x_h \cdot \underline{i}'_r \cdot \underline{i}_s^* \right\} \cong -\frac{x_h^2}{\sigma \cdot x'_r} \cdot i_{s0}^2 \cdot e^{-\tau \cdot \left(\frac{1}{\tau_{r\sigma}} + \frac{1}{\tau_{s\sigma}} \right)} \cdot \sin(\omega_m \tau)$$

$\sigma = 0.0667,\ x_s = 3,\ x'_r = 3,\ r_s = 0.03,\ r'_r = 0.04$

$$m_{e,\max} \approx -\frac{x_h^2}{\sigma \cdot x'_r} \cdot i_{s0}^2 \approx -\frac{3}{0.097} \cdot 0.33^2 \approx -3.3$$

Numerical solution for speed and torque



110 kW, 380 V, 212 A, 720 Nm, 4 pole motor, 50 Hz, $\sigma = 0.097$

7. Dynamics of induction machines

Superconducting rotor in induction machines: $r_r = 0$

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$$\left.\begin{aligned}
\breve{\underline{i}}_s &= \frac{u}{s-j}\cdot\frac{\cancel{r'_r}+(s-j\omega_m)\cdot x'_r}{\sigma\cdot x_s\cdot x'_r\cdot(s-\underline{s}_a)\cdot(s-\underline{s}_b)} = \frac{u}{s-j}\cdot\frac{(s-j\omega_m)\cdot x'_r}{\sigma\cdot x_s\cdot x'_r\cdot(s-\underline{s}_a)\cdot(s-\underline{s}_b)} \\
\breve{\underline{i}}'_r &= -\frac{u}{s-j}\cdot\frac{x_h\cdot(s-j\omega_m)}{\sigma\cdot x_s\cdot x'_r\cdot(s-\underline{s}_a)\cdot(s-\underline{s}_b)} = -\frac{x_h}{x'_r}\cdot\breve{\underline{i}}_s \Rightarrow \breve{\underline{\psi}}'_r = x_h\cdot\breve{\underline{i}}_s + x'_r\cdot\breve{\underline{i}}'_r = 0
\end{aligned}\right\}$$

The rotor current is in phase opposition to the stator current: $\boxed{\breve{\underline{i}}'_r = -(x_h/x'_r)\cdot\breve{\underline{i}}_s \cong -\breve{\underline{i}}_s}$

- No flux can penetrate the superconducting rotor: $\boxed{\breve{\underline{\psi}}'_r = 0 \Rightarrow \underline{\psi}'_r(\tau) = 0}$
- Stator flux linkage is nearly total leakage flux: $\breve{\underline{\psi}}_s = x_s\cdot\breve{\underline{i}}_s + x_h\cdot\breve{\underline{i}}'_r = \sigma\cdot x_s\cdot\breve{\underline{i}}_s \approx x_{s\sigma}\cdot\breve{\underline{i}}_s - x'_{r\sigma}\cdot\breve{\underline{i}}'_r$
- A superconducting induction machine cannot produce any torque: $m_e = -\mathrm{Im}\{\underline{i}'_r\cdot\underline{\psi}'^{*}_r\} = -\mathrm{Im}\{\underline{i}'_r\cdot 0\} = 0$
- A resistive rotor $r_r > 0$ is essentially necessary in induction machines for torque production!
- At $r_r > 0$ the rotor current space vector is NOT shifted by 180° to the stator space current vector. This leads to a torque-producing “normal” current space vector component !
- Result: A superconducting induction machine is useless!

Summary:
Solutions of dynamic equations for constant speed

- Linear voltage & flux linkage equations at constant speed
- *LAPLACE* domain solution with transfer function $\underline{i}_s = F(\underline{u}_s)$
- Time-constants τ_1, τ_2 and natural frequencies $\omega_{d,1}$, $\omega_{d,2}$ depend on speed ω_m
- Homogeneous solution = transient part = DC current component in inductive circuit
- Particular solution for steady-state solution $\underline{i}_{s,p}$ = rotary current space vector
- Examples:

  Switching of voltage on stator winding of running machine

  Sudden short circuit at stator terminals

Solutions of dynamic equations for varying speed

- Solution of all 9 equations simultaneously (e.g. in α-β-frame)
- Equations are non-linear, so numerical solution is necessary:

VOLTAGE: 4 equations

FLUX LINKAGE: 4 equations

TORQUE: 1 equation

9 equations in TOTAL !

- *Example:*

a) No-load start-up of induction motors
and afterwards
b) loading with rated torque
is investigated.

7. Dynamics of induction machines

Data of two example machines

	Induction machine 1 (big)		Induction machine 2 (small)	
Rated power	110.8 kW		1.18 kW	
Rated voltage	380 V		380 V	
Rated current	212 A		2.6 A	
Efficiency	93.4 %		85.5 %	
Power factor	0.85		0.81	
Rated slip	2 %		8 %	
Rated speed	1470/min		1380/min	
Rated torque	720 Nm		8.2 Nm	
R_s	25 mΩ	**0.024 p.u.**	9.5 Ω	**0.113 p.u.**
R'_r	20 mΩ	**0.019 p.u.**	6.2 Ω	**0.073 p.u.**
L_s	9.71 mH	**2.95 p.u.**	668 mH	**2.49 p.u.**
L'_r	9.55 mH	**2.90 p.u.**	662 mH	**2.46 p.u.**
L_h	9.17 mH	**2.78 p.u.**	633 mH	**2.36 p.u.**
σ	0.094		0.094	
J	2.8 kgm^2	τ_J = 155.5	0.00349 kgm^2	τ_J = 15.8
T_J with M_N	611 ms		67 ms	

7. Dynamics of induction machines
Scaling: "Big" vs. "small" induction motor

- "Big" Machine 1 vs. "Small" Machine 2: Power, current & torque rating ratio:

$$P_1 / P_2 \approx 100 \qquad I_1 / I_2 \approx 100 \qquad M_1 / M_2 \approx 100$$

- Same voltage rating: $U_1 / U_2 = 1$

- Big machines have small resistances and inductances, compared to small machines of the same voltage rating:

$$R_1 / R_2 \approx 1/400 \qquad L_1 / L_2 \approx 1/70$$

a) Big machines = big currents = big conductor cross sections = small resistances
b) Big machines = big flux area per pole = small number of turns = small inductances at the same voltage rating

- Big machines have „very" big inertia, compared to small machines:

$$J_1 / J_2 \approx 800 \qquad T_{J1} / T_{J2} \approx 10$$

Scaling of motor data “small / big” machines

- **Rotor inertia:** $J \sim d_{si}^4 \cdot l \sim l^5$
- **Motor power:** $P \sim d_{si}^3 \cdot l \sim l^4$ **,**
- **Scaling ratio:** $J_1 / J_2 = (P_1 / P_2)^{5/4}$

- ***Example:***

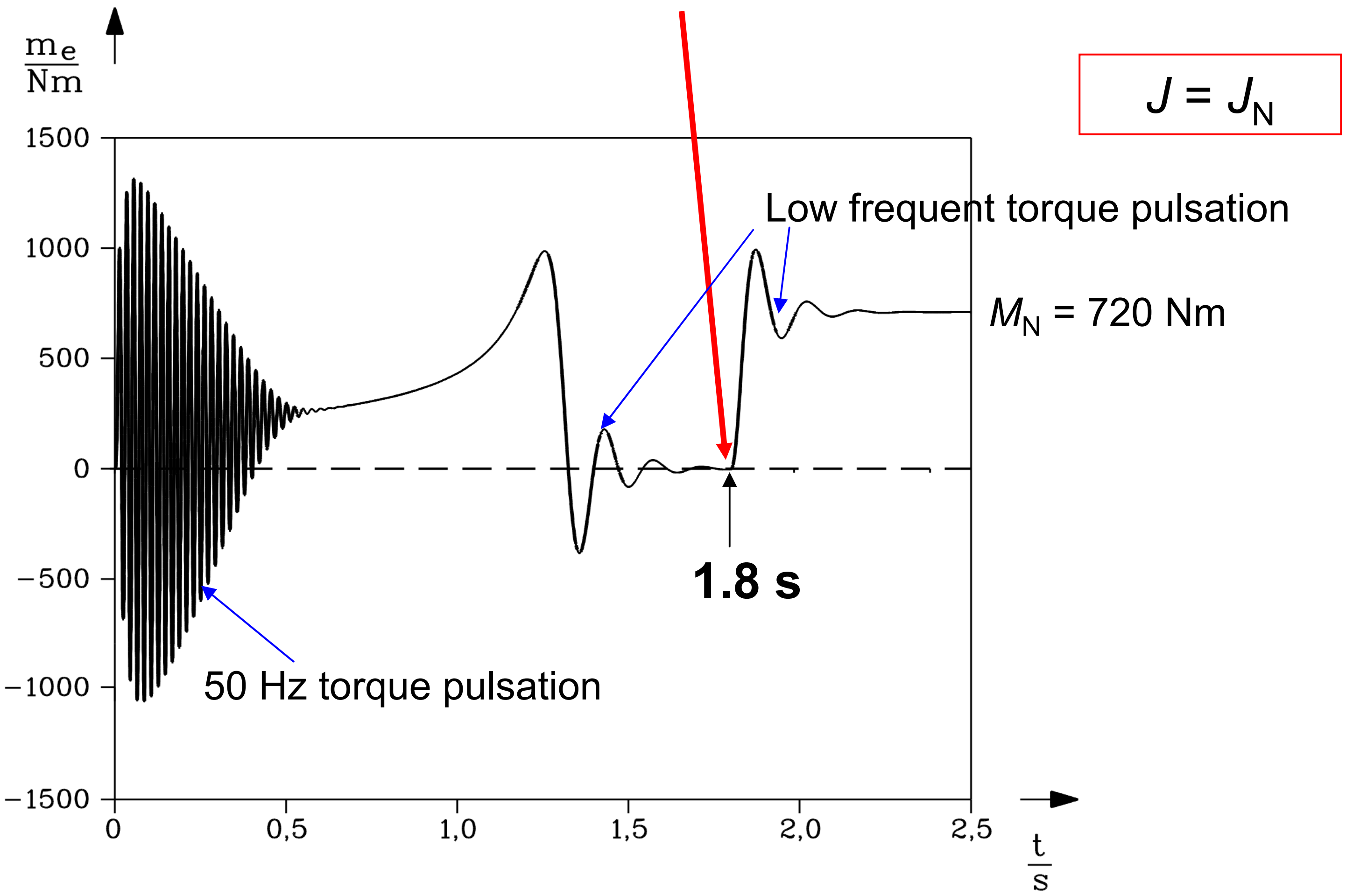
  Big versus small machine: Scaled ratio: $J_1 / J_2 = (110 / 1.1)^{5/4} = 316$ **.**

  Real ratio: $J_1 / J_2 = 2.8 / 0.00349 = 802.$

- The 100 times stronger (bigger) Machine 1 needs due to its about a factor 1000 bigger inertia about 10 times longer to start up.

Calculated electromagnetic torque of “big” induction machine

No-load starting at 50 Hz grid voltage; motor loaded at 1.8 s with rated torque



Calculated rotational speed of induction machine

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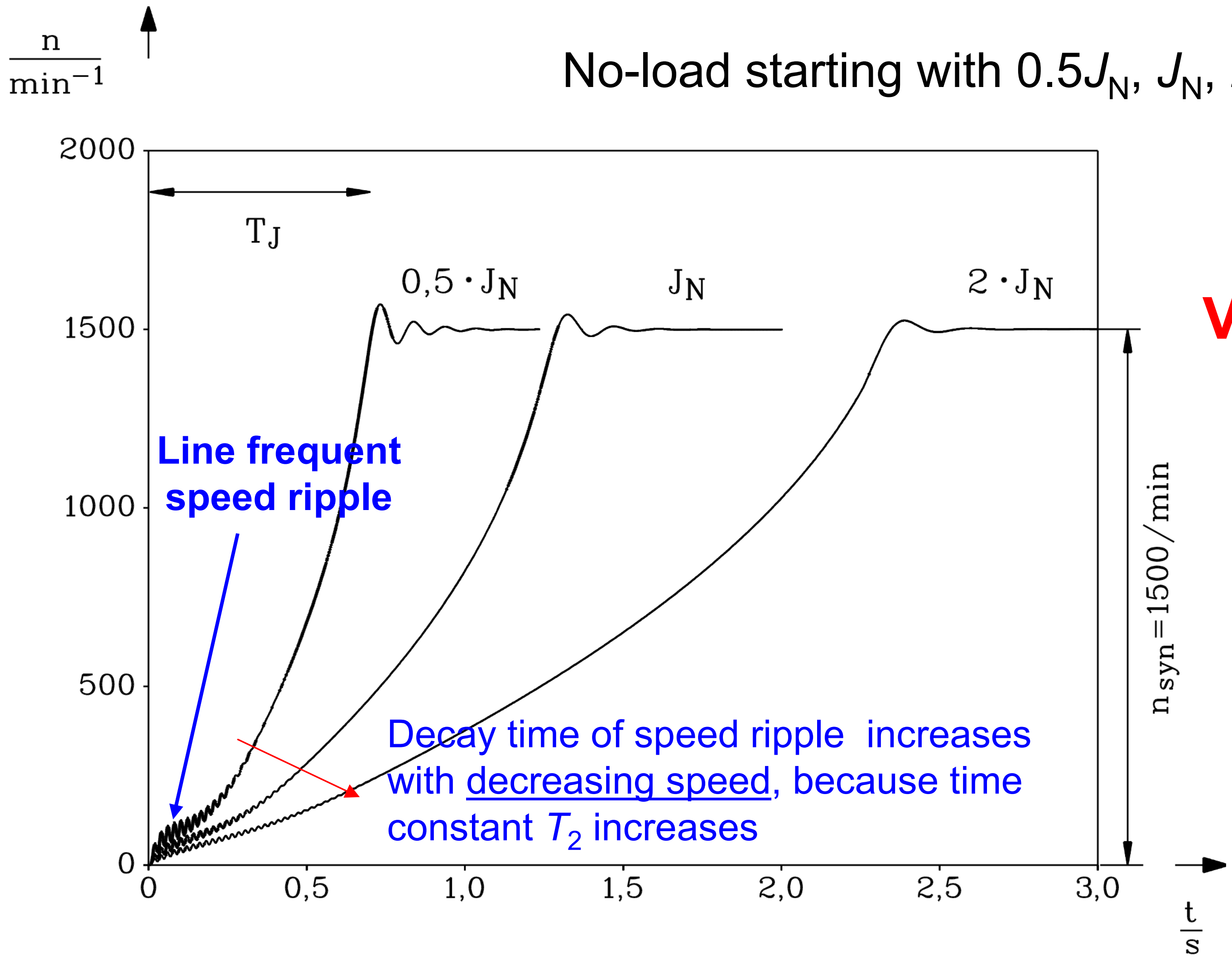


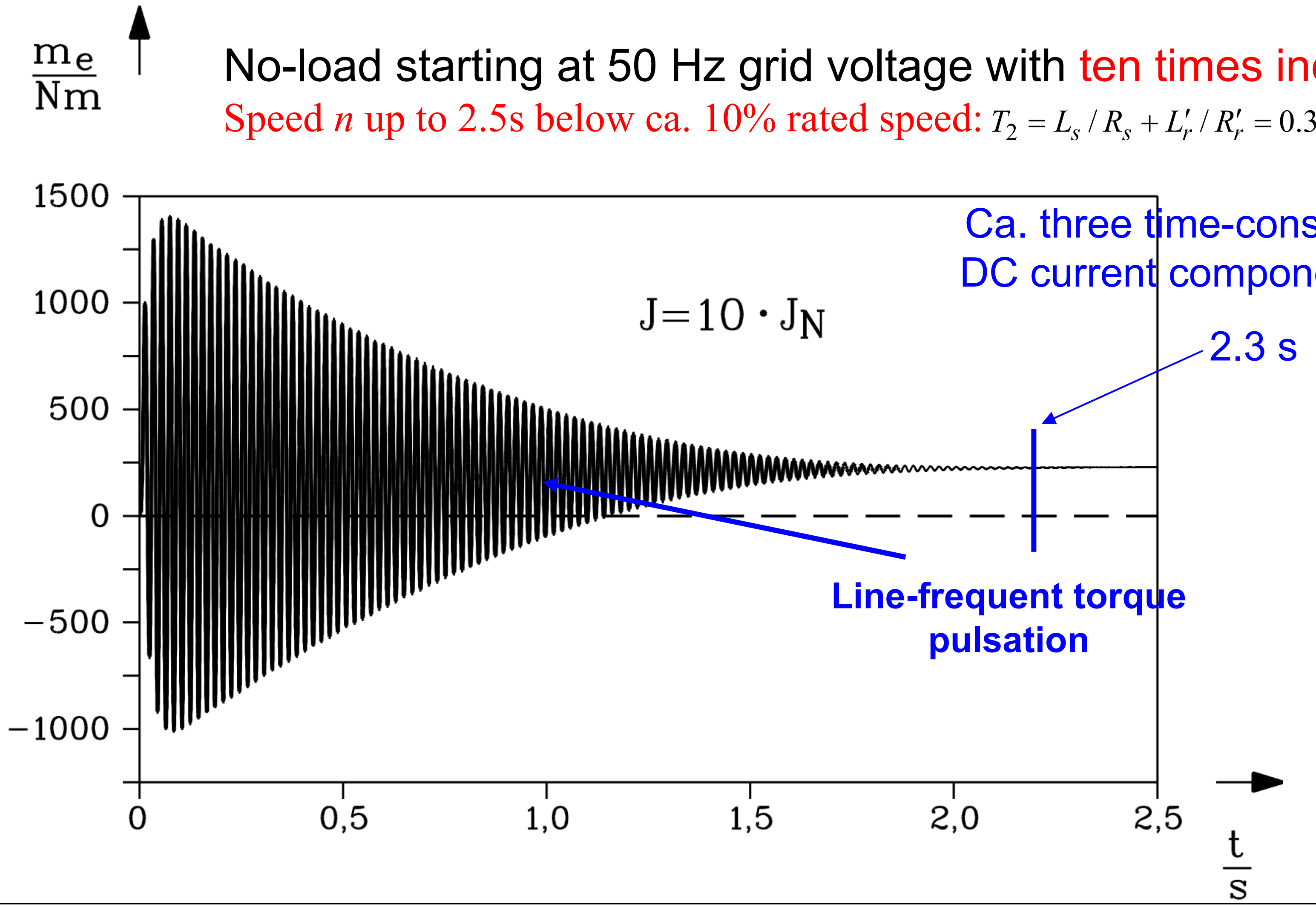
Varying inertia

Starting time at J_N longer as T_J, because $M(t) \neq M_N$

Calculated electromagnetic torque of induction machine

No-load starting at 50 Hz grid voltage with ten times increased inertia

Speed n up to 2.5s below ca. 10% rated speed: $T_2 = L_s / R_s + L'_r / R'_r = 0.388\,\mathrm{s} + 0.478\,\mathrm{s} = 0.866\,\mathrm{s}$



7. Dynamics of induction machines

Influence of mechanical speed *n* on time constant for decay of oscillating starting torque

	a) $J = J_N$	b) $J = 10J_N$
Time of decay of 50 Hz oscillating torque	0.5 s	2.3 s

- Case b): Big inertia $10J_N$ (= slow acceleration) $\Rightarrow$ Decay of transient DC current component occurs at still low speed $n \approx 0$, so "zero speed" formula for time constant $T_2(n=0)$ applies.

$$T_2 = L_s / R_s + L'_r / R'_r = 0.388\ \mathrm{s} + 0.478\ \mathrm{s} = 0.866\ \mathrm{s}$$

$$\left(\tau_2\big|_{\omega_m \approx 0} \approx \frac{x_s}{r_s} + \frac{x_r}{r_r} \approx 2 \cdot \frac{x}{r} \right)$$

- After $3T_2(n=0)$ both DC current & torque oscillation have vanished!

$$3T_2 = 3 \cdot 0.866\ \mathrm{s} = 2.5\ \mathrm{s}$$

$$\left(\tau_2\big|_{|\omega_m| > 0.2} \approx \sigma \cdot \frac{x_s}{r_s} \approx \frac{\sigma}{2} \cdot \tau_2\big|_{\omega_m \approx 0} \right)$$

- Case a): Small inertia J_N (= fast acceleration) $\Rightarrow$ Decay of transient DC current component occurs at already elevated speed $|\omega_m| > 0.2$, so time constant $T_2(n>0) < T_2(n=0)$ is shorter!

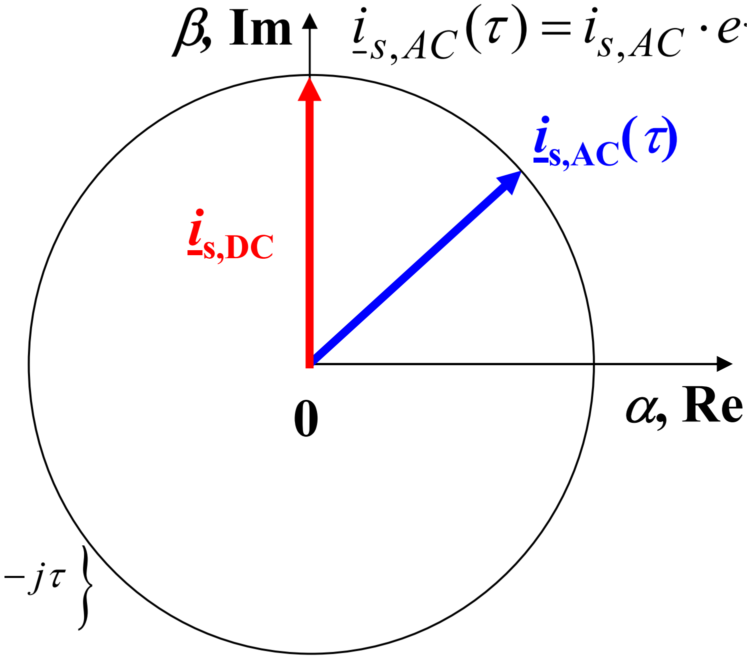
- $T_2(n>0)$ tends TOWARDS "Short circuit time constant !": $T_2(n>0) \to T_{s\sigma} \approx \sigma \cdot T_2(n=0)/2$

$$\sigma \cong 0.1 \Rightarrow 3T_2(n>0) \to 0.05 \cdot 3T_2(n=0) \approx 0.05 \cdot 2.5 = 0.13\ \mathrm{s} \Rightarrow \underbrace{0.13\ \mathrm{s}}_{J \to 0} < \underbrace{0.5\ \mathrm{s}}_{J_N} < \underbrace{2.3\ \mathrm{s}}_{10 \cdot J_N}$$

Line-frequent starting torque oscillation

- **Transient solution:** At speed $n = 0$:
  DC current space vector $\underline{i}_{s,DC}$ does not rotate
- **Stationary solution:** AC current space vector rotates with line frequency $\omega_s = 1$ and its flux induces the rotor cage
- Rotating AC rotor current space vector $\underline{i}'_{r,AC} \cdot e^{j\tau}$

$$\underline{i}_{s,AC}(\tau) = \underline{i}_{s,AC} \cdot e^{j\tau}$$



- Simplified: Undamped: $\underline{i}_{s,DC} = \text{const.}$:

$$\underline{i}_s(\tau) = \underline{i}_{s,AC} \cdot e^{j\tau} + \underline{i}_{s,DC}$$

$$m_e = \mathrm{Im}\left\{x_h \underline{i}_s \cdot \underline{i}'^{*}_r\right\} = \mathrm{Im}\left\{x_h \cdot (\underline{i}_{s,AC} \cdot e^{j\tau} + \underline{i}_{s,DC}) \cdot \underline{i}'^{*}_{r,AC} e^{-j\tau}\right\}$$

$$m_{e,1} = \mathrm{Im}\left\{x_h \underline{i}_{s,AC} \cdot e^{j\tau} \cdot \underline{i}'^{*}_{r,AC} e^{-j\tau}\right\} = \mathrm{Im}\left\{x_h \underline{i}_{s,AC} \cdot \underline{i}'^{*}_{r,AC}\right\} = \text{const.}$$

$$m_{e,2} = \mathrm{Im}\left\{x_h \underline{i}_{s,DC} \cdot \underline{i}'^{*}_{r,AC} \cdot e^{-j\tau}\right\} = \mathrm{Im}\left\{x_h \underline{i}_{s,DC} \cdot \underline{i}'^{*}_{r,AC} \cdot (\cos\tau - j\sin\tau)\right\} = \hat{m}_{e,2} \cdot \cos(\tau - \Delta\varphi)$$

- **Result:**
  Constant starting torque $m_{e,1}$, but torque $m_{e,2}$ pulsates with line frequency $\omega_s = 1$

Phenomena of “dynamic” starting performance (1)

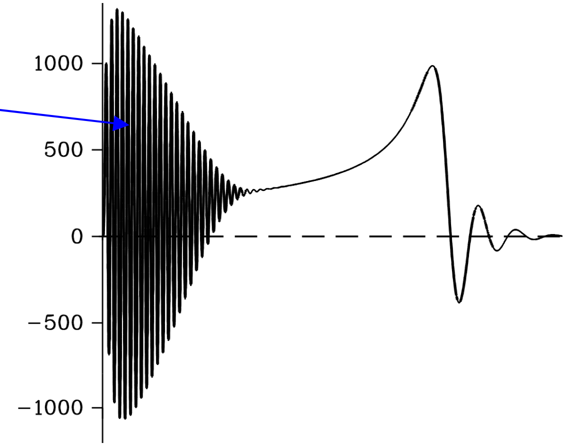
a) Oscillating starting torque:

Switching on of stator voltage:
DC current component $\underline{i}_{DC}$ occurs in stator and rotor winding.

(i)
The 50 Hz AC stator current $\underline{i}_{s,AC}$ reacts with the DC flux of rotor DC current $\underline{i}'_{r,DC}$, yielding a first pulsating 50 Hz-torque component

(ii)
The 50 Hz AC rotor current $\underline{i}'_{r,AC}$ reacts with the DC flux of stator DC current $\underline{i}_{s,DC}$, yielding a second pulsating 50 Hz-torque component

(i) + (ii) constitute the **oscillating starting torque!**

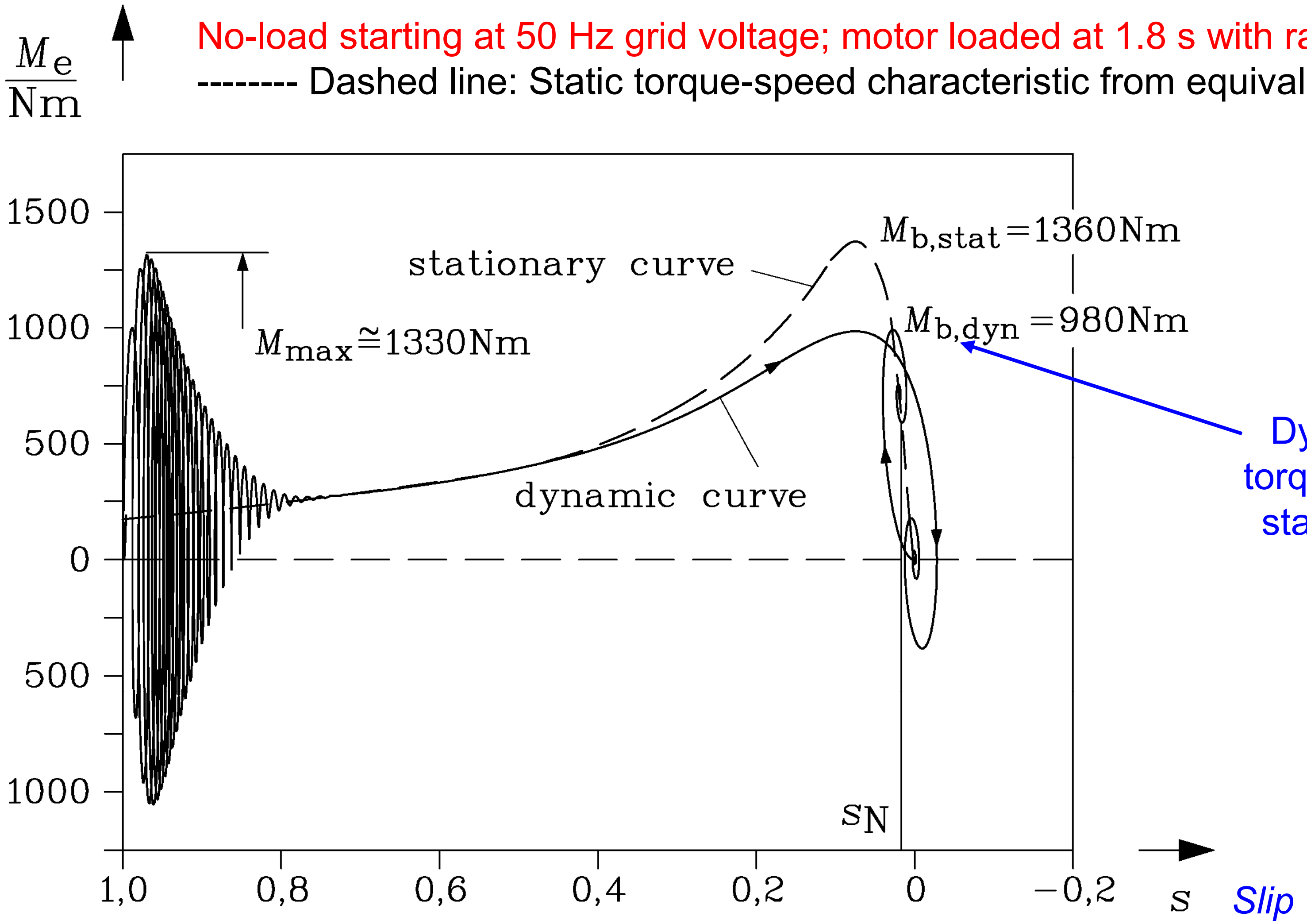


7. Dynamics of induction machines

Calculated dynamic torque-speed characteristic of induction machine

No-load starting at 50 Hz grid voltage; motor loaded at 1.8 s with rated torque.

-------- Dashed line: Static torque-speed characteristic from equivalent circuit diagram



$J = J_N$

Dynamic break-down torque $M_{b,dyn}$, lower than stationary break-down torque $M_{b,stat}$

7. Dynamics of induction machines

Phenomena of "dynamic" starting performance (2)

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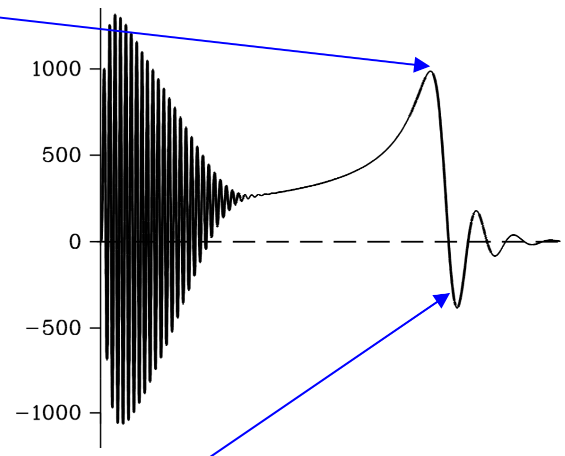
b) Dynamic break-down torque:
Main flux is changing with
big electric time constant T_2 (at $n = 0$: $L_s/R_s + L_r/R_r = 0.866$ s)
Full flux at ca. $3T_2 = 2.5$ s.
Therefore at reaching at 1.2 s break-down slip s_b,
still full flux is missing,
reducing the dynamic break-down torque $M_{b,dyn} < M_{b,stat}$



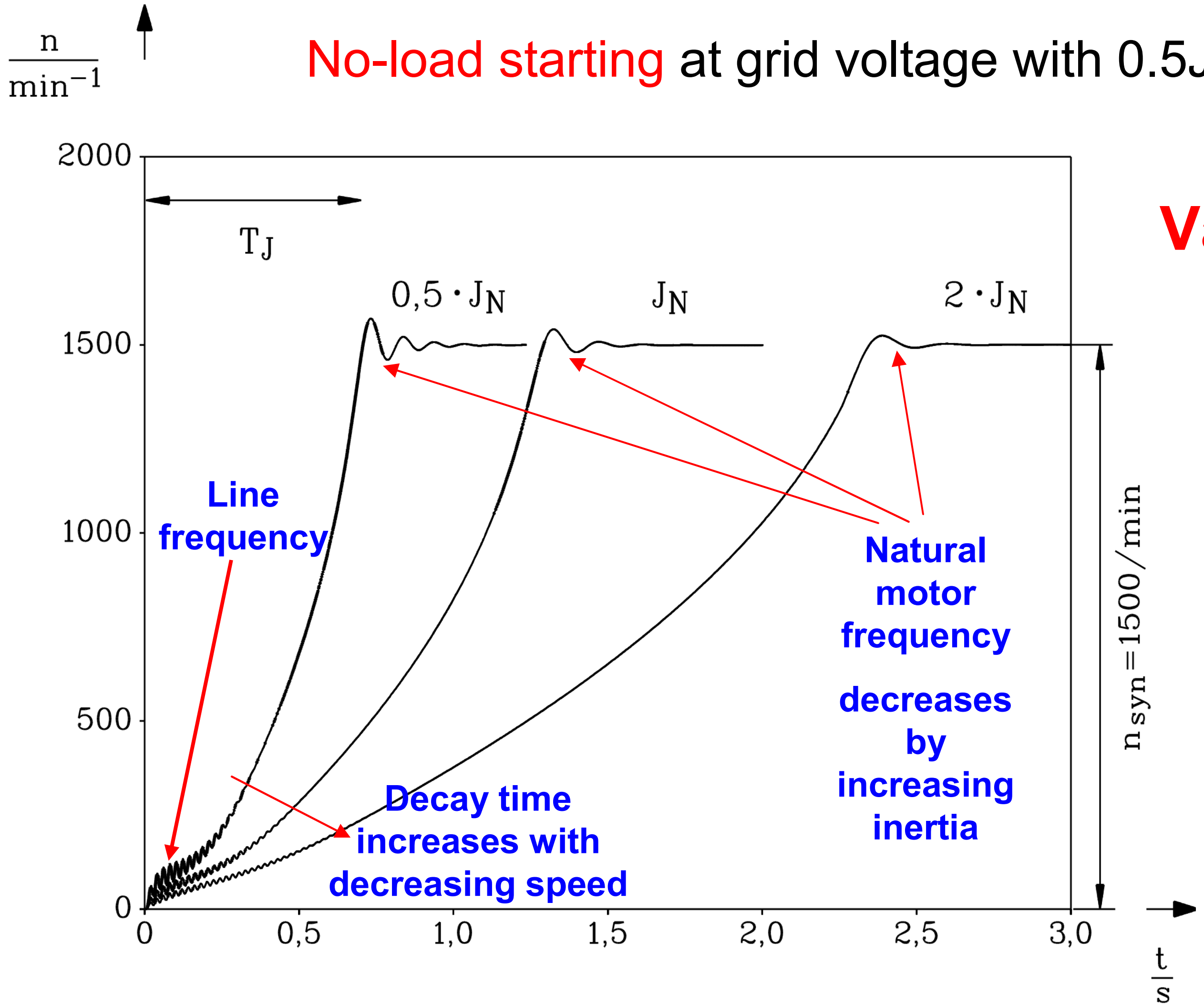
Example:
Reduction by 25% with respect to static break-down torque!

c) Eigen-frequency of induction machine at synchronous and rated speed ("synchronous machine effect in asynchronous machines"):
Low oscillation frequency $f_{d,m}$ at each load step.

Explanation:
Rotor (main) flux changes with big rotor time constant $\tau_r = x_r/r_r$,
may be regarded as "frozen" for a "short" time $\tau << \tau_r \Rightarrow$
It acts like the constant rotor flux in synchronous machines $\Rightarrow$ rotor oscillation possible

Calculated rotational speed of induction machine

No-load starting at grid voltage with $0.5J_N$, J_N, $2J_N$



Varying inertia

$$\boxed{f_{d,m} \sim \frac{1}{\sqrt{J}}}$$

Natural oscillation of induction machine

- Speed oscillation $\Delta\Omega_m(t)$ is superimposed e.g. at synchronous speed:

$$\Omega_m(t) = \Omega_{syn} + \Delta\Omega_m(t)$$

- Magnetic braking force of "frozen" rotor flux on stator current: $M_e(\Delta\vartheta) = -|c_\vartheta| \cdot \Delta\vartheta$.

($\Delta\vartheta$: angle difference between rotor flux axis and stator space current vector)

$$\frac{d\Delta\vartheta}{dt} = p \cdot \Delta\Omega_m$$

- Mechanical equation:

$$J \cdot \frac{d\Omega_m}{dt} = M_e(\vartheta) = -|c_\vartheta| \cdot \Delta\vartheta \qquad \Rightarrow \qquad \frac{d\Omega_m}{dt} = \frac{d\Delta\Omega_m}{dt}$$

$$J \cdot \frac{d^2\Delta\vartheta}{dt^2} + p \cdot |c_\vartheta| \cdot \Delta\vartheta = 0 \quad \Rightarrow \quad \Delta\vartheta(t) \sim \sin(\omega_{d,m} \cdot t), \cos(\omega_{d,m} \cdot t)$$

- **Natural frequency of oscillation:** $\boxed{f_{d,m} = \frac{\omega_{d,m}}{2\pi} = \frac{1}{2\pi} \cdot \sqrt{\frac{p \cdot |c_\vartheta|}{J}}}$

7. Dynamics of induction machines

Calculated rotational speed of induction machine

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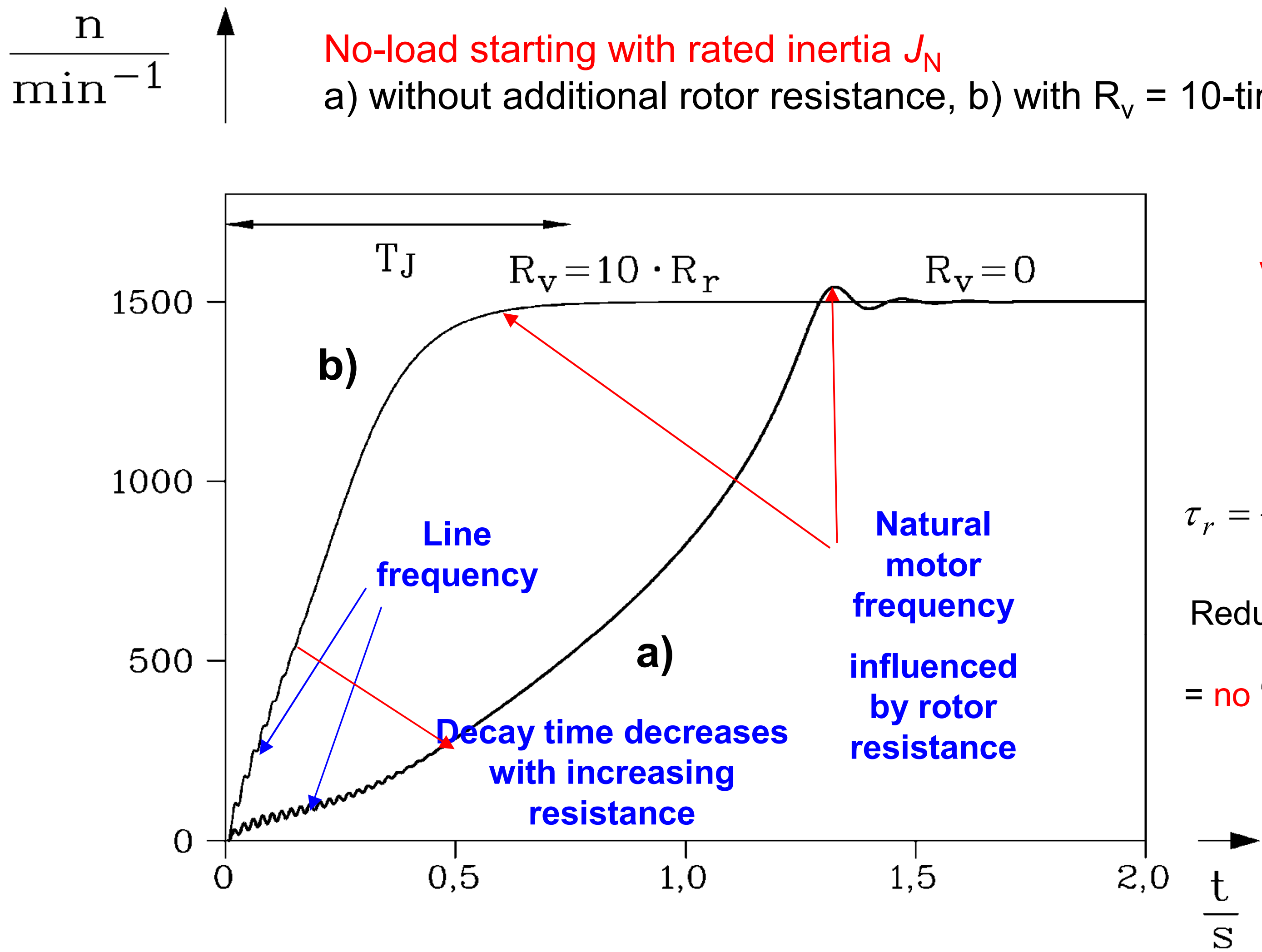
No-load starting with rated inertia J_N

a) without additional rotor resistance, b) with R_v = 10-times rotor resistance



Varying total rotor resistance $R_r + R_v$

$$\tau_r = \frac{x_r}{r_r} \rightarrow \tau_r^* = \frac{x_r}{r_r + r_v} = \frac{\tau_r}{11} << \tau_r$$

Reduced rotor time constant allows fast change of rotor flux = = no "frozen" rotor flux phenomenon

$$\tau >> \tau_r^*$$

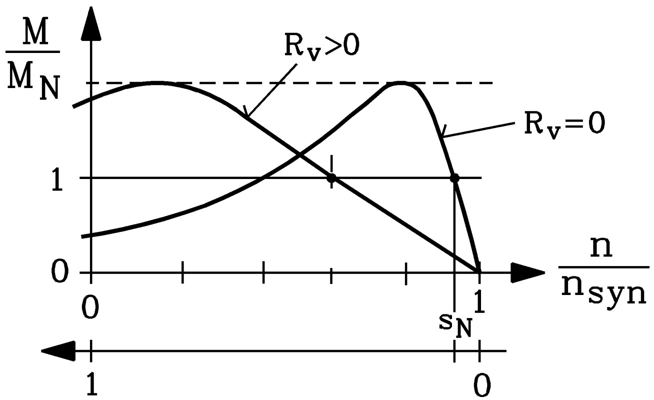
Dynamic starting of slip ring induction machine

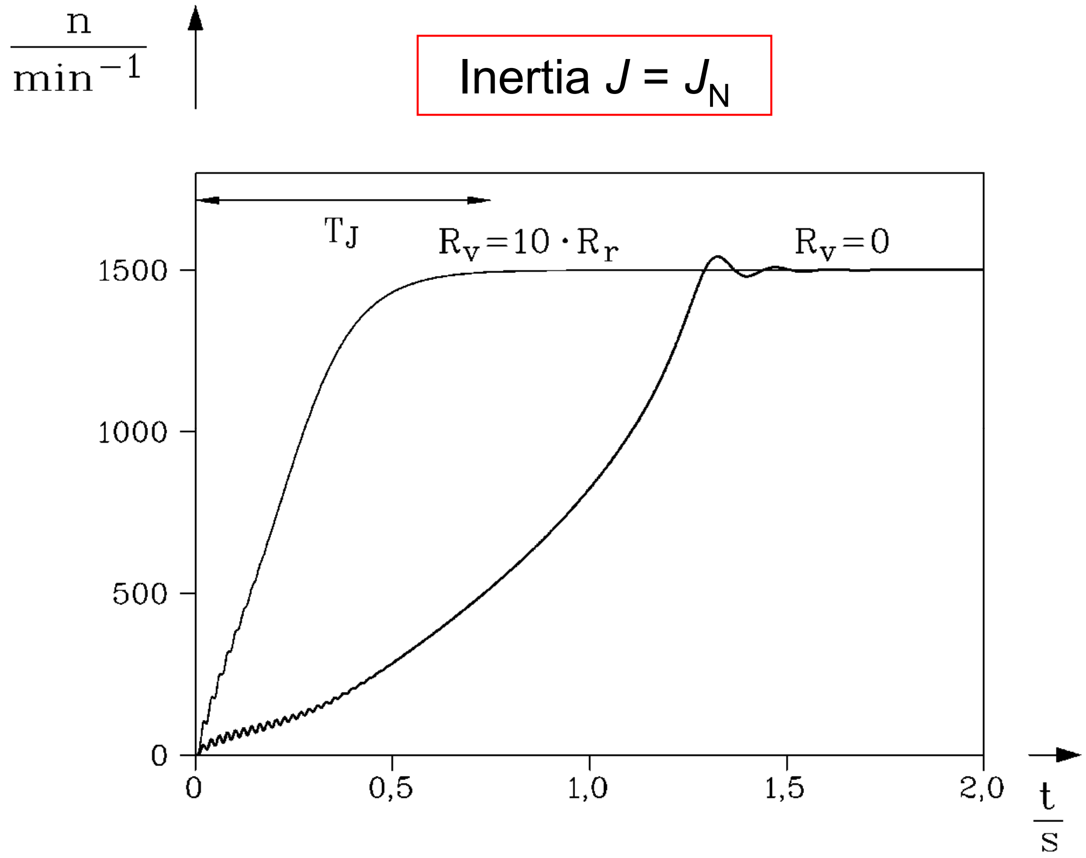
- **Additional rotor resistance:**
  Break-down torque is shifted from slip s_b = 0.08 to 0.88:

$$\frac{R_r}{s_b} = \frac{R_r + 10R_r}{s_b^*} \quad \rightarrow$$

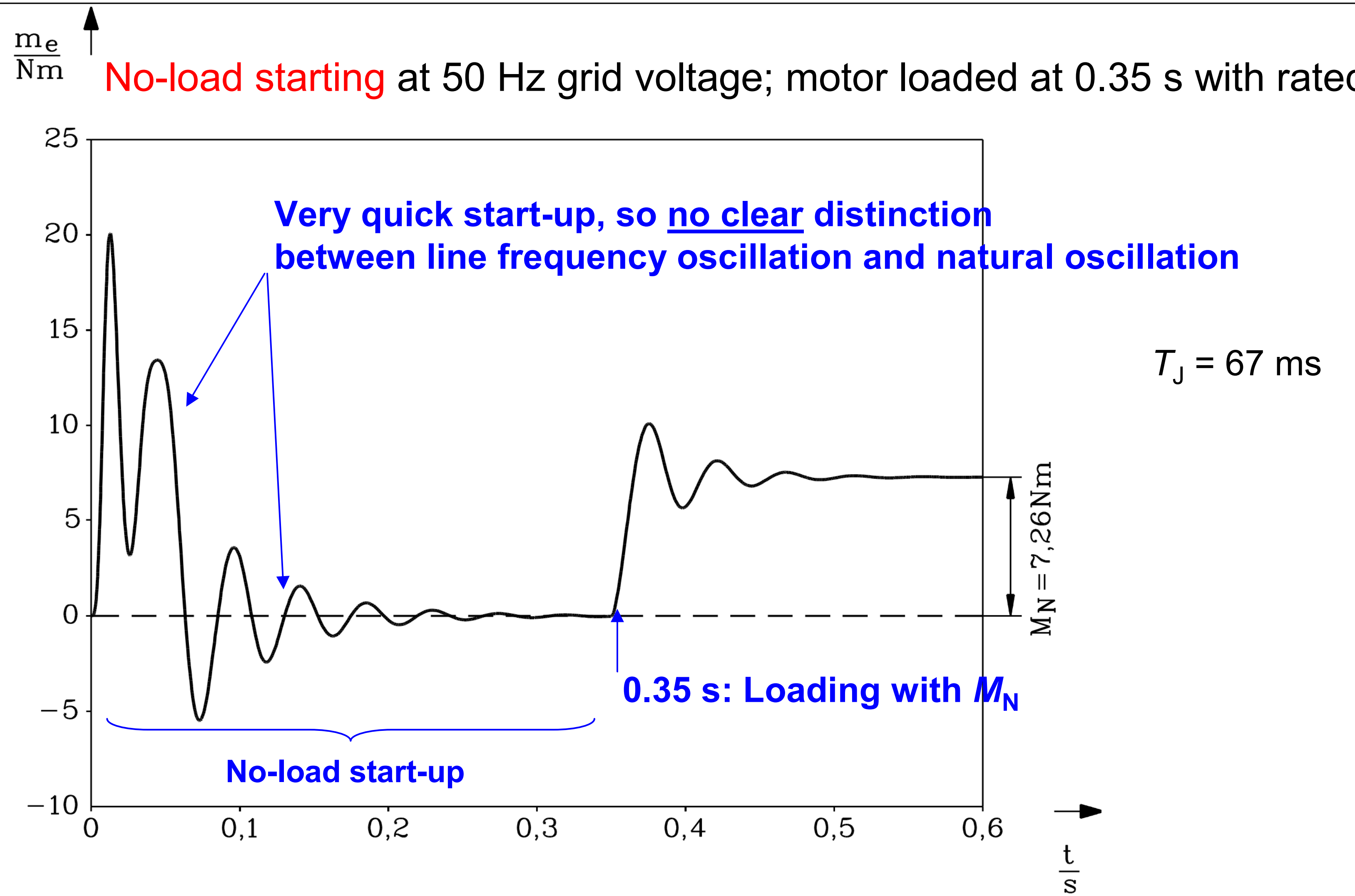
$$\rightarrow \quad s_b^* = 11 \cdot s_b = 11 \cdot 0.08 = 0.88$$

- Motor now starts nearly with break-down torque, hence with reduced start-up time





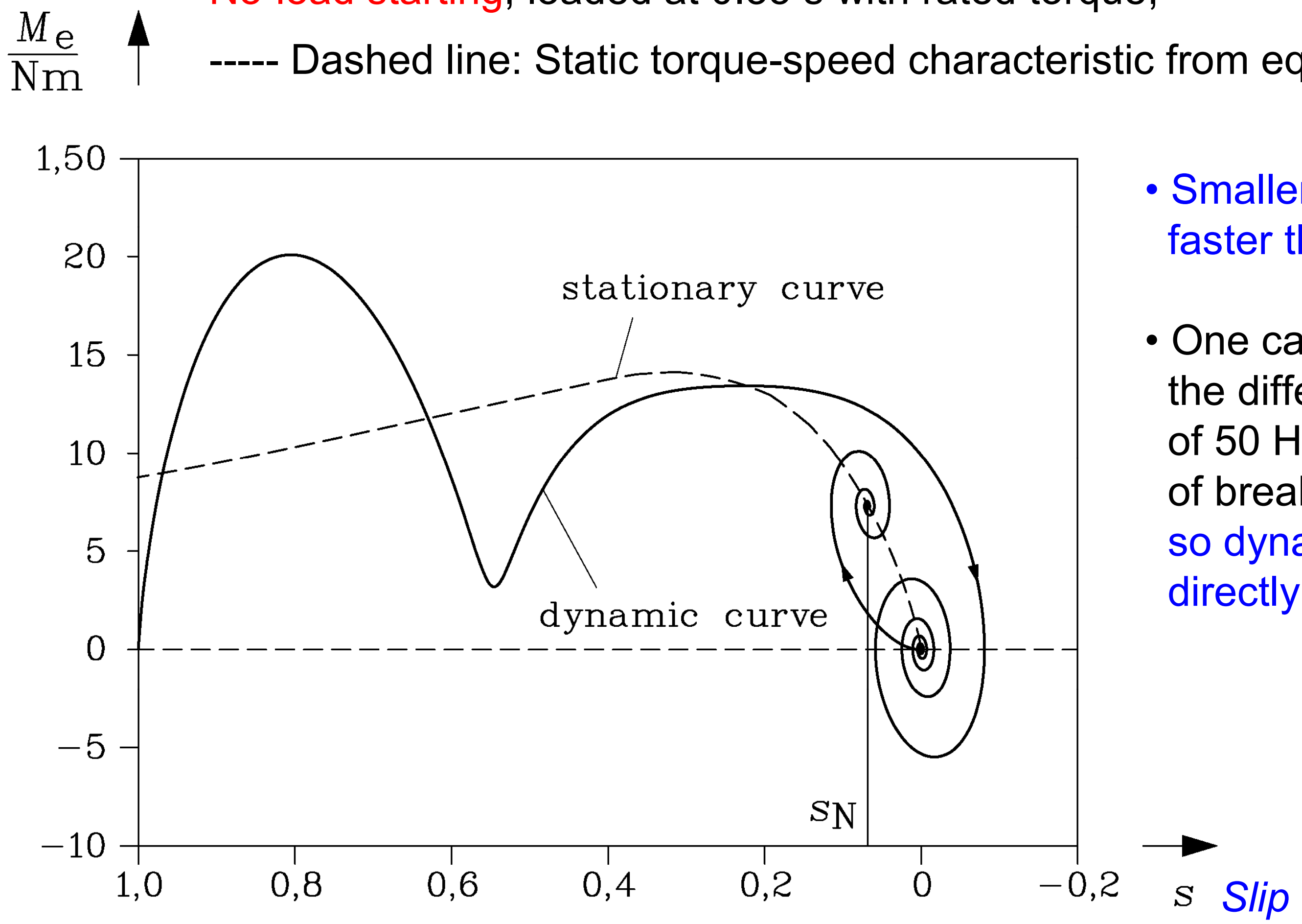
Calculated electromagnetic torque of "small" induction machine



7. Dynamics of induction machines
Starting performance of "small" induction motor

No-load starting, loaded at 0.35 s with rated torque,

----- Dashed line: Static torque-speed characteristic from equivalent circuit diagram

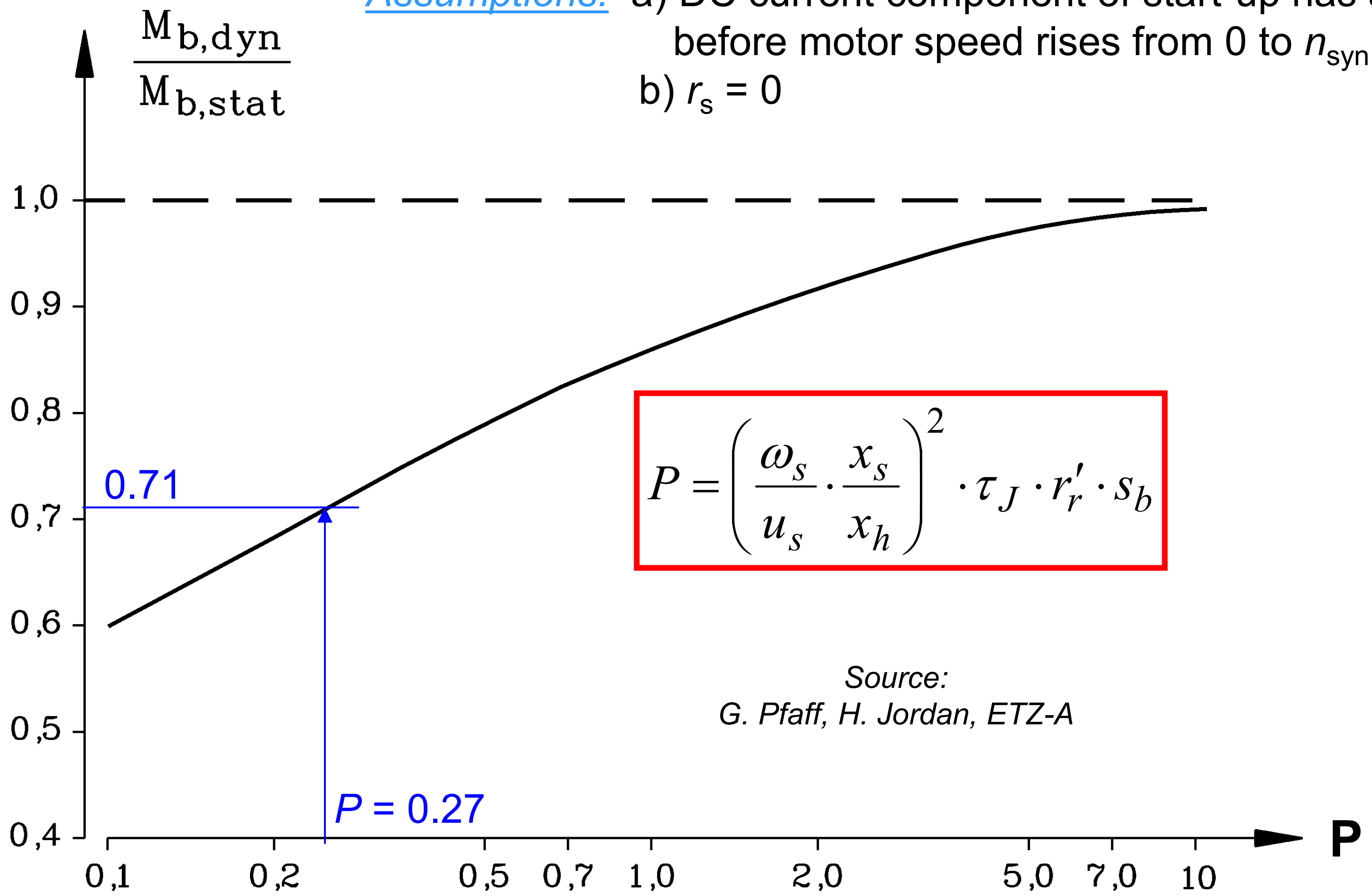


- Smaller Machine 2 accelerates much faster than Machine 1.
- One cannot separate clearly the different stages of 50 Hz oscillating torque and of breakdown torque, so dynamic breakdown torque is not directly visible.

Ratio of dynamic vs. static breakdown torque in dependence of parameter *P* (*Pfaff & Jordan*)

Assumptions: a) DC current component of start-up has already decayed, before motor speed rises from 0 to n_{syn}!

b) $r_s = 0$



$$P = \left(\frac{\omega_s}{u_s} \cdot \frac{x_s}{x_h} \right)^2 \cdot \tau_J \cdot r_r' \cdot s_b$$

Source:
G. Pfaff, H. Jordan, ETZ-A

7. Dynamics of induction machines

Dynamic break down torque: *Pfaff-Jordan* parameter *P*

$$\boxed{P = \left(\frac{\omega_s}{u_s}\cdot\frac{x_s}{x_h}\right)^2 \cdot \tau_J \cdot r_r' \cdot s_b = \left(\frac{2\pi f_s}{U_s}\cdot\frac{L_s}{L_h}\right)^2 \cdot J \cdot \frac{R_r' \cdot s_b \cdot 2\pi f_N}{3p^2}}$$

Example:

Data of induction machine 1: Break down slip s_b = 8%.

Line start at 50 Hz, rated voltage: U_s = 231 V

$$P = \left(\frac{2\pi f_s}{U_s}\cdot\frac{L_s}{L_h}\right)^2 \cdot J \cdot \frac{R_r' \cdot s_b \cdot 2\pi f_N}{3p^2} = \left(\frac{2\pi 50}{380/\sqrt{3}}\cdot\frac{0.00971}{0.00917}\right)^2 \cdot 2.8 \cdot \frac{0.02\cdot 0.08\cdot 2\pi 50}{3\cdot 2^2} = 0.2696$$

Curve with *P*: $M_{b,dyn} / M_{b,stat} = 0.71$

Compare: Numerical solution yields 0.74 ! = sufficient coincidence !

Explanation of *Pfaff-Jordan* parameter *P*

Bigger parameter *P* = less dynamic starting

$P \sim \tau_J$ Bigger inertia = slower motor acceleration = **less dynamic starting**

$$P \sim \left(\frac{x_s}{x_h}\right)^2 = \left(1 + \frac{x_{s\sigma}}{x_h}\right)^2$$

Bigger stator leakage flux = lower motor torque = less motor acceleration = **less dynamic starting**

$$P \sim \left(\frac{\omega_s}{u_s}\right)^2 = \frac{1}{\psi_s^2}$$

Bigger stator flux linkage ψ_s = bigger motor torque = faster motor acceleration = **more dynamic starting**

Summary:
Solutions of dynamic equations for induction machines with varying speed

- Non-linear mechanical equation: Numerical solution necessary
- Three dynamic phenomena:
    - Reduced breakdown torque $M_{b,dyn} < M_{b,stat}$,
    - Line-frequent starting torque oscillation
    - Low-frequent natural oscillation like in synchronous machines
- Numerical example for big and small motor
- Influence of inertia J
- *PFAFF-JORDAN* parameter P for estimation of dynamic breakdown torque from static torque curve

7. Dynamics of induction machines

Decomposition of transfer function of electrical system into α-β-components (n = const.)

$$\underline{\breve{i}}_s / \underline{\breve{u}}_s = \frac{r_r' + (s - j\omega_m)\cdot x_r'}{\sigma \cdot x_s \cdot x_r' \cdot (s - \underline{s}_a)\cdot(s - \underline{s}_b)} = \underline{G}(s) = \mathrm{Re}\{\underline{G}(s)\} + j\cdot \mathrm{Im}\{\underline{G}(s)\}$$

$$\underline{z} = \frac{a + jb}{c + jd} = \frac{a + jb}{c + jd}\cdot\frac{(c + jd)^*}{(c + jd)^*} = \frac{a + jb}{c + jd}\cdot\frac{c - jd}{c - jd} = \frac{ac + bd + j(bc - ad)}{c^2 + d^2}$$

$$\underline{G}(s) = \frac{r_r' + (s - j\omega_m)\cdot x_r'}{\sigma \cdot x_s \cdot x_r' \cdot (s - \underline{s}_a)\cdot(s - \underline{s}_b)} \cdot \frac{(s - \underline{s}_a)^* \cdot (s - \underline{s}_b)^*}{(s - \underline{s}_a)^* \cdot (s - \underline{s}_b)^*} = G_\alpha + jG_\beta$$

$$G_\alpha(s), G_\beta(s) \sim \frac{1}{(s - \underline{s}_a)\cdot(s - \underline{s}_b)\cdot(s - \underline{s}_a)^* \cdot (s - \underline{s}_b)^*}$$

$$s - \underline{s}_a = s - \mathrm{Re}(\underline{s}_a) - j\,\mathrm{Im}(\underline{s}_a)$$

Roots: $\boxed{\underline{s}_a, \underline{s}_a^*, \underline{s}_b, \underline{s}_b^*}$

$$(s - \underline{s}_a)^* = s - \underline{s}_a^* = s - \mathrm{Re}(\underline{s}_a) + j\,\mathrm{Im}(\underline{s}_a)$$

Transfer function of electrical system at *n* = const.

Complex space vector .... Complex transfer function of space vectors $\underline{G}(s)$
Two-axis components .... Real transfer function of components $G_\alpha(s)$, $G_\beta(s)$,

Two roots of complex transfer function $\underline{s}_a$, $\underline{s}_b$ =
= two pairs of conjugate complex roots of real transfer function $\underline{s}_1$, $\underline{s}_4$ & $\underline{s}_2$, $\underline{s}_5$

$$\underline{s}_a = -\frac{1}{\tau_a} + j\omega_{d,a} \quad , \qquad \underline{s}_b = -\frac{1}{\tau_b} + j\omega_{d,b}$$

$$\underline{s}_1 = -\frac{1}{\tau_a} + j\omega_{d,a}, \quad \underline{s}_4 = -\frac{1}{\tau_a} - j\omega_{d,a} \qquad \underline{s}_2 = -\frac{1}{\tau_b} + j\omega_{d,b}, \quad \underline{s}_5 = -\frac{1}{\tau_b} - j\omega_{d,b}$$

- NOTE: With variable speed dynamic equations are non-linear, so no linear transfer function exists.
- BUT: Small signal linearized equations allow transfer function formulation.

Roots of linearized electromechanical transfer function in *s*-plane

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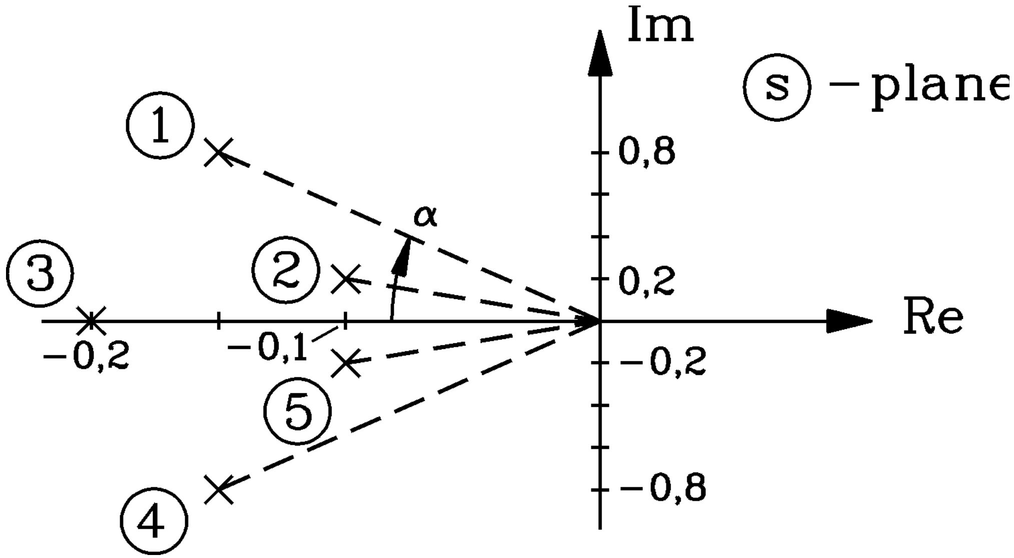
Transfer function of linearized performance (small signal theory) of induction machine for electromechanical performance (variable speed):

$$\underline{s}_1 = -\delta_1 + j\omega_{d,1} \qquad \underline{s}_4 = -\delta_1 - j\omega_{d,1}$$

$$\underline{s}_3 = -\delta_3$$

$$\underline{s}_2 = -\delta_2 + j\omega_{d,2} \qquad \underline{s}_5 = -\delta_2 - j\omega_{d,2}$$



The five roots of induction machine electro-mechanical transfer function comprise two pairs of conjugate complex poles and one real pole (= mechanical influence!)

Compare: *DC machines: Two poles for electromechanical performance.*

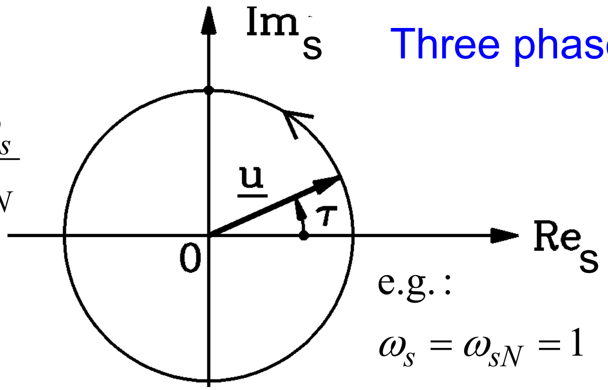
7. Dynamics of induction machines

Use of different co-ordinate systems

Stator reference frame	Rotor reference frame	Synchronous reference frame
Does not rotate	Rotates with $\omega_m = d\gamma / d\tau$	Rotates with $\omega_{syn} = d\delta / d\tau$
Use in induction machines	Use in synchronous machines	Use in induction machines for small signal theory
$\underline{u}_{(s)}(\tau) = u_\alpha(\tau) + ju_\beta(\tau)$	$\underline{u}_{(r)}(\tau) = u_d(\tau) + ju_q(\tau)$	$\underline{u}_{(syn)}(\tau) = u_a(\tau) + ju_b(\tau)$
$\underline{u}_{(s)} = \frac{2}{3}(u_U + \underline{a} \cdot u_V + \underline{a}^2 u_W)$	$\underline{u}_{(r)}(\tau) = \underline{u}_{(s)}(\tau) \cdot e^{-j\cdot\gamma(\tau)}$	$\underline{u}_{(syn)}(\tau) = \underline{u}_{(s)}(\tau) \cdot e^{-j\cdot\delta(\tau)}$

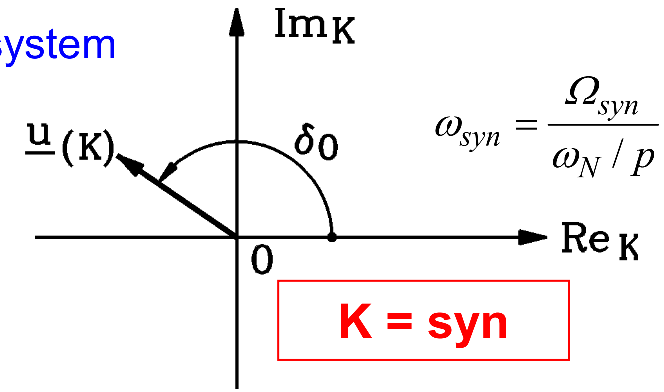
Three phase sinus voltage system

$$\omega_s = \frac{\Omega_s}{\omega_N}$$



e.g.: $\omega_s = \omega_{sN} = 1$

$$\omega_{syn} = \frac{\Omega_{syn}}{\omega_N / p} = \frac{\Omega_s / p}{\omega_N / p} = \omega_s$$



K = syn

Stator reference frame
Space vector rotates

Synchronous reference frame
Space vector does not move at line operation

7. Dynamics of induction machines

Voltage equations in synchronous reference frame

$$\boxed{\begin{aligned}\underline{u}_{s(syn)} &= r_s \cdot \underline{i}_{s(syn)} + \frac{d\underline{\psi}_{s(syn)}}{d\tau} + j \cdot \frac{d\delta}{d\tau} \cdot \underline{\psi}_{s(syn)} \\ \underline{u}'_{r(syn)} &= 0 = r'_r \cdot \underline{i}'_{r(syn)} + \frac{d\underline{\psi}'_{r(syn)}}{d\tau} + j \cdot \frac{d(\delta-\gamma)}{d\tau} \cdot \underline{\psi}'_{r(syn)}\end{aligned}}$$

$$\frac{d\delta}{d\tau} = \omega_s(\tau) \qquad \frac{d(\delta-\gamma)}{d\tau} = \omega_s(\tau) - \omega_m(\tau) \qquad \omega_s = \frac{\Omega_s}{\omega_N}, \quad \omega_{syn} = \frac{\Omega_{syn}}{\omega_N / p} = \frac{\Omega_s / p}{\omega_N / p} = \omega_s$$

Eliminating $\underline{i}_s$, $\underline{i}'_r$ via the flux linkages:

$$\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s + \frac{x_h}{x'_r} \cdot \underline{\psi}'_r \qquad \underline{\psi}'_r = \sigma \cdot x'_r \cdot \underline{i}'_r + \frac{x_h}{x_s} \cdot \underline{\psi}_s$$

$$\boxed{\begin{aligned}\underline{u}_s &= \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_s\right) \cdot \underline{\psi}_s + \frac{d\underline{\psi}_s}{d\tau} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_r \\ 0 &= -\frac{r'_r}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}_s + \left(\frac{r'_r}{\sigma \cdot x'_r} + j(\omega_s - \omega_m)\right) \cdot \underline{\psi}'_r + \frac{d\underline{\psi}'_r}{d\tau}\end{aligned}}$$

Subscript (syn) skipped!

Torque equation with flux linkages

- Eliminating $\underline{i}_s$, $\underline{i}'_r$ via the flux linkages:

$$\underline{\psi}_s = \sigma \cdot x_s \cdot \underline{i}_s + \frac{x_h}{x'_r} \cdot \underline{\psi}'_r \qquad \underline{\psi}'_r = \sigma \cdot x'_r \cdot \underline{i}'_r + \frac{x_h}{x_s} \cdot \underline{\psi}_s$$

$$m_e = -\operatorname{Im}\left\{\underline{i}'_r \cdot \underline{\psi}'^{*}_r\right\} = \frac{x_h^2/(x_s x'_r)}{\sigma \cdot x_h} \cdot \operatorname{Im}\left\{\underline{\psi}_s \cdot \underline{\psi}'^{*}_r\right\} = \frac{1-\sigma}{\sigma \cdot x_h} \cdot \operatorname{Im}\left\{\underline{\psi}_s \cdot \underline{\psi}'^{*}_r\right\}$$

$$\boxed{m_e = \frac{1-\sigma}{\sigma \cdot x_h} \cdot \operatorname{Im}\left\{\underline{\psi}_s \cdot \underline{\psi}'^{*}_r\right\}}$$

7. Dynamics of induction machines
Small signal theory in equilibrium points

- In synchronous reference frame: Current & voltage space vectors in equilibrium points are constant (= steady state operation due to equivalent circuit)
- Electromechanical system of equations in synchronous reference frame (a, b):

$$\left.\begin{aligned}
\underline{u}_s &= \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_s\right)\cdot \underline{\psi}_s + \frac{d\underline{\psi}_s}{d\tau} - \frac{r_s}{x_h}\cdot\frac{1-\sigma}{\sigma}\cdot\underline{\psi}'_r \\
0 &= -\frac{r'_r}{x_h}\cdot\frac{1-\sigma}{\sigma}\cdot\underline{\psi}_s + \left(\frac{r'_r}{\sigma\cdot x'_r} + j(\omega_s - \omega_m)\right)\cdot\underline{\psi}'_r + \frac{d\underline{\psi}'_r}{d\tau} \\
\tau_J \cdot \frac{d\omega_m}{d\tau} &= \frac{1-\sigma}{\sigma\cdot x_h}\cdot \mathrm{Im}\left\{\underline{\psi}_s \cdot \underline{\psi}'^{*}_r\right\} - m_s(\tau)
\end{aligned}\right\}$$

Unknowns:

$\underline{\psi}_s, \underline{\psi}'_r, \omega_m$

Input:

$\underline{u}_s, \omega_s, m_s$

- **Linearization** a) of 3 unknowns: s & r flux linkage, speed (machine performance)

  b) of 3 inputs: stator voltage, stator frequency, load torque

7. Dynamics of induction machines

Linearization of system equations

Linearization of unknowns:

$$\underline{\psi}_s(\tau) = \underline{\psi}_{s0} + \Delta\underline{\psi}_s(\tau)$$

$$\underline{\psi}'_r(\tau) = \underline{\psi}'_{r0} + \Delta\underline{\psi}'_r(\tau)$$

$$\omega_m(\tau) = \omega_{m0} + \Delta\omega_m(\tau)$$

Linearization of known input:

$$\underline{u}_s(\tau) = \underline{u}_{s0} + \Delta\underline{u}_s(\tau)$$

$$\omega_s(\tau) = \omega_{s0} + \Delta\omega_s(\tau)$$

$$m_s(\tau) = m_{s0} + \Delta m_s(\tau)$$

$$\left|\Delta\underline{u}_s / \underline{u}_{s0}\right| << 1, \left|\Delta\omega_s / \omega_{s0}\right| << 1,$$
$$\left|\Delta m_s / m_{s0}\right| << 1, \left|\Delta\omega_m / \omega_{m0}\right| << 1,$$
$$\left|\Delta\underline{\psi}_s / \underline{\psi}_{s0}\right| << 1, \left|\Delta\underline{\psi}'_r / \underline{\psi}'_{r0}\right| << 1$$

Linearization of stator voltage equation is with $d\underline{\psi}_{s0} / d\tau = 0$, $\Delta\underline{\psi}_s(0) = 0$:

$$\underline{u}_{s0} + \Delta\underline{u}_s = \left(\frac{r_s}{\sigma \cdot x_s} + j(\omega_{s0} + \Delta\omega_s)\right) \cdot (\underline{\psi}_{s0} + \Delta\underline{\psi}_s) + \frac{d(\underline{\psi}_{s0} + \Delta\underline{\psi}_s)}{d\tau} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot (\underline{\psi}'_{r0} + \Delta\underline{\psi}'_r)$$

(i) equilibrium point: $\underline{u}_{s0} = \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_{s0}\right) \cdot \underline{\psi}_{s0} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_{r0}$

(ii) deviations: $\Delta\underline{u}_s = \frac{r_s}{\sigma \cdot x_s}\Delta\underline{\psi}_s + j\omega_{s0}\Delta\underline{\psi}_s + j\Delta\omega_s\underline{\psi}_{s0} + \cancel{\underline{\underline{j\Delta\omega_s\Delta\underline{\psi}_s}}} + \frac{d\Delta\underline{\psi}_s}{d\tau} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \Delta\underline{\psi}'_r$

***Laplace* transform:**

$$\boxed{\Delta\breve{\underline{u}}_s \cong \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_{s0} + s\right)\Delta\breve{\underline{\psi}}_s + j\underline{\psi}_{s0}\Delta\breve{\omega}_s - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \Delta\breve{\underline{\psi}}'_r}$$

7. Dynamics of induction machines
Linearization of rotor voltage equation

- Rotor space vector voltage equation in equilibrium point: $\omega_{r0} = \omega_{s0} - \omega_{m0}$

$$0 = -\frac{r'_r}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}_{s0} + \left(\frac{r'_r}{\sigma \cdot x'_r} + j\omega_{r0} \right) \cdot \underline{\psi}'_{r0} \qquad \begin{cases} d\underline{\psi}'_{r0} / d\tau = 0 \\ \Delta\underline{\psi}'_r(0) = 0 \end{cases}$$

- Linearized rotor space vector voltage equation of small deviations from equilibrium:

$$0 \cong -\frac{r'_r}{x_h} \frac{1-\sigma}{\sigma} \Delta\underline{\psi}_s + \frac{r'_r}{\sigma \cdot x'_r} \Delta\underline{\psi}'_r + j\omega_{r0} \cdot \Delta\underline{\psi}'_r + j(\Delta\omega_s - \Delta\omega_m) \cdot \underline{\psi}'_{r0} + \frac{d\Delta\underline{\psi}'_r}{d\tau}$$

- Linearized rotor space vector voltage equation in *LAPLACE* domain:

$$\boxed{0 \cong -\frac{r'_r}{x_h} \frac{1-\sigma}{\sigma} \Delta\underline{\breve{\psi}}_s + \left(\frac{r'_r}{\sigma \cdot x'_r} + j\omega_{r0} + s \right) \Delta\underline{\breve{\psi}}'_r + j\underline{\psi}'_{r0} \Delta\breve{\omega}_s - j\underline{\psi}'_{r0} \Delta\breve{\omega}_m}$$

7. Dynamics of induction machines
Linearization of torque equation

Decomposition of complex space vectors in *a*-*b*-components:

$$\Delta\underline{\psi}_s(\tau) = \Delta\psi_{sa}(\tau) + j\Delta\psi_{sb}(\tau) \qquad \underline{\psi}_{s0} = \psi_{s0a} + j\cdot\psi_{s0b}$$

$$\Delta\underline{\psi}'_r(\tau) = \Delta\psi'_{ra}(\tau) + j\Delta\psi'_{rb}(\tau) \qquad \underline{\psi}'_{r0} = \psi'_{r0a} + j\cdot\psi'_{r0b}$$

$$\Delta\underline{u}_s(\tau) = \Delta u_{sa}(\tau) + j\Delta u_{sb}(\tau)$$

$$\tau_J \frac{d\omega_m}{d\tau} = \frac{1-\sigma}{\sigma\cdot x_h}\cdot\left(\psi_{sb}\cdot\psi'_{ra} - \psi_{sa}\cdot\psi'_{rb}\right) - m_s \qquad 0 = \frac{1-\sigma}{\sigma\cdot x_h}\cdot\left(\psi_{s0b}\cdot\psi'_{r0a} - \psi_{s0a}\cdot\psi'_{r0b}\right) - m_{s0}$$

$\Rightarrow$ Cancelling of equilibrium condition and linearization, e.g.: $\Delta\psi'_{rb}\cdot\Delta\psi_{sa} \approx 0$

$$\tau_J \frac{d\Delta\omega_m}{d\tau} \cong \frac{1-\sigma}{\sigma\cdot x_h}\left(\psi_{s0b}\cdot\Delta\psi'_{ra} + \Delta\psi_{sb}\cdot\psi'_{r0a} - \psi_{s0a}\cdot\Delta\psi'_{rb} - \Delta\psi_{sa}\cdot\psi'_{r0b}\right) - \Delta m_s$$

$\Rightarrow$ *Laplace*-transformation ($\Delta\omega_m(0) = 0$):

$$\boxed{s\cdot\tau_J\cdot\Delta\breve{\omega}_m \cong \frac{1-\sigma}{\sigma\cdot x_h}\cdot\left(\psi_{s0b}\cdot\Delta\breve{\psi}'_{ra} + \psi'_{r0a}\cdot\Delta\breve{\psi}_{sb} - \psi_{s0a}\cdot\Delta\breve{\psi}'_{rb} - \psi'_{r0b}\cdot\Delta\breve{\psi}_{sa}\right) - \Delta\breve{m}_s}$$

7. Dynamics of induction machines

Linearized voltage equations in a-b-components

$$\Delta\underline{\breve{u}}_s \cong \left(\frac{r_s}{\sigma\cdot x_s} + j\omega_{s0} + s\right)\cdot\Delta\underline{\breve{\psi}}_s + j\underline{\psi}_{s0}\cdot\Delta\breve{\omega}_s - \frac{r_s}{x_h}\cdot\frac{1-\sigma}{\sigma}\cdot\Delta\underline{\breve{\psi}}'_r$$

$$0 \cong -\frac{r'_r}{x_h}\frac{1-\sigma}{\sigma}\Delta\underline{\breve{\psi}}_s + \left(\frac{r'_r}{\sigma\cdot x'_r} + j\omega_{r0} + s\right)\cdot\Delta\underline{\breve{\psi}}'_r + j\underline{\psi}'_{r0}\Delta\breve{\omega}_s - j\underline{\psi}'_{r0}\Delta\breve{\omega}_m$$

$$\Delta\breve{u}_{sa} \cong \left(\frac{r_s}{\sigma\cdot x_s} + s\right)\cdot\Delta\breve{\psi}_{sa} - \omega_{s0}\cdot\Delta\breve{\psi}_{sb} - \psi_{s0b}\cdot\Delta\breve{\omega}_s - \frac{r_s}{x_h}\cdot\frac{1-\sigma}{\sigma}\cdot\Delta\breve{\psi}'_{ra}$$

$$\Delta\breve{u}_{sb} \cong \left(\frac{r_s}{\sigma\cdot x_s} + s\right)\cdot\Delta\breve{\psi}_{sb} + \omega_{s0}\cdot\Delta\breve{\psi}_{sa} + \psi_{s0a}\cdot\Delta\breve{\omega}_s - \frac{r_s}{x_h}\cdot\frac{1-\sigma}{\sigma}\cdot\Delta\breve{\psi}'_{rb}$$

$$0 \cong -\frac{r'_r}{x_h}\frac{1-\sigma}{\sigma}\Delta\breve{\psi}_{sa} + \left(\frac{r'_r}{\sigma\cdot x'_r} + s\right)\cdot\Delta\breve{\psi}'_{ra} - \omega_{r0}\cdot\Delta\breve{\psi}'_{rb} - \psi'_{r0b}\cdot\Delta\breve{\omega}_s + \psi'_{r0b}\cdot\Delta\breve{\omega}_m$$

$$0 \cong -\frac{r'_r}{x_h}\frac{1-\sigma}{\sigma}\Delta\breve{\psi}_{sb} + \left(\frac{r'_r}{\sigma\cdot x'_r} + s\right)\cdot\Delta\breve{\psi}'_{rb} + \omega_{r0}\cdot\Delta\breve{\psi}'_{ra} + \psi'_{r0a}\cdot\Delta\breve{\omega}_s - \psi'_{r0a}\cdot\Delta\breve{\omega}_m$$

7. Dynamics of induction machines

Laplace matrix equation system of induction machine: Linearized, in synchronous reference frame

$$
\underbrace{\begin{pmatrix}
s+\dfrac{r_s}{\sigma\cdot x_s} & -\omega_{s0} & -\dfrac{r_s(1-\sigma)}{\sigma\cdot x_h} & 0 & 0 \\
\omega_{s0} & s+\dfrac{r_s}{\sigma\cdot x_s} & 0 & -\dfrac{r_s(1-\sigma)}{\sigma\cdot x_h} & 0 \\
-\dfrac{r'_r(1-\sigma)}{\sigma\cdot x_h} & 0 & s+\dfrac{r'_r}{\sigma\cdot x'_r} & -\omega_{r0} & \psi'_{r0b} \\
0 & -\dfrac{r'_r(1-\sigma)}{\sigma\cdot x_h} & \omega_{r0} & s+\dfrac{r'_r}{\sigma\cdot x'_r} & -\psi'_{r0a} \\
\dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi'_{r0b}}{\tau_J} & -\dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi'_{r0a}}{\tau_J} & -\dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi_{s0b}}{\tau_J} & \dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi_{s0a}}{\tau_J} & s
\end{pmatrix}}_{(N)}
\underbrace{\begin{pmatrix}
\Delta\breve{\psi}_{sa} \\ \Delta\breve{\psi}_{sb} \\ \Delta\breve{\psi}'_{ra} \\ \Delta\breve{\psi}'_{rb} \\ \Delta\breve{\omega}_m
\end{pmatrix}}_{(\Psi)}
=
\underbrace{\begin{pmatrix}
\Delta\breve{u}_{sa}+\psi_{s0b}\cdot\Delta\breve{\omega}_s \\
\Delta\breve{u}_{sb}-\psi_{s0a}\cdot\Delta\breve{\omega}_s \\
\psi'_{r0b}\cdot\Delta\breve{\omega}_s \\
-\psi'_{r0a}\cdot\Delta\breve{\omega}_s \\
-\dfrac{\Delta\breve{m}_s}{\tau_J}
\end{pmatrix}}_{(U)}
$$

$$\boxed{(N)\cdot(\Psi)=(U)}$$

Linearized model: *CRAMER´s* rule for calculation of $\Delta\omega_m(s)$

$$(Z)=\begin{pmatrix} s+\dfrac{r_s}{\sigma\cdot x_s} & -\omega_{s0} & -\dfrac{r_s(1-\sigma)}{\sigma\cdot x_h} & 0 & \Delta\breve{u}_{sa}+\psi_{s0b}\cdot\Delta\breve{\omega}_s \\ \omega_{s0} & s+\dfrac{r_s}{\sigma\cdot x_s} & 0 & -\dfrac{r_s(1-\sigma)}{\sigma\cdot x_h} & \Delta\breve{u}_{sb}-\psi_{s0a}\cdot\Delta\breve{\omega}_s \\ -\dfrac{r'_r(1-\sigma)}{\sigma\cdot x_h} & 0 & s+\dfrac{r'_r}{\sigma\cdot x'_r} & -\omega_{r0} & \psi'_{r0b}\cdot\Delta\breve{\omega}_s \\ 0 & -\dfrac{r'_r(1-\sigma)}{\sigma\cdot x_h} & \omega_{r0} & s+\dfrac{r'_r}{\sigma\cdot x'_r} & -\psi'_{r0a}\cdot\Delta\breve{\omega}_s \\ \dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi'_{r0b}}{\tau_J} & -\dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi'_{r0a}}{\tau_J} & -\dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi_{s0b}}{\tau_J} & \dfrac{1-\sigma}{\sigma\cdot x_h}\dfrac{\psi_{s0a}}{\tau_J} & -\dfrac{\Delta\breve{m}_s}{\tau_J} \end{pmatrix}$$

Induction machine "system response" for speed:

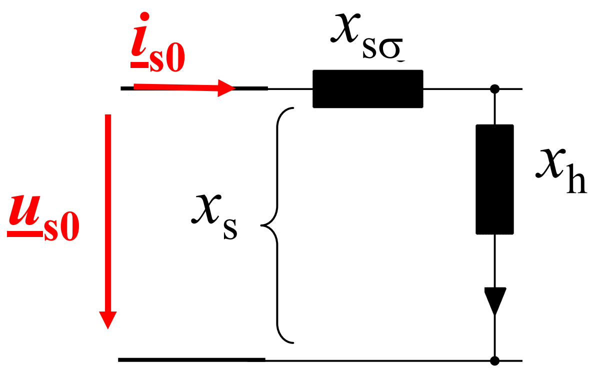
$$\boxed{(N)\cdot(\Psi)=(U)}$$

$$\boxed{\Delta\breve{\omega}_m=\frac{Det(Z)}{Det(N)}=\frac{f(\Delta\breve{u}_{sa},\Delta\breve{u}_{sb},\Delta\breve{\omega}_s,\Delta\breve{m}_s)}{P_5(s)}}$$

7. Dynamics of induction machines

Transfer function at no-load for small resistances & not too small frequencies > 0.5 f_N

$r_s = 0:$



- Small resistances: $r_s << 1, \quad r_r' << 1$
- Not too small stator frequencies: $\omega_{s0} > 0.5...0.6$
- Chosen equilibrium point is no-load operation:

Stator voltage space vector chosen as real: $\underline{u}_{s0} = u_{s0a} = u_{s0}$

$$\omega_{r0} = 0, \underline{i}'_{r0} = 0: \ \underline{\psi}_{s0} = x_s \underline{i}_{s0}, \underline{\psi}'_{r0} = x_h \underline{i}_{s0} = (x_h / x_s) \cdot \underline{\psi}_{s0}$$

$$\underline{u}_{s0} = \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_{s0} \right) \cdot \underline{\psi}_{s0} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_{r0} \qquad 0 = -\frac{r_r'}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}_{s0} + \left(\frac{r_r'}{\sigma \cdot x_r'} + j\omega_{r0} \right) \cdot \underline{\psi}'_{r0}$$

$$r_s = 0: u_{s0} = j\omega_{s0} \underline{\psi}_{s0} \quad \Rightarrow \quad \underline{\psi}_{s0} = -j \frac{u_{s0}}{\omega_{s0}} = \psi_{s0a} + j\,\psi_{s0b} \qquad \underline{\psi}'_{r0} = \frac{x_h}{x_s} \cdot \underline{\psi}_{s0}$$

$$\boxed{\psi_{s0a} = 0, \psi_{s0b} = -\frac{u_{s0}}{\omega_{s0}}} \qquad \boxed{\psi'_{r0a} = 0, \psi'_{r0b} = -\frac{x_h}{\omega_{s0} x_s} u_{s0}}$$

7. Dynamics of induction machines
Characteristic polynomial of linearized transfer function

(1) $r_s = r'_r$

(2) $x_s = x'_r$: **short-circuit time constants**: $\frac{1}{\tau_{s\sigma}} = \frac{r_s}{\sigma \cdot x_s} = \frac{1}{\tau_{r\sigma}} = \frac{r'_r}{\sigma \cdot x'_r} = \frac{1}{\tau_\sigma}$

(3) **not too small stator frequencies** $r_s << \omega_{s0} x_s$ $\quad$ $(\text{e.g.}: \frac{1}{\tau_\sigma} = \frac{0.03}{0.1 \cdot 3} = 0.1)$

(4) **usually small resistance** $r_s << x_s,\ r'_r << x'_r$

- Characteristic polynomial:

$$Det(N) = P_5(s) = (s + \frac{1}{\tau_\sigma}) \cdot \left[\left(s + \frac{1}{\tau_\sigma} \right)^2 + \omega_{s0}^2 \right] \cdot \left[\left(s + \frac{1}{2\tau_\sigma} \right)^2 + \omega_{d,m}^2 \right]$$

$$P_5(s) = (s - \underline{s}_1) \cdot (s - \underline{s}_2) \cdot (s - \underline{s}_3) \cdot (s - \underline{s}_4) \cdot (s - \underline{s}_5)$$

- NOTE: $\left(s + \frac{1}{\tau_\sigma} \right)^2 + \omega_{s0}^2 = \left(s + \frac{1}{\tau_\sigma} + j \cdot \omega_{s0} \right) \cdot \left(s + \frac{1}{\tau_\sigma} - j \cdot \omega_{s0} \right)$ $\quad \begin{cases} \underline{s}_1 = -\frac{1}{\tau_\sigma} - j \cdot \omega_{s0} \\ \underline{s}_4 = -\frac{1}{\tau_\sigma} + j \cdot \omega_{s0} \end{cases}$

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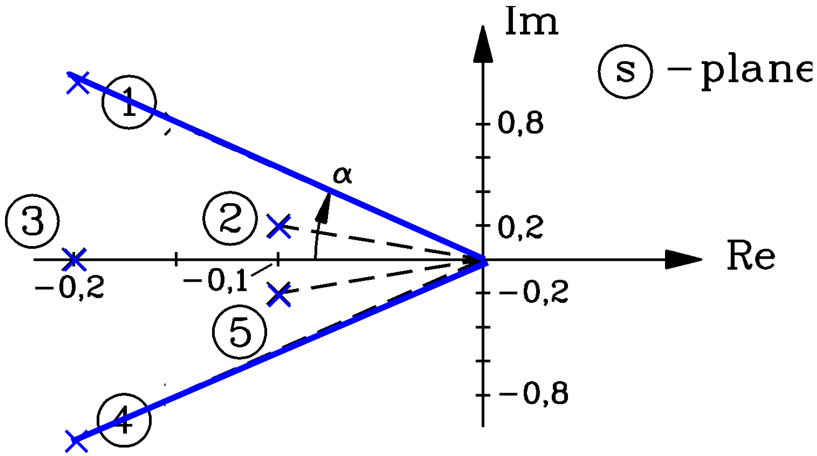
Roots of linearized electromechanical transfer function in *s*-plane

- Five roots of $P_5(s)$ = five poles in *s*-plane: $\underline{s}_1 = -\delta_1 + j\omega_{d,1}$ $\quad \underline{s}_4 = -\delta_1 - j\omega_{d,1}$

$$\underline{s}_3 = -\delta_3$$

$$\underline{s}_2 = -\delta_2 + j\omega_{d,2} \qquad \underline{s}_5 = -\delta_2 - j\omega_{d,2}$$



$$\delta_1 = \delta_3 = \frac{1}{\tau_\sigma} \qquad \delta_2 = \frac{1}{2\tau_\sigma}$$

- Mechanical system natural frequency:

$$\omega_{d,2} = \sqrt{\frac{1}{\tau_J}\cdot\frac{1-\sigma}{\sigma\cdot x_s}\cdot\left(\frac{u_{s0}}{\omega_{s0}}\right)^2 - \frac{1}{(2\tau_\sigma)^2}}$$

$$\omega_{d,1} = \omega_{s0} \qquad \omega_{d,2} = \omega_{d,m}$$

- Poles $\underline{s}_1$, $\underline{s}_3$, $\underline{s}_4$: DC components in current and flux $\Rightarrow$ torque and speed oscillation with stator frequency.
- Poles $\underline{s}_2$, $\underline{s}_5$: “frozen" rotor flux = **"synchronous" machine phenomenon**!

Mechanical system natural frequency

- "Frozen" rotor flux = **"synchronous" machine phenomenon!**

- **Per unit values:** $\omega_{d,m} = \sqrt{\frac{1}{\tau_J}\cdot\frac{1-\sigma}{\sigma\cdot x_s}\cdot\left(\frac{u_{s0}}{\omega_{s0}}\right)^2 - \frac{1}{(2\tau_\sigma)^2}}$

  (No-load, $r_s \approx r_r' << x_s \approx x_r'$ **)**

- **Physical units:** $\Omega_{d,m} = \sqrt{\frac{3p^2}{J}\cdot\frac{1-\sigma}{\sigma\cdot L_s}\cdot\left(\frac{U_{s0}}{\Omega_{s0}}\right)^2 - \frac{1}{(2T_\sigma)^2}}$

- **Breakdown torque at $R_s = 0$:** $M_b = \pm\frac{m_s}{2}\cdot\frac{p}{\Omega_s^2}\cdot U_s^2\cdot\frac{1-\sigma}{\sigma L_s}$ ***m_s* = 3**

  (of *KLOSS*-function)

$$\boxed{\Omega_{d,m} = \sqrt{\frac{2pM_b}{J} - \frac{1}{(2T_\sigma)^2}}}$$

Valid for small signal at no-load operation!

Mechanical system natural frequency for small signals

- "Frozen" rotor flux = **"synchronous" machine phenomenon**!

- **Induction machine:**

(No-load, $r_s \approx r_r' << x_s \approx x_r'$ **)**

$$\boxed{\Omega_{d,m} = \sqrt{\frac{2pM_b}{J} - \frac{1}{(2T_\sigma)^2}}}$$

$$\left.\frac{dM_e}{ds}\right|_{s=0} = \frac{2M_b}{s_b}$$

- **COMPARE: Synchronous machine (without damping):** $\Omega_{d,m} = \sqrt{\frac{p \cdot |c_\vartheta|}{J}}$

At no-load equilibrium: $c_\vartheta = -\partial M_e / \partial \vartheta\big|_{\vartheta=0} = -M_{p0}$

- **Synchronous machine (with damping):** $\Omega_{d,m} = \sqrt{\frac{p \cdot M_{p0}}{J} - \alpha^2}$ $\qquad \alpha = 1/T$

(No-load)

$r_s << 1$

$$\boxed{\Omega_{d,m} = \sqrt{\frac{p \cdot M_{p0}}{J} - \frac{1}{T^2}}}$$

7. Dynamics of induction machines

Calculation of natural frequency of induction machine

Example:

110 kW 4-pole cage induction machine:

$$r_s = 0.024, r_r' = 0.019,\ x_s = 2.95, x_r' = 2.9,\ x_h = 2.78, \sigma = 0.094,$$

$$\tau_J = 155.5$$

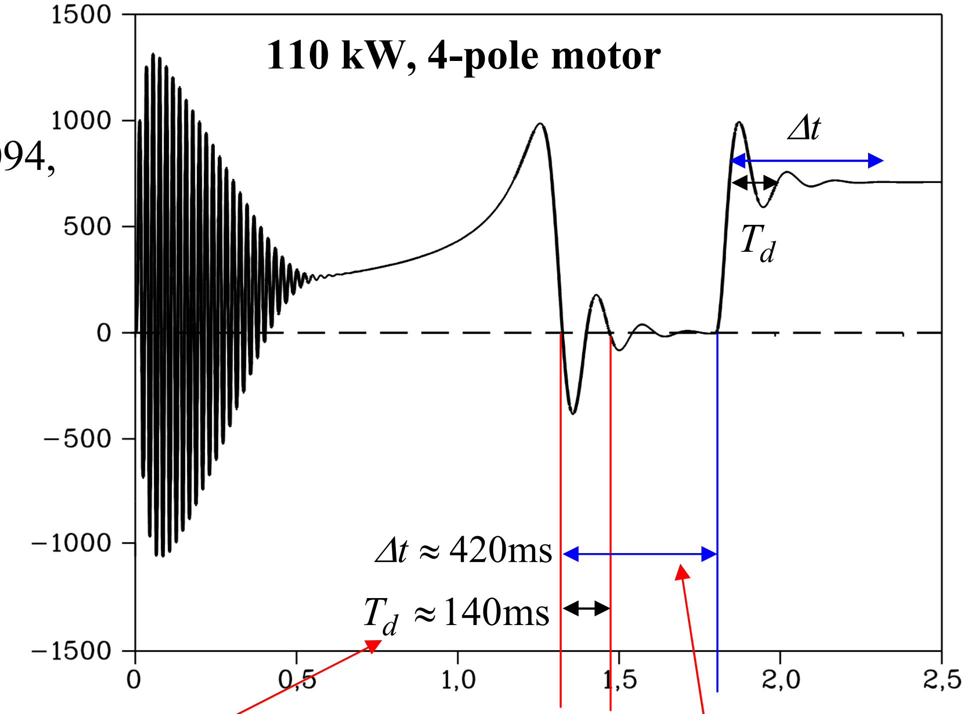
$$r = \frac{r_s + r_r'}{2} = 0.022, x = \frac{x_s + x_r'}{2} = 2.93, \sigma = 1 - \frac{x_h^2}{x^2} = 0.1$$

$$\tau_\sigma = \frac{\sigma \cdot x}{r} = \frac{0.1 \cdot 2.93}{0.022} = 13.3$$

$$T_\sigma = \tau_\sigma / \omega_N = 13.3/(2\pi 50) = \underline{\underline{42.3\,\mathrm{ms}}}$$

$$u_{s0} = 1, \quad \omega_{s0} = 1$$

$$\omega_{d,2} = \sqrt{\frac{1}{155.5} \cdot \frac{1-0.1}{0.1 \cdot 2.93} \cdot \left(\frac{1}{1}\right)^2 - \frac{1}{(2 \cdot 13.3)^2}} = 0.135$$



$$f_{d,m} = f_N \cdot \omega_{d,2} = 50 \cdot 0.135 = \underline{\underline{6.77\ \mathrm{Hz}}}, \quad T_d = 1/6.77 = 147.7\ \mathrm{ms} \quad \Delta t \cong 5 \cdot (2T_\sigma) = 5 \cdot 2 \cdot 42.3\ \mathrm{ms} = 423\ \mathrm{ms}$$

7. Dynamics of induction machines

Equilibrium point: *d./dτ* = 0

$$\underline{u}_{s0} = \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_{s0}\right) \cdot \underline{\psi}_{s0} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_{r0}$$

$$0 = -\frac{r'_r}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}_{s0} + \left(\frac{r'_r}{\sigma \cdot x'_r} + j(\omega_{s0} - \omega_{m0})\right) \cdot \underline{\psi}'_{r0}$$

Rated operation:

$$\underline{u}_{s0} = u_{sN} = 1 \qquad \omega_{s0} = \omega_{sN} = 1$$

$$\omega_{s0} - \omega_{m0} = 1 - \omega_{mN} = Slip_N$$

$$u_{sN} = \left(\frac{r_s}{\sigma \cdot x_s} + j\right) \cdot \underline{\psi}_{sN} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_{rN}$$

$$0 = -\frac{r'_r}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}_{sN} + \left(\frac{r'_r}{\sigma \cdot x'_r} + j \cdot Slip_N\right) \cdot \underline{\psi}'_{rN}$$

Rated rotor flux linkage:

$$\underline{\psi}'_{rN} = \frac{-\frac{r'_r}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot u_{sN}}{\frac{r_s}{x_h} \cdot \frac{r'_r}{x_h} \cdot \left(\frac{1-\sigma}{\sigma}\right)^2 - \left(\frac{r'_r}{\sigma \cdot x'_r} + j \cdot Slip_N\right) \cdot \left(\frac{r_s}{\sigma \cdot x_s} + j\right)}$$

- **Selected equilibrium points at:**

$$\underline{\psi}'_{r0} = \underline{\psi}'_{rN},\ \omega_{m0},\ \omega_{s0} \rightarrow \underline{\psi}_{s0},\ \underline{u}_{s0}$$

$$\underline{\psi}_{s0} = \frac{1}{1-\sigma} \cdot \left(\frac{x_h}{x'_r} + j \cdot \frac{\sigma \cdot x_h}{r'_r} \cdot (\omega_{s0} - \omega_{m0})\right) \cdot \underline{\psi}'_{rN}$$

$$\underline{u}_{s0} = \left(\frac{r_s}{\sigma \cdot x_s} + j\omega_{s0}\right) \cdot \underline{\psi}_{s0} - \frac{r_s}{x_h} \cdot \frac{1-\sigma}{\sigma} \cdot \underline{\psi}'_{rN}$$

- ***Example:*** No-load as equilibrium point with <u>rated rotor flux linkage</u>: $\underline{\psi}'_{r0} = \underline{\psi}'_{rN},\ \omega_{m0} = \omega_{s0}$

$$\underline{\psi}_{s0} = \frac{1}{1-\sigma} \cdot \frac{x_h}{x'_r} \cdot \underline{\psi}'_{rN} = \frac{x_s}{x_h} \cdot \underline{\psi}'_{rN} \qquad \underline{u}_{s0} = \left[\frac{r_s}{\sigma \cdot x_s} + j \cdot \omega_{s0} - \frac{r_s}{x_s} \cdot \frac{1-\sigma}{\sigma}\right] \cdot \underline{\psi}_{s0} \Bigg|_{r_s \ll 1} \approx j \cdot \omega_{s0} \cdot \underline{\psi}_{s0} \sim \omega_{s0}$$

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Example:

30 kW 4-pole cage induction machine:

$$r_s = r_r' = 0.03,\ x_s = x_r' = 3.0,\ \sigma = 0.0667,\ \tau_J = 75$$

$$\tau_{s\sigma} = \tau_{r\sigma} = \frac{\sigma \cdot x_s}{r_s} = \frac{0.0667 \cdot 3}{0.03} = 6.67 \quad T_{s\sigma} = T_{r\sigma} = \tau_{s\sigma} / \omega_N = 6.67/(2\pi 50) = \underline{\underline{21.2}}\ \text{ms}$$

Operation point (= equilibrium point): $u_{s0} = 1, \quad \omega_{s0} = 1:$

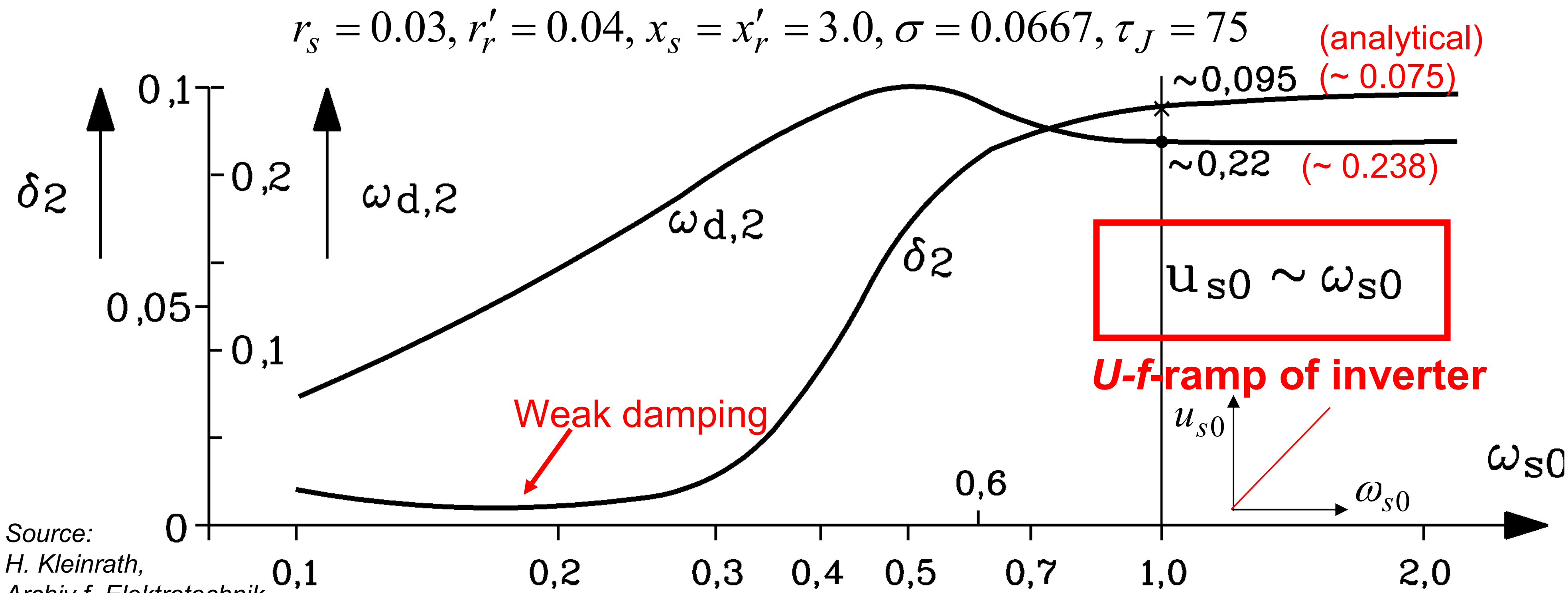
$$\omega_{d,2} = \sqrt{\frac{1}{75} \cdot \frac{1-0.0667}{0.0667 \cdot 3} \cdot \left(\frac{1}{1}\right)^2 - \frac{1}{(2 \cdot 6.67)^2}} = \boxed{0.238}$$

$$f_{d,m} = f_N \cdot \omega_{d,2} = 50 \cdot 0.238 = \underline{\underline{11.9}}\text{Hz}, \quad T_d = 1/11.9 = 84\text{ms}$$

$$\delta_2 = \frac{1}{2 \cdot 6.67} = \boxed{0.075}$$

7. Dynamics of induction machines

Small-signal transfer function for varying stator frequency



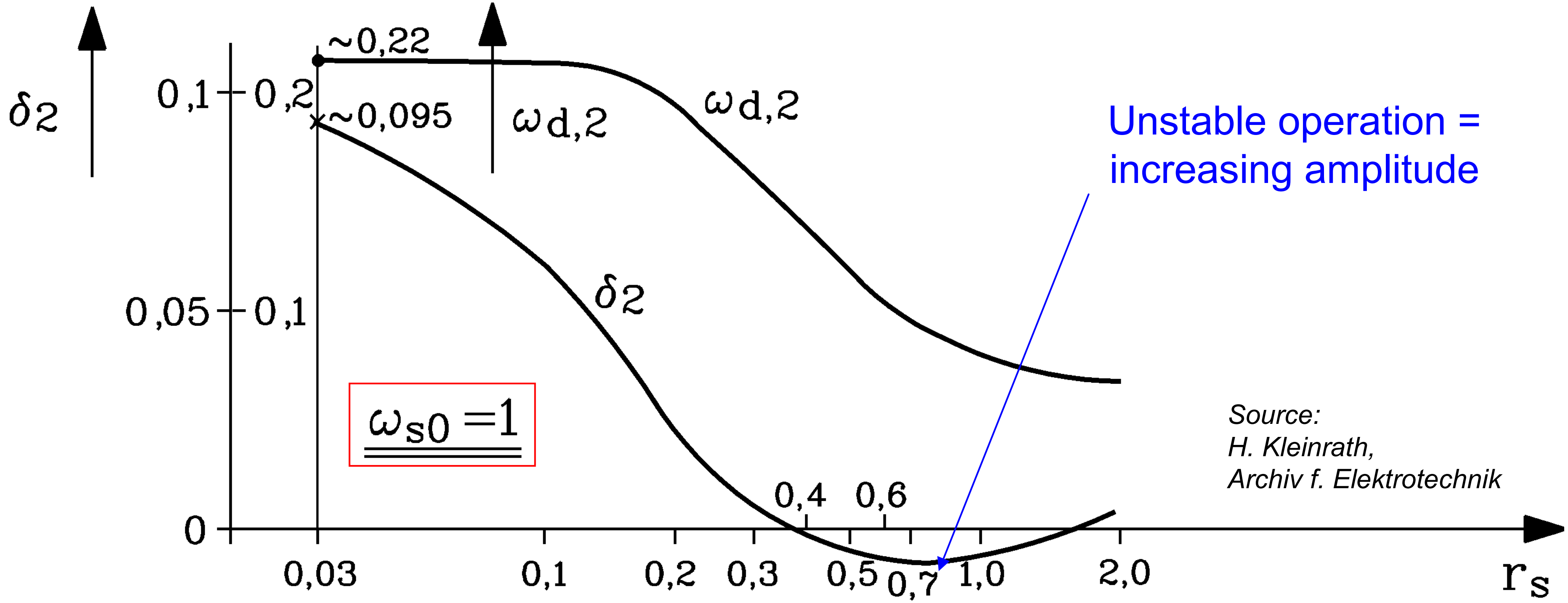
Source: H. Kleinrath, Archiv f. Elektrotechnik

- Variation of damping δ_2 and natural angular frequency $\omega_{d,2}$ with varying feeding stator frequency ω_{s0} and constant rated rotor flux linkage $\underline{\psi}'_{r0} = \underline{\psi}'_{rN}$
- No-load machine operation $m_{s0} = 0,\ \omega_{m0} = \omega_{s0}$, $Slip = 0$.

$\Big\}\ \underline{u}_{s0} \approx j \cdot \omega_{s0} \cdot \underline{\psi}_{s0}$

⇒ WEAK DAMPING AT LOW FREQUENCY, if machine is not controlled !

Small-signal transfer function for increasing stator resistance r_s

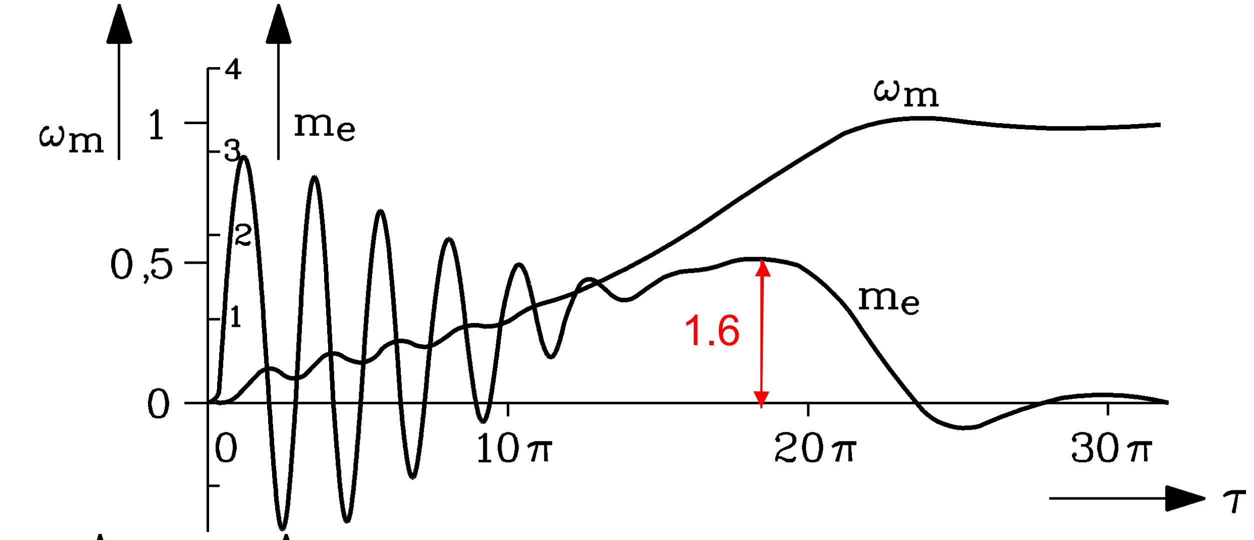


- Variation of damping δ_2 and natural angular frequency $\omega_{d,2}$ with varying stator resistance r_s at rated stator frequency ω_{s0} = 1 and constant rated rotor flux linkage $\underline{\psi}'_{r0} = \underline{\psi}'_{rN}$
- No-load machine operation $m_{s0} = 0,\ \omega_{m0} = \omega_{s0} = 1,\ Slip = 0.$

⇒ UNSTABLE OPERATION AT BIG STATOR RESISTANCE !

7. Dynamics of induction machines

Starting machine at no load, constant stator voltage & frequency



a) directly

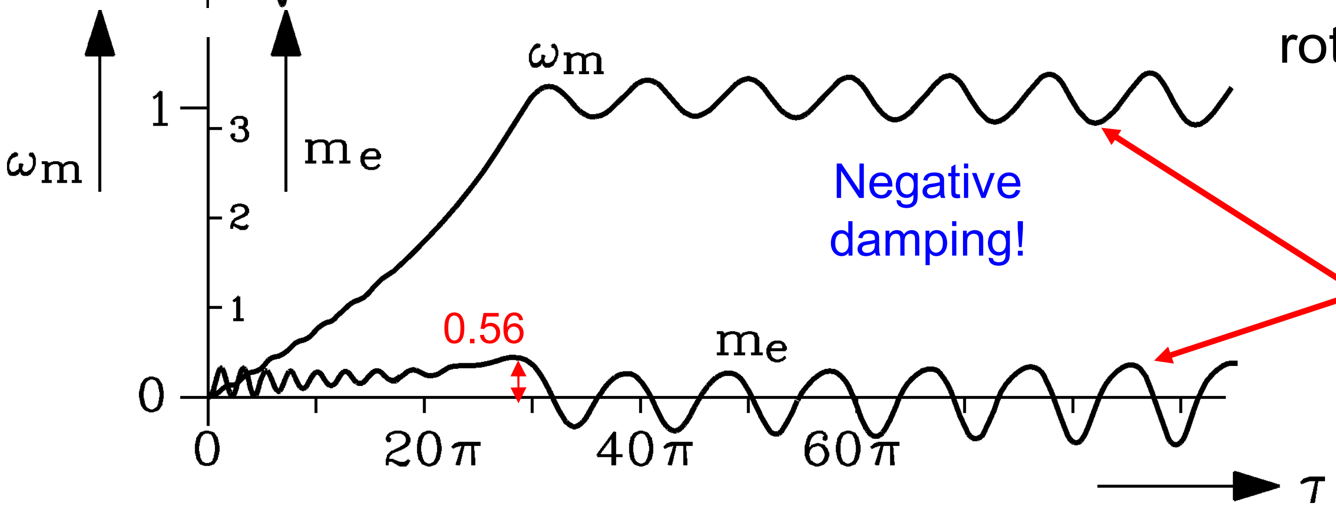
$r_s = 0.03,\ r_r' = 0.04,\ x_s = x_r' = 3.0,$

$\sigma = 0.0667,\ \tau_J = 75$

b) with 13-fold increased stator resistance

$r_s = 13 \cdot 0.03 = 0.39$

and by 1/3 reduced rotor inertia: $\tau_J = 75/3 = 25$.



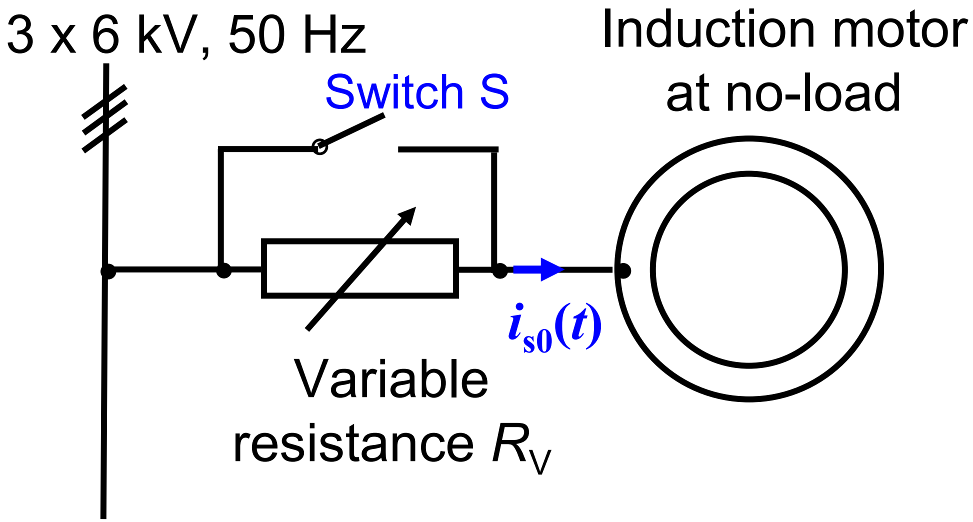
Reduced breakdown torque:

$(1-0.39)^2 \cdot 1.6 \cong 0.56$

Speed oscillation amplitude increases = UNSTABLE operation

7. Dynamics of induction machines

Measurement of instability at big stator resistance r_s



Cage induction motor with deep bar rotor

6 kV Y, 3500 kW, 415 A, 50 Hz

$Z_N = 8.35\ \Omega$, $2p = 4$

$R_s = 0.0426\ \Omega$ (20°C), $r_s = 0.0426/8.35 = 0.5\%$

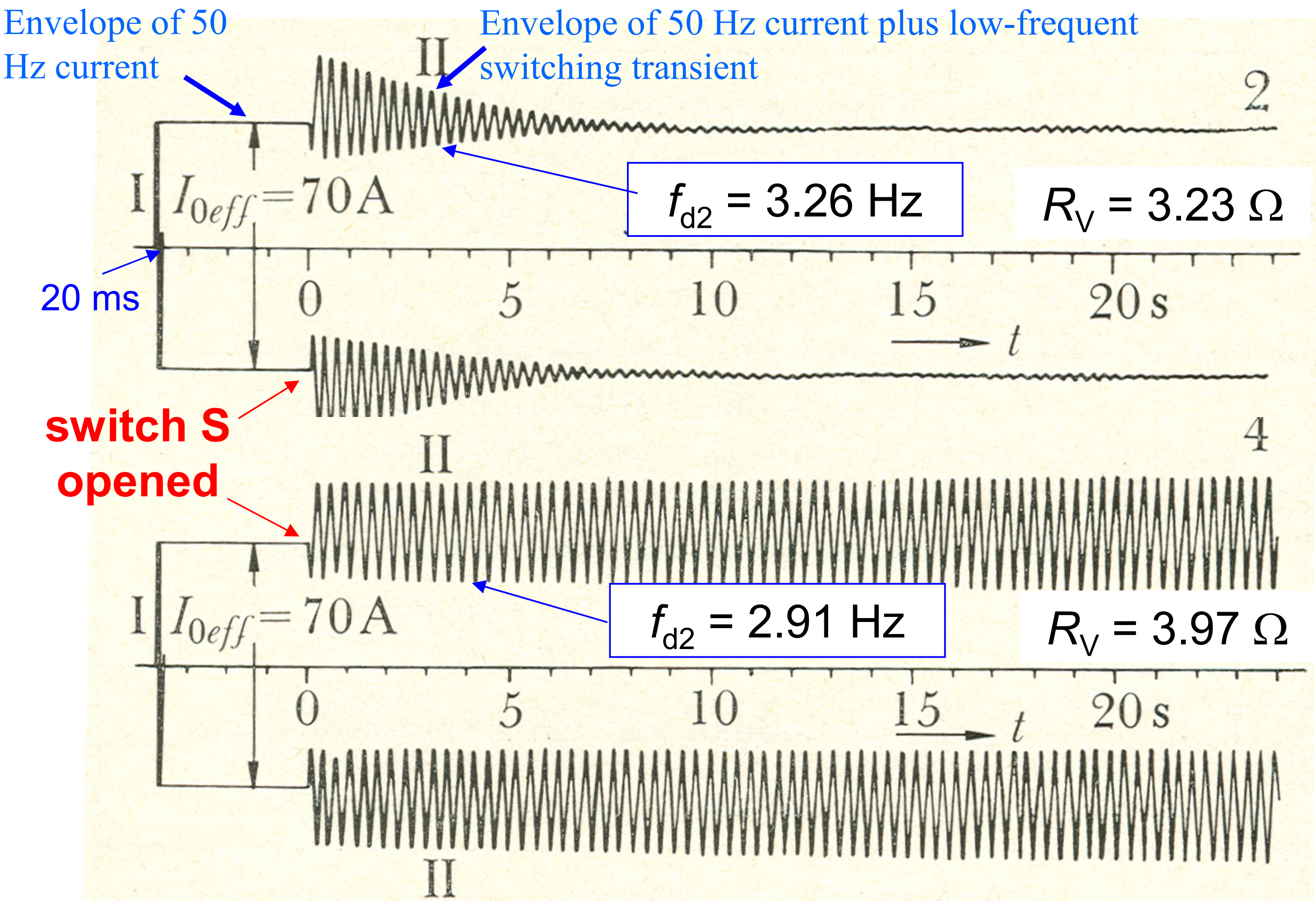
No-load current: $I_{s0} = 74.2$ A

First reported measurement in history of induction machine instability (BBC, Mannheim, Germany, 1969):

- Direct no-load start up of the induction motor at the 20 kV-grid with a 20kV/6 kV-transformer at closed switch *S*. Transformer impedance $X_T \approx X_{s\sigma}$ helps to stabilize!
- Opening of the switch S after completed start-up at no-load speed
- Measurement of the 50 Hz-no-load current $i_{s0}(t)$

7. Dynamics of induction machines

Measured stator current with a big stator resistance r_s



Stable operation

$R_V / R_s = 75.8$

$r_s + r_V = 0.39$

Zero damping = Limit of stability:

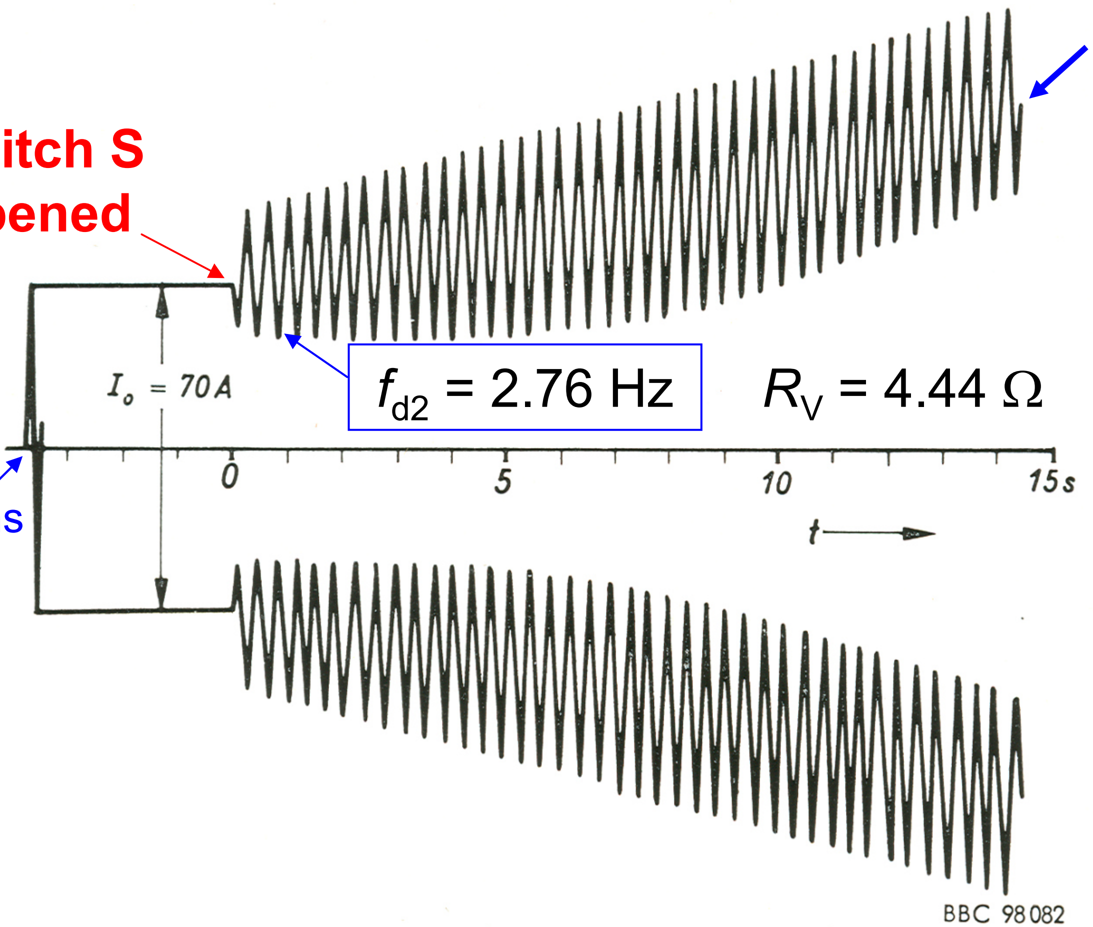
$R_V / R_s = 93.2$

$r_s + r_V = 0.48$

Source: BBC, Mannheim, Germany

Measured unstable no-load stator current with a big stator resistance r_s

switch S opened



20 ms

f_{d2} = 2.76 Hz

R_V = 4.44 Ω

Experiment terminated due to big noise of the motor, caused by mechanical vibrations

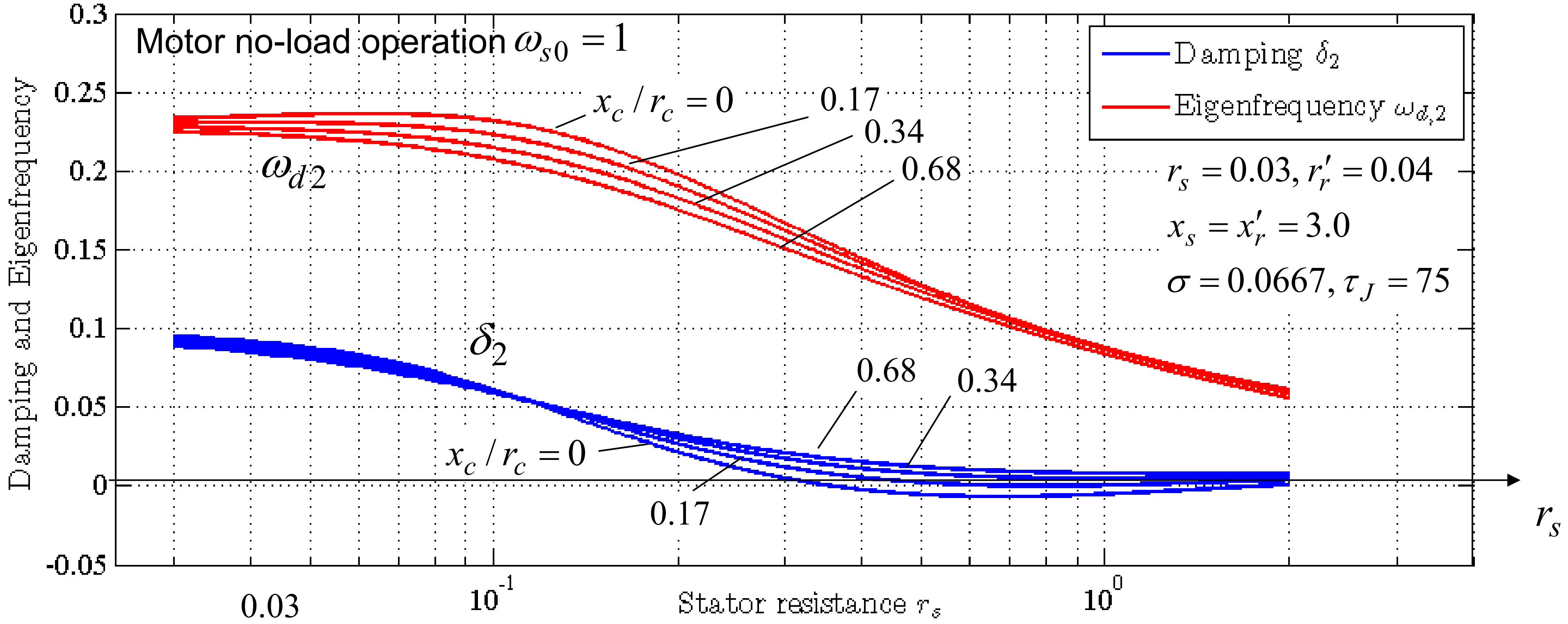
$R_V / R_s = 104.2$

$r_s + r_V = 0.54$

Source: BBC, Mannheim, Germany

7. Dynamics of induction machines

A serial inductance x_c (= a „real“ cable“) helps to stabilize



- Four long cables investigated with parameters: r_c, $\quad x_c = \{0, \underbrace{0.17, 0.34, 0.68}_{\text{stable}}\} \cdot r_c$
- Stator side resistance & inductance vary as:

$$r_s = 0.03 + r_c \quad x_{s\sigma} = 0.102 + x_c$$

- Proportional to cable length: $r_c \sim x_c \sim l$

unstable stable
operation

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Summary:
Linearized transfer function of induction machines in synchronous reference frame

- Small signal theory for transfer function $\Delta\omega_m = F(\Delta u_s, \Delta\omega_s, \Delta m_s)$ in *LAPLACE* domain
- Short-circuit rotor cage and related flux time constant explain "synchronous machine" effect
- Weakly damped oscillation at *U*/*f*-control $u_s \sim f_s$ and low speed $f_s << f_{sN}$
- Unstable operation at strongly increased stator resistance r_s
- Speed control stabilizes the unstable or weakly damped oscillations of the induction machine

7. Dynamics of induction machines

Inverter-fed cage induction machine with air-air cooler

Inverter

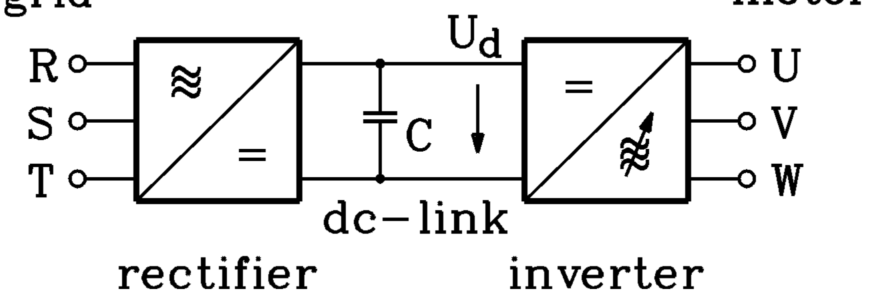
Source:
Siemens AG

Induction motor

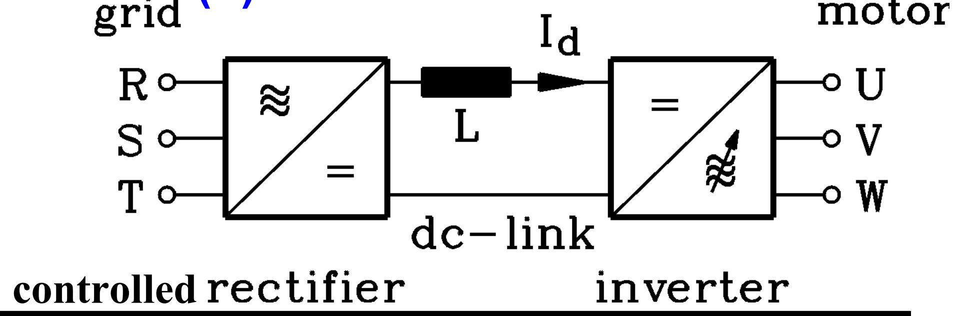
7. Dynamics of induction machines

Variable speed operation of induction machine

(a) voltage source inverter



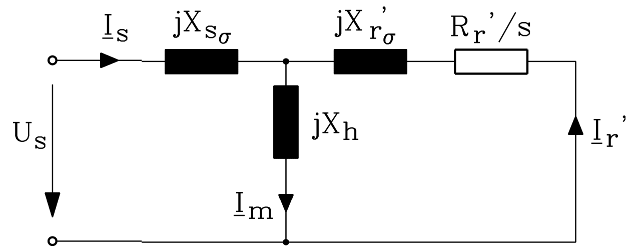
(b) current source inverter



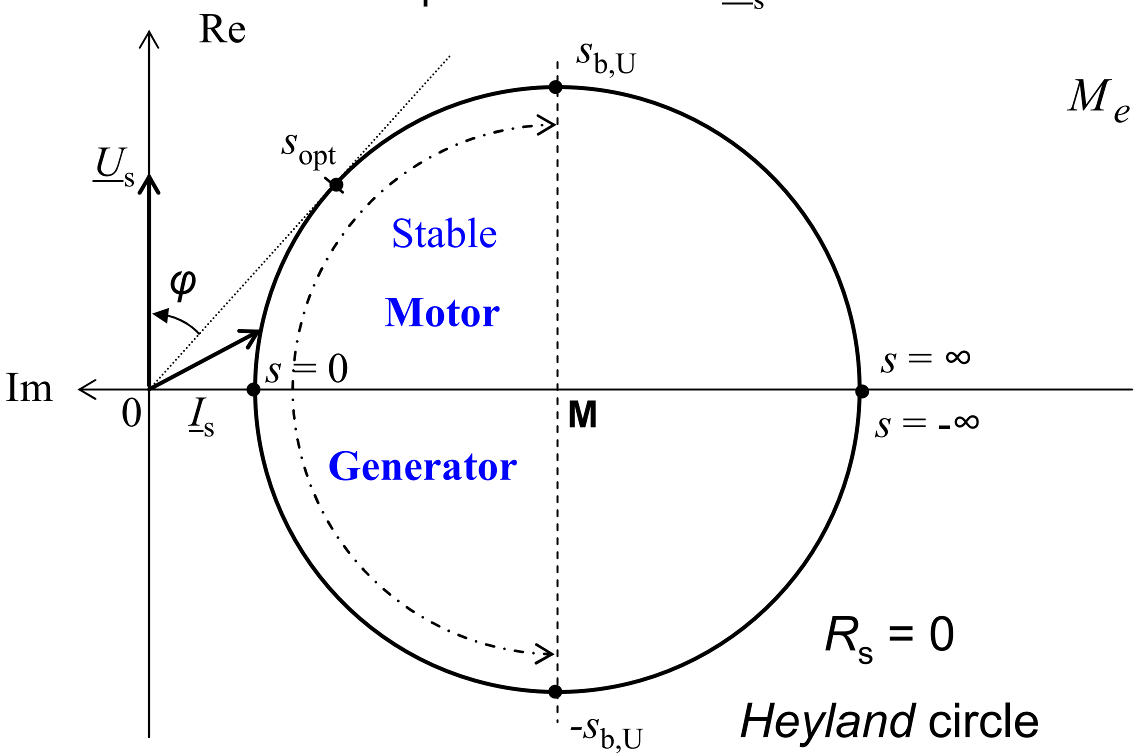
DC link capacitor *C*	DC link inductor (choke) *L*
U_d: DC link voltage	I_d: DC link current
controlled AC voltage, variable frequency	controlled AC current, variable frequency
motor-side inverter operates independently	motor winding part of motor-side inverter
parallel operation of motors possible	each motor needs a separate inverter
motor break down slip, at $r_s = 0$: $s_{b,U} = \frac{r_r}{\sigma \cdot x_r} = 0.1 \dots 0.2$. Motor can operate without control within slip range 0 ... s_b	motor break down slip, independent of r_s: $s_{b,I} = \frac{r_r}{x_r} = 0.005 \dots 0.02$ Motor operating in unstable slip range > s_b. Control is needed for stable performance.

Voltage-fed induction machine at R_s = 0 (*HEYLAND* circle)

$$\underline{I}_s = \frac{\underline{U}_s}{X_s} \cdot \left(\frac{(1 - 1/\sigma) \cdot R'_r}{-s \cdot \sigma \cdot X'_r + jR'_r} - j \cdot \frac{1}{\sigma} \right)$$



Current phasor locus at $\underline{U}_s = \mathrm{const.}$



R_s = 0

Heyland circle

$$M_e = \frac{m_s}{\Omega_{syn}} \cdot U_s^2 \cdot \frac{1-\sigma}{X_s} \cdot \frac{s \cdot R'_r \cdot X'_r}{R'^2_r + (s \cdot \sigma \cdot X'_r)^2}$$

Break-down slip: $s_{b,U} = \dfrac{R'_r}{\sigma \cdot X'_r}$

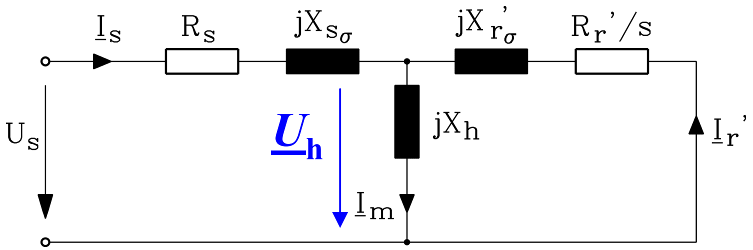
Maximum $\cos\varphi$ at:

$$s_{opt} = \frac{R'_r}{\sqrt{\sigma} \cdot X'_r}$$

is in the stable area of rising torque with slip

7. Dynamics of induction machines

Torque-speed-curve of current-fed induction machine



Impressed current $\underline{I}_s$, so change of voltage drop

$$(R_s + jX_{s\sigma}) \cdot \underline{I}_s$$

of no influence on current locus !

$$\underline{U}_h = \underline{I}_s \cdot \frac{jX_h \cdot \left(\frac{R'_r}{s} + jX'_{r\sigma} \right)}{jX_h + \frac{R'_r}{s} + jX'_{r\sigma}} = \underline{I}_s \cdot \frac{jX_h \cdot (R'_r + j \cdot s \cdot X'_{r\sigma})}{R'_r + j \cdot s \cdot X'_r}$$

$$P_\delta = M_e \cdot \Omega_{syn} = m_s \cdot \mathrm{Re}\{\underline{U}_h \cdot \underline{I}_s^*\}$$

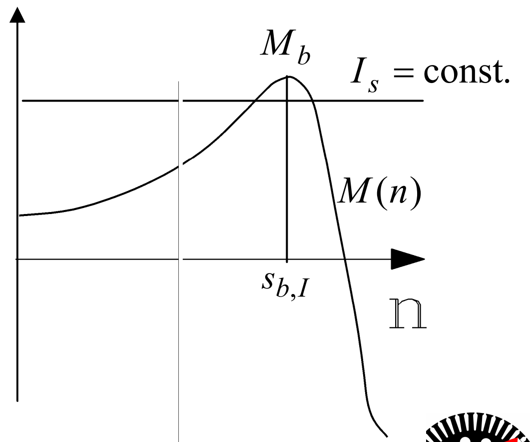
$$\underline{U}_h \cdot \underline{I}_s^* = \left| \underline{I}_s \right|^2 \cdot \frac{X_h \cdot (jR'_r - s \cdot X'_{r\sigma})}{R'_r + j \cdot s \cdot X'_r}$$

$$\mathrm{Re}\{\underline{U}_h \cdot \underline{I}_s^*\} = \frac{I_s^2 \cdot X_h^2 \cdot s \cdot R'_r}{R_r'^2 + (s \cdot X'_r)^2}$$

$$\boxed{M_e = \frac{m_s}{\Omega_{syn}} \cdot \frac{X_h^2 \cdot s \cdot R'_r}{R_r'^2 + (s \cdot X'_r)^2} \cdot I_s^2}$$

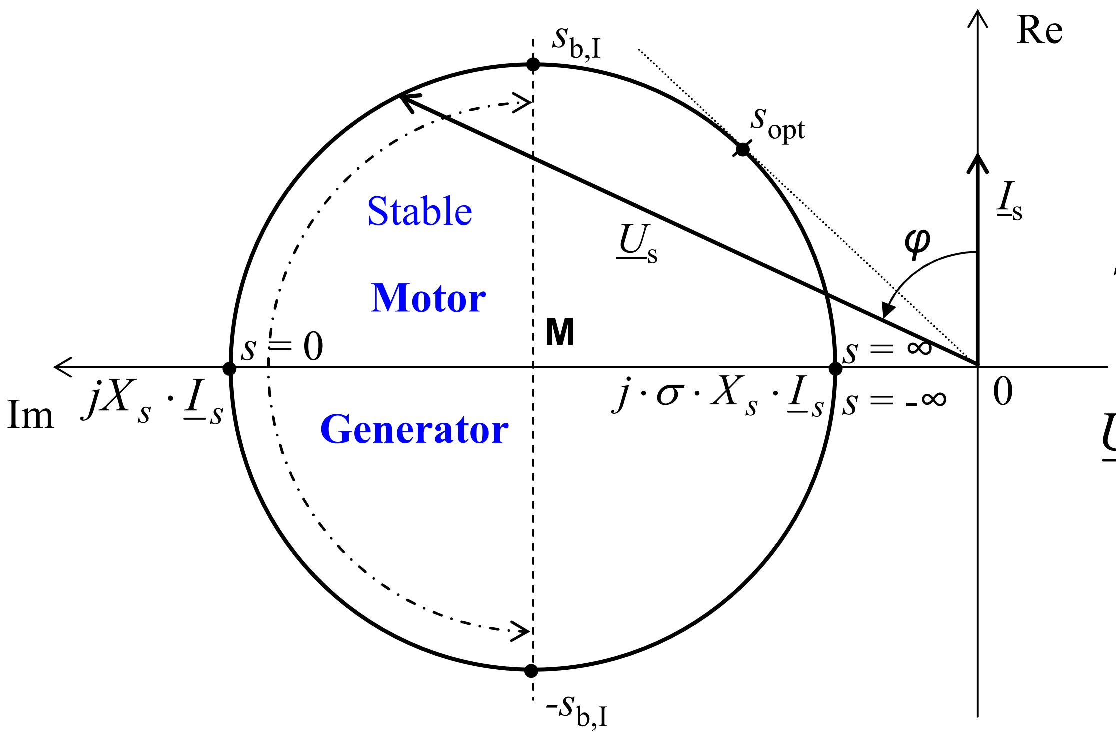
$$\frac{dM_e}{ds} = 0 \Rightarrow s_{b,I} = \frac{R'_r}{X'_r} = \sigma \cdot s_{b,U}$$

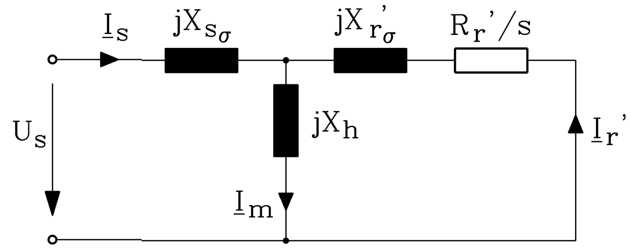
$$\boxed{s_{b,I} \approx 0.1 \cdot s_{b,U} \approx 0.005 \ldots 0.02}$$



Current-fed induction machine ($R_s = 0$)

Voltage phasor locus at $\underline{I}_s = \text{const.}$





$$s_{b,I} = \frac{R_r'}{X_r'} << \frac{R_r'}{\sigma X_r'}$$

$s_{b,I}$ much smaller than at voltage-fed machine

$$\underline{U}_s = X_s \cdot \underline{I}_s \cdot \left(\frac{(\sigma - 1) \cdot R_r'}{-s \cdot X_r' + jR_r'} + j \cdot \sigma \right)$$

Maximum $\cos\varphi$ at:

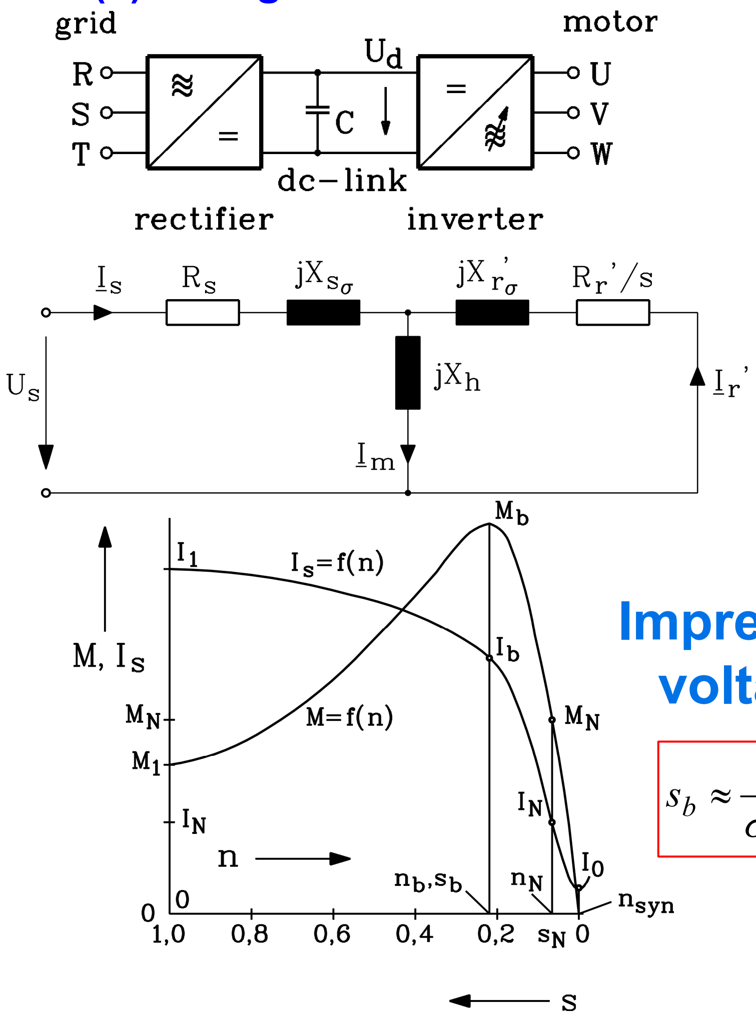
$$s_{opt} = \frac{R_r'}{\sqrt{\sigma} \cdot X_r'}$$

is in the unstable area of falling torque with slip

7. Dynamics of induction machines

Voltage vs. current feeding of induction machine

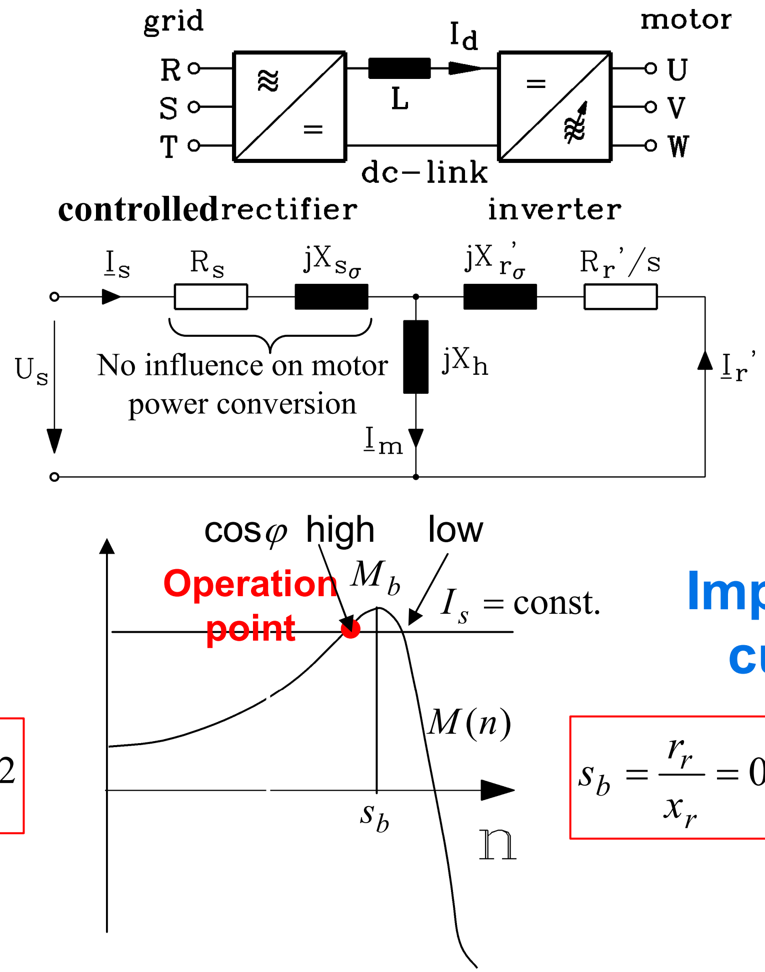
(a) voltage source inverter



Impressed voltage

$$s_b \approx \frac{r_r}{\sigma \cdot x_r} = 0.1 \ldots 0.2$$

(b) current source inverter



Impressed current

$$s_b = \frac{r_r}{x_r} = 0.005 \ldots 0.02$$

7. Dynamics of induction machines
Voltage vs. Current source inverter

Voltage source inverter	*Current source inverter*
Stator voltage is pulse width modulated voltage pattern	Stator voltage: Sinusoidal due to induction by machine flux, which is excited by impressed currents
Stator current nearly sinusoidal with ripple due to voltage switching	Stator current consists of 120° blocks (six step current mode).
Grid side: Diode rectifier - no power flow to grid ! For electric braking chopped DC link brake resistor is needed (Costly alternative: Grid side PWM converter)	Grid side: Controlled thyristor bridge for variable rectified voltage U_d for adjusting positive $I_d > 0$. At $U_d < 0$, $\alpha > 90°$ regenerative brake power flow $U_d I_d < 0$ to grid is possible.
IGBT-power switches: Power range 0.1 kW ... 10 MW, voltage < 6000 V. Bigger power rating with IGCT- or GTO-power switches up to 30 ... 50 MW with medium voltage e.g. 6300 V.	Thyristor power switches: Big induction and synchronous motors (1 ... 100 MW), World-wide biggest motor: 100 MW in NASA centre /*Langley*/*USA*, super wind channel

7. Dynamics of induction machines
Variable speed operation of synchronous motor (1)

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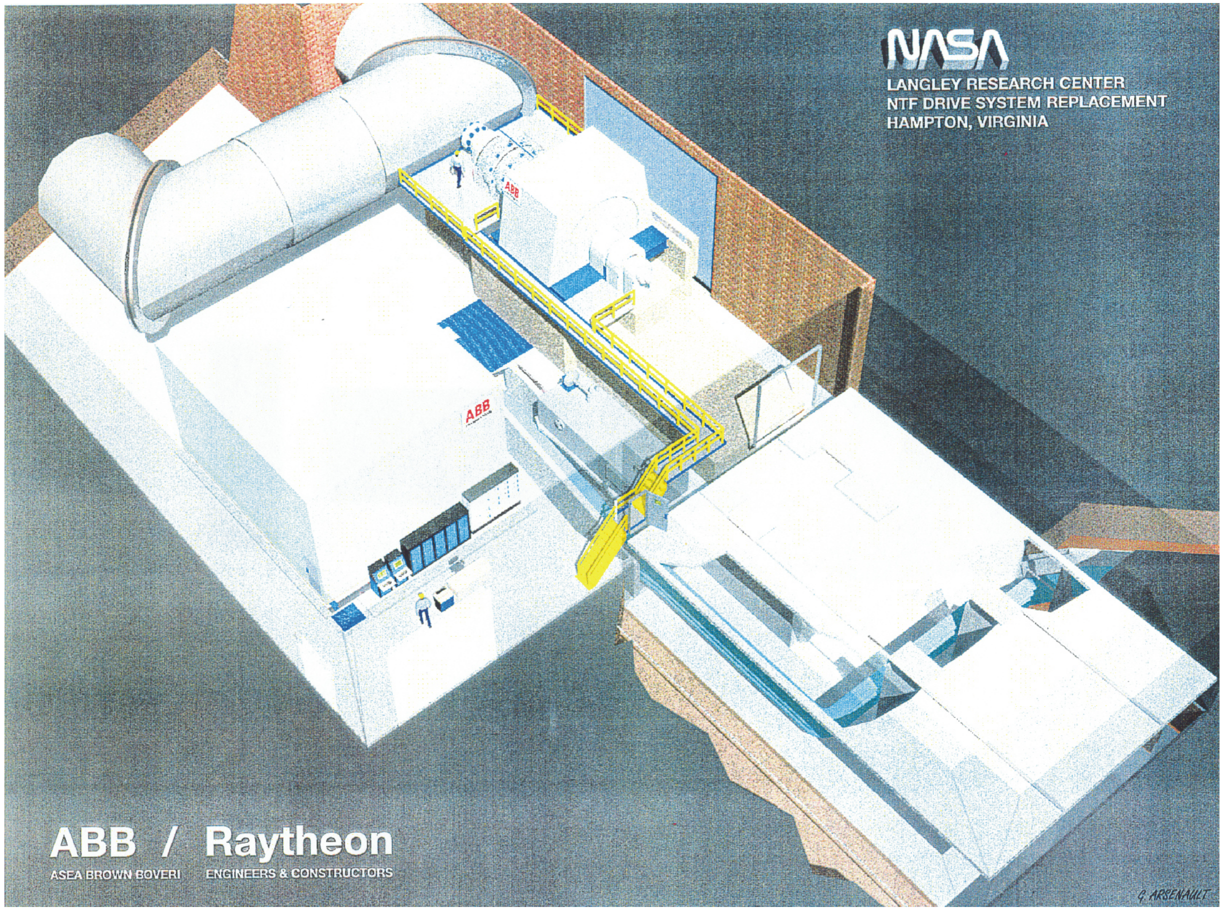
Current source inverter operation



NASA Langley Research Center, Hampton, Virginia, USA

Synchronous 12-pole motor as wind tunnel drive

100 MW, two thyristor current source inverters in parallel: 2 x 12.5 kV

36 … 60 Hz
360 … 600/min

Source: ABB, Switzerland

Variable speed operation of synchronous motor (2)

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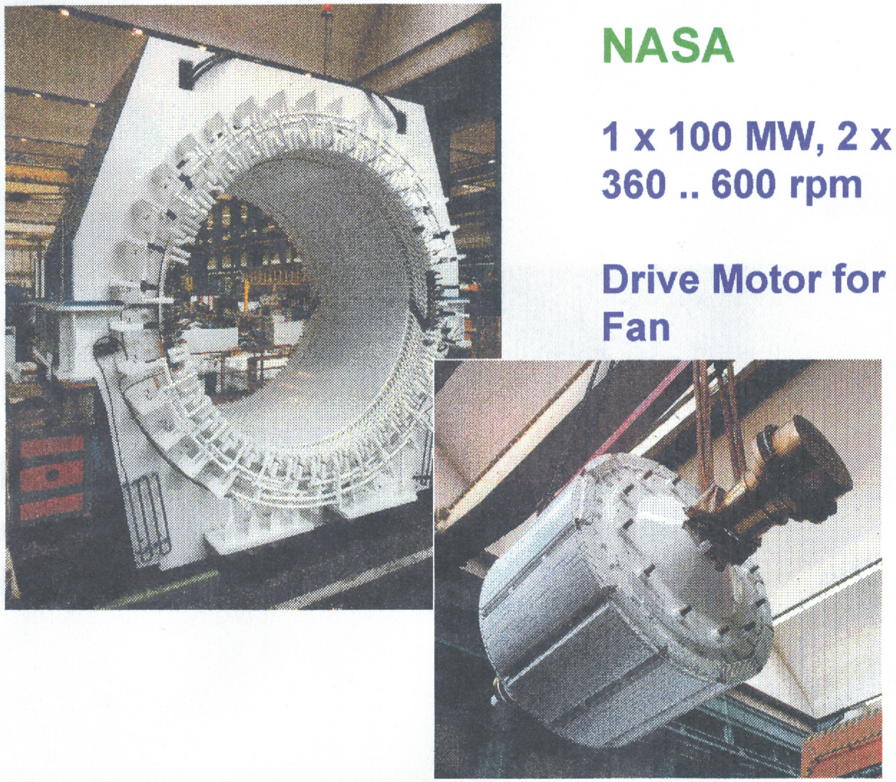
Current source inverter operation

NASA Langley Research Center - National Transonic Facility - Drive System



NASA

1 x 100 MW, 2 x 12.5 kV
360 .. 600 rpm

Drive Motor for a Wind Tunnel Fan

Test Section:		
Length	:	7.6 m
Area	:	2.5 x 2.5 m
Total Length Windtunnel	:	150 m

ABB Power Generation Ltd.
KWHT HYDRO_ 1/ 98-09-16/DS

ABB

12-pole synchronous salient pole motor

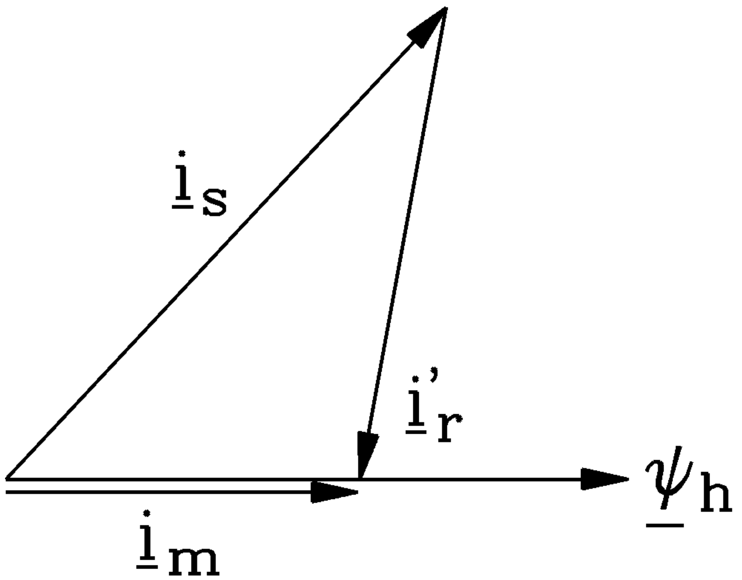
$$n_{syn,max} = f_{s,max}/p = 60/6 = 10/s = 600/\mathrm{min}$$

Source: ABB, Switzerland

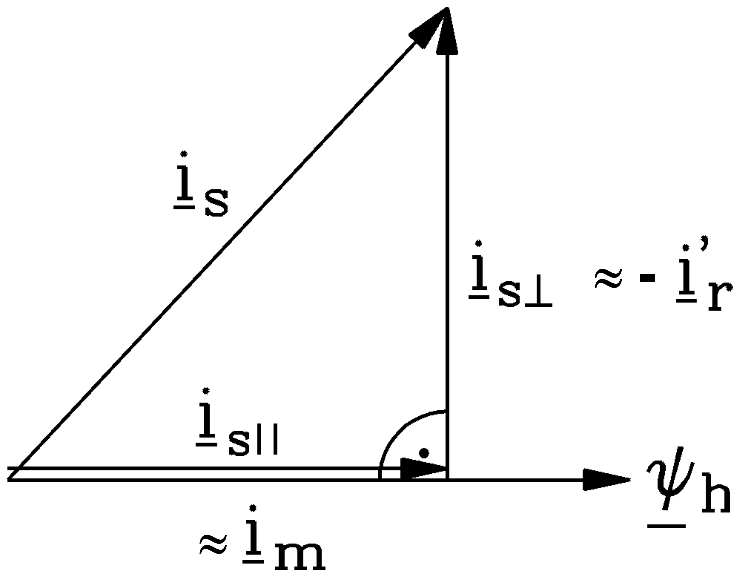
Principle of field-oriented control (1)

Decomposition of stator space current vector $\underline{i}_s$ in torque- and flux-generating component (*F. BLASCHKE, Erlangen & K. HASSE, Darmstadt*, 1969)

Main flux excitation ψ_h by stator <u>and</u> rotor current space vector (= magnetizing current i_m)



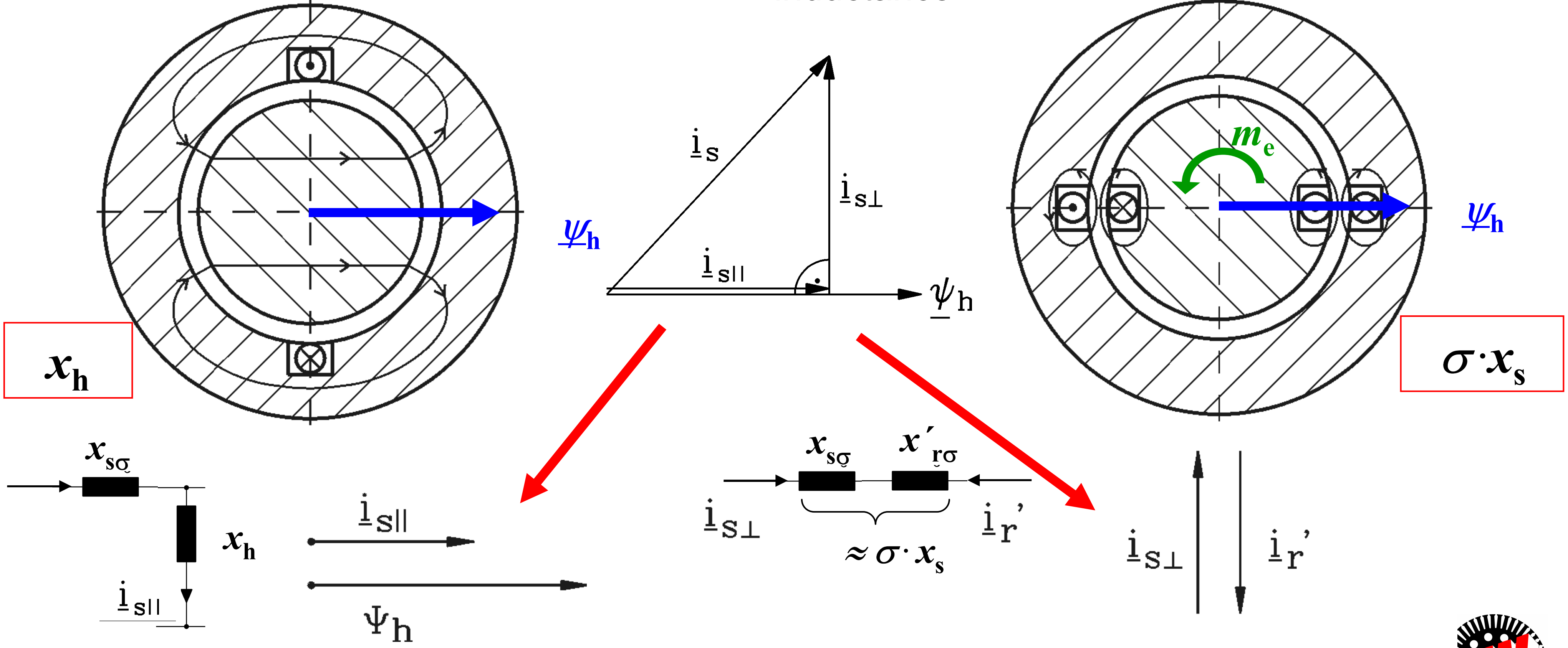
Torque generation in induction machine by main flux and perpendicular current component $i_{s\perp}$



7. Dynamics of induction machines
Principle of field-oriented control (2)

Main flux excitation ψ_h by magnetizing current via main inductance

Torque generation m_e by perpendicular current component $i_{s\perp}$ via total leakage inductance



7. Dynamics of induction machines
Principle of field-oriented control (3)

- **Torque generation m_e :**

$$m_e = \mathrm{Im}\{\underline{i}_s \cdot \underline{\psi}_h^*\} = \mathrm{Im}\{(\underline{i}_{s=} + \underline{i}_{s\perp}) \cdot \underline{\psi}_h^*\} = \mathrm{Im}\{\underline{i}_{s\perp} \cdot \underline{\psi}_h^*\} = i_{s\perp} \cdot \psi_h^*$$

- **Long time constant for changing ψ_h:** $\tau = x_s / r_s = (x_{s\sigma} + x_h)/r_s$

- **Short time constant for changing $i_{s\perp}$:** $\tau_{s\sigma} = \sigma \cdot x_s / r_s$

- By decomposition of the stator current in a flux exciting and a torque generating component, the control via an appropriate stator voltage space vector $\underline{u}_s$

a) can keep the flux exciting component $\underline{i}_{s=}$ constant (= constant main flux ψ_h)

b) can vary the torque generating component $\underline{i}_{s\perp}$ = variable torque.

- As b) changes very fast with short circuit time constant $\tau_{s\sigma}$, we get a dynamic torque variation = Field-oriented control principle of *Blaschke* and *Hasse* (*Germany*, 1969)

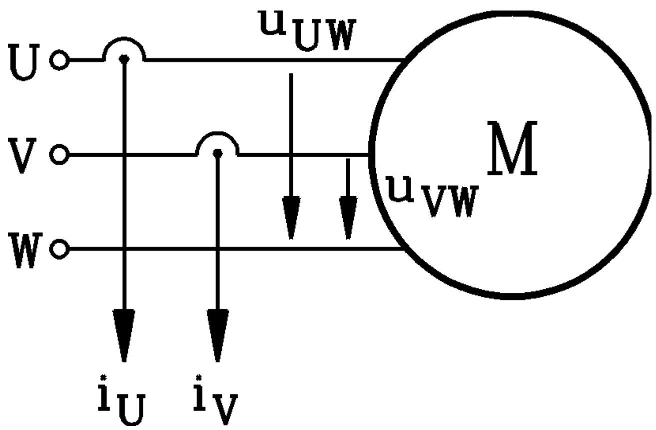
7. Dynamics of induction machines

Comparison of speed control of DC and AC induction machine

Type of machine	*DC machine*	*Cage induction machine*
Control	Armature voltage control	Field oriented control
Guiding variable	Armature voltage	Stator voltage
Fixed main flux	Separately excited main flux Φ	Main flux linkage ψ_h
Magnetization	Field current i_f	Magnetizing current i_m
Long flux time constant	Field time constant $L_\mathrm{f}/R_\mathrm{f}$	Open-circuit time constant x_s / r_s
Torque changed by	Armature current i_a	Flux-perpendicular current space vector component $\underline{i}_{s\perp}$
Short time constant for torque change	Armature time constant $L_\mathrm{a}/R_\mathrm{a}$	Short-circuit time constant $\sigma \cdot x_s / r_s$

7. Dynamics of induction machines

Rotor flux linkage space-vector from stator quantities



$$\left.\begin{aligned}\underline{u}_s &= \frac{2}{3}\cdot\left(u_U + \underline{a}\cdot u_V + \underline{a}^2\cdot u_W\right)\\ 0 &= \frac{2}{3}\cdot\left(u_W + \underline{a}\cdot u_W + \underline{a}^2\cdot u_W\right)\end{aligned}\right\} -$$

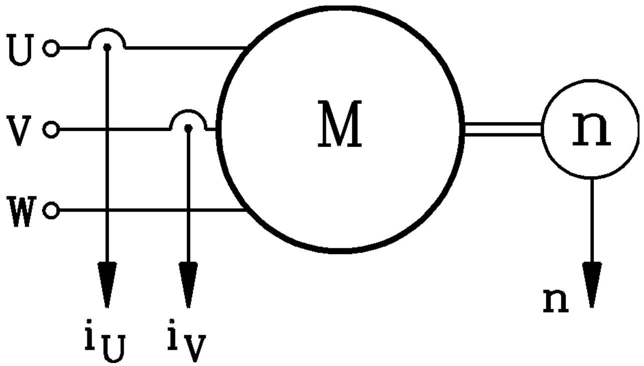
$$\underline{u}_s = \frac{2}{3}\cdot\left(u_U - u_W + \underline{a}\cdot(u_V - u_W)\right) = \underline{\underline{\frac{2}{3}\cdot\left(u_{UW} + \underline{a}\cdot u_{VW}\right)}}$$

$$i_U + i_V + i_W = 0 \qquad \underline{i}_s = \frac{2}{3}\cdot\left(i_U + \underline{a}\cdot i_V + \underline{a}^2\cdot i_W\right) = \underline{\underline{\frac{2}{3}\cdot\left(i_U\cdot(1-\underline{a}^2) + i_V\cdot(\underline{a}-\underline{a}^2)\right)}}$$

From stator voltage and flux linkage equations in stator reference frame we get:

$$\boxed{\left.\begin{aligned}\underline{u}_s &= r_s\underline{i}_s + d\underline{\psi}_s/d\tau\\ \underline{\psi}_s &= x_s\underline{i}_s + x_h\underline{i}'_r\\ \underline{\psi}'_r &= x_h\underline{i}_s + x'_r\underline{i}'_r\end{aligned}\right\} \Rightarrow \underline{\psi}_s = \frac{x_h}{x'_r}\cdot\underline{\psi}'_r + \sigma\cdot x_s\cdot\underline{i}_s \quad \Rightarrow \underline{\underline{\frac{d\underline{\psi}'_r}{d\tau} = \left(\underline{u}_s - r_s\cdot\underline{i}_s - \sigma\cdot x_s\cdot\frac{d\underline{i}_s}{d\tau}\right)\cdot\frac{x'_r}{x_h}}}}$$

Rotor flux linkage space-vector from stator & rotor quantities



Measured stator current space vector is transformed into **rotor reference frame**: Rotor speed sensor is needed !

$$\boxed{\underline{i}_{s(r)} = \underline{i}_{s(s)} \cdot e^{-j\cdot\omega_m \tau}}$$

$$\left.\begin{aligned} 0 &= r_r' \cdot \underline{i}'_{r(r)} + d\underline{\psi}'_{r(r)} / d\tau \\ \underline{\psi}'_{r(r)} &= x_h \cdot \underline{i}_{s(r)} + x_r' \cdot \underline{i}'_{r(r)} \end{aligned}\right\} \Rightarrow \underline{\underline{\frac{d\underline{\psi}'_{r(r)}}{d\tau} + \frac{1}{\tau_r} \cdot \underline{\psi}'_{r(r)} = \frac{x_h}{\tau_r} \cdot \underline{i}_{s(r)}}}$$

- Rotor flux linkage is derived by integrating stator current space vector via rotor open circuit time constant $\tau_r = x_r / r_r$

- Due to PT$_1$-performance no integration error will occur, so rotor flux linkage may be determined at any arbitrary speed, e.g. also n = 0.

Rotor flux linkage space-vector depends on r_r !

We assume stator current space vector to be stationary rotating with stator frequency ω_s

$$\underline{i}_{s(s)} = i_s \cdot e^{j\cdot\omega_s\tau} \Rightarrow \underline{i}_{s(r)} = i_s \cdot e^{j\cdot(\omega_s-\omega_m)\cdot\tau} = i_s \cdot e^{j\cdot\omega_r\tau}$$

Differential equation: $$\frac{d\underline{\psi}'_{r(r)}}{d\tau} + \frac{1}{\tau_r}\cdot\underline{\psi}'_{r(r)} = \frac{x_h}{\tau_r}\cdot i_s \cdot e^{j\omega_r\tau}$$

Solution in rotor reference frame: $$\underline{\psi}'_{r(r)}(\tau) = \left(\underline{\psi}'_{r(r)}(0) - \frac{x_h i_s}{1+j\omega_r\tau_r}\right)\cdot e^{-\tau/\tau_r} + \frac{x_h i_s}{1+j\omega_r\tau_r}\cdot e^{j\omega_r\tau}$$

Stationary solution $\tau \to \infty$: $$\boxed{\underline{\psi}'_{r(r)}(\tau) = \frac{x_h \cdot i_s}{1+j\cdot(\omega_s-\omega_m)\cdot x_r/r_r}\cdot e^{j\cdot(\omega_s-\omega_m)\cdot\tau}}$$

- Stationary solution possible without any integration error also at low speed ω_m << 1
- Flux linkage amplitude and phase angle depend on r_r.
- Only at no-load $\omega_m = \omega_s$ no influence of r_r.

7. Dynamics of induction machines

Determination of rotor flux linkage for field oriented control

a) from stator current and voltage:

- At high speed voltage is big and can be measured with high accuracy.
- At very low speed voltage is very low. Measurement errors such e.g. offset will be integrated to big values, so at low speed this method is NOT useful.

b) from stator current and rotor speed:

- Flux linkage may be determined at any speed, e.g. also $n = 0$.
- In cage induction machines it is difficult to determine on-line rotor resistance, which may change between 20°C and e.g. 150°C by 50%: Thermal model of the machine is needed.

Summary:
Inverter-fed induction machines with field-oriented control

- Voltage source inverter much wider used than current source inverter
- Field-oriented control utilizes separation of current space vector into flux-parallel and flux-orthogonal component
- Flux-parallel component magnetizes main flux = long time constant
- Flux-orthogonal component magnetizes stray flux = short time constant
- Fast change of torque via fast change of flux-orthogonal current
- Dynamic change of torque via field-oriented current space vector separation