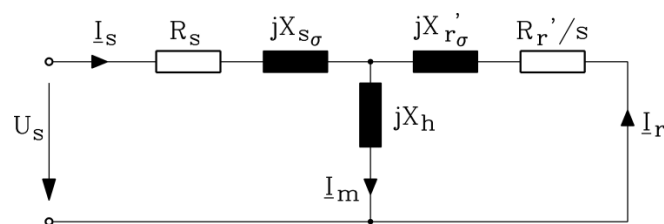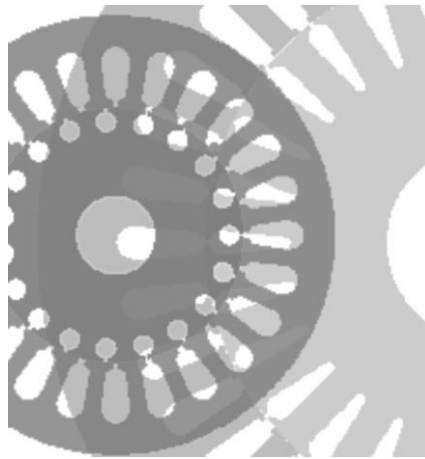# Energy Converters - CAD and System Dynamics

Collection of exercises

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Typical questions for exam preparation
1. The induction machine
2. Cooling of electrical machines
3. The converter-fed DC machine
4. Dynamics of line-operated induction machines
5. Dynamics of inverter-fed induction machines
6. Dynamics of line-operated synchronous machines

Exercise 1.1   1.1/1

Cage induction generator for wind power

A **three-phase 6-pole cage induction generator** for wind power has the following data:

Rated voltage: $U_N = 400$ V Y   Rated phase current: $I_{sN} = 1095$ A
Rated frequency: $f_N = 50$ Hz   Power factor: $\cos\varphi_s = 0.81$ (generator)
Rated shaft power: $P_N = 640$ kW   Bore diameter: $d_{si} = 490$ mm
Number of turns per phase: $N_s = 15$
Amplitude of fundamental field wave: $B_{\delta,1} = 0.994$ T

Stack length: $l_{Fe} = 450$ mm

The stack length l_{Fe} is assumed to be equal to the equivalent iron length l_e for simplification!

1)   Calculate the induced voltage with a winding factor $k_{ws1} = 0.945$! How is this value called in the T-equivalent circuit?

2)   Calculate the internal apparent power S_δ!

3)   Determine *Esson*'s number C in $\dfrac{\text{VAs}}{\text{m}^3}$ and in $\dfrac{\text{kVAmin}}{\text{m}^3}$
        a) via S_δ,
        b) via the current loading A and the fundamental field amplitude $B_{\delta,1}$!

4)   Determine with the rated current density $J = 3.56$ A/mm² the product $A \cdot J$ and discuss, which kind of cooling is recommended, if the stator winding temperature rise at steady state nominal operation shall not exceed $\Delta\vartheta = 105$ K at $\vartheta_{amb} = 40$ °C ambient temperature!

5)   Determine with the product $A \cdot J$ the heat transfer coefficient α_k at nominal temperature rise according to 4)! Which kind of losses is only considered?

6)   Calculate the rated apparent power S_N and the generator efficiency η_N! Is S_N bigger than S_δ and if so, why? Use the consumer reference frame and give the correct sign for the real power flow including the determination of input and output power and overall losses!

7)   Determine the specific thrust for $\cos\phi_s \cong \cos\phi_i$ and the apparent specific thrust $\tau_{AC,app}$! Give the relationship between $\tau_{AC,app}$ and Esson's number (theoretically and calculative)!

8)   Calculate the tangential force F_t and the electromagnetic torque M_e at rated operation from τ_{AC}! Assume, that M_e is equal to the shaft torque M_N and determine the speed and the slip of the generator! If we assume $M_e = M_N$, which kind of losses are neglected?

1) $U_{si} = \sqrt{2}\pi \cdot f_s \cdot N_s \cdot k_{ws1} \cdot \dfrac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1}$

$= \sqrt{2}\pi \cdot 50 \cdot 15 \cdot 0.945 \cdot \dfrac{2}{\pi} \cdot 0.2566 \cdot 0.45 \cdot 0.994 = \underline{\underline{230.09\,\text{V}}}$

$\tau_p = \dfrac{d_{si} \cdot \pi}{2p} = \dfrac{490 \cdot \pi}{6} = 256.6\,\text{mm}$

$U_{si} = U_h$ in the T-equivalent circuit!

2) $S_\delta = 3 \cdot U_{si} \cdot I_s = 3 \cdot 230.09 \cdot 1095 = \underline{\underline{755838\,\text{VA}}}$

3)   a) $C = \dfrac{S_\delta}{d_{si}^2 \cdot l_{Fe} \cdot n_{syn}} = \dfrac{755838}{0.49^2 \cdot 0.45 \cdot 16.67} = \underline{\underline{419735\,\dfrac{\text{VAs}}{\text{m}^3}}} = \underline{\underline{6.99\,\dfrac{\text{kVAmin}}{\text{m}^3}}}$

$n_{syn} = \dfrac{f_s}{p} = \dfrac{50}{3} = 16.67\,\text{s}^{-1}$

b) $C = \dfrac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A \cdot B_{\delta,1} = \dfrac{\pi^2}{\sqrt{2}} \cdot 0.945 \cdot 64000 \cdot 0.994 = \underline{\underline{419549\,\dfrac{\text{VAs}}{\text{m}^3}}} = \underline{\underline{6.99\,\dfrac{\text{kVAmin}}{\text{m}^3}}}$

$A = \dfrac{2m \cdot N_s \cdot I_s}{d_{si} \cdot \pi} = \dfrac{2 \cdot 3 \cdot 15 \cdot 1095}{49 \cdot \pi} = 640\,\dfrac{\text{A}}{\text{cm}} = 64000\,\dfrac{\text{A}}{\text{m}}$

4) $A \cdot J = 640 \cdot 3.56 = \underline{\underline{2278.4\,\dfrac{\text{A}}{\text{cm}} \cdot \dfrac{\text{A}}{\text{mm}^2}}}$

According to Chap. 1 a totally enclosed machine with outside air-cooling and shaft mounted fan might be possible. For the rotor cooling an internal air circulation is necessary, driven by a smaller internal shaft mounted fan!

5) $\Delta\vartheta_w = \dfrac{A \cdot J}{\kappa \cdot \alpha_k}$

$\Rightarrow \alpha_k = \dfrac{A \cdot J}{\kappa \cdot \Delta\vartheta_w} = \dfrac{2278.4 \cdot 10^2 \cdot 10^6}{38.25 \cdot 10^6 \cdot 105} = \underline{\underline{56.7\,\dfrac{\text{W}}{\text{m}^2\text{K}}}}$

$\Delta\vartheta_w = 105\,\text{K} \Rightarrow \vartheta_w = \Delta\vartheta_w + \vartheta_{amb} = 105 + 40 = 145°\text{C}$

$$\kappa = \kappa_{\text{Cu}}(\vartheta_{\text{w}} = 145°\text{C}) = \kappa(20°\text{C}) \cdot \frac{235+20}{235+145} = 57 \cdot 10^6 \cdot \frac{235+20}{235+145} = 38.25 \frac{\text{MS}}{\text{m}}$$

Only the stator I^2R-losses are considered in the product $A \cdot J$!

6) $$S_{\text{N}} = 3 \cdot U_{\text{sN}} \cdot I_{\text{sN}} = 3 \cdot \frac{400}{\sqrt{3}} \cdot 1095 = \underline{\underline{758638\text{VA}}}$$

$$S_{\text{N}} = 758638 \text{ VA} > S_{\delta} = 755838 \text{ VA}$$

The stator voltage drop at R_{s} and $X_{\text{s}\sigma}$ is not included in S_{δ}, but it is included in S_{N}, so usually S_{N} is bigger than S_{δ}, as U_{s} is bigger than $U_{\text{si}} = U_{\text{h}}$!

$$U_{\text{s}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V} > U_{\text{h}} = 230.09 \text{ V} !$$

$$\eta_{\text{N}} = \frac{P_{\text{e, out}}}{P_{\text{m, in}}} = \frac{S_{\text{N}} \cdot \cos\varphi_{\text{s}}}{P_{\text{m, in}}} = \frac{758638 \cdot 0.81}{640000} = \frac{614497}{640000} = \underline{\underline{96\%}}$$

Consumer reference frame:

$$P_{\text{e}} = P_{\text{d}} + P_{\text{m}} \Rightarrow P_{\text{d}} = P_{\text{e}} - P_{\text{m}} = -614.5\text{kW} + 640.0\text{kW} = 25.5\text{kW} > 0$$

$$P_{\text{e}} = -614.5\,\text{kW} = - P_{\text{e,out}}$$
$$P_{\text{m}} = -640.0\,\text{kW} = - P_{\text{m,in}}$$

Losses are always positive, independent of the consumer or generator reference frame!

7)   Specific thrust:
$$\tau_{\text{AC}} = \frac{\hat{A} \cdot \hat{B}_{\delta,1}}{2} \cdot \cos\varphi_{\text{i}} = \frac{85532 \cdot 0.994}{2} \cdot 0.81 = \underline{\underline{34432 \frac{\text{N}}{\text{m}^2}}}$$

$$\hat{A} = \sqrt{2} \cdot k_{\text{ws1}} \cdot A = \sqrt{2} \cdot 0.945 \cdot 64000 = \underline{\underline{85532 \frac{\text{A}}{\text{m}}}}$$

Apparent specific thrust:

$$\tau_{\text{AC, app}} = \frac{\hat{A} \cdot \hat{B}_{\delta,1}}{2} = \frac{85532 \cdot 0.994}{2} = \underline{\underline{42509 \frac{\text{N}}{\text{m}^2}}}$$

Theoretical relationship between $\tau_{AC,app}$ and C:

$$C = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A \cdot B_{\delta,1} = \frac{\pi^2}{2} \cdot \sqrt{2} \cdot k_{ws1} \cdot A \cdot B_{\delta,1} = \frac{\pi^2}{2} \cdot k_{ws1} \cdot \hat{A} \cdot B_{\delta,1} = \pi^2 \cdot \tau_{AC,app}$$

Calculative proof:

$$C = 419549 \frac{VAs}{m^3} \overset{?}{=} \pi^2 42509 \frac{N}{m^2}$$

$$419549 = \pi^2 42509 = 419549!$$

Note: $\dfrac{VAs}{m} = \dfrac{J}{m} = \dfrac{Nm}{m} = N!$

8) $F_t = \tau_{AC} \cdot A = \tau_{AC} \cdot d_{si} \cdot \pi \cdot l_{Fe} = 34432 \cdot 0.49 \cdot \pi \cdot 0{,}45 = \underline{\underline{23852 N}}$

$$M_e = F_t \cdot \frac{d_{si}}{2} = 23852 \cdot \frac{0{,}49}{2} = \underline{\underline{5843.68\,Nm}}$$

$$M_e = M_N : P_{m,in} = 2 \cdot \pi \cdot n_N \cdot M_N$$

$$\Rightarrow n_N = \frac{P_{m,in}}{2\pi \cdot M_N} = \frac{640000}{2\pi \cdot 5843.68} = \underline{\underline{17.43/s}} = \underline{\underline{1046/min}}$$

$$s = \frac{n_{syn} - n}{n_{syn}} = \frac{1000 - 1046}{1000} = -0.046 = \underline{\underline{-4.6\%}}$$

The assumption $M_e = M_N$ means the negligence of all braking torques and corresponding losses:

- friction and windage losses
- additional losses, which are related to the rotor movement like tooth flux pulsation losses (see Chap. 2)

Cage induction motor for a pump drive

A **12-pole high voltage cage induction motor** with open air ventilation is used as a pump drive in a thermal power plant. The three-phase machine has the following data:

Rated power:	$P_N = 1060$ kW	Power factor:	$\cos\varphi_{sN} = 0.83$
Rated voltage:	$U_N = 6600$ V Y	Rated frequency:	$f_N = 50$ Hz
Rated phase current:	$I_{sN} = 117$ A		

Bore diameter:	$d_{si} = 830$ mm
Number of stator slots:	$Q_s = 72$
Number of turns per phase:	$N_s = 156$
Coil span:	$W = 5$ stator slots pitches
Air gap width:	$\delta = 1.4$ mm

The internal voltage U_h is by 5 % smaller than the stator nominal voltage with an internal power factor $\cos\varphi_i = 0.85$. For a good cooling the iron length $l_{Fe} = 800$ mm is separated into 20 packets with radial cooling ducts with a width of $l_k = 8$ mm in between.

1)   Determine the slots per pole and phase q, the distribution factor k_{d1}, the pitch factor k_{p1}, and the winding factor k_{ws1} of the fundamental field wave!

2)   Calculate the overall stack length L, the packet length l_1 and the equivalent iron length l_e! Give the relationship between L, l_{Fe} and l_e!

3)   Evaluate the pole flux Φ of the fundamental field wave and its flux density amplitude $B_{\delta,1}$!

4)   Determine the current loading A and Esson's number C with the length l_e:

        a) via the internal apparent power S_δ,

        b) via A and $B_{\delta,1}$!

5)   The stator winding copper cross sections is $q_{Cu} = 26.5$ mm². Determine the stator current density J and the product $A \cdot J$! Is it valid for open ventilated air cooling?

6)   Calculate the efficiency η_N, the total losses P_d, and the air gap power P_δ via $\cos\varphi_i$! Separate via P_δ the losses P_d into stator-side losses $P_{d,s}$ and rotor-side losses $P_{d,r}$! If rotor friction and windage losses P_{fr+w} and rotor side additional losses $P_{add,r}$ are 5 kW, how big is the slip and the speed of the rotor?

7)   Calculate the specific thrust τ_{AC} and the electromagnetic torque M_e

        a) from τ_{AC},

        b) from P_δ!

1) $q = \dfrac{Q_s}{2p \cdot m} = \dfrac{72}{12 \cdot 3} = \underline{\underline{2}}$

$k_{d1} = \dfrac{\sin\left(\dfrac{\pi}{2m}\right)}{q \cdot \sin\left(\dfrac{\pi}{2mq}\right)} = \dfrac{\sin\left(\dfrac{\pi}{2 \cdot 3}\right)}{2 \cdot \sin\left(\dfrac{\pi}{2 \cdot 3 \cdot 2}\right)} = \underline{\underline{0.9659}}$

Pitching: $\dfrac{W}{\tau_p} = \dfrac{5 \cdot \tau_{Qs}}{m \cdot q \cdot \tau_{Qs}} = \dfrac{5}{m \cdot q} = \dfrac{5}{3 \cdot 2} = \dfrac{5}{6}$

$k_{p1} = \sin\left(\dfrac{\pi}{2} \cdot \dfrac{W}{\tau_p}\right) = \sin\left(\dfrac{\pi}{2} \cdot \dfrac{5}{6}\right) = \underline{\underline{0.9659}}$

$k_{ws1} = k_{d1} \cdot k_{p1} = 0.9659^2 = \underline{\underline{0.933}}$

2) $L = l_{Fe} + (20-1) \cdot l_k = 800 + 19 \cdot 8 = \underline{\underline{952\,\text{mm}}}$

$l_1 = \dfrac{l_{Fe}}{20} = \dfrac{800}{20} = \underline{\underline{40\,\text{mm}}}$

$h' = \dfrac{l_k}{\delta/2} = \dfrac{8}{1.4/2} = 11.43$

$\zeta(h') \cong \dfrac{h'^2}{h'+5} = \dfrac{11.43^2}{11.43+5} = 7.95$

$k_C' = \dfrac{l_1 + l_k}{l_1 + l_k - \zeta(h') \cdot (\delta/2)} = \dfrac{40+8}{40+8-7.95 \cdot (1.4/2)} = 1.13$

$l_e = \dfrac{L}{k_C'} = \dfrac{952}{1.13} = \underline{\underline{841\,\text{mm}}}$

$\underline{\underline{l_{Fe} < l_e < L \Leftrightarrow 800\,\text{mm} < 841\,\text{mm} < 952\,\text{mm}}}$

3) $U_{sN} = \dfrac{U_N}{\sqrt{3}} = \dfrac{6600}{\sqrt{3}} = 3810\,\text{V}$

$U_{si} = U_h = 0.95 \cdot U_{sN} = 0.95 \cdot 3810 = 3620\,\text{V}$

$$\Phi = \frac{U_{si}}{\sqrt{2} \cdot \pi \cdot f_s \cdot k_{ws1} \cdot N_s} = \frac{3620}{\sqrt{2} \cdot \pi \cdot 50 \cdot 0.933 \cdot 156} = 111.96 \, \text{mWb}$$

$$\tau_p = \frac{d_{si} \cdot \pi}{2p} = \frac{830 \cdot \pi}{12} = 217.3 \, \text{mm}$$

$$B_{\delta,1} = \frac{\Phi}{\frac{2}{\pi} \cdot \tau_p \cdot l_e} = \frac{111.96 \cdot 10^{-3}}{\frac{2}{\pi} \cdot 0.2173 \cdot 0.841} = 0.962 \, \text{T}$$

4) $$A = \frac{2 \cdot m \cdot N_s \cdot I_s}{d_{si} \cdot \pi} = \frac{2 \cdot 3 \cdot 156 \cdot 117}{83 \cdot \pi} = 420 \frac{A}{cm}$$

a) $$S_\delta = 3 \cdot U_h \cdot I_s = 3 \cdot 3620 \cdot 117 = 1270620 \, \text{VA}$$

$$n_{syn} = \frac{f}{p} = \frac{50}{6} = 8.33 \, \text{s}^{-1} = 500 \, \text{min}^{-1}$$

$$C = \frac{S_\delta}{d_{si}^2 \cdot l_e \cdot n_{syn}} = \frac{1270620}{0.83^2 \cdot 0.841 \cdot 8.33} = 263280 \frac{\text{VAs}}{\text{m}^3} = 4.39 \frac{\text{kVAmin}}{\text{m}^3}$$

b) $$C = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws1} \cdot A \cdot B_{\delta,1} = \frac{\pi^2}{\sqrt{2}} \cdot 0.933 \cdot 42000 \cdot 0.962 = 263081 \frac{\text{VAs}}{\text{m}^3} = 4.38 \frac{\text{kVAmin}}{\text{m}^3}$$

The difference between a) and b) is due to rounding errors of the numerical calculation.

5) $$J = \frac{I_s}{q_{Cu}} = \frac{117}{26.5} = 4.42 \frac{A}{mm^2}$$

$$A \cdot J = 420 \cdot 4.42 = 1856 \frac{A}{cm} \cdot \frac{A}{mm^2}$$

According to Chap. 1 this is small enough to allow air cooling.

6) $$\eta_N = \frac{P_{m,out}}{P_{e,in}} = \frac{P_N}{\sqrt{3} \cdot U_N \cdot I_N \cdot \cos\varphi_{sN}} = \frac{1060000}{\sqrt{3} \cdot 6600 \cdot 117 \cdot 0.83} = \frac{1060000}{1110116} = 0.95485$$

$$= 95.49\%$$

$$P_d = \left(\frac{1}{\eta_N} - 1 \right) \cdot P_N = \left(\frac{1}{0.95485} - 1 \right) \cdot 1060000 = \underline{\underline{50116 W}}$$

$$P_\delta = 3 \cdot U_h \cdot I_s \cdot \cos\varphi_i = 3 \cdot 3620 \cdot 117 \cdot 0.85 = \underline{\underline{1080027 W}}$$

$$P_{d,s} = P_{e,in} - P_\delta = 1110116 - 1080027 = \underline{\underline{30089 W}}$$

$$P_{d,r} = P_\delta - P_{m,out} = 1080027 - 1060000 = \underline{\underline{20027 W}}$$

$$P_{Cu,r} = s_N \cdot P_\delta = P_{d,r} - \left(P_{fr+w} + P_{ad,r} \right) = 20027 - 5000 = \underline{\underline{15027 W}}$$

$$s_N = \frac{P_{Cu,r}}{P_\delta} = \frac{15027}{1080027} = 0.0139 = \underline{\underline{1.39 \%}}$$

$$n_N = (1 - s_N) \cdot n_{syn} = (1 - 0.0139) \cdot 8.33 = 8.217 / s = \underline{\underline{453 / min}}$$

7) $$\tau_{AC} = \frac{\hat{A} \cdot B_{\delta,1}}{2} \cdot \cos\varphi_i = \frac{\sqrt{2} \cdot k_{ws1} \cdot A \cdot B_{\delta,1} \cdot \cos\varphi_i}{2}$$

$$= \frac{\sqrt{2} \cdot 0.933 \cdot 42000 \cdot 0.962 \cdot 0.85}{2} = \underline{\underline{22657 \frac{N}{m^2}}}$$

a) $$M_e = F \cdot \frac{d_{si}}{2} = \tau_{AC} \cdot d_{si} \cdot \pi \cdot l_e \cdot \frac{d_{si}}{2} = 22657 \cdot 0.83^2 \cdot \pi \cdot 0.841 / 2 = \underline{\underline{20619.7 \, Nm}}$$

b) $$M_e = \frac{P_\delta}{2 \cdot \pi \cdot n_{syn}} = \frac{1080027}{2 \cdot \pi \cdot 8.33} = \underline{\underline{20627 Nm}}$$

The difference between a) and b) is due to rounding errors of the numerical calculation.

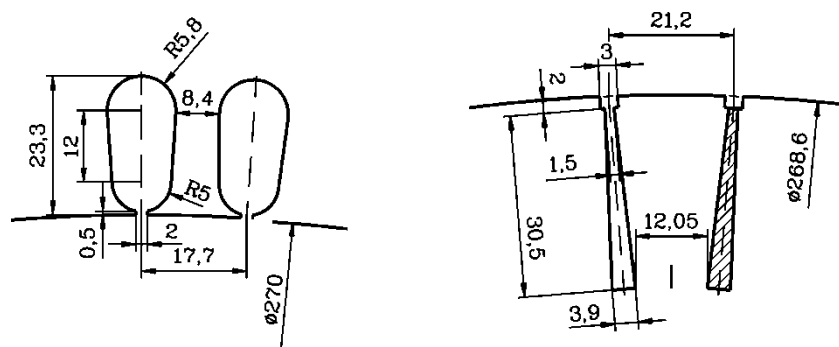**Exercise 2.1**   2.1/1

Magnetic design of an induction motor

A four pole **three-phase squirrel-cage induction motor** (tapered deep-bar cage rotor) has following data:

Nominal power:	$P_N = 55$ kW	Nominal voltage:	$U_N = 380$ V/ 220 V Y/Δ
Rated frequency:	$f_N = 50$ Hz	Power factor:	$\cos\varphi_N = 0.885$
Efficiency:	$\eta_N = 90.8$ %	Stack length:	$l_{Fe} = 220$ mm
Bore diameter:	$d_{si} = 270$ mm	Air gap:	$\delta = 0.7$ mm
Specific iron losses:	$v_{10} = 2.3$ W/kg	Sheet thickness:	$b_{sh} = 0.5$ mm
Iron stack factor:	$k_{Fe} = 0.94$		

The sketch shows the stator and rotor lamination in the slot region. The double-layer stator winding has 11 turns per coil and layer, and four parallel branches. Coils are pitched by 5/6.

1)   Calculate the nominal current I_N as well as the stator and rotor slot number.

2)   How big is the number of turns per phase and the winding factor for the fundamental wave of the magnetic field?

3)   How big is the amplitude of the fundamental wave of the magnetic air-gap field in motor idle operation? The voltage drop across the *ohmic* resistance and the leakage reactance in the stator winding is to be neglected.

4)   How big is the m.m.f. for the air gap related to the amplitude of the field from 3)?

5)   How big is the m.m.f. for the stator tooth related to the amplitude of the field from 3)? Consider only the parallel sided part of the tooth and neglect the magnetic flux through the slot.

6)   As shown in the sketch, the rotor tooth is considerably broader than the stator tooth, so that its influence on saturation – as well as the influence of the rather low yoke flux density – can be neglected. How big is the magnetizing current referred to the rated current in percent?

Dimensions in mm

Fig. 2.1-1: Stator and rotor iron lamination geometry

1) $U_{N,Ph} = \dfrac{U_N}{\sqrt{3}} = \dfrac{380}{\sqrt{3}}\,V = 220\,V$

$I_{N,Ph} = \dfrac{P_N}{3 \cdot U_{N,Ph} \cdot \cos\varphi_N \cdot \eta_N} = \dfrac{55000}{3 \cdot 220 \cdot 0.885 \cdot 0.908}\,A = \underline{\underline{103.7\,A}}$

$\tau_{Qs} = 17{,}7$ mm, $\tau_{Qr} = 21{,}2$ mm, according to sketch dimensions.

$Q_s = \dfrac{d_{si} \cdot \pi}{\tau_{Qs}} = \dfrac{270 \cdot \pi}{17.7} = \underline{\underline{48}}$ $Q_r = \dfrac{d_{si} \cdot \pi}{\tau_{Qr}} = \dfrac{270 \cdot \pi}{21.2} = \underline{\underline{40}}$

$Q_s = 2 \cdot p \cdot q \cdot m, \quad 2p = 4, \quad m = 3 \;\Rightarrow\; q = 4$

2)

- $N_c = 11$ turns per coil and layer
- $a = 4$ parallel circuits (windings of 4 poles connected in parallel)
- $W/\tau_p = 5/6$

$N_s = 2 \cdot p \cdot q \cdot \dfrac{N_c}{a} = 4 \cdot 4 \cdot \dfrac{11}{4} = \underline{\underline{44}}$

$k_{ds} = \dfrac{\sin\dfrac{\pi}{6}}{q \cdot \sin\left(\dfrac{\pi}{6 \cdot q}\right)} = \dfrac{0{,}5}{4 \cdot \sin\left(\dfrac{\pi}{24}\right)} = 0.9566$ $k_{ps} = \sin\left(\dfrac{W}{\tau_p} \cdot \dfrac{\pi}{2}\right) = \sin\left(\dfrac{5 \cdot \pi}{12}\right) = 0.966$

$k_{ws} = k_{ds} \cdot k_{ps} = 0.9566 \cdot 0.966 = \underline{\underline{0.924}}$

3)   no load operation ($R_s \cdot I_s$, $X_{s\sigma} \cdot I_s$ neglected):

$U_i \cong U_{s,Ph} \;\hat{=}\;$ voltage at magnetizing reactance U_h, $f_N = 50$ Hz, $l_{Fe} = 0.22$ m.

$\tau_p = \dfrac{d_{si} \cdot \pi}{2p} = \dfrac{270 \cdot \pi}{4}\,mm = 212{.}1\,mm$

$U_i = \sqrt{2} \cdot \pi \cdot f_N \cdot N_s \cdot k_{ws} \cdot \dfrac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta 1} \Rightarrow \underline{\underline{B_{\delta 1} = 0.82\,T}}$

4) $V_\delta = \dfrac{B_{\delta 1}}{\mu_0} \cdot \delta \cdot k_C$ $k_C = k_{Cs} \cdot k_{Cr} :$ *Carter's* coefficient

Dimensions according to Fig. 2.1-1:

$s_{Qs} = 2$ mm, $s_{Qr} = 3$ mm , $\dfrac{s_{Qs}}{\delta} = \dfrac{2}{0.7} = 2.86$, $\dfrac{s_{Qr}}{\delta} = \dfrac{3}{0.7} = 4.29 \;\Rightarrow$

$\zeta_s = 1.0$; $\zeta_r = 2.0$

$$k_{Cs} = \frac{\tau_{Qs}}{\tau_{Qs} - \zeta_s \cdot \delta} = \frac{17.7}{17.7 - 1 \cdot 0.7} = 1.04 \qquad k_{Cr} = \frac{\tau_{Qr}}{\tau_{Qr} - \zeta_r \cdot \delta} = \frac{21.2}{21.2 - 2 \cdot 0.7} = 1.07$$

$$k_C = 1.04 \cdot 1.07 = 1.11$$

$$\Rightarrow \quad V_\delta = \frac{0.82}{4 \cdot \pi \cdot 10^{-7}} \cdot 0.7 \cdot 10^{-3} \cdot 1.11 \, A = \underline{\underline{580 \, A}}$$

5)   Length of the parallel flank of the stator tooth according to Fig. 2.1-1: $l_d = 12$ mm
        Width of the parallel flank of the stator tooth according to Fig. 2.1-1: $b_d = 8.4$ mm

We assume $B_{\delta 1} \cong B_{\delta max}$.

$$B'_{ds} \cong B_{ds} \qquad B'_{ds} = \frac{B_{\delta 1} \cdot \tau_{Qs}}{k_{Fe} \cdot b_d} = \frac{0.82 \cdot 17.7}{0.94 \cdot 8.4} \cdot T = 1.84 \, T$$

With use of the $B(H)$ curve for the $v_{10} = 2.3$ W/kg sheet according to the textbook, we obtain:

$$H_{ds} = 155 \, A/cm \Rightarrow V_{ds} = H_{ds} \cdot l_d = 155 \cdot 1.2 \, A = \underline{\underline{186 \, A}}$$

6) $V_m = V_\delta + V_{ds} + V_{dr} + V_{ys} + V_{yr}$ with assumption $V_{dr}, V_{ys}, V_{yr} \ll V_{ds}$ we get:

$$V_m \cong V_\delta + V_{ds} = (508 + 186)A = 694A \qquad V_m = \frac{\sqrt{2}}{\pi} \cdot \frac{m}{p} \cdot N_s \cdot k_{ws} \cdot I_m \Rightarrow$$

$$I_m = \frac{V_\delta + V_{ds}}{\frac{\sqrt{2}}{\pi} \cdot \frac{m}{p} \cdot N_s \cdot k_{ws}} = \frac{508 + 186}{\frac{\sqrt{2}}{\pi} \cdot \frac{3}{2} \cdot 44 \cdot 0.924} \, A = 25.3 \, A$$

$$\frac{I_m}{I_{sN}} = \frac{25.3}{103.7} = \underline{\underline{24.4 \, \%}}$$

Design of a squirrel-cage induction machine

A **squirrel-cage induction machine** is to be designed for following rated values:

$$P_N = 200 \text{ kW} \qquad\qquad 2p = 8$$

$$U_N = 400 \text{ V} / 690 \text{ V} , \Delta/\text{Y} \qquad\qquad f_N = 50 \text{ Hz}$$

1)   Determine the main dimensions of "bore diameter" and "length of core" with help of the design curves in the text book. According to these curves, how big ist the efficiency, the power factor, the electric loading as well as the amplitude of the fundamental wave of the magnetic air gap field?

2)   Calculate with help of the values determined in 1) the so called *Esson's* coefficient in units of $\text{kVAmin} / \text{m}^3$.

3)   For verification calculate Esson's coefficient in units of $\text{kVA min}/\text{m}^3$ once more with help of the electric loading and the field amplitude determined in 1). Assume the value of the winding factor for this calculation with $3/\pi$.

4)   Determine the number of turns per phase according to the fundamental wave amplitude of the magnetic field (Neglect the voltage drop across the *ohmic* resistance and the leakage reactance of the stator winding).

5)   How big are the turns per coil if a slot number per phase and pole $q = 3$ and a single layer winding with all coils per phase connected in series is chosen? How big is the winding factor?

1) $P_N = 200 \, \text{kW} \Rightarrow \tau_p = 18 \, \text{cm}$ according to curve $\tau_p(P_N)$, text book Fig. 2.2-3.

$$2p \cdot \tau_p = d_{si} \cdot \pi \Rightarrow d_{si} = \frac{8 \cdot 180}{\pi} \, \text{mm} = \underline{\underline{458 \, \text{mm}}}$$

$l_e \approx l_{Fe} = 37 \, \text{cm} = \underline{370 \, \text{mm}}$ according to curve $l_e(P_N)$, text book Fig. 2.2-4.

$\cos\varphi = \underline{\underline{0{,}815}}, \eta = \underline{\underline{92.1 \, \%}}$ according to curve $\cos\varphi(P_N)$, $\eta(P_N)$, text book Fig. 2.2-9, Fig. 2.2-10.

$A = 400 \, \text{A/cm}$ according to curve $A(\tau_p)$, text book Fig. 2.2-5.
$B_{\delta,av} = 0.6 \, \text{T}$ according to curve $B_{\delta,av}(\tau_p)$, text book Fig. 2.2-5

$$\Rightarrow \quad B_{\delta,1} \cong \frac{\pi}{2} \cdot B_{\delta,av} = \underline{\underline{0.94 \, \text{T}}}$$
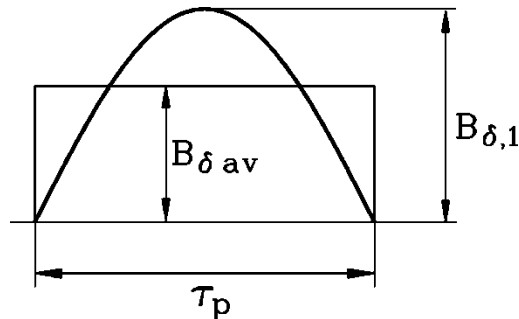
Fig. 2.2-1: Fundamental wave of air gap flux density per pole pitch

2) $C = \dfrac{S_N}{d_{si}{}^2 \cdot l_{Fe} \cdot n}$

$$S_N = \frac{P_N}{\eta \cdot \cos\varphi} = \frac{200}{0{,}921 \cdot 0{,}815} \, \text{kVA} = 266.4 \, \text{kVA}$$

$$n \cong n_{syn} = 750 / \min \quad (n_{syn} = \frac{f}{p} \cdot 60 \, / \min = \frac{50}{4} \cdot 60 \, \min = 750 \, \min)$$

$$C = \frac{266.4}{0.458^2 \cdot 0.37 \cdot 750} \frac{\text{kVAmin}}{\text{m}^3} = \underline{\underline{4.6 \frac{\text{kVAmin}}{\text{m}^3}}}$$

3) $C = \dfrac{\pi^2}{\sqrt{2}} \cdot k_{ws} \cdot A \cdot B_{\delta,1} = \dfrac{\pi^2}{\sqrt{2}} \cdot \dfrac{3}{\pi} \cdot 40000 \cdot 0.94 \dfrac{\text{VAs}}{\text{m}^3} = 2505786 \dfrac{\text{VAs}}{\text{m}^3}$

$$\Rightarrow \quad C = 2505786 \cdot \frac{1}{10^3 \cdot 60} \frac{\text{kVAmin}}{\text{m}^3} = \underline{\underline{4.18 \frac{\text{kVAmin}}{\text{m}^3}}} < 4.6 \frac{\text{kVAmin}}{\text{m}^3}$$

According to the comparison of C in 2) and 3) A and $B_{\delta,1}$ have to be increased slightly, e.g.:

$A = 420$ A/cm, $B_{\delta,1} = 0.99$ T   or $A = 440$ A/cm, $B_{\delta,1} = 0.94$ T.

$$U_i = U_{N,Ph} = \sqrt{2} \cdot \pi \cdot f_N \cdot N_s \cdot k_{ws} \cdot \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1}$$

$$\Rightarrow \quad N_s = \frac{400}{\sqrt{2} \cdot \pi \cdot 50 \cdot \dfrac{3}{\pi} \cdot \dfrac{2}{\pi} \cdot 0.18 \cdot 0.37 \cdot 0.94} = \underline{\underline{47.3}}$$

5) $q = 3 \quad \Rightarrow \quad Q_s = 2p \cdot q \cdot m = 8 \cdot 3 \cdot 3 = 72 \qquad N_c = N_Q$: turns per coil = turns per slot

Single layer winding, $a = 1$ (all poles connected in series):

$$N_s = p \cdot q \cdot \frac{N_c}{a} = 8 \cdot 3 \cdot \frac{N_c}{1} = 47.3 \approx 48 \quad \Rightarrow \quad \underline{\underline{N_c = 4}}$$

$$k_{ds} = \frac{\sin\left(\dfrac{\pi}{6}\right)}{3 \cdot \sin\left(\dfrac{\pi}{6 \cdot 3}\right)} = 0.96$$

A single layer winding is always full-pitched: $k_{ps} = 1 \quad \Rightarrow \quad k_{ws} = k_{ds} \cdot k_{ps} = \underline{0.96}$

(instead of the estimated value of $3/\pi = 0.955$)

Exercise 2.3   2.3/1

Current displacement in deep-bar and keyed-bar

A **rotor of a 55 kW induction motor**, 400 V, Y, 104 A, 1460/min, 50 Hz, is to be equipped
either with deep-bars or keyed-bars, as shown in the sketch below. The length of the
laminated core is 240 mm, the number of turns per phase winding is 44, the winding factor is
0.924 and the number of slots in the rotor is 40. The starting current is five times higher than
rated current.

1)   Calculate the resistance and slot leakage inductance per bar for the keyed as well as
        the deep-bars for no load operation (slip $s = 0$) at 20°C.

2)   Perform the same calculation done in 1) for starting point at $s = 1$ and rated slip.

3)   How big are the bar current densities and the *ohmic* losses per bar for $s = 1$ and rated
        slip? (Note: For $s \geq s_N : I_r' \approx I_s, I_r' = I_r / \ddot{u}_I$)

4)   How big is the loss increase in a bar at $s = 1$ compared to rated slip for both bar types?

Slot sketches:
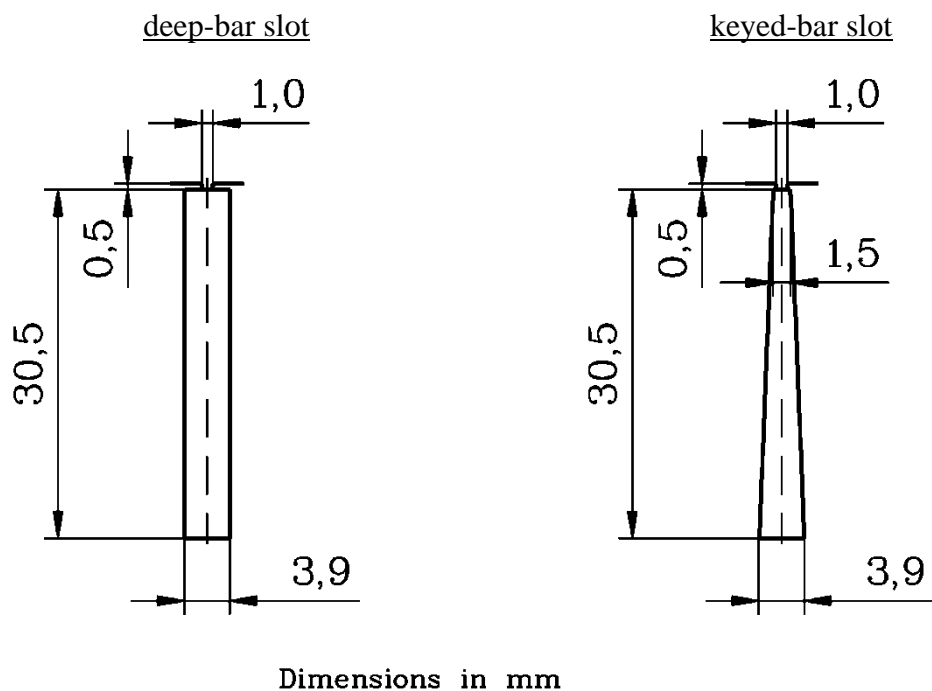
Fig. 2.3-1: Rotor slot geometry of a) deep bar and b) keyed bar

1)   <u>deep-bar slot</u>   <u>keyed-bar slot</u>

$$R_{bar=} = \frac{1}{\kappa} \cdot \frac{l_{Fe}}{q_{bar}}$$

$$s = 0: \quad f_r = s \cdot f_N = 0 \, \text{Hz}$$

a)	b)
$R_{bar=} = \dfrac{10^{-6}}{57} \cdot \dfrac{0.24}{119 \cdot 10^{-6}} \, \Omega = \underline{\underline{35.4 \, \mu\Omega}}$	$R_{bar=} = \dfrac{10^{-6}}{57} \dfrac{0.24}{82.4 \cdot 10^{-6}} \, \Omega = \underline{\underline{51.1 \, \mu\Omega}}$
$\lambda_{Qr} = \dfrac{h_1}{3 \cdot b_Q} \cdot k_L + \dfrac{h_4}{s_Q} = \dfrac{30.5}{3 \cdot 3.9} \cdot 1 + \dfrac{0.5}{1}$ $= 3.107$	$b_{equ(ivalent)} = b_o + \dfrac{b_u - b_o}{10} =$ $= 1.5 + \dfrac{3.9 - 1.5}{10} \, \text{mm} = 1.74 \, \text{mm}$ $\lambda_{Qr} = \dfrac{h_1}{3 \cdot b_{equ}} \cdot k_{L, wedge} + \dfrac{h_4}{s_Q} = \dfrac{30.5}{3 \cdot 1.74} \cdot 1 + \dfrac{0.5}{1}$ $= 6.343$
$L_{\sigma, bar(s=0)} = \underline{\underline{0.937 \, \mu\text{H}}}$	$L_{\sigma, bar(s=0)} = \underline{\underline{1.913 \, \mu\text{H}}}$

2)   <u>$s = 1$:</u> $f_r = s \cdot f_N = 50 \, \text{Hz}$

$$\xi = h \cdot \sqrt{\mu_0 \cdot \pi \cdot f_N \cdot \kappa} \cdot \sqrt{s} = 0.0305 \cdot \sqrt{4\pi \cdot 10^{-7} \cdot \pi \cdot 50 \cdot 57 \cdot 10^6} \cdot \sqrt{s} = 3.23 \cdot \sqrt{s}$$

a)	b)
$k_R = \xi \dfrac{\sinh(2\xi) + \sin(2\xi)}{\cosh(2\xi) - \cos(2\xi)} = 3.26$	$k_R = 4.8$ according to curve in text book
$k_L = \dfrac{3}{2 \cdot \xi} \cdot \dfrac{\sinh(2\xi) - \sin(2\xi)}{\cosh(2\xi) - \cos(2\xi)} = 0.46$	$k_{L, wedge} = 0.49$ according to curve in text book
$\lambda_{Qr} = \dfrac{h_1}{3 \cdot b_Q} \cdot k_L + \dfrac{h_4}{s_Q} = \dfrac{30.5}{3 \cdot 3.9} \cdot 1.699 + \dfrac{0.5}{1}$ $= 1.699$	$\lambda_{Qr} = \dfrac{h_1}{3 \cdot b_{equ}} \cdot k_{L, wedge} + \dfrac{h_4}{s_Q} = \dfrac{30.5}{3 \cdot 1.74} \cdot 0.49 + \dfrac{0.5}{1}$ $= 3.363$
$L_{\sigma, bar(s=1)} = \underline{\underline{0.512 \, \mu\text{H}}}$	$L_{\sigma, bar(s=1)} = \underline{\underline{1.014 \, \mu\text{H}}}$
$R_{bar, s=1} = \underline{\underline{115.4 \, \mu\Omega}}$	$R_{bar, s=1} = \underline{\underline{245.3 \, \mu\Omega}}$

Exercise 2.3   2.3/3

$\underline{s = s_N:}$ $s_N = \dfrac{1500 - 1460}{1500} = 0.027 \Rightarrow \xi = 0.53$

a) $k_R = 1.007$ $k_L = 0.9978$
b) $k_R \cong 1.0$ $k_L \cong 1.0$

$\Rightarrow$ Bar resistance and inductance are identical for s_N and $s = 0$.

3) $I'_{rN} = \dfrac{I_{rN}}{\ddot{u}_I}$ $\ddot{u}_I = \dfrac{m_s \cdot N_s \cdot k_{ws}}{Q_r \cdot 0.5 \cdot 1} = \dfrac{3 \cdot 44 \cdot 0.924}{40 \cdot 0.5 \cdot 1} = 6.098$

$\underline{s_N:}$ $I_{sN} = 104\,\text{A}$ $I'_r \approx I_{sN}$ $I'_r \cdot \ddot{u}_I = 6{,}098 \cdot 104\,\text{A} = 634.2\,\text{A}$

bar current: $I_r = I'_r \cdot \ddot{u}_I = 634.2\,\text{A}$

a)	b)
$J_r = \dfrac{I_r}{q_{bar}} = \dfrac{634.2}{119}\dfrac{\text{A}}{\text{mm}^2} = 5.3\dfrac{\text{A}}{\text{mm}^2}$	$J_r = \dfrac{634.2}{82.4}\dfrac{\text{A}}{\text{mm}^2} = 7.7\dfrac{\text{A}}{\text{mm}^2}$
$P_{d,bar} = R_{bar} \cdot I_r^2 = 35.4 \cdot 10^{-6} \cdot 634.2^2\,\text{W} = 14.2\,\text{W}$	$P_{d,bar} = 51.1 \cdot 10^{-6} \cdot 634.2^2\,\text{W} = 20.6\,\text{W}$

$\underline{s = 1:}$ $I_{s,1} = 5 \cdot 104\,\text{A} = 520\,\text{A}$ $I_{r,1} = 6{,}098 \cdot 520\,\text{A} = 3171\,\text{A}$

a)	b)
$J_r = \dfrac{3171}{119}\dfrac{\text{A}}{\text{mm}^2} = 26.6\dfrac{\text{A}}{\text{mm}^2}$	$J_r = \dfrac{3171}{82.4}\dfrac{\text{A}}{\text{mm}^2} = 38.5\dfrac{\text{A}}{\text{mm}^2}$
$P_{d,bar} = 115.4 \cdot 10^{-6} \cdot 3171^2\,\text{W} = 1160\,\text{W}$	$P_{d,bar} = 245.3 \cdot 10^{-6} \cdot 3171^2\,\text{W} = 2467\,\text{W}$

4)   Increase of losses compared to s_N at $s = 1$:

a) $\dfrac{P_{d,bar}(s = 1)}{P_{d,bar}(s = s_N)} = \dfrac{1160}{14.2} = 81.7$ b) $\dfrac{P_{d,bar}(s = 1)}{P_{d,bar}(s = s_N)} = \dfrac{2467}{20.6} = 119.8$

"Re-winding" of an induction motor

Deep-bar squirrel-cage induction motor:

Rated data:

$P_N = 500$ kW
$U_N = 6$ kV Y
$I_N = 59$ A
$n_N = 1482$/min
$\cos\varphi_N = 0.86$
$f_N = 50$ Hz

$P_{Fe} = 9.4$ kW   (Hysteresis and eddy-current losses)
$P_{Cu} = 15.0$ kW   (*Ohmic* losses)

Starting data:

Starting torque: $M_1 = 0.8 \cdot M_N$
Starting current: $I_1 = 5.2 \cdot I_N$

Stator winding data:

Winding type: double-layer winding
Number of turns per coil and layer: $N_c = 10$
Number of stator slots: $Q_s = 60$
All coils per phase are connected into series!

1) Calculate the number of turns per phase?

2) The number of turns per coil will be decreased from 10 to 9. Decreasing the number of turns per coil allows increasing the cross-section of copper. Calculate the new motor impedances, the starting current and the starting torque! Do the motor impedances change? How do starting torque and current vary in comparison to the given data?

3) Does the total leakage coefficient and the breakdown slip change? In case of changes, what is the percentage of the changes?

4) How do the air-gap induction, the primary current at rated slip and the electric loading change?

5) Compute the no load losses as well as the current-depending losses for the new number of turns per coil!

Exercise 2.4   2.4/2

$$N_s = 2 \cdot p \cdot q \cdot \frac{N_c}{a} = 4 \cdot 5 \cdot \frac{10}{1} = \underline{\underline{200}}$$

1)   Specifications: $a = 1$

$$Q_s = 60 = 2 \cdot p \cdot q \cdot m \Rightarrow q = \frac{Q_s}{2 \cdot p \cdot m} = \frac{60}{2 \cdot 2 \cdot 3} = \underline{\underline{5}}$$

2) $N_c^* = 9 \Rightarrow N_s^* = \frac{9}{10} \cdot 200 = \underline{\underline{180}}$

$$R_s \sim N_s^2 \Rightarrow R_s^* \sim \left(\frac{N_s^*}{N_s} \right)^2 : \underline{\underline{-19\%}} \; ;$$

$$R_r' \sim \ddot{u}^2 \cdot R_r \sim \left(\frac{N_s}{N_r} \right)^2 \cdot N_r^2 \sim N_s^2 \Rightarrow R_r'^* \sim \left(\frac{N_s^*}{N_s} \right)^2 : \underline{\underline{-19\%}} \; ;$$

$$X_s, X_{s\sigma}, X_r', X_{r\sigma}', X_h \sim N_s^2 \Rightarrow X^* \sim \left(\frac{N_s^*}{N_s} \right)^2 : \underline{\underline{-19\%}} \; ;$$

$$I_{s,1} = \frac{U_N}{\sqrt{\left(R_s + R_r' \right)^2 + \left(X_{s\sigma} + X_{r\sigma}' \right)^2}} \Rightarrow \frac{I_{s,1}^*}{I_{s,1}} = \left(\frac{N_s}{N_s^*} \right)^2 = \left(\frac{1}{0.9} \right)^2 = 1.2346$$

$$\Rightarrow I_{s,1}^* = \left(\frac{1}{0.9} \right)^2 \cdot I_{s,1} = \left(\frac{1}{0.9} \right)^2 \cdot 5.2 \cdot 59 \mathrm{A} = \underline{\underline{379\mathrm{A}}}$$

$$s = 1 \Rightarrow P_{Cu,r} = s \cdot P_\delta = P_\delta, \quad I_{s,1} \sim I_{r,1}' \quad (I_m \approx 0)$$

$$\Rightarrow M_1 = \frac{P_\delta}{\Omega_{syn}} = \frac{P_{Cu,r}}{s \cdot \Omega_{syn}} = \frac{m_s \cdot R_r' \cdot I_{r,1}'^2}{\Omega_{syn}} \sim \frac{m_s \cdot R_r' \cdot I_{s,1}^2}{\Omega_{syn}}$$

$$\Rightarrow \frac{M_1^*}{M_1} = \frac{R_r'^*}{R_r'} \cdot \frac{I_{s,1}^{*2}}{I_{s,1}'^2} = \left(\frac{N_s^*}{N_s} \right)^2 \cdot \left(\frac{N_s}{N_s^*} \right)^4 = \left(\frac{N_s}{N_s^*} \right)^2 = \underline{\underline{\left(\frac{1}{0.9} \right)^2}}$$

$$\Rightarrow M_1^* = \left(\frac{1}{0.9} \right)^2 \cdot 0.8 \cdot 3222 \mathrm{Nm} = \underline{\underline{3182\mathrm{Nm}}}$$

$$\left(M_N = \frac{500000\mathrm{W}}{2 \cdot \pi \cdot \frac{1482}{60} \cdot \mathrm{s}^{-1}} = 3222\mathrm{Nm} \right)$$

$\rightarrow$ The increase of starting current and starting torque is 23 %. The decrease of impedances is 19 %!

3) $\sigma = 1 - \frac{X_h^2}{X_s \cdot X_r'} \sim \frac{\left(N_s^{*2} \right)^2}{N_s^{*2} \cdot N_s^{*2}} = 1 \Rightarrow \underline{\underline{\sigma = \sigma^*}}$

$$s_b \approx \frac{R_r^{'}}{\sigma \cdot X_r^{'}} = s_b^{*} \qquad \left(\frac{R_r^{'}}{X_r^{'}} = \frac{R_r^{*'}}{X_r^{*'}}, \quad R_s \ll 1 \right)$$

→   Leakage coefficient and breakdown slip do not vary by changing the number of turns per coil!

4) $U_{N,ph} \approx (1+\sigma_s) \cdot U_i = (1+\sigma_s) \cdot \sqrt{2} \cdot \pi \cdot f \cdot N_s \cdot k_{ws} \cdot \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1} = const.$

$$\sigma_s = \frac{X_{s\sigma}}{X_h} \sim \frac{N_s^2}{N_s^2} = 1 \Rightarrow N_s^{*} \cdot B_{\delta,1}^{*} = N_s \cdot B_{\delta,1} = const. \;\Rightarrow\; \underline{\underline{\frac{B_{\delta,1}^{*}}{B_{\delta,1}} = \frac{N_s}{N_s^{*}}}}$$

$$\Rightarrow B_{\delta,1}^{*} = \frac{10}{9} \cdot B_{\delta,1} = \underline{\underline{1.11 \cdot B_{\delta,1}}}$$

→   The increase of the air-gap induction $B_{\delta 1}$ is 11%.

$$I_s = \frac{U_{N,ph}}{Z_{mot}(s)} \qquad \Rightarrow \underbrace{Z_{mot}(s) \sim N_s^2} \qquad \Rightarrow I_s \sim \frac{U_{N,ph}}{N_s^2}$$

$$\Rightarrow \frac{I_s^{*}}{I_s} \sim \left(\frac{N_s}{N_s^{*}} \right)^2 \qquad \text{(in case of } s = const.)$$

→   At nominal slip, the increase of the nominal current (as well as the starting current) is about $(1/0.9)^2 = 23\%$!

$$A_s = \frac{2 \cdot m_s \cdot N_s \cdot I_s}{2 \cdot p \cdot \tau_p} \Rightarrow \frac{A_s^{*}}{A_s} \sim \frac{N_s^{*} \cdot I_s^{*}}{N_s \cdot I_s} \sim \frac{N_s^{*} \cdot N_s^2}{N_s \cdot N_s^{*2}} \Rightarrow \underline{\underline{\frac{A_s^{*}}{A_s} \sim \frac{N_s}{N_s^{*}}}}$$

→   The increase in the current loading is about $(1/0.9) = 11$ %!

Plausibility check:

$$S \approx S_i = 3 \cdot U_{i,ph} \cdot I_s \quad \Rightarrow S \sim I_s \quad \Rightarrow \frac{S^{*}}{S} = \frac{I_s^{*}}{I_s} = \underline{\underline{\left(\frac{N_s}{N_s^{*}} \right)^2}} \qquad (U_i \approx U_{N,ph} = const.)$$

or

$$S \approx S_i = 3 \cdot U_{i,ph} \cdot I_s = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws} \cdot A_s \cdot B_{\delta,1} \cdot d_{si}^2 \cdot l_{Fe} \cdot n \sim A_s \cdot B_{\delta,1} \; C = \frac{\pi^2}{\sqrt{2}} \cdot k_{ws} \cdot A_s \cdot B_{\delta,1} :$$

Esson's coefficient $\left(\sim \dfrac{N_s}{N_s^{*}} \right)^2$: the electromagnetic utilization increases!

$$\Rightarrow \frac{S^*}{S} = \frac{A_s^* \cdot B_{\delta,1}^*}{A_s \cdot B_{\delta,1}} = \frac{N_s}{N_s^*} \cdot \frac{N_s}{N_s^*} = \underline{\underline{\left(\frac{N_s}{N_s^*}\right)^2}}$$

With assumption $\eta \cdot \cos\varphi = $ const., the output power increases too ($P_m = S \cdot \eta \cdot \cos\varphi$)!

5)　　$P_{Fe} \sim m_{Fe} \cdot v_{10} \cdot B^2,$　　$\frac{P_{Fe}^*}{P_{Fe}} = \left(\frac{B^*}{B}\right)^2 = \underline{\underline{\left(\frac{N_s}{N_s^*}\right)^2}}$

$$\Rightarrow P_{Fe}^* = P_{Fe} \cdot \left(\frac{N_s}{N_s^*}\right)^2 = 9.4\,\text{kW} \cdot \left(\frac{1}{0.9}\right)^2 = \underline{\underline{11.6\,\text{kW}}} \quad (+23\%)$$

$$P_{Cu} \sim R \cdot I^2, \quad \frac{P_{Cu}^*}{P_{Cu}} = \left(\frac{R^*}{R}\right) \cdot \left(\frac{I^*}{I}\right)^2 = \left(\frac{N^*}{N}\right)^2 \cdot \left(\frac{N}{N^*}\right)^4 = \left(\frac{N}{N^*}\right)^2$$

$$\Rightarrow P_{Cu}^* = P_{Cu} \cdot \left(\frac{N}{N^*}\right)^2 = 15\,\text{kW} \cdot \left(\frac{1}{0.9}\right)^2 = 1.23 \cdot 15\,\text{kW} = \underline{\underline{18.5\,\text{kW}}} \quad (+23\%)$$

Note: On the one hand the losses increase by the factor $(N/N^*)^2$, on the other hand, the Esson coefficient C and the output P_m also increase by the factor $(N/N^*)^2$, which leads to a constant efficiency.

Practical application:
By variation of N at $P_m = P_N = $ const. and $U = U_N = $ const., the minimal current consumption can be obtained, as the following pictures show.

Explanation: $P_m \sim A_s \cdot B_{\delta,1} \cdot \cos\varphi \sim I_r' \cdot I_m$　　$(I_r' \approx I_s \cdot \cos\varphi, \ A_s \sim I_s, \ B_{\delta,1} \sim I_m)$

$P_m \sim U_s \cdot I_{s,re} \ (= U_s \cdot I_s \cdot \cos\varphi) \ \sim \ I_{s,im} \cdot I_{s,re} \ \sim \ I_m \cdot I_r'; \ (U_s \sim U_i \sim B_{\delta,1} \sim I_m \approx I_{s,im})$

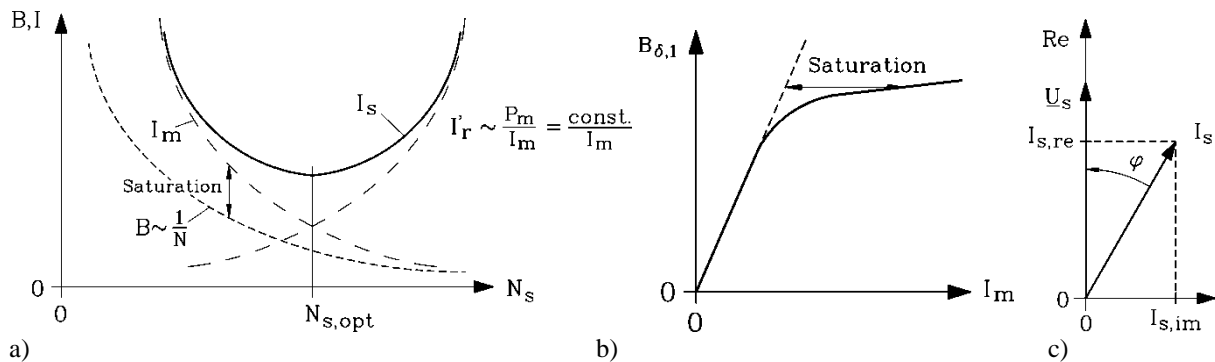$\left(I_s = \sqrt{I_{s,im}^2 + I_{s,re}^2} \ \approx \ \sqrt{I_m^2 + I_r'^2}\right)$

Fig. 2.4-1: a) Variation of flux density amplitude and stator current for variable number of turns per phase at fixed voltage and frequency, b) Fundamental air gap flux density amplitude vs. magnetizing current, c) Stator voltage and current phasors

Exercise 2.5   2.5/1

Hysteresis and eddy-current losses in the stator core

The drawing below shows the cut out of stator lamination. It was made for the stator of a four pole induction motor. The rated power P_N of the motor at 50 Hz is 55 kW. Further data of the motor:

Number of stator slots:	$Q_s = 48$
Lamination factor:	$k_{Fe} = 0.95$
Length of core:	$l_{Fe} = 220$ mm
inner/outer diameter of the laminated core:	$d_{si}/d_{sa} = 270$ mm / 400 mm

The coefficients for the calculation of the hysteresis and eddy current losses of the steel sheet

at 1 T and 50 Hz are: $\sigma_{Hy} = 1.3$ W/kg, $\sigma_{Ft} = 0.4$ W/kg
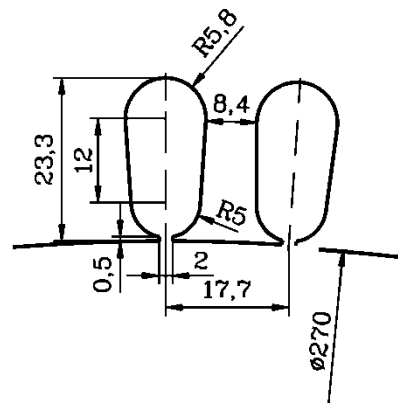
Dimensions in mm

Fig. 2.5-1: Stator iron lamination geometry

1)   Due to the influence of saturation, the distribution of the flux density in the air gap is flat-top. The amplitude of the air gap flux density $B_{\delta,max}$ is 0.8 T, while the amplitude of the fundamental FOURIER-wave $B_{\delta,1}$ is 0.85 T. What is the value of the main flux Φ_h? For calculating Φ_h make use of the value for the flux density of the fundamental wave.

2)   What is the value for the apparent induction B'_{ds} in the teeth and the induction B_{ys} in the yoke. The value of the stator leakage-flux has to be taken into account as $\sigma_s = 0.04$ ($\Phi_{s\sigma} = \sigma_s \cdot \Phi_h$).

3)   Calculate the value for the specific iron loss v_{10} of the magnetic sheet steel for the frequencies of 50 Hz and 100 Hz in the *Epstein* frame !

4)   Under influence of higher harmonics and by stamping the magnetic sheet steel, the loss coefficients for the iron increases compared to *Epstein* frame values. The eddy-current and hysteresis losses in the teeth are 80 % and in the yoke 50 % higher than according to *Epstein* frame. Recalculate the hysteresis and eddy-current losses for 50 and 100 Hz !
      Advice: For simplification make use of B'_{ds} instead of B_{ds}!

Exercise 2.5   2.5/2
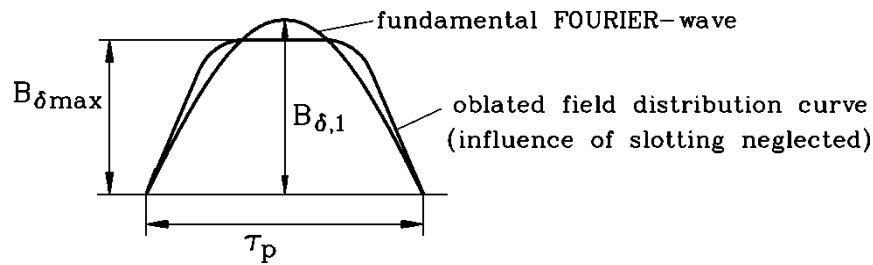
1)

Fig. 2.5-2: Air gap flux density per pole pitch

$$\Phi_h \cong \frac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1}, \qquad \tau_p = d_{si} \cdot \frac{\pi}{2 \cdot p} = 270 \cdot \frac{\pi}{4}\ mm = 212.0\ mm$$

$$= \frac{2}{\pi} \cdot 0.212 \cdot 0.22 \cdot 0.85 = \underline{\underline{0.0252\,Wb}}$$

2)

$\tau_Q = 17.7$ mm (in accordance to the drawing)

$b_{ds} = 8.4$ mm (in accordance to the drawing)

$$B'_{ds} = B_{\delta,max} \cdot \frac{\tau_Q}{b_{ds} \cdot k_{Fe}}$$

$$= 0.8 \cdot \frac{17.7}{8.4 \cdot 0.95} = \underline{\underline{1.77\,T}}$$

Magnetic flux in slot leads to lower tooth flux density $\rightarrow B_{ds} < B'_{ds}$

$$\Phi_{ys} = (1 + \sigma_s) \cdot \Phi_h / 2 = 1.04 \cdot 0.0252 / 2 = 0.0131 Wb$$

$$B_{ys} = \frac{\Phi_{ys}}{h_{ys} \cdot k_{Fe} \cdot l_{Fe}} = \frac{0.0131}{41.7 \cdot 10^{-3} \cdot 0.95 \cdot 0.22} = \underline{\underline{1.51 T}}$$

$$h_{ys} = [400 - (270 + 2 \cdot 23.3)] \cdot \frac{1}{2} = 41.7\,mm$$

$\uparrow$ slot height according to drawing

3)

$$v_{10} = \left(\frac{f}{50}\right) \cdot \sigma_{Hy} + \left(\frac{f}{50}\right)^2 \cdot \sigma_{Ft}$$

$$\rightarrow \text{ at } B = 1\,T \text{ and } f = 50\,Hz: \quad v_{10} = 1.3 + 0.4 = \underline{\underline{1.7 \frac{W}{kg}}}$$

$$\rightarrow \text{ at } B = 1\,T \text{ and } f = 100\,Hz: \quad v_{10} = 2 \cdot 1.3 + 2^2 \cdot 0.4 = \underline{\underline{4.2 \frac{W}{kg}}}$$

4)

mass of teeth m_{ds} $m_{ds} = A_{ds} \cdot l_{Fe} \cdot \gamma_{Fe} \cdot k_{Fe}$

$$= 10813 \cdot 10^{-6} \cdot 0.22 \cdot 7850 \cdot 0.95; \qquad \gamma_{Fe} = 7850 \frac{kg}{m^3}$$

$$= 17.7 \, kg$$

tooth area A_{ds} $A_{ds} = \left[(270 + 2 \cdot 23.3)^2 - 270^2\right] \cdot \frac{\pi}{4} - 48 \cdot 222 = 10813 \, mm^2$

$\underbrace{\qquad\qquad\qquad\qquad\qquad}$

ring area at tooth   number of slots   slot area

mass of yoke m_{ys} $m_{ys} = A_{ys} \cdot l_{Fe} \cdot \gamma_{Fe} \cdot k_{Fe}$

$$= 46938 \cdot 10^{-6} \cdot 0.22 \cdot 7850 \cdot 0.95; \qquad \gamma_{Fe} = 7850 \frac{kg}{m^3}$$

$$= 77.0 \, kg$$

yoke area A_{ys} $A_{ys} = \left[400^2 - (270 + 2 \cdot 23.3)^2\right] \cdot \frac{\pi}{4} = 46938 \, mm^2$

$k_{vd} = 1.8; \qquad k_{vy} = 1.5$

$$P_{Fe,ds} \cong m_{ds} \cdot k_{vd} \cdot \left(\frac{B'_{ds}}{1T}\right)^2 \cdot v_{10} \qquad \text{(by assuming } B'_{ds} \approx B_{ds})$$

$$P_{Fe,ys} \cong m_{ys} \cdot k_{vy} \cdot \left(\frac{B_{ys}}{1T}\right)^2 \cdot v_{10}$$

$f = 50Hz:$	$f = 100Hz:$
$v_{10} = 1.7 \dfrac{W}{kg}$	$v_{10} = 4.2 \dfrac{W}{kg}$
$P_{Fe,ds} = 17.7 \cdot 1.8 \cdot 1.77^2 \cdot 1.7 = \underline{170 \, W}$	$P_{Fe,ds} = \dfrac{4.2}{1.7} \cdot 170 = \underline{\underline{420W}}$
$P_{Fe,ys} = 77.0 \cdot 1.5 \cdot 1.5^2 \cdot 1.7 = \underline{442 \, W}$	$P_{Fe,ys} = \dfrac{4.2}{1.7} \cdot 442 = \underline{\underline{1092W}}$
$P_{Fe,ds+ys} = \underline{612 \, W}$	$P_{Fe,ds+ys} = \underline{\underline{1512W}}$

Cage design of a small induction motor

The data of a totally-enclosed fan-cooled **three-phase 6-pole cage induction motor** with single-layer winding are at a rated slip $s_N = 5.5\%$:

Rated voltage: $U_N = 400$ V Y   Rated efficiency: $\eta_N = 0.79$

Rated frequency: $f_N = 50$ Hz   Power factor: $\cos\varphi_{sN} = 0.75$

Rated shaft power: $P_N = 3.6$ kW   Bore diameter: $d_{si} = 130$ mm

Number of turns per phase: $N_s = 204$,   Stack length: $l_{Fe} = 130$ mm

Fundamental air gap field wave amplitude: $B_{\delta,1} = 0.9$ T, stator/rotor slots: $Q_s / Q_r = 36 / 30$.

1) Calculate the electrical apparent power S_N, the rated current I_{sN}, the *Esson´s* number C in kVA·min/m^3

2) Give the induced voltage per phase U_h, the rated torque M_N and rated speed n_N!

3) Determine the voltage and current transfer ratio $\ddot{u}_U$, $\ddot{u}_I$, the induced voltage per bar $U_{h,bar}$ and the rotor bar current I_r at rated slip via the estimate $I'_{rN} \approx I_{sN} \cdot \cos\varphi_{sN}$! Why is there nor insulation for the rotor bars necessary?

4) Determine for a rated current density $J_r = 4.2$ A/mm² in the Aluminum cage the bar cross section area A_{bar} and the ring cross section area A_{ring}!

5) Give the rotor outer diameter d_{ra} with the air-gap width $\delta = 0.4$ mm! For the rotor slot values $h_4 = 1.0$ mm and $h_{bar} = h_{ring} = 15$ mm calculate the deep bar and ring width b_{bar} and b_{ring}! With the conductivity $\kappa_{Alu}(20°C)$ 34 MS/m give the resistance of one bar (without current displacement) R_{bar}, of one ring segment ΔR_{ring} between two adjacent bars and the value R'_r of the T-bar equivalent circuit per phase!

1) $S_N = P_N /(\cos\varphi_{sN} \cdot \eta_N) = 3600/(0.75 \cdot 0.79) = 6076\,\text{VA}$,

$I_{sN} = S_N /(\sqrt{3} \cdot U_N) = 6076/(\sqrt{3} \cdot 400) = 8.77\,\text{A}$, ,

Stator r.m.s. current loading:

$A_s = \dfrac{2 m_s N_s I_{sN}}{d_{si}\pi} = \dfrac{2 \cdot 3 \cdot 204 \cdot 8.77}{0.13 \cdot \pi} = 26284\,\text{A/m} = 262.84\,\text{A/cm}$,

Single-layer winding = No pitching: $k_{p,1} = 1$,

Slots per pole & phase: $q_s = \dfrac{Q_s}{2p \cdot m_s} = \dfrac{36}{6 \cdot 3} = 2$,

Distribution factor: $k_{d,1} = \dfrac{\sin(\pi /(2m_s))}{q_s \cdot \sin(\pi /(2m_s \cdot q_s))} = \dfrac{\sin(\pi /6)}{2 \cdot \sin(\pi /12)} = 0.966$

Fundamental winding factor: $k_{ws} = k_{d,1} k_{p,1} = 0.966$.

$C = \dfrac{\pi^2}{\sqrt{2}} \cdot k_{ws} \cdot A_s \cdot B_{\delta,1} = \dfrac{\pi^2}{\sqrt{2}} \cdot 0.966 \cdot 26284 \cdot 0.9 = 159477 \dfrac{\text{VAs}}{\text{m}^3} = 2.66 \dfrac{\text{kVAmin}}{\text{m}^3}$.

2)   Pole pitch: $\tau_p = d_{si}\pi /(2p) = 130\pi /6 = 68.1\,\text{mm}$,

Fundamental air-gap flux/pole:

$\Phi_{\delta 1} = \dfrac{2}{\pi} \cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1} = \dfrac{2}{\pi} \cdot 0.0681 \cdot 0.13 \cdot 0.9 = 5.07\,\text{mWb}$,

$U_h = \sqrt{2}\pi \cdot f_s \cdot N_s \cdot k_{ws} \cdot \Phi_{\delta 1} = \sqrt{2}\pi \cdot 50 \cdot 204 \cdot 0.966 \cdot 5.07/10^3 = 221.95\,\text{V}$,

$n_{syn} = f_s / p = 50/3 = 16.67/\text{s} = 1000/\min$,

$n_N = (1 - s_N) \cdot n_{syn} = (1 - 0.055) \cdot 1000 = 945/\min$,

$M_N = P_N /(2\pi \cdot n_N) = 3600/(2\pi \cdot 945/60) = 36.74\,\text{Nm}$.

3) $\ddot{u}_U = \dfrac{k_{ws}N_s}{k_{wr}N_r} = \dfrac{0.966 \cdot 204}{1 \cdot 0.5} = 394.13$, $\ddot{u}_I = \ddot{u}_U \cdot \dfrac{m_s}{m_r} = \ddot{u}_U \cdot \dfrac{3}{Q_r} = 394.13 \cdot \dfrac{3}{30} = 39.41$,

$U_{h,bar} = U_h / \ddot{u}_U = 221.95/394.13 = 0.56\text{V}$:

Due to the very low bar voltage no rotor bar insulation is needed.
Even at stand-still ($s = 1$) the increased bar voltage

$U_{h,bar}(s = 1) = U_{h,bar}(s_N) \cdot (s / s_N)\big|_{s=1} = 0.56 \cdot (1/0.055) = 10.18\,\text{V}$

is very small (below low voltage AC limit of 24 V!).

$I'_{rN} \approx I_{sN} \cdot \cos\varphi_{sN} \Rightarrow I_{rN} = \ddot{u}_I \cdot I'_{rN} = 39.41 \cdot 8.77 \cdot 0.75 = 259.2\,\text{A}$.

4) $A_{bar} = \dfrac{I_{rN}}{J_r} = \dfrac{259.2}{4.2} = 61.7\,\text{mm}^2$, $I_{ring} = \dfrac{I_{rN}}{2 \cdot \sin(p \cdot \pi /Q_r)} = \dfrac{259.2}{2 \cdot \sin(3 \cdot \pi /30)} = 419.4\,\text{A}$,

$A_{ring} = \dfrac{I_{ring}}{J_r} = \dfrac{419.4}{4.2} = 99.9\,\text{mm}^2$.

5)　　$b_{\mathrm{bar}} = A_{\mathrm{bar}} / h_{\mathrm{bar}} = 61.7/15 = 4.1\,\mathrm{mm}$, $b_{\mathrm{ring}} = A_{\mathrm{ring}} / h_{\mathrm{bar}} = 99.9/15 = 6.67\,\mathrm{mm}$,

$b_{\mathrm{ring}} = A_{\mathrm{ring}} / h_{\mathrm{bar}} = 99.9/15 = 6.67\,\mathrm{mm}$, $d_{\mathrm{ra}} = d_{\mathrm{si}} - 2\delta = 130 - 2\cdot 0.4 = 129.2\,\mathrm{mm}$,

$$R_{\mathrm{bar}}(20°\mathrm{C}) = \frac{l_{\mathrm{Fe}}}{\kappa_{\mathrm{Alu},20} \cdot A_{\mathrm{bar}}} = \frac{0.13}{34\cdot 10^{6} \cdot 61.7\cdot 10^{-6}} = 62.0\,\mu\Omega,$$

$d_{\mathrm{ring}} = d_{\mathrm{ra}} - 2h_{4} - h_{\mathrm{ring}} = 129.2 - 2\cdot 1 - 15 = 112.2\,\mathrm{mm}$,

$$\Delta R_{\mathrm{ring}}(20°\mathrm{C}) = \frac{d_{\mathrm{ring}}\pi / Q_{\mathrm{r}}}{\kappa_{\mathrm{Alu},20} \cdot A_{\mathrm{ring}}} = \frac{0.1122 \cdot \pi / 30}{34\cdot 10^{6} \cdot 99.9\cdot 10^{-6}} = 3.46\,\mu\Omega,$$

$$\Delta R_{\mathrm{ring}}^{*}(20°\mathrm{C}) = \frac{\Delta R_{\mathrm{ring}}}{2\cdot \sin^{2}(p\pi / Q_{\mathrm{r}})} = \frac{3.46}{2\cdot \sin^{2}(3\pi / 30)} = 18.1\,\mu\Omega,$$

$R_{\mathrm{r}}(20°\mathrm{C}) = R_{\mathrm{bar}} + \Delta R_{\mathrm{ring}}^{*} = 62.0 + 18.1 = 80.1\,\mu\Omega$,

$R_{\mathrm{r}}'(20°\mathrm{C}) = \ddot{u}_{\mathrm{U}}\ddot{u}_{\mathrm{I}}R_{\mathrm{r}} = 394.13\cdot 39.41\cdot 80.1\cdot 10^{-6} = 1.24\,\Omega$

Stator winding design of a small induction motor

The data of a totally-enclosed fan-cooled (TEFC) **three-phase 6-pole cage induction motor** with single-layer winding are at a rated slip $s_N = 5.5\%$:

Rated voltage:	$U_N = 400$ V Y	Rated current:	$I_{sN} = 8.5$ A
Rated frequency:	$f_N = 50$ Hz	Power factor:	$\cos\varphi_{sN} = 0.75$
Rated shaft power:	$P_N = 3.6$ kW	Bore diameter:	$d_{si} = 130$ mm
Stack length:	$l_{Fe} = 130$ mm	Stator/rotor slots:	$Q_s / Q_r = 36 / 30$

Fundamental air gap field wave amplitude: $B_{\delta,1} = 0.9$ T.

1) Calculate the electrical apparent power S_N and the rated efficiency η_N! Estimate the inner apparent power S_δ and induced voltage per phase U_h the by a stator stray coefficient $\sigma_s = 0.05$! Give with these values the *Esson´s* number C in kVA·min/m^3!

2) Determine the necessary turns per phase N_s for the conditions of 1)! Find the real number of turns per phase and per coil N_c for a series connection of the coil groups!

3) Determine the thermal utilization for a stator current density $J_s = 6.5$ A/mm^2 via the r.m.s. current loading A_s! Is the thermal limit for TEFC motors kept for a temperature rise of 80 K above 40 °C in the stator winding (Thermal Class B)?

4) Choose the round wire copper, if wire diameters $d_{Cu} = 0.6 / 0.7 / 0.8 / 0.9$ mm are available! The slot cross section is trapezoidal with an inner and outer slot breadth $b_{Qi} = 6.5$ mm, $b_{Qa} = 8.7$ mm, and a slot height $h_4 = 0.5$ mm, $h_1 = 12.6$ mm. Check, if the demanded slot fill factor $k_f < 0.45$ for single-layer winding!

5) The winding overhang length is $l_b = 90$ mm! Determine the *ohmic* resistance R_s per phase at 20 °C! Give the corresponding stator winding I^{2}R-losses at rated speed and 120°C hot copper, and their percentage of total losses P_d! Give the needed copper mass $m_{Cu,s}$ for the winding!

6) Calculate the stator tooth width b_{ds}! Is the tooth width constant along the stator tooth? Check, if the apparent stator tooth flux density limit ≤ 1.9 T is fulfilled! Use for the flat-topped real air-gap flux density the value $B_{\delta,max} = 0.8\,T < B_{\delta,1} = 0.9\,T$, and an iron fill factor $k_{Fe} = 0.97$!

1) $S_N = \sqrt{3}\,U_N I_{sN} = \sqrt{3}\cdot 400\cdot 8.5 = 5889\,\text{VA}$, $P_{eN} = S_N \cdot \cos\varphi_{sN} = 5889\cdot 0.75 = 4417\,\text{W}$,

      $\eta_N = P_N / P_{eN} = 3600/4417 = 81.5\%$,

      $U_h \approx U_{sN}/(1+\sigma_s) = (U_N/\sqrt{3})/(1+\sigma_s) = (400/\sqrt{3})/1.05 = 219.9\,\text{V}$,

      $S_\delta = 3\cdot U_h \cdot I_{sN} = 3\cdot 219.9\cdot 8.5 = 5608\,\text{VA}$, $n_{syn} = f_s / p = 50/3 = 16.67/\text{s}$,

      $C = \dfrac{S_\delta}{d_{si}^2 \cdot l_{Fe} \cdot n_{syn}} = \dfrac{5.608}{0.13^2\cdot 0.13\cdot 16.67\cdot 60} = 2.55\dfrac{\text{kVAmin}}{\text{m}^3}$.

2)   Single-layer winding = No pitching: $k_{p,1} = 1$,

      Slots per pole & phase: $q_s = \dfrac{Q_s}{2p\cdot m_s} = \dfrac{36}{6\cdot 3} = 2$,

      Distribution factor: $k_{d,1} = \dfrac{\sin(\pi/(2m_s))}{q_s \cdot \sin(\pi/(2m_s\cdot q_s))} = \dfrac{\sin(\pi/6)}{2\cdot \sin(\pi/12)} = 0.966$,

      Fundamental winding factor: $k_{ws} = k_{d,1}k_{p,1} = 0.966$.

      Pole pitch: $\tau_p = d_{si}\pi/(2p) = 130\pi/6 = 68.1\,\text{mm}$,

      Fundamental air-gap flux/pole:

      $\Phi_{\delta 1} = \dfrac{2}{\pi}\cdot \tau_p \cdot l_{Fe} \cdot B_{\delta,1} = \dfrac{2}{\pi}\cdot 0.0681\cdot 0.13\cdot 0.9 = 5.07\,\text{mWb}$,

      $U_h = \sqrt{2}\pi \cdot f_s \cdot N_s \cdot k_{ws} \cdot \Phi_{\delta 1}$,

      $N_s = \dfrac{U_h}{\sqrt{2}\pi \cdot f_s \cdot k_{ws} \cdot \Phi_{\delta 1}} = \dfrac{219.9}{\sqrt{2}\pi\cdot 50\cdot 0.966\cdot 0.00507} = 202.12$.

      Series connection of all coil groups: $a = a_a = 1$,

      single-layer winding: $N_s = p\cdot q_s \cdot N_c / a$, $N_c = \dfrac{N_s a}{p\cdot q_s} = \dfrac{202.12\cdot 1}{3\cdot 2} = 33.69$,

      next integer choice: $N_c = 34$, real turns per phase: $N_s = 6\cdot 34 = 204$.

3)   Stator r.m.s. current loading:

      $A_s = \dfrac{2m_s N_s I_{sN}}{d_{si}\pi} = \dfrac{2\cdot 3\cdot 204\cdot 8.5}{0.13\cdot \pi} = 25475\,\text{A/m} = 254.75\,\text{A/cm}$,

      $J_s A_s = 6.5\cdot 254.75 = 1655\,(\text{A/mm}^2)\cdot(\text{A/cm})$

      The Thermal Class B temperature rise demands for TEFC motors a thermal limit of

      $(J_s A_s)_{max} \approx 1700\,(\text{A/mm}^2)\cdot(\text{A/cm})$, which is fulfilled!

4) $a = 1$: $I_c = I_{sN}/a = I_{sN}$, $A_L = I_c / J_s = I_{sN}/J_s = 8.5/6.5 = 1.31\,\text{mm}^2$,

      $\pi\cdot d_{Cu}^2/4 = 0.28\,\text{mm}^2/0.38\,\text{mm}^2/0.50\,\text{mm}^2/0.64\,\text{mm}^2$. Closest fitting to the

      demanded value $1.31\,\text{mm}^2$ is $a_i = 2, d_{Cu} = 0.9\,\text{mm}$: $A_L = 1.27\,\text{mm}^2$.

      $J_s = I_{sN}/A_L = 8.5/1.27 = 6.7\,\text{A/mm}^2$, $J_s A_s = 6.7\cdot 254.75 = 1705\,(\text{A/mm}^2)\cdot(\text{A/cm})$.

      Slot cross-section area: $A_Q = h_1 \cdot (b_{Qi} + b_{Qa})/2 = 12.6/(6.5+8.7)/2 = 95.76\,\text{mm}^2$,

$k_f = N_c \cdot A_L / A_Q = 34 \cdot 1.27 / 95.76 = 0.4509 \approx 0.45$. The manufacturing limit for semi-closed slots with random-wound round-wire winding is very tightly fulfilled!

5) $R_s(20°C) = \dfrac{N_s \cdot 2 \cdot (l_{Fe} + l_b)}{\kappa_{Cu,20} \cdot a \cdot A_L} = \dfrac{204 \cdot 2 \cdot (0.13 + 0.09)}{57 \cdot 10^6 \cdot 1 \cdot 1.27 \cdot 10^{-6}} = 1.0\,\Omega$,

$R_s(120°C) = R_s(20°C) \cdot \dfrac{235 + 120}{235 + 20} = R_s(20°C) \cdot 1.39 = 1.39\,\Omega$,

$P_{Cu,s} = 3R_{s,120} \cdot I_{sN}^2 = 3 \cdot 1.39 \cdot 8.5^2 = 302\,W$,

Total losses: $P_d = P_{eN} - P_N = 4417 - 3600 = 817\,W$, $P_{Cu,s} / P_d = 302/817 = 37\%$.

$m_{Cu,s} = \gamma_{Cu} \cdot m_s \cdot N_s \cdot 2 \cdot (l_{Fe} + l_b) \cdot A_L$,

$m_{Cu,s} = 8900 \cdot 3 \cdot 204 \cdot 2 \cdot (0.13 + 0.09) \cdot 1.27 \cdot 10^{-6} = 3.0\,kg$

6) Tooth width b_{di} at b_{Qi}: Diameter: $d_1 = d_{si} + 2h_4 = 130 + 2 \cdot 0.5 = 131\,mm$,

$b_{di} = \dfrac{d_1 \pi}{Q_s} - b_{Qi} = \dfrac{131 \cdot \pi}{36} - 6.5 = 4.93\,mm$,

Tooth width b_{da} at b_{Qa}: Diameter: $d_2 = d_{si} + 2h_4 + 2h_1 = 131 + 2 \cdot 12.6 = 156.2\,mm$,

$b_{da} = \dfrac{d_2 \pi}{Q_s} - b_{Qa} = \dfrac{156.2 \cdot \pi}{36} - 8.7 = 4.93\,mm$. The stator tooth has constant width

$b_d = b_{da} = b_{di} = 4.93\,mm$. Stator slot pitch: $\tau_{Qs} = \dfrac{d_{si} \pi}{Q_s} = \dfrac{130 \cdot \pi}{36} = 11.34\,mm$.

Apparent stator tooth flux density: $B_{ds}' = \dfrac{B_{\delta,max} \cdot \tau_{Qs}}{b_{ds} \cdot k_{Fe}} = \dfrac{0.8 \cdot 11.34}{4.93 \cdot 0.97} = 1.9\,T$.

The saturation condition is tightly fulfilled!

Heating of a three-phase winding

A three-phase a.c. motor has stator coils according to the drawing of the slot as shown below (all conductors are connected in series). The dimensions of the bright shaped wire are 7.1 mm×1.8 mm. The active iron length l_{Fe} is 380 mm, the inner stator diameter d_{si} is 460 mm and the number of stator slots Q_s is 60. Further data of the motor are given.

Data:

> $P_N = 500$ kW
> $U_N = 6$ kV
> $I_N = 59$ A
> $n_N = 1460$/min

1) Calculate the *ohmic* losses per slot and the current density in the conductor at an ambient temperature of 30 °C. Make sure that the temperature-rise limit for a winding of thermal class B is fully utilised!

2) Compute the temperature difference between the copper wire and the stator core, if the heat conductivity of the slot insulation λ is 0.2 W/(m·K). The heat flow via the slot wedge shall be neglected.

3) Calculate the temperature difference between tooth tip (surface along b_d) and the air in the air gap. Assume, that half of the *ohmic* losses is flowing as heat flow directly to the housing via the stator core.
   The heat transfer coefficient α (W/(m²·K)) at the surface of the core assembly is depending on the circumference speed of the rotor v_u as the following equation shows:

   $$\alpha = 30 \cdot \sqrt{0.75 \cdot v_u} - 20 \quad (v_u \text{ in m/s})$$

   What is the temperature difference between the copper of the winding and the cooling air at the air gap, when the temperature gradient of the iron along the tooth is neglected?
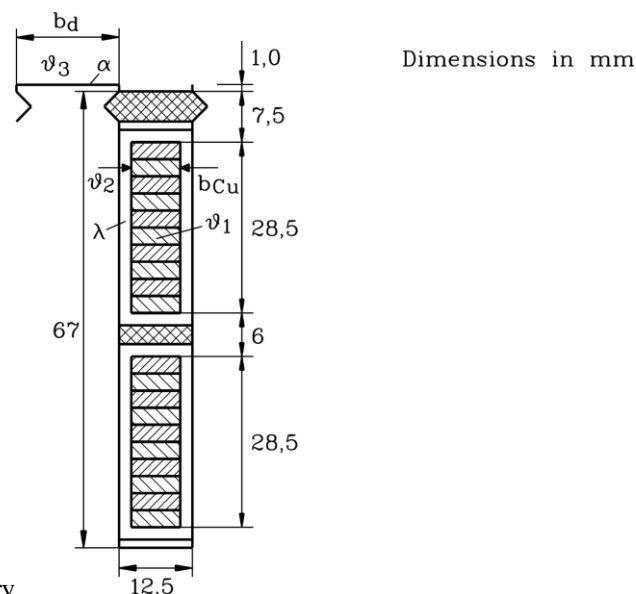

Fig. 3.1-1: Stator slot geometry

1)   *Specifications:*
Thermal class B → temperature-rise limit according to IEC 60034-1:   80 K
Given ambient temperature: 30 °C
→ $\vartheta_{Cu} = (80 + 30)\ °C = 110\ °C$

Temperature coefficient of electrical resistance: $\alpha_\vartheta = 0.0039/K$:

$$\kappa_{Cu} = \kappa_{20°C} \cdot \frac{1}{1 + \alpha_\vartheta \cdot (\vartheta_{Cu} - 20°C)} = \frac{57 \cdot 10^6 \cdot \dfrac{S}{m}}{1 + 0.0039 \cdot (110 - 20)} = 42.2 \cdot 10^6\ \frac{S}{m}$$

Current density of conductor : $J = \dfrac{I}{b_{Cu} \cdot h_{Cu}} = \dfrac{59}{7.1 \cdot 1.8} \cdot \dfrac{A}{mm^2} = \underline{\underline{4.6 \dfrac{A}{mm^2}}}$

$(I_C = I = 59A,\ \text{due to series connection})$

$$P_{d,Conductor} = \frac{1}{\kappa_{Cu}} \cdot \frac{l}{b_{Cu} \cdot h_{Cu}} \cdot I_C^2$$

$$P_{d,Q} = N_Q \cdot P_{d,Conductor} \qquad\qquad N_Q : \text{number of turns per slot}$$

$$= 2 \cdot 10 \cdot \frac{10^{-6}}{42.2} \cdot \frac{0.38}{7.1 \cdot 1.8} \cdot 10^6 \cdot 59^2 = \underline{\underline{49\ W}}$$

2)

Insulation thickness : $\Delta = \dfrac{1}{2}(b_Q - b_{Cu}) = \dfrac{12.5 - 7.1}{2}\ mm = 2.7\ mm,$

A_Q : side surface of slot without area of slot wedge

$A_Q = (2 \cdot 67 + 12.5) \cdot 10^{-3} \cdot 0.38\ m^2 = 55.67 \cdot 10^{-3}\ m^2,$

thermal resistance : $R_d = \dfrac{\Delta}{A_Q \cdot \lambda} = \dfrac{2.7 \cdot 10^3}{55.67 \cdot 0.2}\ \dfrac{K}{W} = 0.243\ \dfrac{K}{W}.$

$\Delta\vartheta = \vartheta_1 - \vartheta_2 = P_{d,Q} \cdot R_d = 49 \cdot 0.243\ K = \underline{\underline{11.9\ K}}$

3)   Circumference speed of the rotor:

$$v_u = d_{si} \cdot \pi \cdot n = 0.46 \cdot \pi \cdot \frac{1460}{60} = 35.2\ \frac{m}{s},$$

$$\tau_Q = \frac{d_{si} \cdot \pi}{Q} = \frac{0.46 \cdot \pi}{60} = 0.0241\ m,$$

$$\alpha = 30 \cdot \sqrt{0.75 \cdot 35.2} - 20 = 134\ \frac{W}{m^2 \cdot K},$$

$$b_d = \tau_Q - b_Q = 0.0241\ m - 0.0125\ m = 0.0116\ m,$$

$$A_{Fe} = Q \cdot b_d \cdot l_{Fe} = 60 \cdot 0.126 \cdot 0.38\ m^2 = 0.265\ m^2,$$

Per slot pitch: $A_{Fe,Q} = b_d \cdot l_{Fe} = 0.0116 \cdot 0.38 = 0.0044\ m^2,$

$$R_\alpha = \frac{1}{A_{Fe,Q} \cdot \alpha} = \frac{1}{0.0044 \cdot 134} = 1.69\ \frac{K}{W} \quad \Rightarrow \quad \Delta\vartheta = \vartheta_2 - \vartheta_3 = \frac{P_{d,Q}}{2} \cdot R_\alpha = \underline{\underline{41.4\ K}},$$

Temperature difference beteen copper and air gap: $\Delta\vartheta_{res} = \vartheta_1 - \vartheta_3 = \underline{\underline{53.3\ K}}.$

Different operation modes of a.c. machines

A **500 kW three-phase asynchronous motor**, with shaft-mounted fan, thermally utilized in accordance with Thermal Class H, with an efficiency of 94.7 %, has a thermal time constant $T_\vartheta = 1$ h with rotating rotor (fan turns) and a time constant $T_{\vartheta St} = 1.5$ h with locked rotor (fan is not moving).

1)   How big is the admissible maximum temperature rise of the stator winding? How big are the losses, which the motor at continuous operation may have?

2)   How big are the admissible total losses and the output power of the motor in the S2-Operation (30 min. operating time)?

3)   How long is the operating time, how big are the admissible total losses and the output power of the machine in the S3-Operation (60 % ON-time, duty cycle time 20 min.)?

4)   Which temperature (°C) and temperature rise (K) are reached within the motor winding at an ambient temperature of 20 °C, if the machine is operated continuously with the admissible output power according to 3)? To what extent is the life span shortening of the used insulation system with this mode of operation?

1)   Thermal Class H: maximum temperature rise is <u>125 K</u> (IEC 60034-1)!

$$P_{\rm d} = \left(\frac{1}{\eta} - 1\right)\cdot P = \left(\frac{1}{0.947} - 1\right)\cdot 500 = \underline{\underline{28\,{\rm kW}}} = P_{\rm d,S1}, \quad \varDelta\vartheta_{\rm S1} = \underline{\underline{125\,{\rm K}}}$$

2)   S2: $\dfrac{P_{\rm d,S2}}{P_{\rm d,S1}} = \dfrac{\varDelta\vartheta_{\rm S2}}{\varDelta\vartheta_{\rm S1}} = \dfrac{1}{1 - e^{-t_{\rm B}/T_\vartheta}} = \dfrac{1}{1 - e^{-30/60}} = \underline{\underline{2.54}}$

$\varDelta\vartheta$: temperature rise ("heating")
$t_{\rm B} = 30\,{\rm min},\qquad T_\vartheta = 60\,{\rm min}$

$P_{\rm d,S2} = 2.54\cdot 28 = \underline{\underline{71\,{\rm kW}}},$

$\dfrac{P_{\rm out,S2}}{P_{\rm out,S1}} \cong \dfrac{U_{\rm N}\cdot I_{\rm S2}}{U_{\rm N}\cdot I_{\rm S1}} = \dfrac{I_{\rm S2}}{I_{\rm S1}} \qquad P_{\rm d} \sim I^2$ (Assumption: current dependent losses dominate)

$\Rightarrow \dfrac{P_{\rm out,S2}}{P_{\rm out,S1}} \cong \sqrt{\dfrac{P_{\rm d,S2}}{P_{\rm d,S1}}} \cong \sqrt{2.54} = 1.6$

$P_{\rm out,S2} = 1.6\cdot P_{\rm out,S1} = \underline{\underline{797\,{\rm kW}}}$

3) $t_{\rm B}/t_{\rm S} = 0.6 \Rightarrow t_{\rm B} = 0.6\cdot 20 = \underline{\underline{12\,{\rm min}}}$

$t_{\rm St} = t_{\rm S} - t_{\rm B} = 8\,{\rm min}$

$\dfrac{P_{\rm d,S3}}{P_{\rm d,S1}} = \dfrac{\varDelta\vartheta_{\rm S3}}{\varDelta\vartheta_{\rm S1}} = 1 + \dfrac{T_\vartheta\cdot t_{\rm St}}{T_{\vartheta {\rm St}}\cdot t_{\rm B}} - \dfrac{t_{\rm St}}{T_{\vartheta {\rm St}}} = 1 + \dfrac{60\cdot 8}{90\cdot 12} - \dfrac{8}{90} = 1.36$

$\rightarrow P_{\rm d,S3} = 1.36\cdot 28 = \underline{\underline{38\,{\rm kW}}}$

$P_{\rm S3}/P_{\rm S1} = \sqrt{1.36} \Rightarrow P_{\rm S3} = \sqrt{1.36}\cdot 500 = \underline{\underline{582\,{\rm kW}}}$

Temperature rise

4) $\varDelta\vartheta_{\rm S3} = \varDelta\vartheta_{\rm S1}\cdot 1.36 = 125\cdot 1.36 = \underline{\underline{170\,{\rm K}}}$:   Temperature rise

$\vartheta_{\rm S3} = 20 + 170 = \underline{\underline{190\,°{\rm C}}}$:   Temperature

MONTSINGER's rule: + 10 K temperature rise $\rightarrow$ half life span L

125 K $\rightarrow$ 135 K $\rightarrow$ 145 K $\rightarrow$ 155 K $\rightarrow$ 165 K $\rightarrow$ 170 K $\rightarrow$ 175 K

L $L/2$ $L/4$ $L/8$ $L/16$ $\downarrow$ $L/32$

$$L \cong \frac{1}{2}\left(\frac{L}{16} + \frac{L}{32}\right) = \frac{L}{21.3}$$

The statistically determined <u>life span</u> of insulation system is shortened down to <u>5 %</u> of nominal value (e.q. 100000 h $\rightarrow$ 5000 h).

Exercise 3.3   3.3/1

Heating up of an asynchronous machine with blocked rotor

A **three-phase asynchronous motor with squirrel-cage rotor** (wedge-bars in the rotor) and following rated data

$P_N = 55$ kW; $U_N = 400$ V Y; $f_N = 50$ Hz; $\cos\varphi_N = 0{,}86$, $\eta_N = 0.9$

will be connected to the mains. Due to a defect in the coupled load machine the motor cannot run up; the rotor is **blocked**. From the calculation sheet of the manufacturer, the following data of the equivalent circuit diagram are known:

$R_s = 0.06\ \Omega$; $X_{s\sigma} = 0.17\ \Omega$; $X_h = 8.63\ \Omega$,
At $s = 1$ (current displacement!): $R_r' = 0.17\ \Omega$; $X_{r\sigma}' = 0.24\ \Omega$.

Answer the following questions:

1.) How big is the stationary current in the stator winding?

2.) How many times is it bigger than the rated current?

3.) How big are the copper losses in the stator winding at $s = 1$ compared to the rated operation?

4.) The mass of the stator winding amounts to 22.9 kg. Determine the final temperature of the stator winding with an ambient temperature (coolant temperature) of $\vartheta_{amb} = 25°C$, if the protection switch disconnects the motor from the mains after 20 s. Neglect due to this short operating time the heat transfer from the winding to the ambient (heat transfer coefficient $\alpha \cong 0$: "adiabatic heating"); specific thermal capacity of copper: $c_{Cu} = 386$ Ws/(kg·K).

5.) The thermal time constant $T_{\vartheta,st}$ of the machine at stand still ($n = 0$) amounts to 40 min. How big is the winding temperature ϑ_{Cu} at the time 25 min after switching the motor off?

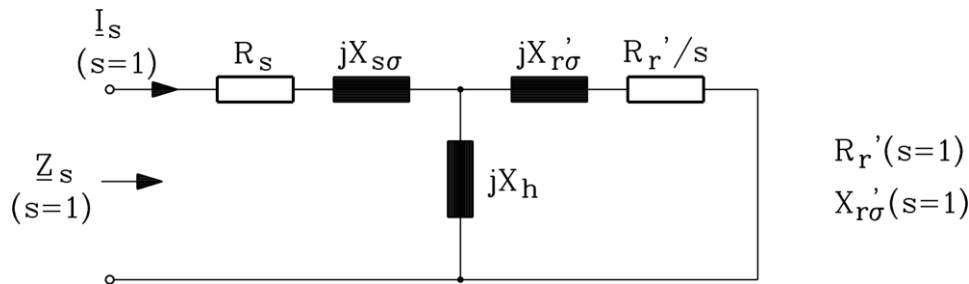Exercise 3.3   3.3/2

1) Circuit diagram for $s = 1$:

Fig. 3.3-1: T-equivalent circuit per phase of the induction machine

$R_r'(s=1) \quad X_{r\sigma}'(s=1)$

$I_s(s=1) = \dfrac{U_{s,\text{phase}}}{Z_s}$ (Y-connection!)

$U_{s,\text{phase}} = \dfrac{U_N}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230$ V,

$\underline{Z}_s = R_s + jX_{s\sigma} + \dfrac{jX_h(R_r'+jX_{r\sigma}')}{R_r'+j(X_h+X_{r\sigma}')} = 0.06 + j0.17 + \dfrac{j8.63 \cdot (0.17 + j0.24)}{0.17 + j(8.63 + 0.24)} =$

$= 0,06 + j0,17 + \dfrac{-2.07 + j1.55}{0.17 + j8.87} = 0,06 + j0,17 + \dfrac{(-2,07 + j1.55)\cdot(0.17 - j8.87)}{0.17^2 + 8.87^2} =$

$= 0.06 + j0.17 + \dfrac{12.66 + j18.61}{78.7} = 0.221 + j0.407 \ \Omega$

$|\underline{Z}_s| = \sqrt{0.221^2 + 0.407^2} = 0.463 \ \Omega, \qquad I_s = \dfrac{230}{0,463} = \underline{\underline{497.1 \ A}}$

Note: Since $X_{r\sigma}', R_r' << X_h$, one can calculate with sufficient accuracy:

$I_s\big|_{s=1} \cong \dfrac{U_s}{(R_s + R_r') + j(X_{s\sigma} + X_{r\sigma}')} = \dfrac{230}{0.23 + j0.41}$

$\Rightarrow \quad I_s = \dfrac{230}{\sqrt{0.23^2 + 0.41^2}} = 489.3 \ A$

(Calculation error: (489.3 – 497.1)/497.1 = -1.6 %)

2) $\quad I_{s,N} = \dfrac{S_N}{\sqrt{3}U_N} = \dfrac{71059}{\sqrt{3}\cdot 400} = 102.57 \ A$

$S_N = \dfrac{P_N}{\cos\varphi_N \cdot \eta_N} = \dfrac{55000}{0.89\cdot 0.9} = 71059$ VA, $\quad \dfrac{I_s(s=1)}{I_{sN}} = \dfrac{497.1}{102.57} = \underline{\underline{4.85}}$

3) $\quad P_{Cu,s}(s=1) = 3\cdot R_s \cdot I_s^2(s=1) = 3\cdot 0.06\cdot 497.1^2 = \underline{\underline{44480 \ W}} \cong 44.5 \ kW$

$P_{Cu,s}(s_N) = 3\cdot R_s \cdot I_{sN}^2 = 3\cdot 0.06\cdot 102.57^2 = \underline{\underline{1894 \ W}} \cong 1.9 \ kW$

$\dfrac{P_{Cu,s}(s=1)}{P_{Cu,s}(s_N)} = \dfrac{44480}{1894} = \left(\dfrac{I_s(s=1)}{I_{sN}}\right)^2 = 4.85^2 = \underline{\underline{23.49}}$

4)   "Homogenous body replica" (the "body" is the stator copper mass):

$$P_V \cdot dt = m_{Cu} \cdot c_{Cu} \cdot d\Delta\vartheta_{Cu,s} + A \cdot \alpha \cdot \Delta\vartheta_{Cu,s} \cdot dt$$

$$\alpha \cong 0 \implies P_V \cdot dt = m_{Cu} \cdot c_{Cu} \cdot d\Delta\vartheta_{Cu,s}, \qquad P_V = P_{Cu,s}(s=1)$$

$$\Delta\vartheta_{Cu,s} = \frac{P_{Cu,s}(s=1)}{m_{Cu} \cdot c_{Cu}} \cdot t + \Delta\vartheta_0(t=0), \qquad \Delta\vartheta_0(t=0) = 0, \ t = 20 \text{ s}$$

$$\Delta\vartheta_{Cu,s} = \frac{44480 \cdot 20}{22,9 \cdot 386} = 100.6 \text{ K}$$

Rotor locked
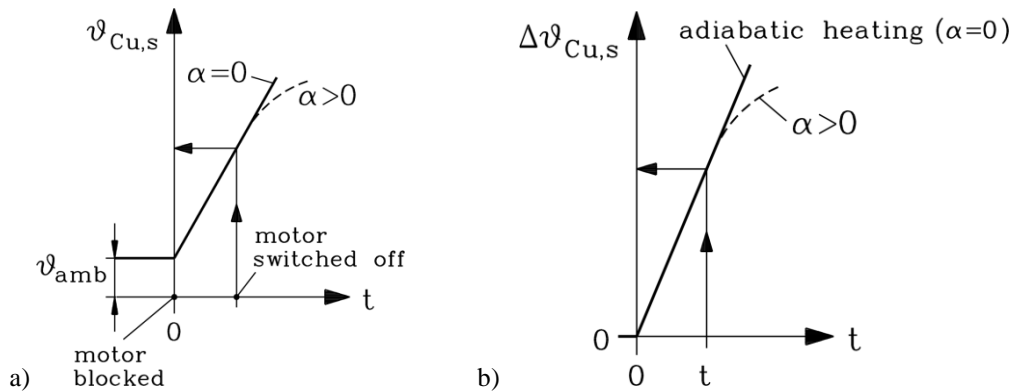
Fig. 3.3-2: Rise of a) the winding temperature, b) the winding temperature difference to ambient temperature

Result: $\vartheta_{Cu,s}(t=20\text{s}) = \Delta\vartheta_{Cu,s} + \vartheta_{amb} = 100{,}6 \text{ K} + 25°C = \underline{\underline{125{,}6°C}}$

5)   Cooling: $\Delta\vartheta_{Cu,s}(t) = \Delta\vartheta_0(t=0) \cdot e^{-t/T}$

New time counting: $t = 0$: Motor switched off $P_V = 0$, $\Delta\vartheta_0(t=0) = 100{,}6$ K

$T = T_{\vartheta st} = 40$ min. (Stand still), $t = 25$ min, $t/T_{\vartheta st} = 25/40 = 0.625$
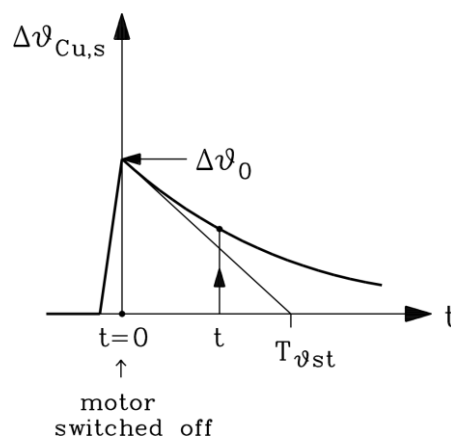

Fig. 3.3-3: Rise and fall of the winding temperature difference to ambient temperature at locked rotor

$$\Delta\vartheta_{Cu,s}(25\text{min}) = 100.6 \cdot e^{-0.625} = 53.9 \text{ K}$$

$$\vartheta_{Cu,s}(25\text{min}) = \Delta\vartheta_{Cu,s}(25\text{min}) + \vartheta_{amb} = 53.9 \text{ K} + 25°C = \underline{78.9°C}$$

Thermal Heating of a Totally Enclosed, Fan Cooled Cage Induction Motor

A totally enclosed, fan cooled, cage induction motor has the following losses and thermal resistances according to Fig. 3.4-1:

- Rotor-end ring via air to end shield: $R_{th1} = 167$ mK/W
- End shield via housing to ambient air: $R_{th2} = 50$ mK/W
- Rotor surface via air gap to stator iron: $R_{th3} = 79$ mK/W
- Stator winding via insulation to stator teeth: $R_{th4} = 21$ mK/W
- Stator teeth to stator yoke: $R_{th5} = 12$ mK/W
- Stator yoke via housing to ambient air $R_{th6} = 20$ mK/W
- Rotor cage losses, including additional losses
  and friction losses: $P_r = P_{cu,r} + P_{ad} + P_{fr+w} = 1300$ W
- Stator winding losses: $P_{Cu,s} = 1200$ W
- Stator iron losses in teeth $P_{Fe,ds} = 200$ W
- Stator iron losses in the yoke $P_{Fe,ys} = 400$ W
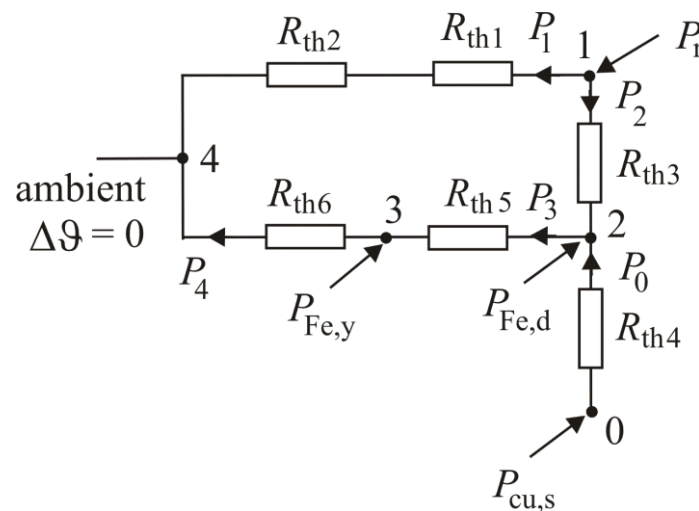
Fig. 3.4-1: Losses and heat sources plot

1) Determine the temperature rise $\Delta\vartheta_{20}$ between stator winding and stator iron!
2) Determine the motor efficiency for an output power of $P_{out} = 60$ kW!
3) Determine the heat flow P_0!
4) Determine the heat flow P_1, P_2, P_3, P_4![1] Give the power flow as a graph like in Fig. 3.4-1.
5) Determine the temperature rise $\Delta\vartheta_1$ in the cage (node 1), $\Delta\vartheta_2$ of the stator iron teeth (node 2) and $\Delta\vartheta_3$ of the stator iron yoke (node 3)!
6) Where is the hot spot in the machine?

1) $\Delta\vartheta_{20} = P_{Cu,s} \cdot R_{th,4} = 1200 \cdot 21 \cdot 10^{-3} = 25.2$ K

2)   Total losses:

$$P_d = P_{Cu,s} + P_r + P_{Fe,ds} + P_{Fe,ys} = 1200 + 1300 + 200 + 400 = 3100\text{W} = 3.1\text{kW}$$

$$\eta = \frac{P_{out}}{P_{out} + P_d} = \frac{60}{60 + 3.1} = 95.09\%$$

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[1] Use the *Kirchhoff*'s circuit laws!

Exercise 3.4

3)   Heat flow: $P_0 = P_{Cu,s} = 1200\text{W}$

4)   Kirchhoff node equations (19, (2), (3):

$$P_{Cu,r} = P_1 + P_2, \quad P_3 = P_2 + P_0 + P_{Fe,ds}, \quad P_4 = P_3 + P_{Fe,ys}$$

Kirchhoff mesh equation (4): $\Delta\vartheta_{14} = \Delta\vartheta_{12} + \Delta\vartheta_{23} + \Delta\vartheta_{34}$

Or: $(R_{th1} + R_{th2}) \cdot P_1 = R_{th3} \cdot P_2 + R_{th5} \cdot P_3 + R_{th6} \cdot P_4$

Solution of the four equations (1) … (4) for the four unknowns P_1, P_2, P_3, P_4

$$P_4 = P_3 + P_{Fe,ys} = P_3 + 400\text{W}, \quad P_2 = P_3 - P_0 - P_{Fe,ds} = P_3 - 1400\text{W}$$

$$\underline{P_1 = P_{Cu,r} - (P_3 - P_0 - P_{Fe,ds}) = -P_3 + 2700\text{W}}$$

$(0.167 + 0.05)P_1 = 0.079 P_2 + 0.012 P_3 + 0.02 P_4$
$0.217 \cdot (2700 - P_3) = 0.079 \cdot (P_3 - 1400) + 0.012 P_3 + 0.02(P_3 + 400)$
$(0.217 + 0.079 + 0.012 + 0.02)P_3 = 0.217 \cdot 2700 + 0.079 \cdot 1400 - 0.02 \cdot 400$
$0.328 P_3 = 688.5\text{W}$

Solution:

$P_3 = 2099\text{W}, \quad P_2 = (2099 - 1400)\text{W} = 699\text{W}, \quad P_1 = (-2099 + 2700)\text{W} = 601\text{W}$
$P_4 = (2099 + 400)\text{W} = 2499\text{W}$
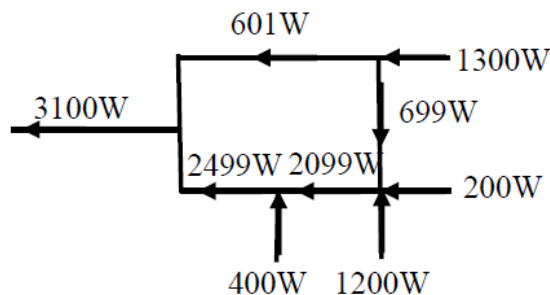
Fig. 3.4-2: Heat flow

5) $\Delta\vartheta_1 = \Delta\vartheta_{14} = (R_{th1} + R_{th2})P_1 = (0.167 + 0.05) \cdot 601 = 130.4\text{K}$

$\Delta\vartheta_3 = \Delta\vartheta_{34} = R_{th6}P_4 = 0.02 \cdot 2499 = 50.0\text{K}$

$\Delta\vartheta_2 = \Delta\vartheta_3 + \Delta\vartheta_{23} = 50.0\text{K} + R_{th5}P_3 = 50 + 0.012 \cdot 2099 = 75.2\text{K}$

Check: $\Delta\vartheta_2 + R_{th3}P_2 = 75.2\text{K} + 0.079 \cdot 699 = 130.4\text{K} = \Delta\vartheta_1$

6) $\Delta\vartheta_1 = 130.4\text{K}, \quad \Delta\vartheta_0 = \Delta\vartheta_2 + \Delta\vartheta_{20} = 75.2\text{K} + 25.2\text{K} = 100.4\text{K} < 130.4\text{K}$

The hot spot in the machine is the rotor cage.

Temperature rise in a copper wire

A cylindrical copper wire with the diameter $d = 0.1$ mm, and the length $l = 10$ cm is fed from $t = 0$ onwards by a DC current $I = 0.8$ A! The resistance of the wire is $R_{20} = 0.223$ Ω. The wire is placed in air and cooled by natural convection and radiation ($\alpha = 15$ W/(m²·K)).

1) Give the differential equation for the wire temperature rise $\Delta\vartheta$, the formula for the thermal time constant T_ϑ and the stationary temperature rise $\Delta\vartheta_\infty$ at the wire loss P_d.

2) Evaluate T_ϑ and $\Delta\vartheta_\infty$ for a conductivity at 20 °C, $\kappa_{20} = 57$ MS/m, a mass density $\gamma = 8900$ kg/m³ and a specific heat capacity $c = 388.5$ J/(kg·K).

3) Sketch the solution with correct scaling and determine $\Delta\vartheta$ (T_ϑ), $\Delta\vartheta$ ($2T_\vartheta$) and $\Delta\vartheta$ ($3T_\vartheta$) and the initial condition $\Delta\vartheta(0) = 0$.

4) Now consider the increase of electric resistance $R(\vartheta)=R_{20}\cdot(1+\alpha_{Cu}\cdot\Delta\vartheta)$. Give the new differential equation for $\Delta\vartheta(t)$, give the new time constant T'_ϑ as formula and in numbers. ($\alpha_{Cu}= 1/255$ K⁻¹)

5) Determine the new solution $\Delta\vartheta(t)$ with $\Delta\vartheta(0) = 0$ with T'_ϑ and determine the values $\Delta\vartheta(t)$ for $t = 0.2\cdot|T'_\vartheta|$, $0.4\cdot|T'_\vartheta|$, and $0.6\cdot|T'_\vartheta|$. Give a qualitative sketch of $\Delta\vartheta(t)$! What is the significant difference to the solution of 3)?

1) $$m\cdot c\frac{d\Delta\vartheta}{dt} + \alpha A\cdot\Delta\vartheta = P_d \, , \quad \frac{d\Delta\vartheta}{dt} + \frac{\alpha A}{m\cdot c}\cdot\Delta\vartheta = \frac{P_d}{m\cdot c} \, , \quad T_\vartheta = \frac{m\cdot c}{\alpha\cdot A} \, , \quad \Delta\vartheta_\infty = \frac{P_d}{\alpha\cdot A}$$

2)   Wire mass: $m = \gamma\dfrac{d^2\pi}{4}l = 8900\cdot\dfrac{(0.1/1000)^2\,\pi}{4}\cdot 0.1 = 6.99\text{mg}$,

   Wire surface: $A = d\pi l = (0.1/1000)\cdot\pi\cdot 0.1 = 31.4\cdot 10^{-6}m^2$

   $$T_\vartheta = \frac{m\cdot c}{\alpha\cdot A} = \frac{6.99\cdot 10^{-6}\cdot 388.5}{15\cdot 31.4\cdot 10^{-6}} = 5.766\text{s}$$

   Wire cross section area: $q = d^2\pi/4 = (0.1/1000)^2\,\pi/4 = 0.0079\text{mm}^2$

   Wire resistance at 20°C: $R_{20} = l/(\kappa_{20}q) = 0.1/(57\cdot 10^6\cdot 0.0079\cdot 10^{-6}) = 0.223\Omega$

   Losses in the wire: $P_d = R_{20}\cdot I^2 = 0.223\cdot 0.8^2 = 0.143\text{W}$

   Steady state temperature rise in the wire: $\Delta\vartheta_\infty = \dfrac{P_d}{\alpha\cdot A} = \dfrac{0.143}{15\cdot 31.4\cdot 10^{-6}} = 303.6\text{K}$

3) $\Delta\vartheta = \Delta\vartheta_\infty\cdot(1-e^{-t/T_\vartheta}) + \Delta\vartheta_0\cdot e^{-t/T_\vartheta} = \Delta\vartheta_\infty\cdot(1-e^{-t/T_\vartheta})$, $\Delta\vartheta_0 = 0$

   $\Delta\vartheta(T_\vartheta) = 0.632\cdot\Delta\vartheta_\infty = 192\text{K}$, $\Delta\vartheta(2T_\vartheta) = 0.865\cdot\Delta\vartheta_\infty = 263\text{K}$,

   $\Delta\vartheta(3T_\vartheta) = 0.950\cdot\Delta\vartheta_\infty = 288\text{K}$

Exercise 3.5   3.5/2
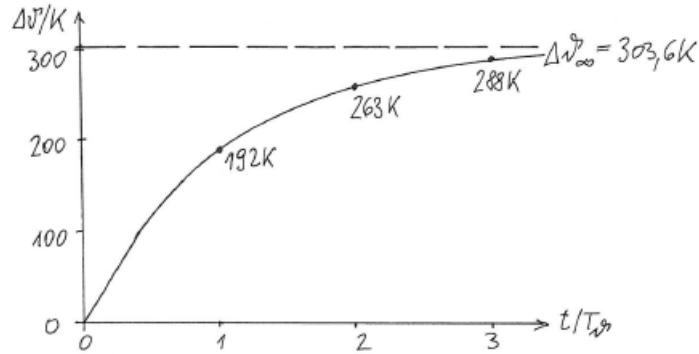

Fig. 3.5-1: Stable temperature rise in the wire

4) $m \cdot c \dfrac{d\Delta\vartheta}{dt} + \alpha A \cdot \Delta\vartheta = P_d = R_{20} \cdot (1 + \alpha_{Cu} \cdot \Delta\vartheta) \cdot I^2$

$m \cdot c \dfrac{d\Delta\vartheta}{dt} + (\alpha A - R_{20} \cdot I^2 \cdot \alpha_{Cu}) \cdot \Delta\vartheta = R_{20} \cdot I^2$

$\dfrac{d\Delta\vartheta}{dt} + \dfrac{\alpha A - R_{20} \cdot I^2 \cdot \alpha_{Cu}}{m \cdot c} \cdot \Delta\vartheta = \dfrac{R_{20} \cdot I^2}{m \cdot c} = \dfrac{d\Delta\vartheta}{dt} + \dfrac{\Delta\vartheta}{T'_\vartheta} = \dfrac{R_{20} \cdot I^2}{m \cdot c}$

$T'_\vartheta = \dfrac{m \cdot c}{\alpha A - R_{20} \cdot I^2 \cdot \alpha_{Cu}} = \dfrac{6.99 \cdot 10^{-6} \cdot 388.5}{15 \cdot 31.4 \cdot 10^{-6} - 0.143/255} = -30.25\text{s} = -\left|T'_\vartheta\right|$

5) $\Delta\vartheta = \Delta\vartheta'_\infty \cdot (1 - e^{-t/T'_\vartheta}) + \Delta\vartheta_0 \cdot e^{-t/T'_\vartheta} = \Delta\vartheta'_\infty \cdot (1 - e^{t/\left|T'_\vartheta\right|})$, $\Delta\vartheta_0 = 0$

$\Delta\vartheta'_\infty = \dfrac{R_{20} \cdot I^2}{\alpha A - R_{20} \cdot I^2 \cdot \alpha_{Cu}} = \dfrac{0.143}{15 \cdot 31.4 \cdot 10^{-6} - 0.143/255} = -1593\text{K} = -\left|\Delta\vartheta'_\infty\right|$

$\Delta\vartheta(t) = \left|\Delta\vartheta'_\infty\right| \cdot (e^{t/\left|T'_\vartheta\right|} - 1)$

There is no stable temperature rise, as the loss increase is faster than the cooling process. The increasing temperature increases the resistance and hence the losses due to the impressed current, which gives a further heating up to infinite values. The wire will reach the melting temperature. This is the basic lay-out principle of a fuse.
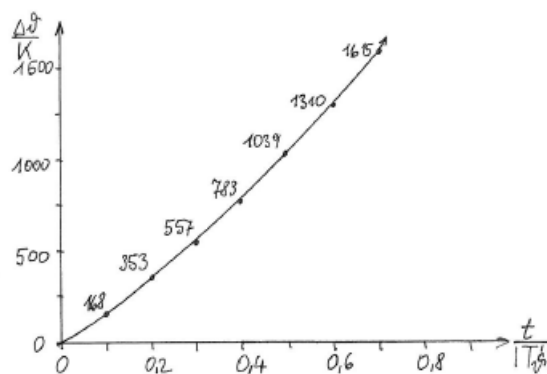

Fig. 3.5-2: Unstable temperature rise in the wire

Exercise 3.6   3.6/1

Transient temperature rise in a motor stator

The temperature rise in the stator iron stack and stator copper winding of a 4-pole 11 kW squirrel cage induction motor for 400 V Y, 50 Hz, has to be calculated for rated steady state operation after turning on the cold machine. The air-cooled motor is totally enclosed with a shaft-mounted radial fan (TEFC). The thermal resistance between the stator iron stack in the housing and the outside air is determined by the cooling fins as $R_{th1} = 0.072$ K/W, and that of the electrical insulation between the stator winding in the slots as $R_{th2} = 0.047$ K/W. The stator winding copper mass and specific heat capacity are $m_{Cu} = 4.8$ kg, $c_{Cu} = 388.5$ Ws/(kgK); the corresponding values for the stator iron stack are $m_{Fe} = 22.5$ kg, $c_{Fe} = 502$ Ws/(kgK). The losses in the stator winding and stator iron stack are constant as $P_{Cu} = 554$ W, $P_{Fe} = 260$ W. Use in the following the simplified thermal equivalent network of Fig. 3.6-1. The masses of the housing and of the end shields and the corresponding heat flow are neglected as well as the rotor copper, friction and additional losses!
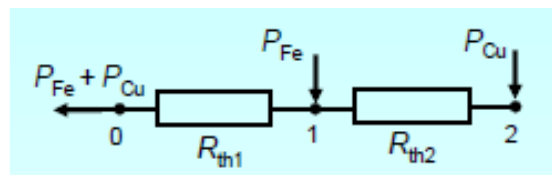

Fig. 3.6-1: Thermal equivalent network of the stator iron stack and stator copper winding

1) Give the set of differential equations for the temperature rise of the iron stack $\Delta\vartheta_{Fe}$ in node 1 and of the stator copper winding $\Delta\vartheta_{Cu}$ in node 2 for general functions $P_{Cu}(t)$, $P_{Fe}(t)$. Assume the temperature rise in node 0 (housing surface) over the air coolant temperature to be zero: $\Delta\vartheta = 0$.
2) Give for the system of Fig. 1 one resulting differential equation a) for $\Delta\vartheta_{Cu}$, b) for $\Delta\vartheta_{Fe}$. Comment the result!
3) Give the relevant thermal time constants.
4) Determine the temperature rise $\Delta\vartheta_{Cu}(t)$ and $\Delta\vartheta_{Fe}(t)$ as general formulas for the initial condition $\Delta\vartheta_{Cu}(0) = 0$, $\Delta\vartheta_{Fe}(0) = 0$ and constant losses $P_{Cu} = $ const., $P_{Fe} = $ const.
5) Give the steady state values $\Delta\vartheta_{Cu}(t \to \infty) = 0$, $\Delta\vartheta_{Fe}(t \to \infty)$.
6) Give the graph $\Delta\vartheta_{Cu}(t)$ and $\Delta\vartheta_{Fe}(t)$ for $0 \le t \le T$ (with T at least as three times the longest time constant of the system).

1)

$$m_{Cu}c_{Cu}\frac{d\Delta\vartheta_{Cu}}{dt}+\frac{\Delta\vartheta_{Cu}-\Delta\vartheta_{Fe}}{R_{th2}}=P_{Cu} \tag{1}$$

$$m_{Fe}c_{Fe}\frac{d\Delta\vartheta_{Fe}}{dt}+\frac{\Delta\vartheta_{Fe}}{R_{th1}}-\frac{\Delta\vartheta_{Cu}-\Delta\vartheta_{Fe}}{R_{th2}}=P_{Fe} \tag{2}$$

2)

$$\frac{d\Delta\vartheta_{Cu}}{dt}+\frac{1}{m_{Cu}c_{Cu}R_{th2}}\cdot\Delta\vartheta_{Cu}-\frac{1}{m_{Cu}c_{Cu}R_{th2}}\cdot\Delta\vartheta_{Fe}=\frac{P_{Cu}}{m_{Cu}c_{Cu}} \tag{1}$$

$$\frac{d\Delta\vartheta_{Fe}}{dt}+\frac{1}{m_{Fe}c_{Fe}}\cdot\left(\frac{1}{R_{th1}}+\frac{1}{R_{th2}}\right)\cdot\Delta\vartheta_{Fe}-\frac{1}{m_{Fe}c_{Fe}R_{th2}}\cdot\Delta\vartheta_{Cu}=\frac{P_{Fe}}{m_{Fe}c_{Fe}} \tag{2}$$

Abbreviations:

$$a=\frac{1}{m_{Cu}c_{Cu}R_{th2}}, \ b=\frac{1}{m_{Fe}c_{Fe}R_{th2}}, \ c=\frac{1}{m_{Fe}c_{Fe}}\cdot\left(\frac{1}{R_{th1}}+\frac{1}{R_{th2}}\right), \ p_{Cu}=\frac{P_{Cu}}{m_{Cu}c_{Cu}}, \ p_{Fe}=\frac{P_{Fe}}{m_{Fe}c_{Fe}}$$

(1): $\Delta\dot{\vartheta}_{Cu}+a\cdot\Delta\vartheta_{Cu}-a\cdot\Delta\vartheta_{Fe}=p_{Cu}$, (2): $\Delta\dot{\vartheta}_{Fe}+c\cdot\Delta\vartheta_{Fe}-b\cdot\Delta\vartheta_{Cu}=p_{Fe}$

a) $\Delta\ddot{\vartheta}_{Cu}+a\cdot\Delta\dot{\vartheta}_{Cu}-a\cdot\Delta\dot{\vartheta}_{Fe}=\dot{p}_{Cu}$

(2)→(1) yields (1´): $\Delta\ddot{\vartheta}_{Cu}+a\cdot\Delta\dot{\vartheta}_{Cu}-a\cdot(p_{Fe}-c\cdot\Delta\vartheta_{Fe}+b\cdot\Delta\vartheta_{Cu})=\dot{p}_{Cu}$

(1)→(1´) yields: $\Delta\ddot{\vartheta}_{Cu}+a\cdot\Delta\dot{\vartheta}_{Cu}-a\cdot b\cdot\Delta\vartheta_{Cu}-c\cdot(p_{Cu}-\Delta\dot{\vartheta}_{Cu}-a\cdot\Delta\vartheta_{Cu})=\dot{p}_{Cu}+a\cdot p_{Fe}$

$$\Delta\ddot{\vartheta}_{Cu}+(a+c)\cdot\Delta\dot{\vartheta}_{Cu}+a\cdot(c-b)\cdot\Delta\vartheta_{Cu}=c\cdot p_{Cu}+\dot{p}_{Cu}+a\cdot p_{Fe} \tag{3´}$$

b): $\Delta\ddot{\vartheta}_{Fe}+c\cdot\Delta\dot{\vartheta}_{Fe}-b\cdot\Delta\dot{\vartheta}_{Cu}=\dot{p}_{Fe}$

(1)→(2) yields (2´): $\Delta\ddot{\vartheta}_{Fe}+c\cdot\Delta\dot{\vartheta}_{Fe}-b\cdot(p_{Cu}-a\cdot\Delta\vartheta_{Cu}+a\cdot\Delta\vartheta_{Fe})=\dot{p}_{Fe}$

(2)→(2´) yields: $\Delta\ddot{\vartheta}_{Fe}+c\cdot\Delta\dot{\vartheta}_{Fe}+a\cdot(\Delta\dot{\vartheta}_{Fe}+c\cdot\Delta\vartheta_{Fe}-p_{Fe})-a\cdot b\cdot\Delta\vartheta_{Fe}=b\cdot p_{Cu}+\dot{p}_{Fe}$

$$\Delta\ddot{\vartheta}_{Fe}+(a+c)\cdot\Delta\dot{\vartheta}_{Fe}+a\cdot(c-b)\cdot\Delta\vartheta_{Fe}=b\cdot p_{Cu}+a\cdot p_{Fe}+\dot{p}_{Fe} \tag{3´´}$$

Results (3´) and (3´´) show, that the homogeneous differential equation, describing the dynamical system behaviour, is identical both for $\Delta\vartheta_{Cu}(t)$ and $\Delta\vartheta_{Fe}(t)$:

$$\Delta\ddot{\vartheta}_{Cu}+(a+c)\cdot\Delta\dot{\vartheta}_{Cu}+a\cdot(c-b)\cdot\Delta\vartheta_{Cu}=0 \quad, \tag{4´}$$

$$\Delta\ddot{\vartheta}_{Fe}+(a+c)\cdot\Delta\dot{\vartheta}_{Fe}+a\cdot(c-b)\cdot\Delta\vartheta_{Fe}=0 \quad. \tag{4´´}$$

The difference is in the right hand side of (3´) and (3´´), describing the steady state solution, which yields different temperatures for copper winding and iron stack.

3)

The solution functions of the homogeneous differential equations (4´), (4´´)

$$\Delta\vartheta_{Cu,h}(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}, \ \Delta\vartheta_{Fe,h}(t)=D_1e^{\lambda_1 t}+D_2e^{\lambda_2 t}$$

yield the characteristic second order polynomial

$$\lambda^2+(a+c)\cdot\lambda+a\cdot(c-b)=(\lambda-\lambda_1)\cdot(\lambda-\lambda_2)=0$$

with its roots λ_1, λ_2 as the negative inverse of the two thermal time constants

$$T_1=-1/\lambda_1, T_2=-1/\lambda_2: \ \lambda_{1,2}=-\frac{a+c}{2}\pm\sqrt{\left(\frac{a+c}{2}\right)^2-a\cdot(c-b)}=-\frac{a+c}{2}\pm\sqrt{\left(\frac{a-c}{2}\right)^2+a\cdot b}\ .$$

Long/short time constant: $T_1 = \dfrac{1}{\dfrac{a+c}{2} - \sqrt{\left(\dfrac{a-c}{2}\right)^2 + a \cdot b}}$, $T_2 = \dfrac{1}{\dfrac{a+c}{2} + \sqrt{\left(\dfrac{a-c}{2}\right)^2 + a \cdot b}}$

$a = \dfrac{1}{m_{Cu} c_{Cu} R_{th2}} = \dfrac{1}{4.8 \cdot 388.5 \cdot 0.047} = 0.0114/s$,

$b = \dfrac{1}{m_{Fe} c_{Fe} R_{th2}} = \dfrac{1}{22.5 \cdot 502 \cdot 0.047} = 0.0019/s$,

$c = \dfrac{1}{m_{Fe} c_{Fe}} \cdot \left(\dfrac{1}{R_{th1}} + \dfrac{1}{R_{th2}}\right) = \dfrac{1}{22.5 \cdot 502} \cdot \left(\dfrac{1}{0.072} + \dfrac{1}{0.047}\right) = 0.0031/s$,

$T_1 = \dfrac{1}{\dfrac{0.0114 + 0.0031}{2} - \sqrt{\left(\dfrac{0.0114 - 0.0031}{2}\right)^2 + 0.0114 \cdot 0.0019}} = 985.8s = 16.4\,min$,

$T_2 = \dfrac{1}{\dfrac{0.0114 + 0.0031}{2} + \sqrt{\left(\dfrac{0.0114 - 0.0031}{2}\right)^2 + 0.0114 \cdot 0.0019}} = 74.2s = 1.2\,min$.

4)

For constant right hand side values ($\dot{p}_{Cu} = 0$, $\dot{p}_{Fe} = 0$) also the particular solutions of (3´), (3´´) are constants: $\Delta\vartheta_{Cu,p}(t) = K_1$, $\Delta\vartheta_{Fe,p}(t) = K_2$.

$K_1 = \dfrac{c \cdot p_{Cu} + a \cdot p_{Fe}}{a \cdot (c - b)} = \Delta\vartheta_{Cu,\infty}$, $K_2 = \dfrac{b \cdot p_{Cu} + a \cdot p_{Fe}}{a \cdot (c - b)} = \Delta\vartheta_{Fe,\infty}$

$K_1 = P_{Cu} \cdot (R_{th1} + R_{th2}) + P_{Fe} \cdot R_{th1} = \Delta\vartheta_{Cu,\infty}$, $K_2 = (P_{Cu} + P_{Fe}) \cdot R_{th1} = \Delta\vartheta_{Fe,\infty}$

Initial conditions:

$\Delta\vartheta_{Cu}(0) = 0$, $\Delta\dot{\vartheta}_{Cu}(0) = p_{Cu}$ (from (1)); $\Delta\vartheta_{Fe}(0) = 0$, $\Delta\dot{\vartheta}_{Fe}(0) = p_{Fe}$ (from (2)).

$\Delta\vartheta_{Cu}(t) = \Delta\vartheta_{Cu,h}(t) + \Delta\vartheta_{Cu,p}(t) = C_1 e^{\lambda_1 t} + C_1 e^{\lambda_2 t} + K_1$

$\Delta\vartheta_{Fe}(t) = \Delta\vartheta_{Fe,h}(t) + \Delta\vartheta_{Fe,p}(t) = D_1 e^{\lambda_1 t} + D_1 e^{\lambda_2 t} + K_2$

$\Delta\vartheta_{Cu}(0) = C_1 + C_2 + K_1 = 0$, $\Delta\dot{\vartheta}_{Cu}(0) = \lambda_1 C_1 + \lambda_2 C_2 = p_{Cu}$

$\Delta\vartheta_{Fe}(0) = D_1 + D_2 + K_2 = 0$, $\Delta\dot{\vartheta}_{Fe}(0) = \lambda_1 D_1 + \lambda_2 D_2 = p_{Fe}$

$C_1 = \dfrac{-K_1 \cdot \lambda_2 - p_{Cu}}{\lambda_2 - \lambda_1}$, $\quad D_1 = \dfrac{-K_2 \cdot \lambda_2 - p_{Fe}}{\lambda_2 - \lambda_1}$

$C_2 = \dfrac{K_1 \cdot \lambda_1 + p_{Cu}}{\lambda_2 - \lambda_1}$, $\quad D_2 = \dfrac{K_2 \cdot \lambda_1 + p_{Fe}}{\lambda_2 - \lambda_1}$

$C_1 = \dfrac{\dfrac{P_{Cu}}{m_{Cu} c_{Cu}} \cdot T_2 - \Delta\vartheta_{Cu,\infty}}{1 - T_2/T_1}$, $\quad D_1 = \dfrac{\dfrac{P_{Fe}}{m_{Fe} c_{Fe}} \cdot T_2 - \Delta\vartheta_{Fe,\infty}}{1 - T_2/T_1}$

$C_2 = \dfrac{\dfrac{P_{Cu}}{m_{Cu} c_{Cu}} \cdot T_1 - \Delta\vartheta_{Cu,\infty}}{1 - T_1/T_2}$, $\quad D_2 = \dfrac{\dfrac{P_{Fe}}{m_{Fe} c_{Fe}} \cdot T_1 - \Delta\vartheta_{Fe,\infty}}{1 - T_1/T_2}$

$$\Delta\vartheta_{\text{Cu}}(t) = C_1 e^{-t/T_1} + C_2 e^{-t/T_2} + \Delta\vartheta_{\text{Cu},\infty}, \ \Delta\vartheta_{\text{Fe}}(t) = D_1 e^{-t/T_1} + D_2 e^{-t/T_2} + \Delta\vartheta_{\text{Fe},\infty}$$

5)
$$\Delta\vartheta_{\text{Cu},\infty} = 554 \cdot (0.072 + 0.047) + 260 \cdot 0.072 = 84.7\text{K}, \ \Delta\vartheta_{\text{Fe},\infty} = (554 + 260) \cdot 0.072 = 58.6\text{K}$$

6)
$$C_1 = \frac{\dfrac{554}{4.8 \cdot 388.5} \cdot 985.8 - 84.7}{1 - 74.2/985.8} = -67.7\text{K}, \ C_2 = \frac{\dfrac{554}{4.8 \cdot 388.5} \cdot 74.2 - 84.7}{1 - 985.8/74.2} = -17.0\text{K}$$

$$D_1 = \frac{\dfrac{260}{22.5 \cdot 502} \cdot 985.8 - 58.6}{1 - 74.2/985.8} = -61.6\text{K}, \ D_2 = \frac{\dfrac{260}{22.5 \cdot 502} \cdot 74.2 - 58.6}{1 - 985.8/74.2} = 3.0\text{K}$$

$$\Delta\vartheta_{\text{Cu}}(t) = -67.7\text{K} \cdot e^{-t/985.8\text{s}} - 17.0\text{K} \cdot e^{-t/74.2\text{s}} + 84.7\text{K},$$

$$\Delta\vartheta_{\text{Fe}}(t) = -61.6\text{K} \cdot e^{-t/985.8\text{s}} + 3.0\text{K} \cdot e^{-t/74.2\text{s}} + 58.6\text{K}$$

Graph for $0 \leq t \leq T = 3000$ s in Fig. 3.6-2! The long time constant is dominating the transient temperature rise.

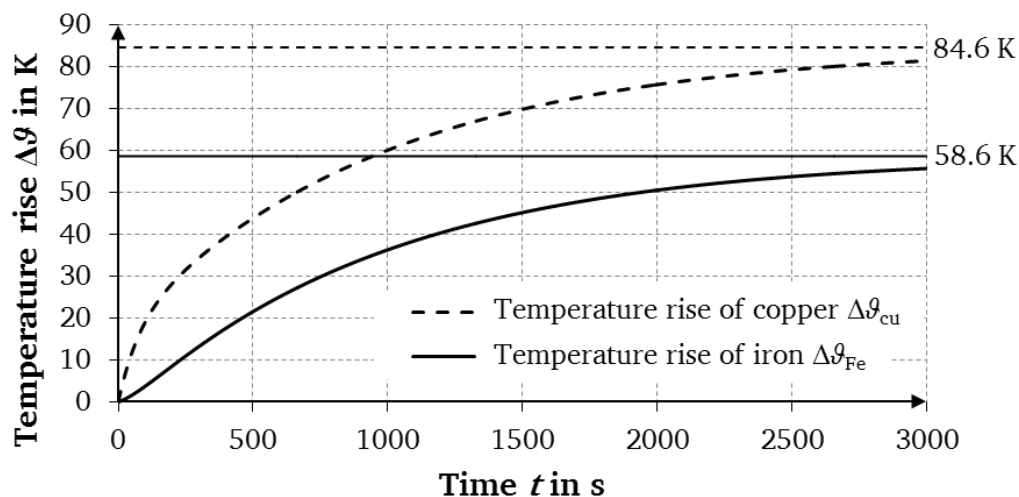Fig. 3.6-2: Calculated temperature rise of copper winding and stator iron stack for constant iron and ohmic losses

Exercise 3.7   3.7/1

Thermal time constants

An open-ventilated four-pole cage induction motor 550 kW, 6.6 kV Y, 50 Hz has $Q_s = 60$ open stator slots (slot height $h_Q = 69$ mm, slot width $b_Q = 12.5$ mm, main insulation thickness $d = 2.7$ mm) and an iron stack length $l_{Fe} = 380$ mm. The double-layer winding overhang is cooled with an air flow velocity $v = 12$ m/s. The coil height is ca. half slot height $h_Q/2$, the coil breadth is ca. slot width b_Q and the length of winding overhang per coil per side is $l_b = 614.8$ mm. The stator housing outer diameter is $D_{sa} = 790$ mm and its axial length is $L = 820$ mm.

1)   Determine the cooling surface of the stator copper winding in the slots A_Q and in the winding overhang A_b! Neglect the hat flow via the stator slot wedges! Calculate the cooling surface of the stator housing A_G! Neglect the housing front sides!

2)   Determine the heat resistance R_{thQ} due to heat conduction from the copper conductors to the stator iron via the main slot insulation thickness d for a thermal conductivity $\lambda_{th} = 0.2$ W/(m·K). Calculate the heat resistance R_{thb} due to heat convection from the insulated winding overhang to the cooling air! Determine the convective heat resistance R_{th1} from the housing surface to the cooling air at an air velocity $v = 12$ m/s.

3)   The I^2R-losses in the stator winding are $P_{Cu,s} = 7739$ W, of which 45 % occur in the slot conductors. Determine the copper temperature rise of the slot conductors $\Delta\vartheta_{Cu,Q} - \Delta\vartheta_{Fe,s}$ above the stator iron temperature, and the copper temperature rise $\Delta\vartheta_{Cu,b} - \Delta\vartheta_{air}$ of the winding overhang conductors above the cooling air temperature!

4)   The stator iron losses at rated operation are $P_{Fe,s} = 6779$ W, the rotor copper losses are $P_{Cu,r} = 4538$ W and the additional losses (occurring mainly in the rotor) are $P_{ad,1} = 2875$ W. Assume that 50% of $P_{Cu,r} + P_{ad,1}$ give a heat flow via the air-gap to the stator iron, whereas the other 50% are transported off via the cooling internal air flow. Determine the steady-state stator iron and slot copper temperature rises $\Delta\vartheta_{Fe,s}$, $\Delta\vartheta_{Cu,Q}$! Is the Thermal Class B limit exceeded?

5)   The mass of the stator iron stack is $m_{Fe,s} = 631$ kg, the stator winding copper mass is $m_{Cu,s} = 142$ kg. The mass of the stator steel housing is $m_G = 427$ kg, which is also heated up and must therefore be added to $m_{Fe,s}$. Give with $c_{Fe} = 502$ J/(kg·K), $c_{Cu} = 389$ J/(kg·K) the long and short time constant $T_{\vartheta 1}$, $T_{\vartheta 2}$ of the stator copper and iron mass! After roughly which time the steady-state over-temperature is reached?

1) Slot cooling surface: No heat flow via stator wedges: $A_Q \sim 2 \cdot h_Q + b_Q$.

$A_Q = Q_s \cdot (2 \cdot h_Q + b_Q) \cdot l_{Fe} = 60 \cdot (2 \cdot 69 + 12.5) \cdot 380 \cdot 10^{-6} = 3.43 \, \text{mm}^2$,

Cooling surface of insulated stator coils in the winding overhang of drive-end and non-drive-end side:

$A_b = 2 \cdot Q_s \cdot 2 \cdot ((h_Q/2) + b_Q) \cdot l_b = 2 \cdot 60 \cdot 2 \cdot ((69/2) + 12.5) \cdot 614.8 \cdot 10^{-6} = 6.93 \, \text{mm}^2$,

Cooling surface of the stator housing:

$A_G = D_{sa} \cdot \pi \cdot L = 0.79 \cdot \pi \cdot 0.82 = 2.04 \, \text{m}^2$.

2) Thermally conductive resistance from slot copper to iron:

$R_{thQ} = \dfrac{d}{\lambda_{th} \cdot A_Q} = \dfrac{0.0027}{0.2 \cdot 3.43} = 3.94 \cdot 10^{-3} \, \text{K/W}$,

Convection from insulated winding to moving air: Heat transfer coefficient:

$\alpha = 8v^{3/4} = 8 \cdot 12^{3/4} = 51.6 \, \text{W/(m}^2 \cdot \text{K)}$,

Thermally convective resistance from insulated overhang to cooling air:

$R_{thb} = \dfrac{1}{\alpha \cdot A_b} = \dfrac{1}{51.6 \cdot 6.93} = 2.59 \cdot 10^{-3} \, \text{K/W}$,

Thermally convective resistance from the housing surface to the cooling air:
Heat transfer coefficient from metal to air:

$\alpha_G = 15 \cdot v^{2/3} = 15 \cdot 12^{2/3} = 78.6 \, \text{W/(m}^2 \cdot \text{K)}$,

$R_{th1} = \dfrac{1}{\alpha_G \cdot A_G} = \dfrac{1}{78.6 \cdot 2.04} = 6.23 \cdot 10^{-3} \, \text{K/W}$.

3) Copper losses in the slot conductors: $P_{Cu,sQ} = 0.45 \cdot 7739 \, \text{W} = 3483 \, \text{W}$,

$\Delta\vartheta_{Cu,Q} - \Delta\vartheta_{Fe,s} = R_{thQ} \cdot P_{Cu,sQ} = 3.94 \cdot 10^{-3} \cdot 3483 = 13.7 \, \text{K}$.

Copper losses in the winding overhang conductors: $P_{Cu,sb} = 0.55 \cdot 7739 \, \text{W} = 4256 \, \text{W}$,

$\Delta\vartheta_{Cu,b} - \Delta\vartheta_{air} = R_{thb} \cdot P_{Cu,sb} = 2.59 \cdot 10^{-3} \cdot 4256 = 11.0 \, \text{K}$.

4) Effective losses for the iron heating: $P_{eff} = P_{Fe,s} + 0.5 \cdot (P_{Cu,r} + P_{ad,1})$,

$P_{eff} = 6779 + 0.5 \cdot (4538 + 2875) = 10486 \, \text{W}$,

$\Delta\vartheta_{Fe,s} = R_{th1} \cdot P_{eff} = 6.23 \cdot 10^{-3} \cdot 10486 = 65.3 \, \text{K}$.

$\Delta\vartheta_{Cu,Q} = (\Delta\vartheta_{Cu,Q} - \Delta\vartheta_{Fe,s}) + \Delta\vartheta_{Fe,s} = 13.7 + 65.3 = 79.0 \, \text{K}$.

Thermal Class B limit (IEC 60034-1): 80 K over 40°C ambient!

$\Delta\vartheta_{Cu,Q} = 79.0 \, \text{K} < 80 \, \text{K}$: Th. Cl. B limit is not exceeded!

5) $\tau_{Fe} = m_{Fe} c_{Fe} \cdot R_{th1} = 1058 \cdot 502 \cdot 6.23 \cdot 10^{-3} = 3308.9 \, \text{s}$,

$m_{Fe} = m_{Fe,s} + m_G = 631 + 427 = 1058 \, \text{kg}$,

$\tau_{Cu} = m_{Cu} c_{Cu} \cdot R_{thQ} = 142 \cdot 389 \cdot 3.94 \cdot 10^{-3} = 217.6 \, \text{s}$,

$\tau = m_{Fe} c_{Fe} \cdot \dfrac{R_{th1} \cdot R_{thQ}}{R_{th1} + R_{thQ}} = 1058 \cdot 502 \cdot \dfrac{6.23 \cdot 3.94}{6.23 + 3.94} \cdot 10^{-3} = 1281.9 \, \text{s}$,

<u>Exercise 3.7</u>   3.7/3

$$T_{\vartheta 1} = \frac{2}{\dfrac{1}{\tau} + \dfrac{1}{\tau_{Cu}} - \sqrt{\left(\dfrac{1}{\tau} + \dfrac{1}{\tau_{Cu}}\right)^2 - \dfrac{4}{\tau_{Cu}\tau_{Fe}}}} \ ,$$

$$T_{\vartheta 1} = \frac{2}{\dfrac{1}{1281.9} + \dfrac{1}{217.6} - \sqrt{\left(\dfrac{1}{1281.9} + \dfrac{1}{217.6}\right)^2 - \dfrac{4}{217.6 \cdot 3308.9}}} = 3674.6 \, \text{s} \, ,$$

$$T_{\vartheta 2} = \frac{2}{\dfrac{1}{\tau} + \dfrac{1}{\tau_{Cu}} + \sqrt{\left(\dfrac{1}{\tau} + \dfrac{1}{\tau_{Cu}}\right)^2 - \dfrac{4}{\tau_{Cu}\tau_{Fe}}}} \ ,$$

$$T_{\vartheta 2} = \frac{2}{\dfrac{1}{1281.9} + \dfrac{1}{217.6} + \sqrt{\left(\dfrac{1}{1281.9} + \dfrac{1}{217.6}\right)^2 - \dfrac{4}{217.6 \cdot 3308.9}}} = 195.9 \, \text{s} \, ,$$

$T_{\vartheta 1} = 3674.6 \, \text{s} > \tau_{Fe} = 3308.9 \, \text{s}$, $T_{\vartheta 2} = 195.9 \, \text{s} < \tau_{Cu} = 217.6 \, \text{s}$!

After ca. 3 long time constants the steady-state over-temperature of copper and iron are reached: $3T_{\vartheta 1} = 3 \cdot 3674.6 \, \text{s} = 11023.8 \, \text{s} = 183.7 \, \text{min} \approx 3 \, \text{hours}$!

Switching of a choke coil

A choke coil with the current-independent inductance L and resistance R is connected at the time $t = 0$ to an AC voltage source

$$u(t) = \hat{U} \cdot \sin(\omega t + \varphi)$$

The following data are given: $\hat{U} = 10\,\text{V}$, $f = \omega/(2\pi) = 100\,\text{Hz}$, $R = 1\,\Omega$, $X = \omega L = 1\,\Omega$.

1)   Using the homogeneous and particular differential equations, calculate analytically the current in the coil and discuss the results.

2)   For the two different switching moments, at $\varphi = 0$ and $\varphi = \pi/2$, give the current expressions! Analyze the special case of $R = 0$!

3)   Using the method of Runge-Kutta, determine for $\varphi = 0$, by numerical integration, the current variation of the coil for the first two periods of the AC voltage source and compare the result with the analytical calculation of 1)!

1)　　The linear differential equation of first order with constant coefficients for $t \geq 0$:

$$L \cdot \frac{di}{dt} + R \cdot i = \hat{U} \cdot \sin(\omega t + \varphi)$$

Initial condition: $i(0) = 0$

Homogeneous differential equation:

$$L \cdot \frac{di_h}{dt} + R \cdot i_h = 0$$

Solution approach: $i_h = C \cdot e^{\lambda \cdot t}$. Inserted in the homogeneous differential equation, this yields: $\lambda = -R/L = -1/T$ with the time constant $T = L/R$.

Particular differential equation:

$$L \cdot \frac{di_p}{dt} + R \cdot i_p = \hat{U} \cdot \left(\sin\omega t \cdot \cos\varphi + \cos\omega t \cdot \sin\varphi \right)$$

Solution approach: $i_p = A \cdot \sin\omega t + B \cdot \cos\omega t$.

Inserted in the particular differential equation, this yields

$$R \cdot \left(A \cdot \sin\omega t + B \cdot \cos\omega t \right) + \omega L \cdot \left(A \cdot \cos\omega t - B \cdot \sin\omega t \right) =$$
$$\hat{U} \cdot \left(\sin\omega t \cdot \cos\varphi + \cos\omega t \cdot \sin\varphi \right)$$

Each of the coefficients of $\sin\omega t$, $\cos\omega t$ of the above equation must be identical on the left and on the right side:

$$\sin\omega t: \ R \cdot A - \omega L \cdot B = \hat{U} \cdot \cos\varphi$$
$$\cos\omega t: \ R \cdot B + \omega L \cdot A = \hat{U} \cdot \sin\varphi$$

The linear system of equations with two unknowns A, B is solved with the *Cramer*'s rule.

$$\begin{pmatrix} R & -\omega L \\ \omega L & R \end{pmatrix} \cdot \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \hat{U} \cdot \cos\varphi \\ \hat{U} \cdot \sin\varphi \end{pmatrix}, \qquad (N) = \begin{pmatrix} R & -\omega L \\ \omega L & R \end{pmatrix}$$

$$Det(N) = \begin{vmatrix} R & -\omega L \\ \omega L & R \end{vmatrix} = R^2 + (\omega L)^2$$

$$A = \frac{1}{Det(N)} \cdot \begin{vmatrix} \hat{U} \cdot \cos\varphi & -\omega L \\ \hat{U} \cdot \sin\varphi & R \end{vmatrix} = \hat{U} \cdot \frac{R \cdot \cos\varphi + \omega L \cdot \sin\varphi}{R^2 + (\omega L)^2}$$

$$B = \frac{1}{Det(N)} \cdot \begin{vmatrix} R & \hat{U} \cdot \cos\varphi \\ \omega L & \hat{U} \cdot \sin\varphi \end{vmatrix} = \hat{U} \cdot \frac{R \cdot \sin\varphi - \omega L \cdot \cos\varphi}{R^2 + (\omega L)^2}$$

The solution for the current is the sum of homogeneous and particular solution.

$$i = i_h + i_p = C \cdot e^{-t/T} + A \cdot \sin\omega t + B \cdot \cos\omega t$$

In order to determine the unknown constants C, the initial condition is used.

$$i(0) = C \cdot e^{0/T} + A \cdot \sin 0 + B \cdot \cos 0 = C + B = 0 \quad \Rightarrow \quad C = -B$$

Thus we obtain the analytical solution for the current flow:

$$i(t) = B \cdot \left(\cos\omega t - e^{-t/T}\right) + A \cdot \sin\omega t$$

Discussion of the result:
The current decays with the time constant $T = L/R$ due to the transient DC component of the homogeneous solution. After about three time constants $3T$, it has almost decreased to zero, and the steady state solution remains as an alternating current $i(t) = B \cdot \cos\omega t + A \cdot \sin\omega t$ of the particular solution. With the approach $A = \hat{I}\cos\psi$, $B = \hat{I}\sin\psi$ we obtain $i(t) = \hat{I} \cdot \sin(\omega t + \psi)$ with the amplitude

$$\hat{I} = \sqrt{A^2 + B^2} = \hat{U}\frac{\sqrt{R^2 + (\omega L)^2}}{R^2 + (\omega L)^2} = \frac{\hat{U}}{\sqrt{R^2 + (\omega L)^2}}$$

and the phase angle $\psi = \arctan(B/A) = \arctan\left(\dfrac{R \cdot \sin\varphi - \omega L \cdot \cos\varphi}{R \cdot \cos\varphi + \omega L \cdot \sin\varphi}\right)$.

2)   a) Switching on at the zero voltage crossing $\varphi = 0$:

$$A = \hat{U} \cdot \frac{R}{R^2 + (\omega L)^2} , \quad B = -\hat{U} \cdot \frac{\omega L}{R^2 + (\omega L)^2}$$

b) Switching on at the maximum voltage $\varphi = \pi/2$:

$$A = \hat{U} \cdot \frac{\omega L}{R^2 + (\omega L)^2} , \quad B = \hat{U} \cdot \frac{R}{R^2 + (\omega L)^2}$$

In the special case $R = 0$, the time constant T is infinite, and the DC component does not decay.

a) For $R = 0$, $\varphi = 0$, with $A = 0$, $B = -\dfrac{\hat{U}}{\omega L}$ the current is

$$i(t) = \frac{\hat{U}}{\omega L} \cdot \left(1 - \cos\omega t\right) ,$$

a superposition of a direct current and an alternating current component of the same amplitude. The first peak value of current occurs at the time $t^* = \pi/\omega$ and with $\hat{i} = \dfrac{2\hat{U}}{\omega L}$, it is twice as large as the AC current amplitude.

b) For $R = 0$, $\varphi = \pi/2$, with $A = \dfrac{\hat{U}}{\omega L}$, $B = 0$ the current is

$$i(t) = \frac{\hat{U}}{\omega L} \cdot \sin \omega t \ ,$$

a pure alternating current. The first peak value of current occurs at the time $t^* = \pi / (2\omega)$, and it is half as big as in the case a).

3)   For the numerical integration, we write the differential equation as follows:

$$\frac{di(t)}{dt} = -\frac{R}{L} \cdot i(t) + \frac{\hat{U}}{L} \cdot \sin(\omega t + \varphi), \ \ i(0) = 0$$

The following values are used:
$\omega = 2\pi \cdot 100 / \text{s}$, $\varphi = 0$, $\hat{U} = 10 \,\text{V}$, $R = 1\,\Omega$,
$L = X / \omega = 1 / (2\pi \cdot 100) = 3.183 \,\text{mH}$.

The voltage period is $1/f = 1/100 = 0.01$ s. The calculation is performed during two periods, so for a period of 20 ms. The integration step size is 1 / 1000 of this period:
$\Delta t = 20\text{ms} / 1000 = 0.02\text{ms}$.
The numerical solution is compared with the analytical solution in Fig. 4.1-1. They are identical, because the numerical deviations from the analytical values are smaller than the line width, used for the graphical representation.
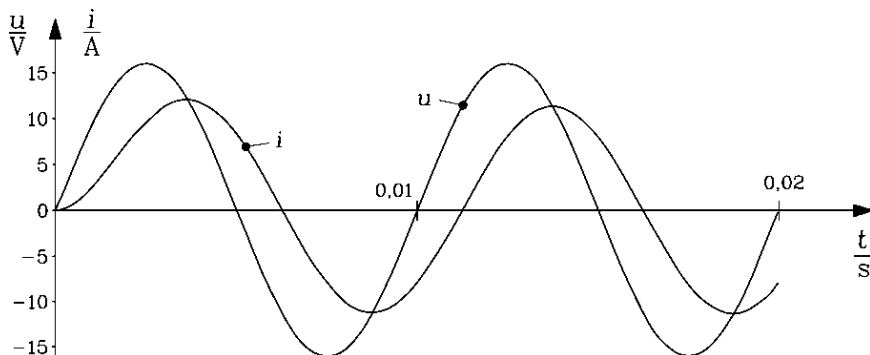
<u>Fig. 4.1-1:</u> Given voltage curve and numerically and analytically calculated current (coincident), flowing in the coil after switching on at $t = 0$ (switching angle $\varphi = 0$, $R = X = 1\,\Omega$, $f = 100$ Hz)

The analytical calculation yields:

$$A = \hat{U} \cdot \frac{R}{R^2 + (\omega L)^2} = 10 \cdot \frac{1}{1^2 + 1^2} = 5 \,\text{A} \ ,$$

$$B = -\hat{U} \cdot \frac{\omega L}{R^2 + (\omega L)^2} = -10 \cdot \frac{1}{1^2 + 1^2} = -5 \,\text{A}$$

$$T = L / R = 3.183\text{mH} / 1\Omega = 3.183 \,\text{ms} \ .$$

$$i(t) = B \cdot \left(\cos \omega t - e^{-t/T} - \sin \omega t \right)$$

After about $3T = 3 \cdot 3.183$ ms ≈ 10 ms resp. the period of the voltage curve, the direct current has almost completely vanished. The current amplitude during the second period reaches the steady state value

$$\hat{I} = \sqrt{5^2 + 5^2} = 7.07 \text{ A}.$$

The steady state phase angle between voltage and current is:

$$\psi = \arctan\left(\frac{B}{A}\right) = -\arctan(1) = -\pi / 4$$

The current lags the voltage by 45 degrees.

Self-excitation of a synchronous generator

A synchronous generator with S_N = 10 MVA, U_N = 3 kV is driven by a turbine at rated speed n_N. Its stator winding is connected via a transformer to a power transmission line, whose terminals are open ("open circuited power transmission line"). Due to the iron remanence, an AC voltage $u_{pR}(t)$ is induced into the stator winding with the rated frequency $f_N = \omega/(2\pi)$= 50 Hz. The resulting reactance per phase X=818.2 mΩ represents the sum of the synchronous reactance of the generator, the short-circuit reactance of the transformer and the reactance of the power transmission line. The capacitance C of the secondary open transmission line is relatively big, because of their relatively big length, so the associated reactance X_C is calculated, relatively to the generator voltage level, which is about 5% larger than X: X_C = 1.0526·X. The ohmic resistances and all other loss components are neglected. The transformer converts the voltage of 3 kV on the generator side to 220 kV on the grid side with the ratio $\ddot{u}$.

1)   For a remanent voltage U_{pR} of 5% of the rated voltage of the generator, calculate the generator charging current $\underline{I}_L = -\underline{I}_s$ of the power transmission line and its rms value as a percentage of the rated current!

2)   Determine the voltage $U´$ at the open terminals of the transmission line in proportion to the rated voltage and the remanent voltage at the generator terminals! Why can we address this as "self-excitation"?

3)   Calculate analytically the stator current per phase $i_s(t)$ and the generator voltage $u(t)$ at the open secondary terminals of the power transmission line, when the transmission line is switched to the generator terminals at the time t = 0. The time function of the remanent voltage in the considered winding phase is:

$$u_{pR}(t) = \hat{U}_{pR} \cdot \sin(\omega t)$$

4)   Calculate numerically $i_s(t)$ and the voltage $u(t)$ for $0 \leq t \leq 0.4\,\text{s}$ with the Runge-Kutta method and compare the resulting curves with the analytically determined values from 3).

1)   Using the complex AC-current calculations, currents and voltages on the grid side
        (transmission line) are calculated relatively to generator voltage level by $\ddot{u}$:
        Charging current of the transmission line, calculated on the generator side $I_L = \ddot{u} \cdot I'_L$
        The voltage at the secondary terminals of the power transmission line, calculated with
        respect to the generator voltage level: $U = U' / \ddot{u}$
        The reactance of the transmission line capacitance, calculated with respect to the
        generator voltage level:

$$X_C = X'_C / \ddot{u}^2, \quad X'_C = 1/(\omega C'), \quad C = \ddot{u}^2 \cdot C'$$

Consumer reference system for the generator ($\underline{I}_s = -\underline{I}_L$, Fig. 4.2-1), determination of
the generator rated current:

$$S_N = \sqrt{3} \cdot U_N I_N, \quad I_N = S_N /(\sqrt{3} U_N) = 10000/(\sqrt{3} \cdot 3) = 1924.5 \text{ A}$$

Stator voltage equation: $jX \cdot \underline{I}_s + \underline{U}_{pR} = \underline{U}$,

Voltage at the capacitance: $\underline{U} = -jX_C \cdot \underline{I}_L$.

Both equations yield the capacitive charging current:

$$\underline{I}_L = j\frac{U_{pR}}{X_C - X} .$$

Due to $X_C > X$ the charging current leads the remanent voltage by 90°. As the rated
voltage is a line-to-line value, the phase value of the remanent voltage is determined
according to:

$$U_{pR} = 0.05 \cdot U_N / \sqrt{3} = 0.05 \cdot 3000/ \sqrt{3} = 86.6 \text{ V} .$$

$$I_L = \frac{U_{pR}}{X_C - X} = \frac{86.6}{(1.0526-1) \cdot 0.8182} = 2012.2 \text{ A} = 1.0456 \cdot I_N$$

Due to the excitation of the electric oscillation system, formed up by L and C, by the
generator remanence voltage with a frequency of 50 Hz, which is close to the
resonance frequency, the charging current is 104.56 % of the rated current and hence
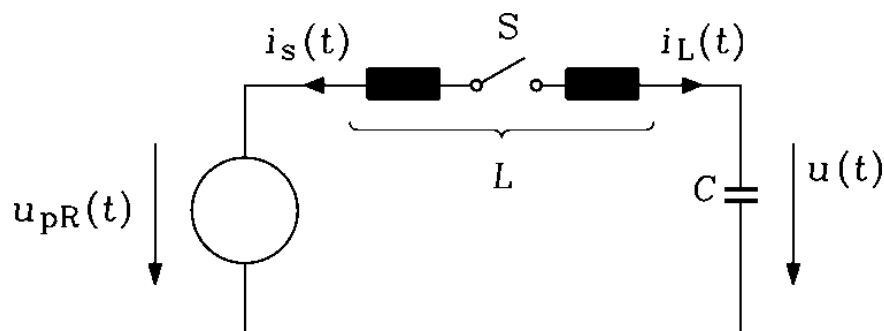too big. The current amplitude is $\hat{I}_s = \sqrt{2} \cdot 2012.2 = 2845.7 \text{ A}$.

Fig. 4.2-1: Single-phase equivalent circuit of the generator and the transmission line, switch S
is closed at $t = 0$.

2)   Voltage at capacitance C:

$$U = X_C \cdot I_L = \frac{X_C \cdot U_{pR}}{X_C - X} .$$

$$U = X_C \cdot I_L = 1.0526 \cdot 0.8182 \cdot 2012.2 = 1733.0 \text{ V} .$$

$$U / U_{pR} = 1733.0 / 86.6 = 20$$

$$U\sqrt{3}/U_N = 1733.0 \cdot \sqrt{3}/3000 = 1.0005$$

$$\ddot{u} = 230/3 = 76.67$$

$$\ddot{u}U\sqrt{3}/U_{N,grid} = 76.67 \cdot 1733.0 \cdot \sqrt{3}/230000 = 1.0005$$

$$U' = \ddot{u}U = 76.67 \cdot 1733.0 = 132.87\,\text{kV}$$

Although the synchronous machine is running unexcited, a voltage higher than the rated voltage occurs at the open secondary terminals of the power transmission line. The machine has excited „itself" via the iron remanence of the rotor and the capacitive loading. The voltage amplitude at the open terminals of the transmission line is, with respect to the generator voltage level, $\hat{U} = \sqrt{2} \cdot 1733.0 = 2450.8\,\text{V}$.

3)　　Before switching the power transmission line the current i_s and the voltage u at the capacitance is zero. Linear integro-differential equation with constant coefficients for $t \geq 0$:

$$L \cdot \frac{di_s}{dt} + \frac{1}{C}\int_0^t i_s \cdot dt + \hat{U}_{pR} \cdot \sin\omega t = 0$$

Differentiating once yields the linear differential equation of second order with constant coefficients for $t \geq 0$:

$$\frac{d^2 i_s}{dt^2} + \frac{i_s}{LC} = -\frac{\omega\hat{U}_{pR}}{L} \cdot \cos\omega t$$

As the magnetic energy $L i_s^2 / 2$ cannot change abruptly, the current must have shortly after switching the same value as before switching, so it has to be zero. As the electric energy $C u^2 / 2$ also cannot change abruptly, the voltage u must have shortly after switching the same value as before switching, so it has to be zero. Hence the initial conditions result: $i_s(0) = 0, u(0) = 0$

Homogeneous differential equation: $\dfrac{d^2 i_{s,h}}{dt^2} + \dfrac{i_{s,h}}{LC} = 0$

Approach to solution: $i_h = C_1 \cdot \sin\omega_e t + C_2 \cdot \cos\omega_e t$.

i_h inserted into the homogeneous differential equation, yields the eigen-frequency of the oscillating system: $\omega_e = 1/\sqrt{LC}$.

Particular differential equation:

$$\frac{d^2 i_{s,p}}{dt^2} + \omega_e^2 \cdot i_{s,p} = -\frac{\omega\hat{U}_{pR}}{L} \cdot \cos\omega t$$

Approach to solution: $i_{s,p} = B \cdot \cos\omega t$.

$i_{s,p}$, inserted into the particular differential equation yields:

$$-\omega^2 \cdot B \cdot \cos\omega t + \omega_e^2 \cdot B \cdot \cos\omega t = -\frac{\omega\hat{U}_{pR}}{L} \cdot \cos\omega t$$

$$B = -\frac{\omega^2 \hat{U}_{pR}}{\omega L \cdot (\omega_e^2 - \omega^2)}$$

The solution for the current is the sum of the homogeneous and the particular solution.

$$i_s(t) = i_{s,h} + i_{s,p} = C_1 \cdot \sin\omega_e t + C_2 \cdot \cos\omega_e t + B \cdot \cos\omega t$$

The voltage at the capacity is:

$$u(t) = u_{pR}(t) + L \cdot di_s / dt$$

$$u(t) = \hat{U}_{pR} \cdot \sin\omega t + L \cdot \{\omega_e C_1 \cdot \cos\omega_e t - \omega_e C_2 \cdot \sin\omega_e t - \omega B \cdot \sin\omega t\}$$

Matching the initial conditions:

$$i_s(0) = C_1 \cdot \sin 0 + C_2 \cdot \cos 0 + B \cdot \cos 0 = 0 \quad \Rightarrow \quad C_2 = -B$$

$$u(0) = \hat{U}_{pR}\sin 0 + L \cdot \{\omega_e C_1 \cos 0 - \omega_e C_2 \sin 0 - \omega B \sin 0\} = 0 \quad \Rightarrow \quad C_1 = 0$$

Analytical solution for the time functions of the phase current and the voltage at the capacity:

$$i_s(t) = -\frac{\omega^2 \hat{U}_{pR}}{\omega L \cdot (\omega_e^2 - \omega^2)} \cdot (\cos\omega t - \cos\omega_e t)$$

$$u(t) = -\frac{\hat{U}_{pR}}{\omega_e^2 - \omega^2} \cdot \left(-\omega_e^2 \cdot \sin\omega t + \omega\omega_e \cdot \sin\omega_e t\right)$$

As the damping was neglected, the transient inrush current, which oscillates with the eigen-frequency $f = \omega_e/2\pi$, does not decay. Therefore the solution is a superposition of two signals with almost the same angular frequency (with $\omega = 314.16/s$ and $\omega_e = 322.3/s$).

$$L = X/\omega = 0.8182/(2\pi 50) = 2.6044\,\text{mH},$$

$$C = \frac{1}{\omega X_C} = \frac{1}{\omega \cdot 1.0526X} = \frac{1}{2\pi 50 \cdot 1.0526 \cdot 0.8182} = 3.696\,\text{mF}$$

$$\omega_e = 1/\sqrt{LC} = 1/\sqrt{2.6044 \cdot 3.696 \cdot 10^{-6}} = 322.3/s, \quad f_e = \omega_e/(2\pi) = 322.3/(2\pi) = 51.3\,\text{Hz}$$

For real existing damping, the homogeneous part of the solution, which oscillates with ω_e, decays with a certain time constant. The particular solution remains as the steady state solution with the current amplitude

$$\hat{I}_s = \frac{\omega^2 \hat{U}_{pR}}{\omega L \cdot (\omega_e^2 - \omega^2)} = \frac{\hat{U}_{pR}}{X \cdot (\omega_e^2/\omega^2 - 1)} = \frac{\hat{U}_{pR}}{X_C - X} = 2845.7\,\text{A},$$

which was already determined in 1) by the complex AC-current calculation.

The steady state voltage amplitude

$$\hat{U} = -\frac{\omega_e^2 \cdot \hat{U}_{pR}}{\omega_e^2 - \omega^2} = \frac{X_C \cdot \hat{U}_{pR}}{X_C - X} = 2450.8\,\text{V}$$

has already been calculated in 2).

Due to the superposition of two undamped signals with the same amplitude, current amplitudes of double steady state values occur ($2 \cdot 2845.7 = 5691.4\,\text{A}$), like shown in Fig. 4.2-2. Similarly the maximum value of the voltage is (Fig. 4.2-2)

$$u_{max} = \frac{\hat{U}_{pR} \cdot (\omega_e^2 + \omega\omega_e)}{\omega_e^2 - \omega^2} = 4850\,\text{V}.$$

4)　　For the numerical integration, the second order differential equation is written as two first order differential equations:

$$\frac{du(t)}{dt} = -\frac{1}{C} \cdot i_s(t), \quad u(0) = 0$$

$$\frac{di_s(t)}{dt} = \frac{1}{L} \cdot u(t) - \frac{\hat{U}_{pR}}{L} \cdot \sin\omega t, \quad i(0) = 0$$

The following values are used:

$$\omega = 2\pi \cdot 50/\text{s}, \ \hat{U}_{pR} = \sqrt{2} \cdot 86.6\,\text{V} = 122.5\,\text{V}, L = 2.6044\,\text{mH}, \ C = 3.696\,\text{mF}$$

The calculation is performed for 0.4 s. An integration step size 1/10000 of this duration is used:

$$\Delta t = 400\text{ms}/10000 = 0.04\,\text{ms}.$$

The numerical solution for the phase current and voltage at the secondary terminals of the power transmission line (with respect to the generator voltage level) are compared with the analytical solution in Fig. 4.2-2, they are coincident, because the numerical deviations from the analytical values are smaller than the line width, used for the graphical representation.
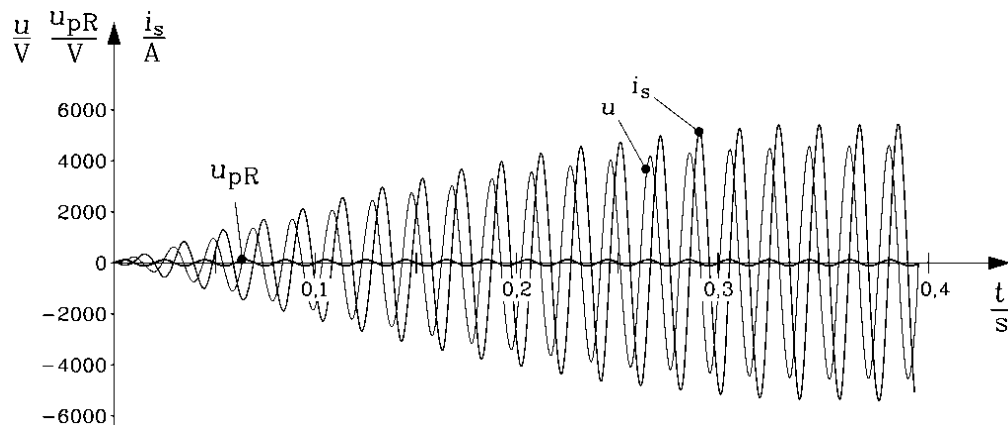
Fig. 4.2-2: Self-excitation of a synchronous machine: Analytical and coincident numerical undamped solution for the phase current $i_s(t)$ and the voltage $u(t)$ at the secondary terminals of the power transmission (with respect to the generator voltage level) line after connecting the power transmission line at the time $t = 0$ to the remanent generator voltage $u_{pR}(t) = 122.5\ \text{V}\cdot\sin(\omega t)$ ($\omega = 314.16/\text{s}$).

Exercise 4.3   4.3/1

Braking during the run-out of a rotating machine

The mechanically braked run-out of a rotating machine with the inertia J is to be calculated, starting with on the mechanic angular speed Ω_{m0}. The braking mechanic torque M_s has the following three different dependencies on the mechanic angular speed Ω_m:

a) $M_s(\Omega_m) = M_{s0}$

b) $M_s(\Omega_m) = M_{s0} \cdot (\Omega_m / \Omega_{m0})$

c) $M_s(\Omega_m) = M_{s0} \cdot (\Omega_m / \Omega_{m0})^2$

1)   Calculate analytically the mechanic angular speed $\Omega_m(t)$ as a function of time, starting with the value Ω_{m0}, for the three braking torques a), b), c). Which one of the three braking torques gives the strongest braking?

2)   Calculate numerically the mechanic angular speed $\Omega_m(t)$ as a function of time for the time range $0 \leq t \leq 100\,\text{s}$ for the following data: $J / M_{s0} = 0.0637\,\text{kgm}^2/(\text{Nm})$, $n(t = 0) = 1500/\text{min}$. Compare the numerical solution with the analytical one obtained from 1)!

Exercise 4.3   4.3/2

1)   *Newton*'s equation of motion:

$$J \cdot \frac{d\Omega_{\mathrm{m}}(t)}{dt} = -M_{\mathrm{s}}(\Omega_{\mathrm{m}}(t)), \text{ initial condition: } \Omega_{\mathrm{m}}(0) = \Omega_{\mathrm{m}0}$$

a) $J \cdot \dfrac{d\Omega_{\mathrm{m}}(t)}{dt} = -M_{\mathrm{s}0}, \ \Omega_{\mathrm{m}}(t) = \displaystyle\int_0^t \left(-\dfrac{M_{\mathrm{s}0}}{J} \right) \cdot dt = -\dfrac{M_{\mathrm{s}0}}{J} \cdot t + C$,

$$\Omega_{\mathrm{m}}(0) = C = \Omega_{\mathrm{m}0}, \ \Omega_{\mathrm{m}}(t) = -\frac{M_{\mathrm{s}0}}{J} \cdot t + \Omega_{\mathrm{m}0}$$

$$\Omega_{\mathrm{m}}(T) = 0 = -\frac{M_{\mathrm{s}0}}{J} \cdot T + \Omega_{\mathrm{m}0} \Rightarrow T = \Omega_{\mathrm{m}0} \cdot J / M_{\mathrm{s}0}$$

The mechanical angular speed decreases linearly with the time and is zero at the time $T = \Omega_{\mathrm{m}0} \cdot J / M_{\mathrm{s}0}$.

b) $J \cdot \dfrac{d\Omega_{\mathrm{m}}(t)}{dt} = -M_{\mathrm{s}0} \cdot (\Omega_{\mathrm{m}}(t)/\Omega_{\mathrm{m}0}), \ \dfrac{d\Omega_{\mathrm{m}}(t)}{dt} + \dfrac{M_{\mathrm{s}0}}{J \cdot \Omega_{\mathrm{m}0}} \cdot \Omega_{\mathrm{m}}(t) = 0$

This is a linear differential equation of first order with constant coefficients. Solution by Laplace-transformation:

$$s \cdot \breve{\Omega}_{\mathrm{m}} - \Omega_{\mathrm{m}0} + \frac{M_{\mathrm{s}0}}{J \cdot \Omega_{\mathrm{m}0}} \cdot \breve{\Omega}_{\mathrm{m}} = 0, \ \breve{\Omega}_{\mathrm{m}} \cdot \left(s + \frac{M_{\mathrm{s}0}}{J \cdot \Omega_{\mathrm{m}0}} \right) = \Omega_{\mathrm{m}0} \ ,$$

$$\breve{\Omega}_{\mathrm{m}} = \frac{\Omega_{\mathrm{m}0}}{s + \dfrac{M_{\mathrm{s}0}}{J \cdot \Omega_{\mathrm{m}0}}}, \ \Omega_{\mathrm{m}}(t) = \Omega_{\mathrm{m}0} \cdot e^{-\frac{M_{\mathrm{s}0}}{J \cdot \Omega_{\mathrm{m}0}} \cdot t} = \Omega_{\mathrm{m}0} \cdot e^{-t/T}$$

The mechanic angular speed decreases exponentially with the time and has decayed to the value $\Omega_{\mathrm{m}}(T) = \Omega_{\mathrm{m}0}/e$ after the time constant $T = \Omega_{\mathrm{m}0} \cdot J / M_{\mathrm{s}0}$. The zero value is reached after infinite long time.

c) $J \cdot \dfrac{d\Omega_{\mathrm{m}}(t)}{dt} = -M_{\mathrm{s}0} \cdot \left(\dfrac{\Omega_{\mathrm{m}}(t)}{\Omega_{\mathrm{m}0}} \right)^2, \ \dfrac{d\Omega_{\mathrm{m}}(t)}{dt} + \dfrac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot \Omega_{\mathrm{m}}^2(t) = 0$

This is a non-linear differential equation of first order with constant coefficients. The solution is possible by separation of variables Ω_{m} and t:

$$\frac{d\Omega_{\mathrm{m}}}{\Omega_{\mathrm{m}}^2} = -\frac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot dt, \ \int \frac{d\Omega_{\mathrm{m}}}{\Omega_{\mathrm{m}}^2} = -\int \frac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot dt, \ -\frac{1}{\Omega_{\mathrm{m}}} = -\frac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot t + K$$

Determination of the integration constant K via the initial conditions:

$$-\frac{1}{\Omega_{\mathrm{m}0}} = -\frac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot 0 + K \Rightarrow K = -\frac{1}{\Omega_{\mathrm{m}0}}$$

$$\Omega_{\mathrm{m}}(t) = \frac{1}{\dfrac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}^2} \cdot t + \dfrac{1}{\Omega_{\mathrm{m}0}}} = \frac{\Omega_{\mathrm{m}0}}{\dfrac{M_{\mathrm{s}0}}{J\Omega_{\mathrm{m}0}} \cdot t + 1} = \frac{\Omega_{\mathrm{m}0}}{\dfrac{t}{T} + 1}$$

The mechanical angular speed decreases inverse proportionally with time and has decayed to the value $\Omega_{\mathrm{m}}(T) = \Omega_{\mathrm{m}0}/2$ after the time $T = \Omega_{\mathrm{m}0} \cdot J / M_{\mathrm{s}0}$. The zero value is reached after infinite long time.

The braking torque a) brakes the most, the braking torque c) brakes the least.

2)   For the numerical integration the first order differential equations for the three cases a), b), c) are written as:

a) $\dfrac{d\Omega_m(t)}{dt} = -\dfrac{M_{s0}}{J}$, $\Omega_m(0) = \Omega_{m0}$

b) $\dfrac{d\Omega_m(t)}{dt} = -\dfrac{M_{s0}}{J \cdot \Omega_{m0}} \cdot \Omega_m(t)$, $\Omega_m(0) = \Omega_{m0}$

c) $\dfrac{d\Omega_m(t)}{dt} = -\dfrac{M_{s0}}{J \cdot \Omega_{m0}^2} \cdot \Omega_m^2(t)$, $\Omega_m(0) = \Omega_{m0}$

The following values are used:

$J / M_{s0} = 0.0637 \,\mathrm{kgm^2/(Nm)}$, $\Omega_m(0) = \Omega_{m0} = 2\pi \cdot n(0) = 2\pi \cdot 1500/60 = 157.08/\mathrm{s}$

The calculation is done for 100 s. An integration step size 1/1000 of this duration is used: $\Delta t = 100\mathrm{s}/1000 = 0.1\,\mathrm{s}$.

The numerical solution is compared with the analytical solution in Fig. 4.3-1 and is coincident, because the numerical deviations from the analytical values are smaller than the line width, used for the graphical representation.

The characteristic time is $T = \Omega_{m0} \cdot \dfrac{J}{M_{s0}} = 157.08 \cdot 0.0637 = 10\,\mathrm{s}$.

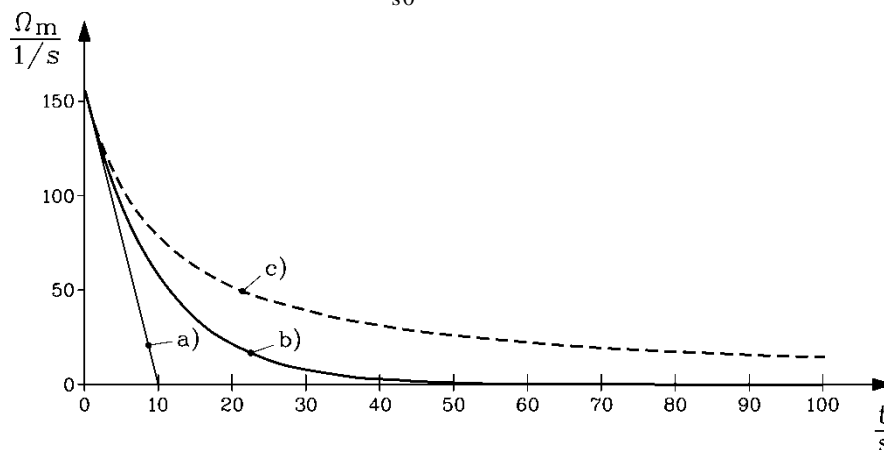Fig. 4.3-1: Mechanic angular speed as a function of time for mechanical braking of a rotating machine from $t = 0$ on. Analytical and coincident numerical solution for a) constant braking torque, b) linear with speed decaying braking torque, c) quadratic with speed decaying braking torque. At the characteristic time $T = 10$ s the speed has decayed for a) to zero, for b) to $1/e = 0.37$, and for c) to 0.5 of the initial value.

Exercise 4.4   4.4/1

Asynchronous no-load start-up of an induction machine
The electromagnetic torque of a two-pole induction machine ($P_N = 500$ kW, $U_N = 690$ V Y, $f_N = 50$ Hz, $n_N = 2982$/min) with the inertia $J = 5.9$ kg $\cdot$ m^2 is described with dependence on the slip s by *Kloss* function (see EMA and EMD lecture notes, Chap. 5)

$$\frac{M_e(s)}{M_b} = \frac{2}{\dfrac{s}{s_b} + \dfrac{s_b}{s}} \quad ,$$

where $s_b = 7.5 s_N$ is the corresponding breakdown slip.

1)   What simplifications are valid for the use of *Kloss'* function? Calculate the breakdown slip, the breakdown torque and the startup torque!

2)   Calculate analytically the mechanical angular speed $\Omega_m(t)$ as a function of time, when the machine is uncoupled (unloaded) and starts-up from zero speed (flywheel mass start-up)!

3)   Calculate numerically the mechanical angular speed $\Omega_m(t)$ as a function of time for the time range $0 \leq t \leq 2.0$ s ! Compare the numerical solution with the analytical one, obtained from 2)!

1)   Simplifications for the use of *Kloss*' function:
-     Negligence of stator resistance
-     Constant rotor resistance = no influence of slip-dependent current displacement
-     Constant reactances = no influence of variable saturation of main- and stray flux, as well as no influence of slip-dependent current displacement on stray flux.

Synchronous speed: $n_{syn} = f / p = 50/1 = 50/s = 3000/min$

Synchronous mechanical angular speed:
$\Omega_{syn} = 2\pi \cdot n_{syn} = 2\pi \cdot 50/s = 314.16/s$

Rated slip: $s_N = \dfrac{n_{syn} - n}{n_{syn}} = \dfrac{3000 - 2982}{3000} = 0.006$

Breakdown slip: $s_b = 7.5 s_N = 7.5 \cdot 0.006 = \underline{\underline{0.045}}$

Rated torque: $M_N = \dfrac{P_N}{2\pi n_N} = \dfrac{500000}{2\pi \cdot 2982/60} = 1601.16 \text{ Nm}$

Breakdown torque: $M_b = \dfrac{M_N}{2} \cdot \left(\dfrac{s_N}{s_b} + \dfrac{s_b}{s_N} \right) = \dfrac{1601.16}{2} \cdot \left(\dfrac{1}{7.5} + \dfrac{7.5}{1} \right) = 6111.08 \text{ Nm}$

Start-up torque: $M_e(s=1) = \dfrac{2M_b}{\dfrac{1}{s_b} + \dfrac{s_b}{1}} = \dfrac{2 \cdot 6111.08}{\dfrac{1}{0.045} + 0.045} = 548.9 \text{ Nm}$

2)   Flywheel mass start-up: Braking load torque at the shaft $M_s = 0$.
$$J \cdot \frac{d\Omega_m(t)}{dt} = M_e = \frac{2M_b}{\dfrac{s}{s_b} + \dfrac{s_b}{s}}, \quad s = 1 - \frac{\Omega_m}{\Omega_{syn}}$$

$$\frac{d\Omega_m(t)}{dt} = \Omega_{syn} \cdot d(1-s)/dt = -\Omega_{syn} \cdot ds/dt$$

$$-J\Omega_{syn} \cdot \frac{ds}{dt} - \frac{2M_b}{\dfrac{s}{s_b} + \dfrac{s_b}{s}} = 0$$

This is a non-linear differential equation of first order with constant coefficients. The solution is accomplished by separation of variables s and t:

$$-\frac{J\Omega_{syn}}{2M_b} \cdot \left(\frac{s}{s_b} + \frac{s_b}{s} \right) \cdot ds = dt \quad , \quad -\frac{J\Omega_{syn}}{2M_b} \cdot \int \left(\frac{s}{s_b} + \frac{s_b}{s} \right) \cdot ds = \int dt = t + K$$

$$-\frac{J\Omega_{syn}}{2M_b} \cdot \left(\frac{s^2}{2s_b} + s_b \cdot \ln(s) \right) = t + K$$

Determination of the integration constants K via the initial conditions:

$$t = 0: \quad s = 1: \quad -\frac{J\Omega_{syn}}{2M_b} \cdot \left(\frac{1}{2s_b} + s_b \cdot \ln(1) \right) = 0 + K \Rightarrow K = -\frac{J\Omega_{syn}}{4s_b M_b}$$

$$-\frac{J\Omega_{syn}}{2M_b} \cdot \left(\frac{s^2 - 1}{2s_b} + s_b \cdot \ln(s) \right) = t$$

$$\frac{J\Omega_{syn}}{2s_b M_b} \cdot \left[\frac{\Omega_m}{\Omega_{syn}} - \frac{1}{2} \cdot \left(\frac{\Omega_m}{\Omega_{syn}} \right)^2 - s_b^2 \cdot \ln\left(1 - \frac{\Omega_m}{\Omega_{syn}} \right) \right] = t$$

The analytical solution is a function $t(\Omega_m)$ instead of $\Omega_m(t)$.

3)   For the numerical integration the first order differential equation is written as:

$$\frac{d\Omega_m(t)}{dt} = \frac{2M_b s_b s}{J \cdot (s^2 + s_b^2)} = \frac{2M_b s_b \cdot (1 - \Omega_m / \Omega_{syn})}{J \cdot \left((1 - \Omega_m / \Omega_{syn})^2 + s_b^2 \right)}, \quad \Omega_m(0) = 0$$

The under 1) determined values are used. The calculation is performed for 2.0 s. An integration step size $\Delta t = 1.6\,\text{ms}$ is used. The numerical solution is compared with the analytical solution in Fig. 4.4-1. It is coincident, because the numerical deviations from the analytical values are smaller than the line width, used in the graphical representation.
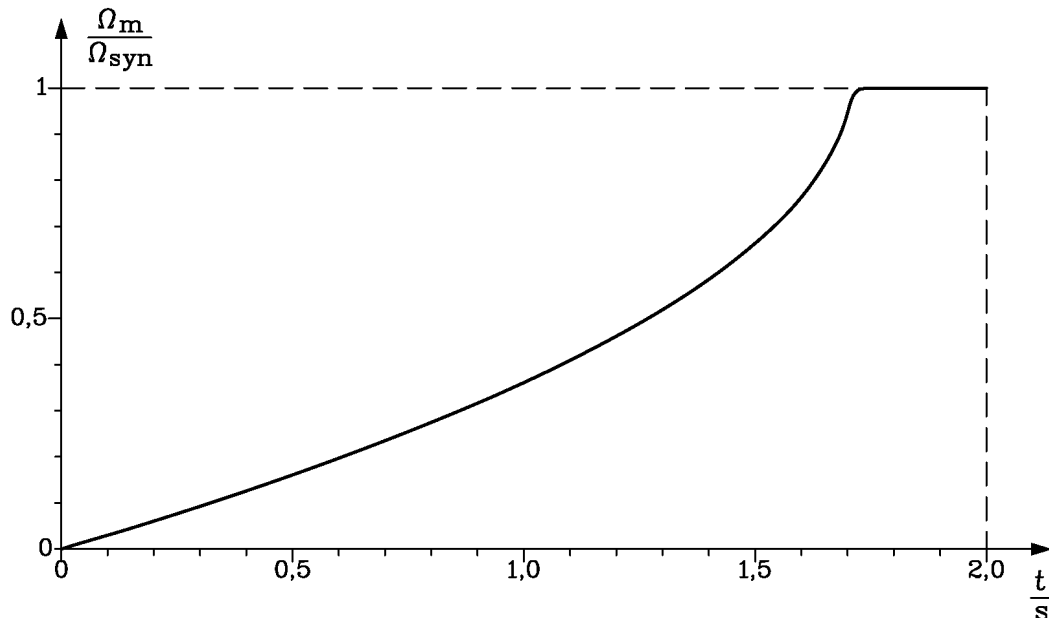

Fig. 4.4-1: Normalized analytically and numerically calculated mechanical angular speed as a function of time for a flywheel mass start-up of a two-pole 500 kW-induction motor on the 50 Hz-grid, when the *Kloss'* formula is used for the $M(n)$-curve (breakdown torque $M_b = 6111.08$ Nm, breakdown slip $s_b = 0.045$, synchronous angular speed $\Omega_{syn} = 314.16/\text{s}$)

D.C. machine as controlled drive

A **separately excited**, **compensated four-pole d.c. machine** is speed-controlled operated with a 4-quadrant-converter.

Machine data: frame size of 132 mm, externally ventilated, 13.8 kW, 220V, 74.5 A, rated speed 1500/min, maximum speed 5000/min. Motor moment of inertia: 0.22 kgm^2.
Inductances: armature: 3.3 mH, excitation: 21.0 H.
Resistances: armature: 0.35 Ohm, excitation: 82.1 Ohm.
At 1500/min with an excitation current of 1.8 A, a no-load voltage of 194 V is measured. The calculated reactance voltage u_r amounts to 1.6 V at 1500/min and rated current.

Calculate:

1)   the motor efficiency (inclusive excitation losses), the rated torque and the no-load speed n_0. The voltage drop at the brush contacts for the n_0 calculation can be neglected.

2)   the electrical and mechanical time constants T_f, T_a, T_{J0}, T_m, if the drive is coupled with a 0.30 kgm^2 load moment of inertia. Is the drive oscillating in uncontrolled operation a) de-coupled b) coupled with load?

3)   Indicate the structural diagram of the coupled machine for the design of a controller at rated operation.

4)   Draw the operating characteristics of armature voltage, armature current, main flux and power output in the entire speed range for converter supply, if a maximum reactance voltage of 3.5 V is admitted.

1) $\eta_a = \dfrac{P_N}{U_N \cdot I_N} = \dfrac{13800}{220 \cdot 74.5} = 0.842$ without excitation losses

$U_N = U_i + R_a \cdot I_N \Rightarrow \quad U_i = 220 - 0.35 \cdot 74.5 = \underline{194\,V}$

$n = 1500\,/\min \rightarrow I_f = 1.8\,A$

$P_f = R_f \cdot I_f^{\,2} = 82.1 \cdot 1.8^2 = 266\,W$

$\eta = \dfrac{P_N}{U_N \cdot I_N + P_f} = \dfrac{13800}{220 \cdot 74.5 + 266} = 0.8285 \cong \underline{\underline{82.9\,\%}}$

$M_N = \dfrac{P_N}{2\pi \cdot n_N} = \dfrac{13800}{2\pi \cdot \dfrac{1500}{60}} = \underline{\underline{87.9\,Nm}}$

$U_i = k_2 \Phi_N \cdot \Omega_{mN} \quad \Rightarrow \quad k_2 \Phi_N = \dfrac{194}{2\pi \cdot \dfrac{1500}{60}} = 1.235\,Vs$

$U_N = U_0 = k_2 \Phi_N \cdot \Omega_{m0} \quad \Rightarrow \quad \Omega_{m0} = \dfrac{220}{1.235} = 178.1\,s^{-1}$

$n_0 = \dfrac{178.1}{2\pi} 60 = \underline{\underline{1701/\min}}$

2) $T_a = \dfrac{L_a}{R_a} = \dfrac{3.3 \cdot 10^{-3}}{0.35} = \underline{\underline{9.4\,ms}}$,

a) $J_m = 0.22\,kgm^2$,   b) $J_{m+L} = 0.22 + 0.30 = 0.52\,kgm^2$

$T_m = \dfrac{J \cdot R_a}{(k_2 \Phi_N)^2} = \begin{cases} \text{a)} \quad \dfrac{0.22 \cdot 0.35}{1.235^2} = \underline{\underline{50.5\,ms}} > 4T_a \\[3mm] \text{b)} \quad \dfrac{0.52 \cdot 0.35}{1.235^2} = \underline{\underline{119.3\,ms}} > 4T_a \end{cases} \Bigg\}$ **not** oscillating

$4T_a = 4 \cdot 9.4 = 37.6\,ms$

The drive is in both uncoupled and coupled state **not** oscillating.

$T_f = \dfrac{L_f}{R_f} = \dfrac{21}{82.1} = \underline{\underline{255.8\,ms}}, \quad T_{J0} = \dfrac{J \cdot \Omega_{m0}}{M_N}, \qquad \Omega_{m0} = 178.1\,s^{-1}$

a) uncoupled: $T_J = \underline{\underline{445.8\,ms}}$ b) coupled: $T_J = \dfrac{0.52 \cdot 178.1}{87.9} = \underline{\underline{1.05\,s}}$,

3)

$$\Delta \breve{m}_S(s)$$

$$\Delta \breve{u}_a(s) \rightarrow \boxed{\frac{106,4}{s+106,4} \cdot 3,53} \rightarrow \bigcirc \rightarrow \boxed{\frac{1,92(s+106,4)}{s^2+106,4s+892}} \rightarrow \Delta \breve{\Omega}_m(s)$$
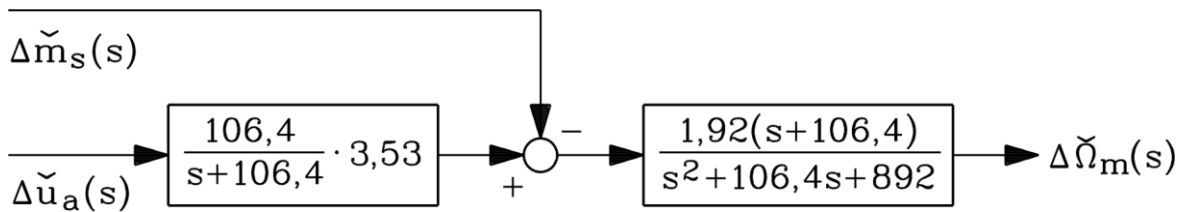
Fig. 5.1-1: Structural diagram of the coupled DC machine for the design of a controller at rated operation

$$\frac{k_2\Phi_N}{R_a} = \frac{1.235}{0.35} = 3.53\,\text{As}\,, \qquad \gamma = \frac{1}{T_a} = \frac{10^3}{9.4} = 106.4\,\text{s}^{-1}$$

$$\frac{1}{T_m}\bigg| = \frac{10^3}{119.3} = 8.38\,\text{s}^{-1}\,, \qquad \frac{\gamma}{T_m} = \frac{106.4}{1/8.38} = 892$$

coupled:

$$\frac{1}{J_{m+L}} = \frac{1}{0.52} = 1.92\,\frac{1}{\text{kgm}^2}$$

4) $u_{R,max} = 3.5\,\text{V}$ at I_N

$$u_{R,N} = 1.6\,\text{V}\ \text{ at }\ I_N,\,n_N \qquad u_R \sim I \cdot n \Rightarrow n_f = n_N \cdot \frac{u_{R,max}}{u_{R,N}} = 1500 \cdot \frac{3.5}{1.6} = 3280/\text{min}$$

Fig. 5.1-2: Operating characteristics of armature voltage, armature current, main flux and power output

Exercise 5.2   5.2/1

Oscillation behaviour of an uncontrolled d.c. drive

A **separately excited, four-pole d.c. machine** is speed-controlled operated with a 4-quadrant-converter as high-speed dynamomenter machine for a test-rig.
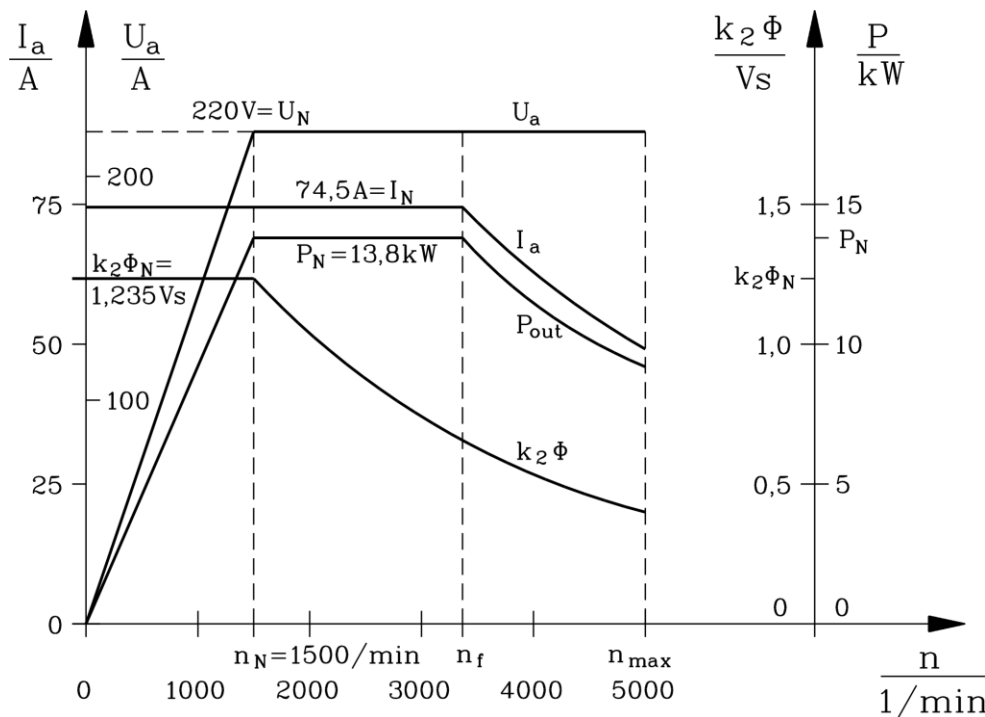
Machine data: externally ventilated, 35.5 kW, 290V, 145 A, rated speed 3000/min, maximum speed 9000/min. Machine moment of inertia: 0.31 kgm^2, inductance of the armature winding: 1.87 mH, *ohmic* resistance of the armature winding: 0.051 Ohm.

1)   Sketch the circuit of the drive (B6C)A(B6C).

2)   Calculate the armature efficiency (= motor efficiency without excitation losses), the rated torque and the no-load speed n_0. The voltage drop at the brush contacts for the n_0 calculation can be neglected.

3)   The machine is coupled with a load moment of inertia of 0.27 kgm^2. Is the drive a) uncoupled b) coupled in the uncontrolled case oscillating? If yes, determine the natural frequency and damping of the oscillation.

4)   At 6000/min (field weakening) the oscillating behaviour is to be examined again similar to 3).
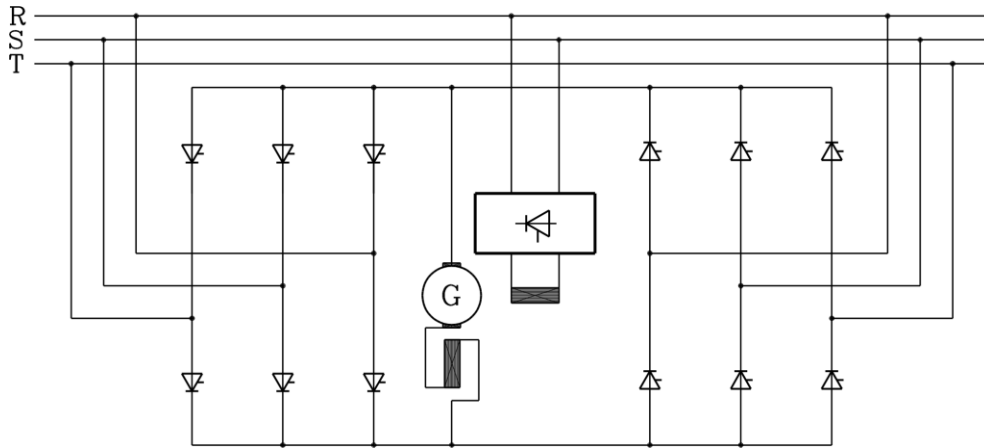
1)

<u>Fig. 5.2-1:</u> Circuit of the 4-quadrant drive (B6C)A(B6C)

Both B6C-bridges are operated only, if the other bridge is switched off! So no circulating parasitic current will occur. Otherwise additional chokes would be necessary.

2) $\eta_a = \dfrac{P_N}{U_N I_N} = \dfrac{35500}{290 \cdot 145} = \underline{\underline{84.4\,\%}}$ $M_N = \dfrac{P_N}{2\pi \cdot n_N} = \dfrac{35500}{2\pi \cdot \dfrac{3000}{60}} = \underline{\underline{113\,\text{Nm}}}$

$\underbrace{-I_N R_a}_{145 \cdot 0.051} + \underbrace{U_N}_{290\,\text{V}} = U_i = k_2 \Phi_N \cdot \underbrace{\Omega_{mN}}_{2\pi \cdot \dfrac{3000}{60} = 314.16\,\text{s}^{-1}}$ $\Rightarrow \quad k_2 \Phi_N = 0.9\,\text{Vs}$

$\Omega_{m0} = \dfrac{U_N}{k_2 \Phi_N} = \dfrac{290}{0.9} = 322.2\,\text{s}^{-1} \quad \Rightarrow \quad n_0 = \dfrac{322.2}{2\pi} \cdot 60 = \underline{\underline{3077/\text{min}}}$

3) $J_m + J_L = 0.31 + 0.27 = 0.58\,\text{kgm}^2$

$T_a = \dfrac{L_a}{R_a} = \dfrac{1.87 \cdot 10^{-3}}{0.051} = \underline{\underline{36.67\,\text{ms}}}, \qquad 4T_a = 4 \cdot 36.67 = 146.7\,\text{ms}$

$T_m = \dfrac{J \cdot R_a}{(k_2 \Phi_N)^2} = \begin{cases} \text{a)} \quad \dfrac{0.31 \cdot 0.051}{0.9^2} = \underline{\underline{19.5\,\text{ms}}} \; < 4T_a \\[3mm] \text{b)} \quad \dfrac{0.58 \cdot 0.051}{0.9^2} = \underline{\underline{36.5\,\text{ms}}} \; < 4T_a \end{cases}$ At 3000 /min and full excitation,

the drive tends to oscillate.

$\omega_d = 2\pi f_d = \dfrac{1}{T_a} \sqrt{\dfrac{T_a}{T_m} - \dfrac{1}{4}}, \qquad \text{damping:} \;\; \delta = \dfrac{1}{2T_a}$

a) uncoupled: $f_d = \dfrac{1}{2\pi} \cdot \dfrac{10^3}{36.67} \sqrt{\dfrac{36.67}{19.5} - \dfrac{1}{4}} = \underline{\underline{5.54\,\text{Hz}}}, \qquad \delta = \dfrac{10^3}{2 \cdot 36.67} = \underline{\underline{13.64\,\text{s}^{-1}}}$

b) coupled: $f_d = \dfrac{1}{2\pi} \cdot \dfrac{10^3}{36.67} \sqrt{\dfrac{36.67}{36.5} - \dfrac{1}{4}} = \underline{\underline{3.77\,\text{Hz}}}, \qquad \delta = \dfrac{10^3}{2 \cdot 36.67} = \underline{\underline{13.64\,\text{s}^{-1}}}$

4) $n = 6000/\text{min}, \quad I_a = I_N \quad \Rightarrow \quad U_N = U_i + R_a I_N \quad \Rightarrow \quad U_i = 282.6\,\text{V}$

$\qquad\qquad\qquad\qquad\qquad\qquad\nearrow$

$\qquad\qquad\qquad\qquad\qquad 290\,\text{V} \qquad 0.051 \cdot 145$

$\Rightarrow \quad k_2 \Phi_N = \underline{\underline{0.45\,\text{Vs}}}$

$$T_m = \dfrac{J \cdot R_a}{\left(k_2 \Phi_N\right)^2} = \begin{cases} \text{a)} \quad \dfrac{0.31 \cdot 0.051}{0.45^2} = \underline{\underline{78.1\,\text{ms}}} \;\; < 4T_a \;:\; \text{tends to oscillate} \\[4mm] \text{b)} \quad \dfrac{0.58 \cdot 0.051}{0.45^2} = \underline{\underline{146.1\,\text{ms}}} \;\approx 4T_a \;:\; \text{a - periodic limit} \end{cases}$$

In <u>uncoupled</u> operation, the drive tends to oscillate:

$$f_d = \dfrac{1}{2\pi} \cdot \dfrac{10^3}{36.67} \sqrt{\dfrac{36.67}{78.1} - \dfrac{1}{4}} = \underline{\underline{2.03\,\text{Hz}}}, \qquad\qquad \delta = \dfrac{10^3}{2 \cdot 36.67} = \underline{\underline{13.64\,\text{s}^{-1}}}$$

In <u>coupled</u> operation, <u>no</u> (or an extremely slow) oscillations occur.

The damping δ of the vibration <u>is not</u> be influenced by the operating condition (J, Φ).

Small DC-motor with permanent magnet excitation and chopper controller

An experimental cross-country vehicle used on *Mars* mission for surface survey should be equipped with DC-motors excited by permanent magnets. The motors serve as wheel drives similar to those in three-axis *Mars*-vehicle *Sojourner*. In a pilot survey, a DC-motor will be used with variable armature voltage generated by a MOSFET-chopper from a 48 V battery. Pulse frequency of the MOSFET-chopper equals 16 kHz. Operation with variable speed will be investigated.
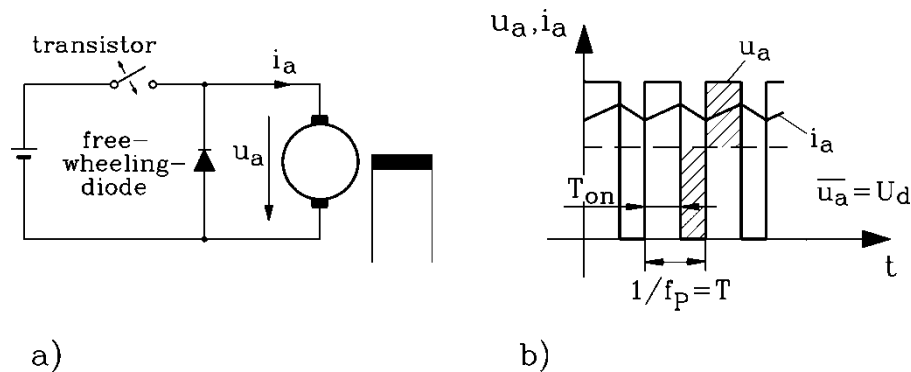

Fig. 5.3-1: 1-quadrant operation of DC-chopper: a) Basic circuit scheme, b) armature voltage pulse pattern

Motor specification:

U_N = 48 V, I_N = 2.1 A, no-load speed n_0 = 1020/min, R_a = 5.44 Ω, L_a = 2.34 mH (Voltage drop across brushes is neglected).

1)   What is electrical time constant of the armature in ms and how much is it in relation to period T of one chopper switching cycle of the battery voltage ?

2)   What is the value of rated speed and efficiency of the motor (if considering only *ohmic* losses)?

3)   How long is duration T_{on} of voltage pulse to operate at half of no-load speed? In this case, what is the speed at rated torque?

4)   Due to the switching modulation as depicted in Fig. 5.3-1 b), the armature current $i_a(t) = I_a + \Delta i_a(t)$ comprises an average value $I_a = \bar{i}_a$ and a small ripple of $\Delta i_a(t)$. Calculate this ripple with following approximation for the operating point according to 3)!

$$u_a(t) = L_a \cdot \frac{di_a(t)}{dt} + R_a \cdot i_a(t) + U_i = L_a \cdot \frac{d\Delta i_a(t)}{dt} + R_a \cdot (I_a + \Delta i_a(t)) + U_i \approx$$

$$\approx L_a \cdot \frac{d\Delta i_a(t)}{dt} + R_a \cdot I_a + U_i$$

1) $T_a = \dfrac{L_a}{R_a} = \dfrac{2.34 \cdot 10^{-3}}{5.44} \, \text{s} = 0.43 \, \text{ms}$

$T = \dfrac{1}{f_p} = \dfrac{1}{16000} \, \text{s} = 0.0625 \, \text{ms} \quad \Rightarrow \quad \dfrac{T_a}{T} = \dfrac{0.343}{0.0625} = 6.88$

2) $U_0 = U_i = k \cdot n_0 \cdot \Phi = U_{\text{batt}} \quad \Rightarrow \quad k \cdot \Phi = \dfrac{U_{\text{batt}}}{n_0} = \dfrac{48}{\dfrac{1020}{60}} = 2.824 \, \text{Vs}$

$n_N = \dfrac{U_a - I_{aN} \cdot R_a}{k \cdot \Phi} = \dfrac{48 - 2.1 \cdot 5.44}{2.824} \, \text{s}^{-1} = 12.95 \, \text{s}^{-1} = 777.2 \, \text{min}^{-1}$

$P_{\text{in}} = U_a \cdot I_{aN} = 48 \cdot 2.1 \, \text{W} = 100.8 \, \text{W}, \qquad P_d = R_a \cdot I_{aN}^{\,2} = 5.44 \cdot 2.1^2 \, \text{W} = 24 \, \text{W}$

$P_{\text{out}} = P_{\text{in}} - P_d = 100.8 - 24 \, \text{W} = 76.8 \, \text{W} \;\Rightarrow\; \eta = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{76.8}{100.8} = 76.2 \, \%$

3) $n_0 \;\rightarrow\; \dfrac{n_0}{2} \;\Leftrightarrow\; U_i \;\rightarrow\; \dfrac{U_i}{2} \;\Rightarrow\; \dfrac{U_0}{2} = \dfrac{U_{\text{batt}}}{2}$

$U_d = \overline{u_a} = \dfrac{U_{\text{batt}}}{2} \quad \Rightarrow \quad \overline{u_a} = \dfrac{T_{\text{on}}}{T} \cdot U_{\text{batt}} \quad \Rightarrow \quad T_{\text{on}} = \dfrac{T}{2} = 0.0313 \, \text{ms}$

Rated torque $\Leftrightarrow$ Rated current (field excited by permanent magnet)

$n^* = \dfrac{U_a - I_{aN} \cdot R_a}{k \cdot \Phi} = \dfrac{\dfrac{U_N}{2} - I_{aN} \cdot R_a}{k \cdot \Phi} = \dfrac{24 - 2.1 \cdot 5.44}{2.824} \, \text{s}^{-1} = 4.45 \, \text{s}^{-1} = 267.2 \, \text{min}^{-1}$

4)   a) $0 \le t \le T_{\text{on}}:$ $u_a(t) = U_{\text{batt}}, \quad I_a = I_N, \quad U_i = k \cdot \Phi \cdot n^* = 12.576 \, \text{V}$

$U_{\text{batt}} - R_a \cdot I_N - U_i \cong L_a \cdot \dfrac{d\Delta i_a(t)}{dt}, \qquad \Delta i_a(t) = \underbrace{\dfrac{U_{\text{batt}} - R_a \cdot I_N - U_i}{L_a}} \cdot t$

$= \dfrac{48 - 5.44 \cdot 2.1 - 12.576}{2.34 \cdot 10^{-3}} \, \dfrac{\text{V}}{\text{H}} = \dfrac{24 \, \text{V}}{2.34 \cdot 10^{-3} \, \text{H}}$

$= 10256 \, \dfrac{\text{A}}{\text{s}}$

$\Delta i_a(t = T_{\text{on}}) = 10256 \cdot 0.0313 \cdot 10^{-3} \, \text{A} = 0.32 \, \text{A}$

b) $T_{\text{on}} \le t \le T:$ $u_a(t) = 0, \quad I_a = I_N, \quad U_i = k \cdot \Phi \cdot n^*$

$0 - R_a \cdot I_N - U_i \cong L_a \cdot \dfrac{d\Delta i_a(t)}{dt} \;\Rightarrow\; t' = t - T_{\text{on}} : 0 \le t' \le T - T_{\text{on}} \underset{T_{\text{on}} = T/2}{=} T_{\text{on}} \;\Rightarrow$

$\Rightarrow \Delta i_a'(t') = \dfrac{-R_a \cdot I_N - U_i}{L_a} = -10256 \, \dfrac{\text{A}}{\text{s}}, \quad \Delta i_a(t' = T - T_{\text{on}}) = -0.32 \, \text{A}$

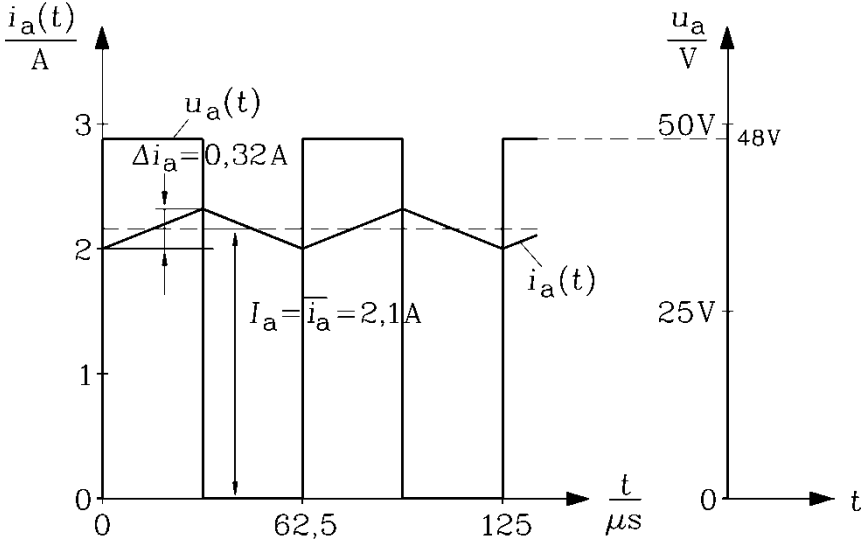Fig. 5.3-2: 1-quadrant operation of DC-chopper: Armature voltage and armature current

Exercise 5.4   5.4/1

Starting DC Motor Directly from DC Grid

A permanent-magnet excited small DC motor shall be started at a DC voltage $U = U_N$ by closing the switch S at $t = 0$ (Fig. 5.4-1)! The total rotor inertia is J.
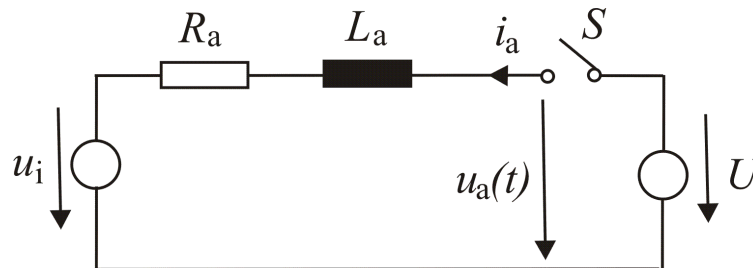

Fig. 5.4-1: Equivalent circuit of the PM excited DC machine

1) Give the formula for u_i!
2) What are the initial conditions for angular speed $\Omega_m(t)$ and armature current $i_a(t)$?
3) What basic mathematical property has the stator flux per pole with respect to time?
4) Give the qualitative graph for the armature voltage $u_a(t)$ as a time function!
5) Give the differential equations for calculating the armature current $i_a(t)$ and the angular speed $\Omega_m(t)$, if the load torque at the shaft is zero ($m_s = 0$)!
6) Transform the differential equations into Laplace domain and determine the current $\breve{I}_a = L\{i_a(t)\}$! [2]
7) Calculate the formula $i_a(t)$ in the time domain during start up. Use for the calculation the fact that the DC motor's time constants T_a=10 ms, $T_m = 100$ ms hold the relationship $T_m > 4T_a$! [3]
8) Define the time constants T_1 and T_2 and calculate them!
9) Calculate the function $\Omega_m(t)$! Sketch qualitatively the functions $i_a(t)$ and $\Omega_m(t)$ between $-\infty \leq t \leq +\infty$. How do the current $i_a(t)$ and $\Omega_m(t)$ behave for $t \to +\infty$ and why? Give the peak current value!

1) $u_i(t) = k_2 \cdot \Omega_m(t) \cdot \Phi$

2) $\Omega_m(0) = 0$, $i_a(0) = 0$

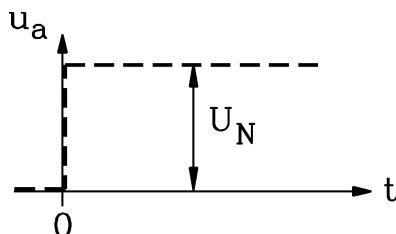3) $\Phi = const.$, because it is a permanent magnet excited flux

4)

Fig. 5.4-2: Armature voltage step: $t \leq 0$: Switch S is open; $t > 0$: Switch S is closed

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[2] Use the DC motor's time constants in the expression of $\breve{I}_a$

[3] Give the poles of the transfer function of 6). Then, decompose the transfer function of 6) in a sum of 1st order partial functions and transform them in the time domain!

Exercise 5.4   5.4/2

$$u_a(t) = U_N \cdot \varepsilon(t) \qquad \varepsilon(t): \text{Heaviside's step function}$$

5) $$u_a(t) = R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + k_2 \cdot \Omega_m(t) \cdot \Phi$$

$$J \cdot \frac{d\Omega_m}{dt} = k_2 \cdot i_a(t) \cdot \Phi - m_s(t) = k_2 \cdot i_a(t) \cdot \Phi$$

6) $$J \cdot \left(s\breve{\Omega}_m - \Omega_m(0) \right) = k_2 \cdot \Phi \cdot \breve{I}_a, \; \breve{U}_a = \frac{U}{s} = R_a \cdot \breve{I}_a + L_a \cdot (s\breve{I}_a - i_a(0)) + k_2 \cdot \Phi \cdot \breve{\Omega}_m$$

$$J \cdot s\breve{\Omega}_m = k_2 \cdot \Phi \cdot \breve{I}_a, \; \frac{U}{s} = R_a \cdot \breve{I}_a + L_a \cdot s\breve{I}_a + k_2 \cdot \Phi \cdot \breve{\Omega}_m$$

$$\breve{\Omega}_m = \frac{k_2 \cdot \Phi}{J \cdot s} \cdot \breve{I}_a, \; \breve{I}_a = \frac{U}{s} \cdot \frac{1}{sL_a + R_a + (k_2 \cdot \Phi)^2 / (s \cdot J)} = \frac{U}{L_a \cdot \left(s^2 + \dfrac{s}{T_a} + \dfrac{1}{T_a T_m} \right)}$$

7) $$s^2 + \frac{s}{T_a} + \frac{1}{T_a T_m} = (s - s_1) \cdot (s - s_2)$$

With $T_m > 4T_a$ we get: $s_{1,2} = -\dfrac{1}{2T_a} \pm \dfrac{1}{2T_a} \sqrt{1 - \dfrac{4T_a}{T_m}} < 0, \; |s_2| > |s_1|$

Long time constant: $T_1 = -1/s_1$, short time constant: $T_2 = -1/s_2$

$$\breve{I}_a = \frac{U}{L_a \cdot (s - s_1) \cdot (s - s_2)} = \frac{U}{L_a} \cdot \left[\frac{A}{s - s_1} + \frac{B}{s - s_2} \right], \; A = \frac{1}{s_1 - s_2} = -B$$

$$i_a(t) = \frac{U}{L_a} \cdot A \cdot \left[e^{s_1 t} - e^{s_2 t} \right] = \frac{U}{R_a} \cdot \frac{1}{\sqrt{1 - \dfrac{4T_a}{T_m}}} \cdot \left[e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}} \right]$$

8) $$T_1 = \frac{1}{\dfrac{1}{2T_a} - \dfrac{1}{2T_a}\sqrt{1 - \dfrac{4T_a}{T_m}}} = \frac{1}{\dfrac{10^3}{2 \cdot 10} - \dfrac{10^3}{2 \cdot 10} \cdot \sqrt{1 - \dfrac{4 \cdot 10}{100}}} = 88.7 \text{ms}$$

$$T_2 = \frac{1}{\dfrac{1}{2T_a} + \dfrac{1}{2T_a}\sqrt{1 - \dfrac{4T_a}{T_m}}} = \frac{1}{\dfrac{10^3}{2 \cdot 10} + \dfrac{10^3}{2 \cdot 10} \cdot \sqrt{1 - \dfrac{4 \cdot 10}{100}}} = 11.27 \text{ms}$$

9) $$\Omega_m(t) = (k_2 \cdot \Phi / J) \cdot \int_0^t i_a(t)dt = \frac{k_2 \Phi}{J} \cdot \frac{U}{R_a} \cdot \frac{1}{\sqrt{1 - \dfrac{4T_a}{T_m}}} \cdot \left[T_2 e^{-\frac{t}{T_2}} - T_1 e^{-\frac{t}{T_1}} + T_1 - T_2 \right]$$

$$\Omega_m(t \to \infty) = \frac{k_2\Phi}{J} \cdot \frac{U}{R_a} \cdot \frac{T_1 - T_2}{\sqrt{1 - \frac{4T_a}{T_m}}} = \frac{k_2\Phi}{J} \cdot \frac{U}{R_a} \cdot \frac{T_m \cdot \sqrt{1 - \frac{4T_a}{T_m}}}{\sqrt{1 - \frac{4T_a}{T_m}}} = \frac{k_2\Phi}{J} \cdot \frac{U}{R_a} \cdot \frac{JR_a}{(k_2\Phi)^2} = \frac{U}{k_2\Phi}$$

For infinte time the armature current decreases to zero, because the speed rises to no- load speed $\Omega_m(t \to \infty) = U/(k_2\Phi)$, so $u_i = U$ and $i_a = 0$.
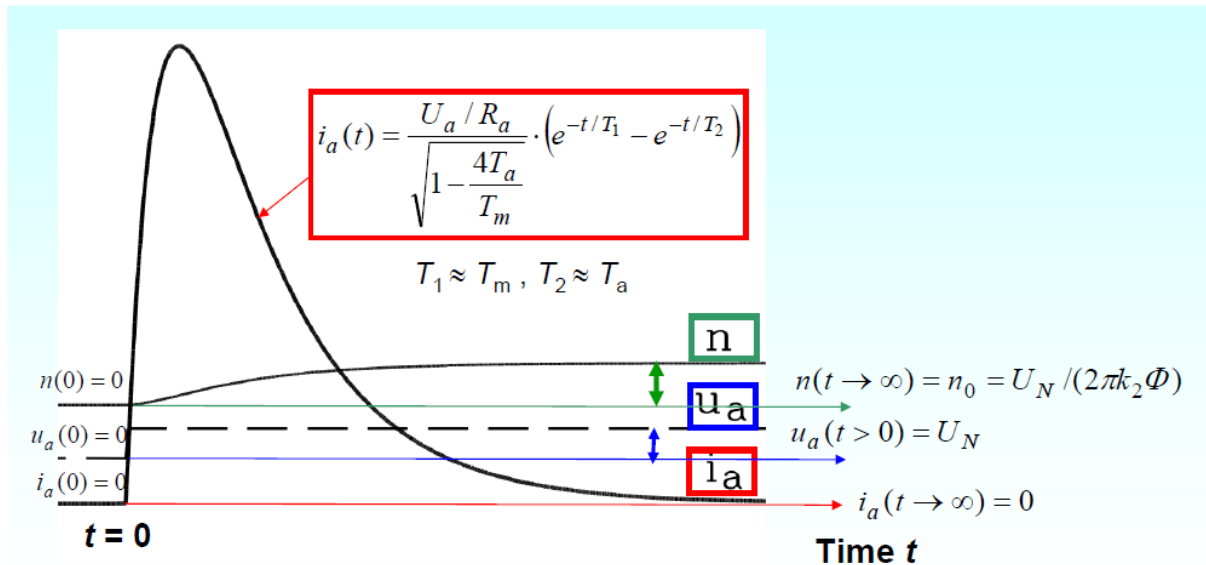
Fig. 5.4-3: Armature voltage step: $t \le 0$: Switch S is open; $t > 0$: Switch S is closed, and armature current and speed during start-up

Peak current: at time t^*:

$$di_a(t)/dt = 0 : \frac{d}{dt}\left[e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}} \right] = 0 \Rightarrow -\frac{e^{-\frac{t}{T_1}}}{T_1} + \frac{e^{-\frac{t}{T_2}}}{T_2} = 0 \Rightarrow t^* = \frac{\ln(T_1/T_2)}{\frac{1}{T_2} - \frac{1}{T_1}}$$

$$t^* = \frac{\ln(T_1/T_2)}{\frac{1}{T_2} - \frac{1}{T_1}} = \frac{\ln(88.7/11.27)}{\frac{1}{0.01127} - \frac{1}{0.0887}} = 26.6ms$$

Separately excited DC motor, operated at the six-pulse converter bridge B6C

A separately excited DC machine is operated at the B6C thyristor bridge, which is fed from the public grid 400 V, $f = 50$ Hz. The machine data are $U_N = 460$ V, $I_N = 320$ A, rated speed $n_N = 625$/min, $R_a = 0.05\ \Omega$, $L_a = 1.5$ mH. The small voltage drop of 2 V across the brushes may be neglected due to 2 V / 460 V = 0.4% $\ll 1$. The time function of the field current may be assumed to be constant due to the big inductance of the field winding. Hence the main flux per pole Φ_N is constant with the data $k_2\Phi_N = 6.7838$ Vs. Calculate for rated motor operation with controlled armature voltage

1) the necessary control angle α for rated armature voltage,
2) the time function of the armature current for 1/6 of the grid period $T/6 = 1/(6f)$. Due to the large inertia of the rotor we assume that the motor speed is ideally constant (n = const.).

1)
The grid frequency is $f = \omega/(2\pi)$. The grid line-to-line voltage U_{UV} is leading the corresponding grid phase voltage U_U by 30°: $u_{UV}(t') = \hat{U}_{LL}\cos(\omega t' - \pi/6)$. The control angle α leads to conducting thyristors of one bridge branch during the interval $\alpha \le \omega t' \le \alpha + \pi/3$ of 1/6 of grid period. During that time interval the bridge output voltage $u_{UV}(t') = \hat{U}_{LL}\cos(\omega t' - \pi/6)$ feeds the armature of the DC machine. By shifting the time origin $\omega t' - \alpha = \omega t$ we consider the voltage $u_{UV}(t) = \hat{U}_{LL}\cos(\omega t - \pi/6 + \alpha)$ during the time interval $0 \le \omega t \le \pi/3$. The average armature voltage is:

$$U_d = \frac{3}{\pi}\int_0^{\pi/3}\hat{U}_{LL}\cos(\omega t - \pi/6 + \alpha)\cdot d(\omega t) = \frac{3}{\pi}\cdot\hat{U}_{LL}\sin(\omega t - \pi/6 + \alpha)\Big|_0^{\pi/3},$$

$$U_d = \frac{3}{\pi}\cdot\hat{U}_{LL}\cos\alpha = U_{d0}\cos\alpha.$$

At rated operation the armature voltage is $U_d = U_N = R_a\cdot I_N + U_i = R_a\cdot I_N + k_2\Phi_N 2\pi n_N$, hence satisfying $U_d = (0.05\cdot 320 + 6.7838\cdot 2\pi\cdot 625/60)\,\text{V} = 460\,\text{V} = U_N$. For this voltage we need a control angle according to $U_N = \sqrt{2}\cdot U_{LL}\cdot\frac{3}{\pi}\cdot\cos\alpha$.

$$460\,\text{V} = \sqrt{2}\cdot 400\,\text{V}\cdot\frac{3}{\pi}\cdot\cos\alpha,$$

$$\cos\alpha = \frac{460}{\sqrt{2}\cdot 400\cdot(3/\pi)} = 0.8516,\ \alpha = 0.5519\text{rad} = 31.62°.$$

2)
The armature voltage is $u_{UV}(t) = \hat{U}_{LL}\cos(\omega t - \pi/6 + \alpha)$ during the time interval $0 \le \omega t \le \pi/3$. Hence the armature voltage equation is
$\hat{U}_{LL}\cos(\omega t - \pi/6 + \alpha) = R_a i_a(t) + L_a di_a(t)/dt + U_i$,
which is a first order linear differential equation with constant coefficients

$$\frac{di_a}{dt} + \frac{R_a}{L_a}\cdot i_a = \frac{\hat{U}_{LL}}{L_a}\cdot\cos(\omega t - \pi/6 + \alpha) - \frac{U_i}{L_a}\ .$$

<u>Exercise 5.5</u>   5.5/2

The initial condition must fulfill the periodicity of armature current during 1/6 of grid period $i_a(0) = i_a(\pi/(3\omega))$. We assume, that the machine is loaded with rated armature current, so that this current is always non-zero: $i_a(0) = i_a(\pi/(3\omega)) \geq 0$.

a) Solution of the homogenenous differential equation $\dfrac{di_{a,h}}{dt} + \dfrac{R_a}{L_a} \cdot i_{a,h} = 0$ with $i_{a,h} = D \cdot e^{\lambda \cdot t}$,

yielding $\lambda = -R_a / L_a = -1/T_a$ with the armature time constant $T_a = \dfrac{L_a}{R_a} = \dfrac{1.5 \cdot 10^{-3}}{0.05} \, \text{s} = 30 \, \text{ms}$.

b) Solution of the particular differential equation
$$\frac{di_{a,p}}{dt} + \frac{R_a}{L_a} \cdot i_{a,p} = \frac{\hat{U}_{LL}}{L_a} \cdot \left(\cos(\omega t)\cos(\alpha - \frac{\pi}{6}) - \sin(\omega t)\sin(\alpha - \frac{\pi}{6}) \right) - \frac{U_i}{L_a}$$
with $i_p = A \cdot \sin\omega t + B \cdot \cos\omega t + C$, yielding

$$\omega(A\cos\omega t - B\sin\omega t) + \frac{R_a}{L_a} \cdot (A\sin\omega t + B\cos\omega t + C) =$$

$$= \frac{\hat{U}_{LL}}{L_a} \cdot \left(\cos(\omega t)\cos(\alpha - \frac{\pi}{6}) - \sin(\omega t)\sin(\alpha - \frac{\pi}{6}) \right) - \frac{U_i}{L_a}.$$

The unknown coefficients A, B are determined by comparing the above noted expressions for $\sin\omega t$, $\cos\omega t$ and the constant, yielding

$$C = -\frac{U_i}{R_a} \text{ and}$$

$$\sin\omega t: \frac{R_a}{L_a} \cdot A - \omega \cdot B = -\frac{\hat{U}_{LL}}{L_a} \cdot \sin(\alpha - \frac{\pi}{6}) \quad , \qquad \cos\omega t: \frac{R_a}{L_a} \cdot B + \omega \cdot A = \frac{\hat{U}_{LL}}{L_a} \cdot \cos(\alpha - \frac{\pi}{6}).$$

The linear equation system for A, B is solved with *Cramer*'s rule.

$$\begin{pmatrix} \omega & 1/T_a \\ 1/T_a & -\omega \end{pmatrix} \cdot \begin{pmatrix} A \\ B \end{pmatrix} = \frac{\hat{U}_{LL}}{L_a} \cdot \begin{pmatrix} \cos(\alpha - \pi/6) \\ -\sin(\alpha - \pi/6) \end{pmatrix},$$

$$(N) = \begin{pmatrix} \omega & 1/T_a \\ 1/T_a & -\omega \end{pmatrix} \quad , \qquad Det(N) = \begin{vmatrix} \omega & 1/T_a \\ 1/T_a & -\omega \end{vmatrix} = -\left(\omega^2 + \frac{1}{T_a^2} \right)$$

$$A = \frac{1}{Det(N)} \cdot \frac{\hat{U}_{LL}}{L_a} \cdot \begin{vmatrix} \cos(\alpha - \pi/6) & 1/T_a \\ -\sin(\alpha - \pi/6) & -\omega \end{vmatrix} \quad , \quad A = \frac{\hat{U}_{LL}}{L_a} \cdot \frac{\omega \cdot \cos(\alpha - \pi/6) - \sin(\alpha - \pi/6)/T_a}{\omega^2 + \dfrac{1}{T_a^2}}$$

$$B = \frac{1}{Det(N)} \cdot \frac{\hat{U}_{LL}}{L_a} \cdot \begin{vmatrix} \omega & \cos(\alpha - \pi/6) \\ 1/T_a & -\sin(\alpha - \pi/6) \end{vmatrix} \quad , \quad B = \frac{\hat{U}_{LL}}{L_a} \cdot \frac{\omega \cdot \sin(\alpha - \pi/6) + \cos(\alpha - \pi/6)/T_a}{\omega^2 + \dfrac{1}{T_a^2}}$$

The final solution for the armature current is the sum of homogeneous and particular solution:
$$i_a = i_{a,h} + i_{a,p} = D \cdot e^{-t/T_a} + A \cdot \sin\omega t + B \cdot \cos\omega t + C \ .$$

The unknown constant D is determined via the initial condition:
$$i_a(0) = D \cdot e^{0/T_a} + A \cdot \sin 0 + B \cdot \cos 0 + C = D + B + C \ ,$$

$$i_a(\pi/(3\omega)) = D \cdot e^{-\pi/(3\omega T_a)} + A \cdot \sin(\pi/3) + B \cdot \cos(\pi/3) + C \ .$$

With $i_a(0) = i_a(\pi/(3\omega))$ we get: $D = \dfrac{A \cdot \sin(\pi/3) + B \cdot (\cos(\pi/3) - 1)}{1 - e^{-\pi/(3\omega T_a)}}$.

Inserting A and B in D and using the trigonometric relationships

$$\cos(\alpha - \frac{\pi}{6})\sin(\frac{\pi}{3}) + \sin(\alpha - \frac{\pi}{6})\cos(\frac{\pi}{3}) - \sin(\alpha - \frac{\pi}{6}) = 2\cos\alpha \sin(\frac{\pi}{6}) \ ,$$

$$\sin(\alpha - \frac{\pi}{6})\sin(\frac{\pi}{3}) - \cos(\alpha - \frac{\pi}{6})\cos(\frac{\pi}{3}) + \sin(\alpha - \frac{\pi}{6}) = 2\sin\alpha \sin(\frac{\pi}{6}) \text{ we end up with}$$

$$D = \frac{\hat{U}_{LL}}{R_a^2 + (\omega L_a)^2} \cdot \frac{\omega L_a \cos(\alpha) - R_a \sin(\alpha)}{1 - e^{-\pi/(3\omega T_a)}} \ .$$

Hence the analytically determined solution for the armature current is:

$$i_a(t) = \frac{\hat{U}_{LL}}{R_a^2 + (\omega L_a)^2} \cdot \left\{ \frac{\omega L_a \cos\alpha - R_a \sin\alpha}{1 - e^{-\pi/(3\omega T_a)}} e^{-\frac{t}{T_a}} + \omega L_a \sin(\omega t + \alpha - \frac{\pi}{6}) + \right.$$

$$\left. + R_a \cos(\omega t + \alpha - \frac{\pi}{6}) \right\} - \frac{U_i}{R_a} \ .$$

A transient DC component occurs due to the homogenoeus part of the solution, which decays with the armature time constant T_a. With the induced voltage at rated operation

$$U_i = k_2 \Phi_N 2\pi n_N = 6.7838 \cdot 2\pi \cdot 625/60 = 444 \text{ V}$$

and the values from 1) the time function of the armature current is given in Fig. 5.5-1 for 1/3 of the grid period. Averaging this function we get very close to the rated value 320 A of 1).
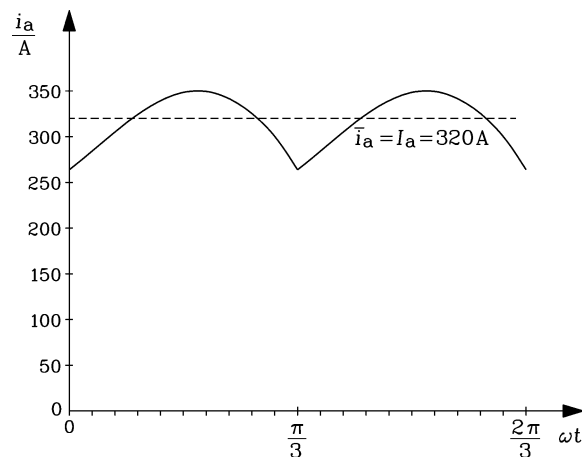
Fig. 5.5-1: Calculated armature current i_a of a separately excited DC motor, fed by a B6C-converter at rated speed $n_N = 625$/min, operating at rated average armature current 320 A and rated average armature voltage 460V. The control angle of the thyristors is $\alpha = 31.62°$.

Note that the limit, where the armature current gets zero during 1/6 of grid period, is given by the condition

$$i_a(0) = D \cdot e^{0/T_a} + A \cdot \sin 0 + B \cdot \cos 0 + C = D + B + C = 0 \ ,$$

$$\frac{\hat{U}_{LL}}{R_a^2 + (\omega L_a)^2} \cdot \left\{ \frac{\omega L_a \cos(\alpha) - R_a \sin(\alpha)}{1 - e^{-\pi R_a/(3\omega L_a)}} + \omega L_a \sin(\alpha - \frac{\pi}{6}) + R_a \cos(\alpha - \frac{\pi}{6}) \right\} - \frac{U_i}{R_a} = 0 \ .$$

It depends on α, on the induced voltage U_i, the grid voltage and frequency and on the machine parameters L_a, R_a. Starting with rated current at constant rated speed and hence constant U_i, a reduction of the average armature current I_a according to

$$U_d = R_a \cdot I_a + U_i = \sqrt{2} \cdot U_{LL} \cdot \frac{3}{\pi} \cdot \cos\alpha$$

yields a lower average armature voltage U_d, so that α has to be increased, until at a certain angle the condition $i_a(0) = 0$ occurs, which is the limit for non-zero current operation. The corresponding current value for this limit is

$$I_{aL\ddot{u}ck} \sim U_{d0} / L_a = \sqrt{2} \cdot U_{LL} \cdot \frac{3}{\pi L_a} \quad .$$

Exercise 5.6   5.6/1

Short-circuit braking of a DC generator

A separately excited DC generator G is driven by an *Ossberger* water turbine T in a small hydro power plant at n_N and supplies a local DC grid in a rural district.

DC Generator data: $U_{aN} = 460\,\text{V}$, $n_N = 645/\min$, $I_{aN} = 320\,\text{A}$, $J_{G+T} = 8\,\text{kg}\cdot\text{m}^2$,

$R_a = 0.14\,\Omega$, $L_a = 5\,\text{mH}$, efficiency without exciter power: $\eta_N = 86\%$.
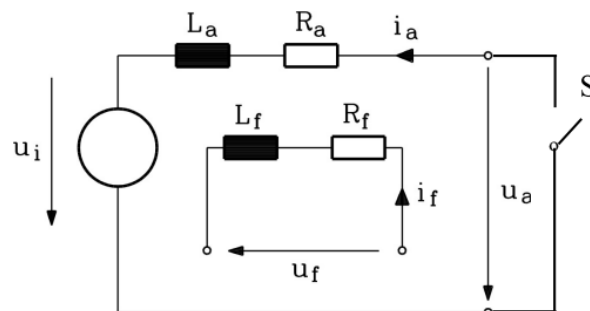

Fig. 5.6-1: Electrical equivalent circuit of a separately excited DC generator, whose armature winding is short-circuited at the brushes via the switch S.

1)   When the generator is accidently switched off the DC grid at rated electrical power, the turbine accelerates the DC generator to the turbine no-load speed. Therefore an additional flywheel inertia J_F has to be added to reduce the dn/dt. The closing of the turbine water main valve takes 5 s to put the turbine driving torque at the generator shaft to $M_s = 0$. Give the needed J_F, so that a $T = 5$ s the speed is limited to $1.5 \cdot n_N$!

2)   Determine the diameter d and mass m of the flywheel J_F as a steel disc with an axial length $l = d/2$ for J_F! ($\gamma_{Fe} = 7850\ \text{kg/m}^3$, $J = \gamma \cdot (d/2)^4 \cdot l \cdot (\pi/2)$)

3)   For a fast speed down of the turbine-generator set the generator "short-circuit brake" is used. For that first the turbine water main valve is closed to put the driving torque $M_s = 0$ and the generator is switched off the grid: $I_a = 0$. Due to the small friction torque the speed n is (nearly) not changed $n = n_N$. Then at $t = 0$ the DC generator armature winding is short-circuited via a short-circuit switch S (Fig. 5.6-1), while the separately excited main flux Φ is still active. It induces the armature winding, so that the short-circuit armature current i_a flows. It generates with Φ the electrical braking torque $M_e < 0$, which decelerates the turbine-generator unit fast to zero speed. Give the dynamic equations for $i_a(t)$, $\Omega_m(t)$ and the initial conditions $i_a(0)$, $\Omega_m(0)$! Tends the generator-turbine set to oscillate under dynamical load changes?

4)   Determine $i_a(t)$ with the method of homogeneous and particular solution generally with the conditions of 3). What is its stationary value $i_a(t \to \infty)$?

5)   Determine $\Omega_m(t)$ from 4) as a general formula! Check, if the "short-circuit brake" puts the generator-turbine set to stand-still!

6)   Determine the time constants! Give the peak armature current $i_{a,max}$ with respect to rated current I_{aN}! Give $i_a(t)$, $\Omega_m(t)$ in correct scaling for three long time constants!

1)   Generator switched off the grid, so braking electromagnetic generator torque $M_e = 0$.
$J \cdot d\Omega_m / dt = M_{sN} - M_e = M_{sN}$, $J = J_F + J_{G+T}$,
$\Delta n = (1.5 - 1) \cdot n_N = 0.5 \cdot 645 / 60 = 5.375 / s$, $(J_F + J_{G+T}) \cdot 2\pi \cdot \Delta n / T = M_{sN}$,
$M_{sN} = M_{eN} = P_{G,N} / (\eta_N \cdot 2\pi \cdot n_N) = U_{aN} \cdot I_{aN} / (\eta_N \cdot 2\pi \cdot n_N)$,
$M_{sN} = (460 \cdot 320) / (0.86 \cdot 2\pi \cdot 645 / 60) = 2534\,\text{Nm}$,
$$J_F = \frac{M_{sN} \cdot T}{2\pi \cdot \Delta n} - J_{G+T} = \frac{2534 \cdot 5}{2\pi \cdot 5.375} - 8 = 367\,\text{kg} \cdot \text{m}^2,$$
$J = J_F + J_{G+T} = 367 + 8 = 375\,\text{kg} \cdot \text{m}^2$

2) $d = 2 \cdot 5\sqrt{\dfrac{2 J_F}{\pi \cdot \gamma_{Fe}}} = 2 \cdot 5\sqrt{\dfrac{2 \cdot 367}{\pi \cdot 7850}} = 0.99\,\text{m}$,

$m = \gamma_{Fe} \cdot \dfrac{d^2 \pi}{4} \cdot l = 7850 \cdot \dfrac{0.99^2 \pi}{4} \cdot \dfrac{0.99}{2} = 2991\,\text{kg}$.

3) $0 = u_a(t) = R_a \cdot i_a(t) + L_a \cdot \dfrac{di_a(t)}{dt} + u_i(t) = R_a \cdot i_a(t) + L_a \cdot \dfrac{di_a(t)}{dt} + k_2 \cdot \Phi \cdot \Omega_m(t)$,

$\Omega_m(t) = 2\pi \cdot n(t)$, $J \cdot \dfrac{d\Omega_m}{dt} = \underbrace{m_e(t)}_{<0} - \underbrace{m_s(t)}_{=0} = k_2 \cdot \Phi \cdot i_a(t)$, $\Omega_m(0) = 2\pi \cdot n_N$, $i_a(0) = 0$.

$U_i = U_{aN} + I_{aN} \cdot R_a = 460 + 320 \cdot 0.14 = 504.8\,\text{V}$,

$k_2\Phi = \dfrac{U_i}{\Omega_{mN}} = \dfrac{504.8}{2\pi \cdot (645 / 60)} = 7.474\,\text{Vs}$, $T_a = \dfrac{L_a}{R_a} = \dfrac{0.005}{0.14} = 35.7\,\text{ms}$,

$T_m = \dfrac{J \cdot R_a}{(k_2 \cdot \Phi)^2} = \dfrac{375 \cdot 0.14}{7.474^2} = 939.8\,\text{ms}$, $T_m > 4 T_a = 142.8\,\text{ms}$. No oscillations will

occur, but two time constants T_1, T_2!

4)   Derivative of electrical equation: $R_a \cdot \dfrac{di_a(t)}{dt} + L_a \cdot \dfrac{d^2 i_a(t)}{dt^2} + k_2 \cdot \Phi \cdot \underbrace{\dfrac{d\Omega_m(t)}{dt}}_{k_2 \cdot \Phi \cdot i_a(t) / J} = 0$,

$\dfrac{d^2 i_a(t)}{dt^2} + \dfrac{R_a}{L_a} \cdot \dfrac{di_a(t)}{dt} + \dfrac{(k_2 \cdot \Phi)^2}{J \cdot R_a} \cdot \dfrac{R_a}{L_a} \cdot i_a(t) = 0$, $i_a'' + \dfrac{i_a'}{T_a} + \dfrac{i_a}{T_a T_m} = 0$,

$i_{ah} = C_1 \cdot e^{\lambda_1 t} + C_2 \cdot e^{\lambda_2 t}$, $i_{ap} = 0$, $\lambda^2 + \dfrac{\lambda}{T_a} + \dfrac{1}{T_a T_m} = 0$, $1 - \dfrac{4 T_a}{T_m} > 0$,

$\lambda_{1,2} = -\dfrac{1}{2 T_a} \pm \dfrac{1}{2 T_a} \cdot \sqrt{1 - \dfrac{4 T_a}{T_m}}$, $T_1 = -\dfrac{1}{\lambda_1} = \dfrac{2 T_a}{1 - \sqrt{1 - \dfrac{4 T_a}{T_m}}}$, $T_2 = -\dfrac{1}{\lambda_2} = \dfrac{2 T_a}{1 + \sqrt{1 - \dfrac{4 T_a}{T_m}}}$,

$i_a = i_{ah} + i_{ap}$, $i_a(0) = 0 = C_1 + C_2 \Rightarrow C_1 = -C_2$,

$0 = R_a \cdot \underbrace{i_a(0)}_{=0} + L_a \cdot \dfrac{di_a(0)}{dt} + k_2 \cdot \Phi \cdot \underbrace{\Omega_m(0)}_{\Omega_{mN}} \to i_a'(0) = -k_2 \cdot \Phi \cdot \Omega_{mN} / L_a$,

$$i_a' = \lambda_1 C_1 \cdot e^{\lambda_1 t} + \lambda_2 C_2 \cdot e^{\lambda_2 t} \rightarrow i_a'(0) = \lambda_1 C_1 + \lambda_2 C_2 = C_1 \cdot (\lambda_1 - \lambda_2),$$

$$C_1 = \frac{-k_2 \cdot \Phi \cdot \Omega_{mN} / L_a}{\lambda_1 - \lambda_2} = \frac{-k_2 \cdot \Phi \cdot \Omega_{mN} / L_a}{\frac{1}{T_a} \cdot \sqrt{1 - \frac{4T_a}{T_m}}} = -\frac{U_i / R_a}{\sqrt{1 - \frac{4T_a}{T_m}}} = -I_{a,th},$$

$$i_a = -I_{a,th} \cdot \left(e^{-t/T_1} - e^{-t/T_2}\right), \text{ Check: } i_a(0) \sim e^{-0/T_1} - e^{-0/T_2} = 1 - 1 = 0.$$

$$i_a(t \rightarrow \infty) \sim e^{-\infty/T_1} - e^{-\infty/T_2} = 0 - 0 = 0: \text{ The stationary current value is zero!}$$

5) $$R_a \cdot i_a(t) + L_a \cdot \frac{di_a(t)}{dt} + k_2 \cdot \Phi \cdot \Omega_m(t) = 0, \quad \Omega_m(t) = \frac{-1}{k_2 \cdot \Phi} \cdot \left(R_a \cdot i_a + L_a \cdot i_a'\right),$$

$$\Omega_m(t) = \frac{\Omega_{mN}}{\sqrt{1 - (4T_a / T_m)}} \cdot \left(e^{-t/T_1} - e^{-t/T_2} - T_a \cdot (\frac{e^{-t/T_1}}{T_1} - \frac{e^{-t/T_2}}{T_2})\right),$$

$$\Omega_m(t) = \frac{\Omega_{mN}}{\sqrt{1 - (4T_a / T_m)}} \cdot \left(e^{-t/T_1} \cdot (1 - \frac{T_a}{T_1}) - e^{-t/T_2} \cdot (1 - \frac{T_a}{T_2})\right),$$

$$\frac{\Omega_m(t)}{\Omega_{mN}} = \frac{e^{-t/T_1}}{2} \cdot (\frac{1}{\sqrt{1 - (4T_a / T_m)}} + 1) - \frac{e^{-t/T_2}}{2} \cdot (\frac{1}{\sqrt{1 - (4T_a / T_m)}} - 1),$$

$$\text{Check: } \Omega_m(0) = \frac{\Omega_{mN}}{2} \cdot \left(\frac{1}{\sqrt{1 - (4T_a / T_m)}} + 1 - \frac{1}{\sqrt{1 - (4T_a / T_m)}} + 1\right) = \Omega_{mN},$$

$$\Omega_m(t \rightarrow \infty) = \Omega_{mN} \cdot \left(e^{-\infty} \cdot (\frac{1}{\sqrt{1 - (4T_a / T_m)}} + 1) - e^{-\infty} \cdot (\frac{1}{\sqrt{1 - (4T_a / T_m)}} - 1)\right) = 0.$$

The stationary speed value is zero, so the turbine-generator has been put to stand-still!

6) $$T_1 / T_2 = \frac{2T_a}{1 \mp \sqrt{1 - \frac{4T_a}{T_m}}} = \frac{2 \cdot 35.7}{1 \mp \sqrt{1 - \frac{4 \cdot 35.7}{939.8}}} = 902.63 \text{ ms} / 37.17 \text{ ms}.$$

Braking time is ca. five long time constants $5T_1 \approx 4.5$ s, so very short. This needs a huge current peak $i_{a,max}$ for a big braking torque.

$$i_a'(t) = I_{a,th} \cdot \left(\frac{e^{-t*/T_1}}{T_1} - \frac{e^{-t*/T_2}}{T_2}\right) = 0 \Rightarrow t* = \frac{\ln(T_1 / T_2)}{\frac{1}{T_2} - \frac{1}{T_1}} = 123.66 \text{ ms},$$

$$\frac{i_a(t*)}{I_{a,th}} = \frac{i_{a,max}}{I_{a,th}} = -\left(e^{-t*/T_1} - e^{-t*/T_2}\right) = -\left(e^{-0.137} - e^{-3.327}\right) = -0.836,$$

$$I_{a,th} = \frac{U_i / R_a}{\sqrt{1 - \frac{4T_a}{T_m}}} = \frac{504.8 / 0.14}{\sqrt{1 - \frac{4 \cdot 35.7}{939.8}}} = 3915.4 \text{ A}.$$

Current peak at $t*$ with $i_{a,max} = -0.836 \cdot I_{a,th} = -3273.5 \text{ A} = -10.23 \cdot I_{aN}$. The current (Fig. 5.6-2a) is negative with respect to the positive direction in Fig. 5.6-1, as it is a generating current. The kinetic energy is transformed into heat in R_a. Plot in Fig. 5.6-2 for three long time constants $3T_1 = 3 \cdot 902.63 \text{ ms} = 2708 \text{ ms}$.

a)

b)
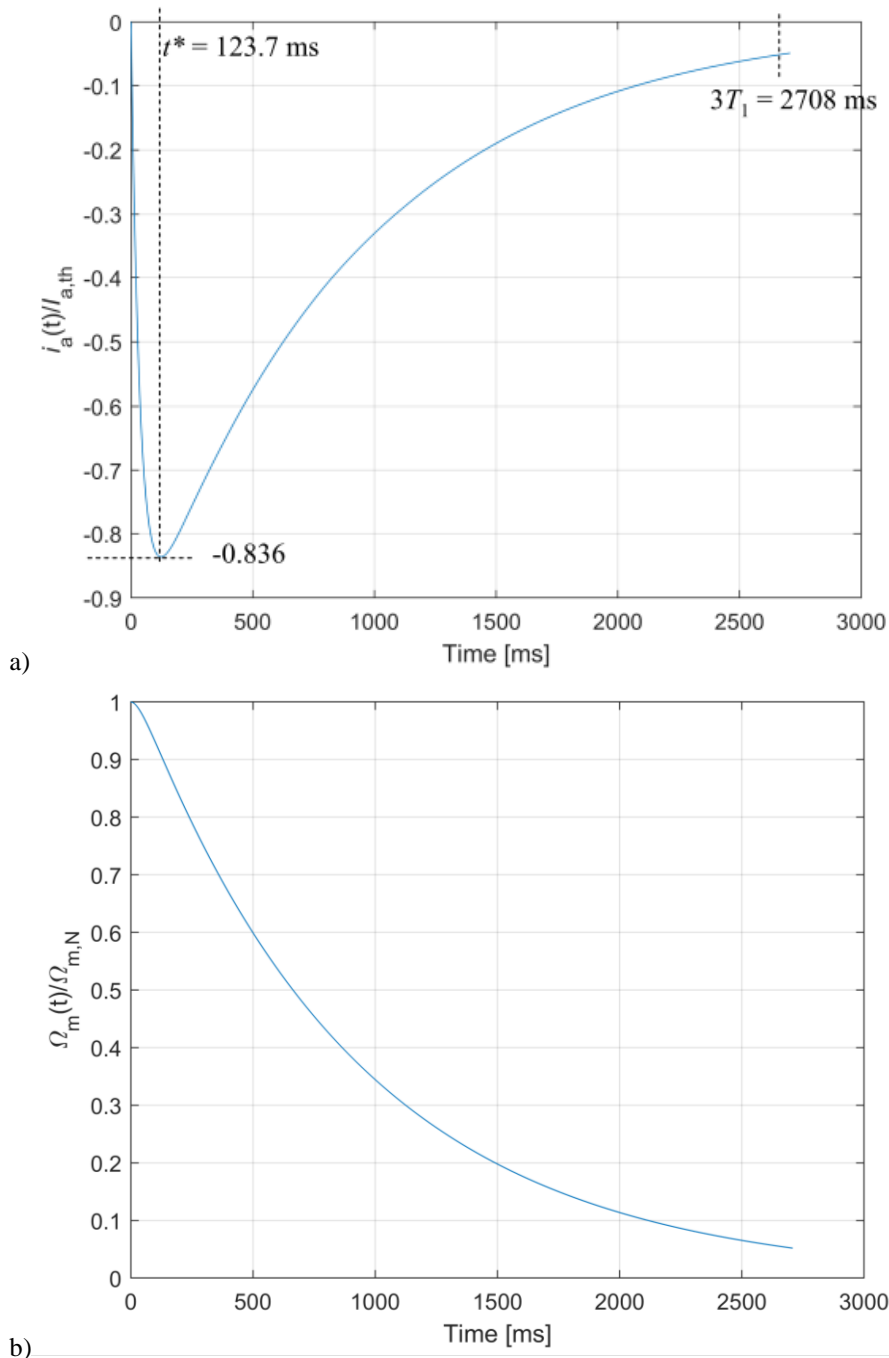
Fig. 5.6-2: a) Armature current during braking. b) Speed of generator-turbine set during braking.

Practical advice:

$i_{a,max} = -10.23 \cdot I_{aN}$ is too big. It stresses the generator both mechanically and thermally too much. Therefore the short-circuit is done with an additional series resistance R_v to limit current peak to $-2 \cdot I_{aN}$:

$R_v = 4R_a : R_a \rightarrow R = R_v + R_a = 5R_a = 0.7\,\Omega$, $T_a \rightarrow T_a/5$, $T_m \rightarrow 5T_m$.

$T_1/T_2 = 4691.8\,\text{ms}/7.14\,\text{ms}$, $t* = 46.45\,\text{ms}$, $U_i = 504.8\,\text{V} : I_{a,th} = 723.3\,\text{A}$,

$i_a(t*)/I_{a,th} = i_{a,max}/I_{a,th} = -\left(e^{-0.0099} - e^{-6.4962}\right) = -0.9886$,

$i_{a,max} = -0.9886 \cdot 723.3 = -715.08\,\text{A} = -2.23 \cdot I_{aN}$.

<u>Result:</u> A five times smaller current brakes the set with five times less torque. So the long time constant T_1 increases by $4691.8/902.63 = 5.2$ to take for braking ca. $5T_1 \approx 23.46\,\text{s} \approx 0.39\,\text{min}$, which is a realistic value and still much faster than braking only by friction, which would take several hours to transform the stored kinetic energy $W_k = J \cdot (2\pi \cdot n_N)^2/2 = 375 \cdot (2\pi \cdot (645/60))^2/2 = 855.4\,\text{kJ} = 0.238\,\text{kWh}$ into friction heat.

Simplified treatment of short-circuit braking of a DC generator

A separately excited DC generator G is driven by an *Ossberger* water turbine T in a small hydro power plant at n_N and supplies a local DC grid in a rural district (Fig. 5.6-1).

DC Generator data: $U_{aN} = 460\,\text{V}$, $n_N = 645/\text{min}$, $I_{aN} = 320\,\text{A}$, $J_{G+T} = 8\,\text{kg}\cdot\text{m}^2$,

$R_a = 0.14\,\Omega$, $L_a = 5\,\text{mH}$, efficiency without exciter power: $\eta_N = 86\%$.

An additional flywheel inertia $J_F = 367\,\text{kg}\cdot\text{m}^2$ is coupled the generator set to limit its speeding up, if the generator is accidently switched off the DC grid ($M_e = 0$), and the still operating turbine is speeding via its turbine shaft torque M_s.

For a fast speed down of the turbine-generator set the generator "short-circuit brake" is used. For that first the turbine water main valve is closed to put the driving torque $M_s = 0$, and the generator is switched off the grid: $I_a = 0$, $M_e = 0$. Due to the small friction torque the speed n is (nearly) not changed $n = n_N$. Then at $t = 0$ the DC generator armature winding is short-circuited via a short-circuit switch S (Fig. 5.6-1), while the separately excited main flux Φ is still active. It induces the armature winding, so that the short-circuit armature current i_a flows. It generates with Φ the electrical braking torque $M_e < 0$, which decelerates the turbine-generator unit fast to zero speed.

1)   Give the dynamic equations for $i_a(t)$, $\Omega_m(t)$ and the initial conditions $i_a(0)$, $\Omega_m(0)$! Tends the generator-turbine set to oscillate under dynamical load changes?

2)   Neglect in the following L_a and determine $\Omega_m(t)$ and $i_a(t)$ with the method of homogeneous and particular solution generally with the conditions of 3). What is its stationary value $i_a(t \to \infty)$?

3)   Determine the time constant! Give the peak armature current $i_{a,max}$ with respect to rated current I_{aN}!

4)   Give the sketch $i_a(t)$, $\Omega_m(t)$ in correct scaling for three time constants! Show that the "short-circuit brake" puts the generator-turbine set to stand-still in roughly three time constants!

5)   Which additional armature series resistance R_v is necessary in the short-circuit to limit the current peak $i_{a,max}$ to 2.5·I_{aN}? How much is the braking time increased by R_v?

6)   Give the kinetic energy W_k in the rotating generator-turbine set. Show that it is dissipated during braking completely as I^2R-heat in the resistances $R_v + R_a$!

1) $\quad 0 = u_a(t) = R_a \cdot i_a(t) + L_a \cdot \dfrac{di_a(t)}{dt} + u_i(t) = R_a \cdot i_a(t) + L_a \cdot \dfrac{di_a(t)}{dt} + k_2 \cdot \Phi \cdot \Omega_m(t)$,

$\quad \Omega_m(t) = 2\pi \cdot n(t)$, $\quad J \cdot \dfrac{d\Omega_m}{dt} = \underbrace{m_e(t)}_{<0} - \underbrace{m_s(t)}_{=0} = k_2 \cdot \Phi \cdot i_a(t)$, $\quad \Omega_m(0) = 2\pi \cdot n_N$, $\quad i_a(0) = 0$.

$\quad U_i = U_{aN} + I_{aN} \cdot R_a = 460 + 320 \cdot 0.14 = 504.8 \text{ V}$,

$\quad k_2\Phi = \dfrac{U_i}{\Omega_{mN}} = \dfrac{504.8}{2\pi \cdot (645/60)} = 7.474 \text{ Vs}$, $\quad T_a = \dfrac{L_a}{R_a} = \dfrac{0.005}{0.14} = 35.7 \text{ ms}$,

$\quad T_m = \dfrac{J \cdot R_a}{(k_2 \cdot \Phi)^2} = \dfrac{375 \cdot 0.14}{7.474^2} = 939.8 \text{ ms}$, $\quad T_m > 4T_a = 142.8 \text{ ms}$. No oscillations will

occur, but two time constants T_1, T_2!

2) $\quad$ Neglecting L_a:

$\quad 0 = R_a \cdot i_a(t) + k_2 \cdot \Phi \cdot \Omega_m(t) = R_a \cdot i_a(t) + k_2 \cdot \Phi \cdot \Omega_m(t) \Rightarrow i_a = -k_2 \cdot \Phi / R_a$,

$\quad J \cdot \dfrac{d\Omega_m}{dt} + k_2 \cdot \Phi \cdot \dfrac{k_2 \cdot \Phi}{R_a} \cdot \Omega_m = 0$, $\quad \dfrac{d\Omega_m}{dt} + \dfrac{\Omega_m}{T_m} = 0$.

$\quad \Omega_m = \Omega_{mh} + \Omega_{mp}$, $\quad \Omega_{mh} = C \cdot e^{\lambda \cdot t}$, $\quad \Omega_{mp} = 0$, $\quad \lambda = -1/T_m$, $\quad \Omega_m = \Omega_{mh}$,

$\quad \Omega_m(0) = C = \Omega_{mN}$, $\quad \Omega_m(t) = \Omega_{mN} \cdot e^{-t/T_m}$,

$\quad i_a(t) = \underbrace{-\dfrac{k_2 \cdot \Phi \cdot \Omega_{mN}}{R_a}}_{-U_i/R_a} \cdot e^{-t/T_m}$ for $t > 0$, $\quad i_a(t) = -\dfrac{U_i}{R_a} \cdot e^{-t/T_m}$.

3) $\quad$ Due to neglecting L_a, only one time constant occurs, which is the mechanical time

constant of 1): $T_m = \dfrac{J \cdot R_a}{(k_2 \cdot \Phi)^2} = \dfrac{375 \cdot 0.14}{7.474^2} = 939.8 \text{ ms}$.

Due to neglecting L_a the current may jump from 0 at $t = 0$- to $-U_i/R_a$ at $t = 0$+,

which is the peak current: $i_a(0+) = -U_i/R_a = -504.8/0.14 = -3605.7 \text{ A} = -11.27 \cdot I_{aN}$,

$i_{a,max} = |-11.27 \cdot I_{aN}| = 11.27 \cdot I_{aN}$.

4) $\quad \Omega_m(3T_m) = \Omega_{mN} \cdot e^{-3} = 0.05 \cdot \Omega_{mN} \approx 0$.

Sketches of $i_a(t)$, $\Omega_m(t)$ for three time constants see Fig. 5.7-1.

5) $\quad 2.5 \cdot I_{aN} = |-U_i/(R_a + R_v)| = U_i/(R_a + R_v)$,

$\quad R_v = \dfrac{U_i}{2.5 \cdot I_{aN}} - R_a = \dfrac{504.8}{2.5 \cdot 320} - 0.14 = 0.491 \Omega \Rightarrow$ Time constant $T_m^* = \dfrac{J \cdot (R_a + R_v)}{(k_2 \cdot \Phi)^2}$.

Braking time $\approx 3T_m^* \sim R_a + R_v$. Increase of braking time $\dfrac{R_a + R_v}{R_a} = \dfrac{0.14 + 0.491}{0.14} = 4.5$.

6) $\quad W_k = J \cdot (2\pi \cdot n_N)^2 / 2 = 375 \cdot (2\pi \cdot (645/60))^2 / 2 = 855.4 \text{ kJ} = 0.238 \text{ kWh}$,

<u>Exercise 5.7</u>   5.7/3

$$W_{\mathrm{Cu}} = \int_0^\infty (R_{\mathrm{a}} + R_{\mathrm{v}}) \cdot i_{\mathrm{a}}^2 \cdot dt = \int_0^\infty \frac{U_{\mathrm{i}}^2 \cdot e^{-2t/T_{\mathrm{m}}^*}}{R_{\mathrm{a}} + R_{\mathrm{v}}} \cdot dt = -\frac{U_{\mathrm{i}}^2 \cdot T_{\mathrm{m}}^* \cdot e^{-2t/T_{\mathrm{m}}^*}}{2 \cdot (R_{\mathrm{a}} + R_{\mathrm{v}})}\bigg|_0^\infty = \frac{U_{\mathrm{i}}^2 \cdot T_{\mathrm{m}}^*}{2 \cdot (R_{\mathrm{a}} + R_{\mathrm{v}})} ,$$

$$W_{\mathrm{Cu}} = \frac{U_{\mathrm{i}}^2}{2 \cdot (R_{\mathrm{a}} + R_{\mathrm{v}})} \cdot \frac{J \cdot (R_{\mathrm{a}} + R_{\mathrm{v}})}{(k_2 \cdot \Phi)^2} = \frac{J \cdot \Omega_{\mathrm{mN}}^2}{2} = W_{\mathrm{k}} .$$

a)

b)
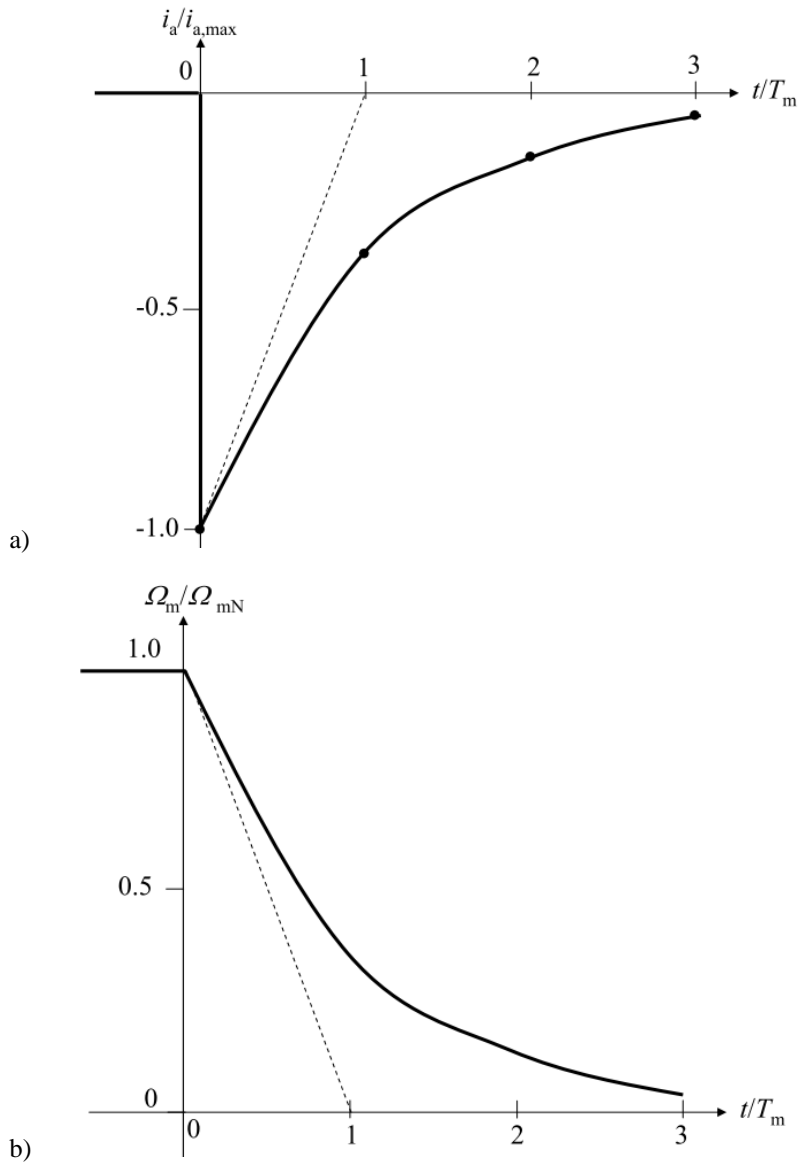
<u>Fig. 5.7-1:</u> $L_{\mathrm{a}} = 0$: a) Armature current during braking. b) Speed of generator-turbine set during braking.

Space vector representation of an induction machine:

An induction machine is supplied by a voltage-source inverter (with DC voltage link) and is operated with field – oriented control. The machine is running at nominal values, this means, that the inverter provides nominal voltage and nominal frequency, the machine is loaded with the rated torque. Due to the high motor frequency, the stator resistance is negligible.

Motor data:

$x_s = 3 \text{p.u.}$; $x_r' = 2.95 \text{p.u.}$; *Blondel's* leakage coefficient $\sigma = 0.08$.

rotor time constant: $\tau_r' = \dfrac{x_r'}{r_r'} = 100$; $\cos\varphi_N = 0.87$;

nominal frequency 75 Hz, 6-pole machine

1.) Calculate and draw the space vector of the stator voltage, of the stator and rotor current, of the stator and rotor flux for the nominal values in the synchronous reference frame.

2.) Calculate the torque that is produced by the motor in p.u.. Draw the proper rectangle area in the space vector diagram, which represents that torque!

3.) What are the values for the slip and the speed of the motor?

1) In the synchronous reference frame the space vector is not moving in stationary condition.

Nominal voltage: $\underline{u}_s = 1\,\text{p.u.}$ (voltage space vector is assumed to lie on real axis of reference frame).

Nominal frequency: $\omega_s = 1$

$$\underline{u}_s = r_s \cdot \underline{i}_s + \frac{d\underline{\psi}_s}{d\tau} + j\omega_s \underline{\psi}_s$$

Stationary: $\frac{d\underline{\psi}_s}{d\tau} = 0$; $r_s \cdot \underline{i}_s = 0$ is negligible because of the high frequency; $\omega_s = 1 \Rightarrow$

$u_s = j\omega_s \psi_s = j\psi_s$; $\psi_s = -ju_s = -j\,\text{p.u.}$

$\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}_r'$; $x_h = \sqrt{(1-\sigma)x_s x_r'} = \sqrt{(1-0.08)\cdot 3 \cdot 2.95} = 2.85\,\text{p.u.}$

Rated operation: $\left|\underline{i}_s\right| = 1\,\text{p.u.}$ with phase angle according to $\cos\varphi_N$

$\underline{i}_s = \left|\underline{i}_s\right| \cdot e^{-j\varphi_N} = 1 \cdot e^{-j\varphi_N} = \cos\varphi_N - j\sin\varphi_N = 0.87 - j0.493$

$\sin\varphi_N = \sqrt{-\cos^2\varphi_N + 1} = 0.493$

$\underline{u}_s = u_s = j(x_s \cdot \underline{i}_s + x_h \cdot \underline{i}_r') \Rightarrow \left(-ju_s - x_s\underline{i}_s\right)\frac{1}{x_h} = \underline{i}_r'$

$\underline{i}_r' = \left(-j\cdot 1 - 3\cdot(0.87 - j0.493)\right)\frac{1}{2.85} = -0.916 + j0.168\,\text{p.u.}$

$\underline{\psi}_r = x_h \cdot \underline{i}_s + x_r' \cdot \underline{i}_r' = 2.85\cdot(0.87 - j0.493) + 2.95\cdot(-0.916 + j0.168) =$

$= -0.223 - j0.909\,\text{p.u.}$

$\left|\psi_s\right| = 1\,\text{p.u.}$; $\left|\underline{i}_r'\right| = 0.93\,\text{p.u.}$; $\left|\underline{\psi}_r'\right| = 0.936\,\text{p.u.}$; $\left|\underline{u}_s\right| = \left|\underline{i}_s\right| = 1\,\text{p.u.}$
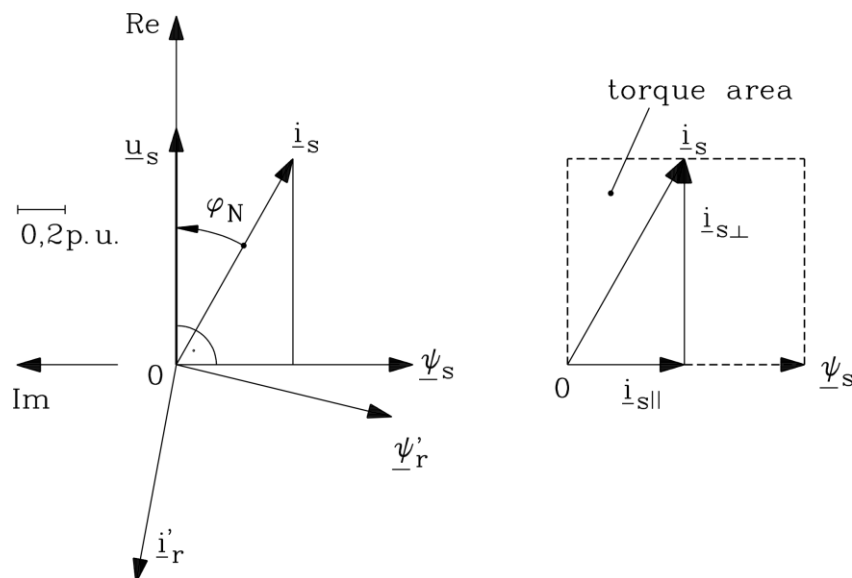

Fig. 6.1-1: Induction machine: Left: Space vectors at steady state motor operation, Right: Corresponding torque

2) $m_e = \text{Im}(\underline{i}_s \cdot \underline{\psi}_s^*) = i_{s\perp} \cdot \psi_s = 0.87 \cdot 1 = 0.87\,\text{p.u.}$

$i_{s\perp} = 0.87\,\text{p.u.}$

Note: $\dfrac{M_{eN}}{M_{ref}} \neq 1$, because $M_{ref} = \dfrac{3 \cdot U_{NPh} \cdot I_N}{\dfrac{\omega_N}{p}}$; $P_\delta = \Omega_{syn} \cdot M_{eN}$

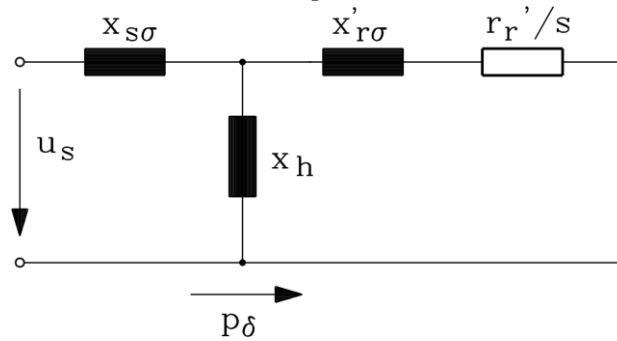

Fig. 6.1-2: Induction machine: Equivalent circuit with neglected stator resistance

$$\frac{P_\delta}{S_N} = p_\delta = \frac{\Omega_{syn} \cdot M_{eN}}{3 \cdot U_{PhN} \cdot I_N} = \frac{M_{eN}}{\dfrac{3 \cdot U_{NPh} \cdot I_N}{\omega_N / p}} = \frac{M_{eN}}{M_{ref}} => m_e = p_\delta$$

$$P_\delta = 3 \cdot U_{NPh} \cdot I_N \cdot \cos\varphi_N => \left(R_s \cdot I_s^2 \sim 0\right) => p_\delta = \cos\varphi_N = m_e = 0.87\,!$$

3) $3 \cdot R_r' \cdot I_r'^2 = P_{Cu,r} = s \cdot P_\delta => \dfrac{3 \cdot R_r' \cdot I_r'^2}{S_N} = r_r' \cdot i_r'^2 = s \cdot p_\delta$

$$s = \frac{r_r' \cdot i_r'^2}{p_\delta} = \frac{0.0295 \cdot 0.932}{0.87} = 0.0293$$

$$\tau_r = \frac{x_r'}{r_r'} => r_r' = \frac{x_r'}{\tau_r} = \frac{2.95}{100} = 0.0295\,\text{p.u.}$$

$$\frac{\Omega_{syn} - \Omega_m}{\Omega_{syn}} = s, \quad n = \frac{\Omega_m}{2\pi} = \frac{152.48}{2\pi} = 24.27\,\text{s}^{-1}$$

$$\Omega_{syn} = \frac{2\pi f_N}{p} = 2\pi \cdot \frac{75}{3} = 157.08\,\text{s}^{-1}; \quad \Omega_m = 152.48\,\text{s}^{-1}$$

Rotational speed: $n = 1456/\min$

Exercise 6.2   6.2/1

Current feeding of a locked rotor induction machine:

A locked rotor induction machine is fed by a current controlled inverter. At time $t = 0$ the de-energized machine is abruptly fed by a stator current space vector $\underline{i}_s(\tau) = i_s \cdot e^{j0°}$, $i_s = 1$ p.u.

Motor data: (p.u. values, phase values, star-connected stator winding)

Stator resistance: $r_s = 0.04$ p.u.

Stator inductance: $x_s = 3.0$ p.u.

Rotor inductance: $x_r^{'} = 3.0$ p.u.

Rotor open-circuit time constant: $\tau_r = \dfrac{x_r^{'}}{r_r^{'}} = 60$

Blondel's leakage coefficient: $\sigma = 0.07$

1.)   Draw the circuit state of the inverter and the current path through the machine for the given current space vector.

2.)   Calculate the time characteristic of the rotor flux linkage space vector after the current step and draw the time function of the current. Use the stator reference frame!
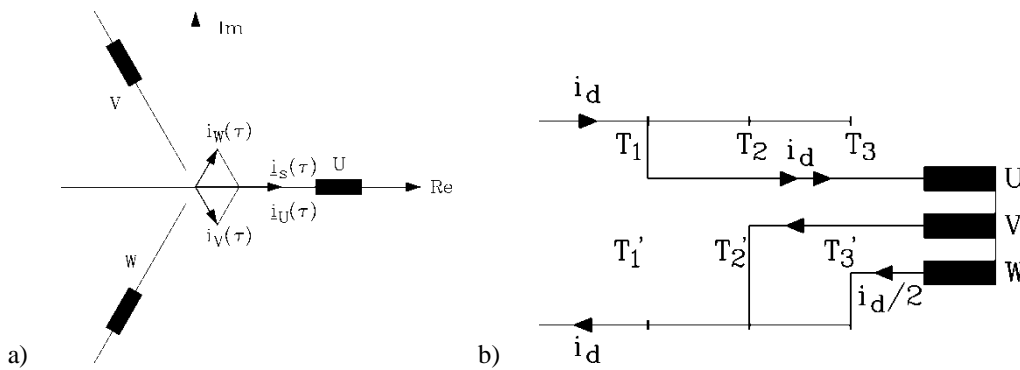
1.)

<u>Fig. 6.2-1:</u> a) Space vector decomposition in phase current vectors, b) Current flow at inverter operation

$$\underline{i}_s = \left|\underline{i}_s\right| \cdot e^{j0°}, \quad \underline{i}_U = \mathrm{Re}\left(\underline{i}_s\right) = i_s, \quad \underline{i}_V = \mathrm{Re}\left(\underline{a}^2 \cdot \underline{i}_s\right) = \mathrm{Re}\left(e^{j\frac{4\pi}{3}} \cdot \underline{i}_s\right) = -0.5 \cdot i_s$$

$$\underline{i}_W = \mathrm{Re}\left(\underline{a} \cdot \underline{i}_s\right) = \mathrm{Re}\left(e^{j\frac{2\pi}{3}} \cdot \underline{i}_s\right) = -0.5 \cdot i_s$$

Transistors T1, T2', T3' conducting, T1', T2, T3 switched off:

$i_s = i_d$: DC-current:

$$\underline{i}_s = \frac{2}{3} \cdot \left(i_d + \underline{a} \cdot \left(-\frac{1}{2}i_d\right) + \underline{a}^2 \cdot \left(-\frac{1}{2}i_d\right) \right) = \frac{2}{3} \cdot i_d\left(1 - \frac{1}{2}\left(-0.5 + j\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(-0.5 - j\frac{\sqrt{3}}{2}\right)\right)$$

$$\underline{i}_s = \frac{2}{3} \cdot i_d\left(1 + \frac{1}{4} + \frac{1}{4}\right) = \frac{2}{3} \cdot \frac{3}{2} i_d = i_d$$

2.)   Stator reference frame is used, because the rotor is locked ($\omega_m = 0$)

$$\underline{u}_s = r_s \cdot \underline{i}_s + \frac{d\underline{\psi}_s}{d\tau}; \quad 0 = r_r' \cdot \underline{i}_r' + \frac{d\underline{\psi}_r'}{d\tau} - j\omega\underline{\psi}_r' \quad (0 = j\omega\underline{\psi}_r')$$

$$\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}_r'; \quad \underline{\psi}_r' = x_h \cdot \underline{i}_s + x_r' \cdot \underline{i}_r'$$

$$x_h = \sqrt{(1-\sigma)x_s x_r'} = \sqrt{(1-0.07)} \cdot 3 = 2.89 \mathrm{p.u.}$$

The interest is on the rotor voltage equation to calculate of the rotor flux.

$$0 = r_r' \cdot \underline{i}_r' + \frac{d\underline{\psi}_r'}{d\tau}; \quad \underline{i}_r' = (\underline{\psi}_r' - x_h \cdot \underline{i}_s)\frac{1}{x_r'}$$

$$0 = r_r' \cdot (\underline{\psi}_r' - x_h \cdot \underline{i}_s)\frac{1}{x_r'} + \frac{d\underline{\psi}_r'}{d\tau}; \quad \tau_r = \frac{x_r'}{r_r'}$$

$$\frac{d\underline{\psi}'_r}{d\tau} + \frac{1}{\tau_r}\underline{\psi}'_r = \frac{1}{\tau_r}x_h \cdot \underline{i}_s \quad \text{first order differential equation}$$

Initial conditions: $\tau = 0$; $\underline{\psi}'_r = 0$

Homogeneous solution: $\underline{\psi}'_{r,hom}(\tau) = \underline{C} \cdot e^{-\frac{\tau}{\tau_r}}$

Particular solution: $\underline{\psi}'_{r,part} = x_h \cdot \underline{i}_s$

$$\Rightarrow \underline{C} = -x_h \cdot \underline{i}_s : \underline{\psi}'_r(\tau) = \underline{\psi}'_{r,hom} + \underline{\psi}'_{r,part} = x_h \cdot \underline{i}_s \left(1 - e^{-\frac{\tau}{\tau_r}}\right)$$
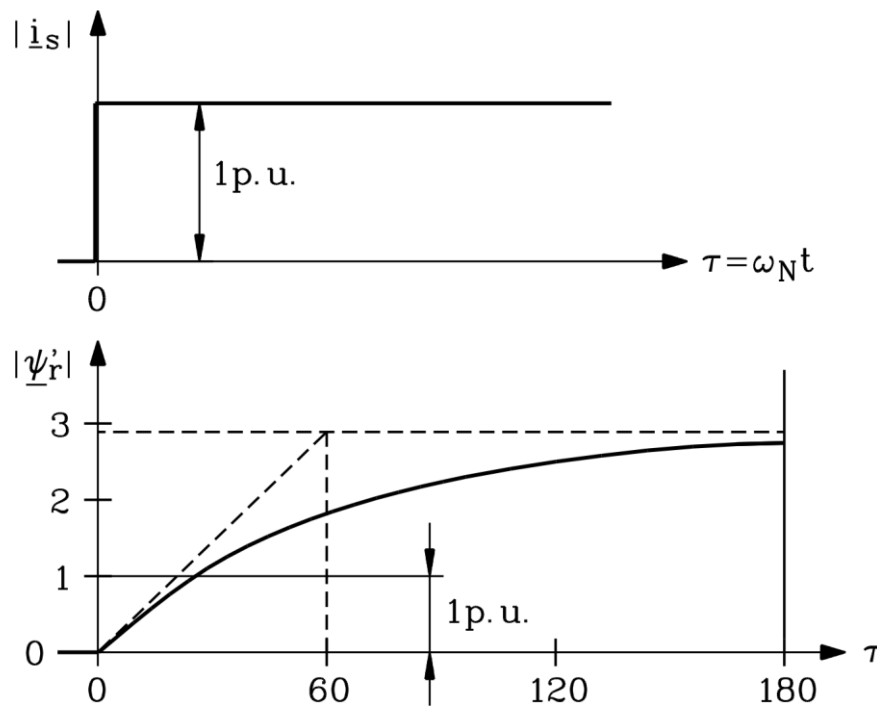
Fig. 6.2-2: Phase current (above) and rotor flux (below) at rotor stand still

Note: The nominal current is injected, therefore the machine is magnetised to 289 % of the nominal flux. A real machine would be high saturated. In this case it is assumed, that the saturation is independent of the current. A current controlled machine is normally at locked rotor only current fed to the nominal flux. This means, current has to be reduced down to: $|\underline{i}_s| \cong \frac{1}{x_s} = 0.33 \, \text{p.u.}$!

Squirrel-cage induction motor at voltage source inverter supply (no-load operation)

A squirrel-cage induction motor with stator winding delta connected is operated from voltage source inverter with block mode (see picture).

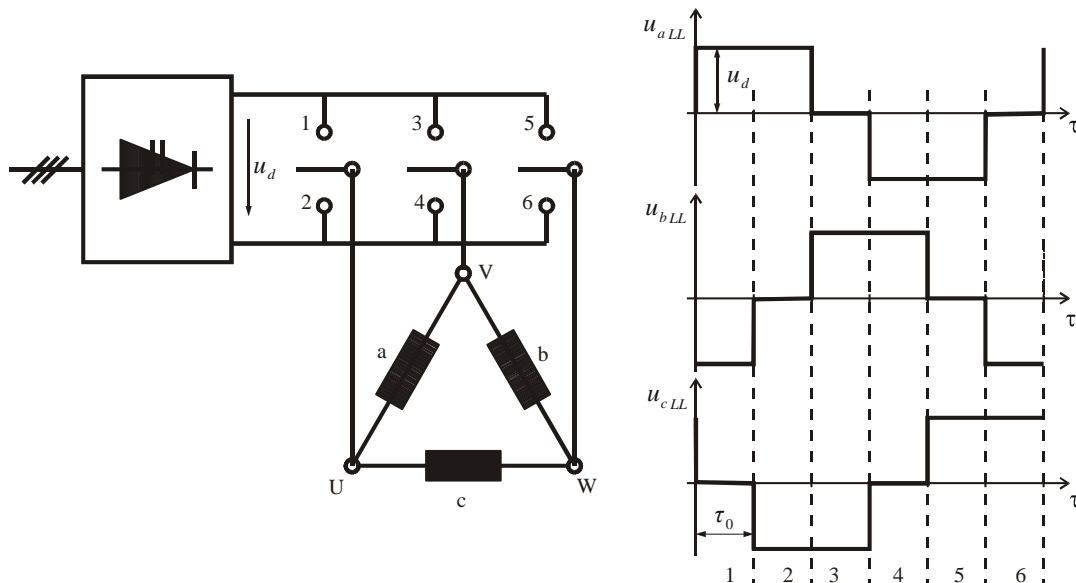Fig. 6.3-1: Left: Schematic sketch of the inverter and stator winding, right: Inverter output line-to-line voltage

Motor rated data:
$P_N = 110$ kW, $U_N = 400$ V, $I_N = 212$ A, $\cos\varphi_N = 0.85$, $f_N = 50$ Hz, $2p = 4$
Stator inductance per phase: $L_s = 29.13$ mH, stator resistance can be neglected ($R_s = 0$).

Inverter data: DC link voltage $U_d = 565$ V, output frequency 50 Hz

1)   Calculate per unit value (p.u.) the DC link voltage $u_d = U_d/U_{ref}$ (U_{ref}: reference voltage) and the stator inductance x_s. Determine in p.u. the time interval τ_0 at 50 Hz and 100 Hz output frequency!

2)   Calculate and draw the position of the stator voltage space vector $\underline{u}_s$ for the six time intervals per period at 50 Hz and determine the absolute value of the complex vector.
       Note: Calculate $\underline{u}_s$ only for the time interval 1, 2 and 3. Determine $\underline{u}_s$ for the time interval 4, 5 and 6 from symmetry of the voltage waveform.

3)   Determine the equations of stator voltage $\underline{u}_s$ and flux linkage $\underline{\psi}_s$ in stator fixed co-ordinate system for the space vector representation.

4)   The motor is operated at no-load. Calculate the time function of stator phase current in steady state condition for the time intervals 1, 2 and 3. For the determination of the time interval 4, 5 and 6 use the symmetry of voltage waveform.
       Note: The current cannot change as abruptly as the voltage does.

5)   Draw to scale (in p.u. and in SI-units) the time function of the phase voltage u_a and the phase current i_a ! Determine the phase angle between the Fourier fundamental of current and voltage time function.

1) $U_{\text{ref}} = \sqrt{2} \cdot U_{\text{N,ph}};$ delta connection: $U_{\text{N,ph}} = U_{\text{N}} = 400\,\text{V} \Rightarrow U_{\text{ref}} = 565\,\text{V}$

$$I_{\text{N,ph}} = I_{\text{N}} / \sqrt{3} = 212 / \sqrt{3} = 122.4\,\text{V}$$

$$u_{\text{d}} = \frac{U_{\text{d}}}{U_{\text{ref}}} = \frac{565}{\sqrt{2} \cdot 400} = \underline{\underline{1}}$$

$$Z_{\text{N}} = \frac{U_{\text{N,ph}}}{I_{\text{N,ph}}} = \frac{400}{122.4} = 3.27\,\Omega$$

$$x_{\text{s}} = \frac{\omega_{\text{N}} \cdot L_{\text{s}}}{Z_{\text{N}}} = \frac{2 \cdot \pi \cdot 50 \cdot 29.13 \cdot 10^{-3}}{3.27} = \underline{\underline{2.8 \ \text{p.u.}}}$$

$$\tau = \omega_{\text{N}} \cdot t: \text{ at } f = 50\,\text{Hz} \Rightarrow T = \frac{1}{f} = \frac{1}{50} = 20\,\text{ms}: \ \tau_0 = \omega_{\text{N}} \cdot \frac{T}{6} = \frac{2 \cdot \pi \cdot 50 \cdot 1}{50 \cdot 6} = \underline{\underline{\frac{\pi}{3}}}$$

$$\text{at } f = 100\,\text{Hz} \Rightarrow T = \frac{1}{f} = \frac{1}{100} = 10\,\text{ms}: \ \tau_0 = \omega_{\text{N}} \cdot \frac{T}{6} = \frac{2 \cdot \pi \cdot 50 \cdot 1}{100 \cdot 6} = \underline{\underline{\frac{\pi}{6}}}$$

2) $u_{\text{s}} = \frac{2}{3} \cdot (u_{\text{a}} + \underline{a} \cdot u_{\text{b}} + \underline{a}^2 \cdot u_{\text{c}}), \quad \underline{a} = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, \quad \underline{a}^2 = e^{j\frac{4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

time interval	1	2	3	4	5	6
u_{a}	U_{d}	U_{d}	0	$-U_{\text{d}}$	$-U_{\text{d}}$	0
u_{b}	$-U_{\text{d}}$	0	U_{d}	U_{d}	0	$-U_{\text{d}}$
u_{c}	0	$-U_{\text{d}}$	$-U_{\text{d}}$	0	U_{d}	U_{d}

<u>time interval 1</u> (in p.u.): $\underline{u}_{\text{s1}} = \frac{2}{3} \cdot u_{\text{d}} \cdot (1 - \underline{a}) = \frac{2}{3} \cdot u_{\text{d}} \cdot (1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}) = \frac{2}{3} \cdot u_{\text{d}} \cdot (\frac{3}{2} - j\frac{\sqrt{3}}{2})$

$$= \frac{2}{\sqrt{3}} \cdot u_{\text{d}} \cdot (\frac{\sqrt{3}}{2} - j\frac{1}{2}) = \underline{\underline{\frac{2}{\sqrt{3}} \cdot u_{\text{d}} \cdot e^{-j30°}}}$$

<u>time interval 2</u> (in p.u.): $\underline{u}_{\text{s2}} = \frac{2}{3} \cdot u_{\text{d}} \cdot (1 - \underline{a}^2) = \frac{2}{3} \cdot u_{\text{d}} \cdot (1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}) = \frac{2}{3} \cdot u_{\text{d}} \cdot (\frac{3}{2} + j\frac{\sqrt{3}}{2})$

$$= \frac{2}{\sqrt{3}} \cdot u_{\text{d}} \cdot (\frac{\sqrt{3}}{2} + j\frac{1}{2}) = \underline{\underline{\frac{2}{\sqrt{3}} \cdot u_d \cdot e^{j30°}}}$$

<u>time interval 3</u> (in p.u.): $\underline{u}_{\text{s3}} = \frac{2}{3} \cdot u_{\text{d}} \cdot (\underline{a} - \underline{a}^2) = \frac{2}{3} \cdot u_{\text{d}} \cdot \underline{a} \cdot (1 - \underline{a}) = \underline{a} \cdot \underline{u}_{\text{s1}}$

$$= e^{j120°} \cdot \frac{2}{\sqrt{3}} \cdot u_{\text{d}} \cdot e^{-j30°} = \underline{\underline{\frac{2}{\sqrt{3}} \cdot u_{\text{d}} \cdot e^{j90°}}}$$

<u>time interval 4</u> (in p.u.): $\underline{u}_{\text{s4}} = \frac{2}{3} \cdot u_{\text{d}} \cdot (-1 + \underline{a}) = \underline{\underline{-\underline{u}_{\text{s1}}}}$

<u>time interval 5</u> (in p.u.) : $\underline{u}_{s5} = \dfrac{2}{3} \cdot u_d \cdot (-1 + \underline{a}^2) = \underline{\underline{-\underline{u}_{s2}}}$

<u>time interval 6</u> (in p.u.) : $\underline{u}_{s6} = \dfrac{2}{3} \cdot u_d \cdot (-\underline{a} + \underline{a}^2) = \underline{\underline{-\underline{u}_{s3}}}$

Absolute value of the vector $\underline{u}_s$: $|\underline{u}_s| = \dfrac{2}{\sqrt{3}} \cdot u_d = \dfrac{2}{\sqrt{3}} = \underline{\underline{1.155 \text{ p.u.}}}$
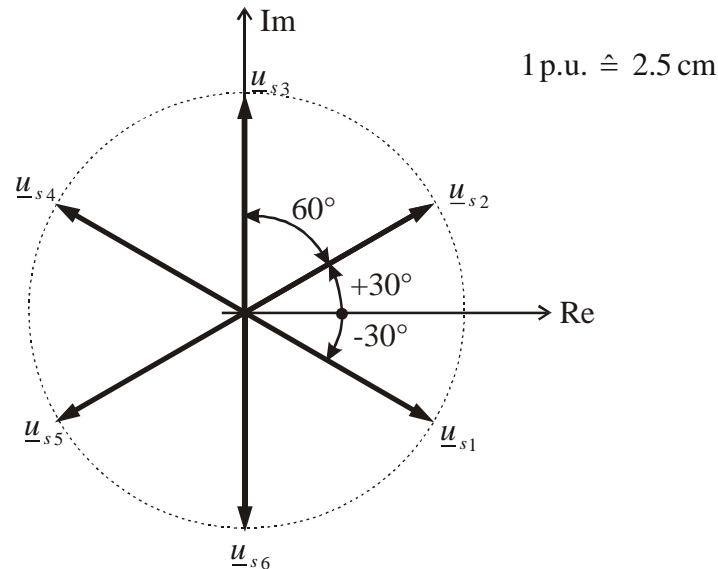

Fig. 6.3-2: Stator voltage space vectors at the different inverter switching states

3) $\underline{u}_s = r_s \cdot \underline{i}_s + \dfrac{d\underline{\psi}_s}{d\tau}$

       $\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r$

4)   No-load: $\underline{i}'_r = 0, \ r_s \approx 0 \text{ (neglected)}, \ \underline{u}_s = x_s \cdot \dfrac{d\underline{i}_s}{d\tau} \Rightarrow \underline{i}_s = \dfrac{1}{x_s} \int \underline{u}_s \, d\tau$

         phase current $i_a = \mathrm{Re}\{\underline{i}_s\}$

<u>time interval 1</u>: $\underline{i}_{s1}(\tau) = \dfrac{1}{x_s} \int \underline{u}_{s1}(\tau) \cdot d\tau = \dfrac{\underline{u}_{s1}}{x_s} \int d\tau = \dfrac{\underline{u}_{s1}}{x_s} \cdot \tau + \underline{i}_{s1}(0)$

<u>time interval 2</u>: $\underline{i}_{s2}(\tau') = \dfrac{1}{x_s} \int \underline{u}_{s2}(\tau') \cdot d\tau' = \dfrac{\underline{u}_{s2}}{x_s} \cdot \tau' + \underline{i}_{s2}(0)$

time counting : $\tau' = \tau - \tau_0$

The current cannot change abruptly: $\underline{i}_{s1}(\tau = \dfrac{\pi}{3}) = \underline{i}_{s2}(\tau' = 0)$

$i_a(\tau)$: real part of $\underline{i}_s$

<u>time interval 1</u>: $i_{a1}(\tau) = \mathrm{Re}\left\{\underline{u}_{s1}\right\} \cdot \dfrac{\tau}{x_s} + \mathrm{Re}\left\{\underline{i}_{s1}(0)\right\} =$

$= u_d \cdot \dfrac{\tau}{x_s} + K_1$; $u_d = \mathrm{Re}\left\{\underline{u}_{s1}\right\}, \; K_1 = \mathrm{Re}\left\{\underline{i}_{s1}(0)\right\}$

<u>time interval 2</u>: $i_{a2}(\tau) = \mathrm{Re}\left\{\underline{u}_{s2}\right\} \cdot \dfrac{\tau'}{x_s} + \mathrm{Re}\left\{\underline{i}_{s2}(0)\right\} = u_d \cdot \dfrac{\tau'}{x_s} + K_2$;

$u_d = \mathrm{Re}\left\{\underline{u}_{s2}\right\}, \; K_2 = \mathrm{Re}\left\{\underline{i}_{s2}(0)\right\}$

<u>time interval 3</u>: $i_{a3}(\tau) = \mathrm{Re}\left\{\underline{u}_{s3}\right\} \cdot \dfrac{\tau''}{x_s} + \mathrm{Re}\left\{\underline{i}_{s3}(0)\right\} = K_3$; $\mathrm{Re}\left\{\underline{u}_{s3}\right\} = 0, \; K_3 = Re\left\{\underline{i}_{s3}(0)\right\}$

<u>time counting</u>: $\tau'' = \tau' - \tau_0 = \tau - 2 \cdot \tau_0$

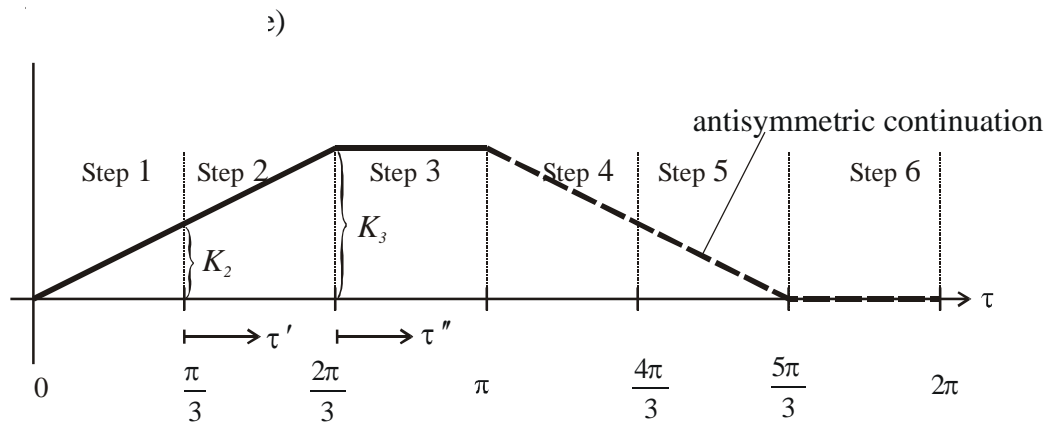<u>Assume</u>:

$K_1 = 0$

<u>Fig. 6.3-3:</u> Stator phase current at inverter operation, calculated with simplifying assumptions

The currents start from zero and have a dc offset. The DC offset decays with time due to losses in the resistances!

$K_2 = \mathrm{Re}\left\{\underline{i}_{s1}(\tau = \dfrac{\pi}{3})\right\} = u_d \cdot \dfrac{\pi}{3} \cdot \dfrac{1}{x_s}$

$K_3 = \mathrm{Re}\left\{\underline{i}_{s2}(\tau' = \dfrac{\pi}{3})\right\} = 2 \cdot u_d \cdot \dfrac{\pi}{3} \cdot \dfrac{1}{x_s}$

Steady state condition: The currents are alternating quantities as the voltages, therefore the CORRECTION of K_1 is so, that $i_a(\tau)$ becomes alternating current = Average value of $i_a = 0$.

$\Rightarrow \; K_1 = -K_3 = -u_d \cdot \dfrac{\pi}{3} \cdot \dfrac{1}{x_s}, \quad K_2 = 0$

5)
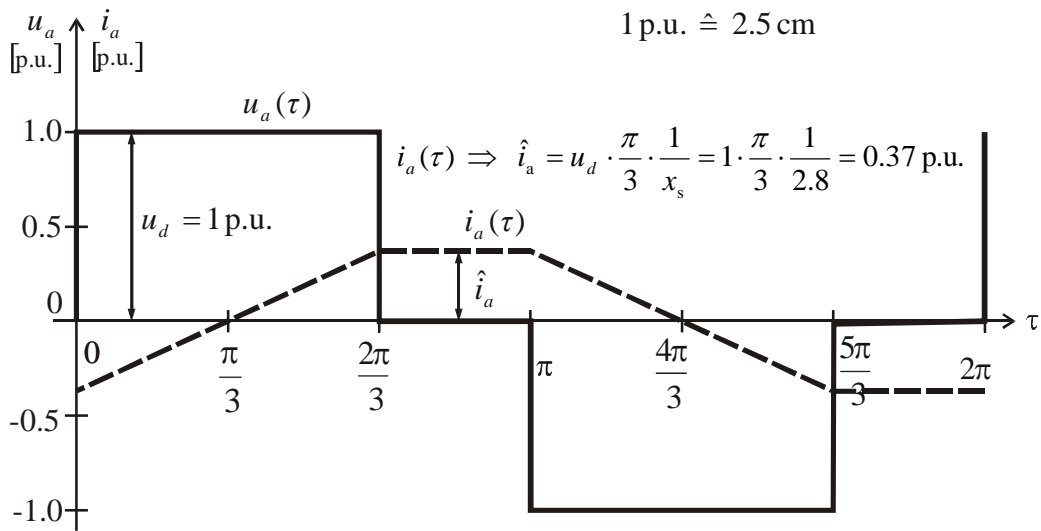
Fig. 6.3-4: Stator phase current and phase voltage at inverter operation. The current is calculated with simplifying assumptions

Line-to-line voltage = phase voltage because of delta connection.

Because of the symmetry of $u_a(\tau)$ and $i_a(\tau)$ concerning the quarterly periods the position of the fundamental-frequency is simply determinable.

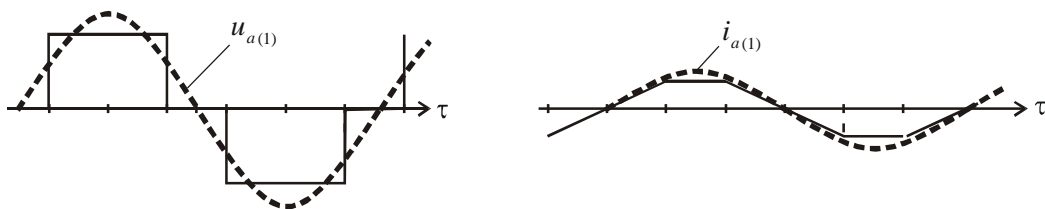Fig. 6.3-5: Time signal and corresponding fundamental of (left) stator phase voltage, (right) stator phase current at inverter operation, the latter calculated with simplifying assumptions

$\Rightarrow$ Phase angle $\underline{\underline{\varphi}}$ between $u_{a(1)}(\tau)$ and $i_{a(1)}(\tau)$ is $\underline{90° \text{ inductive}}$ (since r_s was neglected).

Stator space vector of an induction machine with removed rotor:
The three-phase stator winding of an induction motor is tested during manufacturing with removed rotor for winding tests. The stator winding is connected in star to the three-phase voltage sinusoidal source U, V, W with connected star point to the neutral N, but the connection W is broken (Fig. 6.4-1). Hence an asymmetric feeding with a zero sequence system occurs. Due to the removed rotor the zero sequence phase impedance x_0 is roughly the same value as the stator phase impedance x_s with symmetric feeding (intact connection W). For that special case $x_s = x_0$ the two remaining phases U and V can be treated with independent voltage equations, as parts of the flux linkages cancel. The feeding phase voltages from the grid are:

$$u_{s,U}(\tau) = u \cdot \cos(\omega_s \tau), \quad u_{s,V}(\tau) = u \cdot \cos\left(\omega_s(\tau - \frac{2\pi}{3})\right).$$

The stator resistance is neglected. Due to the removed rotor (air gap = half stator bore diameter) the impedance x_s is rather small, containing only the stator stray flux and the small stator bore field: $x_s = 0.2$ p.u.

1) Give the independent voltage equation per phase and calculate the phase currents i_U and i_V in p.u. at rated frequency $\omega_s = 1$. Give the value u for phase current to be rated current $i_U = 1$.

2) Calculate the current in the neutral connection i_N and the zero-sequence current i_0.

3) Calculate the current space vector $\underline{i}_s(\tau)$. Use the formula $\sin x = (e^{jx} - e^{-jx})/(2j)$.

4) Calculate the voltage space vector $\underline{u}_s(\tau)$!

5) Check, if the phase voltage $u_U(\tau)$, derived from the space vector, is identical with the originally applied function $u_U(\tau) = u \cdot \cos(\omega_s \tau)$! Do not forget to consider the zero sequence system!

6) Calculate the, by the stator bore field induced phase voltage $u_W(\tau)$ from $\underline{u}_s(\tau)$.

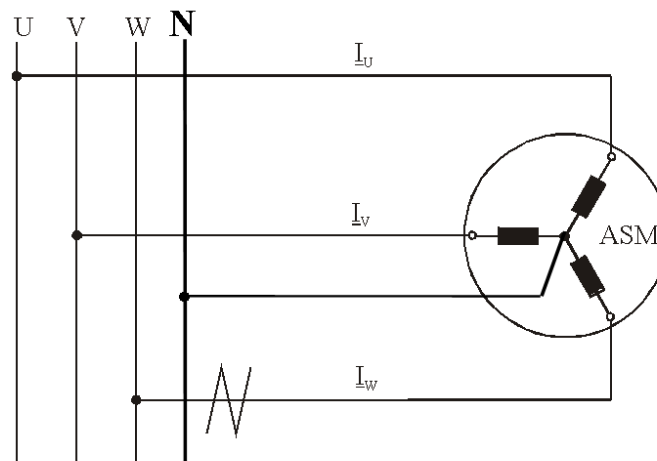7) Draw to scale the trajectory of $\underline{u}_s(\tau)$ (e.g. 0.2 p.u. = 1 cm) for $u = 1$ p.u.

Fig. 6.4-1: Electric stator winding circuit with connected neutral and broken W connection

Exercise 6.4   6.4/2

1) $u_{s,U}(\tau) = r_s \cdot i_{s,U}(\tau) + \dfrac{d\psi_{s,U}(\tau)}{d\tau} \cong \dfrac{d\psi_{s,U}(\tau)}{d\tau}$ $u_{s,V}(\tau) \cong \dfrac{d\psi_{s,V}(\tau)}{d\tau}$

Independent flux linkages: $\psi_{s,U} = x_s \cdot i_{s,U}$ $\psi_{s,V} = x_s \cdot i_{s,V}$

$i_{s,U}(\tau) = 1/x_s \cdot \int u_{s,U} d\tau = \underline{(u/x_s) \cdot \sin\tau}$, $i_{s,V}(\tau) = \underline{(u/x_s) \cdot \sin(\tau - 2\pi/3)}$

$u = x_s \cdot i_{sU} = 0.2 \cdot 1 = \underline{\underline{0.2}}$

2) $i_N = i_{s,U} + i_{s,V} = (u/x_s) \cdot (\sin\tau + \sin(\tau - 2\pi/3)) =$

$= (u/x_s) \cdot (\sin\tau + \sin\tau \cdot \cos(2\pi/3) - \cos\tau \cdot \sin(2\pi/3)) =$

$= (u/x_s) \cdot ((1/2) \cdot \sin\tau - (\sqrt{3}/2) \cdot \cos\tau) = (u/x_s) \cdot \sqrt{\dfrac{1}{2^2} + \dfrac{3}{2^2}} \cdot \sin(\tau + \alpha)$

$\tan\alpha = -(\sqrt{3}/2)/(1/2) = -\sqrt{3} \Rightarrow \alpha = \arctan(-\sqrt{3}) = -\pi/3$

$\underline{\underline{i_N(\tau) = (u/x_s) \cdot (\sin(\tau - \pi/3))}}$

The amplitude of the neutral current is rated current $i_N = 0.2/0.2 = 1$p.u.

Zero-sequence current: $i_0 = (i_{s,U} + i_{s,V} + i_{s,W})/3 = (i_{s,U} + i_{s,V})/3 = \underline{\underline{i_N(\tau)/3}}$.

The neutral current per phase is one third of the neutral current $i_0 = 0.33$ p.u.

3) $\underline{i}_s(\tau) = (2/3) \cdot (i_U + \underline{a} \cdot i_V) = (2u/3x_s) \cdot (\sin\tau + e^{j \cdot 2\pi/3} \cdot \sin(\tau - 2\pi/3))$

$\underline{i}_s(\tau) = \dfrac{2u}{3x_s} \cdot \left(\dfrac{e^{j\tau} - e^{-j\tau}}{2j} + e^{j \cdot 2\pi/3} \cdot \dfrac{e^{j(\tau - 2\pi/3)} - e^{-j(\tau - 2\pi/3)}}{2j} \right) =$

$= \dfrac{2u}{3 \cdot jx_s} \cdot \left(e^{j\tau} - \dfrac{e^{-j\tau}}{2} \cdot (1 + e^{j \cdot 4\pi/3}) \right) = \underline{\underline{\dfrac{2u}{3 \cdot jx_s} \cdot \left(e^{j\tau} + \dfrac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3} \right)}}$

4) $\underline{u}_s = x_s \cdot d\underline{i}_s / d\tau = \dfrac{2u}{3j} \cdot \left(je^{j\tau} - j\dfrac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3} \right) = \underline{\underline{\dfrac{2u}{3} \cdot \left(e^{j\tau} - \dfrac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3} \right)}}$

5)   If a zero-sequence system occurs, its values must always be added to the phase voltages and currents, because the space vector theory does not include zero-sequence effects: $u_{s,U}(\tau) = \text{Re}\{\underline{u}_s(\tau)\} + u_0$

$u_0 = x_0 \cdot di_0 / d\tau \cong x_s \cdot di_0 / d\tau = (x_s/3) \cdot di_N / d\tau =$

$= x_s \cdot (u/x_s) \cdot \dfrac{1}{3} \dfrac{d(\sin(\tau - \pi/3))}{d\tau} = (u/3) \cdot (\cos(\tau - \pi/3))$

$\text{Re}\{\underline{u}_s(\tau)\} = \text{Re}\left\{ \dfrac{2u}{3} \cdot \left(e^{j\tau} - \dfrac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3} \right) \right\} = \dfrac{2u}{3} \cdot \left(\cos\tau - \dfrac{\cos(\tau - 2\pi/3)}{2} \right)$

$u_{s,U}(\tau) = \text{Re}\{\underline{u}_s(\tau)\} + u_0 = \dfrac{2u}{3} \cdot \left(\cos\tau - \dfrac{\cos(\tau - 2\pi/3)}{2} \right) + (u/3) \cdot (\cos(\tau - \pi/3)) =$

$= \dfrac{2u}{3} \cdot \cos\tau + (u/3) \cdot \cos\tau = \underline{\underline{u \cdot \cos\tau}}$

The calculation delivers correctly the feeding voltage.

6) $u_{s,W}(\tau) = \mathrm{Re}\{\underline{a} \cdot \underline{u}_s(\tau)\} + u_0$

$$\mathrm{Re}\{\underline{a} \cdot \underline{u}_s(\tau)\} = \mathrm{Re}\left\{\frac{2u}{3} \cdot \left(e^{j \cdot 2\pi/3} e^{j\tau} - e^{j \cdot 2\pi/3} \cdot \frac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3}\right)\right\} =$$

$$= \mathrm{Re}\left\{\frac{2u}{3} \cdot \left(e^{j(\tau + 2\pi/3)} - \frac{e^{-j(\tau - 4\pi/3)}}{2}\right)\right\} = \frac{2u}{3} \cdot \left(\cos(\tau + 2\pi/3) - \frac{\cos(\tau - 4\pi/3)}{2}\right)$$

$$u_{s,W}(\tau) = \mathrm{Re}\{\underline{a} \cdot \underline{u}_s(\tau)\} + u_0$$

$$= \frac{2u}{3} \cdot \left(\cos(\tau + 2\pi/3) - \frac{\cos(\tau - 4\pi/3)}{2}\right) + (u/3) \cdot (\cos(\tau - \pi/3)) =$$

$$= \frac{u}{3} \cdot \cos(\tau - 4\pi/3) + \frac{u}{3} \cdot (\cos(\tau - \pi/3)) = \underline{\underline{0}}$$

The total induced voltage in phase W is zero.

7)   The voltage space vector is a superposition of two vector components, which rotate on circles with different radius and opposite rotation direction, yielding a resultant ellipse.

$$\underline{u}_s = \frac{2u}{3} \cdot \left(e^{j\tau} - \frac{e^{-j\tau}}{2} \cdot e^{j \cdot 2\pi/3}\right) = \frac{2u}{3} \cdot \left(e^{j\tau} + \frac{e^{-j\tau}}{2} \cdot e^{-j \cdot \pi/3}\right)$$

a) Positive sequence component: $\underline{u}_{s,1}(\tau) = \frac{2u}{3} \cdot e^{j\tau}$

b) Negative sequence component: $\underline{u}_{s,2}(\tau) = \frac{u}{3} \cdot e^{-j\tau} \cdot e^{-j \cdot \pi/3}$

At $\tau = 0$ the angle between the two vector components is 60° (Fig. 6.4-2). At $\tau = -\pi/6$ and $\tau = 5\pi/6$ both vector components are aligned with identical direction and give the resultant:

$$\underline{u}_s(-\pi/6) = \frac{2u}{3} \cdot \left(e^{-j\pi/6} + 0.5 \cdot e^{-j\pi/6}\right) = u \cdot e^{-j\pi/6} \quad \text{or} \quad \underline{u}_s(5\pi/6) = u \cdot e^{j \cdot 5\pi/6}. \text{ This}$$

defines the major half axes of the ellipse (length u), being inclined by 30° to the abscissa. At $\tau = 2\pi/6$ and $\tau = 8\pi/6$ both vector components are aligned with opposite direction and give the resultant:

$$\underline{u}_s(2\pi/6) = \frac{2u}{3} \cdot \left(e^{j \cdot 2\pi/6} + 0.5 \cdot e^{-j \cdot 4\pi/6}\right) = (u/3) \cdot e^{j \cdot 2\pi/6} \qquad \text{or}$$

$\underline{u}_s(8\pi/6) = (u/3) \cdot e^{j \cdot 8\pi/6}$, which defines the minor half axes of the ellipse with the length $u/3$. They are positioned in a right angle to the major half axes.
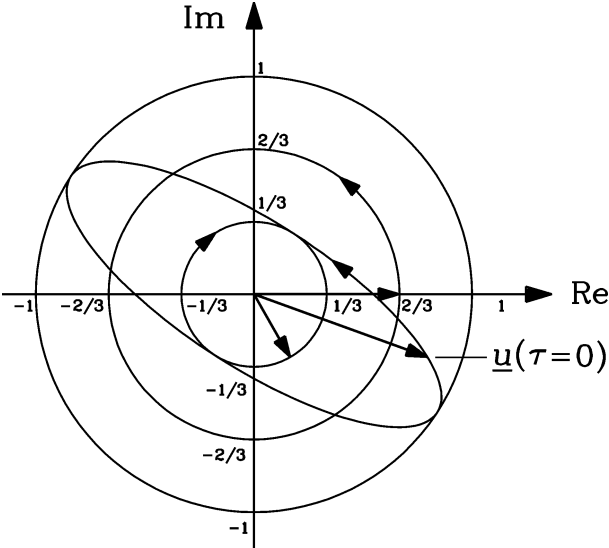
Fig. 6.4-2: The space vector trajectory of the voltage space vector at broken phase W and connected neutral is an ellipse.

Exercise 6.5   6.5/1

Stator current space vector of an asymmetrically fed AC machine:
A three phase induction machine with star-connected winding is operated in the test field of the manufacturer with removed rotor to measure the stator losses only. Due to the removed rotor the impedance per phase is only $x_s = 0.2$ p.u. The symmetrical feeding grid voltage system is given as ($u = 0.2$ p.u., $\omega_s = 1$ p.u.):

$$u_R(\tau) = u \cdot \cos(\omega_s \tau), \quad u_S(\tau) = u \cdot \cos\left(\omega_s(\tau - \frac{2\pi}{3})\right), \quad u_T(\tau) = u \cdot \cos\left(\omega_s(\tau - \frac{4\pi}{3})\right)$$

Due to a failure the phase connection W of the induction machine breaks (Fig. 6.5-1).

1)   Calculate the feeding voltage between terminals U and V.

2)   How big is the zero sequence current system?

3)   Calculate the phase currents i_U, i_V and space vector $\underline{i}_s(\tau)$ in p.u. after the failure.

4)   Calculate the phase voltages u_U, u_V and space vector $\underline{u}_s(\tau)$ in p.u. after the failure.

5)   Draw a vector diagram of $\underline{i}_s(\tau)$ and of $\underline{u}_s(\tau)$ to scale (e.g. 0.1 p.u. = 1 cm).
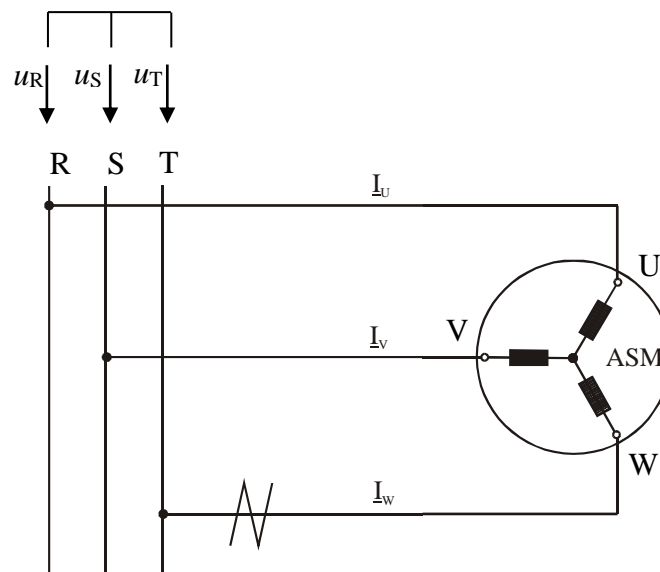

Fig. 6.5-1: Electric circuit with broken W connection

Exercise 6.5   6.5/2

1) $\quad u_{UV} = u_R - u_S = u \cdot (\cos\tau - \cos(\tau - 2\pi/3)) =$

$= u \cdot (\cos\tau - \cos(\tau - 2\pi/3)) = u \cdot (\cos\tau - \cos\tau \cdot \cos(2\pi/3) - \sin\tau \cdot \sin(2\pi/3)) =$

$= u \cdot \left((3/2) \cdot \cos\tau - (\sqrt{3}/2) \cdot \sin\tau\right) = u \cdot \sqrt{(3/2)^2 + (\sqrt{3}/2)^2} \cdot \cos(\tau + \alpha)$

$\tan\alpha = (\sqrt{3}/2)/(3/2) = 1/\sqrt{3} \Rightarrow \alpha = \arctan(1/\sqrt{3}) = \pi/6$

$\underline{\underline{u_{UV}(\tau) = u \cdot \sqrt{3} \cdot \cos(\tau + \pi/6)}}$

The feeding voltage between U and V is the line-to-line voltage, which is by $\sqrt{3}$ bigger than the feeding phase voltage and by 30° leading to the phase voltage u_R.

2) $\quad i_W = 0, \, i_U = - i_V, \, i_0 = (i_U + i_V + i_W)/3 = \underline{\underline{0}}$

No zero-sequence system occurs!

$\underline{i}_s = (2/3) \cdot (i_U + \underline{a} \cdot i_V) = (2/3) \cdot i_U \cdot (1 - \underline{a})$

The current space vector does not rotate, but is pulsating with a fixed orientation.

$\underline{u}_s = x_s \cdot d\underline{i}_s / d\tau = (2/3) \cdot x_s \cdot (1 - \underline{a}) \cdot di_U / d\tau$

Also the voltage space vector does not rotate, but is pulsating with the same fixed orientation as the current space vector.

$u_U = \text{Re}\{\underline{u}_s\} = \text{Re}\{(2/3) \cdot x_s \cdot (di_U/d\tau) \cdot (1 - \underline{a})\} = (2/3) \cdot x_s \cdot (di_U/d\tau) \cdot (3/2) =$

$= x_s \cdot (di_U/d\tau)$

$u_V = \text{Re}\{\underline{a}^2 \cdot \underline{u}_s\} = \text{Re}\{(2/3) \cdot x_s \cdot (di_U/d\tau) \cdot (\underline{a}^2 - \underline{a}^3)\} =$

$= \text{Re}\{(2/3) \cdot x_s \cdot (di_U/d\tau) \cdot (\underline{a}^2 - 1)\} = -(2/3) \cdot x_s \cdot (di_U/d\tau) \cdot (3/2) = -u_U$

The phase voltages u_U, u_V are not shifted by 120° like the feeding u_R, u_S, but are opposite.

$u_U - u_V = u_{UV} = u \cdot \sqrt{3} \cdot \cos(\tau + \pi/6) = 2u_U = 2x_s \cdot di_U / d\tau$

$i_U(\tau) = 1/(2x_s) \cdot \int u_{UV} d\tau = \underline{\underline{\frac{\sqrt{3}u}{2x_s} \cdot \sin(\tau + \pi/6)}}, \quad i_V(\tau) = \underline{\underline{-\frac{\sqrt{3}u}{2x_s} \cdot \sin(\tau + \pi/6)}}$

$\hat{i}_U = \hat{i}_V = \frac{\sqrt{3}u}{2x_s} = \frac{\sqrt{3} \cdot 0.2}{2 \cdot 0.2} = \underline{\underline{0.866}},$

$\underline{i}_s(\tau) = \frac{u \cdot (1 - \underline{a})}{\sqrt{3} \cdot x_s} \cdot \sin(\tau + \pi/6) = \underline{\underline{\frac{1 - \underline{a}}{\sqrt{3}} \cdot \sin(\tau + \pi/6)}}$

Amplitude of current space vector: $|\underline{i}_s| = \left| \frac{u}{x_s} \cdot \frac{1 - \underline{a}}{\sqrt{3}} \right| = \frac{u}{x_s} = \underline{\underline{1}} \text{p.u.}$

$u_U = x_s \cdot (di_U/d\tau) = x_s \cdot \frac{\sqrt{3}u}{2x_s} \cdot \cos(\tau + \pi/6) = \underline{\underline{\frac{\sqrt{3}u}{2} \cdot \cos(\tau + \pi/6) = -u_V}}$

3) Amplitude of u_U and u_V: $(\sqrt{3}/2) \cdot u = 0.866 \cdot 0.2 = \underline{\underline{0.173}}$

$\underline{u}_s(\tau) = (2/3) \cdot x_s \cdot (1 - \underline{a}) \cdot \frac{d}{d\tau}\left(\frac{\sqrt{3}u}{2x_s} \cdot \sin(\tau + \pi/6) \right) = \underline{\underline{\frac{u}{\sqrt{3}} \cdot (1 - \underline{a}) \cdot \cos(\tau + \pi/6)}}$

<u>Exercise 6.5</u>   6.5/3

Amplitude of voltage space vector: $\left|\underline{u}_s\right| = \left|u \cdot \dfrac{1-\underline{a}}{\sqrt{3}}\right| = u = \underline{\underline{0.2}}\,\text{p.u.}$

4) $i_V = -i_U = \underline{\underline{-0.866}}\,\text{p.u.}$ $\underline{i}_s(\tau) = \dfrac{2}{3} \cdot \left(i_U(\tau) + \underline{a} \cdot i_V(\tau) + \underline{a}^2 \cdot i_W(\tau)\right);$ $\underline{a} = -\dfrac{1}{2} + j \cdot \dfrac{\sqrt{3}}{2}$

$\underline{i}_s(\tau) = \dfrac{2}{3} \cdot \left(0.866 \cdot \sin(\tau + \pi/6) - (-\dfrac{1}{2} + j \cdot \dfrac{\sqrt{3}}{2}) \cdot 0.866 \cdot \sin(\tau + \pi/6) + 0\right) =$

$= \underline{\underline{(0.866 - j \cdot 0.5) \cdot \sin(\tau + \pi/6)}}$

$\underline{u}_s(\tau) = \dfrac{2}{3} \cdot \left(\dfrac{\sqrt{3} \cdot u}{2} \cdot \cos(\tau + \pi/6) - (-\dfrac{1}{2} + j \cdot \dfrac{\sqrt{3}}{2}) \cdot \dfrac{\sqrt{3} \cdot u}{2} \cdot \cos(\tau + \pi/6) + 0\right) =$

$= \underline{\underline{(0.173 - j \cdot 0.1) \cdot \cos(\tau + \pi/6)}}$

The voltage and current space vector orientation is inclined at an angle of -30° to the abscissa.
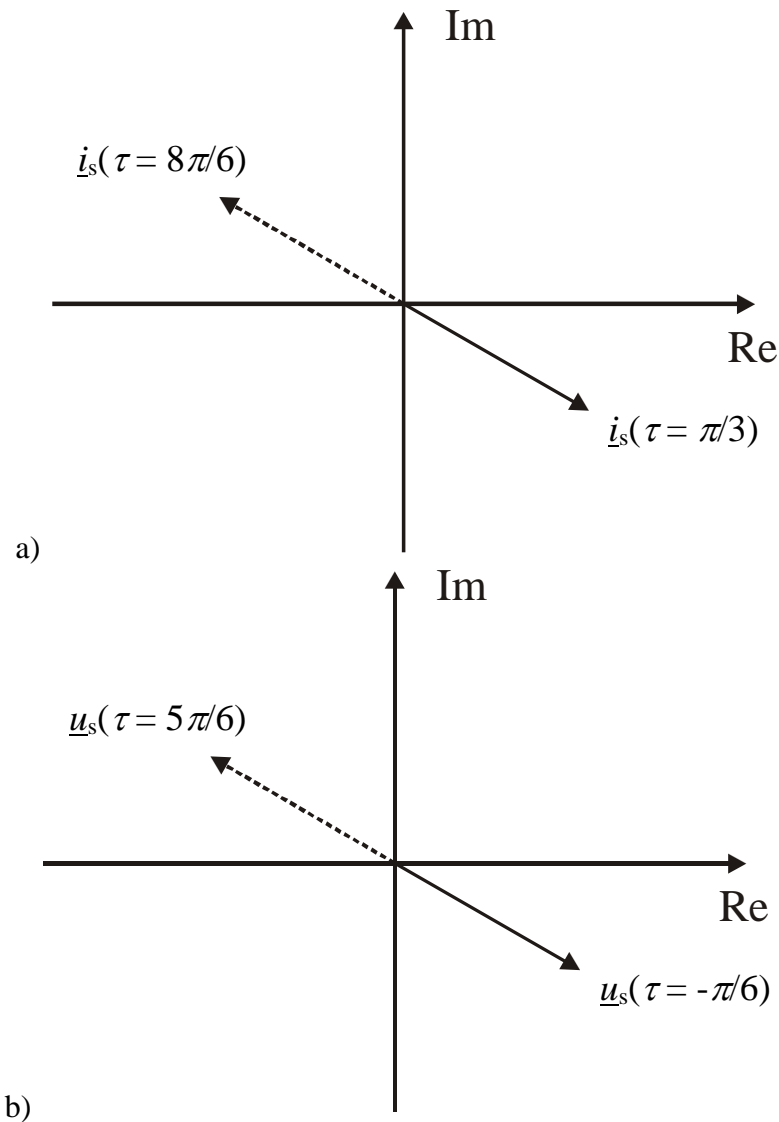

a)

b)

<u>Fig. 6.5-2:</u> a) Current space vector (amplitude: 1 p.u.), b) Voltage space vector (Amplitude: 0.2)

Exercise 6.6   6.6/1
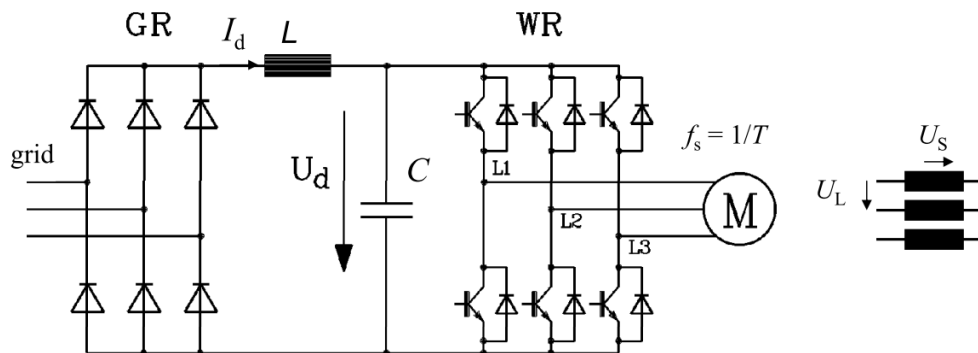
Zero sequence system of voltage source inverter-output voltage

Fig. 6.6-1: Voltage-source inverter with diode rectifier (GR), DC link with capacitor C and filter choke L, transistor converter (WR). A three-phase AC motor with star-connected stator winding is fed by the inverter line-to-line voltage U_L.

1)   Show, that a three-phase symmetrical voltage system has no zero-sequence phase and line-to-line system!

2)   Sketch to scale the line-to-line and the line-to-phase voltage time function $U_L(t)$ and $U_S(t)$ per period T at a start-connected three-phase stator winding, when being fed by a voltage source inverter (Fig. 6.6-1) with the DC link voltage U_d at block commutation (six-step commutation). Assume an ideally constant DC link voltage $U_d = $ const. (C is assumed to be infinitely big).

3)   Give the *Fourier* series of the three phase voltages! Comment the result!

4)   Determine the zero-sequence of the line and phase voltage $U_{S0}(t)$, $U_{L0}(t)$ and of the terminal-to-ground potential $\varphi_{L0}(t)$ per period! Give the graphs!

Exercise 6.6   6.6/2

1)   Three-phase symmetrical voltage system (either phase or line voltage system):
$U_U(t) = \hat{U} \cdot \cos\omega t$, $U_V(t) = \hat{U} \cdot \cos(\omega t - (2\pi/3))$, $U_W(t) = \hat{U} \cdot \cos(\omega t - (4\pi/3))$.

$U_0(t) = (U_U(t) + U_V(t) + U_W(t))/3$, abbreviation: $\omega t = \varepsilon$,

$$U_U + U_V + U_W = \hat{U} \cdot (\cos\varepsilon + \underbrace{\cos\varepsilon \cdot \cos\frac{2\pi}{3}}_{-1/2} + \underbrace{\sin\varepsilon \cdot \sin\frac{2\pi}{3}}_{\sqrt{3}/2} + \underbrace{\cos\varepsilon \cdot \cos\frac{4\pi}{3}}_{-1/2} + \underbrace{\sin\varepsilon \cdot \sin\frac{4\pi}{3}}_{-\sqrt{3}/2},$$

$U_U + U_V + U_W = \hat{U} \cdot ((1 - \frac{1}{2} - \frac{1}{2}) \cdot \cos\varepsilon + (\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}) \cdot \sin\varepsilon) = 0$. This is true for the phase
and the line voltage system!

2) $U_{L1-L2}(t) = \varphi_{L1}(t) - \varphi_{L2}(t)$, $U_{L2-L3}(t) = \varphi_{L2}(t) - \varphi_{L3}(t)$, $U_{L3-L1}(t) = \varphi_{L3}(t) - \varphi_{L1}(t)$:
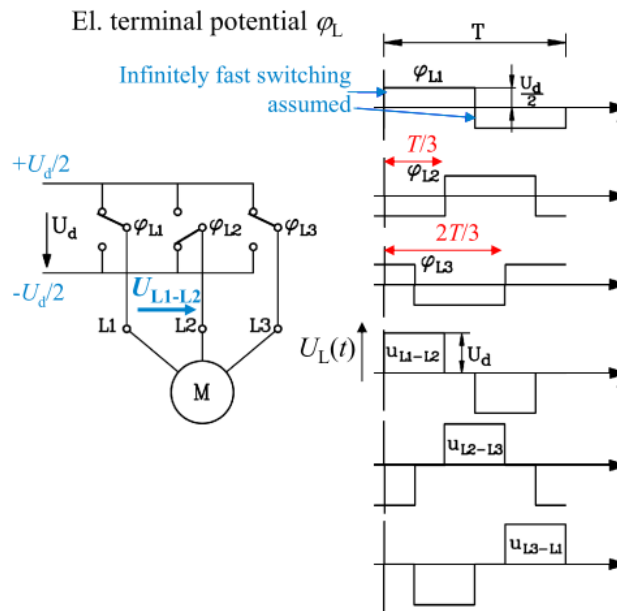

Fig. 6.6-2: Six-step operation of voltage source inverter: El. terminal potential φ_L and line-to-line
inverter output voltage U_L.

Each **phase voltage** can be given as a *Fourier*-Series: $U_{S1}(t) = \sum_{l=1}^{\infty} \hat{U}_l \cdot \cos(l \cdot \omega t)$.

$\omega = 2\pi \cdot f = 2\pi/T$. Due to the abscissa-symmetric line-to-line voltages also the phase
voltages must be <u>abscissa-symmetric</u>, hence <u>no even ordinal</u> numbers l occur! The
line-to-line voltages are shifted acc. to Fig. 6.6-2 by $T/3$ (resp. $2\pi/3$), so also the phase
voltages are shifted by $2\pi/3$. For that case according to 1) we get, that each *Fourier*
harmonic system adds <u>up to zero</u>: $U_{S1,l}(t) + U_{S2,l}(t) + U_{S3,l}(t) = 0$, if ordinal numbers
are <u>not</u> divisible by 3.
Ordinal numbers l^* divisible by 3 would lead to <u>identical</u> (in-phase) voltages:
$U_{S1,l^*}(t) = \hat{U}_{l^*} \cdot \cos(l^* \cdot \omega t)$, $U_{S2,l^*}(t) = \hat{U}_{l^*} \cdot \cos(l^* \cdot \omega t - (l^* \cdot 2\pi/3)) = U_{S1,l^*}(t)$,

$U_{S3,l^*}(t) = \hat{U}_{l^*} \cdot \cos(l^* \cdot \omega t - (l^* \cdot 4\pi/3)) = U_{S1,l^*}(t)$. They would cause identical
harmonic currents $I_{S1,l^*}(t) = I_{S2,l^*}(t) = I_{S3,l^*}(t)$, which <u>cannot flow</u> due to the start-
point *Kirchhoff*'s node law, $I_{S1,l^*}(t) + I_{S2,l^*}(t) + I_{S3,l^*}(t) = 3I_{S1,l^*}(t) = 0 \Rightarrow I_{S,l^*} = 0$.
So no voltage harmonics with ordinal numbers l^* divisible by 3 occur.

Exercise 6.6   6.6/3

<u>Result:</u> $U_{S1}(t) + U_{S2}(t) + U_{S3}(t) = \sum\limits_{l=1,5,7,...}^{\infty} U_{S1,l}(t) + U_{S2,l}(t) + U_{S3,l}(t) = 0$.

Calculation of phase voltages: (Fig. 6.6-3):

$U_{S1} - U_{S2} = U_{L1-L2}, U_{S2} - U_{S3} = U_{L2-L3}, U_{S1} + U_{S2} + U_{S3} = 0$:

$U_{S1} = \dfrac{2U_{L1-L2} + U_{L2-L3}}{3}$, $U_{S2} = \dfrac{2U_{L2-L3} + U_{L3-L1}}{3}$, $U_{S3} = \dfrac{2U_{L3-L1} + U_{L1-L2}}{3}$
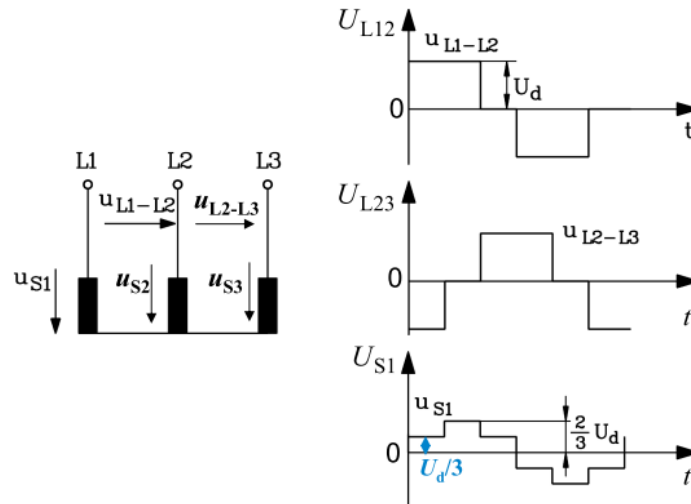

Fig. 6.6-3: Six-step operation of voltage source inverter: Line-to-line and phase voltages at star-connection of stator winding U_L, U_S.

3)   Using abscissa- symmetry and using cos-series as even function:

$\hat{U}_{S1,l} = \dfrac{4}{\pi} \cdot \int\limits_{0}^{\pi/2} U_{S1}(\varepsilon) \cdot \cos(l \cdot \varepsilon) \cdot d\varepsilon = \dfrac{4}{\pi} \cdot \dfrac{U_d}{3} \cdot (\int\limits_{0}^{\pi/6} 2 \cdot \cos(l \cdot \varepsilon) \cdot d\varepsilon + \int\limits_{\pi/6}^{\pi/2} \cos(l \cdot \varepsilon) \cdot d\varepsilon)$,

$\hat{U}_{S1,l} = \dfrac{8}{3\pi} \cdot \dfrac{U_d}{l} \cdot \sin\dfrac{l\pi}{3} \cdot \cos\dfrac{l\pi}{6}$, $l = 1,3,5,7,...$

For $l^* = 3n, \quad n = 1,2,3,...$: $\hat{U}_{S1,l^*} = \dfrac{8}{3\pi} \cdot \dfrac{U_d}{3n} \cdot \underbrace{\sin n\pi}_{0} \cdot \cos(n\pi/2) = 0$,

so we get $l = |1 + 6g| = 1,5,7,11,13,...$ $g = 0,\pm1,\pm2,...$: $\hat{U}_{S1,l} = \dfrac{2}{\pi} \cdot \dfrac{U_d}{l} \cdot \sin\dfrac{l\pi}{2}$.

$U_{S1}(t) = \sum\limits_{l=1,5,7,...}^{\infty} \dfrac{2U_d}{\pi} \cdot \dfrac{\sin(l\pi/2)}{l} \cdot \cos(l \cdot \omega t) = \sum\limits_{l=1,5,7,...}^{\infty} \hat{U}_l \cdot \cos(l \cdot \omega t)$,

$U_{S2}(t) = \sum\limits_{l=1,5,7,...}^{\infty} \hat{U}_l \cdot \cos(l \cdot \omega t - (l \cdot 2\pi/3))$, $U_{S3}(t) = \sum\limits_{l=1,5,7,...}^{\infty} \hat{U}_l \cdot \cos(l \cdot \omega t - (l \cdot 4\pi/3))$,

Comment: The *Fourier* series proves that no harmonics with ordinal numbers, divisible by 3, occur.

4)   Acc. To 2): $U_{S0}(t) = (U_{S1} + U_{S2} + U_{S3})/3 = 0$,

$\varphi_{L0}(t) = (\varphi_{L1}(t) + \varphi_{L2}(t) + \varphi_{L3}(t))/3 = \pm U_d/6 \neq 0$, $U_{L0}(t) = \varphi_{L0}(t) - \varphi_{L0}(t) = 0 \Rightarrow$

$U_{L0}(t) = (U_{L1-L2}(t) + U_{L2-L3}(t) + U_{L3-L1}(t))/3 = 0$.

See Fig. 6.6-4!

$$
\left.\begin{array}{l}
U_{L1-L2}(t)/3 = (\varphi_{L1}(t) - \varphi_{L2}(t))/3 \\[4pt]
U_{L2-L3}(t)/3 = (\varphi_{L2}(t) - \varphi_{L3}(t))/3 \\[4pt]
\underline{U_{L3-L1}(t)/3 = (\varphi_{L3}(t) - \varphi_{L1}(t))/3}
\end{array}\right\} +
$$

$$
U_{L0}(t) = \qquad \varphi_{L0}(t) - \varphi_{L0}(t) = 0
$$

The whole winding systems varies its electrical potential to ground with $3f$. This causes via the winding insulation capacity C_{wg} from the slot copper to the grounded stator iron stack a capacitive ground current flow $i_g(t) = C_{wg} \cdot d\varphi_{L0}/dt$ as a disturbing common-mode current. It flows via the PE grounding connector from the stator iron stack and metallic housing to the earth and back via the PE connector of the inverter housing to the ground connection of the DC link.
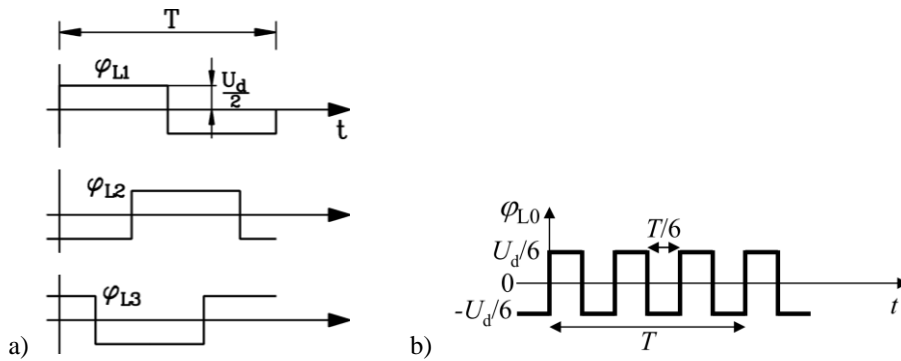

Fig. 6.6-4: Six-step operation of voltage source inverter: a) Terminal-to-ground el. potentials for the three phase terminals 1, 2, 3 and b) its zero-sequence system.

Exercise 6.7   6.7/1
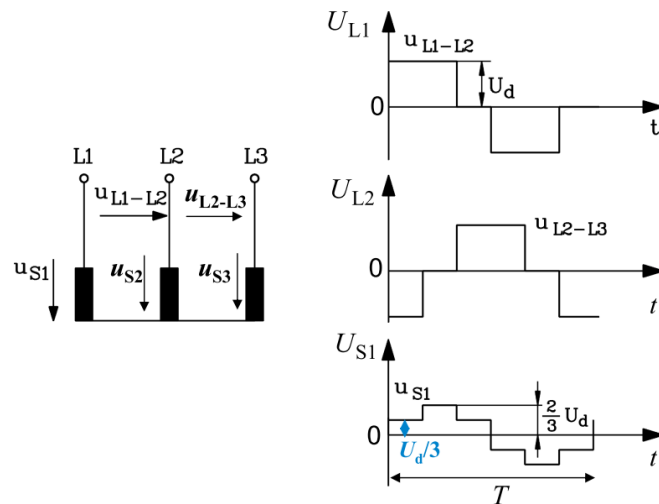
Voltage space vector at voltage-source inverter block-commutation

Fig. 6.7-1: Block-commutation (Six-step operation) of a voltage source inverter: Line-to-line and phase voltages at star-connection of stator winding U_L, U_S. The maximum voltage U_{S1} is shifted with respect to the maximum of U_{L1} by $T/12$ lagging.

1)   Use the <u>line-to-line</u> inverter output voltage U_L at block-commutation of Exercise 6.7 (Fig. 6.7-1) to give the voltage space vector $\underline{u}_L$ in per unit of the rated phase voltage!

2)   Use the <u>phase</u> inverter output voltage U_S at block-commutation of Exercise 6.7 (Fig. 6.7-1) to give the voltage space vector $\underline{u}_S$ in per unit of the rated phase voltage!

3)   Give the *Fourier* series of the three phase voltages! Comment the result!

4)   Determine the zero-sequence of the line and phase voltage $U_{S0}(t)$, $U_{L0}(t)$ and of the terminal-to-ground potential $\varphi_{L0}(t)$ per period! Give the graphs!

5)   Give the graph of the phase voltage space vector $u_S(t)$ in p.u. per period!

1)   Line-to-line output inverter voltage according to Fig. 6.7-1:

A	B	C	D	E	F
$0 \leq t \leq \dfrac{T}{6}$	$\dfrac{T}{6} \leq t \leq \dfrac{T}{3}$	$\dfrac{T}{3} \leq t \leq \dfrac{T}{2}$	$\dfrac{T}{2} \leq t \leq \dfrac{2T}{3}$	$\dfrac{2T}{3} \leq t \leq \dfrac{5T}{6}$	$\dfrac{5T}{6} \leq t \leq T$
$U_{L1} = U_d$	U_d	0	$-U_d$	$-U_d$	0
$U_{L2} = -U_d$	0	U_d	U_d	0	$-U_d$
$U_{L3} = 0$	$-U_d$	$-U_d$	0	U_d	U_d

Phase voltage fundamental amplitude $l = 1$ according to Exercise 6.6:

$$\hat{U}_{S1,l=1} = \frac{2}{\pi} \cdot \frac{U_d}{l} \cdot \sin\frac{l\pi}{2}\bigg|_{l=1} = \frac{2}{\pi} \cdot U_d \cdot \sin\frac{\pi}{2} = \frac{2}{\pi} \cdot U_d .$$

$$\underline{u}_L(t) = \frac{2}{3} \cdot \frac{1}{\hat{U}_{S1,l=1}} (U_{L1}(t) + \underline{a} \cdot U_{L2}(t) + \underline{a}^2 \cdot U_{L3}(t)),$$

$$0 \leq t \leq \frac{T}{6}: \ \underline{u}_{LA}(t) = \frac{2/3}{2U_d/\pi} \cdot (U_d - \underline{a} \cdot U_d) = \frac{\pi}{3} \cdot (1 - \underline{a}) = \frac{\pi}{3} \cdot (1 - (-\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2})),$$

$$\underline{u}_{LA}(t) = \frac{\pi}{3} \cdot (\frac{3}{2} - j \cdot \frac{\sqrt{3}}{2}) = \frac{\pi}{2} \cdot (1 - j \cdot \frac{1}{\sqrt{3}}) = \frac{\pi}{2} \cdot \frac{2}{\sqrt{3}} \cdot e^{-j\pi/6} = \frac{\pi}{\sqrt{3}} \cdot e^{-j\pi/6},$$

$$\frac{T}{6} \leq t \leq \frac{T}{3}: \ \underline{u}_{LB}(t) = \frac{2/3}{2U_d/\pi} \cdot (U_d - \underline{a}^2 \cdot U_d) = \frac{\pi}{3} \cdot (1 - \underline{a}^2) = \frac{\pi}{3} \cdot (1 - (-\frac{1}{2} - j \cdot \frac{\sqrt{3}}{2})),$$

$$\underline{u}_{LB}(t) = \frac{\pi}{3} \cdot (\frac{3}{2} + j \cdot \frac{\sqrt{3}}{2}) = \frac{\pi}{2} \cdot (1 + j \cdot \frac{1}{\sqrt{3}}) = \frac{\pi}{2} \cdot \frac{2}{\sqrt{3}} \cdot e^{j\pi/6} = \frac{\pi}{\sqrt{3}} \cdot e^{j\pi/6}$$

For each time interval A, B, C, D, E, F one voltage space vector exists with the p.u. value $|\underline{u}_L| = \dfrac{\pi}{\sqrt{3}} = 1.81$, which jumps due to the infinite fast voltage switching with the angle $\pi/3$ in counter-clockwise direction (Fig. 6.7-2). The p.u.-value is roughly by $\sqrt{3}(\approx 1.81)$ bigger than the phase voltage nominal peak value.
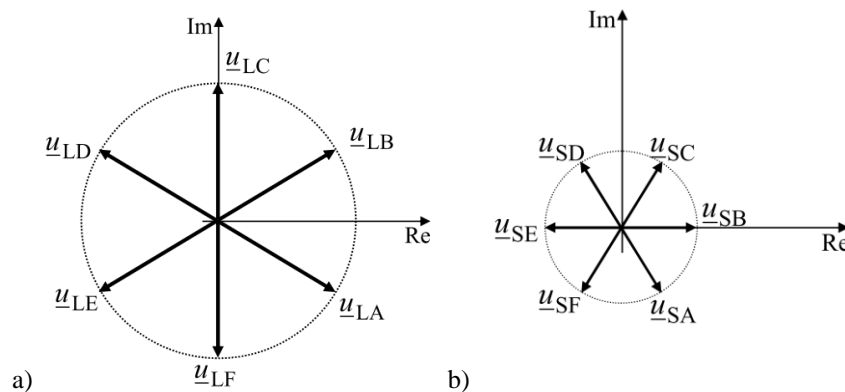

Fig. 6.7-2: Six-step operation of voltage source inverter: a) Line-to-line p.u. voltage space vector (length 1.81 p.u.), b) Phase p.u. voltage space vector (length 1.047 p.u.)

2) $$\underline{u}_S(t) = \frac{2}{3} \cdot \frac{1}{\hat{U}_{S1,l=1}} (U_{S1}(t) + \underline{a} \cdot U_{S2}(t) + \underline{a}^2 \cdot U_{S3}(t)),$$

Phase output inverter voltage according to Fig. 6.7-1:

A	B	C	D	E	F
$0 \le t \le \dfrac{T}{6}$	$\dfrac{T}{6} \le t \le \dfrac{T}{3}$	$\dfrac{T}{3} \le t \le \dfrac{T}{2}$	$\dfrac{T}{2} \le t \le \dfrac{2T}{3}$	$\dfrac{2T}{3} \le t \le \dfrac{5T}{6}$	$\dfrac{5T}{6} \le t \le T$
$U_{S1} = U_d/3$	$2U_d/3$	$U_d/3$	$-U_d/3$	$-2U_d/3$	$-U_d/3$
$U_{S2} = -2U_d/3$	$-U_d/3$	$U_d/3$	$2U_d/3$	$U_d/3$	$-U_d/3$
$U_{S3} = U_d/3$	$-U_d/3$	$-2U_d/3$	$-U_d/3$	$U_d/3$	$2U_d/3$

$$0 \le t \le \frac{T}{6}: \ \underline{u}_{SA}(t) = \frac{2/3}{2U_d/\pi} \cdot (\frac{U_d}{3} - \underline{a} \cdot \frac{2U_d}{3} + \underline{a}^2 \cdot \frac{U_d}{3}) = \frac{\pi}{9} \cdot (1 - 2\underline{a} + \underline{a}^2),$$

$$\underline{u}_{SA}(t) = \frac{\pi}{9} \cdot (1 + 1 - j \cdot \frac{2\sqrt{3}}{2} - \frac{1}{2} - j \cdot \frac{\sqrt{3}}{2}) = \frac{\pi}{9} \cdot (\frac{3}{2} - j \cdot \frac{3\sqrt{3}}{2}) = \frac{\pi}{6} \cdot (1 - j \cdot \sqrt{3}),$$

$$\underline{u}_{SA}(t) = \frac{\pi}{6} \cdot 2 \cdot e^{-j\pi/3} = \frac{\pi}{3} \cdot e^{-j\pi/3}.$$

$$\frac{T}{6} \le t \le \frac{T}{3}: \ \underline{u}_{SB}(t) = \frac{2/3}{2U_d/\pi} \cdot (\frac{2U_d}{3} - \underline{a} \cdot \frac{U_d}{3} - \underline{a}^2 \cdot \frac{U_d}{3}) = \frac{\pi}{9} \cdot (2 - \underline{a} - \underline{a}^2) = \frac{\pi}{3},$$

For each time interval A, B, C, D, E, F one phase voltage space vector exists with the p.u. value $|\underline{u}_S| = \frac{\pi}{3} = 1.047$, which jumps due to the infinite fast voltage switching with the angle $\pi/3$ in counter-clockwise direction (Fig. 6.7-2). The phase voltage value is exactly by $\sqrt{3}$ smaller than the line voltage. It is lagging with $\pi/6$ in space, as its time signal is lagging also with $\pi/6$ resp. $T/12$ acc. to Fig. 6.7-1.

Turn-off of a 3-phase induction machine

A 3-phase induction motor is running at no-load in steady state, connected to a 50 Hz grid.

Motor data: $x_s = x'_r = 2.6$ p.u., $x_h = 2.5$ p.u., $x_s = x'_s = x'_r = 2.6$ p.u., $x_h = 2.5$ p.u. rotor open-circuit time constant $\tau_r = x'_r / r'_r = 100$. The stator resistance is neglected. At time $\tau = 0$, the motor will be disconnected from the grid. Determine time characteristic of decaying stator voltage of phase U, which is induced by decaying rotor flux. You can solve this task by following calculation steps which are given below. Speed of the motor can be assumed constant during the electrical transient process.

Note: We can see practical use of this example on switching major drives in power plant (e. g. coolant pumps) to an emergency power supply during a power failure. In case of successful switching (i. e. decaying voltage is still sufficient high), no dangerous high current will be occurring. Generally, high current could occur during a restart of a de-energized motor (6 until 7 times of rated current!).

1)   In rotating-field oriented reference frame, you should determine per-unit equation of rotor flux linkage after the disconnection of the motor from the grid.

2)   Solve this 1st order differential equation. Initial condition: Magnetic energy of main flux can not change suddenly before the turn-off.

3)   Using the rotor flux linkage, calculate space vector of stator voltage.

4)   Calculate the phase voltage u_U by transforming the space vector of the stator voltage into stator-oriented reference frame, assuming that rotation angle δ_0 between stator-oriented and rotating-field oriented reference frame equals zero at time $\tau = 0$. Assume, that at $\tau = 0$, terminal voltage gains its maximum value.

5)   Draw time characteristic of decaying phase voltage u_U.

Specifications: $x_s = x_r' = 2.6$ p.u., $\qquad x_h = 2.5$ p.u., $\qquad \dfrac{r_r'}{x_r'} = \dfrac{1}{100}$

1) Reference frame is oriented to rotating-field: $\omega_{syn} = \omega_s$ (per unit!)

Equation of rotor: $\quad 0 = r_r' \cdot \underline{i}_r' + \dfrac{d\underline{\psi}_r'}{d\tau} + j(\omega_s - \omega_m) \cdot \underline{\psi}_r'$

No load: $s = 0$, $\qquad \omega_m = \omega_s$, $\quad \rightarrow \quad \omega_s - \omega_m = 0$

After disconnection from the grid: $\quad \underline{i}_s = 0$

Rotor flux linkage: $\underline{\psi}_r' = x_h \cdot \underline{i}_s + x_r' \cdot \underline{i}_r' = x_r' \cdot \underline{i}_r' \quad$ after disconnection from the grid

$\Rightarrow$ Equation of the rotor flux linkage: $t > 0 \Rightarrow \tau = \omega_N \cdot t > 0$:

$\dfrac{d\underline{\psi}_r'}{d\tau} + \dfrac{r_r'}{x_r'} \cdot \underline{\psi}_r' = 0 \quad$ 1st order differential equation

2) Initial condition:

Main flux linkage BEFORE the turn-off:

Motor operation at no load: $\underline{i}_s \neq 0$, $\underline{i}_r' = 0 \Rightarrow \underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}_r' = x_s \cdot \underline{i}_s$
Equation of stator voltage: $\tau < 0$:

$$\underline{u}_{s0} = \underbrace{r_s \cdot \underline{i}_{s0}}_{\approx 0} + \underbrace{\dfrac{d\underline{\psi}_{s0}}{d\tau}}_{=0} + \underbrace{j\omega_s}_{1} \cdot \underline{\psi}_{s0} \qquad \Rightarrow \qquad \underline{\psi}_{s0} = \dfrac{\underline{u}_{s0}}{j} = x_s \cdot \underline{i}_{s0}$$

$$\Rightarrow \underline{\psi}_r' = x_h \cdot \underline{i}_{s0} + x_r' \cdot \underline{i}_r' = x_h \cdot \underline{i}_{s0}$$

Directly AFTER the turn-off: $\tau = 0$: $\quad \underline{\psi}_r' = x_h \cdot \underline{i}_{s0}(\tau = 0) = \underline{\psi}_{r0}'$

$$\underline{\psi}_r' = \underline{u}_{s0} \cdot \dfrac{x_h}{jx_s}$$

Solution of the differential equation:

$$\underline{\psi}_r'(\tau) = \underline{\psi}_r'(\tau = 0) \cdot e^{\frac{-\tau}{\tau_r}}, \qquad \tau_r = \dfrac{x_r'}{r_r'} : \quad \text{rotor open-circuit time constant}$$

$$\underline{\psi}_r'(\tau) = -j \cdot \underline{u}_{s0} \cdot \dfrac{x_h}{x_s} \cdot e^{\frac{-\tau}{\tau_r}}$$

3)   Equation of stator voltage: $\tau > 0$, $\quad r_s = 0$, $\quad \omega_s = 1$

$$\underline{u}_s = \frac{d\underline{\psi}_s}{d\tau} + j\underline{\psi}_s = \frac{x_h}{x_r'}\cdot\left(\frac{d\underline{\psi}_r'}{d\tau} + j\underline{\psi}_r'\right) = \frac{x_h}{x_r'}\cdot\left(-j\underline{u}_{s0}\cdot\frac{x_h}{x_s}\right)\cdot\left(-\frac{1}{\tau_r} + j\right)\cdot e^{-\frac{\tau}{\tau_r}}$$

$$\underline{\psi}_s = x_s\cdot\underline{i}_s + x_h\cdot\underline{i}_r' = x_h\cdot\underline{i}_r' = \frac{x_h}{x_r'}\cdot\underline{\psi}_r' \qquad \left(\underline{\psi}_r' = x_h\cdot\underline{i}_s + x_r\cdot\underline{i}_r'\right)$$
$$\underset{=0}{} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underset{=0}{}$$

$$\underline{u}_s(\tau) = -j(1-\sigma)\cdot\underline{u}_{s0}\cdot\left(-\frac{1}{\tau_r} + j\right)\cdot e^{\frac{-\tau}{\tau_r}}$$

$$\frac{x_h^2}{x_s\cdot x_r'} = 1-\sigma$$

4) $\left|\underline{u}_s(\tau)\right|\cdot e^{j\delta} = \left|\underline{u}_s(\tau)\right|, \qquad \delta = \underbrace{\omega_s}_{=1}\cdot\tau + \delta_0, \qquad \delta_0 = 0$

Rotating-field   Stator field
reference frame   reference frame

$$\underline{u}_s(\tau) = -j(1-\sigma)\cdot\underline{u}_{s0}\cdot\left(\frac{-1}{\tau_r} + j\right)\cdot e^{-\frac{\tau}{\tau_r}}\cdot e^{j\tau}$$

$$u_U(\tau) = \mathrm{Re}\left\{\underline{u}_s(\tau)\right\} = \mathrm{Re}\left\{\underline{u}_{s0}\cdot(1-\sigma)\cdot\left(1 + j\cdot\frac{1}{\tau_r}\right)\cdot e^{-\frac{\tau}{\tau_r}}\cdot(\cos\tau + j\sin\tau)\right\}, \quad \frac{1}{\tau_r} = 0.01 \ll 1$$

$$\cong \mathrm{Re}\left\{\underline{u}_{s0}\cdot(1-\sigma)\cdot e^{-\frac{\tau}{\tau_r}}\cdot(\cos\tau + j\sin\tau)\right\},$$

$$\underline{u}_{s0} = u_{s0} \Rightarrow \tau = 0 : u_U(0) : \text{maximum value}$$

$$u_U(\tau) = u_{s0}\cdot(1-\sigma)\cdot e^{-\frac{\tau}{\tau_r}}\cdot\cos\tau$$

5) $\omega_N\cdot t = \tau :$ $\ 1\,\text{period} \hat{=} \tau = 2\pi = 6.28$, $\quad \sigma = 1 - \frac{2.5^2}{2.6^2} \Rightarrow 1-\sigma = 0.92$

At $\tau = 0$, amplitude of stator voltage jumps from 100 % down to 92 %, as stator current is zero, thus stator leakage flux is zero, too.
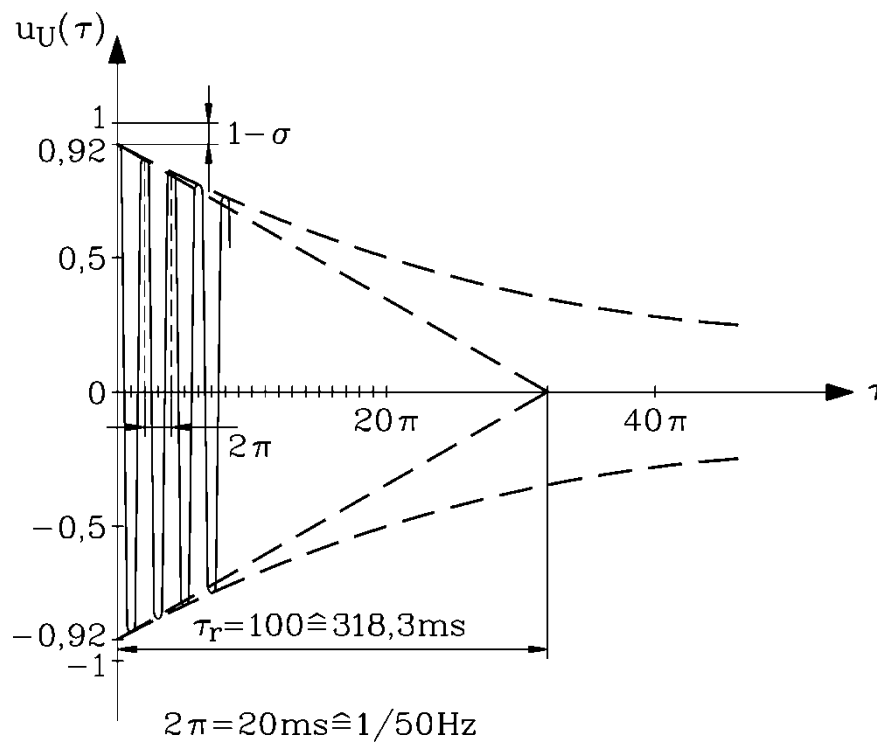
$$u_U(\tau)$$

Fig. 7.1-1: Decaying stator phase voltage in phase U after separating the induction machine at no-load operation from the grid

Practical use of the result:
By taking the time function via storage oscilloscope of the decaying stator voltage,

-     the leakage coefficient σ and
-     the rotor open-circuit time constant τ_r

may be determined from experiment.

Exercise 7.2　　　　　　　　　　　　　　　　　　　　　　　　　　　7.2/1

Sudden short circuit of an induction machine

A **3-phase induction motor** is running at no-load connected to a 60 Hz grid.

Motor specification: U_N = 460 V Y, 60 Hz, rated current 22 A, no-load current 9 A, *Blondel's* leakage coefficient σ = 0.07. Measurement of resistance between two stator terminals results in 0.4 Ω (rotor open-circuit time constant $\tau_r = x'_r / r'_r = 70$).

Calculate

1)　　the rated impedance and per-unit value of the stator resistance.

2)　　the primary reactance X_s, its per-unit value x_s and corresponding inductance L_s.

3)　　the space vector of stator and rotor flux linkage in rotating-field oriented reference frame. Assume that $x_s = x'_r$!

4)　　A short-circuit occurs at the terminals suddenly. How big is the maximum value of the sudden short circuit current in the worst case, when neglecting any damping of current? The short circuit current decays to the steady state value. What is the value of this steady state level? With which typical time constant does associated short-circuit alternating torque decay and to which steady state value? How much time is needed approximately to reach this steady state value?
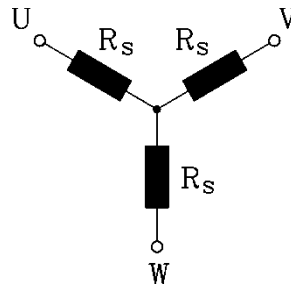
1)

Fig. 7.2-1: Star-connected stator winding

$$Z_N = \frac{U_N}{\sqrt{3} \cdot I_N} = \frac{460\,\text{V}}{\sqrt{3} \cdot 22\,\text{A}} = 12.07\,\Omega, \quad 2R_s = R_{UV} \Rightarrow R_s = 0.2\,\Omega,$$

$$\frac{R_s}{Z_N} = r_s = \frac{0.2}{12.07} = 0.0166\,\text{p.u.}$$

2)   Reference frame is oriented to rotating-field: $\omega_{syn} = \omega_s = 1$

Space vector is fixed in steady-state operation $\left(\dfrac{d}{d\tau} = 0\right)$ in rotating-field reference frame.

Equation of stator voltage: $\underline{u}_s = r_s \cdot \underline{i}_s + \underbrace{\dfrac{d\underline{\psi}_s}{d\tau}}_{=0} + j\omega_s \cdot \underline{\psi}_s$

$$\underline{\psi}_s = x_s \cdot \underline{i}_s + x_h \cdot \underline{i}'_r = \underset{(\text{No load:}\ \underline{i}'_r = 0)}{x_s \cdot \underline{i}_s}$$

$$\underline{u}_s = r_s \cdot \underline{i}_s + jx_s \cdot \underline{i}_s \Rightarrow \frac{u_s}{i_{s0}} = r_s + jx_s$$

$$\left(\frac{u_s}{i_{s0}}\right)^2 = r_s^2 + x_s^2 \Rightarrow x_s = \sqrt{\left(\frac{1}{0.41}\right)^2 - 0.0166^2} = 2.44\,\text{p.u.}, \quad \frac{I_0}{I_N} = i_{s0} = \frac{9}{22} = 0.41\,\text{p.u.}$$

$$X_s = x_s \cdot Z_N = 2.44 \cdot 12.07\,\Omega = 29.5\,\Omega, \qquad L_s = \frac{X_s}{2\pi \cdot f_N} = \frac{29.5}{2\pi \cdot 60}\,\text{H} = 78.3\,\text{mH}$$

3) $\underline{u}_s = r_s \cdot \underline{i}_{s0} + j\omega_s \cdot \underline{\psi}_s, \qquad \underline{\psi}_s = x_s \cdot \underline{i}_{s0}, \quad \omega_s = 1, \qquad u_s = 1\,\text{p.u.} \quad (\text{real axis} \parallel \underline{u}_s)$

$$\underline{i}_{s0} = \frac{u_s}{r_s + jx_s}, \qquad \underline{\psi}_s = x_s \cdot \underline{i}_{s0} = \frac{x_s \cdot u_s}{r_s + jx_s} = \frac{2.44 \cdot 1}{0.0166 + j2.44} = 0.0068 - j \cdot 1.0 \quad \text{p.u.}$$

$$\underline{\psi}'_r = x_h \cdot \underline{i}_s + x'_r \cdot \underline{i}'_r = x_h \cdot \underline{i}_{s0} = \frac{x_h \cdot u_s}{r_s + jx_s},$$

$$x_h = \sqrt{(1-\sigma) \cdot x_s \cdot x'_r} = \underbrace{\sqrt{1-\sigma} \cdot x_s}_{x_s^2} = 2.35\,\text{p.u.}$$

$$\underline{\psi}'_r = \frac{2.35 \cdot (0.0166 - j2.44)}{0.0166^2 + 2.44^2} = 0.0066 - j0.96 \quad \text{p.u.}$$

4)   Peak current of sudden short-circuit:

Worst-case at short-circuit during zero crossing of voltage: $\hat{I}_{sc} = \dfrac{u_s}{x_s} \cdot \left(\dfrac{2}{\sigma} - 1\right) \cdot \sqrt{2} \cdot I_N$

$\hat{I}_{sc} = \dfrac{1}{2.44} \cdot \left(\dfrac{2}{0.07} - 1\right) \cdot \sqrt{2} \cdot I_N = 16.0 \cdot I_N = 351.6$ A : This current decays to zero!

Short-circuit time constant: $\tau_{sc} = \dfrac{1}{1/\tau_{s\sigma} + 1/\tau_{r\sigma}} \cong \dfrac{\sigma x_s}{r_s + r_r'}$ for the torque.

The sudden short-circuit torque decays with T_{sc} to the final value <u>zero</u>.

The final value is attained after approximately 3 T_{sc}, due to $e^{-\frac{3T_{sc}}{T_{sc}}} = \dfrac{1}{e^3} = 0.05$. The

torque of the short-circuit has already decayed to 5 % of the peak value after 26.3 ms

$\left(0.05 \cdot 351.6 = 17.6 \text{ A}, \quad \dfrac{17.6}{22 \cdot \sqrt{2}} = 0.57 \cdot I_N\right)$.

$T_{sc} = \dfrac{\tau_{sc}}{\omega_N} \cong \dfrac{\sigma x_s}{(r_s + r_r') \cdot \omega_N} = \dfrac{0.07 \cdot 2.44}{(0.0166 + 0.035) \cdot 2\pi 60} = 8.78$ ms, $3 \cdot T_{sc} = 26.3$ ms

$\tau_r = \dfrac{x_r'}{r_r'} = \dfrac{x_s}{r_r'} = 70 \Rightarrow r_r' = \dfrac{x_s}{\tau_r} = \dfrac{2.44}{70} = 0.035$ p.u.

$\quad x_s = x_r' \qquad R_r' = Z_N \cdot r_r' = 0.035 \cdot 12.07 = 0.42\,\Omega$

Exercise 7.3   7.3/1

Starting of an induction machine:

A wedge bar cage induction machine with the following nameplate data is given:

Frame size 250 mm, 55 kW, 220V/380V, Δ/Y, 178 A/103 A, 50 Hz, 1455/min, $\cos\varphi = 0.885$, continuous duty, Thermal Class B, mounting IM B3

From the catalogue additional details are given: starting current 4.4-times, starting torque 1.55-times, break down torque 1.94-times, inertia 0.9 kgm^2, machine mass 440 kg

From the calculation sheet of the manufacturer there are the addition data available:
Break down slip 13.3 %, at 75° according to Thermal Class B, stator resistance $R_s = 0.06$ Ω, rotor resistance $R'_r = 0.0643$ Ω, stator reactance $X_s = 8.8$ Ω,
rotor reactance $X'_r = 9.03$ Ω, leakage coefficient $\sigma = 0.067$.

Calculate:

1.)   the nominal torque, breakdown torque, starting torque in Nm, starting current at Δ- and Y- connection and the motor efficiency,

2.)   the nominal impedance and the motor equivalent circuit parameters for rated slip in per unit (p.u.). Calculate also in p.u.: the nominal torque, the nominal voltage (peak value), the nominal current (peak value), the nominal frequency, the nominal apparent power, the nominal power,

3.)   the start-up time constant in seconds and in p.u.

4.)   The motor is starting with no load. Is the static break down torque reached? If not, please calculate the maximum occurring torque. In a second test the machine starts up with no load at half of the nominal voltage. Is static break down torque reached?

1.) $M_N = \dfrac{P_N}{2\pi \cdot n_N} = \dfrac{55000}{2\pi \dfrac{1455}{60}} = 361 \text{Nm}$

$M_b = 1.94 M_N = 700 \text{Nm} ; \quad M_1 = 1.55 M_N = 560 \text{Nm}$

$I_{1Y} = 4.4 \cdot 103 = 453 \text{A} ; \qquad I_{1\Delta} = 4.4 \cdot 178 = 783 \text{A}$

$\eta = \dfrac{P_N}{\sqrt{3} \cdot U_N \cdot I_N \cdot \cos\varphi} = \dfrac{55000}{\sqrt{3} \cdot 380 \cdot 103 \cdot 0.885} = 91.67\%$

2.) $Z_N = \dfrac{U_{N,ph}}{I_{N,ph}} = \dfrac{220}{103} = 2.136\,\Omega$

$r_s = \dfrac{R_s}{Z_N} = \dfrac{0.06}{2.136} = 0.028 \text{p.u.} ; \qquad r_r' = \dfrac{R_r'}{Z_N} = \dfrac{0.064}{2.136} = 0.03 \text{p.u.}$

$x_s = \dfrac{X_s}{Z_N} = \dfrac{8.8}{2.136} = 4.12 \text{p.u.} ; \qquad x_r' = \dfrac{X_r'}{Z_N} = \dfrac{9.03}{2.136} = 4.23 \text{p.u.}$

$x_h = \sqrt{(1-\sigma)x_s x_r'} = \sqrt{(1-0.067) \cdot 4.12 \cdot 4.23} = 4.032 \text{p.u.}$

$m_N = \dfrac{M_N}{M_{ref}} = \dfrac{361}{431.6} = 0.836 \text{p.u.} ; \quad M_{ref} = \dfrac{S_N}{\dfrac{\omega_{syn}}{p}} = \dfrac{\sqrt{3} \cdot 380 \cdot 103}{\dfrac{2\pi \cdot 50}{2}} = 431.6 \text{Nm}$

Check: $m_N = \dfrac{\cos\varphi \cdot \eta}{1 - s_N} = \dfrac{0.885 \cdot 0.9167}{1 - 0.03} = 0.836 ; \quad s_N = \dfrac{1500 - 1455}{1500} = 0.03$

$u_N = \dfrac{U_{N,ph} \cdot \sqrt{2}}{U_{N,ph} \cdot \sqrt{2}} = 1 \text{p.u.} ; \quad i_N = \dfrac{I_N \cdot \sqrt{2}}{I_N \cdot \sqrt{2}} = 1 \text{p.u.} ; \quad \dfrac{f_N}{f_N} = 1 \text{p.u.} ;$

apparent power $s_N = \dfrac{S_N}{\dfrac{3}{2} \cdot U_{N,ph} \cdot \sqrt{2} \cdot I_{N,ph} \cdot \sqrt{2}} = 1 \text{p.u.}$

$p_N = \dfrac{P_N}{\dfrac{3}{2} \cdot U_{N,ph} \cdot \sqrt{2} \cdot I_{N,ph} \cdot \sqrt{2}} = \dfrac{55000}{\sqrt{3} \cdot 380 \cdot 103} = 0.81 \text{p.u.}$

Check: $p_N = \cos\varphi \cdot \eta = 0.885 \cdot 0.9167 = 0.81 \text{p.u.}$

3.) $T_J = J \dfrac{\overset{\omega_N}{\overbrace{\dfrac{p}{}}}}{M_{ref}} = 0.9 \cdot \dfrac{\dfrac{2\pi \cdot 50}{2}}{431.6} = 0.328\text{s} \; ; \; \tau_J = \omega_N \cdot T_J = 2\pi \cdot 50 \cdot 0.328 = 103$

4.)   *Pfaff-Jordan* parameter P:

$$P = \left(\frac{\omega_s}{u_s} \cdot \frac{x_s}{x_h} \right)^2 \cdot \tau_J \cdot r'_r \cdot s_b \Rightarrow$$

a.) $\omega_s = \omega_N$; $u_s = u_N = 1$

$$P = \left(\frac{1}{1} \cdot \frac{4.12}{4.032} \right)^2 \cdot 103 \cdot 0.03 \cdot 0.133 = 0.43$$

according to the P-Curve $\dfrac{M_{b,dyn}}{M_{b,stat}} = f(P)$ is: $\dfrac{M_{b,dyn}}{M_{b,stat}} = 0.785$

$M_{b,dyn} = 0.785 \cdot 700 = 550\text{Nm}$ The static break down torque is not reached ("fast" start-up)! The maximum torque that occurs is the dynamic break-down torque $M_{b,dyn}$.

b.) $\omega_s = \omega_N = 1$; $u_s = u_N / 2 = 0.5 \Rightarrow P = 0.43 \left[\left(\dfrac{1}{2} \right)^{-1} \right]^2 = 1.72$

$\dfrac{M_{b,dyn}}{M_{b,stat}} = 0.9, \; M_{b,stat} \square \, u_s^2 \; \Rightarrow M_{b,dyn} = 157.5 \text{ Nm}$

Due to the lower voltage the start up is slower, so the dynamic breakdown torque is close to the static breakdown torque.

Exercise 8.1   8.1/1

Sudden short-circuit of an hydro-electric synchronous salient pole generator

The machine has following specifications:
20 poles, 32.2 MVA, 11 kV, $\cos\varphi = 0.9$, bore diameter = 3.8 m, $X_{dh} = 3.42\ \Omega$, $X_{qh} = 0.6 \cdot X_{dh}$, $X_{s\sigma} = 0.52\ \Omega$, $X_{f\sigma} = 0.484\ \Omega$, $X_{D\sigma} = 1.73\ \Omega$, $X_{Q\sigma} = 0.56\ \Omega$, $R_s = 15.2\ m\Omega$, $R_f = 0.002\ \Omega$, $R_D = 0.31\ \Omega$, $R_Q = 0.08\ \Omega$.
The reactances and resistances are already converted to the stator side at 50 Hz and 75°C (Thermal Class B).

Note: Above rotor values are already converted with the respective transformation ratio to the stator side. The "true", measurable value of the ohmic resistance of the field winding is for example $R_f = 0.23\ \Omega$. For this reason the converted values of the field winding are smaller than those of the damper winding, because the transformation ratio "stator- to- field" is very small ($\ddot{u} \sim N_s/N_f$, N_f is large, since all 20 poles are connected in series: $N_s = 96$, $N_f = 20 \times 55 = 1100$).

Calculate

1)   rated current, rated impedance and per-unit value of above resistances and reactances.

2)   the reactances: direct-axis X_d, quadrature-axis X_q, transient X_d', direct-axis subtransient X_d'' and quadrature-axis subtransient X_q'' in per-unit value. Is the machine "subtransient symmetrical"?

3)   the time constants: open-circuit field T_f, direct-axis open-circuit damper T_D, quadrature- axis open-circuit damper T_Q, armature time constant T_a, transient T_d' and subtransient   time constants T_d''.

4)   The worst case of sudden short-circuit at the generator terminals shall be calculated! Use   the formula indicated in the lecture notes for the calculation of the sudden short-circuit   current in phase U in case, when the short circuit happens at zero voltage. How much   smaller/bigger is the value compared to the simplified relation

$$\hat{i}_{sc} = 0.8 \cdot \frac{2 \cdot \sqrt{2}}{x_d''} \cdot I_N\ ?$$

1) $I_N = \dfrac{S_N}{\sqrt{3} \cdot U_N} = \dfrac{32200}{\sqrt{3} \cdot 11} = \underline{\underline{1691 A}}$

   $Z_N = \dfrac{U_N}{\sqrt{3} \cdot I_N} = \dfrac{11000}{\sqrt{3} \cdot 1691} = \underline{\underline{3.76 \,\Omega}}$

   $x_{dh} = \dfrac{X_{dh}}{Z_N} = \dfrac{3.42}{3.76} = \underline{\underline{0.91 \text{ p.u.}}}, \qquad X_{dh} = \dfrac{X_{qh}}{0.6} \;\Rightarrow\; x_{qh} = 0.6 \cdot 0.91 = \underline{\underline{0.55 \text{ p.u.}}}$

   $x_{d\sigma} = x_{q\sigma} = x_{s\sigma} = \dfrac{X_{s\sigma}}{Z_N} = \dfrac{0.52}{3.76} = \underline{\underline{0.138 \text{ p.u.}}}, \qquad x_{f\sigma} = \dfrac{0.484}{3.76} = \underline{\underline{0.129 \text{ p.u.}}}$

   $x_{D\sigma} = \dfrac{1.73}{3.76} = \underline{\underline{0.46 \text{ p.u.}}}, \qquad x_{Q\sigma} = \dfrac{0.56}{3.76} = \underline{\underline{0.15 \text{ p.u.}}}$

   $r_s = \dfrac{R_s}{Z_N} = \dfrac{15.2}{3.76} \cdot 10^{-3} = \underline{\underline{0.004 \text{ p.u.}}}, \qquad r_f = \dfrac{0.002}{3.76} = \underline{\underline{0.00053 \text{ p.u.}}}$

   $r_D = \dfrac{0.31}{3.76} = \underline{\underline{0.082 \text{ p.u.}}}, \qquad r_Q = \dfrac{0.08}{3.76} = \underline{\underline{0.021 \text{ p.u.}}}$

2) $x_d = x_{dh} + x_{s\sigma} = 0.91 + 0.138 = \underline{\underline{1.048 \text{ p.u.}}}, \qquad x_q = x_{qh} + x_{s\sigma} = 0.55 + 0.138 = \underline{\underline{0.688 \text{ p.u.}}}$

   $x_d' = x_{s\sigma} + \dfrac{x_{dh} \cdot x_{f\sigma}}{x_{dh} + x_{f\sigma}} = 0.138 + \dfrac{0.91 \cdot 0.129}{0.91 + 0.129} = \underline{\underline{0.25 \text{ p.u.}}}$

   $x_d'' = x_{s\sigma} + \dfrac{x_{dh} \cdot x_{f\sigma} \cdot x_{D\sigma}}{x_{dh} \cdot x_{f\sigma} + x_{dh} \cdot x_{D\sigma} + x_{f\sigma} \cdot x_{D\sigma}} =$

   $= 0.138 + \dfrac{0.91 \cdot 0.129 \cdot 0.46}{0.91 \cdot (0.129 + 0.46) + 0.129 \cdot 0.46} = \underline{\underline{0.229 \text{ p.u.}}}$

   $x_q'' = x_{s\sigma} + \dfrac{x_{qh} \cdot x_{Q\sigma}}{x_{qh} + x_{Q\sigma}} = 0.138 + \dfrac{0.55 \cdot 0.15}{0.55 + 0.15} = \underline{\underline{0.256 \text{ p.u.}}}$

   $x_q'' = 1.12 \cdot x_d'' : \;\Rightarrow\; x_q'' \neq x_d'' : \quad$ subtransient asymmetrically, but asymmetry is <u>small</u> !

3) $T_f = T_{d0}' = \dfrac{\tau_f}{\omega_N}, \quad \tau_f = \dfrac{x_f}{r_f} = \dfrac{x_{dh} + x_{f\sigma}}{r_f} = \dfrac{0.91 + 0.129}{0.00053} = 1960.4$

   $T_f = \dfrac{1}{2 \cdot \pi \cdot 50} \cdot 1960.4 = \underline{\underline{6.24 \text{ s}}}$

   $\tau_D = \dfrac{x_D}{r_D} = \dfrac{x_{dh} + x_{D\sigma}}{r_D} = \dfrac{0.91 + 0.46}{0.082} = 16.7, \qquad T_D = \dfrac{16.7}{2 \cdot \pi \cdot 50} = \underline{\underline{0.053 \text{ s}}}$

   $\tau_Q = \dfrac{x_Q}{r_Q} = \dfrac{x_{qh} + x_{Q\sigma}}{r_Q} = \dfrac{0.55 + 0.15}{0.021} = 33.3, \qquad T_Q = \dfrac{33.3}{2 \cdot \pi \cdot 50} = \underline{\underline{0.106 \text{ s}}}$

The <u>damper time constants are clearly shorter</u> than the field time constant (factor: 100!). This only makes the <u>definition of a transient reactance</u> x'_d meaningful. It describes that condition, where the damper-currents are already decayed after e.g. a "load step", whereas the induced current in the field winding flows nearly unchanged.

$$T_a = \frac{\tau_a}{\omega_N}, \quad \tau_a = \frac{1}{\alpha_r} = \frac{2 \cdot x''_d \cdot x''_q}{x''_d + x''_q} \cdot \frac{1}{r_s} = \frac{2 \cdot 0.229 \cdot 0.256}{0.229 + 0.256} \cdot \frac{1}{0.004} = 60.4, \quad T_a = \frac{60.4}{2 \cdot \pi \cdot 50} = \underline{\underline{0.192\,s}}$$

$$T'_d = \frac{\tau'_d}{\omega_N}, \quad \tau'_d = \frac{x'_d}{x_d} \cdot \tau_f = \frac{0.25}{1.048} \cdot 1960.4 = 467.65, \quad T'_d = \frac{467.65}{2 \cdot \pi \cdot 50} = \underline{\underline{1.49\,s}}$$

$$T''_d = \frac{\tau''_d}{\omega_N}, \quad \tau''_d = \frac{x''_d}{x'_d} \cdot \tau''_{d0}, \quad \tau''_{d0} = \sigma_{fD} \cdot \frac{x_D}{r_D} = \sigma_{fD} \cdot \tau_D = 0.42 \cdot 16.7 = 7.0$$

$$\sigma_{fD} = 1 - \frac{x^2_{dh}}{(x_{dh} + x_{f\sigma})(x_{dh} + x_{D\sigma})} = 1 - \frac{0.91^2}{(0.91 + 0.129)(0.91 + 0.46)} = 0.42$$

$$\tau''_d = \frac{0.229}{0.25} \cdot 7.0 = 6.4 \implies T''_d = \frac{6.4}{2 \cdot \pi \cdot 50} = \underline{\underline{0.0204\,s}}$$

4)   <u>Sudden short-circuit current</u>: short-circuit at the generator terminals :

$$\hat{i}_{sc} \cong 0.8 \cdot \frac{2 \cdot \sqrt{2}}{0.229} \cdot 1691 = 16709\,A = \underline{\underline{9.9 \text{ - fold}}}$$

Sudden short-circuit at voltage zero: $\gamma_0 = 0$, maximum current occurs at $\omega_N \cdot t = \pi$!
Sudden short-circuit after no-load with rate no-load voltage: $u_0 = 1$.

$$i_U(\tau) = i_U(\pi) = -u_0 \left[\frac{1}{x_d} + \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \cdot e^{-\frac{\pi}{\tau'_d}} + \left(\frac{1}{x''_d} - \frac{1}{x'_d} \right) \cdot e^{-\frac{\pi}{\tau''_d}} \right] \cdot \cos(\pi + 0) +$$

$$+ u_0 \left[\frac{1}{2} \cdot \left(\frac{1}{x''_d} + \frac{1}{x''_q} \right) \cdot \cos(0) + \frac{1}{2} \cdot \left(\frac{1}{x''_d} - \frac{1}{x''_q} \right) \cdot \cos(2\pi + 0) \right] \cdot e^{-\pi \cdot \alpha_r} =$$

$$= - \left[\frac{1}{1.048} + \left(\frac{1}{0.25} - \frac{1}{1.048} \right) \cdot e^{-\frac{\pi}{467.65}} + \left(\frac{1}{0.229} - \frac{1}{0.25} \right) \cdot e^{-\frac{\pi}{6.4}} \right] \cdot (-1) +$$

$$+ \left[\frac{1}{2} \cdot \left(\frac{1}{0.229} + \frac{1}{0.256} \right) \cdot 1 + \frac{1}{2} \cdot \left(\frac{1}{0.229} - \frac{1}{0.256} \right) \cdot 1 \right] \cdot e^{-\frac{\pi}{60.4}} =$$

$$= [0.954 + 3.025 + 0.225] + 4.145 = 8.349$$

$$i_{\mathrm{U}}(\pi) = \frac{\hat{i}_{\mathrm{sc}}}{\sqrt{2} \cdot I_{\mathrm{N}}} = 8.349, \quad \hat{i}_{\mathrm{sc}} = \sqrt{2} \cdot 1691 \cdot 8.349 = 11.8 \cdot I_{\mathrm{N}} = 19967\,\mathrm{A}$$

The accurate peak value is higher by around 19 %, since the DC-component decays slower than was assumed in simplified equation.

The theoretical maximum value without decay of the DC-component:

$$\hat{i}_{\mathrm{sc}} = \frac{2 \cdot \sqrt{2}}{x_{\mathrm{d}}''} \cdot I_{\mathrm{N}} = \frac{2 \cdot \sqrt{2}}{0.229} \cdot 1691 = 12.35 \cdot I_{\mathrm{N}}$$
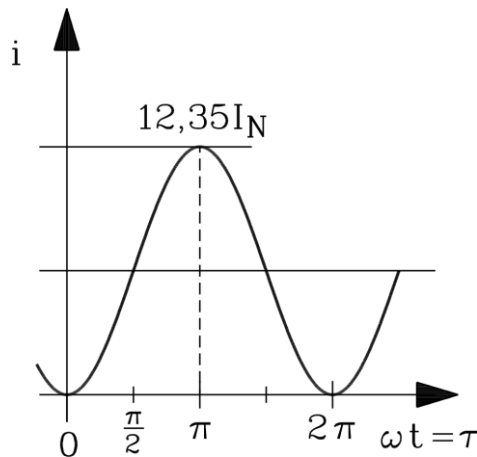
Fig. 8.1-1: Short circuit phase current with neglected damping for the worst case (short circuit at zero voltage crossing)

Exercise 8.2   8.2/1

Sudden short-circuit of a cylindrical-rotor generator (Turbine generator)

A two-pole turbine generator in a steam power station with the data: 400 MVA, 21 kV, Y, 60 Hz, $\cos\varphi = 0.75$, $x_d = 2.2$ p.u., $x_q \approx x_d$, $x_d'' = x_q'' = 0.17$ p.u., $x_d' = 0.28$ p.u., $T_d'' = 0.2$ s, $T_d' = 1.1$ s, $T_a = 0.3$ s
suffers a sudden terminal short-circuit, after having been operated at rated conditions.

At rated operation the internal voltage U_h - induced by air gap flux - is 1.1-fold rated voltage. This voltage feeds on the short-circuit.

1) How big is the sudden short-circuit current in phase U
    a) in case the short-circuit happens at voltage maximum,
    b) in case the short-circuit happens at voltage zero ?

2) Calculate and draw the time function of short-circuit torque neglecting damping.

$$I_N = \frac{S_N}{\sqrt{3} \cdot U_N} = \frac{400000}{\sqrt{3} \cdot 21} = 10997 \, \text{A}$$

$$M_{\text{ref}} = \frac{S_N}{\omega_{\text{syn}}} = \frac{400000 \cdot 10^3}{2 \cdot \pi \cdot \dfrac{3600}{60}} = 1061 \, \text{kNm}, \qquad u_0 = 1.1 \, \text{p.u.}$$

<u>1a) Sudden short-circuit at voltage maximum:</u> $\gamma_0 = \dfrac{\pi}{2}$, maximum current arises at $\tau = \dfrac{\pi}{2}$.
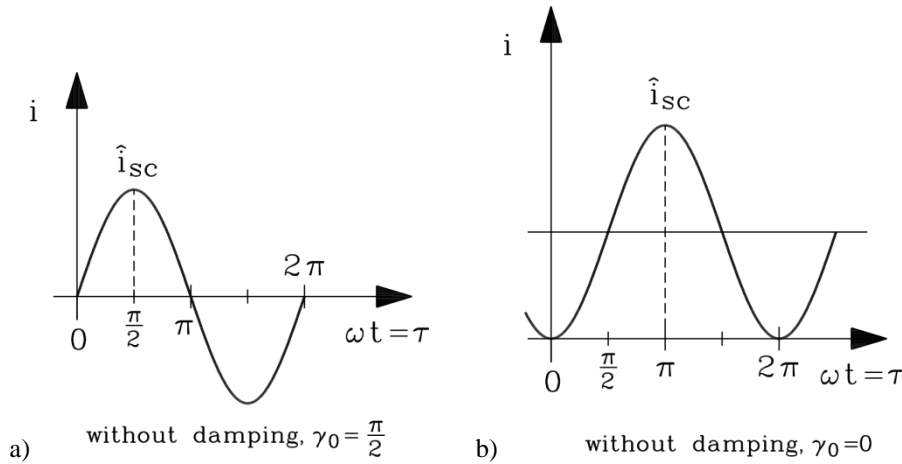

a)   without damping, $\gamma_0 = \dfrac{\pi}{2}$ b)   without damping, $\gamma_0 = 0$

<u>Fig. 8.2-1:</u> Short circuit phase current with neglected damping a) for best case: Short circuit at maximum voltage, b) for worst case: Short circuit at zero voltage crossing

$$i_U\left(\frac{\pi}{2}\right) = -u_0\left[\frac{1}{x_d} + \left(\frac{1}{x_d'} - \frac{1}{x_d}\right) \cdot e^{-\frac{\pi/2}{\tau_d'}} + \left(\frac{1}{x_d''} - \frac{1}{x_d'}\right) \cdot e^{-\frac{\pi/2}{\tau_d''}}\right] \cdot \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right) +$$

$$+ u_0\left[\frac{1}{2} \cdot \left(\frac{1}{x_d''} + \frac{1}{x_q''}\right) \cdot \cos\left(\frac{\pi}{2}\right) + \frac{1}{2} \cdot \left(\frac{1}{x_d''} - \frac{1}{x_q''}\right) \cdot \cos\left(2\frac{\pi}{2} + \frac{\pi}{2}\right)\right] \cdot e^{\frac{-\pi}{2} \cdot \alpha_r} =$$

$$= 1.1 \cdot \left[\frac{1}{2.2} + \left(\frac{1}{0.28} - \frac{1}{2.2}\right) \cdot e^{-\frac{\pi/2}{414.7}} + \left(\frac{1}{0.17} - \frac{1}{0.28}\right) \cdot e^{-\frac{\pi/2}{75.4}}\right] + 0 + 0 = 6.4 \, \text{p.u.}$$

$$\cos\left(\frac{\pi}{2}\right) = 0, \qquad \frac{1}{x_d''} - \frac{1}{x_q''} = 0, \quad \tau_d' = 2 \cdot \pi \cdot f_N \cdot T_d' = 2 \cdot \pi \cdot 60 \cdot 1.1 = 414.7,$$

$$\tau_d'' = 2 \cdot \pi \cdot 60 \cdot 0.2 = 75.4$$

$$\frac{1}{\alpha_r} = \tau_a = 2 \cdot \pi \cdot f_N \cdot T_a = 113.1,$$

$$\hat{i}_{sc} = 6.4 \cdot \sqrt{2} \cdot I_N = 6.4 \cdot \sqrt{2} \cdot 10997 = \underline{99559 \cdot \text{A}}$$

Exercise 8.2   8.2/3

1b) Sudden short-circuit at voltage zero: $\gamma_0 = 0$, maximum current arises at $\tau = \pi$:

$$i_U(\pi) = -1.1 \cdot \left[\frac{1}{2.2} + \left(\frac{1}{0.28} - \frac{1}{2.2} \right) \cdot e^{-\frac{\pi}{414.7}} + \left(\frac{1}{0.17} - \frac{1}{0.28} \right) \cdot e^{-\frac{\pi}{75.4}} \right] \cdot \cos(\pi + 0) +$$

$$+ 1.1 \cdot \left[\frac{1}{2} \cdot \left(\frac{1}{0.17} \cdot 2 \right) \cdot \cos(0) + 0 \right] \cdot e^{\frac{-\pi}{113.1}}$$

$$= 1.1 \cdot [0.455 + 3.093 + 2.2166 + 5.72] = 12.63 \, \text{p.u.}$$

$$\hat{i}_{sc} = 12.63 \cdot \sqrt{2} \cdot I_N = 12.63 \cdot \sqrt{2} \cdot 10997 = \underline{196491 \text{A}}$$

2)   Short-circuit torque: $m_{sc} = -\frac{u_0^2}{x_d''} \cdot \sin(\tau) + \frac{1}{2} \cdot \left(\frac{1}{x_d''} - \frac{1}{x_q''} \right) \cdot \sin(2\tau)$

$$\hat{m}_{sc} = \frac{1.1^2}{0.17} = 7.1 \, \text{p.u.}$$

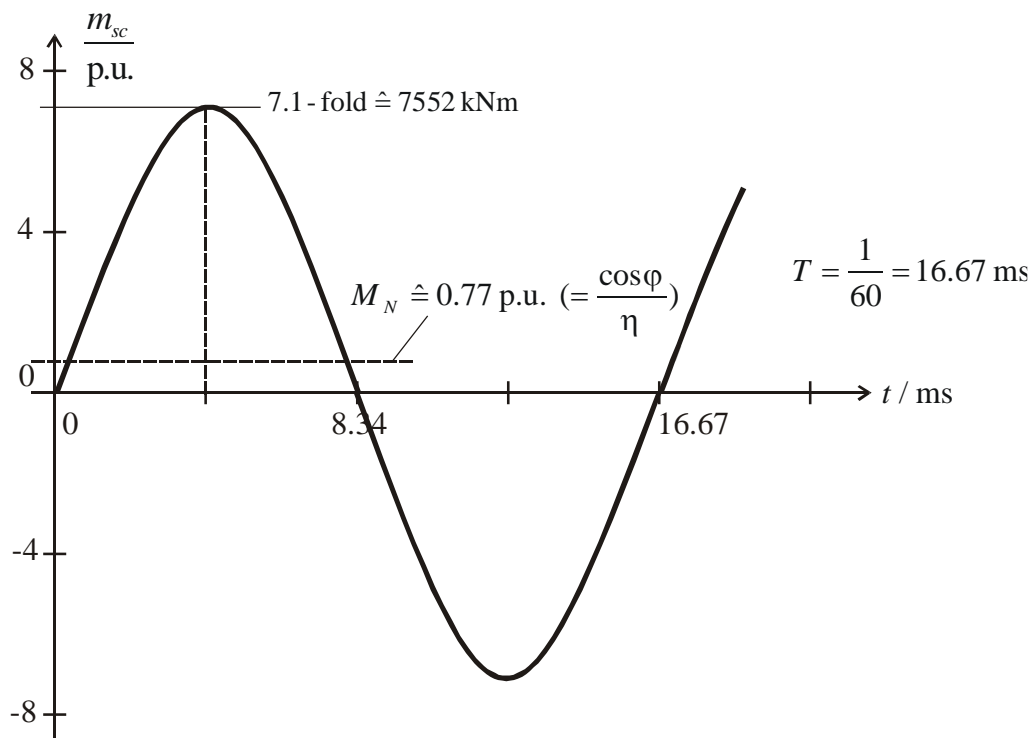$$\hat{M}_{sc} = 7.1 \cdot 1061 = 7552 \, \text{kNm}$$

Fig. 8.2-2: Short circuit torque with neglected damping

$$M_N = \frac{P_N}{2 \cdot \pi \cdot n_N \cdot \eta_N} \cong \frac{300000}{2 \cdot \pi \cdot \frac{3600}{60} \cdot 0.98} = 812 \, \text{Nm} \,\hat{=}\, 0.765 \, \text{p.u.}$$

$$\eta_N \cong 0.98, \qquad P_N = \sqrt{3} \cdot U_N \cdot I_N \cdot \cos\varphi_N = 400 \cdot 0.75 = 300 \, \text{MW}$$

Steady state short-circuit of a wind generator

A **small permanent magnet wind generator** ($x_d = x_q$) with damping cage for asynchronous starting, shows by rotation with half of rated speed at his terminals an open-circuit voltage, which amounts 50 % of the rated voltage at nominal speed. After the short-circuit of the stator windings for the same constant speed a steady state short-circuit current of 3 p.u. is flowing. The stator resistance is 0.07 p.u.

1)   How big is the p.u.- value of stator reactance x_d ?

2)   How big is the steady-state short-circuit current, when the rotating speed is increased to the rated value?

3)   How big is the steady-state short-circuit current, when the rotating speed is decreased to 10 % of rated value? Sketch the variation of the steady-state short circuit current for different stator frequency!

4)   How big is in the peak short-circuit current for 2) at sudden short-circuit at zero voltage. Neglect damping and assume $x_{D\sigma} = x_{Q\sigma} = 0.09$ p.u. and $x_{s\sigma} = 0.1$ p.u..

1)   Synchronous reference frame
      PM rotor main reactance in d- and q-axis are identical: $x_{dh} = x_h = x_{qh}, \quad x_d = x_f = x_s$

   (1) $\underline{u}_s = r_s \cdot \underline{i}_s + \dfrac{d\underline{\psi}_s}{d\tau} + j\omega_s \cdot \underline{\psi}_s$: stator winding

   (2) $0 = r'_r \cdot \underline{i}'_r + \dfrac{d\underline{\psi}'_r}{d\tau} + j(\omega_s - \omega_m) \cdot \underline{\psi}'_r$: damper cage

   (3) $\underline{\psi}_s = x_d \cdot \underline{i}_s + x_h \cdot \underline{i}'_r + \underline{\psi}_f$: stator flux linkage

   • No load: $\omega_s = \omega_m = \dfrac{1}{2}$, steady state operation $\dfrac{d}{d\tau} = 0$

   $i_1 = 0 \Rightarrow$ (1) $\underline{u}_s = 0 + 0 + j\omega_s \cdot \underline{\psi}_s = j\omega_s \cdot \underline{\psi}_f$

   (2) $\underline{\psi}_s = \underline{\psi}_f$

   (3) $0 = r'_r \cdot \underline{i}'_r + 0 + 0 \Rightarrow \underline{i}'_r = 0$; the damper current is zero.

   $\underline{u}_{s0} = j\omega_s \underline{\psi}_f :\ u_{s0} = 0.5\,\text{p.u.} = 0.5 \cdot \psi_f \Rightarrow \psi_f = 1\,\text{p.u.}$

• Short circuit: steady state, $\omega_s = \omega_m = 1/2$, $d/d\tau = 0$.

   (1) $0 = r_s \cdot \underline{i}_{s,sc} + j\omega_s \cdot \underline{\psi}_s$

   (2) $0 = r'_r \cdot \underline{i}'_r + 0 + 0 \Rightarrow \underline{i}'_r = 0$; the damper current is zero.

   (3) $\underline{\psi}_s = x_d \cdot \underline{i}_{s,sc} + 0 + \underline{\psi}_f$

   Substituting (3) in (1):
   $0 = r_s \cdot \underline{i}_{s,sc} + j\omega_s \cdot x_d \cdot \underline{i}_{s,sc} + j\omega_s \cdot \underline{\psi}_f$

   $\underline{i}_{s,sc} = \dfrac{-j\omega_s\underline{\psi}_f}{r_s + j\omega_s x_d} \Rightarrow i_{s,sc} = \dfrac{\omega_s\psi_f}{\sqrt{r_s^2 + \omega_s^2 x_d^2}} = \underline{\underline{3\,\text{p.u.}}}$

   $\sqrt{\left(\dfrac{\omega_s\psi_f}{i_{s,sc}}\right)^2 - r_s^2} \cdot \dfrac{1}{\omega_s} = x_d = \sqrt{\left(\dfrac{0.5 \cdot 1}{3}\right)^2 - 0.07^2} \cdot \dfrac{1}{0.5} = \underline{\underline{0.303\,\text{p.u.}}}$

2) $i_{s,sc} = \dfrac{1 \cdot 1.0}{\sqrt{0.07^2 + (1 \cdot 0.303)^2}} = \underline{\underline{3.216\,\text{p.u.}}}$ The steady state short-circuit current is slightly

   increased by 7.2% compared with 1) $(3.216/3 = 1.072)$.

   <u>Note</u>: For high frequencies $(\omega_s \cdot x_d \gg r_s)$ the short circuit current is independent of speed.

$$i_{s,sc} \approx \frac{\omega_s \cdot \psi_f}{\omega_s \cdot x_d} = \frac{\psi_f}{x_d} = \text{constant.}$$

3) $\omega_s = 0.1$: $i_{s,sc} = \dfrac{0.1 \cdot 1.0}{\sqrt{0.07^2 + (0.1 \cdot 0.303)^2}} = 1.31 \,\text{p.u.}$

For low frequencies $(\omega_s \cdot x_d \ll r_s)$ the influence of r_s is dominant. In the limit case $\omega_s \to 0$ the current $i_{s,sc} \to 0$.
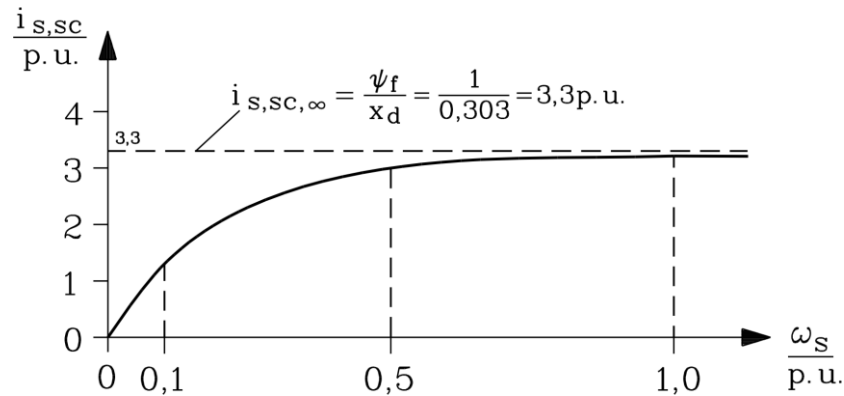

Fig. 8.3-1: Short circuit current for different stator frequency

4)   The subtransient reactance is, due to the missing of the rotor field winding, easy to calculate:
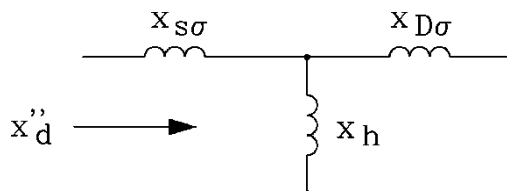
Fig. 8.3-2: Equivalent circuit per phase for calculating the stator subtransient inductance resp. reactance.

$x_h = x_s - x_{s\sigma} = 0.303 - 0.1 = 0.203 \,\text{p.u.}$

$x_d'' = x_{s\sigma} + \dfrac{x_h \cdot x_{D\sigma}}{x_h + x_{D\sigma}} = 0.1 + \dfrac{0.203 \cdot 0.09}{0.203 + 0.09} = 0.162 \,\text{p.u.}$

$\hat{i}_{sc} = \dfrac{2\sqrt{2}}{x_d''} u_0.$ For $\omega_s = 1$: $u_0 = \omega_s \cdot \psi_f = 1 \cdot 1 = 1 \,\text{p.u.}$

$\hat{i}_{sc} = \dfrac{2\sqrt{2}}{0.162} 1 = 17.4 \,\text{p.u.}$, then 17.4 times the rated current.

Synchronous servomotor for a machine tool drive (Brushless DC motor)

A **permanent magnet excited synchronous motor** is operated from voltage source inverter. Rotor position is measured by an encoder and controls phase angle of stator voltage (field oriented control = brushless DC drive). No damper cage is needed in the rotor.
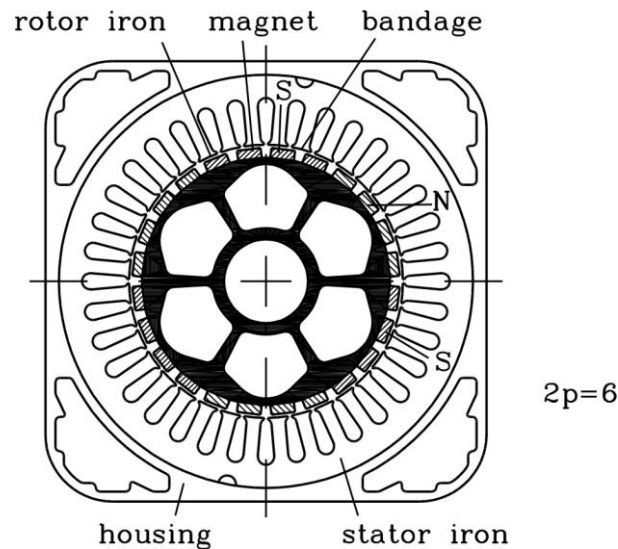

Fig. 8.4-1: Axial section of a permanent magnet synchronous machine without damper cage

Given: $x_d = 0.35$ p.u., $r_s \approx 0$, back EMF $u_p = 0.71$ p.u. at nominal speed.

1)   Indicate the dynamic set of equations of the machine.

2)   Draw the space vector diagram for stationary operation in rotor reference frame for rated speed. With which angular position relative to q-axis the stator current space vector has to be operated, so that the machine develops maximum torque?

3)   Indicate for the results of 2) the space vector diagram for rated current and speed in rotor reference frame. How big is the amplitude of the stator voltage space vector?

4)   How big is the maximum torque for field-oriented control operation in accordance with 2) for rated speed, if the inverter maximum voltage U_{max} is rated voltage U_N?

1) Equations system: No damper and no excitation winding $\to i_D = 0$, $i_Q = 0$, $i_f = i_{f0}$
permanent magnetic flux corresponds with constant equivalent current i_{f0}.
It remains:

$$u_d = r_s i_d + \frac{d\psi_d}{d\tau} - \omega_m \psi_q$$

$$u_q = r_s i_q + \frac{d\psi_q}{d\tau} + \omega_m \psi_d$$

$$\tau_J \frac{d\omega_m}{d\tau} = -i_d \psi_q + i_q \psi_d - m_L$$

$$\psi_d = x_d i_d + \psi_f$$

$$\psi_q = x_q i_q = x_d i_q \quad (x_d = x_q)$$

($\psi_f = x_{dh} \cdot i_{f0}$: equivalent excitation current)

2) Steady state operation $\Rightarrow d/d\tau = 0$, rated speed: $\omega_m = 1$, $r_s \cong 0$, $u_d = -\psi_q$, $u_q = \psi_d$,
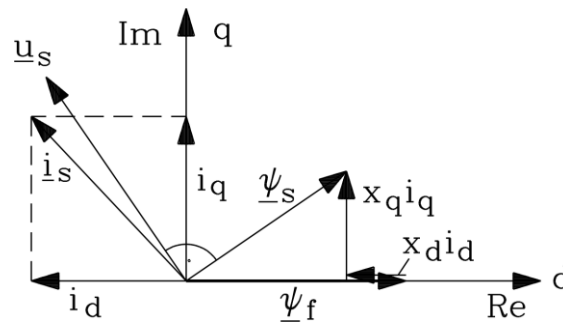$\psi_d = x_d i_d + \psi_f$, $\psi_q = x_d i_q$.

Fig. 8.4-2: Space vector diagram for steady state operation

For example: $i_d < 0$, $i_q > 0$, $\underline{\psi}_s = \psi_d + j\psi_q$, $\underline{u}_s = j\omega_m \underline{\psi}_s$!

Adjustment of notation according to electrically excited synchronous machine:

$$u_d = -\omega_m x_q i_q, \quad u_q = \omega_m x_d i_d + \underbrace{\omega_m \psi_f}_{u_p}$$

Torque:

$$m_e = -i_d \psi_q + i_q \psi_d = -i_d x_q i_q + i_q x_d i_d + i_q \psi_f =$$
$$= x_d \left(-i_d i_q + i_q i_d \right) + i_q \psi_f = i_q \psi_f$$

$i_s = i_q$. A pure q-current must be impressed, in order that at $x_d = x_q$ for given current
the maximum torque is obtained. The d-current does not generate any torque.

3) $i_s = i_q = 1.0 \,\text{p.u.}$, $\omega_m = 1$, $u_p = 0.71 \,\text{p.u.}$, $x_d i_s = 0.35 \,\text{p.u.}$

$$u_s = \sqrt{(x_d i_s)^2 + u_p^2} = \sqrt{(0.35 \cdot 1)^2 + 0.71^2} = \underline{\underline{0.79 \,\text{p.u.}}}$$
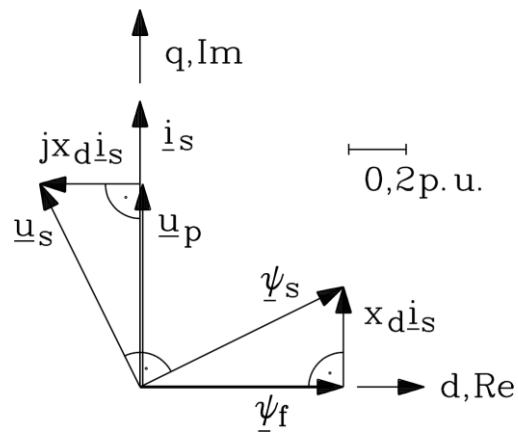
Fig. 8.4-3: Space vector diagram for steady state q-current operation

4) $u_s = 1\,\text{p.u.} \Rightarrow u_s^2 - u_p^2 = (x_d i_s)^2 \Rightarrow i_s = \dfrac{\sqrt{u_s^2 - u_p^2}}{x_d} = \dfrac{\sqrt{1^2 - 0.71^2}}{0.35} = 2\,\text{p.u.} = |\underline{i}_s| = i_q$,

$m_e = i_q \psi_f = 2\psi_f = 2\underbrace{(i_N \psi_f)}_{m_{eN}}$: The maximum torque is <u>two times bigger</u> than the rated

torque.

Exercise 8.5   8.5/1

Locked Rotor Synchronous Drive

A 6-pole permanent magnet synchronous machine without damper cage, which is controlled by a rotor position encoder, is fed by a converter in field oriented operation. In locked rotor operation the machine is supplied from two phases of the converter with a voltage space vector $\underline{u}_s = u_s \cdot e^{j \cdot 120°}$.
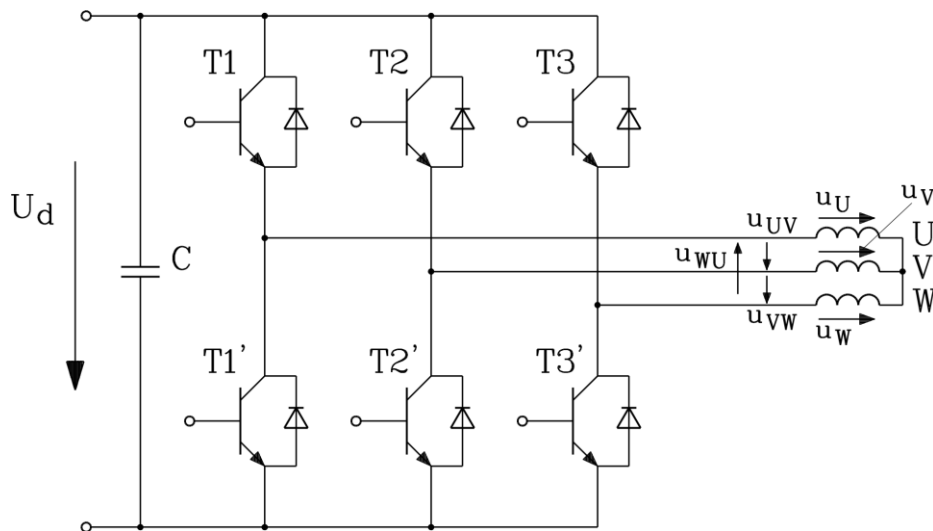

Fig. 8.5-1: The electric circuit of the inverter with the star connected stator winding

Motor parameters: stator winding star connection, $r_s = 0.05$ p.u., $x_d = 0.3$ p.u., rated voltage 231 V, rated current: 10 A (r.m.s.), rated speed 3000/min. The six-pole rotor has surface mounted permanent magnets, so the direct and the quadrature axis inductance are identical: $x_d = x_q$.

1)   Which transistors must conduct in Fig. 8.5-1 to get the wanted position of the stator voltage space vector? Describe the corresponding current flow!

2)   Compute and draw for 1) the time function of current in the phase W (in Ampere), if the current was zero at the beginning. Since *d-* and *q*-axis are identical and the machine is not rotating (no rotation voltage is induced by the permanent magnets), you can use the asynchronous machine system of equations without secondary (no damper) e. g. in synchronous reference frame. The dc link voltage is $U_d = 489$ V. Is this point of operation admissible for the drive system?

3)   Give the six switching states of the inverter and the corresponding positions of the stator voltage space vector for voltage six step operation.

1)   The voltage space vector may be given depending on the line-to-line voltages:

$$\underline{u}_s = \frac{2}{3}\left(u_U + \underline{a}\cdot u_V + \underline{a}^2\cdot u_W\right) = \frac{2}{3}\left(-u_{WU} + \underline{a}\cdot u_{VW}\right)$$

This expression is derived as follows:

$$+\begin{cases} \underline{u}_s = \frac{2}{3}\left(u_U + \underline{a}\cdot u_V + \underline{a}^2\cdot u_W\right) \\[2mm] 0 = \frac{2}{3}\left(u_W + \underline{a}\cdot u_W + \underline{a}^2\cdot u_W\right) \end{cases} \qquad \text{and} \qquad u_{UW} = -u_{WU}$$

$$u_s = \frac{2}{3}\left(\underbrace{u_U - u_W}_{u_{UW}} + \underline{a}\cdot\underbrace{\left(u_V - u_W\right)}_{u_{VW}} \right)$$

For $\underline{u}_s = u_s e^{j120^o} = u_s\underline{a}$ according to Fig.8.5-2 the transistors T2, T1' and T3' must be conducting; the others must block. The per-unit DC link voltage is $u = U_d/(\sqrt{2}U_N)$.

Hence we have $u_{VW} = u$ and $u_{WU} = 0$, which results in $u_s = \frac{2}{3}u\cdot\underline{a}$. The current flows with 100% positive in phase V and with 50% negative in phase U and W: $i_W = -\frac{i_V}{2}$,
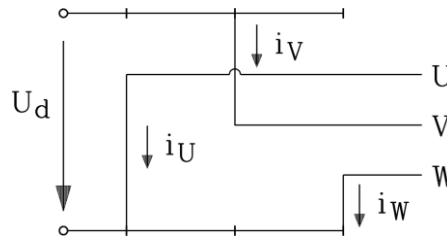
$i_U = -\frac{i_V}{2}$.

Fig. 8.5-2: Conduction scheme for $\underline{u}_s = u_s e^{j120^o} = u_s\underline{a}$

2)   We first calculate the stator current space vector for stand still: $\omega_s = 0$. Note that we have $x_s = x_d = x_q, \psi_f$ is constant.

Stator flux linkage equation: $\underline{\psi}_s = x_s\underline{i}_s + \underline{\psi}_f$

Stator voltage space vector equation: $\underline{u}_s = r_s\underline{i}_s + d\underline{\psi}_s/d\tau + j\omega_s\underline{\psi}_s = r_s\underline{i}_s + x_s d\underline{i}_s/d\tau$

Linear 1st order differential equation:

$$x_d\frac{d\underline{i}_s}{d\tau} + r_s\underline{i}_s = \underline{u}_s = (2/3)\cdot u\cdot\underline{a} \qquad \text{Initial condition: } \underline{i}_s(\tau = 0) = 0$$

Homogeneous solution: $\underline{i}_{sh}(\tau) = \underline{C}e^{-\tau/\tau_s}$ ($\tau_s = \frac{x_d}{r_s}$: Stator winding time constant)

Particular solution: $\underline{i}_{sp} = \underline{i}_{s\infty} = \frac{(2/3)\cdot u\cdot\underline{a}}{r_s}$ (stationary DC current)

Satisfying the initial condition leads to: $\underline{C} = -\underline{i}_{s\infty}$

The stator current space vector $\underline{i}_s(\tau) = \underline{i}_{s\infty}\left(1 - e^{-\tau/\tau_s}\right)$ excites the stator magnetic field in direction of the stator voltage space vector (V axis!). The phase current W is given as $i_W(\tau) = Re\left(\underline{a} \cdot \underline{i}_s(\tau)\right)$:

$$i_W(\tau) = Re\left(\underline{a} \cdot \underline{a}\frac{(2/3)\cdot u}{r_s}\left(1 - e^{-\tau/\tau_s}\right)\right), \text{ which is with } \underline{a}^2 = e^{j\frac{4\pi}{3}}, \quad Re\left(\underline{a}^2\right) = -0.5:$$

$$\underline{\underline{i_W(\tau) = -\frac{u}{3r_s}\left(1 - e^{-\tau/\tau_s}\right) \text{ in per unit}}}$$

$$\underline{\underline{i_W(t) = -\frac{U_d}{3R_s}\cdot\left(1 - e^{-t/T_s}\right) \text{ in physical units.}}}$$

Calculating the electrical stator winding time constant via the rated frequency:

$$n_N = 3000\,\text{min}^{-1} = 50\,\text{s}^{-1} = \frac{f_N}{p} \quad \Rightarrow \quad f_N = 3 \cdot 50 = 150\,\text{Hz} \ (p = 3)$$

$$\omega_N = 2\pi f_N = 2\pi \cdot 150 = 942.5\,\text{s}^{-1}, \ T_s = \frac{1}{\omega_N}\tau_s = \frac{1}{942.5}\cdot\frac{0.3}{0.05} = 6.37\,\text{ms}$$

$$R_s = r_s \cdot (U_N / I_N) = 0.05 \cdot (231/10) = 1.155\,\Omega$$

$$\underline{\underline{i_W(t)}} = -489/(3\cdot 1.155)\cdot\left(1 - e^{-t/6.37\,\text{ms}}\right) = \underline{\underline{-141.1\cdot\left(1 - e^{-t/6.37\,\text{ms}}\right)\text{A}}}$$
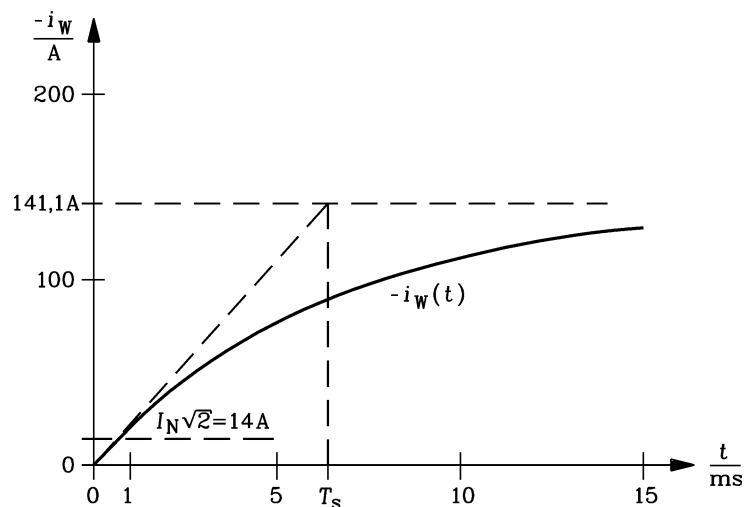

Fig.8.5-3: Current in phase W at rotor stand still and voltage feeding according to Fig. 8.5-2

The current in phase W is 50% of the phase current V and negative. The resulting stationary DC current in phase V may be calculated due to $n = 0$ (back EMF is zero!) also simply according to Fig. 8.5-2 via the phase resistances (phase U and W in parallel, in series with phase V):

$$i_{V,\infty} = i_{s\infty} = U_d /(1.5R_s) = 489/(1.5 \cdot 1.155) = 282.2\,\text{A}.$$

$$\left|i_{W\infty}\right| = \left|\frac{i_{s\infty}}{2}\right| = 141.1\,\text{A}.$$

The phase current is 28.2-times the rated current, which is much too high. The inverter over-current protection must switch off the machine.

3)   The sequence of conducting transistors (three at a time interval $T/6$) for six-step operation according to Fig. 8.5-4a is given in Table 8.2-1.

Table 8.5-1: On-state transistors in Fig. 8.2-1 for six-step operation according to Fig. 8.2-4a

Time interval	1	2	3	4	5	6
On-state	T1	T1	T1´	T1´	T1´	T1
On-state	T2´	T2	T2	T2	T2´	T2´
On-state	T3´	T3´	T3´	T3	T3	T3

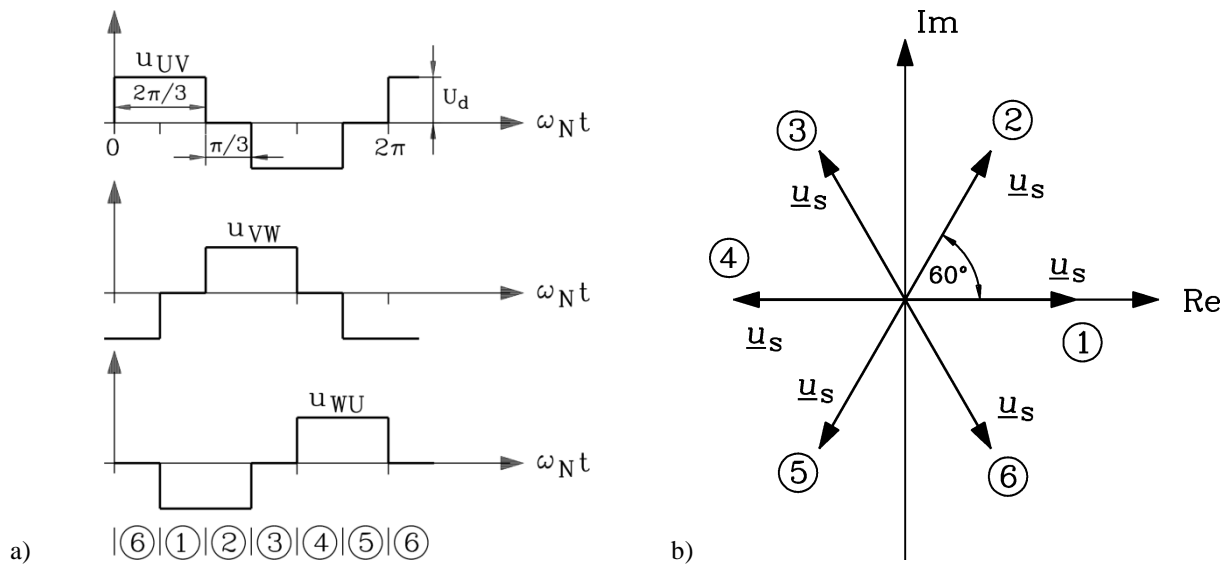a)   b)

Fig. 8.5-4: a) Voltage six step operation diagram of the three line-to-line voltages, b) Voltage space vector diagram for the six-step operation

With the on-states of the transistors according to Table 8.2-1 for the six time intervals 1 … 6 per period T (duration of each interval $T/6$) the three line-to-line voltage time functions are derived: e.g. for time interval 3 see Fig. 8.5-2. With these time functions of u_{WU} and u_{VW} with the formula $\underline{u}_s = \frac{2}{3}\left(-u_{WU} + \underline{a} \cdot u_{VW}\right)$ the six stator voltage space vectors are calculated in physical units in Fig. 8.5-4b, for example in time interval 2:

$$\underline{u}_s = \frac{2}{3}\left(\underbrace{-u_{WU}}_{-U_d} + \underline{a} \cdot \underbrace{u_{VW}}_{U_d}\right) = \frac{2}{3}U_d(1+\underline{a}), \ |\underline{U}_s| = \frac{2}{3}U_d$$

Power synchronous drive for a cooling compressor

An **electrically excited 17 MW synchronous motor** used for a cooling compressor in an petrochemical factory is supplied by a current-source inverter.
Motor parameters:
$P_N = 17$ MW; $n_N = 4850$/min; $2p = 2$
$U_N = 7{,}2$ kV, Y; $I_N = 1435$ A; (fundamental voltage and current, r.m.s.-value);
The inverter DC-link current is $I_d = 1850$ A

The time function of the phase currents in stator winding and the phase voltages are shown in
Fig. 8.6-1 and Fig. 8.6-2 respectively.
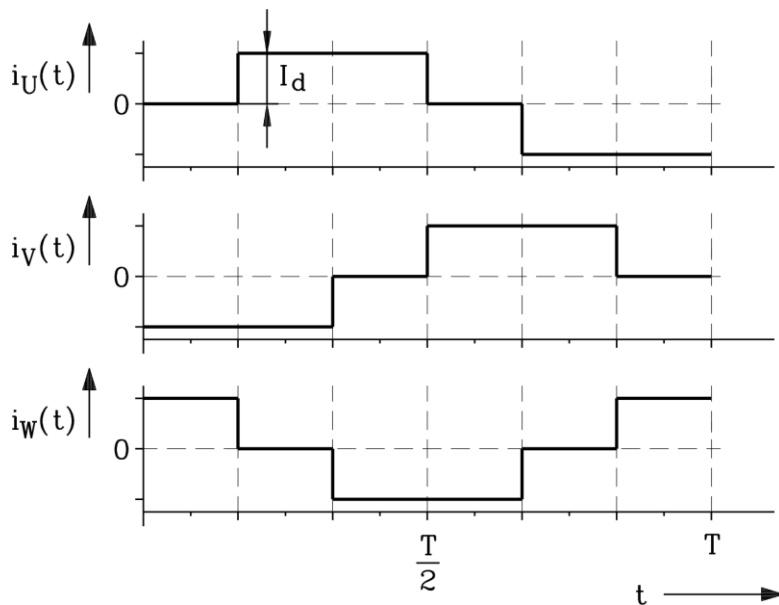

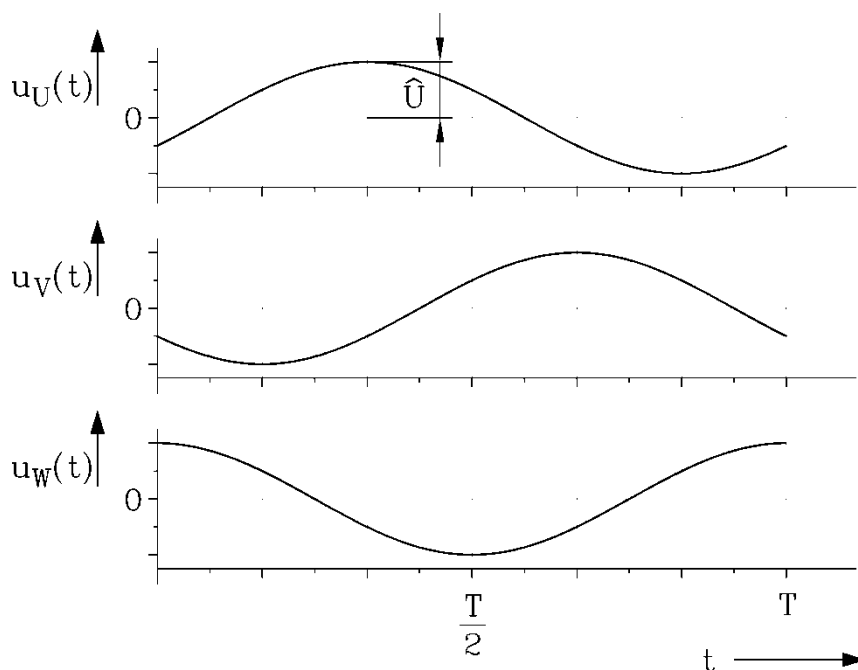Fig. 8.6-1: Phase currents time function in the phases: U, V, W

Fig. 8.6-2: Phase voltage time function in phases: U, V, W

Exercise 8.6   8.6/2

Answer the following questions:

1.)   How big is the time period T and the frequency of $i(t)$ and $u(t)$ for rated operation ?

2.)   With which power factor of fundamental voltage and current $\cos\varphi$ is the motor operated (Sketch fundamental of $u(t)$, $i(t)$ according to Fig. 8.6-1 and 8.6-2) ?

3.)   Calculate the current space vector in physical units (A) and as p.u.–values for $I_d = 1850$ A for the following instants (time t):

        a) $t = T/12$ and   b) $t = T/4$.

4.)   Draw the current space vector $\underline{i}(T/12)$, $\underline{i}(T/4)$ of 3), using scale: 200 A/cm, in the cross-section plane of the machine in complex reference frame (Fig. 8.6-3). How big is the angle between the two vectors?

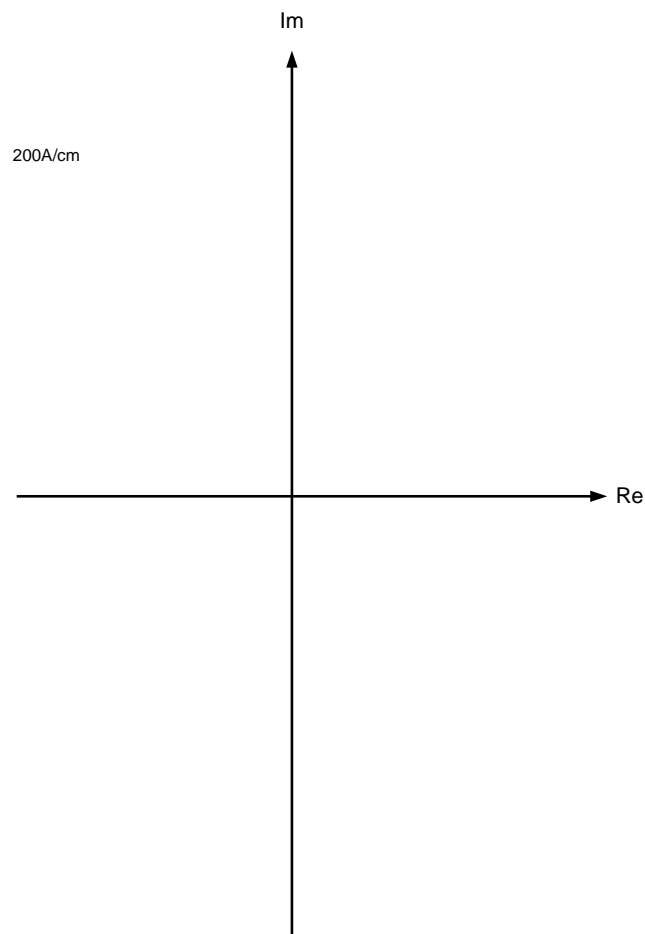5.)   How long does the position of the current space vector remain unchanged?

Fig. 8.6-3: Chart 1 for the space vector diagram

Exercise 8.6   8.6/3

1) $2 \cdot p = 2 \Rightarrow f_N = p \cdot n_N \Rightarrow f_N = 1 \cdot n_N = 1 \cdot \dfrac{4850}{60} = \underline{\underline{80,83\,\text{Hz}}}$

$T = \dfrac{1}{f_N} = \dfrac{1}{80,83} = \underline{\underline{12,37\,\text{ms}}}$

2)   Power factor $\cos\varphi$ is defined by phase shift angle φ <u>only</u> between sinusoidal time
functions with the same frequency.
$\Rightarrow$ The motor phase voltage is sinusoidal (Fig. 8.6-2).
$\Rightarrow$ The motor current fundamental harmonic (Fig. 8.6-4: ordinal number $k = 1$) has the
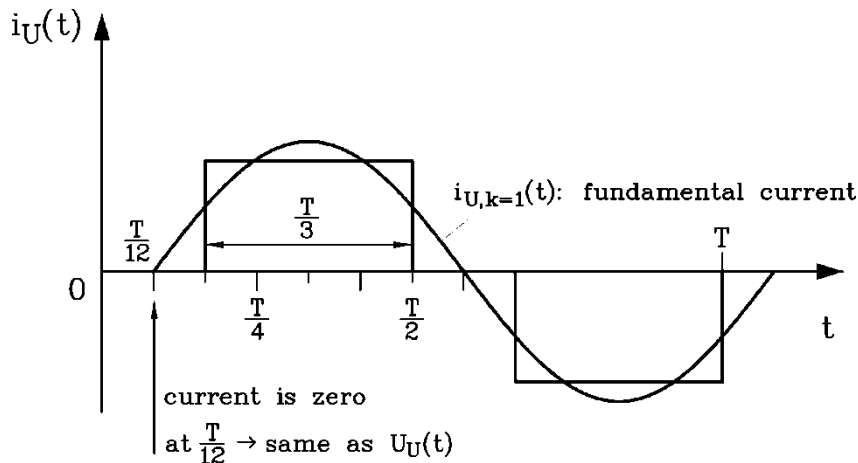same frequency as the motor voltage.

Fig. 8.6-4: Block-shaped phase current and its fundamental

$\Rightarrow \varphi = 0^0, \underline{\underline{\cos\varphi = 1}}$.   The motor is operated with unity power factor.

3) $\underline{i}(t) = \dfrac{2}{3}\left(i_U(t) + \underline{a} \cdot i_V(t) + \underline{a}^2 \cdot i_W(t)\right),$ $\underline{a} = e^{j\frac{2\pi}{3}}$

a) $\underline{t = T/12}:\ i_U = 0, i_V = -I_d, i_W = +I_d$

$\underline{i}(T/12) = \dfrac{2}{3}\left(0 + e^{j \cdot 2\pi/3} \cdot (-I_d) + e^{j \cdot 4\pi/3} \cdot I_d\right) = \dfrac{2}{3}\left(\dfrac{I_d}{2} - j\dfrac{\sqrt{3}}{2}I_d - \dfrac{I_d}{2} - j\dfrac{\sqrt{3}}{2}I_d\right) =$

$= -j\dfrac{2}{\sqrt{3}}I_d = -j\dfrac{2}{\sqrt{3}} \cdot 1850\,\text{A} = \underline{\underline{-j \cdot 2136\,\text{A}}}$

Normalisation with $\sqrt{2} \cdot I_N = \sqrt{2} \cdot 1435\,\text{A} : \underline{i}(T/12) = -j\dfrac{2136}{\sqrt{2} \cdot 1435} = \underline{\underline{-j \cdot 1,05\,\text{p.u.}}}$

b) $\underline{t = T/4}:\ i_U = +I_d, i_V = -I_d, i_W = 0$

$\underline{i}(T/4) = \dfrac{2}{3}\left(I_d + (-I_d) \cdot e^{j\frac{2\pi}{3}} + 0\right) = \dfrac{2}{3}\left(I_d + \dfrac{1}{2}I_d - j\dfrac{\sqrt{3}}{2}I_d\right) =$

$= I_d - j\dfrac{1}{\sqrt{3}}I_d = \underline{\underline{(1850 - j \cdot 1068)\,\text{A}}}$

Note: The absolute value $\left|\underline{i}(T/4)\right| = \sqrt{1850^2 + 1068^2} = 2136\,\text{A}$ is identical with the absolute value $\left|\underline{i}(T/12)\right|$.

Conclusion: The current space vector is changing the <u>direction,</u> but <u>not</u> its absolute value (length of vector) (Fig. 8.6-5).

Normalisation: $\dfrac{1850 - j \cdot 1068}{\sqrt{2} \cdot 1435} = (0.912 - j \cdot 0.526)\,\text{p.u.} = \underline{i}(T/4)$

Absolute value: $\left|\underline{i}(T/4)\right| = \sqrt{0.912^2 + 0.526^2} = 1.05\,\text{p.u.}$

4)
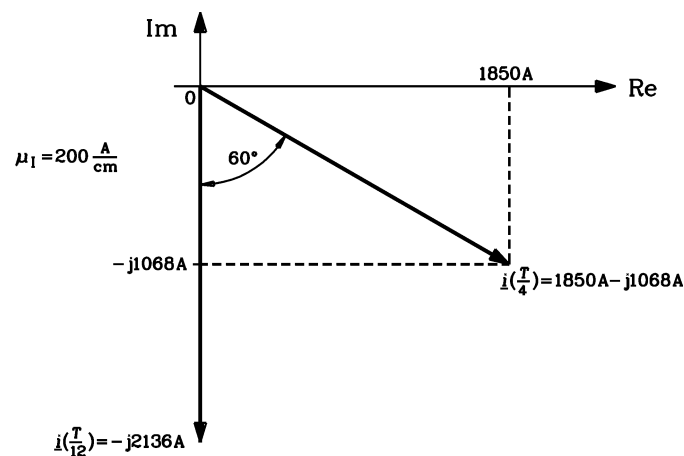
Fig. 8.6-5: Stator current space vector at a) $t = T/12$, b) $t = T/4$. The angle between the two space vectors is <u>60^0</u>.

5)   For a time interval of $\dfrac{T}{6} = \dfrac{12.37}{6} = 2.06\,\text{ms}$ the direction of the current space vector remains unchanged, then it "jumps" 60° in anti-clockwise direction (Fig. 8.6-6).

Note: During one time period, the current space vector $\underline{i}$ has 6 different positions.

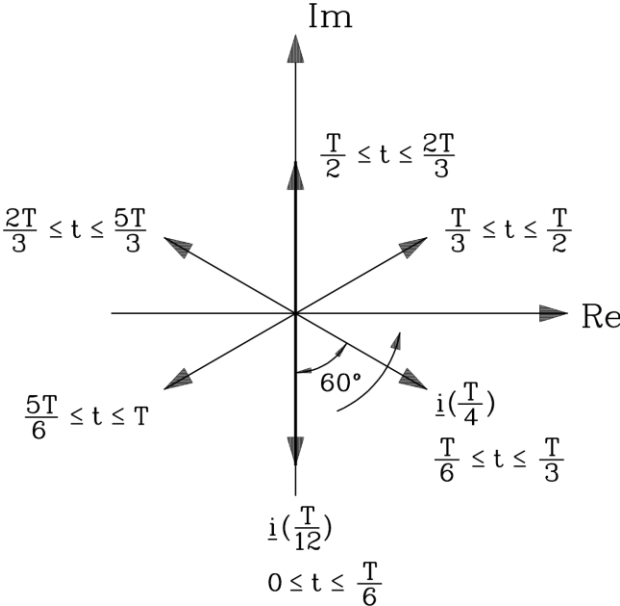Fig. 8.6-6: The stator current space vectors for the six states during one fundamental cycle T

Exercise 8.7   8.7/1

Sudden short circuit of a marine generator

On a cruise ship with electric propulsion the power supply is provided by 3 diesel-powered on-board generators with 32 MVA each. Data of each of the synchronous generators:
U_N = 10.5 kV Y, S_N = 32 MVA, $\cos\varphi_N$ = 0.8, overexcited, f_N = 50 Hz, reactances: x_d = 1.1 p.u., x_d'' = 0.18 p.u.. The stator resistance is neglected.

1) One of the generators, which was operating at no-load ("stand-by"), suffers from a sudden terminal short circuit after a system failure. The short circuit happens during the zero-crossing of phase V. How big is the peak value of the sudden short circuit current in phase V, if a decay rate of the dc-component of 0.8 is assumed?

2) How big is the amplitude of the steady-state short circuit current, after all transient currents have vanished, if this current is not switched off immediately by a power switch?

3) The generator needs to be sent back to the manufacturer to be repaired. After successful recovery, the generator shall be asynchronously started in the manufacturer test bay via the damper cage. How big is the stator current per phase assuming a direct connection of the machine to the grid in the worst case?

4) To limit the asynchronous starting current according to 3) a series inductor shall be dimensioned. How big must this inductance be to reduce the starting current to 70 %?

5) Which further advantages are obtained by using a starting-transformer, which feeds with its secondary winding the stator winding, with a transfer ratio $\ddot{u}$ = 2? Calculate the AC starting current on grid and secondary side, if the primary and secondary leakage reactances are $x_{1\sigma} = x'_{2\sigma}$ = 0.03 p.u. and the rated primary voltage is 10.5 kV, and the rated transformer power is 6.5 MVA!

1) Rated current: $I_N = S_N / (\sqrt{3} \cdot U_N) = 32000 / (\sqrt{3} \cdot 10.5) = 1670\text{A}$

$$\hat{I}_{k,V} = \frac{2\sqrt{2} \cdot I_N}{x_d''} \cdot 0.8 = \frac{2\sqrt{2} \cdot 1760}{0.18} \cdot 0.8 = 22119\text{A}$$

$\hat{I}_{k,V} = 22119\text{A} = 12.6\text{-times the rated current}$

2) Short circuit after no-load operation at rated voltage: Short-circuited voltage = no-load voltage = back EMF: $U_{s0} = U_p = U_{Nph}$. Limiting impedance = synchronous reactance X_d. Stator resistance neglected!

$$Z_N = U_{N,ph} / I_N = (10500 / \sqrt{3}) / 1670 = 3.44\Omega, \quad X_d'' = x_d'' \cdot Z_N = 0.18 \cdot 3.44 = 0.62\Omega$$

$$X_d = x_d \cdot Z_N = 1.1 \cdot 3.44 = 3.78\Omega, \quad \hat{I}_k = \frac{\sqrt{2} \cdot U_p}{X_d} = \frac{\sqrt{2} \cdot (10500 / \sqrt{3})}{3.78} = 2268\text{A}$$

Amplitude of the steady-state short circuit current: $\hat{I}_k = 2268\text{A} = 1.29\text{-times rated}$ current.

3) Starting of the synchronous machine via its rotor cage = T-equivalent circuit! Starting current = slip is unity!
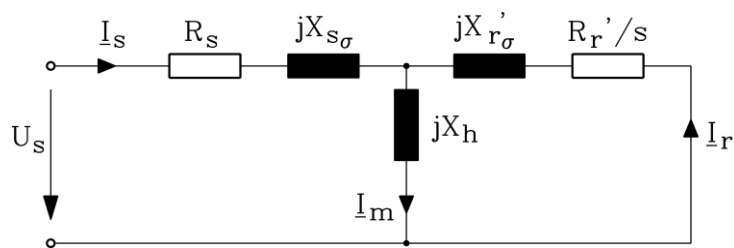

Fig. 8.7-1: T-equivalent circuit per phase for the field fundamental of a synchronous machine without consideration of short-circuited excitation winding. Rotor quantities are damper cage quantities (subscript "r" is "D" for d-axis)

The excitation winding is short-circuited to avoid secondary induced high over-voltages during asynchronous start up! The induced AC current in the short-circuited excitation winding contributes to the asynchronous starting torque, giving instead of Fig. 8.7-1 a three-winding transformer Fig. 8.7-2!
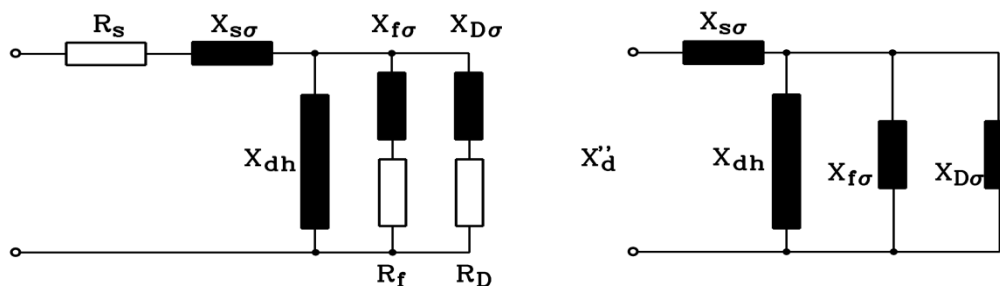

Fig. 8.7-2: T-equivalent circuit per phase for the three-winding transformer (stator winding, rotor cage and field winding) at slip s = 1: a) with resistances, b) without resistances

According to Fig. 8.7-2 the stator reactance at starting ($s = 1$) is x_d'' in the d-axis, and x_q'' in the q-axis. Hence the average value is active: $x_s(s=1) = (x_d'' + x_q'') / 2$.

Exercise 8.7　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　8.7/3

As $x_d'' \approx x_q''$, we take $x_s(s=1) \cong x_d''$. For calculating the starting current, when switching the stator winding to the grid, it is done like for the induction machine in CAD textbook, Chapter 7 (Fig. 8.7-3). The maximum starting current occurs in the worst case of zero crossing of the phase voltage! With $x_s(s=1) \cong x_d''$ the worst-case starting current is with damping due to the resistances (damping factor 0.8 like in case of short-circuit)

$$\hat{I}_{k,U} = \frac{2\sqrt{2} \cdot U_N}{X_d''} \cdot 0.8 = \frac{2\sqrt{2} \cdot I_N}{x_d''} \cdot 0.8 = \frac{2\sqrt{2} \cdot 1760}{0.18} \cdot 0.8 = 22119 A \quad .$$
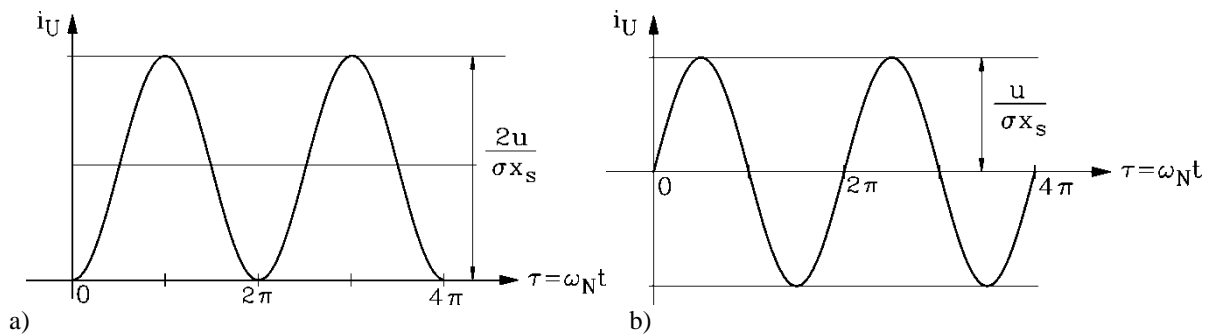

Fig. 8.7-3: Phase current in phase U after switching a sinus voltage u to the stator winding of an induction machine, damping neglected: a) Switching on at zero crossing of U phase voltage, b) Switching on at maximum U phase voltage. For a synchronous machine the same equivalent circuit may be taken with $x_s(s = 1) = x_d''$ instead of $x_s(s = 1) = \sigma x_s$.

The worst-case starting current amplitude occurs, when connecting the phase to the grid during voltage zero-crossing: $\hat{I}_1 = 22119 A$. It is identical in value with the sudden short-circuit current of 1).

4)　　Additional series inductor impedance x_D increases the stator winding reactance per phase at $s = 1$: $x_s(s=1) \cong x_d'' + x_D$.

Condition for designing the size of x_D: $\hat{I}_s = 0.7 \cdot 22119 = 15483 A$.

$$\frac{x_d''}{x_d'' + x_D} = 0.7, \quad x_D = (\frac{1}{0.7} - 1) \cdot x_d'' = 0.077 \text{p.u.} ,$$

$$L_D = X_D / \omega_s = x_D \cdot Z_N / \omega_s = 0.077 \cdot 3.44 / (2\pi \cdot 50) = 8.45 \text{mH}$$

A series inductor $L_D = 8.54$ mH limits the peak starting current to 15.5 kA.

5)　　A starting transformer does not only act as a series impedance, but via the transfer ratio it is also reducing the starting current $I_2 = I_1 \cdot \ddot{u}$ due to a smaller secondary voltage $U_2 = U_1 / \ddot{u} < U_1$, which feeds the stator winding (Fig. 8.7-4). The transformer series impedance is mainly its total leakage reactance.
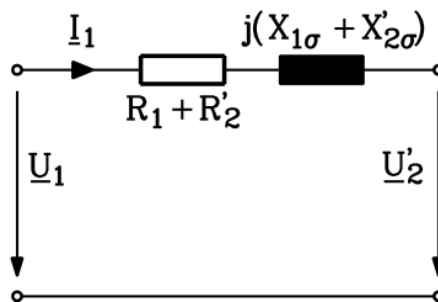
<u>Fig. 8.7-4:</u> Transformer equivalent circuit per phase with infinite big magnetizing reactance (= neglected magnetizing current).

The grid-side AC starting current (without the DC link component) is calculated with neglected resistances.

Transformer rated voltages: $U_{1N} = 10.5\,\mathrm{kV}$, $\quad U_{2N} = 10.5\,\mathrm{kV}/\ddot{u} = 5.25\,\mathrm{kV}$

Rated transformer primary current: $I_{1N} = S_{NT}/(\sqrt{3}\cdot U_{1N}) = 6500/(\sqrt{3}\cdot 10.5) = 357\,\mathrm{A}$

Rated transformer impedance: $Z_{1N} = U_{1N,ph}/I_{1N} = (10500/\sqrt{3})/357 = 16.96\,\Omega$

Transformer leakage reactance: $X_{1\sigma} = 0.03\cdot 16.96\,\Omega = 0.509\,\Omega = X'_{2\sigma}$

Starting AC current at the primary transformer winding:

$$I_1 = \frac{U_1}{X_{1\sigma} + \ddot{u}^2\cdot(X_{2\sigma} + X''_d)} = \frac{U_1}{X_{1\sigma} + X'_{2\sigma} + \ddot{u}^2\cdot X''_d} =$$

$$= \frac{10500/\sqrt{3}}{0.509 + 0.509 + 2^2\cdot 0.62} = 1735\,\mathrm{A}$$

The starting current is 4.85-times the rated primary transformer current!

For comparison: AC starting current without transformer:

$$I_1 = \frac{U_1}{X''_d} = \frac{10500/\sqrt{3}}{0.62} = 9778\,\mathrm{A}$$

Due to the step-down transformer (50% secondary starting voltage) and the transformer leakage reactance the line-side starting current is reduced by a factor 9778/1735 = 5.6!

Synchronous generators for power plant
Following salient-pole synchronous generators have been offered by a manufacturer to be installed at a hydro power plant in *Matala* in *Angola*:

S_N = 17.8 MVA (rated apparent power) U_N = 10.5 kV Y (rated line-to-line voltage)
$2p$ = 28 f_N = 50 Hz
η = 96.6 %

Mechanical input power of the FRANCIS turbine unit P_m = 14.8 MW.

Following calculated values have been provided to the planning consortium by the manufacturer:
X_d = 5.4 Ω (synchronous reactance)
R_s = 0.33 Ω (winding resistance per phase at 20°C)
X_d'' = 1.3 Ω (subtransient reactance of the direct axis)

1)   How big are the rated current and the rated power factor of the generators?

2)   Calculate the rated impedance of the generators!

3)   How big is the worst-case amplitude of the short circuit current, if a three pole sudden short circuit occurs? Generators were running prior to short circuit at no-load rated voltage! Describe the worst-case conditions!

4)   How much higher are the current forces during short circuit in the generator as well as in the switch-gear compared to rated operation?

5)   How much higher is the sudden short circuit alternating torque amplitude compared to static pull-out torque?

Exercise 8.8   8.8/2

1) $I_N = \dfrac{S_N}{\sqrt{3} \cdot U_N} = \dfrac{17.8 \cdot 10^6}{\sqrt{3} \cdot 10500} \, A = \underline{\underline{978.7 \, A}}$

$\cos\varphi = \dfrac{P_e}{S_N}$, generator operation: $P_e = P_m \cdot \eta \Rightarrow \cos\varphi = \dfrac{P_m \cdot \eta}{S_N} = \dfrac{14.8 \cdot 0.966}{17.8} = \underline{\underline{0.803}}$

2) $Z_N = \dfrac{U_{sN}}{I_{sN}}$, Y-connection of the windings:

$I_{sN} = I_N, U_{sN} = \dfrac{U_N}{\sqrt{3}} : \qquad Z_N = \dfrac{10500}{\sqrt{3} \cdot 978.7} = \underline{\underline{6.19 \, \Omega}}$

3)   Worst-case for sudden short circuit:
       - sudden shortcircuit occurs at zero-crossing of voltage $\Rightarrow$ full DC component occurs.
       - short circuit direct at terminals: no additional resistance for current decay, no
       additional reactance for current limitation

       Machine is running at no-load, rated voltage: $U_s = U_p$.
       If no decay of DC component considered, we get:

$\hat{I}_k = 2\dfrac{\hat{U}_{sN}}{X_d''} = 2 \cdot \sqrt{2} \cdot \dfrac{10500}{\sqrt{3} \cdot 1.3} \, A = \underline{\underline{13189.6A}}$

       Remark: $\hat{I}_k = 13.5 \cdot I_{sN}$!

4)   The forces between two conductors are proportional to $I_1 \cdot I_2$, in this case I^2.

$\dfrac{\hat{F}_k}{\hat{F}_N} = \left(\dfrac{\hat{I}_k}{\hat{I}_{sN}}\right)^2 = \left(\dfrac{13189.6}{\sqrt{2} \cdot 978.7}\right)^2 = \underline{\underline{90.8}}$!

       The current forces at sudden short circuit compared to the current forces at rated
       operation are higher by a factor of 90.8!

5) $\dfrac{\hat{M}_k}{\hat{M}_{p0}} = 2 \cdot \dfrac{U_s}{U_p} \cdot \dfrac{X_d}{X_d''} = 2 \cdot \dfrac{5,4}{1.3} = \underline{\underline{8.3}}$

Steady state operation and transient stability of a synchronous motor

A 2-pole 20 MW synchronous motor with cylindrical rotor is used for a pipe-line compressor drive. The main motor data are $x_{S\sigma} = 0.1$, $x_{dh} = x_{qh} = 1.5$, $x_{f\sigma} = 0.15$. The stator winding resistance is neglected $r_s \cong 0$.
1) Give the *p.u.* steady state voltage (u_d, u_q), flux linkage (ψ_d, ψ_q) and torque (m_s) equations as a special case of the general dynamic equation in the *d-q*-reference frame.
2) The motor is operated at rated torque $m_s = 1$ and rated speed $\omega_m = 1$, with $i_f = 1.5$ at under-voltage $u_s = 0.9$ and $u_q > 0$. Determine u_d, u_q, i_d, i_q ! Is the motor current exceeding the rated value $i_s = 1$?
3) Give the phasor diagram, scale 0.1 *p.u.* = 0.5 cm for phase voltage, phase current, the back EMF $x_{dh} \cdot i_f$ and the self-induced voltage $x_d \cdot i_s$. Introduce a complex coordinate system (Re-axis = d-axis, Im-axis = q-axis) and give the correct complex phasors in the diagram. Show the load angle ϑ and phase angle φ in the diagram. Discuss the result.
4) Give the power equation $p_e(\vartheta)$ at transient condition as a formula in *p.u.*!
5) Determine x'_d and u'_p and give the graph $p_e(\vartheta)$ for motor operation in the load angle range $-\pi \le \vartheta \le 0$! Determine the values $p_e(\vartheta)$ for $\vartheta = -\pi$, $-3\pi/4$, $-5\pi/8$, $-\pi/2$, $-\pi/4$, and 0.
6) How big is the transient motor overload capability (= transient pull-out power) in *p.u.* and in terms of the electrical real power! You can do this either graphically or analytically!

1)　　Steady state: $d./d\tau = 0$:

$$u_d = r_s \cdot i_d - \omega_m \cdot \psi_q \qquad\qquad u_q = r_s \cdot i_q + \omega_m \cdot \psi_d$$

$$\psi_d = x_d \cdot i_d + x_{dh} i_f \qquad\qquad \psi_q = x_q \cdot i_q$$

$$0 = i_q \cdot \psi_d - i_d \cdot \psi_q - m_s$$

2)　　$x_d = x_q = x_{dh} + x_{s\sigma} = 1.6$, $r_s = 0$

$$u_d = -\omega_m x_q \cdot i_q, \quad u_q = \omega_m \cdot x_d i_d + \omega_m \cdot x_{dh} i_f$$

$$m_s = i_q \cdot \psi_d - i_d \cdot \psi_q = x_{df} i_q i_f \Rightarrow i_q = \frac{m_s}{x_{df} i_f} = \frac{1}{1.5 \cdot 1.5} = 0.44$$

$$u_d = -\omega_m x_q \cdot i_q = -1 \cdot 1.6 \cdot 0.44 = -0.711$$

$$u_q = \pm\sqrt{u_s^2 - u_d^2} = \pm\sqrt{0.9^2 - (-0.711)^2} = +0.55$$

The negative sign yields a too big load angle > 90°, so that the operation is not stable.

$$i_d = \frac{u_q - \omega_m \cdot x_{dh} i_f}{\omega_m \cdot x_d} = \frac{0.55 - 1 \cdot 1.5 \cdot 1.5}{1 \cdot 1.6} = -1.063$$

$$i_s = \sqrt{i_d^2 + i_q^2} = \sqrt{1.063^2 + 0.44^2} = 1.15 > 1$$

The motor stator phase current exceeds the rated value.

3)　　See Fig. 8.9-1.

4)　　$$P_{e,dyn} = -m_s \cdot \left(\frac{U_s U'_p}{X'_d} \sin\vartheta - \frac{U_s^2}{2} \left(\frac{1}{X'_d} - \frac{1}{X_d} \right) \sin(2\vartheta) \right)$$
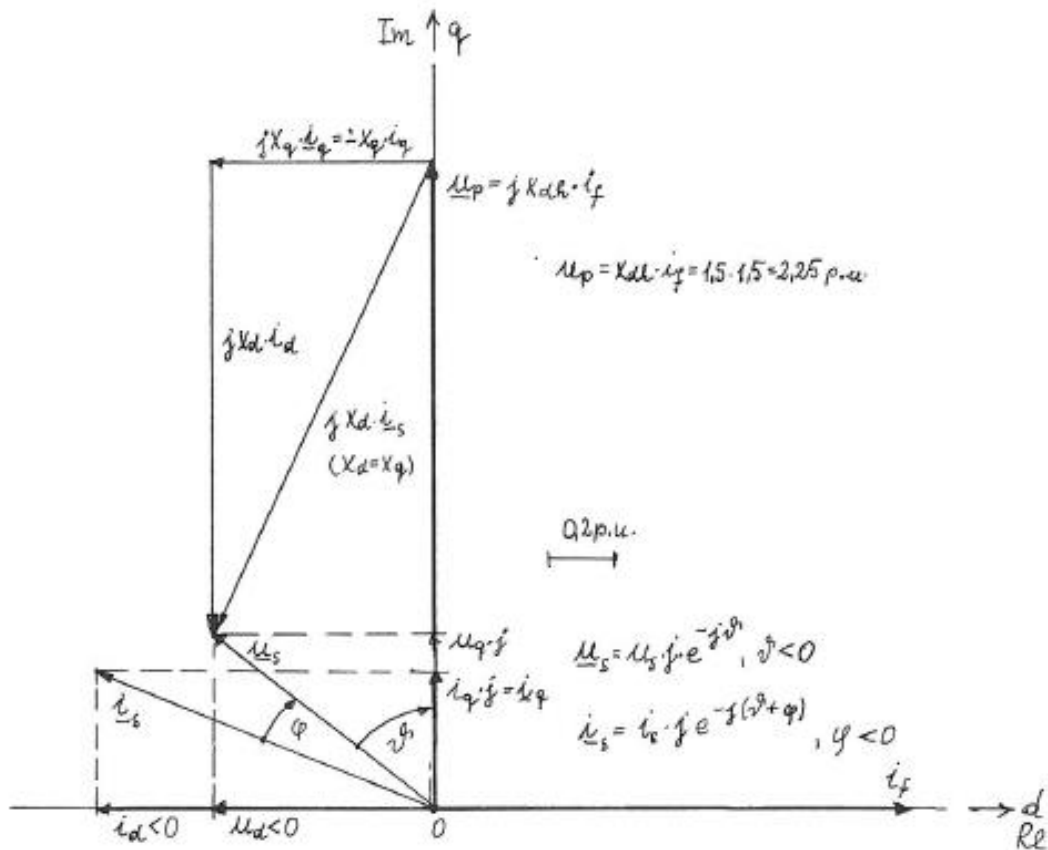
Fig. 8.9-1: Steady state phasor diagram for motor operation: The load angle is negative (motor operation). The phase angle is leading (negative phase angle): The motor is over-excited.

$$p_{e,dyn} = \frac{P_{e,dyn}}{m_s U_N I_N}$$

$$p_{e,dyn} = -\frac{1}{U_N I_N} \cdot \left(\frac{\dfrac{U_s U'_p}{U_N U_N}}{\dfrac{X'_d}{U_N / I_N}} U_N I_N \sin\vartheta - \frac{\left(\dfrac{U_s}{U_N}\right)^2}{2} \cdot U_N I_N \cdot \left(\frac{\dfrac{1}{X'_d}}{U_N / I_N} - \frac{\dfrac{1}{X_d}}{U_N / I_N} \right) \sin(2\vartheta) \right)$$

$$p_{e,dyn} = -\frac{u_s u'_p}{x'_d} \cdot \sin\vartheta + \frac{u_s^2}{2} \cdot \left(\frac{1}{x'_d} - \frac{1}{x_d} \right) \sin(2\vartheta)$$

5) $x'_d = x_{s\sigma} + \dfrac{x_{dh} x_{f\sigma}}{x_{dh} + x_{f\sigma}} = 0.1 + \dfrac{1.5 \cdot 0.15}{1.65} = \underline{\underline{0.236}}$

According to Fig. 8.9-2: $u'_p = u_q - x'_d i_d = 0.55 + 0.236 \cdot 1.063 = 0.8$
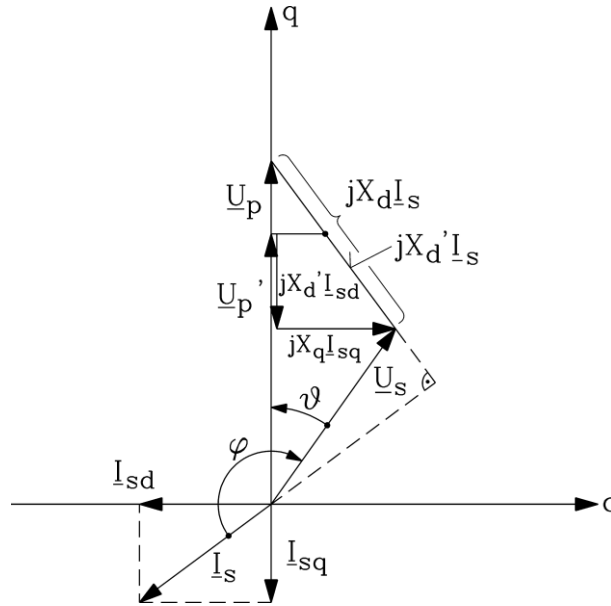
Fig. 8.9-2: Cylindrical rotor synchronous machine: Typical phasor diagram per phase in physical units in rotor reference frame during transient state, stator resistance neglected ($R_s = 0$):

Alternative calculation: $x_f = x_{f\sigma} + x_{dh} = 0.15 + 1.5 = 1.65$

$\psi_f = x_{dh}i_d + x_f i_f = 1.5 \cdot (-1.063) + 1.65 \cdot 1.5 = 0.8805$

$\omega_s = \omega_m = 1 : u'_p = (x_{dh} / x_f) \cdot \psi_f = (1.5 / 1.65) \cdot 0.8805 = 0.8$

$\dfrac{u_s u'_p}{x'_d} = \dfrac{0.9 \cdot 0.8}{0.236} = 3.05$, $\dfrac{u_s^2}{2} \cdot \left(\dfrac{1}{x'_d} - \dfrac{1}{x_d} \right) = \dfrac{0.9^2}{2} \cdot \left(\dfrac{1}{0.236} - \dfrac{1}{1.6} \right) = 1.46$

$p_{e,dyn} = -3.05 \cdot \sin\vartheta + 1.46 \cdot \sin(2\vartheta)$ see Fig. 8.8-3!

ϑ	$-\pi$	$-3\pi/4$	$-5\pi/8$	$-\pi/2$	$-\pi/4$	0
$p_{e,dyn}(\vartheta)$	0	3.62	3.85	3.05	0.7	0

6) $A = 3.05$, $B = 1.46$, $p_{e,dyn} = -A \cdot \sin\vartheta + B \cdot \sin(2\vartheta)$

$$dp_{e,dyn} / d\vartheta = 0 \Rightarrow \cos(\vartheta_{p,dyn}) = \frac{A}{8B} \pm \sqrt{\left(\frac{A}{8B}\right)^2 + \frac{1}{2}} = 1.015, -0.493 \qquad \text{The value}$$

must be less than or equal to unity because of the cosine function! $\Rightarrow$

$\vartheta_{p,dyn} = \pm 119.5°$, for motor operation: $\vartheta_{p,dyn} = -119.5°$

Dynamic pull-out power:

$p_{e,p,dyn} = -A \cdot \sin\vartheta_{p,dyn} + B \cdot \sin(2\vartheta_{p,dyn}) = 3.92$ p.u. This value may be also derives from the graph of Fig. 8.9-3.

Exercise 8.9   8.9/4

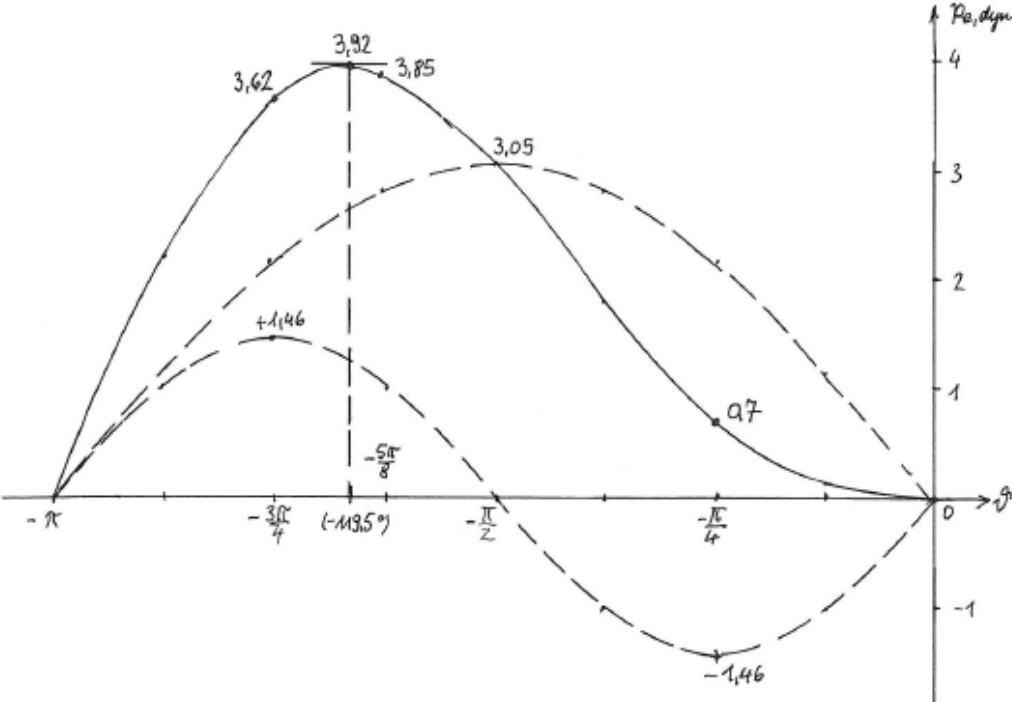Fig. 8.9-3: Sketch of the transient power capability of the cylindrical rotor synchronous machine at motor operation ($R_s = 0$)

Exercise 8.10   8.10/1

Mechanical natural frequency for small speed deviations

1) Give for a permanent magnet excited cylindrical pole synchronous machine without damper cage the dynamical equations in p. u. for stator current per phase and speed in the rotor reference frame and specialize them for steady state operation at a given three-phase stator voltage system a constant staor angular frequency ω_0. Neglect any damping!
2) Specifiy the steady-state condition for stator voltage and current for a) mechanical no-load and b) electrical no-load condition!
3) Linearize the voltage and mechanical differential equation for a given no-load steady-state condition!
4) Consider a small load torque disturbance Δm_s. Which of the two stator current components will change slightly? Determine the corresponding voltage components!
5) Solve the mechanical linearized equation for a small step Δm in load torque for the initial conditions $\Delta i_d(0) = 0$, $\Delta i_q(0) = 0$, $\Delta \omega_m(0) = 0$!
6) Show that the occurring undamped angular eigenfrequency $\omega_{d,m}$ of speed in physical units is identical with the undamped rotor oscillation frequency of a synchronous cylindrical rotor machine at constant voltage and excitation, given in Chapter 7.7 as

$$\Omega_{d,m} = \sqrt{\frac{p \cdot |c_\vartheta|}{J}} \; .$$

7) Give the time function of the stator current in the d-q-reference frame at this undamped oscillation and make a sketch of $i_q(\tau)$ together with $m_s(\tau)$, $\Delta\omega_m(\tau)$!
8) Give the time function of the stator current per phase U $i_{s,U}(\tau)$ at this undamped oscillation.

1) Rotor reference frame d-q:

$$u_d = r_s \cdot i_d + \frac{d\psi_d}{d\tau} - \omega_m \cdot \psi_q, \quad u_q = r_s \cdot i_q + \frac{d\psi_q}{d\tau} + \omega_m \cdot \psi_d \, ,$$

Permanent magnet rotor: $\psi_f = \psi_p = const. \Rightarrow \psi_d = x_d i_d + \psi_p, \quad \psi_q = x_q i_q :$

$$u_d = r_s \cdot i_d + x_d \frac{di_d}{d\tau} - \omega_m \cdot x_q i_q, \quad u_q = r_s \cdot i_q + x_q \frac{di_q}{d\tau} + \omega_m \cdot x_d i_d + \omega_m \cdot \psi_p \, ,$$

$$\tau_J \cdot \frac{d\omega_m}{d\tau} = i_q \cdot \psi_d - i_d \cdot \psi_q - m_s = i_q \cdot x_d i_d + i_q \cdot \psi_f - i_d \cdot x_q i_q - m_s$$

Cylindrical rotor: $x_d = x_q$, neglected damping: $r_s = 0$.

$$u_d = x_d \frac{di_d}{d\tau} - \omega_m \cdot x_d i_q, \quad u_q = x_d \frac{di_q}{d\tau} + \omega_m \cdot x_d i_d + \omega_m \cdot \psi_p \, ,$$

$$\tau_J \cdot \frac{d\omega_m}{d\tau} = i_q \cdot \psi_p - m_s \, .$$

Steady state: $d./d\tau = 0$: $0 = i_{q0} \cdot \psi_p - m_{s0} \Rightarrow \omega_m = const. = \omega_{m0} = \omega_s \, ,$

$$u_{d0} = -\omega_{m0} \cdot x_d i_{q0}, \quad u_{q0} = \omega_{m0} \cdot x_d i_{d0} + \omega_{m0} \cdot \psi_p \, .$$

2) No-load: a) Mechanical: $m_{s0} = 0 \Rightarrow i_{q0} = 0, u_{d0} = 0, u_{q0} = \omega_{m0} \cdot x_d i_{d0} + \omega_{m0} \cdot \psi_p \, ,$
         b) Electrical: $\underline{i}_{s0} = 0 \Rightarrow i_{d0} = 0, i_{q0} = 0 : u_{d0} = 0, u_{q0} = \omega_{m0} \cdot \psi_p \, .$

3) $u_d = u_{d0} + \Delta u_d = \Delta u_{d0}, \; u_q = u_{q0} + \Delta u_q, \; \omega_m = \omega_{m0} + \Delta \omega_m \, ,$

$i_d = i_{d0} + \Delta i_d = \Delta i_d$, $i_q = i_{q0} + \Delta i_q = \Delta i_q$, $m_s = m_{s0} + \Delta m_s = \Delta m_s$,

(3.1): $\Delta u_d = x_d \dfrac{d\Delta i_d}{d\tau} - (\omega_{m0} + \Delta\omega_m) \cdot x_d \Delta i_q \approx x_d \dfrac{d\Delta i_d}{d\tau} - \omega_{m0} \cdot x_d \Delta i_q$,

$u_{q0} + \Delta u_q = x_d \dfrac{d\Delta i_q}{d\tau} + (\omega_{m0} + \Delta\omega_m) \cdot x_d \cdot \Delta i_d + (\omega_{m0} + \Delta\omega_m) \cdot \psi_p$,

(3.2): $\Delta u_q = x_d \dfrac{d\Delta i_q}{d\tau} + \omega_{m0} \cdot x_d \cdot \Delta i_d + \Delta\omega_m \cdot \psi_p$,

(3.3): $\tau_J \cdot \dfrac{d\Delta\omega_m}{d\tau} = \Delta i_q \cdot \psi_p - \Delta m_s$

4) From (3.3): A load torque disturbance will cause a current Δi_q and a change in speed $\Delta\omega_m$, whereas $\Delta i_d = 0$.

$\Delta u_d \approx x_d \dfrac{d\Delta i_d}{d\tau} - \omega_{m0} \cdot x_d \Delta i_q = -\omega_{m0} \cdot x_d \Delta i_q$,

$\Delta u_q = x_d \dfrac{d\Delta i_q}{d\tau} + \omega_{m0} \cdot x_d \cdot \Delta i_d + \Delta\omega_m \cdot \psi_p = x_d \dfrac{d\Delta i_q}{d\tau} + \Delta\omega_m \cdot \psi_p = 0$.

The voltage u_d must change slightly, whereas u_q may be kept constant.

5)   (3.3): $\tau_J \cdot \dfrac{d^2\Delta\omega_m}{d\tau^2} = \dfrac{d\Delta i_q}{d\tau} \cdot \psi_p - \dfrac{d\Delta m_s}{d\tau} = -\dfrac{\Delta\omega_m}{x_d} \cdot (\psi_p)^2 - \dfrac{d\Delta m_s}{d\tau}$,

Torque step with *Heavisides*'s step function $\varepsilon(\tau)$:

$\Delta m_s(\tau) = 0$, $\tau < 0$; $\Delta m_s(\tau) = \Delta m$, $\tau \geq 0$: $\dfrac{d\Delta m_s}{d\tau} = 0$, $\tau > 0$.

(3.3): $\tau_J \cdot \dfrac{d^2\Delta\omega_m}{d\tau^2} + \Delta\omega_m \cdot \dfrac{\psi_p^2}{x_d} = 0$, $\Delta\omega_m = A \cdot \sin(\omega_{d,m}\tau) + B \cdot \cos(\omega_{d,m}\tau)$,

$\omega_{d,m}^2 = \dfrac{\psi_p^2}{\tau_J \cdot x_d}$, $\Delta\omega_m(0) = B = 0$, $\Delta\dot\omega_m(0) = \omega_{d,m} A \cdot \cos(\omega_{d,m} \cdot 0) = \omega_{d,m} \cdot A$,

$\Delta\omega_m(\tau) = \dfrac{\Delta\dot\omega_m(0)}{\omega_{d,m}} \cdot \sin(\omega_{d,m}\tau)$, $\tau = 0+ : \tau_J \cdot \dfrac{d\Delta\omega_m(0+)}{d\tau} = \Delta i_q(0+) \cdot \psi_p - \Delta m_s(0+)$,

$\tau_J \cdot \dfrac{d\Delta\omega_m(0+)}{d\tau} = \tau_J \cdot \Delta\dot\omega_m(0) = -\Delta m$, $\Delta\omega_m(\tau) = \dfrac{-\Delta m}{\tau_J \cdot \omega_{d,m}} \cdot \sin(\omega_{d,m}\tau)$.

6) $\omega_{d,m} = \dfrac{\Omega_{d,m}}{\omega_N} = \sqrt{\dfrac{\psi_p^2}{\tau_J \cdot x_d}}$, $\psi_p = \dfrac{\hat\Psi_p}{\hat U_N / \omega_N} = \dfrac{\hat U_p}{\hat U_N}$, $\hat U_p = \omega_N \hat\Psi_p$, $x_d = \dfrac{\omega_N L_d}{\hat U_N / \hat I_N}$,

No-load: $u_{d0} = 0$, $u_{q0} = \omega_{m0} \cdot \psi_p \Rightarrow u_s = \sqrt{u_{d0}^2 + u_{q0}^2} = u_{q0} = \omega_{m0} \cdot \psi_p = \dfrac{\Omega_s / p}{\omega_N / p} \cdot \dfrac{\hat U_p}{\hat U_N}$

Operation at rated frequency: $\Omega_s = \omega_N$, $u_s = \dfrac{\hat U_s}{\hat U_N} = \dfrac{\hat U_p}{\hat U_N}$, $\tau_J = \dfrac{2J \cdot \omega_N^3}{3p^2 \cdot \hat U_N \hat I_N}$,

Exercise 8.10

$$\Omega_{d,m} = \omega_N \cdot \sqrt{\frac{\hat{U}_s}{\hat{U}_N} \cdot \frac{\hat{U}_p}{\hat{U}_N} \cdot \frac{3p^2 \cdot \hat{U}_N \hat{I}_N}{2J \cdot \omega_N^3} \cdot \frac{\hat{U}_N / \hat{I}_N}{\omega_N L_d}} = \sqrt{\frac{\hat{U}_s \hat{U}_p}{X_d} \cdot \frac{3p}{2J \cdot \Omega_{syn}}} \ ,$$

Pull-out torque at $r_s = 0$: $M_{p,0} = \dfrac{3U_s U_p}{\Omega_{syn} X_d}$, $\Omega_{d,m} = \sqrt{\dfrac{M_{p0} \cdot p}{J}}$.

Synchronous steady-state torque in dependence of load angle ϑ: $\dfrac{M_e}{M_{p,0}} = -\sin\vartheta$.

Magnetic spring constant $c_\vartheta = \dfrac{dM_e}{d\vartheta} = -M_{p0} \cdot \cos\vartheta$, at no-load: $\vartheta = 0$, $c_\vartheta = -M_{p0}$.

At no-load: $\Omega_{d,m} = \sqrt{\dfrac{M_{p0} \cdot p}{J}} = \sqrt{\dfrac{|c_\vartheta| \cdot p}{J}}$ in concordance with Chapter 7.7.

7) $x_d \dfrac{d\Delta i_q}{d\tau} = -\Delta\omega_m \cdot \psi_p$, $\Delta i_q = -\dfrac{\psi_p}{x_d} \cdot \int \Delta\omega_m \cdot d\tau = -\dfrac{\psi_p}{x_d} \cdot \dfrac{\Delta m}{\tau_J \cdot \omega_{d,m}^2} \cdot \cos(\omega_{d,m}\tau) + K$,

$$\Delta i_q(0) = 0 = -\frac{\psi_p}{x_d} \cdot \frac{\Delta m}{\tau_J \cdot \omega_{d,m}^2} + K \rightarrow K = \frac{\psi_p}{x_d} \cdot \frac{\Delta m \cdot \tau_J \cdot x_d}{\tau_J \cdot \psi_p^2} = \frac{\Delta m}{\psi_p} \ ,$$

$$\Delta i_q(\tau) = \frac{\Delta m}{\psi_p} \cdot (1 - \cos(\omega_{d,m}\tau)), \ \Delta i_d(\tau) = 0, \ \Delta i_{s(r)}(\tau) = j \cdot \Delta i_q(\tau) .$$
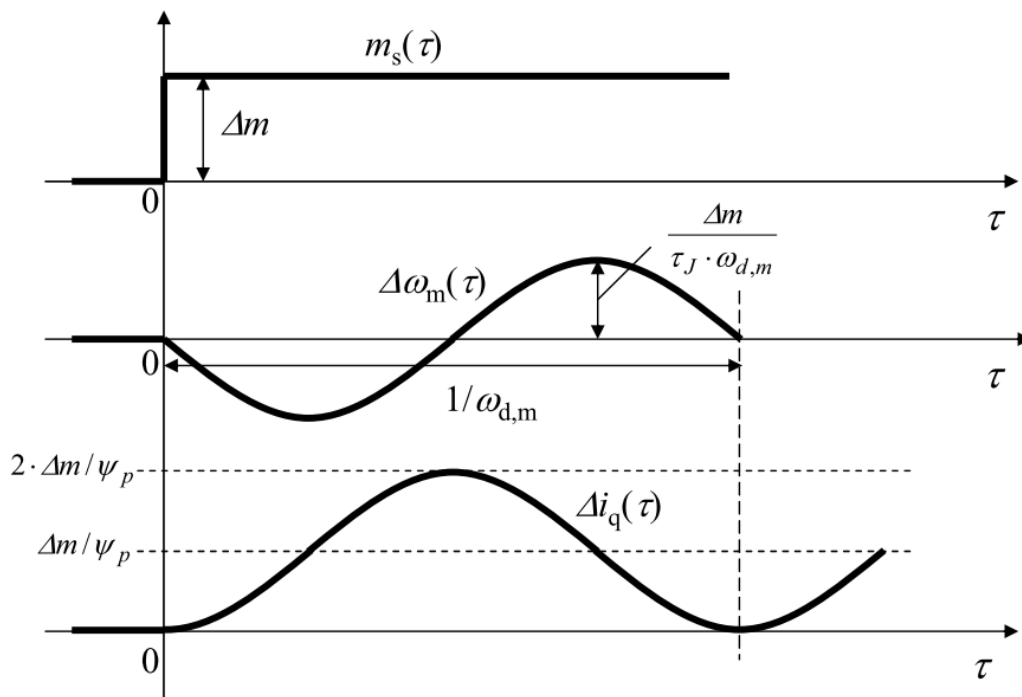

Fig. 8.10-1: Sketch of load torque step, speed and q-current variation due to natural rotor oscillation frequency $\omega_{d,m}$ at no-load and $R_s = 0$

8) $\underline{i}_{s(r)} = j \cdot \Delta i_q(\tau) = j \cdot \dfrac{\Delta m}{\psi_p} \cdot (1 - \cos(\omega_{d,m}\tau))$, $\underline{i}_{s(s)} = \underline{i}_{s(r)} \cdot e^{j\gamma}$, $\gamma = \omega_m \tau + \gamma_0$,

$i_{s,U} = \mathrm{Re}\{\underline{i}_{s(s)}\} = -\dfrac{\Delta m}{\psi_p} \cdot (1 - \cos(\omega_{d,m}\tau)) \cdot \sin(\omega_m \tau + \gamma_0)$.

As $\omega_m \sim 50\,\mathrm{Hz} \gg \omega_{d,m} \sim 1\ldots2\,\mathrm{Hz}$, the stator phase current pulsates with 50 Hz within the envelope of amplitude, given by $\dfrac{\Delta m}{\psi_p} \cdot (1 - \cos(\omega_{d,m}\tau))$, showing the rotor oscillation also in the "breathing" stator current amplitude.

Energy Converters - CAD and System Dynamics

Typical questions for exam preparation

1. The induction machine

1.1 Name typical utilisation factors according to *ESSON* for dc, synchronous and induction machines. Why do large machines have the highest utilisation factors?

1.2 Which electrical quantities determine the thermal utilisation of electrical machines? Why?

1.3 How do current loading *A* and magnetic air-gap flux density *B* determine the overload behaviour of rotating field machines?

1.4 How does the number of turns per phase of a rotating field winding determine the air-gap flux density?

1.5 Assume that the hysteresis and eddy-current losses in the iron core of a rotating field machine are too high. How can they be reduced by changing the number of turns at a fixed voltage supply?

1.6 How do the parameters of the equivalent circuit change with the number of turns? Assume that the starting current of an induction machine is 15 % too high. How can it be reduced by changing the number of turns at a fixed voltage supply?

1.7 Assume the starting torque to be by 20 % too small. How can it be increased?

1.8 Explain the basic approach of designing magnetic circuits with saturated iron.

1.9 What is *Carter's* coefficient?

1.10 Explain the method of calculating the ampere-turns for magnetising the teeth and yokes of rotating field machines.

1.11 How does the stray flux of a slot conductor look like? Which components of the total stray flux do you know?

1.12 Describe the losses occurring during energy conversion in induction motors? Give formulas!

1.13 What are "surface" and "pulsation" losses? Describe their generation in case of an induction machine under no load conditions.

1.14 According to the standards, how big must the overload capability of a line-fed induction motor be at least? What is the minimum allowed torque in the *M*(*n*) characteristic? Why are parameters like these standardised?

1.15 How big should the starting current be at most?

1.16 Give formulas for the "specific thrust"! How is it related to *Esson's* utilisation factor?

1.17 With what mathematical power (x^n) do the stack length, the electric loading A and the flux density increase with increasing output power?

1.18 Why do we have additional losses inside the slot conductors of the stator winding? How can they be reduced?

1.19 Which conductor height should not be exceeded at 50 Hz in order to limit the additional losses due to current displacement to reasonably small values?

1.20 A typical value of breakdown strength of air is 40 kV/cm. Compared to this value, what is the breakdown strength of modern insulating materials which are nowadays used in electrical machines?

1.21 Why is the rated voltage of electrical machines currently limited to about 30 kV, whereas transformers can be built with rated voltages of up to 750 kV and more?

1.22 Define the "locked-rotor voltage" of slip-ring induction machines! How can it be measured directly? How can the transformation ratio be derived from this measurement?

1.23 In which parts of the iron core are hysteresis and eddy-current losses mainly located?

1.24 Explain the specific losses in electric steel sheets? How can they be determined experimentally?

1.25 How do we choose the air gap of an induction motor? Should it be rather big or rather small? Give an explanation!

1.26 Describe the influence of open slots on the magnetic air-gap flux density! Why do we use open slots?

1.27 What is the "equivalent length" of the iron stack?

1.28 Assume the magnetising current to be too big! How can it be reduced for a given stator voltage supply?

1.29 How does the pole number, at a given bore diameter, influence the magnetising inductance?

1.30 How does saturation influence the magnetising inductance?

1.31 Explain the "current" and "voltage transformation ratio" of a squirrel cage and a slipring induction motor.

1.32 How big is the increase of ohmic resistance of a 5 cm high copper deep-bar at 50 Hz and slip $s = 1$? How big is it when the machine is supplied by a 60 Hz grid? How does an aluminium rotor behave compared to a copper rotor (aluminium conductivity smaller by a ratio of 34/57 compared to copper)?

1.33 How can iron losses and additional stray load losses be included in the equivalent circuit diagram?

2. Cooling of electrical machines

2.1 Give an overview of typical cooling systems for rotating electric machines.

2.2 What is the Thermal Class of insulation systems? Give some examples. Where are these classes defined?

2.3 What is "*Montsinger's* rule"?

2.4 Outline an equivalent scheme of heat sources and heat flow inside an induction motor.

2.5 Which physical principles are responsible for heat transport in electrical machines? What are "hot spots"?

2.6 Describe the thermal time constant of a homogeneous body. Discuss the influences of the body parameters on this time constant and the special case of "adiabatic" heating.

2.7 Where and how are "duty types" of electrical machines defined? Give some examples.

2.8 Why can the thermal utilisation of an electrical machine be increased in case of a short-term or intermittent periodic duty?

3. The converter-fed dc machine

3.1 Which equations describe the dynamic behaviour of a separately exited dc machine?

3.2 Name the time constants of a separately excited dc machine. Which ones are short, which ones are long?

3.3 Which transient phenomenon is described by the mechanical time constant, which one is described by the electrical time constant?

3.4 How is the mechanical time constant influenced by field weakening?

3.5 Explain the correlation between starting time constant and mechanical time constant. What is the starting time constant?

3.6 When is a dc drive capable of oscillating? Which parameters have an influence on the tendency to oscillate? Which two kinds of energy storage determine the oscillation of a dc drive?

3.7 How does the oscillation frequency change in case of field weakening?

3.8 Of which order is the control transfer function of a dc drive?

3.9 Sketch and discuss the operating limit characteristics of a converter-fed dc drive, depending on speed!

3.10 Sketch the converter circuit of a four-quadrant dc drive! Explain the generation of circulating current in converter-fed dc drives! Discuss advantages/disadvantages of this kind of drive operation.

3.11 Which are the effects of the converter supply on the operating behaviour of the dc machine?

3.12 Specify the dynamic characteristics of controlled dc drives in terms of rise time, time constants, and dynamic performance.

3.13 Explain the generation of circulating parasitic currents in anti parallel dc converters.

4. Dynamics of line-operated induction machines

4.1 What is a space vector? Give both a descriptive explanation and an exact mathematical definition of a space vector.

4.2 Are field space harmonics caused by a) the distribution of coils in slots, b) limited number of phase belts and c) due to slot openings included in the definition of space vectors?

4.3 How does a zero sequence current system influence the performance of electrical machines? How do positive phase-sequence and negative phase-sequence systems determine motor operation?

4.4 Does the space vector definition include the positive, negative and zero sequence current systems?

4.5 Explain graphically the transformation of phase currents into current space vector and vice versa.

4.6 Give the definition of flux linkage space vectors of main and leakage flux!

4.7 Describe the method of calculation with per-unit quantities! Name advantages and disadvantages! Give the definition of the reference impedance Z_N!

4.8 Are the reference voltage for calculating with p.u. values and the rated voltage of a rotating field machine identical? Give an example!

4.9 Which reference frames of induction machines are used with space vector calculation? Why do we use different reference frames?

4.10 Give an explanation of electromagnetic torque generation using space vectors.

4.11 How many and which equations do we need to completely describe the dynamic behaviour of an induction machine using a) complex space vectors, b) α- and β- components of space vectors?

4.12 Where do we include the effect of space harmonics in the space vector calculation theory?

4.13 Explain the starting of a mains-connected induction motor at voltage zero-crossing and maximum voltage.

4.14 Mains-connected induction motor in steady state conditions: Describe the locus of the moving current space vector, voltage space vector and flux linkage space vector performed in the stator reference frame!

4.15 Which effects can be distinguished at a "quick" start-up of an induction machine, compared to the steady state $M(n)$-characteristic? What is the "dynamic" breakdown torque?

4.16 Describe the influence a dc component of the starting current on the torque?

4.17 Give a descriptive explanation of the induction machine as an oscillating system.

4.18 Discuss the time function of the sudden short-circuit current of an induction machine with respect to the instant, when the sudden short circuit occurs.

5. Dynamics of inverter-fed induction machines

5.1 Discuss time constants, damping and natural frequencies of an induction motor with variable voltage/frequency operation. Why are these values only valid for small deviations from the considered points of operation?

5.2 Inverter supply of an induction machine with open-loop variable U/f operation: Describe the dynamic performance at low frequencies!

5.3 How does a big stator series resistor influence the dynamic performance of an induction machine with open-loop variable U/f operation?

5.4 Explain the basic principle of field oriented control. Why does this control strategy allow good dynamic performance of an induction machine?

5.5 For field oriented control we need to know the spatial location of the field axis of the flux space vector. Why? How can it be determined?

5.6 Which kinds of inverters are used with variable speed induction machines? Discuss advantages/disadvantages!

6. Dynamics of line-operated synchronous machines

6.1 Which reference frame is commonly used for the mathematical description of synchronous machines? Why?

6.2 Why is the complex space vector equations decomposed into its d- and q-components?

6.3 How many and which equations do we need to describe the dynamic behaviour of synchronous machines? How many main equations do we have?

6.4 Explain *Park's* transformation!

6.5 Discuss the transient and sub-transient reactance of *d*- and *q*-axis and give a physically descriptive explanation.

6.6 What is a "reactance operator" ? In which way is it used?

6.7 Explain the damper no-load and the damper short-circuit time constant!

6.8 Give an overview on different time constants of a synchronous machine and show their interrelation.

6.9 Explain the influence of the sub-transient reactance on the magnitude of the sudden short-circuit current of a synchronous machine!

6.10 Why can it happen, that the sudden short-circuit current does not have any zero crossings during the first oscillation periods?

6.11 Discuss the current components of the sudden short-circuit current! What is the influence of the instant of time of the short circuit on the magnitude of the peak current with respect to the phase angle of the stator voltage?

6.12 How does the transient torque occurring during a sudden short-circuit look like? May this torque be dangerous for the operation of electrical machines?

6.13 For rapidly changing currents (loads) at constant speed the synchronous machine is described as 3 winding transformer in the *d*-axis and as a 2 winding transformer in the *q*-axis, additionally to the rotary voltage induction.

6.14 Why can this transformer effect be omitted during steady-state synchronous operation, where currents are changing sinusoidal with synchronous frequency?

6.15 Which windings are "involved" in the 3-, which are involved in the 2-winding transformer? Draw equivalent circuits for these transformer configurations!

6.16 Explain the terms "transient" and "sub-transient" reactance using the *d*-axis transformer equivalent representation of the synchronous machine? Give typical per unit values!

6.17 Why does the *q*-axis only have a sub-transient, but no transient reactance? Determine the sub-transient reactance of the *q*-axis from the 2-winding equivalent transformer!

6.18 Name typical orders of magnitude of the different reactances of synchronous machines in p.u.-values! Which reactances are the smallest and why?

6.19 Give an approximate formula for the calculation of the magnitude of the sudden short-circuit current of synchronous machines with damper cage! Is the sudden short-circuit current larger than the steady-state short circuit current? If so, why and by typically which value?

6.20 When has the sudden short-circuit to happen - with respect to the terminal voltage - to cause the maximum current amplitude? When does it need to happen to cause the

minimum current amplitude? Why is it inevitable in case of three-phase machines that at least one phase will suffer from (nearly) maximum short circuit current amplitude?