7. Inverter operated induction machines

Source:

Siemens AG, Germany

Inverter-fed induction machine

- **Frequency converter (inverter)** generates three-phase voltage system with variable frequency f_s and variable amplitude U_s (rms). Hence synchronous speed is continuously variable. With that induction machine is **continuously variable in speed.**

- **Reversal of speed =** Changing of two phases of stator winding. Changing of energy flow (motor / generator) by decreasing / increasing phase shift between voltage and current :

motor $\varphi < \pi / 2$ generator $\varphi > \pi / 2$

- Voltage amplitude U_s must be changed in proportion to f_s **to keep the flux in the machine constant**. Thus torque will stay constant, if the the same current is used.

For $R_s = 0$:

$$\underline{U}_s = j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

$$\frac{\underline{U}_s}{\omega_s} = jL_{s\sigma} \underline{I}_s + jL_h (\underline{I}_s + \underline{I}'_r) = j(\underline{\hat{\Psi}}_{s\sigma} + \underline{\hat{\Psi}}_h) / \sqrt{2} = j\underline{\hat{\Psi}}_s / \sqrt{2} = const.$$

Rule for controlling the inverter: $U_s \sim \omega_s$

- Slip: $s = f_r / f_s = \omega_r / \omega_s \quad \Rightarrow \quad \Omega_m = \frac{\omega_s}{p} - \frac{\omega_r}{p}$

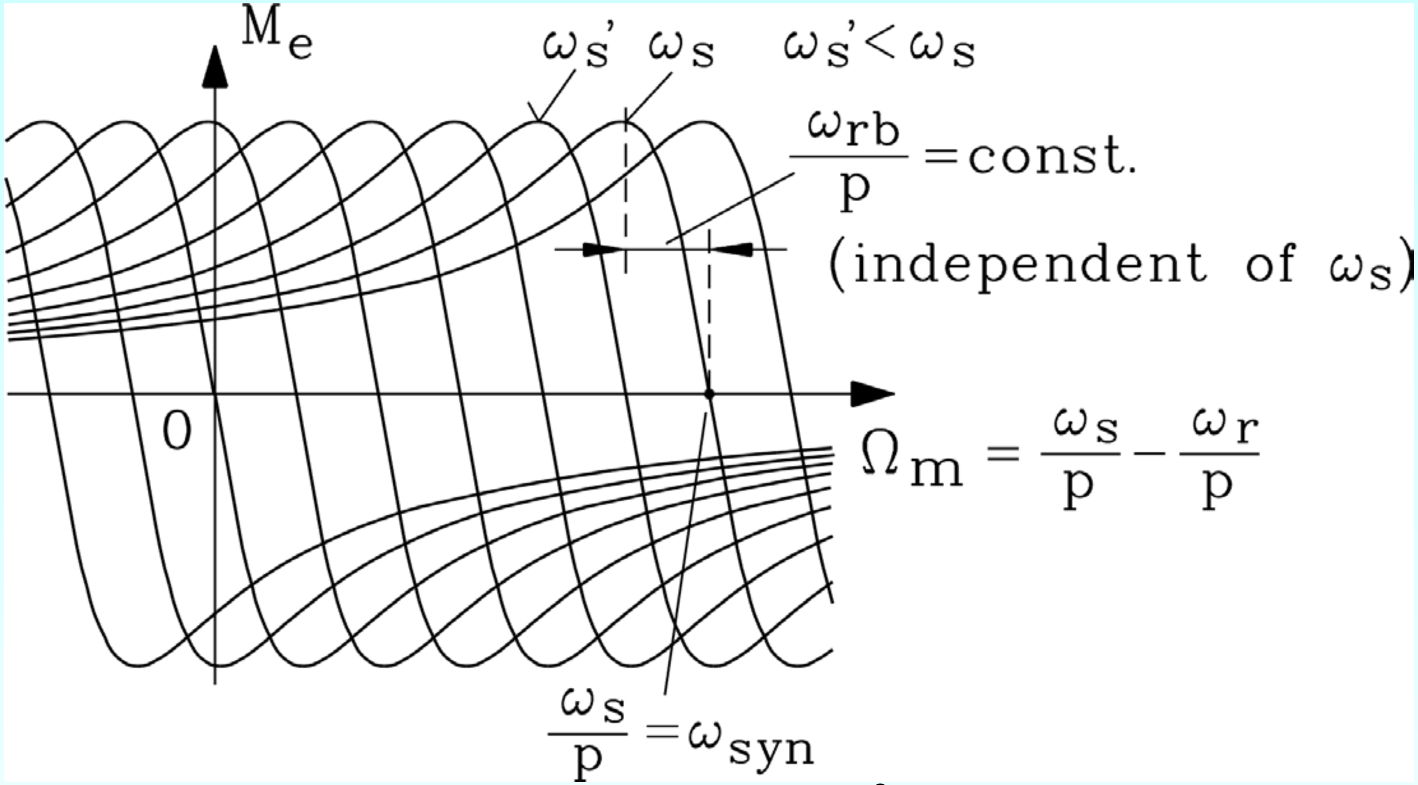
Curve $M_e(n) = M_e(\Omega_m)$ as **Curve** $M_e(\omega_r)$ for varying ω_s **is shifted in parallel !**

$M(n)$-Characteristic for inverter-fed induction machine

- $R_s = 0$:

KLOSS formula:

$$M_e = \frac{2M_b}{\frac{s}{s_b} + \frac{s_b}{s}} = \frac{2M_b}{\frac{\omega_r}{\omega_{rb}} + \frac{\omega_{rb}}{\omega_r}}$$



Break down torque M_b : $$M_b = \frac{m_s U_s^2}{\omega_s / p} \cdot \frac{1}{X_s} \cdot \frac{1-\sigma}{2\sigma} = \frac{m_s p}{2} \cdot \left(\frac{U_s}{\omega_s}\right)^2 \cdot \frac{1-\sigma}{\sigma L_s} = const.$$

Break down slip: $$s_b / s = \frac{R_r'}{s \sigma X_r'} = \frac{(\omega_s / \omega_r) \cdot R_r'}{\sigma \cdot \omega_s L_r'} = \frac{\omega_{rb}}{\omega_r}$$ with $$\omega_{rb} = \frac{R_r'}{\sigma L_r'}$$ **Slip frequency**

Influence of stator winding resistance R_s

- Voltage drop at stator resistance in stator voltage equation **MUST NOT be neglected at small angular frequency** ω_s

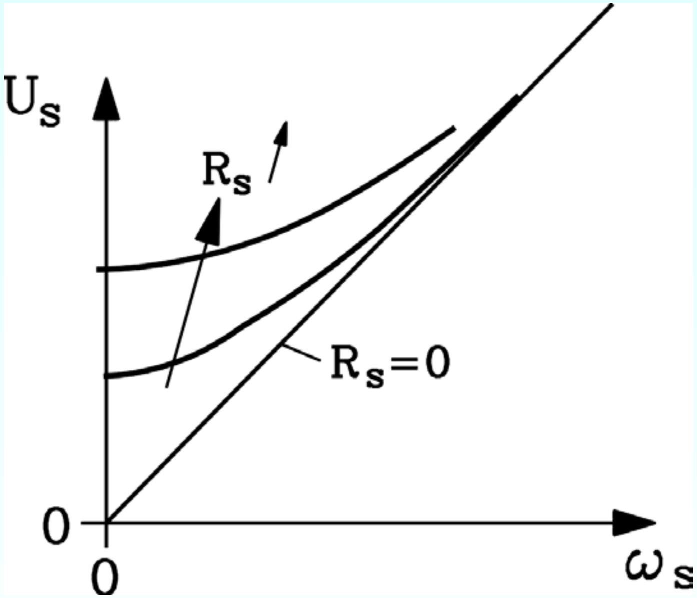
$$\underline{U}_s = R_s \underline{I}_s + j\omega_s L_{s\sigma} \underline{I}_s + j\omega_s L_h (\underline{I}_s + \underline{I}'_r)$$

- *Example:* Induction machine:
  Rated data: f_{sN} = 50 Hz, U_{sN} = 230 V:

$$f_s = 50Hz : \frac{R_s}{\omega_s L_s} = \frac{0.06\Omega}{3.0\Omega} = 0.02$$

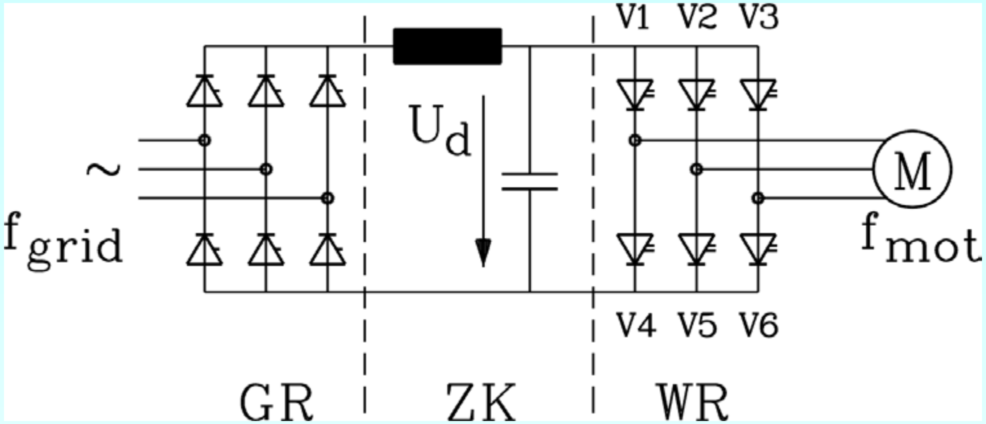
NOTE: At small f_s resistance R_s **must not be** neglected.

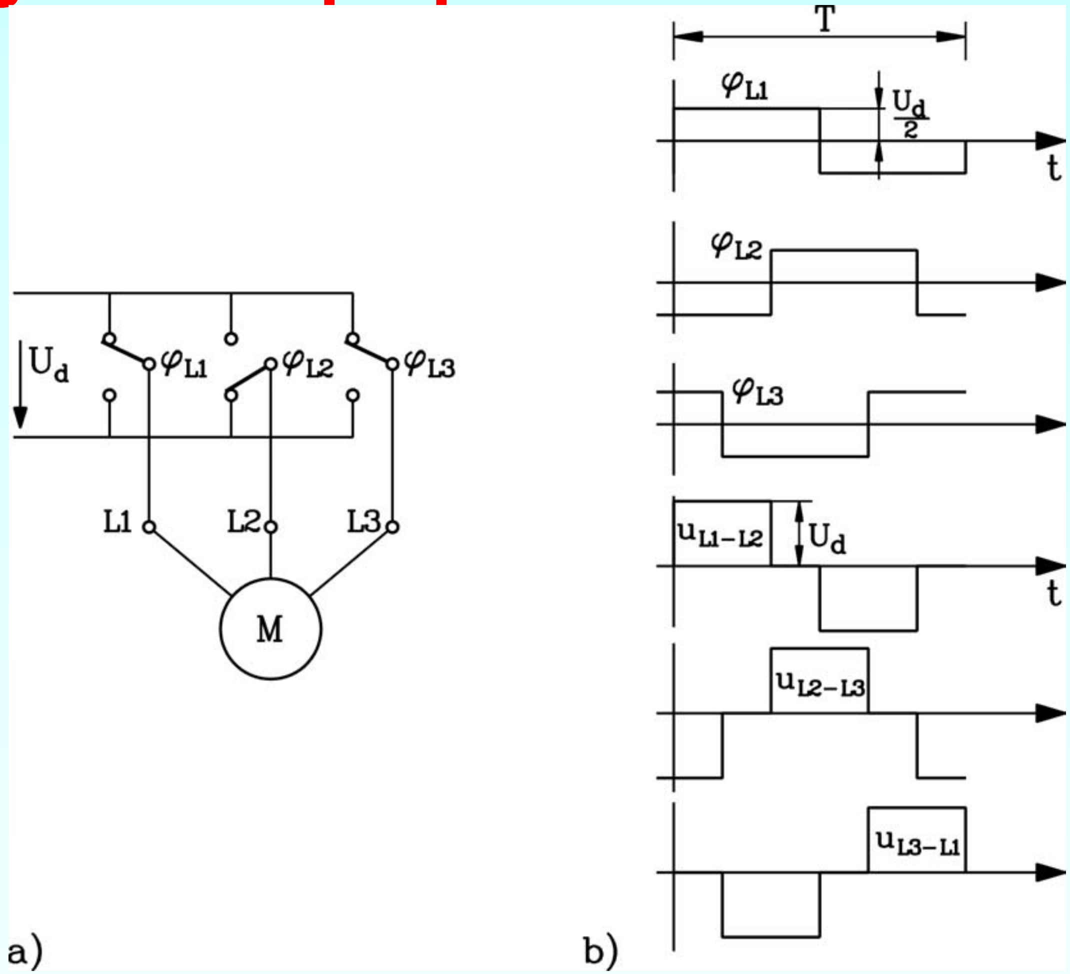
$$f_s = 5Hz : \frac{R_s}{\omega_s L_s} = \frac{6}{\frac{5}{50} \cdot 300} = \underline{\underline{0.2}}$$



- Voltage drop at stator resistance reduces at constant stator phase voltage U_s the internal voltage U_h. Hence break down torque decreases with square of internal voltage !
- By **increasing of U_s by $R_s I_s$ internal voltage** U_h must be kept constant for constant M_b.

Inverter with voltage six step operation

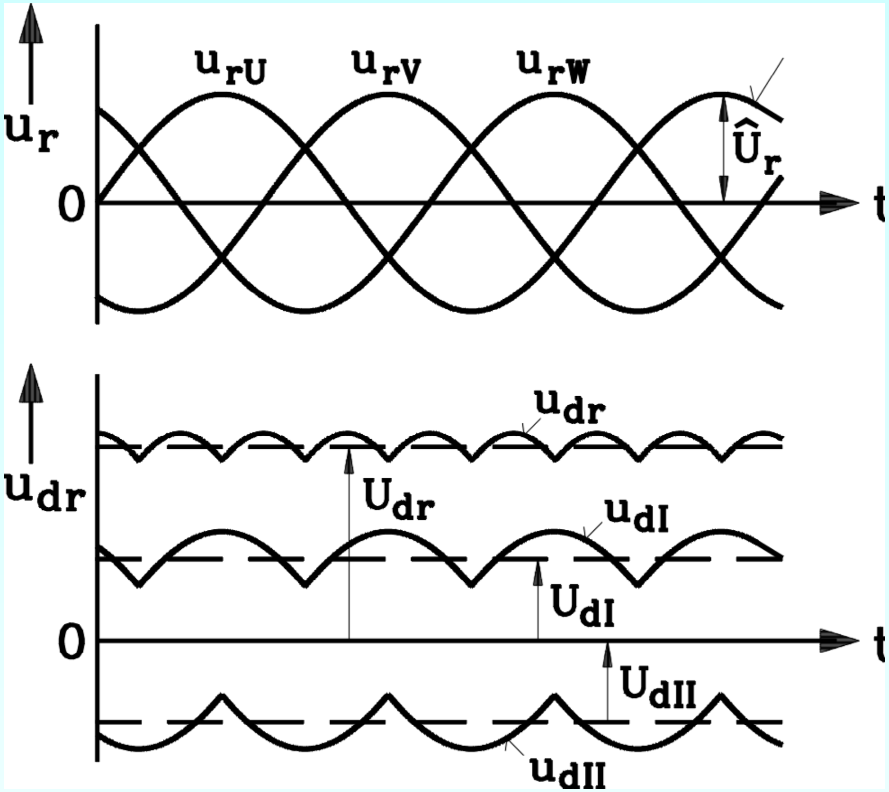


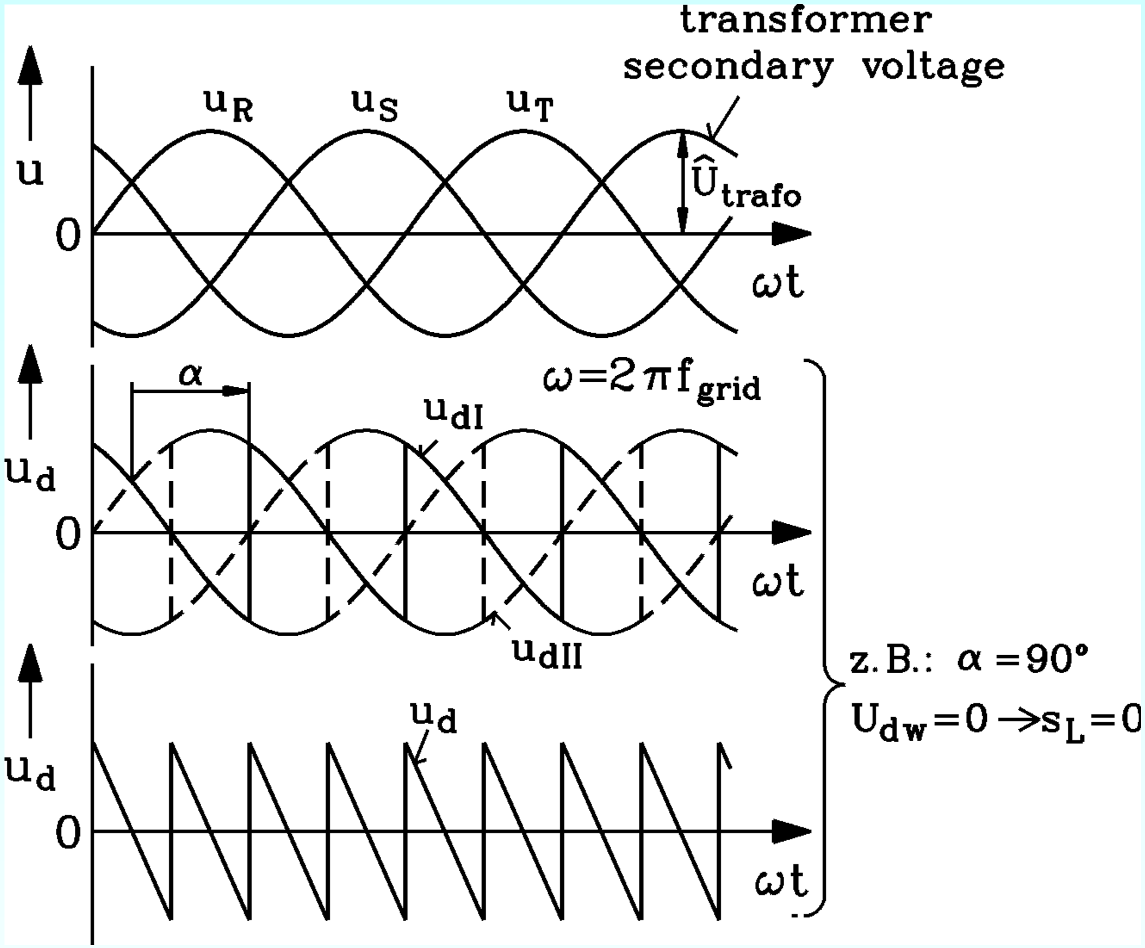


- Bridge rectifier with thyristors on grid side GR (firing angle α) generates variable DC voltage U_d in DC link ZK; voltage smoothed by capacitor.
- Inverter WR generates by six-step switching from U_d a **block shaped** line-to-line output voltage between terminals L1, L2, L3.

- DC link voltage U_d is changed by α **proportional** with output frequency f_{mot} .
- Grid side: At $\alpha > 90^\circ$: U_d and hence dc link power is negative = power flow back to the grid (generator braking).

Grid-side rectification



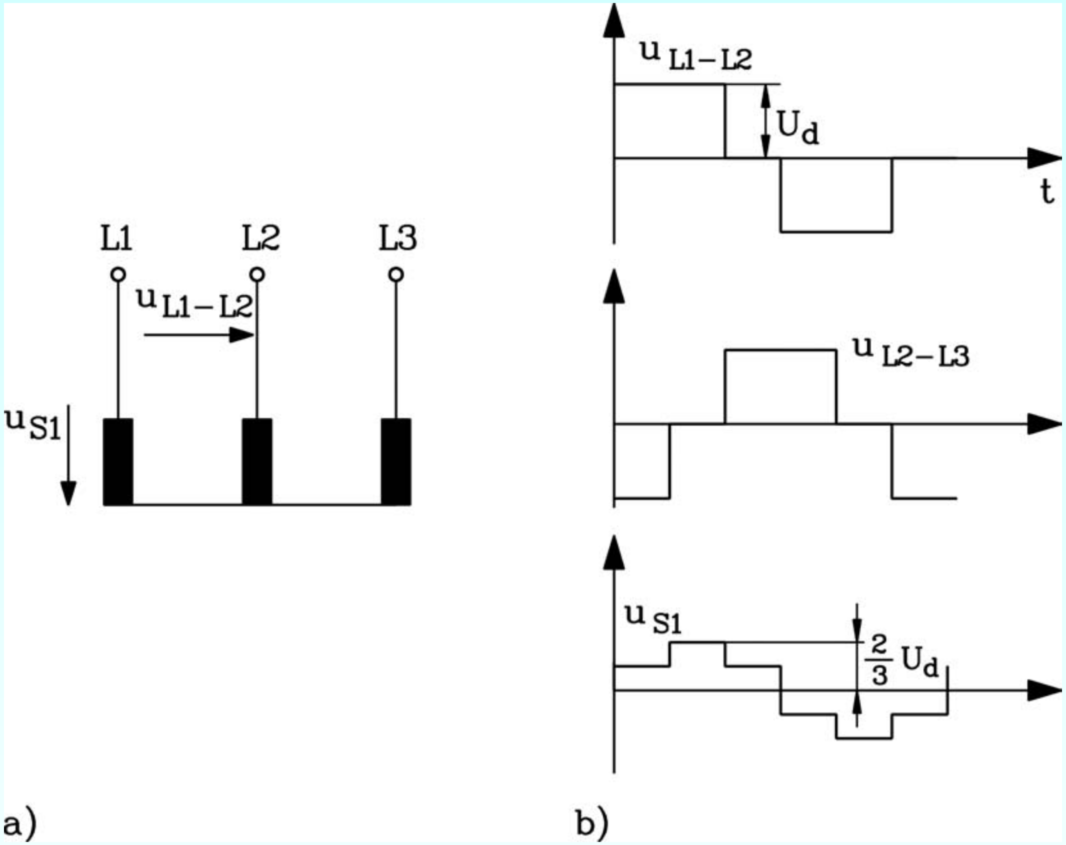


- Bridge rectifier on grid side GR:
  Maximum rectified voltage U_{d0} at firing angle $\alpha = 0$.
- Zero rectified voltage U_d at firing angle $\alpha = 90^\circ$

$$U_{dw} = \frac{3}{\pi}\sqrt{3}\cdot\hat{U}_{Trafo}\cdot\cos\alpha = U_{dw,\max}\cos\alpha$$

Voltage harmonics at six-step operation

- **Inverter output phase voltage:** $u_{S1} - u_{S2} = u_{L1-L2}; \quad u_{S2} - u_{S3} = u_{L2-L3}; \quad u_{S1} + u_{S2} + u_{S3} = 0;$



we get: $u_{S1} = \dfrac{2u_{L1-L2} + u_{L2-L3}}{3}$

- Block shaped line-to-line voltage, expanded as *FOURIER*-series:

$$u_L(t) = \sum_{k=1,-5,7,..}^{\infty} \hat{U}_{L,k} \cdot \cos(k \cdot \omega_s t)$$

$$k = 1 + 6g, \qquad g = 0, \pm 1, \pm 2, ...$$

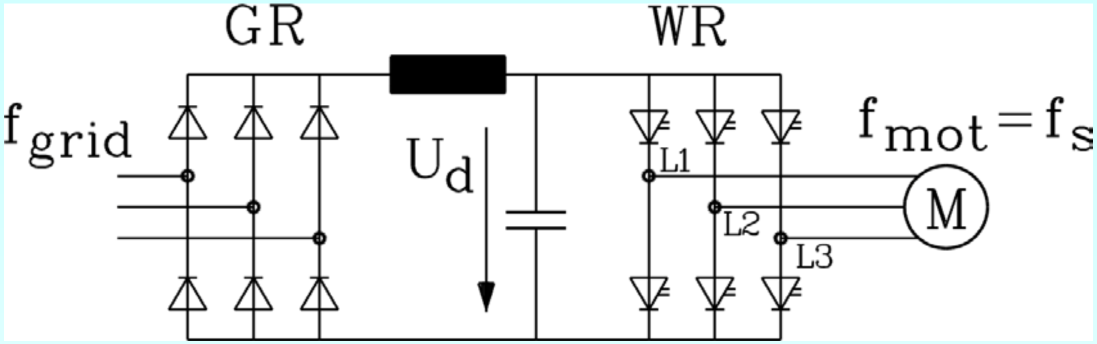
$\Rightarrow$ **k = 1, -5, 7, -11, 13, ...**

$$\hat{U}_{L,k} = \frac{2}{\pi} \sqrt{3} \frac{U_d}{k}$$

Electrical machine is fed with a blend of harmonic voltages of different amplitude, frequency and phase angle. Only fundamental (ordinal number k = 1) is desired. Voltage harmonics (|k| > 1) cause harmonic currents in electric machine with additional losses, torque pulsation, vibrations and acoustic noise.

Pulse width modulation (PWM)

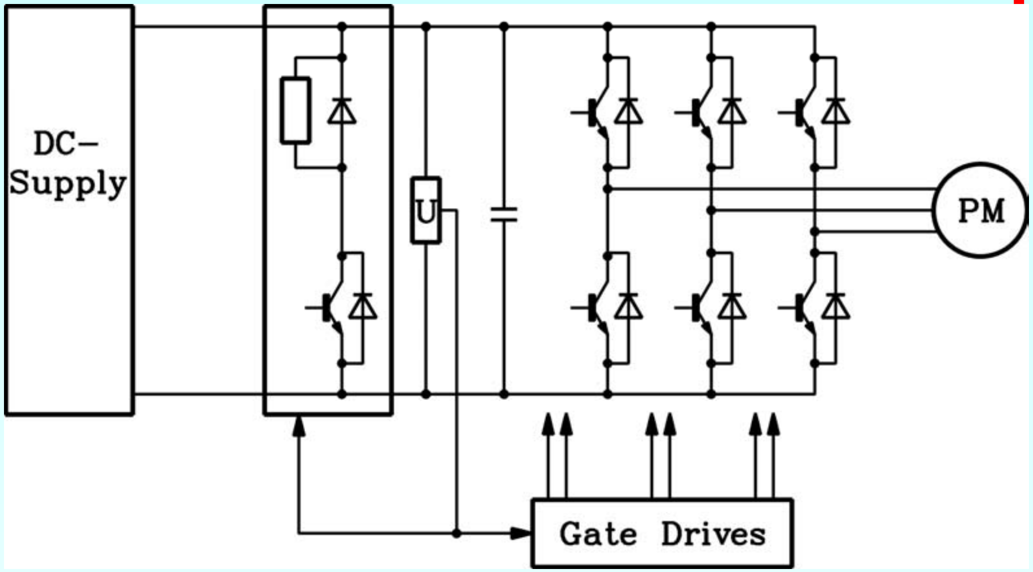
- At grid side: Diode rectifier GR (= firing angle α = 0): generates constant DC link voltage U_d , which is smoothed by capacitor: $U_d \sim U_{grid} = const.$



- Motor sinde inverter WR generates from U_d **by pulse width modulation** a line-to-line voltage between L1, L2, L3. Width of pulses is defined by comparison of **saw tooth signal** u_{SZ} (switching frequency f_{sch}) with AC reference signal u_{ref} , which pulsates with desired stator frequency f_s . With comparator a **PWM-signal** is generated to control power switches. *Reference signal is most often sine wave.*
- Amplitude A1 of u_{ref} defines amplitude of fundamental of PWM voltage at motor terminal. So it is varied **proportional** to f_{mot} .

- Grid side: $\cos\varphi = 1$. No power flow back into grid possible. (For that an anti-parallel second diode bridge or a line-side inverter is necessary !). Generator braking power has to be dissipated in ” brake"-resistors, which are connected in parallel with capacitor in DC link.

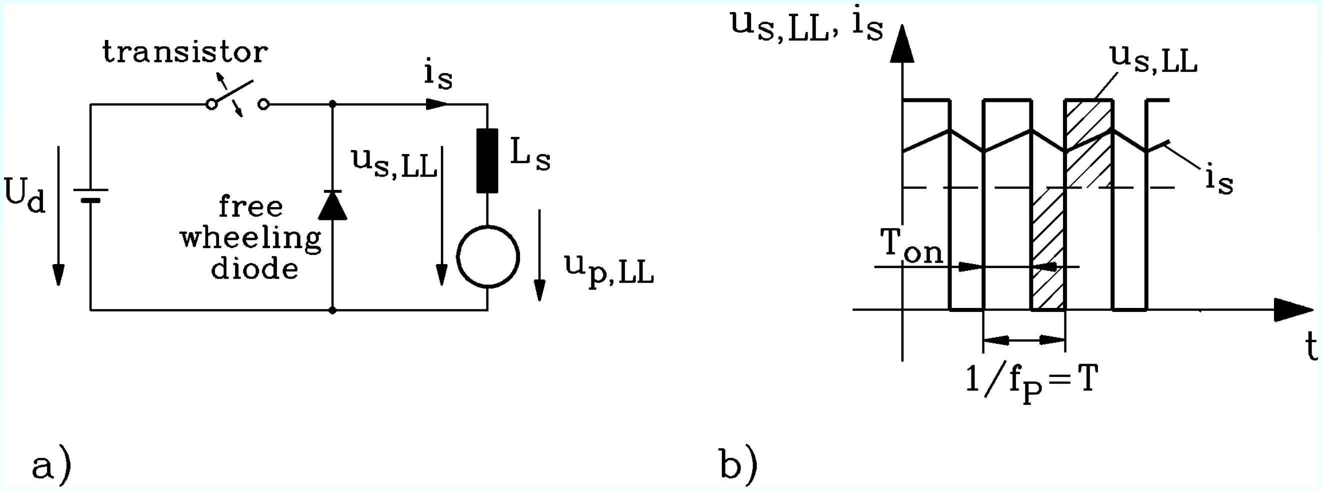
Stator current ripple generation



DC link voltage source inverter with switching transistors and free-wheeling diodes

R_s neglected:

$$U_d - U_{p,LL} \approx L_s \cdot di_s / dt$$



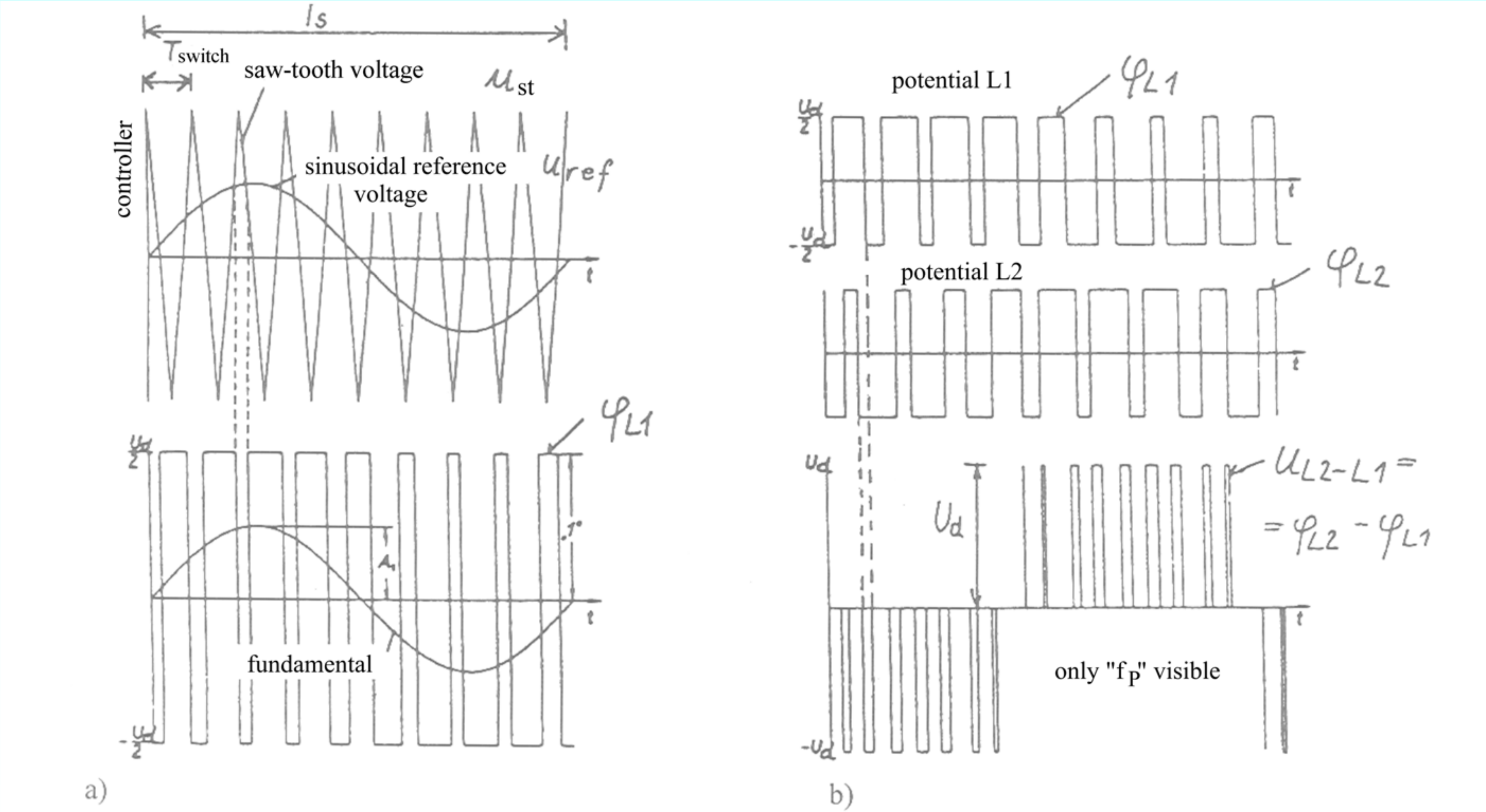
a) Equivalent switching scheme of DC link voltage source inverter, connected to the two phases with switching transistor and free-wheeling diode,

b) Current ripple and chopped inverter voltage

Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\varphi_{L1}(t)$ at terminal L1 varies with that PWM signal

b) Difference of two terminal potentials delivers line-to-line voltage $u_{L1-L2}(t)$



Voltage harmonics: Six-step and PWM

- ***Six-step modulation:*** FOURIER spectrum of line-to-line inverter output voltage:

k	1	-5	7	-11	13
$\hat{U}_{Lk} / \hat{U}_{L1}$	1	-0.2	0.14	-0.1	0.08

- ***PWM:*** FOURIER spectrum of terminal electric potential $\varphi_{L1}(t)$ and of line-to-line voltage $u_{L1-L2}(t)$ (at modulation degree A_1 = 0.5 and switching frequency ration f_{sch}/f_s = 9)

$\lvert k \rvert$	1	3	5	7	9	11	13	15	17	19
$\hat{\varphi}_k / (U_d / 2)$	0.5	$<10^{-5}$	0.001	0.09	1.08	0.09	0.002	0.04	0.36	0.36
$\hat{U}_{L,k} / \hat{U}_{L,k=1}$	1	0	0.002	0.18	0	0.18	0.004	0	0.72	0.72

Spectrum of terminal potential φ_L shows big amplitude of fundamental, of switching harmonic (k = 9) and at ***about twice switching frequency f_p = 2 f_{sch}*** *(k = 17 and 19).*

$$k = \left| \frac{f_p}{f_s} \pm 1 \right| \Rightarrow \quad k = |18 \pm 1| = 17, 19$$

Voltage harmonics with ordinal numbers, divisible by 3, do ***not*** *occur in line-to-line voltage ! At high switching frequency f_{sch} the amplitudes of all low frequency harmonics are small.*

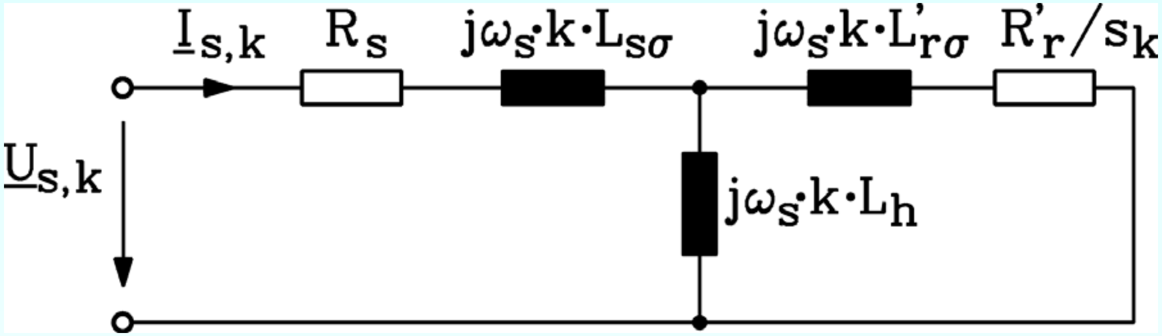
Voltage harmonics cause current harmonics

- The *voltage harmonics per phase* $U_{s,k}$ (frequency k-times fundamental frequency kf_s) cause current harmonics per phase $I_{s,k}$ in stator winding. These 3-phase harmonic current systems excite in air gap "high-speed" magnetic field wave (with pole count $2p$ due to winding):

k^{th} synchronous velocity ("high speed"): $n_{syn,k} = k \cdot f_s / p$

- **Rotor slip with k^{th} high-speed field s_k :** $s_k = \frac{n_{syn,k} - n}{n_{syn,k}} = \frac{kn_{syn} - n}{kn_{syn}} = 1 - \frac{1}{k} \cdot \frac{n}{n_{syn}} = 1 - \frac{1}{k} \cdot (1-s) \approx 1$

As harmonic slip s_k is nearly unity, independent of base slip s, harmonic currents amplitude $I_{s,k}$ is nearly independent from load. Current harmonics are already present at no-load to full extent at s = 0 .



High speed fields induce rotor, causing rotor current harmonics with high frequency:
$f_{rk} = s_k f_{s,k} \approx f_{s,k}$; causing big eddy current sin rotor bars and **big additional rotor losses !**

$$s_k \approx 1 \quad \Rightarrow \quad I_{s,k} \approx \frac{U_{s,k}}{\sqrt{(R_s + R_r')^2 + (k\omega_s)^2 \cdot (L_{s\sigma} + L_{r\sigma}')^2}} \approx \frac{U_{s,k}}{|k|\omega_s (L_{s\sigma} + L_{r\sigma}')}$$

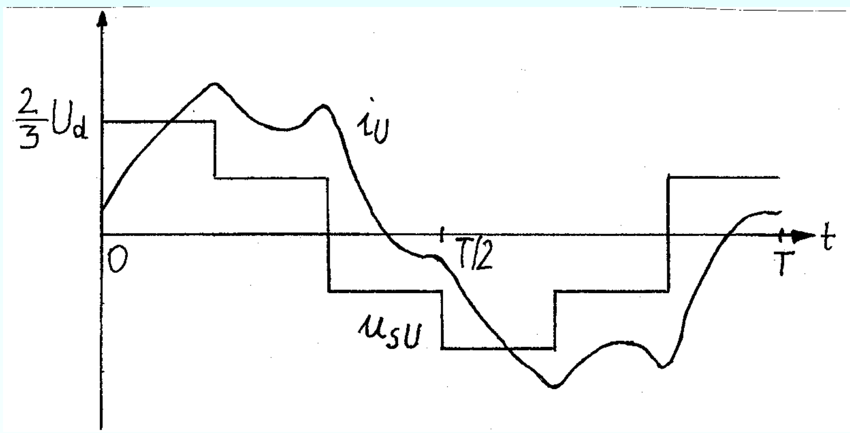
Example: Current harmonics at six-step modulation

- Amplitudes of current harmonics at six step operation:

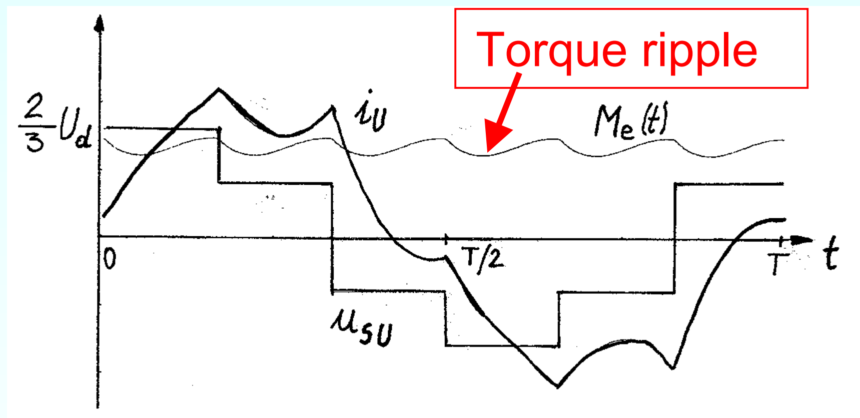
$$I_{s,k} \approx \frac{U_{s,k}}{|k|\omega_s(L_{s\sigma} + L'_{r\sigma})} \sim \frac{1}{|k|^2}$$

k	1	-5	7	-11	13
$\left\lvert \hat{U}_{Lk} / \hat{U}_{L1} \right\rvert$	**1**	**0.2**	**0.14**	**0.1**	**0.08**
$I_{s,k} / I_{s,k=1}$	**1**	**0.04**	**0.02**	**0.008**	**0.006**

- *Amplitudes of current harmonics decrease with inverse of square of ordinal number k, because leakage inductance* ***smoothes*** *the shape of current (= reduces the current harmonics !)*



FOURIER sum of 25 current harmonics

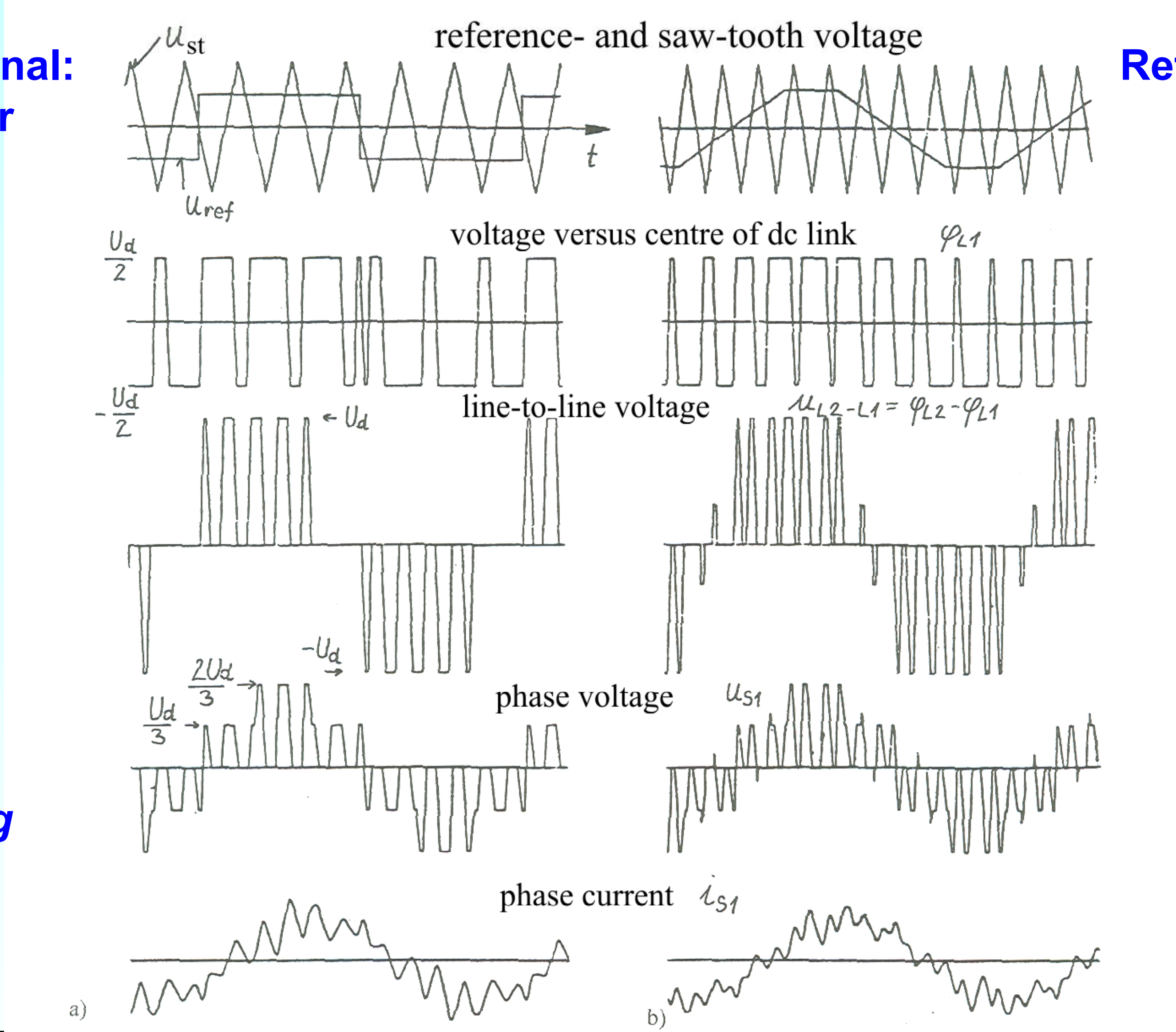


Exact solution of dynamic machine equation

Example: Current harmonics at PWM

Reference signal: rectangular

Reference signal: trapezoidal



Switching ratio:

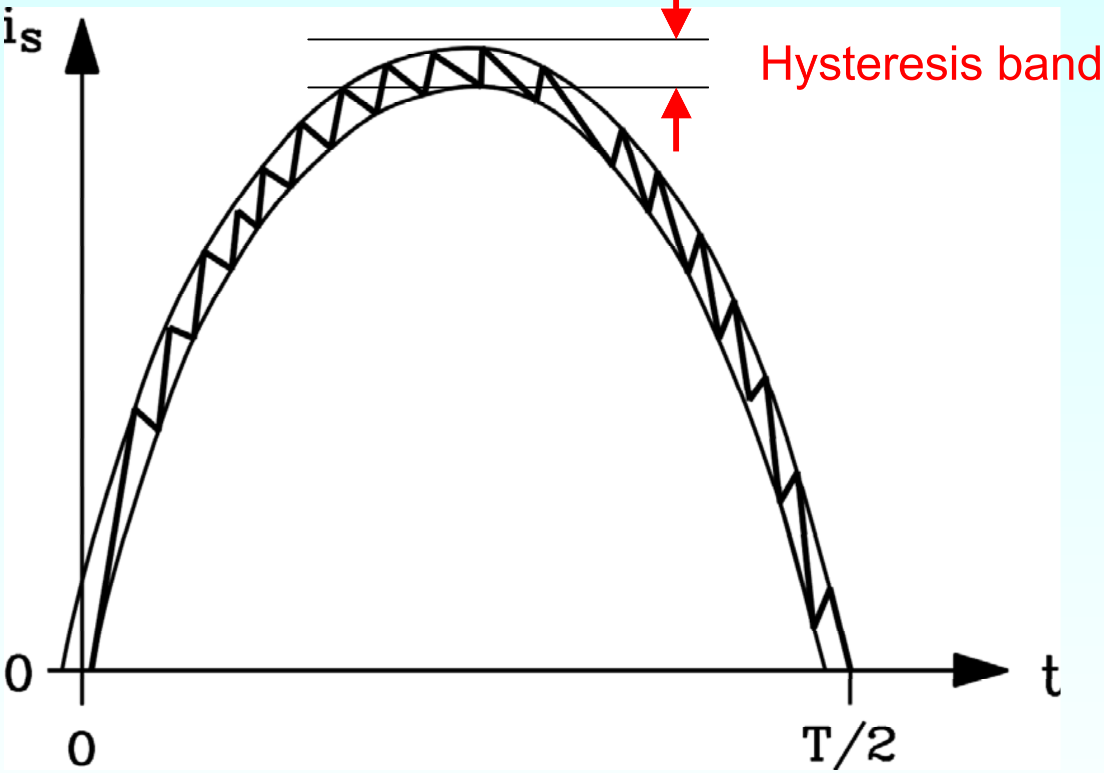
$f_{sch}/f_s = 6$

Switching ratio:

$f_{sch}/f_s = 9$

Hysteresis band control for stator current

- No fixed pulse pattern of switched DC link voltage, but switching instants are determined by comparison of measured current with hysteresis band limits.
- No fixed switching frequency !

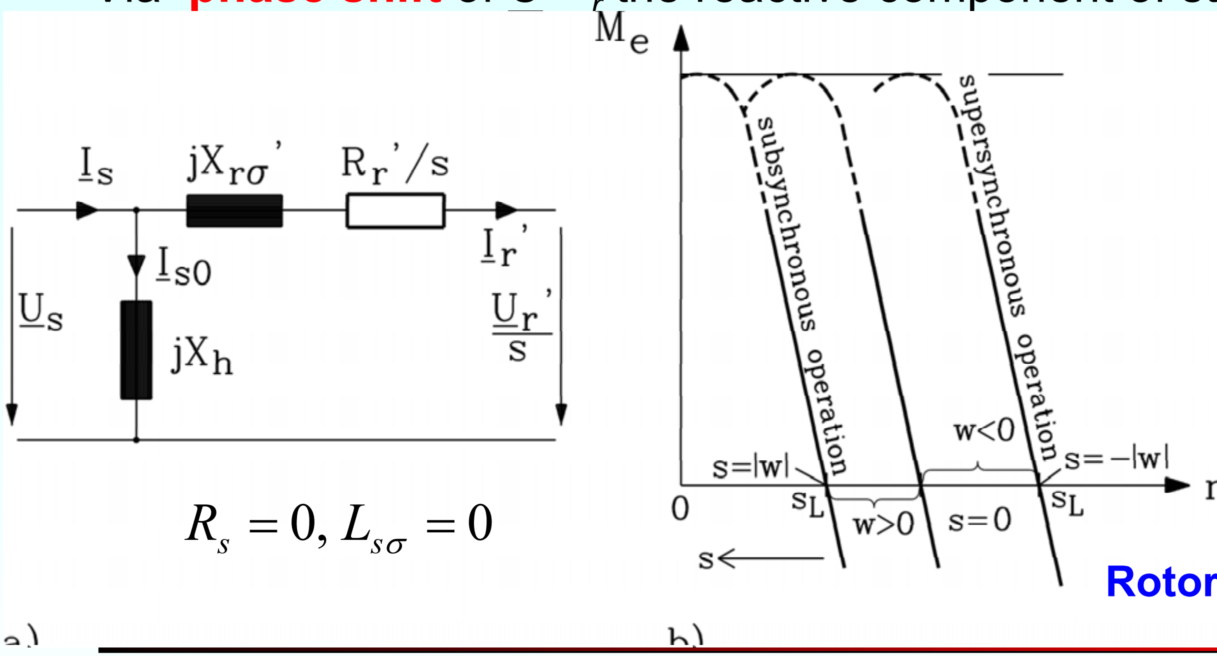


Doubly fed induction machine

- *Aim*: Speed variable operation with small inverter:
  inverter rating less than motor rating $S_{Umr} < S_{Mot}$
- *Solution:* Line-fed slip-ring induction machine, fed by small inverter in the rotor via slip rings
- **but:** Speed range $n_{min} \le n_{syn} \le n_{max}$ small. If we want $n_{min} = 0$, we get $S_{Umr} = S_{Mot}$.
- **Inverter feeds with rotor frequency an additional rotor voltage $\underline{U}'_r$ into rotor winding.**

  - Via variable **amplitude** of $\underline{U}'_r$ the speed is changed,

  - Via **phase shift** of $\underline{U}'_r$ the reactive component of stator current I_s is changed



Explanation with simplified T-equivalent circuit per phase:

$$\underline{U}_s = jX_h(\underline{I}_s - \underline{I}'_r) = jX_h \underline{I}_{s0}$$

$$\underline{U}'_r = -(R'_r + jsX'_r)\underline{I}'_r + jsX_h \underline{I}_s$$

Rotor current: $$\underline{I}'_r = \frac{\underline{U}_s - \frac{\underline{U}'_r}{s}}{\frac{R'_r}{s} + jX'_{r\sigma}}$$

Rotor additional voltage: $\underline{U}'_r = \underline{U}_s \cdot (w - jb)$

Simplified torque-speed curve of doubly fed machine

- **Electromagnetic torque M_e** : Approximation for small slip s << 1:

$$\underline{I}'_r = \frac{s\underline{U}_s - \underline{U}'_r}{R'_r + jsX'_{r\sigma}} \approx \frac{s\underline{U}_s - \underline{U}'_r}{R'_r} = \frac{U_s}{R'_r}(s - w + jb) \qquad s << 1$$

$$P_{in} = P_\delta = m_s \operatorname{Re}\{\underline{U}_s \cdot \underline{I}'^{*}_r\} = m_s \frac{U_s^2}{R'_r}(s - w) \Rightarrow \boxed{M_e = \frac{P_\delta}{\Omega_{syn}} = \frac{m_s U_s^2}{\Omega_{syn} R'_r}(s - w)}$$

*By real part of additional rotor voltage w the **M_e-n-curves are shifted in parallel !***

- Torque is ZERO at **no-load slip $s_L = w$**.
  - If no-load slip s_L is positive (**SUB-synchronous no-load points**) ⇔ Active component of additional rotor voltage IN PHASE with stator voltage
  - If s_L is negative (**SUPER-synchronous no-load points**) ⇔ Active component of additional rotor voltage is in PHASE OPPOSITION with stator voltage

$$M_e = 0 \quad \Rightarrow \quad s - w = 0 \quad \Rightarrow \quad \boxed{s_L = w = \frac{U'_{r,active}}{U_s}}$$

- **Inverter rating:** $S_{Inv} = 3U_r I_r$
- At n_{min} (⇔ $s_{L,max}$) both U_r and S_{Umr} are at maximum, thus defining inverter rating.

Doubly-fed wind generator

- Wind turbine **with variable speed** allows to extract **maximum possible wind power** at each wind velocity ***v*** .
- $P_{Wind} \sim v^3 \Rightarrow \boxed{P_{Turbine} \sim n^3}$
- Doubly fed induction machine used as **variable speed generator, operating at grid with constant grid frequency !**
- Additional rotor voltage with rotor frequency generated by 4-quadrant PWM inverter via slip ring fed into rotor winding.

- *Example:* Wind velocity varies between $0.65v_{max}$ and v_{max}:
- Generator and gear to turbine are designed hence for speed range$n_{syn} \pm 20\%$ ($s = \pm 0.2$):

Wind velocity	Generator speed	slip	add. voltage	power
v_{max}	$n = 1.2n_{syn} = n_{max}$	$s = -0.2$	$w = -0.2$	$P = 100\%$
$v_{min} = 0.65v_{max}$	$n = 0.8n_{syn} = 0.65n_{max}$	$s = +0.2$	$w = +0.2$	$P = 30\%$

- Rated power of inverter at steady state operation and rated torque:

$$P_{Inverter} = sP_\delta \approx sP_N = 0.2P_N$$

Here inverter rating is only 20% of generator rating, thus it is a very cheap solution, which is used nowadays widely at big wind turbines 1.5 ... 5 MW.

Inverter operated cage induction machine

IGBT inverter with grid rectifier and machine-side converter, feeding stator winding:

- power electronics
- control unit
- grid filter

Air-air cooled cage induction machine with heat top-mounted exchanger

Source:

Siemens AG, Germany

Rotor side voltage source PWM inverter for doubly-fed 3 MW induction generator

Source:

Winergy, Germany