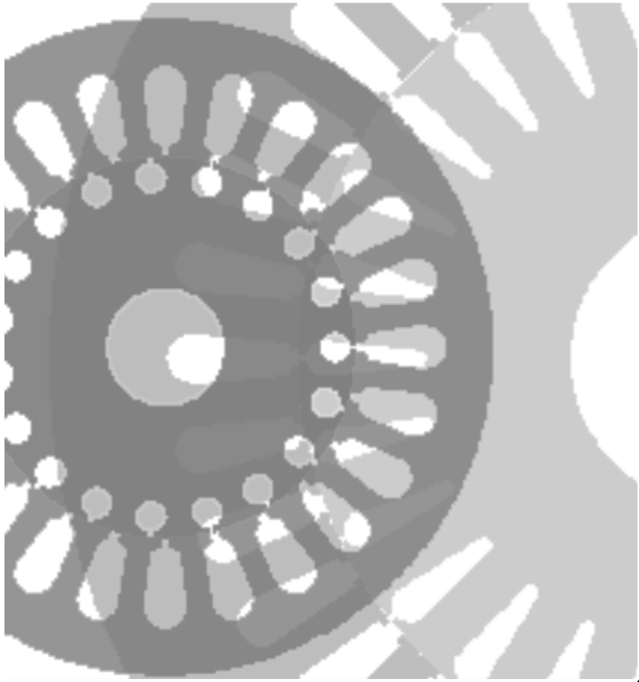
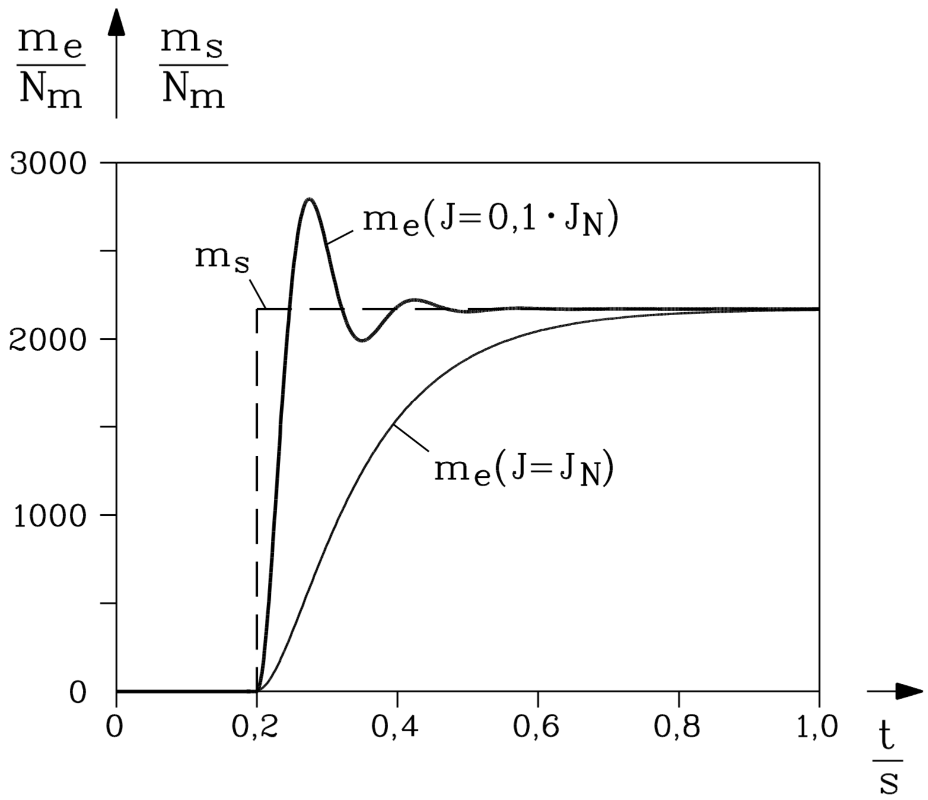
Energy Converters – CAD and System Dynamics

1. Basic design rules for electrical machines
2. Design of Induction Machines
3. Heat transfer and cooling of electrical machines
4. Dynamics of electrical machines
5. Dynamics of DC machines
6. Space vector theory
7. Dynamics of induction machines
8. Dynamics of synchronous machines



Source:
SPEED program

4. Dynamics of electrical machines



4. Dynamics of electric machines

4.1 Motivation: Why do we need dynamic theory of electric machines ?

4.2 Methods for calculation of transient machine operation

Motivation for dynamic simulations

Why do we need dynamic theory of electric machines ?

- *Controlled drives*
- *Switching of electric machines in normal operation*
- *Failures in electric machines*
- *Inverter operation*
- *Stability of operation*

For switching of motors, sudden failures, for response of machines to controller operation, for inverter operation and for investigation of stability dynamic modelling of electric machines is necessary.

4. Dynamics of electric machines

4.1 Motivation: Why do we need dynamic theory of electric machines ?

4.2 Methods for calculation of transient machine operation

4. Dynamics of electrical machines

Methods for calculation of transient machine operation

Differential equations instead of algebraic equations !

DC machinery: $U = R \cdot I \quad \rightarrow \quad u(t) = R \cdot i(t) + L \cdot di(t)/dt$

AC machinery: $\underline{U} = R \cdot \underline{I} + j\omega L \cdot \underline{I} \quad \rightarrow \quad u(t) = R \cdot i(t) + L \cdot di(t)/dt$

Mechanical system: $F_e = F_s \quad \rightarrow m \cdot dv(t)/dt = f_e(t) - f_s(t)$

Solving of differential equations:

- Dynamic models have only **time *t* as variable**
- **Initial conditions** are necessary: e.g. current $i(t = 0) = I_0$

a) Linear differential equations

b) Non-linear differential equations

Solving of linear differential equations

Linear superposition of solutions:

If $i_1(t)$ and $i_2(t)$ are solutions, then also $i_3(t) = i_1(t) + i_2(t)$ is a solution !

Example:

$$R(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t) \neq 0$$

a) Homogenous equation: $R(t) \cdot i(t) + L(t) \cdot di(t)/dt = 0$

homogenous solution $i_h(t)$

b) Inhomogenous equation: $R(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t)$

particular solution $i_p(t)$

c) Resulting solution: $i(t) = i_h(t) + i_p(t)$

Linear differential equations with constant coefficients

Example: $R(t) \cdot i(t) + L(t) \cdot di(t)/dt = u(t)$

Coefficients are constant: $R(t) = R$, $L(t) = L$ $\Rightarrow$ $\boxed{R \cdot i(t) + L \cdot di(t)/dt = u(t)}$

Example:

Separately excited DC machine

DC supply voltage U_f switched on: $u_f(t) = U_f$

Calculate field current increase $i_f(t)$

$W_m = L_f i_f^2 / 2$

Energy does not "jump",

(otherwise: infinite power,

$P = dW_m / dt = "\Delta W_m / 0" \to \infty$)

$W_m(0-) = W_m(0+)$

- Differential equation: $R_f \cdot i_f(t) + L_f \cdot di_f(t)/dt = u_f(t) = U_f$
- Initial condition: $i_f(0) = 0$.
- Homogenous solution: $i_h(t) = C \cdot e^{\lambda t} \to R_f + L_f \cdot \lambda = 0 \Rightarrow \lambda = -R_f / L_f$
- Particular solution: $i_p(t) = K \to R_f \cdot K + L_f \cdot dK/dt = U_f \quad \to \quad K = U_f / R_f$
- Initial condition determines constant C:

$$i_f(0) = i_h(0) + i_p(0) = C \cdot e^0 + K = 0: \quad C = -K$$

Solution: $i_f(t) = i_h(t) + i_p(t) = \frac{U_f}{R_f} \cdot \left(1 - e^{-\frac{t}{T_f}}\right)$, time constant: $T_f = L_f / R_f$.

Example:

Separately excited DC machine

DC supply voltage U_f switched on: $u_f(t) = U_f$

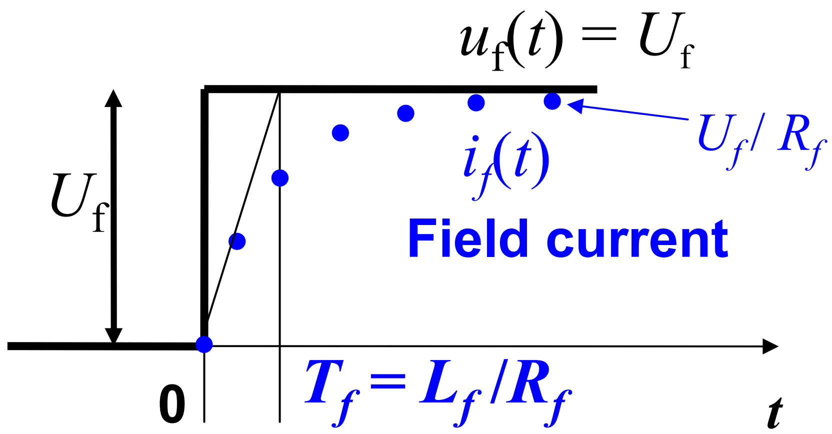
Calculate field current increase $i_f(t)$

Field current

$$i_f(t) = \frac{U_f}{R_f} \cdot \left(1 - e^{-t/T_f}\right)$$

After infinitely long time
(in reality after ca. $3T_f$)
the field current is a DC current:

$$i_f = I_f = U_f / R_f .$$



Laplace transform

$$F(\underline{s}) = L\{f(t)\} = \int_{t=0}^{\infty} f(t) \cdot e^{-\underline{s} \cdot t} \cdot dt$$

- Linear transformation of an arbitrary time function *f*(*t*), which is zero for $t \leq 0$

- Used for solving linear differential equations

Laplace transform table (1)

$f(t)$, $t > 0$ and zero, $t \le 0$	$F(\underline{s})$
K	$K/\underline{s}$
t	$1/\underline{s}^2$
t^n , $n = 1, 2, 3, ...$	$\frac{n!}{\underline{s}^{n+1}}$
$e^{b \cdot t}$	$\frac{1}{\underline{s} - b}$
$\sin(b \cdot t)$	$\frac{b}{\underline{s}^2 + b^2}$
$\cos(b \cdot t)$	$\frac{\underline{s}}{\underline{s}^2 + b^2}$
$\sinh(b \cdot t)$	$\frac{b}{\underline{s}^2 - b^2}$
$\cosh(b \cdot t)$	$\frac{\underline{s}}{\underline{s}^2 - b^2}$

Laplace transform table (2)

Linearity	$f_1(t)+f_2(t)$	$F_1(\underline{s})+F_2(\underline{s})$
	$k\cdot f(t)$	$k\cdot F(\underline{s})$
Similarity	$f(t/b)$	$b\cdot F(b\cdot\underline{s})$
	$f(t\cdot c)$	$\frac{1}{c}\cdot F(\underline{s}/c)$
Shifting	$f(t-\tau)$	$e^{-\underline{s}\cdot\tau}\cdot F(\underline{s})$
	$f(t)\cdot e^{-b\cdot t}$	$F(\underline{s}+b)$
Derivation	$df/dt=f'$	$\underline{s}\cdot F(\underline{s})-f(0)$
	$d^n f/dt^n=f^{(n)}$, n = 1, 2, ...	$\underline{s}^n F(\underline{s})-\underline{s}^{n-1}f(0)-\underline{s}^{n-2}f'(0)-$ $-\ldots-f^{(n-1)}(0)$

Laplace transform table (3)

Integration	$\int_0^t \int_0^t \ldots f(t) \cdot dt \cdot dt \cdot \ldots$, *n* times	$\frac{1}{\underline{s}^n} \cdot F(\underline{s})$
Convolution	$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau$	$F_1(\underline{s}) \cdot F_2(\underline{s})$
Limits	$\lim_{t \to 0+} f(t)$	$\lim_{\underline{s} \to \infty} \underline{s} \cdot F(\underline{s})$
	$\lim_{t \to \infty} f(t)$	$\lim_{\underline{s} \to 0} \underline{s} \cdot F(\underline{s})$

Convolution

Note: $$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau) \cdot d\tau$$

Proof: $$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau : \quad t - \tau = x, -dx = d\tau$$

$$-\infty < \tau < \infty \Leftrightarrow \infty > x > -\infty$$

$$\int_{\tau=-\infty}^{\tau=\infty} f_1(t-\tau) \cdot f_2(\tau) \cdot d\tau = \int_{x=\infty}^{x=-\infty} f_1(x) \cdot f_2(t-x) \cdot (-dx) =$$

$$= \int_{x=-\infty}^{x=\infty} f_1(x) \cdot f_2(t-x) \cdot dx$$

4. Dynamics of electrical machines

Example:

Switching on DC excitation voltage $U_f : u_f(t) = U_f$

Calculate current increase $i_f(t)$:

- **Differential equation:** $R_f \cdot i_f(t) + L_f \cdot di_f(t)/dt = u_f(t) = U_f$
- **Initial condition:** $i_f(0) = 0$.

- *Laplace* transformation:

$$L(i_f(t)) = I(\underline{s})$$

$$L\left(R_f \cdot i_f(t) + L_f \cdot di_f(t)/dt\right) = R_f \cdot I(\underline{s}) + L_f \cdot \left(\underline{s} \cdot I(\underline{s}) - i_f(0)\right) = L(U_f) = U_f / \underline{s}$$

- Solution of algebraic equation:

$$R_f I(\underline{s}) + L_f \cdot \underline{s} I(\underline{s}) = \frac{U_f}{\underline{s}} \quad \rightarrow \quad I(\underline{s}) = \frac{U_f}{\underline{s}} \cdot \frac{1}{R_f + \underline{s} L_f} = \frac{U_f}{R_f} \cdot \left(\frac{1}{\underline{s}} - \frac{T}{1 + \underline{s}T}\right)$$

with $T = \frac{L_f}{R_f}$. Initial condition is already implemented in algebraic equation !

- **Inverse transformation, using Tables, yields with**
- $1/\underline{s} \leftrightarrow 1$ **and** $\frac{1}{1+\underline{s}T} \leftrightarrow \frac{e^{-t/T}}{T}$ **the solution:**

$$L^{-1}\left(\underline{I}(\underline{s})\right) = i_f(t) = \frac{U_f}{R_f} \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

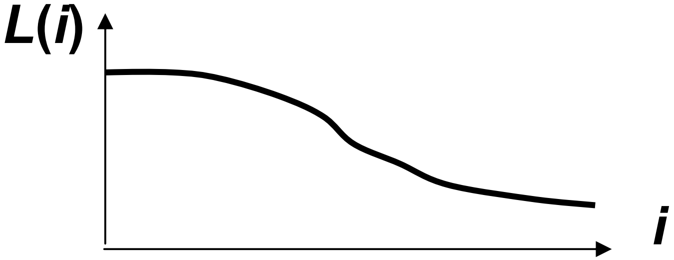
Solving of non-linear differential equations

- Superposition of two solutions **does not yield** another solution of that equation !
- Soving usually must be done **numerically** in step-by-step integration with finite step length, starting from $t = 0$ with the value for initial condition.
- Integration method: *Euler´s algorithm, preferred:* method of *Runge-Kutta*.
- Optimum step length exists !
- Commercial software:
  - *MATLAB/Simulink®,*
  - *DYMOLA/Modelica®,*
  - SIMPLORER, ...

Example:

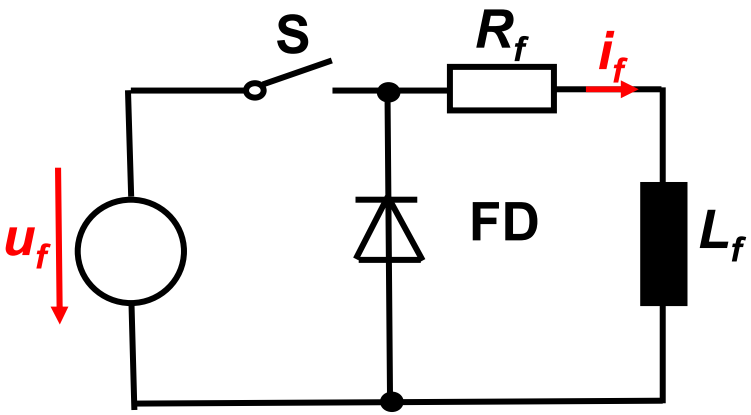
Iron saturation decreases inductance

$$R \cdot i(t) + L(i) \cdot di(t)/dt = u(t)$$



Example:
Solving of differential equation with *Euler´s* methods

Switching off at t = 0 a DC current i_f via a switch S and a free-wheeling diode FD

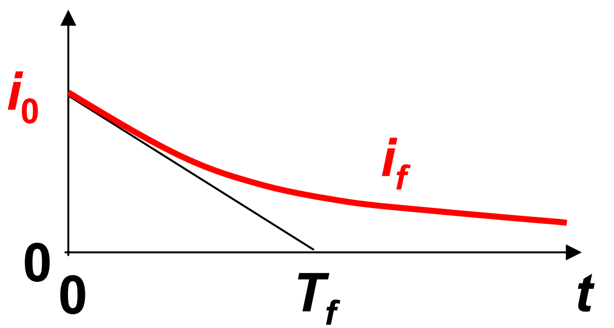


Initial condition:

$$\text{Magnetic energy does not "jump"}: W_m(0-) = W_m(0+)$$

$$W_m = L_f i_f^2(t=0-)/2 = L_f i_f^2(t=0+)/2$$

$$i_f(0) = i_0$$



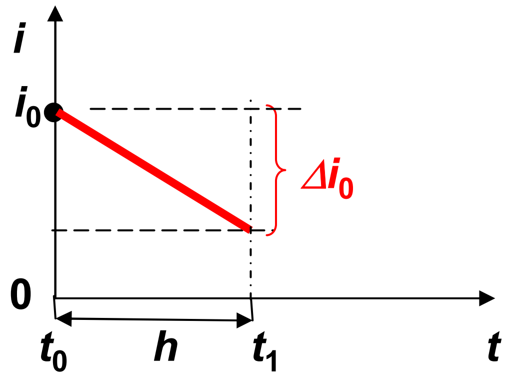
Differential equation:

$$R_f \cdot i_f(t) + L_f(i_f) \cdot di_f(t)/dt = 0$$

$$\text{If } L_f = \text{const.}: i(t) = i_0 \cdot e^{-t/T_f}, T_f = L_f / R_f$$

Example:

- NON-LINEAR differential equation $R \cdot i(t) + L(i) \cdot di(t)/dt = 0$
- Solved numerically with ***Euler´s* method**:
- $\frac{di}{dt} = -\frac{R \cdot i(t)}{L(i)}$ or $i' = f(i,t)$ with initial condition $i(t_0) = i_0$.
- Integration step length $\Delta t = h$: $\Delta i_0 = f(i_0, t_0) \cdot h$.
- Point by point solution: $t_1 = t_0 + h$: $i_1 = i(t_0 + h) \approx i_0 + \Delta i_0$.
- Next point $t = t_0 + h + h = t_1 + h$: $\Delta i_1 = f(i_1, t_1) \cdot h$
- $i_2 = i(t_1 + h) \approx i_1 + \Delta i_1$ and so on.



General rule is $i_{n+1} = i_n + h \cdot f(i_n, t_n)$ with *n* = 1, 2, 3, ... to be calculated in recursive way.
Thus the values i_n at t_n are taken instead of the exact (but unknown) function $i(t)$.

Example:

Comparison of *Euler* and *Runge-Kutta* method for -R = L = 1: "negative damping"

$$\frac{di}{dt} = i(t), \quad i(0) = 1$$

Source:
H.-J. Dirschmid, Springer-Verlag

- Exact solution is known $i(t) = e^{t}$, $T = -1$

t	$i(t)$ ($h = \Delta t = 0.2$)	$i(t)$ ($h = \Delta t = 0.2$)	$i(t)$
	Euler	***Runge-Kutta***	***exact solution***
0	1.0	1.0	1.0
0.2	1.2	1.2214	1.2214027
0.4	1.44	1.49182	1.4918247
0.6	1.728	1.82211	1.8221188
0.8	2.0736	2.22552	2.2255409
1.0	2.48832	2.71825	2.7182818
1.2	2.98598	3.32007	3.3201169
...	...	...	...

The deviation from exact solution is much smaller with *Runge-Kutta* in comparison with *Euler´s* method.

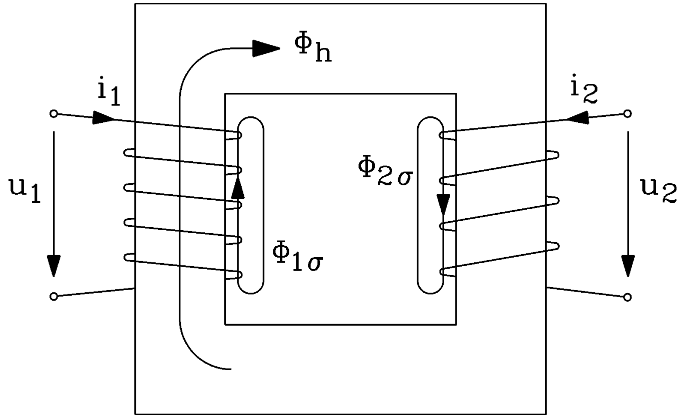
Summary:
Methods for calculation of transient machine operation

- Linear differential equations allow superposition of solutions
- Linear diff. equ. with constant coefficients: homogeneous and particular solution
- *Laplace* transform used for solving linear differential equations
- Non-linear ordinary differential equations solved by time-stepping
- RUNGE-KUTTA time stepping numerical solution method widely used

Appendix: Transformer in-rush current (1)

Switching the primary winding of a single-phase transformer to the grid, secondary winding is open.

Worst case: Zero crossing of voltage at switching



$$u_1(t) = \hat{U} \cdot \sin(\omega \cdot t) = R_1 i_1(t) + d\psi_1(t)/dt \quad t \geq 0$$

$$u_1(t) = \hat{U} \cdot \sin(\omega \cdot t) \approx d\psi_1(t)/dt$$

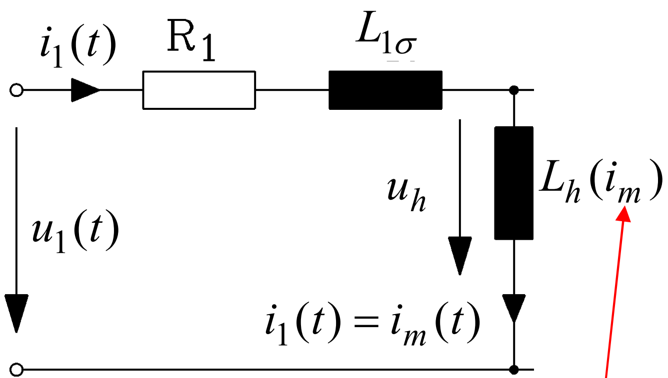
$$\psi_1(t) \approx \int_0^t \hat{U} \cdot \sin(\omega \cdot t) \cdot dt = -\frac{\hat{U} \cdot \cos(\omega \cdot t)}{\omega} + C$$

$$\psi_1(0) = 0 \Rightarrow C = \frac{\hat{U}}{\omega}$$

$$\psi_1(t) = \frac{\hat{U}}{\omega} \cdot (1 - \cos(\omega t))$$

DC flux component

AC flux component



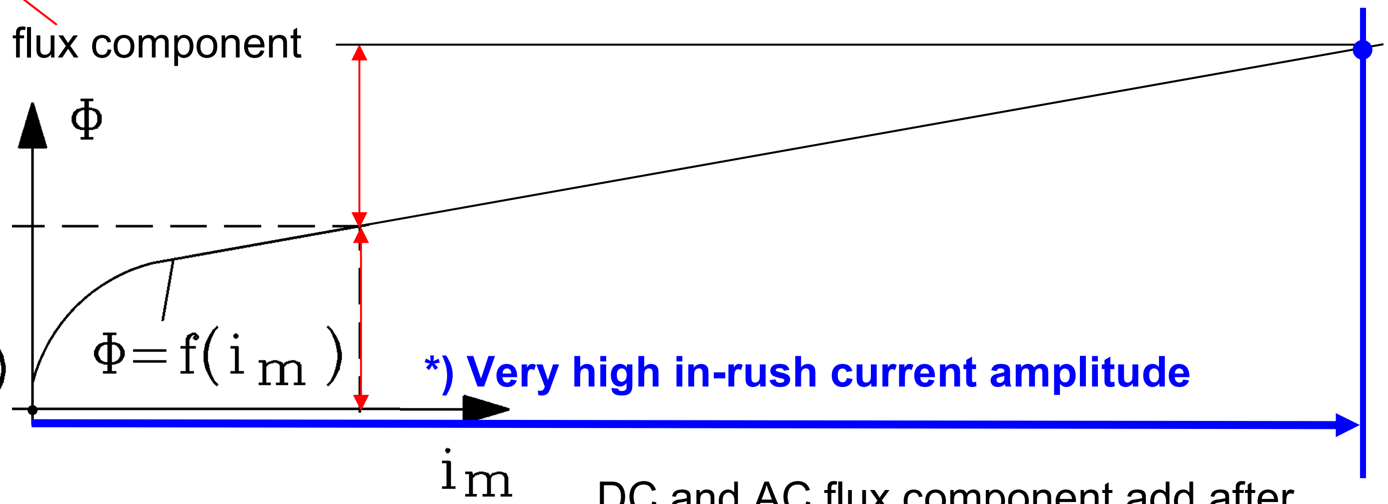
Non-linear main inductance: Decreases strongly with current due to iron core saturation!

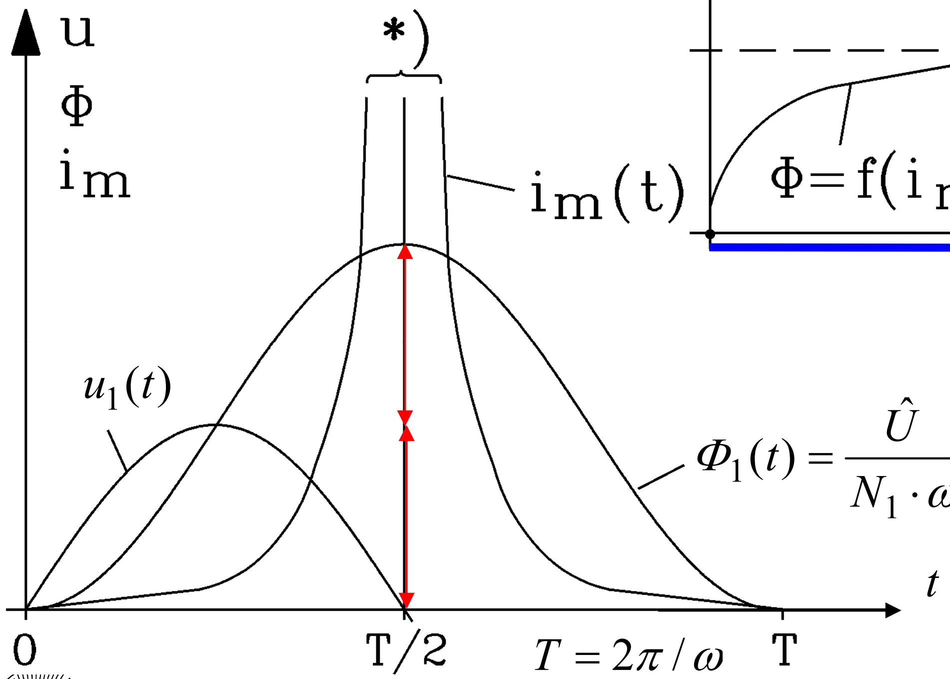
Transformer in-rush current (2)

$$\psi_1(t) = N_1 \cdot \Phi_1(t) = \frac{\hat{U}}{\omega} \cdot (1 - \cos(\omega t))$$

$$i_1(t) = i_m(t)$$

DC flux component — AC flux component

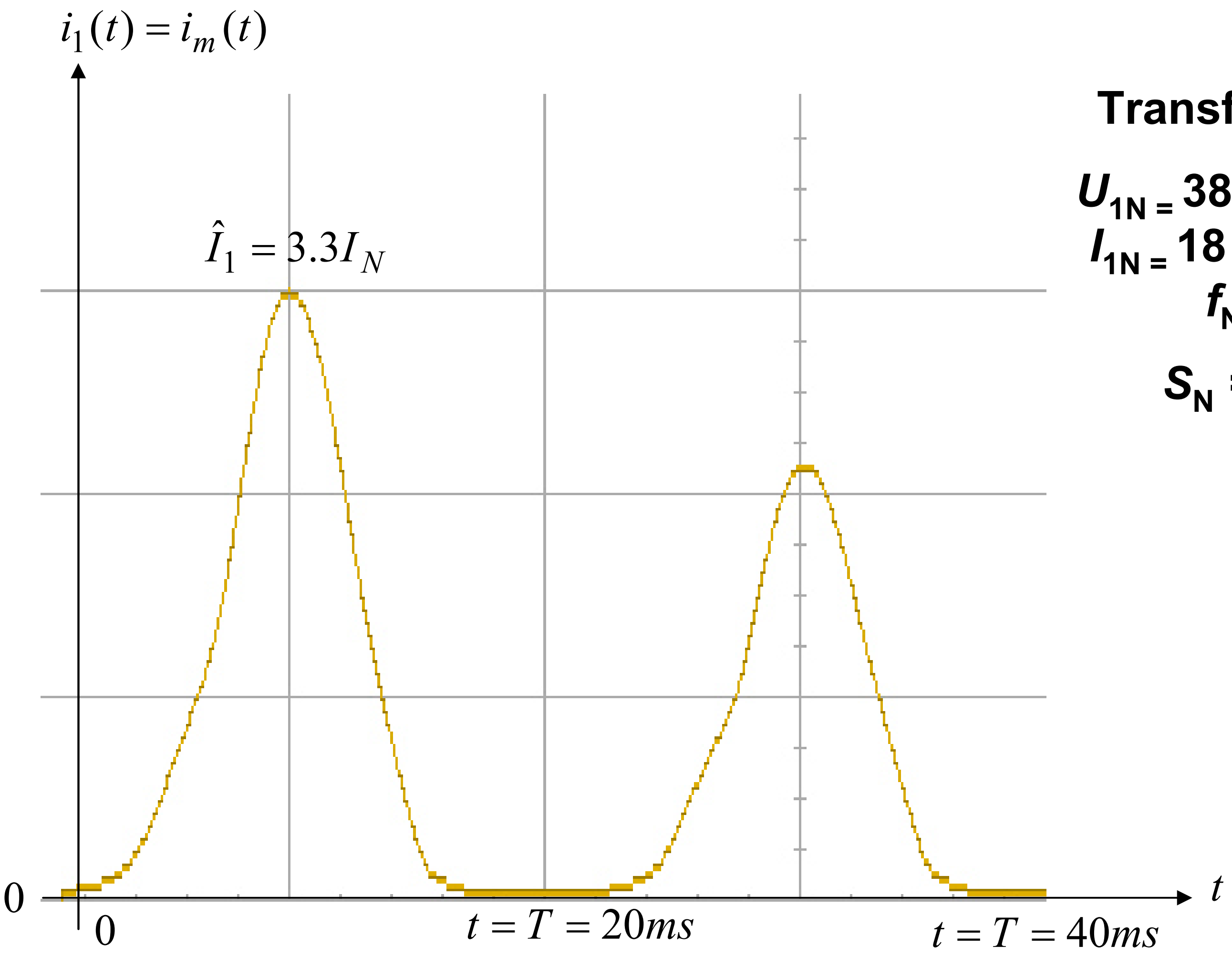




DC and AC flux component add after half period *T*/2: = double rated flux occurs = very high saturation = very low main inductance L_h = very high magnetizing and no-load primary current = **IN-RUSH CURRENT!**

Up to 15-times rated current!

Measured in-rush current of a small transformer



Transformer rating:

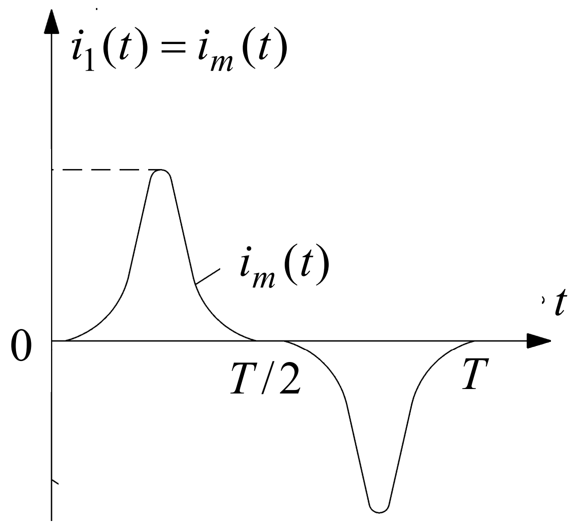
U_{1N} = 380 V Y / 220 V Δ
I_{1N} = 18 A Y / 31,1 A Δ
f_N = 50 Hz

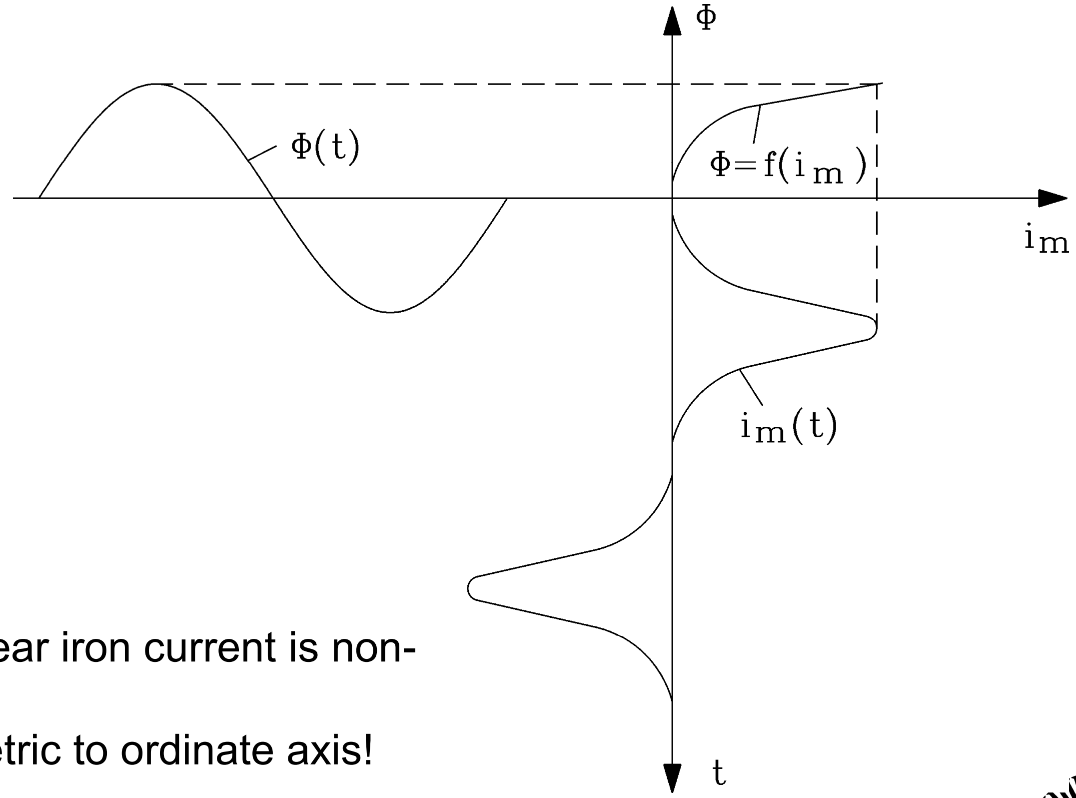
S_N = 11.8 kVA

Steady-state transformer primary no-load current

- Due to I^2R losses the DC current component decays!
- The DC flux component vanishes, only the sinusoidal AC flux component remains.
- The AC no-load current is very small, but non-sinusoidal due to the big, but non-linear $L_h(i_m)$.

$$\Phi_1(t) = -\frac{\hat{U}}{N_1 \cdot \omega} \cdot \cos(\omega t)$$

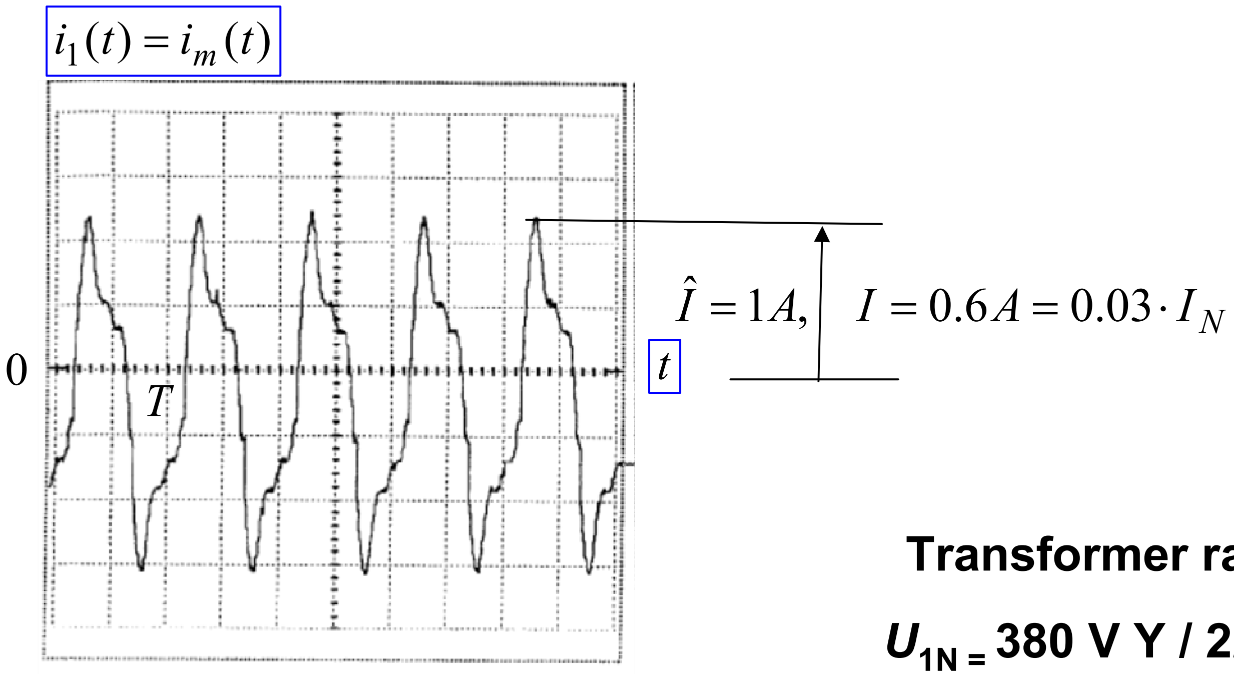


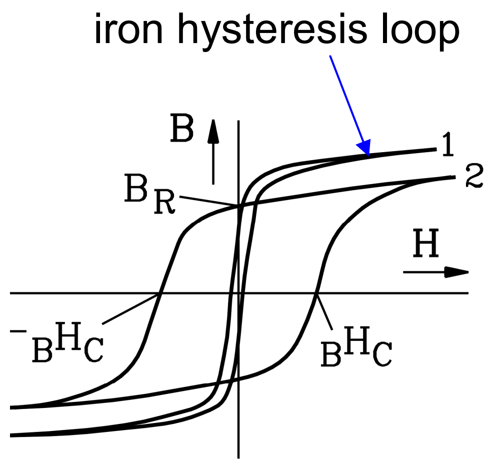


Note:
Due to non-linear iron current is non-sinusoidal,
but still symmetric to ordinate axis!

Measured steady-state transformer primary no-load current

- Due to iron hysteresis loop the no-load current is not ordinate-symmetric, but still abscissa-symmetric!





Transformer rating:

U_{1N} = 380 V Y / 220 V Δ
I_{1N} = 18 A Y / 31,1 A Δ
f_N = 50 Hz

S_N = 11.8 kVA