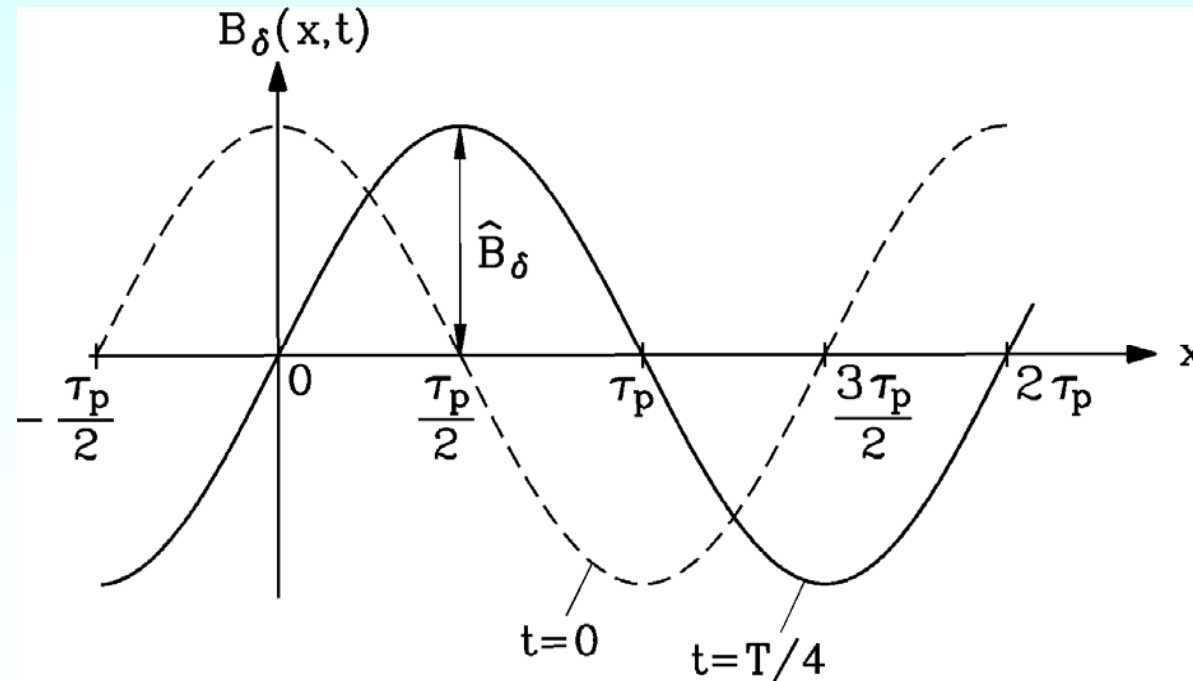
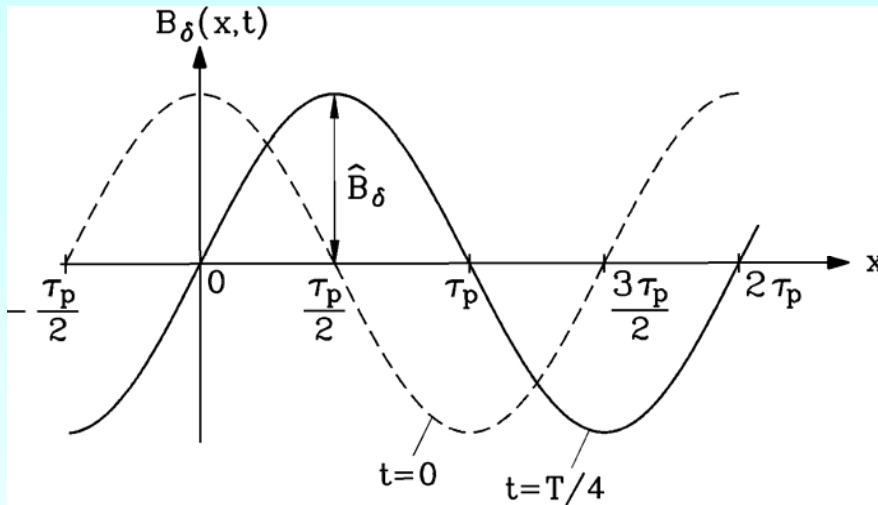


3. Mathematical Analysis of Air-Gap Fields



Rotating waves - Travelling waves

- **Rotating wave:** x is stator circumference co-ordinate (*rotating machine*)
- **Travelling wave:** x is stator linear co-ordinate (*linear machine*)



$$B_{\delta 1}(x, t) = \hat{B}_{\delta 1} \cos\left(\frac{x \cdot \pi}{\tau_p} - 2\pi f \cdot t\right)$$

Wave velocity: Observer, who is moving with the wave, sees constant argument of $\cos() = const.$

$$v_{syn} = \frac{dx}{dt} = \frac{d}{dt} (const. + 2\pi f t) \frac{\tau_p}{\pi} = \underline{\underline{2 f \tau_p}}$$

- **Wave in opposite direction:** $B_{\delta 1}(x, t) = \hat{B}_{\delta 1} \cos\left(\frac{x \cdot \pi}{\tau_p} + 2\pi f \cdot t\right) \Rightarrow v_{syn} = -2 f \tau_p$

• Example:

At frequency $f = 50$ Hz: v_{syn} in m/s is **SAME** number as pole pitch in cm : $v_{syn}^{[m/s]} = \tau_p^{[cm]}$

e.g. 2-pole turbine generator ($2p = 2$) in thermal power plant: $n_{syn} = 3000/\text{min}$:

- stator bore diameter $d_{si} = 1.2$ m, pole pitch $\tau_p = 1.2\pi/2 = 1.88$ m = **188 cm**

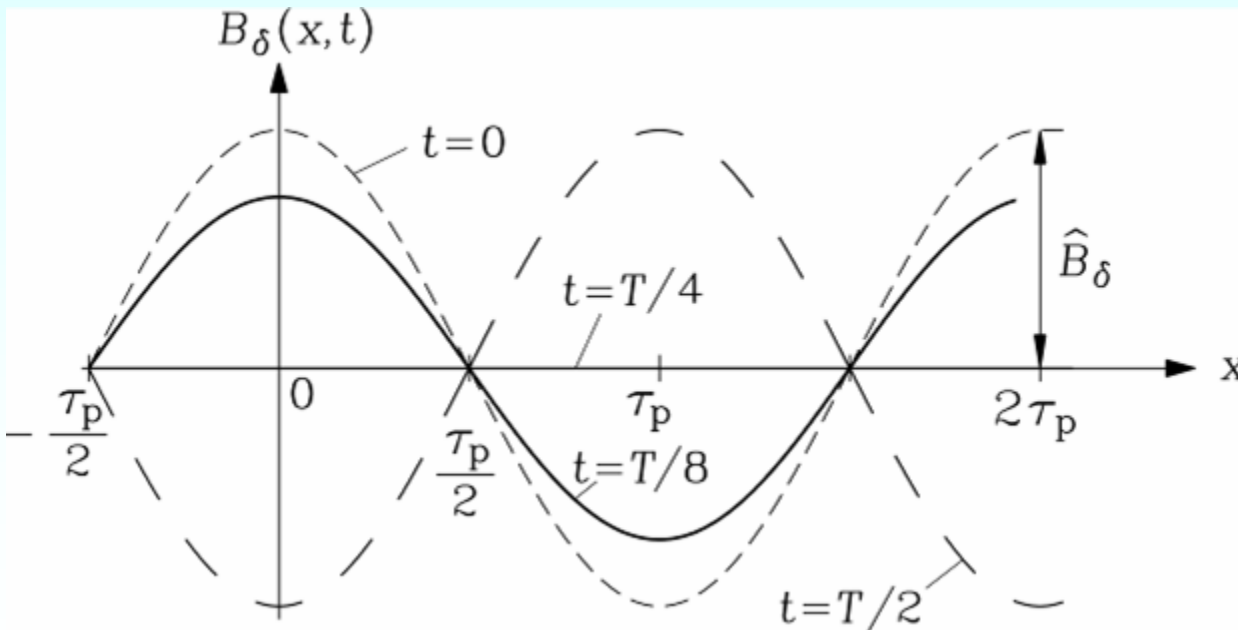
- $v_{syn} = \underline{\underline{188 m/s}} = 676$ km/h = rotor surface velocity, as rotor is spinning synchronously with rotating stator magnetic field wave (synchronous machine !)



Pulsating fields – standing waves

- **Standing wave is pulsating field:** The location of wave nodes and wave amplitudes is **fixed**, but the amplitudes are **pulsating**.
- At time $t = 0$: Maximum of wave at $x = 0$ has amplitude $\hat{B}_{\delta 1}$
 at $t = T/8$: $\hat{B}_{\delta 1} / \sqrt{2}$,
 at $t = T/4$: amplitude is zero !
 at $t = T/2$: $-\hat{B}_{\delta 1}$ and so on.

$$B_{\delta 1}(x, t) = \hat{B}_{\delta 1} \cos\left(\frac{x \cdot \pi}{\tau_p}\right) \cdot \cos(2\pi f \cdot t)$$



FOURIER-Analysis: Determining fundamental & harmonic waves

- **FOURIER-series:** A periodical function $V(\gamma)$ with period 2π may be described by an infinite sum of sine & co-sine functions with decreasing wave length.

$$V(\gamma) = V_0 + \sum_{\nu=1,2,3,\dots}^{\infty} [\hat{V}_{\nu,a} \cdot \cos(\nu \cdot \gamma) + \hat{V}_{\nu,b} \cdot \sin(\nu \cdot \gamma)]$$

- **Ordinal numbers:** $\nu = 1, 2, 3, \dots$

- **Amplitudes:** $\hat{V}_{\nu,a} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \cos(\nu \cdot \gamma) \cdot d\gamma$, $\hat{V}_{\nu,b} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \sin(\nu \cdot \gamma) \cdot d\gamma$

- **Average value:** $V_0 = \frac{1}{2\pi} \int_0^{2\pi} V(\gamma) \cdot d\gamma$

- **Magnetomotive force (MMF) of air gap field:**

a) NO UNIPOLAR flux: $V_0 = 0$

b) MMF *V symmetrical to abscissa*: NO even ordinal numbers

c) By choosing origin so, that MMF V is even function $V(\gamma) = V(-\gamma)$:

NO sine-wave functions occur in **FOURIER** sum.

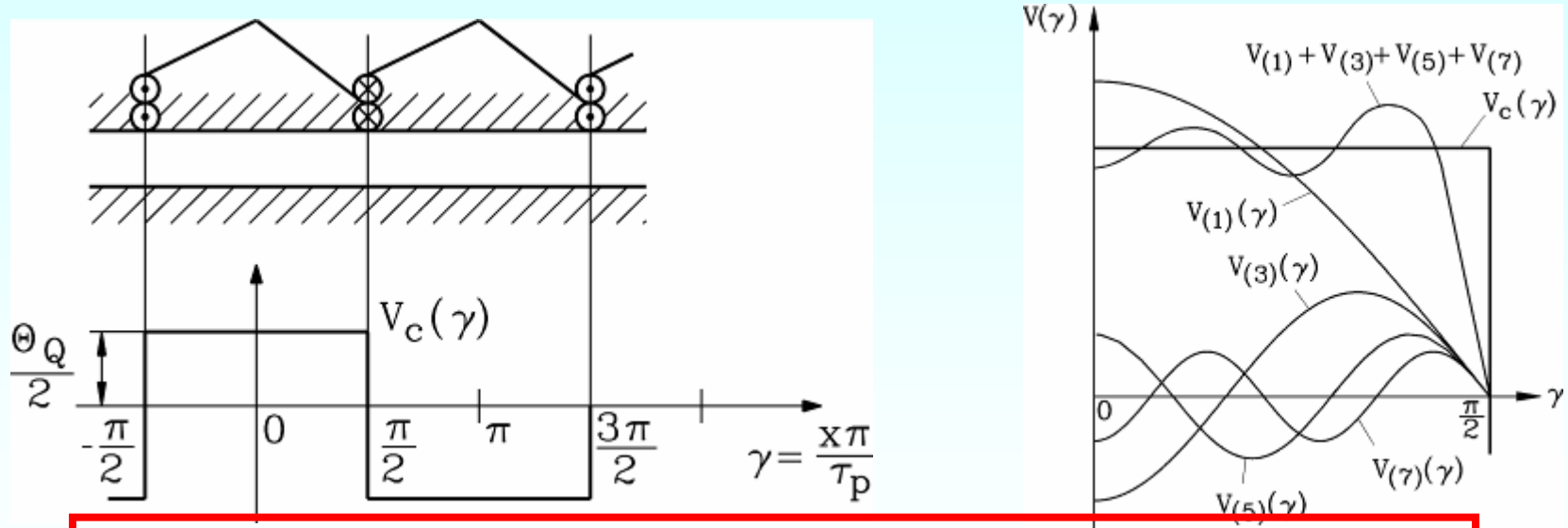


FOURIER-Analysis of field of one full-pitched coil ($q = 1$)

- Magnetomotive force $V_c(x)$ is **block-shaped function**, slot-Ampere-turns $\Theta_Q = 2N_c i_c$
- Circumference angle : $\gamma = x \cdot \pi / \tau_p$:

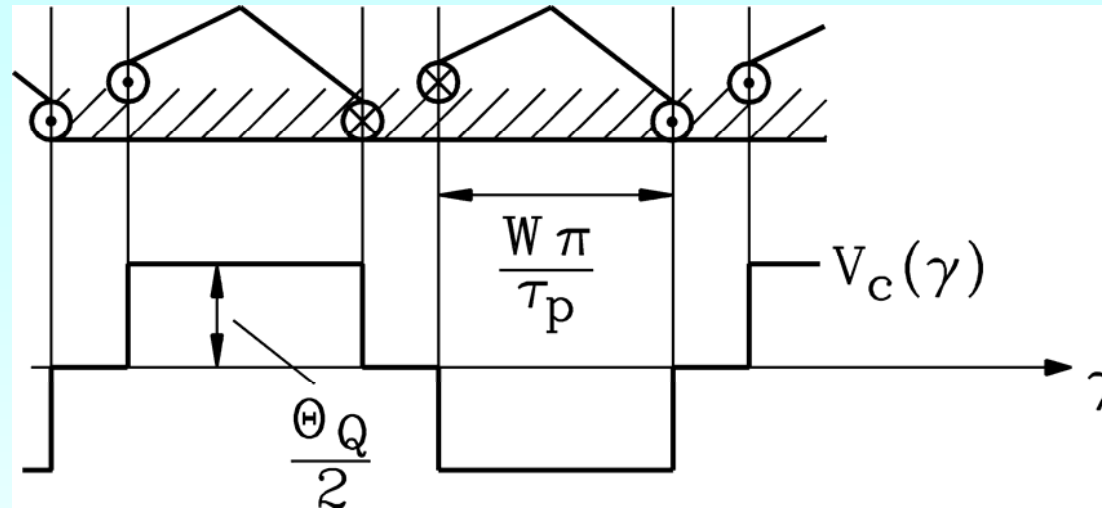
$$V_c(\gamma) = \sum_{v=1,3,5,\dots}^{\infty} \hat{V}_{c,v} \cos(v\gamma)$$

Example: Four pole machine: Half circumference = π in "mechanical degrees", equals two pole pitches $x = 2\tau_p$, thus $\gamma = 2\pi$, in "electrical degrees"



$$\hat{V}_{c,v} = \frac{2}{\pi} \int_0^{\pi} V_c(\gamma) \cos(v\gamma) d\gamma = \frac{\Theta_Q}{2} \cdot \frac{4}{v\pi} \cdot \sin\left(\frac{v\pi}{2}\right), \quad v = 1, 3, 5, \dots$$

FOURIER-Analysis of field of one pitched coil ($q = 1$)



- $$\hat{V}_{c,\nu} = \frac{2}{\pi} \int_0^{\pi} V_c(\gamma) \cos(\nu\gamma) d\gamma = \frac{\Theta_Q}{2} \cdot \frac{4}{\nu\pi} \cdot \sin\left(\frac{W}{\tau_p} \cdot \frac{\nu\pi}{2}\right), \quad \nu = 1, 3, 5, \dots$$

- Compare MMF function's *FOURIER* series with that of full-pitched coils: Amplitudes are smaller by the "**pitch coefficient**" $k_{p,\nu}$:

$$k_{p,\nu} = \sin\left(\frac{W}{\tau_p} \cdot \frac{\nu\pi}{2}\right)$$

Example: $W/\tau_p = 5/6$ ($q = 2$): $\nu = 1$: $k_{p1} = 0.966$, $\nu = 5$: $k_{p5} = 0.259$

FOURIER-Analysis of field of a full-pitched coil group $q > 1$

- Magnetomotive force of field of full-pitched coil group: $\nu = 1, 3, 5, \dots$

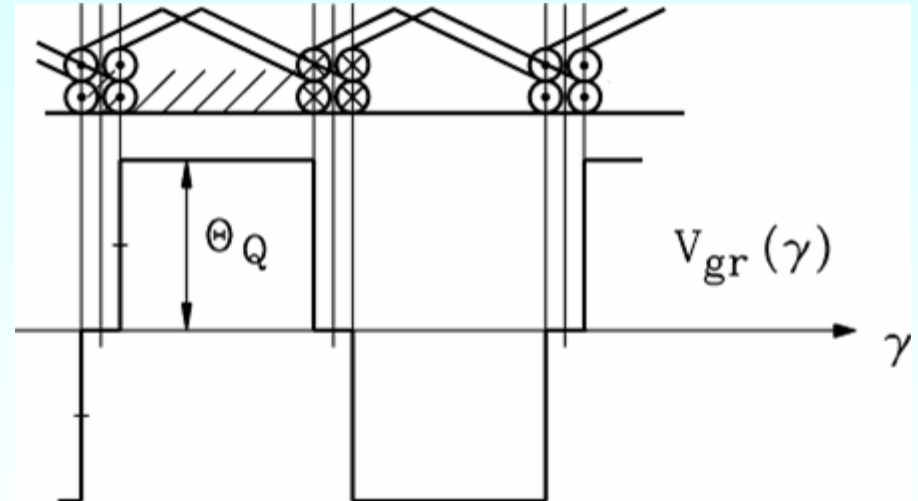
$$\hat{V}_{gr,\nu} = \frac{2}{\pi} \int_0^{\pi} V_{gr}(\gamma) \cos(\nu\gamma) d\gamma = \frac{q\Theta_Q}{2} \cdot \frac{4}{\nu\pi} \cdot \sin\left(\frac{\nu\pi}{2}\right) \cdot \frac{\sin\left(\frac{\nu\pi}{2m}\right)}{q \cdot \sin\left(\frac{\nu\pi}{2mq}\right)}$$

- "Distribution coefficient,,

$$k_{d,\nu} = \frac{\sin\left(\frac{\nu\pi}{2m}\right)}{q \cdot \sin\left(\frac{\nu\pi}{2mq}\right)}$$

Example: $q = 2$

MMF amplitude: $q\Theta_Q/2 = \Theta_Q$



- With relationship we get

$$\frac{q\Theta_Q}{2} = qN_c i_c = \frac{2pqN_c}{a} \cdot \frac{ai_c}{2p} = N \cdot \frac{i}{2p} \Rightarrow \hat{V}_{gr,\nu} = N \cdot \frac{i}{2p} \cdot \frac{4}{\nu\pi} \cdot \sin\left(\frac{\nu\pi}{2}\right) \cdot k_{d,\nu}$$

FOURIER-series of field of one phase with pitched coil groups

$q > 1, W/\tau_p < 1$

- FOURIER-analysis of MMF of one phase $V_{phase}(\gamma)$:

$$\hat{V}_{phase,v} = N \cdot \frac{i}{2p} \cdot \frac{4}{v\pi} \cdot k_{p,v} \cdot k_{d,v}, \quad v = 1, 3, 5, \dots$$

- Phase current $i = I \cdot \sqrt{2} \cdot \cos(\omega t)$
- The MMF distribution (and hence the air gap field) is a sum of pulsating, standing waves („Pulsating field“).

$$\hat{V}_{phase,v} = \frac{2 \cdot \sqrt{2}}{\pi} \cdot \frac{N}{p} \cdot \frac{1}{v} \cdot k_{p,v} \cdot k_{d,v} \cdot I, \quad v = 1, 3, 5, \dots$$

$$V_{phase}(\gamma, t) = \sum_{v=1,3,5,\dots}^{\infty} \hat{V}_{phase,v} \cdot \cos(v\gamma) \cdot \cos(\omega t)$$

- "Winding coefficient" $k_{w,v} = k_{p,v} \cdot k_{d,v}$

Example of typical three phase windings A, B, C

- Pitch coefficient, distribution coefficient and winding coefficient for pitched winding with 6, 12 and 18 slots per pole pair Q/p

	A			B			C		
	$q = 1, W/\tau_p = 2/3$			$q = 2, W/\tau_p = 5/6$			$q = 3, W/\tau_p = 7/9$		
	$Q/p = 6$			$Q/p = 12$			$Q/p = 18$		
ν	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$	$k_{p,\nu}$	$k_{d,\nu}$	$k_{w,\nu}$
1	0.866	1	0.866	0.966	0.966	0.933	0.940	0.960	0.902
5	-0.866	1	-0.866	0.259	0.259	0.067	-0.174	0.218	-0.038
7	0.866	1	0.866	0.259	-0.259	-0.067	0.766	-0.177	-0.136
11	-0.866	1	-0.866	0.966	-0.966	<u>-0.933</u>	0.766	-0.177	-0.136
13	0.866	1	0.866	-0.966	-0.966	<u>0.933</u>	-0.174	0.218	-0.038
17	-0.866	1	-0.866	-0.259	-0.259	0.067	0.940	0.960	<u>0.902</u>
19	0.866	1	0.866	-0.259	0.259	-0.067	-0.940	0.960	<u>-0.902</u>

- Pitch and distribution coefficient REDUCE certain field harmonic amplitudes.
- Winding coefficient is a PERIODICAL function of ordinal number ν .

Inserting form-wound two-layer winding in induction generator stator

Winding overhang

$Q = 72$ slots

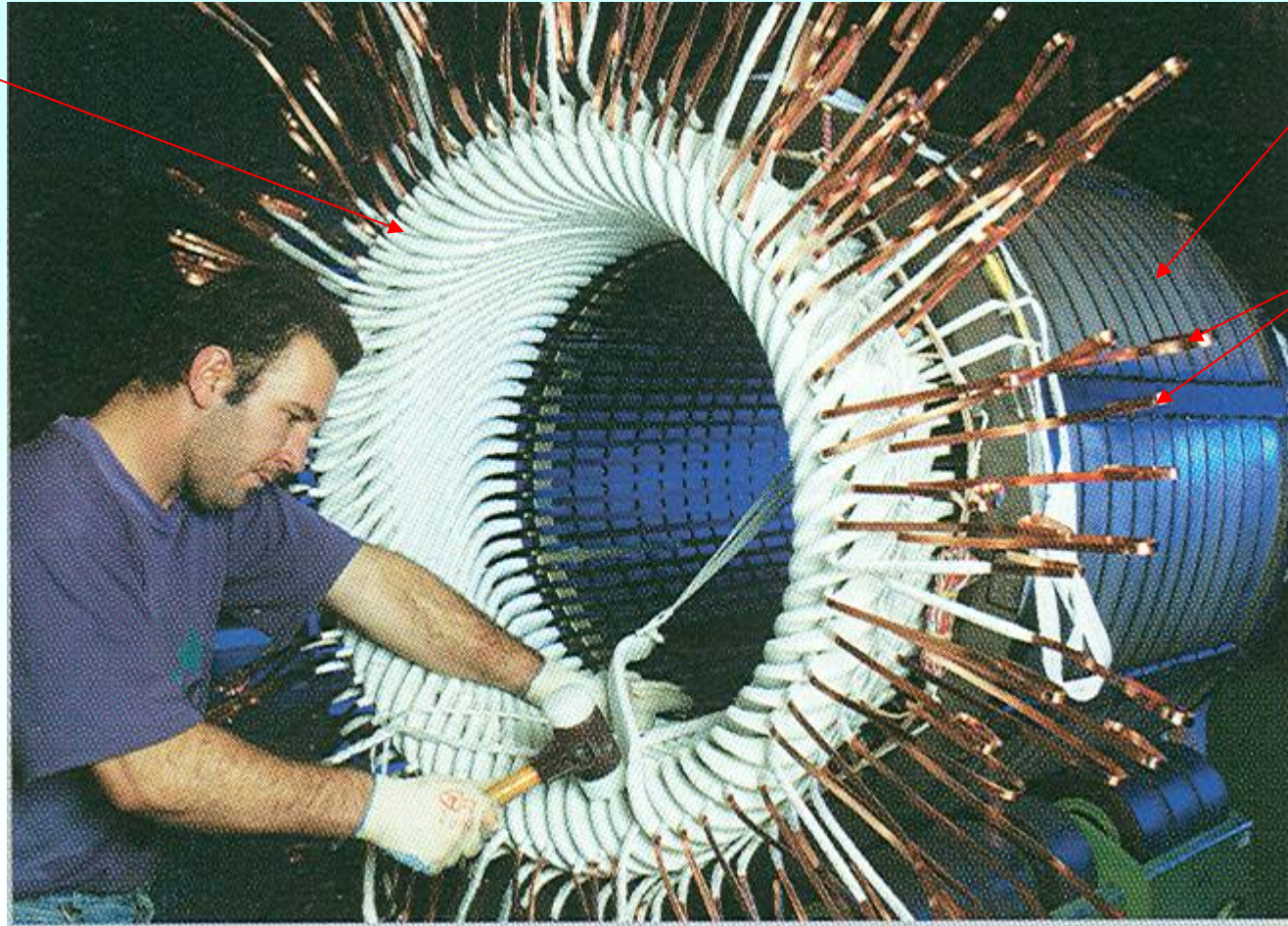
72 coils

$m = 3$ phases

$2p = 4$ poles

$q = 6$ slots per pole and phase

$$Q = 2p \cdot m \cdot q = 4 \cdot 3 \cdot 6 = 72$$



Stator iron stack

coil ends

Source:
Winergy
Germany



Inserting form-wound two-layer winding in stator slots

Winding overhang

Stator iron stack



60 slots for a 4-pole induction machine

$$q = 5:$$

$$Q = 2p \cdot m \cdot q = 4 \cdot 3 \cdot 5 = 60$$

Source:

ABB, Switzerland



Stator three phase two-layer winding of induction

$Q = 72$ slots

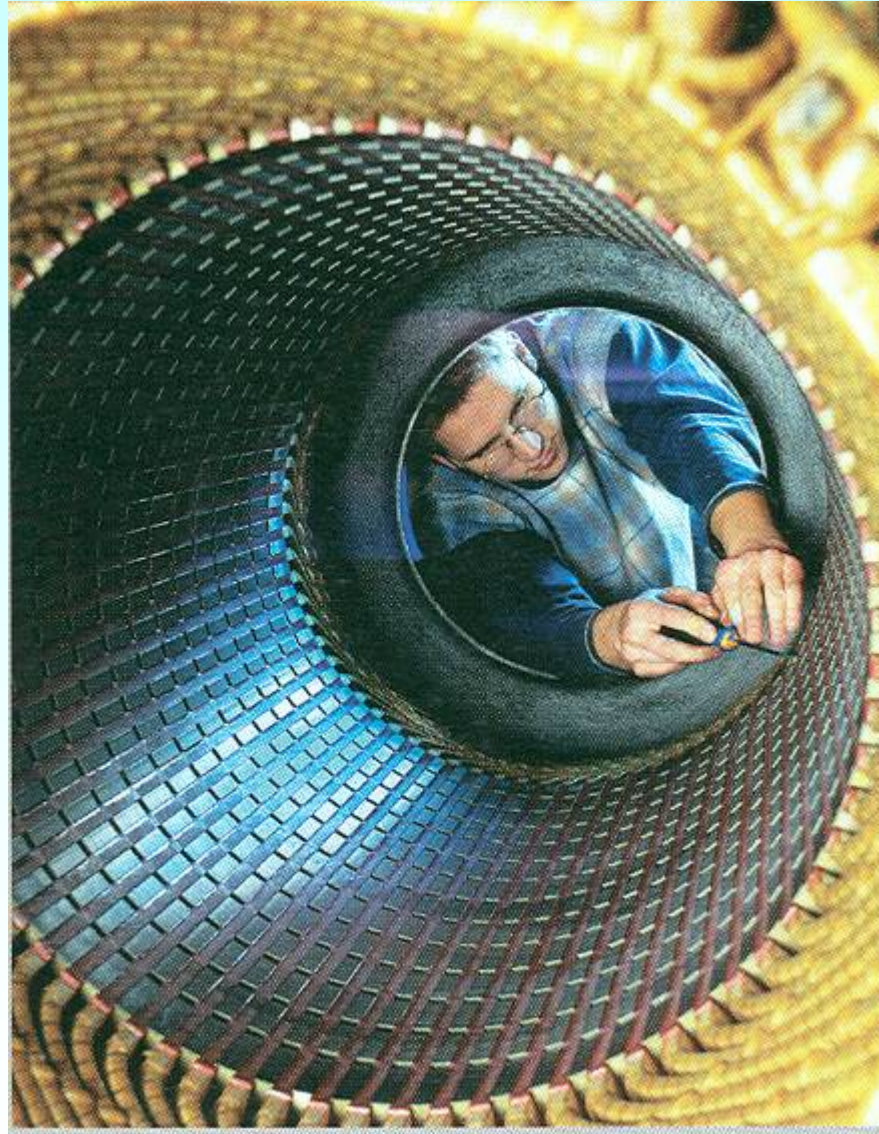
72 coils

$m = 3$ phases

$2p = 4$ poles

$q = 6$ slots per
pole and phase

$$Q = 2p \cdot m \cdot q =$$
$$= 4 \cdot 3 \cdot 6 = 72$$



- Completed two-layer winding after the impregnation with resin

- Some resin “noses” are removed after the impregnation to get a smooth air gap

Source:
Winergy
Germany



FOURIER-analysis of field of a 3-phase winding

- Pulsating fields per phase** are shifted spatially by $2\tau_p/3$ as phases U, V, W. These 3 phases are fed by sinusoidal AC current with phase shift of $T/3$.

$$V_{U\nu}(\gamma, t) = \hat{V}_{phase,\nu} \cdot \cos(\nu\gamma) \cdot \cos(\omega t)$$

$$V_{V\nu}(\gamma, t) = \hat{V}_{phase,\nu} \cdot \cos(\nu(\gamma - 2\pi/3)) \cdot \cos(\omega t - 2\pi/3)$$

$$V_{W\nu}(\gamma, t) = \hat{V}_{phase,\nu} \cdot \cos(\nu(\gamma - 4\pi/3)) \cdot \cos(\omega t - 4\pi/3)$$

- Pulsating field per phase** is decomposed in **negative** and **positive sequence** rotating field waves (clockwise and counter-clockwise rotation) by use of trigonometric summation law: $\cos\alpha \cdot \cos\beta = \frac{1}{2} \cdot [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$

$$V_{U\nu}(\gamma, t) = \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma + \omega t) + \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma - \omega t)$$

$$V_{V\nu}(\gamma, t) = \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma - \frac{\nu 2\pi}{3} + \omega t - \frac{2\pi}{3}) + \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma - \frac{\nu 2\pi}{3} - \omega t + \frac{2\pi}{3})$$

$$V_{W\nu}(\gamma, t) = \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma - \frac{\nu 4\pi}{3} + \omega t - \frac{4\pi}{3}) + \frac{\hat{V}_{phase,\nu}}{2} \cos(\nu\gamma - \frac{\nu 4\pi}{3} - \omega t + \frac{4\pi}{3})$$



Fundamental of air gap MMF (and field)

- **Resulting effect (summation)** of all 3 pulsating phase fields U, V, W:

$$V(\gamma, t) = \sum_{\nu=1,3,5,\dots}^{\infty} (V_{U\nu}(\gamma, t) + V_{V\nu}(\gamma, t) + V_{W\nu}(\gamma, t)) = \sum_{\nu=1,3,5,\dots}^{\infty} V_{\nu}(\gamma, t)$$

- Summation for ordinal number $\nu = 1$ (“**Fundamental**”):

$$V_{U1}(\gamma, t) = \frac{\hat{V}_{phase,1}}{2} \cos(\gamma + \omega t) + \frac{\hat{V}_{phase,1}}{2} \cos(\gamma - \omega t)$$

$$V_{V1}(\gamma, t) = \frac{\hat{V}_{phase,1}}{2} \cos(\gamma + \omega t - \frac{4\pi}{3}) + \frac{\hat{V}_{phase,1}}{2} \cos(\gamma - \omega t)$$

$$V_{W1}(\gamma, t) = \frac{\hat{V}_{phase,1}}{2} \cos(\gamma + \omega t - \frac{8\pi}{3}) + \frac{\hat{V}_{phase,1}}{2} \cos(\gamma - \omega t)$$

With $\cos(\lambda) + \cos(\lambda - \frac{4\pi}{3}) + \cos(\lambda - \frac{8\pi}{3}) = 0$ we get:

- Result: **rotating wave**:

$$V_1(x, t) = \frac{3}{2} \hat{V}_{phase,1} \cos(\gamma - \omega t) \Rightarrow B_{\delta} = \mu_0 V / \delta$$



Harmonic waves of air gap MMF and field

- Summation of pulsating phase fields for $\nu > 1$:

$$V_3(\gamma, t) = 0 + 0 = 0 \qquad V_5(\gamma, t) = \frac{3}{2} \hat{V}_{phase,5} \cos(5\gamma + \omega t)$$
$$V_7(\gamma, t) = \frac{3}{2} \hat{V}_{phase,7} \cos(7\gamma - \omega t) \qquad V_9(\gamma, t) = 0 + 0 = 0$$

- For ordinal numbers ν , **which are divisible by 3**, the summation of harmonic phase fields U, V, W is **zero**: The 3 positive and negative sequence phase waves cancel.

For **$\nu = 7, 13, 19, \dots$** **cancel** the 3 **negative sequence** phase waves. The 3 **positive sequence** phase waves add with the SAME phase angle and rotate WITH the fundamental wave.

For **$\nu = 5, 11, 17, \dots$** **cancel** the 3 **positive sequence** phase waves. The 3 **negative sequence** phase waves add with the SAME phase angle, rotating inverse to fundamental.

- Facit:** A 3-phase winding, fed by symmetrical 3-phase sinusoidal current system, excites a step-like MMF distribution in air gap $V(x,t)$, which may be decomposed into fundamental and harmonic rotating waves. Wave lengths are determined by ordinal numbers. Only odd ordinal numbers exist $\nu = 1, 5, 7, 11, 13, 17, 19, \dots$, which are indivisible by 3.



Fundamental and harmonic air gap field waves

$$V(x,t) = \sum_{\nu=1,-5,7,\dots}^{\infty} V_{\nu}(x,t) = \sum_{\nu=1,-5,7,\dots}^{\infty} \frac{\sqrt{2}}{\pi} \frac{m}{p} N \frac{k_{w,\nu}}{\nu} I \cdot \cos\left(\frac{\nu\pi x}{\tau_p} - \omega t\right)$$

$\nu = 1, -5, 7, -11, 13, -17, \dots$

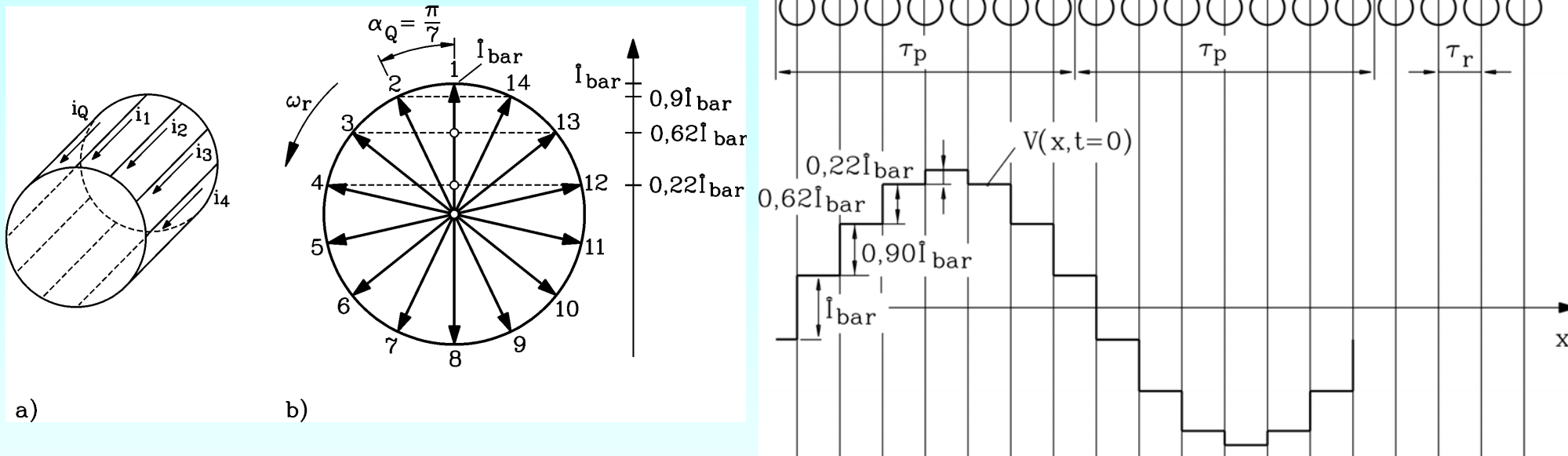
Phase number m : is usually 3

- **Positive and negative ordinal numbers:** $\nu = 1 + 2mg$ $g = 0, \pm 1, \pm 2, \pm 3, \dots$
- Velocity of harmonic waves **decreases** with $1/\nu$: $v_{syn,\nu} = 2f\tau_p / \nu$
- **Wave amplitudes (% of fundamental):** $\hat{B}_{\delta\nu} / \hat{B}_{\delta 1}$ (%) **Underlined: “slot harmonics” !**

ν	$q = 1, W/\tau_p = 2/3, Q/p = 6$	$q = 2, W/\tau_p = 5/6, Q/p = 12$	$q = 3, W/\tau_p = 7/9, Q/p = 18$
1	100	100	100
-5	-20	1.4	-0.8
7	14.3	-1.0	-2.2
-11	-9.1	<u>-9.1</u>	-1.4
13	7.7	<u>7.7</u>	-0.3
-17	-5.6	-0.4	<u>5.9</u>
19	5.3	0.38	<u>-5.3</u>



Magnetomotive force and air gap field of squirrel cage winding



- a) **Rotor: Squirrel cage:** Q_r conductive bars (copper, aluminium) in Q_r slots. Bars are short-circuited by 2 conductive rings at the front ends.
- b) **Symmetrical rotor current system:** In each bar flows a **sinusoidal bar current** with a constant phase shift to the current of the adjacent bar. Thus each bar **is a phase of a Q_r -phase system**.

Example: $Q_r = 28$ bars, $2p = 4$: Bar current system repeats after $Q_r/p = 14$ bars. Phase shift is "**slot angle**" $\alpha_Q = 2\pi p/Q_r = \pi/7$.

FOURIER-Analysis of MMF (and field) of squirrel cage

- Each bar = 1 phase AND each bar = half turn of a coil. Hence:
Pitch and distribution coefficient = 1.

μ : Ordinal number of harmonic waves of rotor field of cage winding

- Relationships: $N \rightarrow 1/2$, $m \rightarrow Q_r$, $k_{w,v} \rightarrow 1$, $I \rightarrow I_{bar}$, $v \rightarrow \mu$

- **FOURIER-series:**

$$V(x,t) = \sum_{\mu=1,\dots}^{\infty} \frac{\sqrt{2}}{\pi} \frac{Q_r}{p} \frac{1}{2} \frac{1}{\mu} I_{bar} \cdot \cos\left(\frac{\mu\pi x}{\tau_p} - 2\pi f_r t\right)$$

with ordinal numbers

$$\mu = 1 + \frac{Q_r}{p} g_r \quad g_r = 0, \pm 1, \pm 2, \dots$$

- **Example:** $Q_r = 28$ bars, $2p = 4$: Ordinal numbers $\mu = 1, -13, 15, -27, 29, \dots$
Wave amplitudes **decrease** with $1/\mu$!

Copper cage rotor of an induction machines



Source:
Breuer Motors,
Germany



TECHNISCHE
UNIVERSITÄT
DARMSTADT

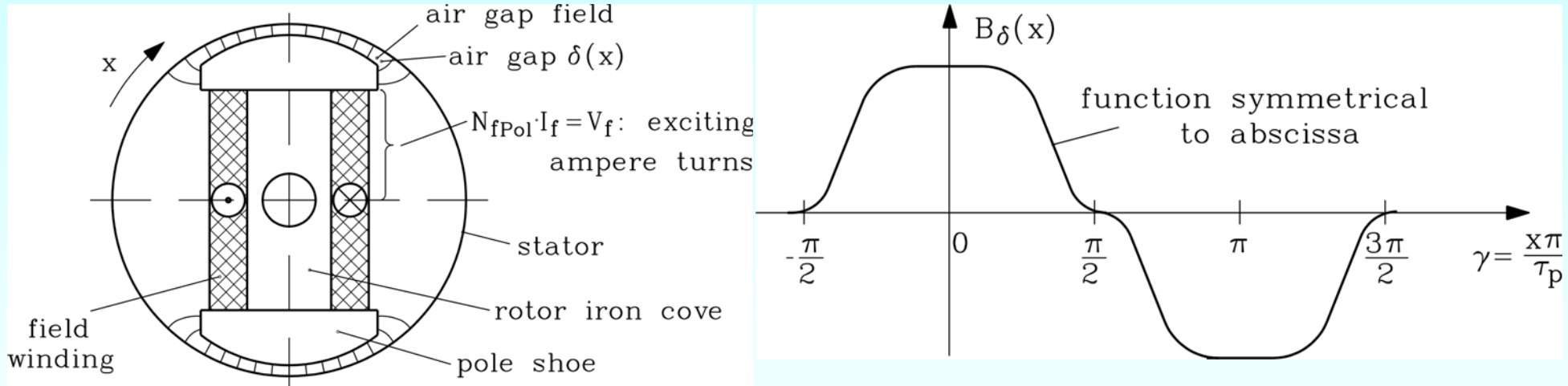
Prof. A. Binder : Electrical Machines and Drives
3/19

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Energiewandlung • FB 18



Salient pole air gap field, excited by DC current

- **Air gap field** H_δ : e.g. 2-pole rotor of salient pole synchronous machine: Rotor winding ampere-turns $N_{fPol} I_f$, (N_{fPol} : number of turns per pole)
- **Air gap** enlarged at inter-pole gaps, hence air gap is function of circumference coordinate $\delta(x)$.



- Calculation of **radial component** B_δ of rotor air gap field by *AMPERE*'s law:

$$\oint_C \vec{H} \cdot d\vec{s} = 2N_{fPol} I_f = 2V_f = 2H_\delta \delta(x) + 2H_{Fe} \Delta_{Fe} = 2H_\delta \delta(x)$$

$$B_\delta(x) = \mu_0 H_\delta(x) = \mu_0 \frac{V_f}{\delta(x)} \quad (\text{Iron: } \mu_{Fe} \rightarrow \infty)$$

Completed salient pole before mounting



Pump storage
hydro power plant
Vianden/Belgium

Refurbishment

Three-fold hammer
head fixation

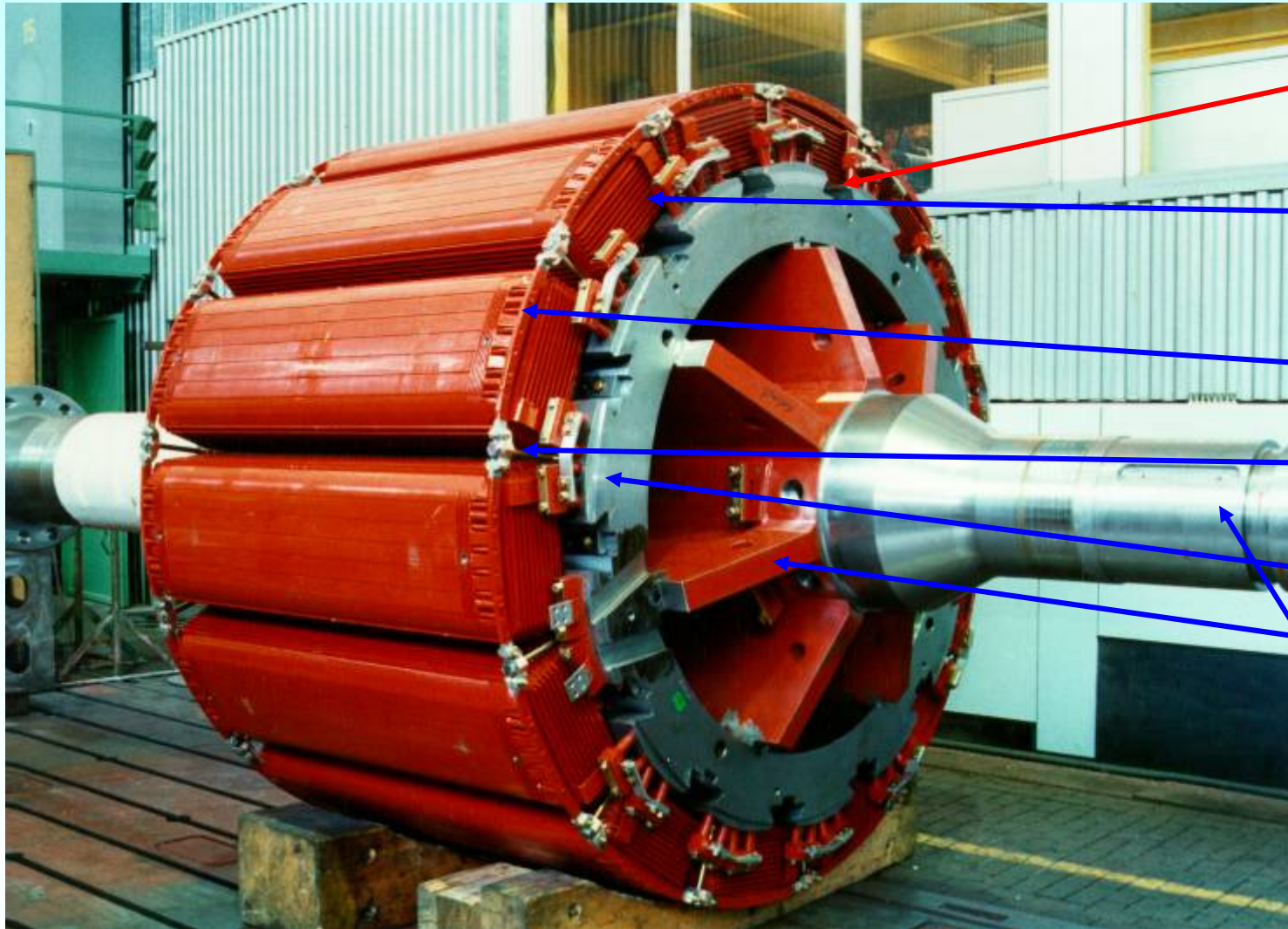
“Cooling fins” by
increased copper
width

Damper ring
segments

Source:

*VATech Hydro,
Austria*

Completed “big hydro” salient pole synchronous rotor for high centrifugal force at over-speed, 14 poles



Dove tail fixation of rotor poles

“Cooling fins” by increased copper width

Damper ring

Damper retaining bolts

Rotor back iron

Rotor spider

Generator shaft

Source:

VATech Hydro,
Austria



FOURIER-Analysis of air gap field of DC-excited salient poles

- Field curve is symmetrical to abscissa: $B_\delta(\gamma) = -B_\delta(\gamma + \pi)$
only harmonics with **odd ordinal numbers**
- Field curve as **FOURIER-Cosine-series**: $B_\delta(\gamma) = \sum_{\mu=1,3,5,\dots}^{\infty} \hat{B}_{\delta\mu} \cos(\mu\gamma) \quad \mu = 1, 3, 5, 7, 9, \dots$
- **Shape of field curve** is determined by air gap function $\delta(x)$ (= contour of pole shoe);
hence also ordinal numbers, divisible by 3, are existing!
- Poles rotate with **constant rotational speed n** : $\Omega_m = 2\pi n = 2v_m/d_{si}$ Hence all rotor waves (fundamental and harmonics) B_μ move with same speed v_m with respect to stator winding. Stator induced voltage of μ -th harmonic has μ -times higher frequency than that of fundamental $\mu = 1$.

$$f_\mu = \frac{\omega_\mu}{2\pi} = \frac{\mu p \Omega_m}{2\pi} = \mu \cdot p \cdot n \quad \gamma_s(t) = \gamma_r + \gamma(t) = \gamma_r + p \cdot \Omega_m \cdot t$$

$$B_{\delta\mu}(\gamma_r) = \hat{B}_{\delta\mu} \cdot \cos(\mu\gamma_r) = \hat{B}_{\delta\mu} \cdot \cos(\mu\gamma_s - \mu p \Omega_m t) = \hat{B}_{\delta\mu} \cdot \cos\left(\frac{\mu\pi x}{\tau_p} - \omega_\mu t\right)$$

Facit: In stator winding sinusoidal voltage due to fundamental rotor wave ($\mu = 1$) is induced, but ALSO **harmonic voltages** with **higher frequencies**, whose harmonic stator currents, excite magnetic fields, which may interfere from power line to parallel telephone cables.

