8. The Synchronous Machine
Synchronous machine with round rotor and salient pole rotor

- **Synchronous machine**: Rotor field winding excites static magnetic rotor field with DC field current $I_f$.

- **MOTOR-operation**: Stator 3-phase ac current system $I_s$ excites stator rotating air gap field. This field rotates with $n = f_s/p$ and attracts rotor magnetic field, which has same number of poles. So rotor will rotate *synchronously* with stator field.

- **GENERATOR-mode**: Rotor is driven mechanically, and induces with rotor field in the stator winding a 3-phase voltage system with frequency $f_s = n \cdot p$. Stator current due to this voltage excites stator field, which rotates *synchronously with rotor*. 

**ROUND ROTOR**: Field winding distributed in rotor slots; constant air gap

**SALIENT POLE ROTOR**: concentrated field winding on rotor poles; air gap is minimum at pole centre
Round rotor synchronous machine, 8 poles

Three field coils per pole: $q_r = 3$
Damper cage with 9 bars per pole
Radial ventilation ducts
Glass fibre bandage for fixing rotor coil overhang

Source:
VA Tech Hydro, Bhopal, India
Rotor air gap field and stator back EMF of round rotor synchronous machine

- Rotor m.m.f. and air gap field distribution have steps due to slots and contain fundamental ($\mu = 1$):

  \[
  \hat{V}_f = \frac{2}{\pi} \cdot \frac{N_f}{p} \cdot (k_{p,f} k_{d,f}) \cdot I_f
  \]

  \[
  \hat{B}_p = \mu_0 \frac{\hat{V}_f}{\delta}, \quad N_f = 2p \cdot q_r \cdot N_{fc}
  \]

  \[
  k_{p,f} = \sin\left(\frac{W \cdot \pi}{\tau_p \cdot 2}\right) = \sin(\pi / 3) = \frac{\sqrt{3}}{2}
  \]

  \[
  k_{d,f} = \frac{\sin(\pi / 6)}{q_r \cdot \sin(\pi / (6q_r))}, \quad k_{wf} = k_{pf} k_{df}
  \]

- Back EMF $U_p$ (synchronously induced stator voltage): Rotor field fundamental $B_p$ induces in 3-phase stator winding at speed $n$ a 3-phase voltage system $U_p$:

  \[
  U_p = \omega \cdot \Psi_p / \sqrt{2} = \omega \cdot N_s k_{w,s} \cdot \Phi_p / \sqrt{2} = \sqrt{2} \pi f_s \cdot N_s k_{w,s} \cdot \frac{2}{\pi} l \tau_p \hat{B}_p
  \]

  with frequency $f_s = n p \Rightarrow$ Current $I_s$ will flow in stator winding.
Round rotor synchronous machine: Equivalent circuit

- **Stator winding:** Three phase AC winding like in induction machines with *self-induced voltage* due to stator rotating magnetic field, described by stator air gap field main reactance $X_h$ and stator leakage flux reactance $X_{ss}$. With stator phase resistance $R_s$ we get stator voltage equation per phase:

$$U_s = U_p + jX_h I_s + jX_{ss} I_s + R_s I_s$$

"synchronous reactance": $X_d = X_{ss} + X_h$ contains effect of total stator magnetic field!

- **Equivalent circuit per stator phase:** for stator voltage equation (ac voltage and current). In rotor winding only DC voltage and current: $U_f = R_f \cdot I_f$

- **Rotor electric circuit:**

  - $U_i$: Rotor dc field voltage: (exciter voltage):
    $U_i$ impresses via 2 slip rings and carbon brushes a rotor DC current (*field current $I_f$) into rotor field winding. Field winding resistance is $R_f$. 
Transfer ratio for rotor field current

- **Stator self-induced voltage:** \( U_{s,s} = jX_h I_s \) by stator air-gap field

- **Back EMF \( U_p \):** Induced by rotor air gap field. It may be changed by field current \( I_f \) arbitrarily DURING OPERATION = "synchronous machine is controlled voltage source".
  
  a) Amplitude of \( U_p \) is determined via \( I_f \).
  
  b) Phase shift of \( U_p \) with respect to stator voltage \( U_s \) is determined by relative position of rotor north pole axis with respect to stator north pole axis. Rotor pole position is described by load angle \( \delta \).

- **Amplitude and phase shift of \( U_p \):** may be described in equivalent circuit by fictive AC stator current \( I'_f \):
  \[
  U_p = jX_h I'_f
  \]

- **This defines transfer ratio of field current \( \dot{I}_{lf} \):**
  \[
  I'_f = \frac{1}{\dot{I}_{lf}} I_f
  \]

\[
I'_f = \frac{U_p}{U_{s,s}} I_s = \frac{B_p}{B_{s,\delta}} I_s = \hat{V}_f \cdot \hat{V}_s : \text{shall be} \quad \frac{1}{\dot{I}_{lf}} I_f
\]

With \( \hat{V}_f = \frac{2 \cdot N_f}{\pi} k_{wf} \cdot I_f \), \( \hat{V}_s = \frac{\sqrt{2}}{\pi} \cdot \frac{m_s N_s}{p} k_{ws} I_s \) we get:

\[
\dot{I}_{lf} = \frac{m_s N_s k_{ws} \sqrt{2}}{2N_f k_{wf}}
\]
Alternative equivalent circuit: Current source for equivalent field current $I'_f$

$$U_h = U_p + jX_h I_s$$

Fictitious AC current source $I'_f$ generates synchronous back EMF $U_p$ as voltage drop at magnetizing reactance

$$U_p = jX_h I'_f$$
Phasor diagram of round rotor synchronous machine

- **Example:** Generator, over-excited:
  - a) electrical active power:
    
    $P_e = m_s U_s I_s \cos \varphi$

    Phase angle $\varphi$ between -90° and -180°:
    Hence $\cos \varphi$ negative: $P_e$ is negative = power delivered to the grid (GENERATOR).
    
    $P_e < 0$: **Generator**,
    $P_e > 0$: **Motor**.

  - b) electrical reactive power:
    
    $Q = m_s U_s I_s \sin \varphi$

    Phase angle $\varphi$ negative = stator current leads ahead stator voltage:
    $\sin \varphi$ negative: $Q$ is negative = capacitive reactive power: Machine is **capacitive consumer**.
    
    $Q < 0$: **over-excited**, capacitive consumer.
    $Q > 0$: **under-excited**, inductive consumer.
Load angle $\vartheta$, internal voltage $U_h$, magnetising current $I_m$

- **Load angle $\vartheta$** between stator phase voltage $U_s$ and back EMF phasor $U_p$. Counted in mathematical positive sense (counter-clockwise).

- **Internal voltage $U_h$** is induced in stator winding by resulting air gap field (rotor and stator field):
  \[ U_h = U_p + jX_h I_s \]

- **Magnetising current $I_m$**: Fictitious stator current to excite resulting air gap field (rotor and stator field):
  \[ I_m = I'_f + I_s \]

- Voltage triangle $U_p, jX_h I_s, U_h$ and current triangle $I'_f, I_s, I_m$ are of the same shape, but shifted by 90°.
Load angle $\vartheta$ - internal voltage $U_h$

Load angle $\vartheta$ (exactly at $R_s, X_{sc} = 0$)

- $U_p$
- $U_h$

Diagram showing stator, rotating field, rotor.
Over-/under-excitation, generator/motor-mode

<table>
<thead>
<tr>
<th>GENERATOR: rotor leading</th>
<th>MOTOR: rotor lagging</th>
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<tr>
<td>$U_p$</td>
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<td>$I_s$</td>
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<td>$\varphi &lt; 0$</td>
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- **Generator mode**: $\theta > 0$: Rotor LEADS ahead of resulting rotating magnetic field = Phasor $U_p$ LEADS ahead of $U_h$.

- **Motor mode**: $\theta < 0$: Rotor LAGS behind resulting magnetic field = Phasor $U_p$ LAGS behind $U_h$.

- **Over-excitation**: Machine is capacitive consumer: Phasor $U_p$ is longer than phasor $U_h$: big field current $I_f$ is needed.

- **Under-excitation**: Machine is inductive consumer: Phasor $U_p$ is shorter than phasor $U_h$: small field current $I_f$ is needed.

- **Facit**: Stator- and rotor field rotate always synchronously. Generator- and motor mode are only defined by sign of load angle $\theta$. 

Round rotor synchronous machine: Magnetic field at no-load

Rotor cross section without field winding:
- Slots per pole $2q_r = 10$, 2-pole rotor
- Rotor may be constructed of massive iron, as rotor contains only static magnetic field!

Magnetic field at no-load ($I_s = 0$, $I_r > 0$):
- Field winding excited by $I_f$
- Stator winding without current (no-load)
- Field lines in air gap in radial direction = no tangential magnetic pull = torque is zero!
  
  *(Example: $2p = 2$, $q_s = 6$, $q_r = 6$)*
Round rotor synchronous machine: Magnetic field at load

- Magnetic field at load ($I_s > 0$, $I_f > 0$): Rotor pole axis = Direction of $U_p$, resulting field axis = Direction of $U_h$
- Field lines in air gap have also tangential component = tangential magnetic pull = torque!
Torque of round rotor synchronous machine at \( U_s = \text{const.} \) & \( R_s = 0 \)

- **Machine operates at RIGID grid:** \( U_s = \text{constant} = U_s \) (= phasor put in real axis of complex plane):
  \[
  U_p = U_p (\cos \vartheta + j \cdot \sin \vartheta) \quad \text{and} \quad \bar{I_s} = (U_s - U_p)/(jX_d) \quad \Rightarrow \quad \bar{I_s}^* = (U_s - U_p^*)/(-jX_d)
  \]

- **Active power** \( P_e \):
  \[
  P_e = m_s U_s I_s \cos \varphi = m_s \cdot \text{Re}\left\{U_s \bar{I_s}^*\right\}
  \]

  \[
  P_e = m_s \cdot \text{Re}\left\{U_s \cdot \frac{U_s - U_p (\cos \vartheta - j \cdot \sin \vartheta)}{-jX_d}\right\} = -m_s \frac{U_s U_p}{X_d} \sin \vartheta
  \]

- **Electromagnetic torque:**
  \[
  M_e = \frac{P_m}{\Omega_{\text{syn}}} = \frac{P_e}{\Omega_{\text{syn}}} = -m_s \cdot \frac{U_s U_p}{X_d} \sin \vartheta = -M_{p0} \sin \vartheta
  \]

**Note:**
All losses neglected ("unity" efficiency).

*Negative torque:* Generator: \( M_e \) is braking

*Positive torque:* Motor: \( M_e \) is driving

Machine speed is always synchronous speed!
Stable points of operation

- **Example:** Torque-load angle curve $M(\vartheta)$: in generator mode the mechanical driving shaft torque $M_s$ is determining operation points 1 and 2.
- Operation point 1 is **stable**, operation point 2 is **unstable**. The **stability limit** is at load angle $\pi/2$ (generator limit) and $-\pi/2$ (motor limit).

**Facit:** Synchronous motor and generator pull-out torque $\pm M_{p0}$ occurs at pull-out load angle $\pm \pi/2$. Rotor is "pulled out" of synchronism, if load torque exceeds pull-out torque. Result: Pulled-out rotor does not run synchronously with stator magnetic field, which is determined by the grid voltage. The rotor slips! **No** active power is converted any longer.
Stability analysis of operation points

- **Torque-load angle curve** $M_e(\vartheta)$ linearized in operation point $\vartheta_0$: **Tangent** as linearization: $$M_e(\vartheta) \approx M_e(\vartheta_0) + \frac{\partial M_e}{\partial \vartheta} \Delta \vartheta$$ with $$\Delta \vartheta = \vartheta - \vartheta_0$$

$$c_\vartheta(\vartheta_0) = \left. \frac{\partial M_e}{\partial \vartheta} \right|_{\vartheta_0}$$: Equivalent spring constant $\Leftrightarrow \Delta M_e = c_\vartheta \cdot \Delta \vartheta$

- **Change of load angle** with time causes **change of speed** $\Delta \Omega_m$:

$$\frac{d\Delta \vartheta}{dt} = p \cdot \Delta \Omega_m \Rightarrow \Omega_m(t) = \Omega_{syn} + \Delta \Omega_m(t)$$

- **NEWTON´s law of motion**: $$J \frac{d^2 \Delta \vartheta}{dt^2} = M_e(\vartheta) - M_s(\vartheta) = c_\vartheta \cdot \Delta \vartheta = J \frac{d\Delta \Omega_m}{dt}$$ leads to

$$J \frac{d^2 \Delta \vartheta}{dt^2} - p \cdot c_\vartheta \cdot \Delta \vartheta = 0$$

a) $|\vartheta| < \pi/2$ : $c_\vartheta = -|c_\vartheta| < 0$

b) $|\vartheta| > \pi/2$ : $c_\vartheta = |c_\vartheta| > 0$

a) $|\vartheta| < \pi/2$ : $\Delta \dot{\vartheta} + (p \cdot |c_\vartheta| / J) \cdot \Delta \vartheta = 0 \Rightarrow \Delta \ddot{\vartheta} + \omega_e^2 \Delta \vartheta = 0 \Rightarrow \Delta \vartheta(t) \sim \sin(\omega_e t)$

Deviation of load angle from steady state point of operation remains limited: **STABLE** operation

b) $|\vartheta| > \pi/2$ : $\Delta \dot{\vartheta} - (p \cdot |c_\vartheta| / J) \cdot \Delta \vartheta = 0 \Rightarrow \Delta \ddot{\vartheta} - \omega_e^2 \Delta \vartheta = 0$

$\Rightarrow \Delta \vartheta(t) \sim \sinh(\omega_e t)$

Deviation of load angle from operation point increases: **UNSTABLE**
Torsional oscillations of synchronous machine

- Deviation of load angle in stable point of operation due to disturbance:
  \[
  |\mathcal{\Phi}| < \pi / 2 : \quad \Delta \ddot{\mathcal{\Phi}} + (p \cdot |c_{\mathcal{\Phi}}| / J) \cdot \Delta \mathcal{\Phi} = 0 \Rightarrow \Delta \ddot{\mathcal{\Phi}} + \omega_e^2 \Delta \mathcal{\Phi} = 0 \Rightarrow \Delta \mathcal{\Phi}(t) \sim \sin(\omega_e t)
  \]
  leads to differential equation with oscillation as solution. Rotor oscillates around steady state point of operation \( \mathcal{\Phi}_0 \), which is defined by the stator field, that is generated by the “rigid” grid.

Natural frequency of oscillation (eigen-frequency):

\[
f_e = \frac{\omega_e}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{p |c_{\mathcal{\Phi}}|}{J}}
\]

Facit: The synchronous machine is performing like a (non-linear) torsional spring.

Example: Operation at no-load \( (M_e = 0, \mathcal{\Phi}_0 = 0) \):
\[
|c_{\mathcal{\Phi}}| = | -M_{p0} \cdot \cos(0) | = M_{p0}
\]

With \( p \Omega_{\text{syn}} = \omega_N \) and rated acceleration time \( T_J = \frac{J \cdot \Omega_{\text{syn}}}{M_N} \) we get: \[
f_e = \frac{1}{2\pi} \sqrt{\frac{\omega_N}{T_J} \cdot \frac{M_{p0}}{M_N}}
\]

Synchronous motor driving a fan blower for a wind tunnel:
\[
P_N = 50 \text{ MW}, \quad f_N = 50 \text{ Hz}, \quad T_J = 10 \text{ s}, \quad M_{p0}/M_N = 1.5,
\]
\[
f_e = \frac{1}{2\pi} \sqrt{\frac{2\pi 50}{10} \cdot 1.5} = 1.09 \text{ Hz}
\]
Round rotor synchronous machine – Stator 1

Turbo-Generator

Stator stacking

Source: ALSTOM

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Round rotor synchronous machine – Stator 2

Turbo-Generator
Stator complete

4-pole Turbogenerator

Source: ALSTOM

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Round rotor synchronous machine – Rotor 1

Turbo-Generator

Rotor wound

Source: ALSTOM

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Round rotor synchronous machine – Rotor 2

Turbo-Generator

Rotor insertion in stator

Source: ALSTOM

4-pole Turbogenerator
Rotor field and back EMF of salient pole synchronous machine

- **Bell shaped rotor air gap field curve** $B_\delta(x)$: A constant m.m.f. $V_f$ excites with a variable air gap $\delta(x)$ a bell shaped field curve. Fundamental of this “bell-shape” ($\mu = 1$):

$$B_\delta(x) = \mu_0 \frac{V_f}{\delta(x)} \quad \rightarrow \quad \text{FOURIER-fundamental wave: Amplitude } \hat{B}_p \text{ proportional to } I_f$$

- **Back EMF $U_p$**: Sinusoidal rotor field fundamental wave $B_p$ induces in three-phase stator winding at speed $n$ a three-phase voltage system $U_p$

$$U_p = \omega \cdot \Psi_p / \sqrt{2} = \omega \cdot N_{s} k_{w,s} \cdot \Phi_p / \sqrt{2} = \sqrt{2} \pi f \cdot N_{s} k_{w,s} \cdot \frac{2}{\pi} l_{p} \hat{B}_p$$

with **frequency** $f = n \cdot p \quad \Rightarrow \text{Stator current } I_s \text{ is flowing in stator winding.}$
Rotor salient pole manufacturing

Massive pole pressing plates

Dove tail rotor pole

Lamination pressed by pressing plates

Source:
VA Tech Hydro, Bhopal, India
Completed salient pole synchronous rotor, 8 poles

Fly wheel to increase rotor inertia to limit acceleration in case of load drop

Shaft mounted fan with backward bent radial blades for rotation in clockwise direction at fixed speed

Source:
VA Tech Hydro, Bhopal, India

Kauli power plant
Four-pole salient pole rotor with massive pole shoes for asynchronous line start up as motor

- At asynchronous line start up the stator field induces eddy currents in the rotor massive pole shoes.
- These eddy currents develop with the stator field the needed starting torque.

50 Hz, \(2p = 4\), \(n = 1500/\text{min}\)

Source:
VA Tech Hydro, Austria
Stator winding is three-phase winding like in induction machines, BUT the air gap is LARGER in neutral zone (inter-pole gap of $q$-axis) than in pole axis ($d$-axis). Hence for equal m.m.f. $V_s$ (sinus fundamental $\nu = 1$) the corresponding air gap field is SMALLER in $q$-axis than in $d$-axis and NOT SINUSOIDAL.

Stator field in $d$-axis (direct axis): Fundamental of field a little bit smaller than for constant air gap $\delta_0$: $c_d = \hat{B}_{d1} / \hat{B}_s < 1$ ca. 0.95, thus: $L_{dh} = c_d \cdot L_h$

Stator field in $q$-axis (quadrature axis): Fundamental of field significantly smaller than at constant air gap $\delta_0$: $c_q = \hat{B}_{q1} / \hat{B}_s << 1$ ca. 0.4 ... 0.5, thus $L_{qh} = c_q \cdot L_h$
Stator current $I_s$: $d$- and $q$-component

- **Stator current phasor $I_s$** decomposed into $d$- and $q$-component:
  \[
  I_s = I_{sd} + I_{sq}
  \]
  $I_{sd}$ is in phase or opposite phase with fictitious current $I'_f$. So it excites a stator air gap field in $d$-axis (in rotor pole axis), which together with rotor field gives $d$-axis air gap flux $\Phi_{dh}$.

  $I_{sq}$ is phase-shifted by 90° to $I_{sd}$ and excites therefore a stator air gap field in $q$-axis (inter-pole gap). The corresponding air gap flux is $\Phi_{qh}$.

- **Stator self-induced voltage** consists of two, by 90° phase shifted components:
  \[
  \Psi_{dh} / \sqrt{2} = L_{dh} \cdot (I'_f + I_{sd}) \rightarrow \Phi_{dh} = \Psi_{dh} / (k_{ws} N_s)
  \]
  \[
  \Psi_{qh} / \sqrt{2} = L_{qh} \cdot I_{sq} \rightarrow \Phi_{qh} = \Psi_{qh} / (k_{ws} N_s)
  \]
  \[
  j \omega_s L_{dh} I_{sd}
  \]
  \[
  j \omega_s L_{qh} I_{sq}
  \]
  and of self-induced voltage of stator leakage flux:
  \[
  j \omega_s L_s \sigma I_s
  \]
Stator voltage equation of salient pole synchronous machine

- **Stator voltage equation per phase:** Considering self-induction of main and leakage flux $L_{dh}$, $L_{qh}$, $L_{s\sigma}$ and of rotor phase resistance $R_s$ we get:

$$U_s = R_s I_s + j\omega_s L_{s\sigma} I_s + j\omega_s L_{qh} I_{sq} + j\omega_s L_{dh} I_{sd} + U_p \quad U_p = j\omega_s L_{dh} I'_f$$

$$U_s = R_s I_s + j\omega_s L_{s\sigma} (I_{sd} + I_{sq}) + j\omega_s (L_{qh} I_{sq} + L_{dh} I_{sd}) + U_p$$

- **$X_d$: "synchronous d-axis reactance"**:

$X_d = X_{s\sigma} + X_{dh} = \omega_s L_{s\sigma} + \omega_s L_{dh}$

- **$X_q$: "synchronous q-axis reactance"**:

$X_q = X_{s\sigma} + X_{qh} = \omega_s L_{s\sigma} + \omega_s L_{qh}$

- **Typical values:** Due to inter-pole gap it is $X_d > X_q$ (typically: $X_q = (0.5 \ldots 0.6) \cdot X_d$) e.g. salient pole hydro-generators, diesel engine generators, reluctance machines, ...

- **Note:** Round rotor synchronous machine may be regarded as "special case" of salient pole machine for $X_d = X_q$.

The slot openings of rotor field winding in round rotor machines may also be regarded as non-constant air gap, yielding also $X_d > X_q$ (typically: $X_q = (0.8 \ldots 0.9) \cdot X_d$)
Phasor diagram of salient pole machine

- **Example**: Generator, over-excited:
  a) $I'_f$ and $I_{sd}$ lie in d-axis, $I_{sq}$ lies in q-axis
  b) $U_p \sim jI'_f$ and $jX_{dh}I_{sd}$ lie in q-axis, $jX_{qh}I_{sq}$ lies in d-axis (!)

- **Induced internal voltage $U_h$**: 
  $$U_h = j\omega_s \Psi_h = U_{qh} + U_{dh}$$
  is decomposed into components
  $$U_{qh} = j\omega_s L_{dh}I_{sd} + U_p$$
  $$U_{dh} = j\omega_s L_{qh}I_{sq}$$

- **Stator voltage and current**: 
  - Load angle $\vartheta$,
  - Phase angle $\varphi$
  are defined in the same way as with round rotor synchronous machines!
**Torque of salient pole machine at** $U_s = \text{const. \& } R_s = 0$

- **OPERATION at "rigid" grid:** $U_s = \text{constant}$

  *We choose: d-axis = Re-axis, q-axis = Im-axis of complex plane:*

  \[
  U_s = U_{sd} + jU_{sq} \quad I_s = I_{sd} + jI_{sq} \quad U_p = jU_p
  \]

  $R_s = 0$: \[
  U_s = jX_d I_{sd} + jX_q I_{sq} + U_p \quad \Rightarrow \quad U_s = jX_d I_{sd} - X_q I_{sq} + jU_p
  \]

- **Active power** $P_e$:

  \[
  P_e = m_s U_s I_s \cos \varphi = m_s \cdot \text{Re}\left\{U_s I_s^*\right\} = m_s \left( U_{sd} I_{sd} + U_{sq} I_{sq} \right)
  
  P_e = m_s \left( -X_q I_{sq} I_{sd} + X_d I_{sd} I_{sq} + U_p I_{sq} \right)
  \]

- **Electromagnetical torque**:

  \[
  M_e = \frac{P_m}{\Omega_{\text{syn}}} = \frac{P_e}{\Omega_{\text{syn}}} = \frac{m_s}{\Omega_{\text{syn}}} \left( U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right)
  \]

  - **Two torque components:**
    a) prop. $U_p$ as with round rotor machines
    b) "**Reluctance**”torque due to $X_d \neq X_q$. NO rotor excitation is necessary!

  **Synchronous reluctance machine:** **Reluctance torque** = robust rotor WITHOUT ANY winding, but DEEP inter-pole gaps.
Torque as function of load angle $\vartheta$

\[ U_s = jX_d I_{sd} - X_q I_{sq} + jU_p \]

\[ \Rightarrow \begin{cases} U_{sd} = -X_q I_{sq} \\ jU_{sq} = jX_d I_{sd} + jU_p \Rightarrow I_{sd} = \frac{U_{sq} - U_p}{X_d} \end{cases} \]

\[ \begin{align*} \ \ & U_s = U_{sd} + jU_{sq} \\ & \left\{ \begin{array}{l} U_{sd} = U_s \sin \vartheta \\ U_{sq} = U_s \cos \vartheta \end{array} \right. \end{align*} \]

\[ M_e = \frac{m_s}{\Omega_{syn}} \cdot \left( U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right) = \]

\[ = \frac{m_s}{\Omega_{syn}} \cdot \left( -\frac{U_p U_s \sin \vartheta}{X_q} - \frac{X_d - X_q}{X_d X_q} \cdot U_s \sin \vartheta \cdot (U_s \cos \vartheta - U_p) \right) \]

\[ M_e = -\frac{p \cdot m_s}{\omega_s} \left( \frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right) \]
Torque-load angle curve $M_e(\vartheta)$

- Torque is expressed by stator voltage, back EMF and load angle: $I_{sd}, I_{sq}$ are expressed by $U_s, \vartheta$:

$$M_e = -\frac{p \cdot m_s}{\omega_s} \left( \frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$$

Absolute value of pull-out load angle is $< 90^\circ$, as pull-out torque of reluctance torque occurs at load angle $\pm 45^\circ$.

Pull-out torque is increased by reluctance torque.

Equivalent spring constant $c_\vartheta$ bigger than in round rotor machines, as reluctance torque adds (“stiffer” $M_e(\vartheta)$-curve).
Synchronous reluctance machine

- Rotor without any winding, but with deep inter-pole gaps: $X_d > X_q$.

- Rotor $d$-axis wants to move into stator field axis, to allow field lines to cross air gap via the MOST SHORTEST distance possible, thus yielding the reluctance torque.
Salient pole synchronous machine - Stator

Source: ALSTOM
Salient pole synchronous machine - Rotor

Source: Hydro-Generator

Rotor insertion

Three Gorges (China)
840 MVA
80 poles

Source: ALSTOM
Hydro Power Plant – 1

Hydro-Generator

Rotor insertion

Three Gorges
(China)
840 MVA
80 poles

Source: ALSTOM
Hydro Power Plant – 2

Hydro-Generator

Rotor assembly

Karakaya
(Turkey)
315 MVA
40 poles

Source:

ALSTOM
Generator no-load

Rotor with excited field winding is driven, field current is $I_{f0}$, stator terminals are not connected (no stator current flow): At open stator terminals $U_p$ is measured.

$$U_p = jX_{dh} I'_{f0}$$
Synchronous machine as „phase shifter“, \( (R_s \approx 0) \)

**Diagram:**
- **Overexcited:** \( U_p - jX_d I_s \) \( \sin \varphi = -1, \ \varphi = -\frac{\pi}{2} \)
  - Big \( I_f \): machine is capacitive consumer
- **Underexcited:** \( U_p + jX_d I_s \) \( \sin \varphi = 1, \ \varphi = \frac{\pi}{2} \)
  - Small \( I_f \): machine is inductive consumer

\[
R_s = 0 : \quad U_s = U_p + jX_d I_s
\]

Stator connected to grid, no active power transfer, but phase angle either inductive or capacitive!
Special operating points of synchronous machines ($R_s \approx 0$)

**Non-excited at the grid:** $I_f = 0$.
Stator winding current $I_s$ acts as magnetizing current

$$R_s = 0 : \quad U_s = jX_d I_s$$

**Steady state short circuit:**
Stator winding short-circuit: $U_s = 0$,
Rotor is driven, $U_p$ is induced, causes short circuit current $I_{sk}$, which is limited by $X_d$ and $R_s$:

$$I_{sk} \approx U_p / X_d$$

$$R_s = 0 : \quad 0 = U_p + jX_d I_s$$
Synchronous machine in stand-alone operation

- **Examples:** Automotive generator, Airplane/ship generator, generator stations on islands, off-shore platforms, oasis, mountainous regions, emergency generators in hospitals, military use (e.g., radar supply)

- **No ”rigid” grid available:** $U_s$ is **not** constant: Rotor is excited and driven, field current $I_f$, back EMF $U_p$ is induced as “voltage source”, $U_s$ is **depending on load**. E.g.: round rotor synchronous machine:
  - No $M_e \sim \sin \vartheta$ - curve,
  - No rotor pull out at $\vartheta = \pm 90^\circ$

- **Example:** Mixed OHM’ic-inductive load $Z_L$ (Load current $I_L = -I_s$)

Load impedance: $Z_L$ (here: $Z_L = R_L + jX_L$)
Stand-alone voltage-current characteristic $U_s(I_s)$ at $R_s = 0$

- **No-load:** $I_s = 0 \Rightarrow U_s = U_p = U_{s0}$;
- **Short-circuit:** $Z_L = 0$: $U_s = 0 \Rightarrow I_s = U_p/X_d = I_{sk}$

- **Inductive load:** $Z_L = j \omega L_L = jX_L$:
  Phasor diagram: voltage drops are in line: $U_p = U_s + X_d I_s$
  Voltage decreases linear with increasing load current!
  $\frac{U_s}{U_p} = 1 - \frac{I_s}{(U_p / X_d)}$
  $u = 1 - i$

- **Resistive load:** $Z_L = R_L$:
  Phasor diagram: right angle triangle. *Pythagoras:* $U_p^2 = U_s^2 + (X_d I_s)^2$
  $(U_s / U_p)^2 = 1 - \frac{I_s^2}{(U_p / X_d)^2}$
  $u^2 = 1 - i^2$

Voltage-current curve in per-unit of no-load voltage and short-circuit current is segment of circle!
At mixed resistive-inductive and resistive-capacitive load the per unit voltage-current curves are sections of ellipses.
Stand-alone capacitive load curve $U_s(I_s)$ for $R_s = 0$

**Capacitive load:** $Z_L = 1/(j\omega C_L) = -jX_C$

- From phasor diagram we see, that voltage drops are in line!
- Two cases are to be considered:
  a) $U_p$ in phase opposition to $U_s$: $X_C < X_d$ COUNTER-EXCITATION

$$-U_p = U_s - X_d I_s$$

$$U_s / U_p = I_s / (U_p / X_d) - 1$$

$$u = -1 + i$$

b) $U_p$ in phase with $U_s$: $X_C > X_d$

$$U_p = U_s - X_d I_s$$

$$U_s / U_p = 1 + I_s / (U_p / X_d)$$

$$u = 1 + i$$

*Usually* case b) occurs!
May also start from un-excited generator, where remanence of rotor induces a (small) back EMF in stator winding ("self-excitation of synchronous generator").

Voltage RISES with increasing load: FERRANTI-phenomenon
Example – stand alone operation: Automotive synchronous generator

- **Automotive synchronous generator:**
  - three phases, \( q = 1 \), single layer wave wound winding, 12 poles
  - Claw pole rotor, electrically excited
  - Driven via belt with variable speed from internal combustion engine
  - Diode rectifier for stator power at 12 V or 24 V DC voltage
  - Diode rectifier for rotor field winding excitation
  - Transistor controller keeps stator voltage constant - independently from varying speed \( n \) and stator load current \( I_s \) – via variable exciter current \( I_f \)

- **Data:** e. g.: 12…14 V, 90 A, 1 kW, 3000 … 6500/min

Source: Bosch, Germany