2. Reluctance motors

2.1 Switched reluctance drives

2.1.1 Basic function

Stator and rotor of switched reluctance machines consist of different number of teeth and slots, e.g.: stator: 8 teeth, rotor: 6 teeth (Fig.2.1.1-1). Stator teeth bear tooth coils, which are connected in $m$ different phases. Usually three phases are used for medium power motors.

![Fig. 2.1.1-1: Two pole, four phase switched reluctance machine (cross section): (i) Phase “4” is energized by a H-bridge inverter, fed from DC link $U_d$. The magnetic pull of the flux lines drags the next rotor teeth into aligned position with the energized stator teeth, thus creating a torque (E. Hopper, Maccon, Germany), (ii) numerical field calculation shows the flux pattern with energized phase “4” (Motor data: outer stator diameter: 320 mm, air gap: 1 mm, iron stack length: 320 mm, shaft diameter: 70 mm, coil turns per tooth: 10, current per turn: 10 A DC), (Source: A. Omekanda).](image)

The rotor teeth contain no winding, so it is a very robust construction. Each phase winding is energized independently from the next. By using a rotor position sensor, that phase is chosen to be energized, which may pull the next rotor tooth into an aligned position (Fig.2.1.1-1; Phase “4”). In Fig.2.1.1-1 the flux lines show a tangential and a radial direction. Thus magnetic pull is both tangential to generate torque and radial, attracting the stator teeth versus the rotor teeth (radial pull). When the rotor is in an aligned position for phase “4”, it is switched off, and phase “1” is energized to generate torque.

Thus it is sufficient to impress unipolar current (= block shaped current of one polarity) into the coils, as the magnetic pull is independent of the sign of the current flow. By switching one phase after the other in that mode, the rotor keeps turning and explains the name “switched” reluctance motors (motor mode). The flux lines try to pass through the iron teeth with their high permeability ($\mu_{Fe} \gg \mu_0$) and avoid the slot region. We say: The iron teeth have a low magnetic resistance, and the slots a big one. This rotor structure of high and low magnetic resistance is called a reluctance structure. By switching the phases, the rotor moves stepwise, therefore this principle is also used for small reluctance stepper motors, but then without a position sensor to get a cheap drive. With position sensor the rotor movement is completely controllable. No pull-out at overload is possible, as long as the inverter is able to impress current. Speed can be measured by using the rotor position sensor as a speed sensor. Thus, a variable speed drive is easily realized (speed control). By measuring the current, its amplitude may be controlled by chopping the DC link voltage with PWM.
In Fig.2.1.1-1 the switching on/off sequence of the phases “1”, “2”, “3”, “4”, “1”, ... gives clockwise rotation, whereas “4”, “3”, “2”, “1”, “4”, ... gives counter-clockwise rotation.

If the rotor is driven mechanically and the stator coils are energized when the rotor moves from aligned to unaligned position, then the magnetic pull is braking the rotor. As the rotor movement causes a flux change in the stator coils, a voltage is induced, which along with the stator current gives generated electric power, which is fed to the inverter (generator mode).

The switched reluctance (SR) machine Fig.2.1.1-1 is a 2-pole, 4-phase machine, as each of the phases excites one N- and S-pole (notation: per pole pair: 8/6-stator/rotor teeth). In Fig.2.1.1-2 the cross section of a 4-pole, 3-phase machine is shown. Each of the 3 phases excites 2 N- and 2 S-poles (notation: per pole pair: 6/4-stator/rotor teeth).

![Cross section of a 4-pole SR machine](image)

As teeth numbers of stator and rotor must be different, usually the number of rotor teeth is chosen smaller than the stator teeth number: \( Q_r < Q_s \). Stator teeth number is

\[
Q_s = 2p \cdot m \tag{2.1.1-1}
\]

and rotor teeth number is often chosen as

\[
Q_r = Q_s - 2p \tag{2.1.1-2}
\]

Example 2.1.1-1:
Stator and rotor teeth numbers
a) Three phase machine: per pole pair \((2p = 2)\): \( m = 3 \):
\[
Q_s = 2p \cdot m = 2 \cdot 3 = 6, \ Q_r = Q_s - 2p = 6 - 2 = 4
\]
So for higher number of pole pairs the 6/4 arrangement is repeated \(p\)-times at the motor circumference. In Fig.2.1.1-2 a four pole machine yields 12/8 as teeth numbers.

b) Four phase machine: per pole pair \((2p = 2)\): \( m = 4 \):
\[
Q_s = 2p \cdot m = 2 \cdot 4 = 8, \ Q_r = Q_s - 2p = 8 - 2 = 6 \tag{Fig.2.1.1-1}
\]
From Figs.2.1.1-1 and 2.1.1-2 we see, that the motors can start from any position, as there are always some rotor and stator teeth non-aligned and will exert a tangential magnetic pull, when the corresponding coils are energized. Note: For 2-phase machines self-starting is not possible from any rotor position (Fig.2.1.1-3a). By putting a step into the rotor tooth surface, the rotor will be asymmetric and then self-starting again is possible (Fig.2.1.1-3b).

**Example 2.1.1-2:**
Stator and rotor teeth numbers, two phase machine: per pole pair \((2p = 2)\): \(m = 2\):
\[
Q_s = 2p \cdot m = 2 \cdot 2 = 4, \quad Q_r = Q_s - 2p = 4 - 2 = 2
\]
Self starting is only assured, if some special asymmetry is put into the machine e.g. asymmetric rotor teeth, additional permanent magnet in one stator tooth etc.

![Cross section of a two phase, two pole SR Machine](image)

**2.1.2 Flux linkage per phase**

![Flux linkage](image)

The magnetic flux \(\Phi\) in the air gap is the same as in iron, if the coil stray flux is neglected.
Motor development

\[ \Phi = A \cdot B_\delta = A \cdot B_{Fe} \Rightarrow B_\delta = B_{Fe} \]  \hfill (2.1.2-1)

The flux linkage with the tooth coil (\( N = N_c \); number of turns per coil) is given by

\[ \Psi = N \cdot \Phi = N_c \cdot A \cdot B_\delta \quad \text{with} \quad A = b \cdot l \]  \hfill (2.1.2-2)

with the permeability of iron and air according to \( \mu_{Fe} \) and \( \mu_0 \).

\[ H_{Fe} = \frac{B_{Fe}}{\mu_{Fe}}, \quad H_\delta = \frac{B_\delta}{\mu_0} \]  \hfill (2.1.2-3)

Ampère’s law yields the field strength \( H \) along the air gap and iron path:

\[ \oint_H \cdot d\delta = H_\delta \cdot \delta + H_{Fe} \cdot s_{Fe} = N_c \cdot i \]  \hfill (2.1.2-4)

Thus we get

\[ B_\delta = \frac{\mu_0 \cdot N_c \cdot i}{\delta + s_{Fe} \cdot (\mu_0 / \mu_{Fe})} \]  \hfill (2.1.2-5)

And the self inductance per coil as:

\[ L_c = \frac{\Psi}{i} = \frac{N_c \cdot A \cdot B_\delta}{i} = \frac{\mu_0 \cdot N_c^2 \cdot A}{\delta + s_{Fe} \cdot (\mu_0 / \mu_{Fe})} = \frac{\mu_0 \cdot N_c^2 \cdot b}{\delta \cdot k_s} \cdot l = \mu_0 \cdot N_c^2 \cdot \lambda \cdot l \]  \hfill (2.1.2-6)

The self inductance is defined by the square of the number of turns and by the "geometric parameter":

\[ \lambda = \frac{b}{\delta \cdot k_s} \quad \text{with} \quad k_s = 1 + (s_{Fe} / \delta) \cdot (\mu_0 / \mu_{Fe}) \]  \hfill (2.1.2-7)

**Conclusions:**

With an increasing air gap the inductance is decreasing. In case of ideally unsaturated iron (\( \mu_{Fe} \to \infty \)) the saturation factor \( k_s = 1 \). With an increasing current \( i \) the flux increases and iron saturation occurs: \( \mu_{Fe} \) decreases and \( k_s \) increases. Thus the coil inductance \( L_c(i) \) decreases with an increasing current.

The inductance per phase includes all coil inductances along with the stray flux in the slots and the winding overhangs. If all coils per phase are connected in series, the total number of turns is \( N = 2p \cdot N_c \). The inductance \( L = 2p \cdot L_c \) is biggest in aligned position (\( d \)-position), as the flux lines then only have to cross the small air gap \( \delta \). In the unaligned position a rotor slot opposes a stator tooth, thus the air gap increases and is equal to the slot depth (\( q \)-position). In that case the inductance is smallest. Thus the inductance varies with moving rotor between \( L_d \) and \( L_q \).

**Conclusions:**

The inductance depends on current and rotor position \( L(i, \gamma) \).
Motor development

2.5

Reluctance machines

Fig. 2.1.2-1b shows the flux linkage depending on the coil current for different rotor positions. At low current no saturation occurs and assuming $\mu r_0 >> \mu_0$, the factor $k_s = 1$. Then the characteristics are linear rising. With beginning saturation $k_s > 1$ the curves are bent (“saturation region”). In d-position the air gap is $\delta_d = \delta$, and in q-position the air gap is equal to rotor tooth length $\delta_q = \delta + l_{dr}$. Therefore the q-axis flux linkage is much smaller than the flux of d-axis. In order to get a big torque the difference between d-axis and q-axis flux linkage must be very big, as shown in Section 2.1.4.

$$\Psi = N \cdot A \cdot B_d = \mu_0 \cdot \frac{N^2}{2p} \cdot i \cdot \frac{b}{\delta \cdot k_s} \cdot l$$  \hspace{1cm} (2.1.2-8)

![Fig.2.1.2-2: Flux linkage for different rotor positions, calculated with Finite Element numerical field calculation for motor of Fig.2.1.1-1b, compared with measurement]

2.1.3 Voltage and torque equation

Each phase is fed independently from an H-bridge, which is considered here as a voltage source $u$. Each phase has a resistance $R$ and an inductance $L$. With a big current the excited flux is so big that the iron in teeth and yokes is saturating. Rotor movement is described by the rotor position angle $\gamma$, leading to the rotor angular speed

$$\Omega_m = \frac{d\gamma}{dt}.$$  \hspace{1cm} (2.1.3-1)

The flux linkage per phase is given by the inductance

$$\psi(\gamma,i) = L(\gamma,i) \cdot i$$  \hspace{1cm} (2.1.3-2)

and without saturation: $L(\gamma)$, leading to

$$\frac{d\psi}{dt} = L \cdot \frac{di}{dt} + i \cdot \frac{dL}{d\gamma} \cdot \frac{d\gamma}{dt},$$  \hspace{1cm} (2.1.3-3)

and the voltage equation is

$$u = R \cdot i + \frac{d\psi}{dt} = R \cdot i + L \cdot \frac{di}{dt} + i \cdot \frac{dL}{d\gamma} \cdot \Omega_m$$  \hspace{1cm} (2.1.3-4)
Motor development

2.1.3 Reluctance machines

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with the “rotational induced voltage” (where \( \gamma \) is considered in mech. degrees)

\[
  u_i = i \cdot \frac{dL}{dy} \cdot \Omega_m
\]

(2.1.3-5)

which may be regarded as back EMF. The electric input power \( p_e \) per phase has to balance the change of the stored magnetic energy \( W_{mag} \), the resistive losses and the internal \( p_\delta \), which is converted into mechanical output power. Here we neglect the stator iron losses.

\[
  p_e = p_{Cu} + \frac{dW_{mag}}{dt} + p_\delta
\]

(2.1.3-6)

The magnetic energy per phase and its derivative are

\[
  W_{mag} = \frac{L(\gamma, i) i^2}{2} \Rightarrow \frac{dW_{mag}}{dt} = i \cdot L \cdot \frac{di}{dt} + \frac{1}{2} \cdot i^2 \cdot \frac{dL}{dy} \cdot \Omega_m
\]

(2.1.3-7)

By multiplying the voltage equation with the current these different parts of the power balance are determined.

\[
  p_e = u \cdot i = R \cdot i^2 + i \cdot L \cdot \frac{di}{dt} + \frac{1}{2} \cdot i^2 \cdot \frac{dL}{dy} \cdot \Omega_m
\]

(2.1.3-8)

Comparing (2.1.3-6), (2.1.3-7), (2.1.3-8) we get the internal power

\[
  p_\delta = \frac{1}{2} \cdot i^2 \cdot \frac{dL}{dy} \cdot \Omega_m
\]

(2.1.3-9)

and finally the electromagnetic torque without considering iron saturation

\[
  M_e = p_\delta / \Omega_m = \frac{1}{2} \cdot i^2 \cdot \frac{dL}{dy}
\]

(2.1.3-10)

2.1.4 SR machine operation at ideal conditions

In Fig.2.1.4-1 the change of the inductance is shown, which may be considered almost linear as the overlapping region of stator and rotor tooth is decreasing linear, when the rotor is moving along the angle \( \alpha \) (here considered in mechanical degrees: 1 rotor revolution = \( 2\pi \)).

Stator and rotor tooth width \( b_{ds} \) and \( b_{dr} \) correspond with the circumference angles \( \alpha_s \) and \( \alpha_r \). For \( Q_s > Q_r \), a stator tooth is usually smaller than a rotor tooth, hence there is an angle \( \tilde{\alpha} = |\alpha_s - \alpha_r| \), where the complete stator tooth width is facing either a rotor tooth or a rotor slot opening. In that region the stator inductance will not change. By impressing ideal constant current \( i = \bar{I} \) into the considered phase during the movement of the rotor from an unaligned to an aligned position only for the section \( \alpha \), motor torque is produced.

\[
  M_e = \frac{1}{2} \cdot i^2 \cdot \frac{dL}{dy} = \frac{1}{2} \cdot i^2 \cdot \frac{L_d - L_q}{\alpha}
\]

(2.1.4-1)
Conclusions:
In order to get a big torque the difference between d-axis and q-axis flux linkage (inductance) must be very big, which holds true for all kinds of reluctance machines. The sign of the current polarity does not influence the sign of the torque, so unidirectional current feeding is sufficient. This corresponds with the fact that the direction of magnetic pull does not depend on the polarity of flux density. If no saturation occurs, torque rises with the square of current.

In order to ensure a big d-axis and a small q-axis inductance to get maximum torque, the air gap $\delta$ must be very small and the rotor slot depth rather big. Further, the stator tooth width $\alpha_s$ must be smaller than the rotor slot opening $2\pi/Q_r - \alpha_r$, so that the stator tooth in the q-position is completely facing a rotor slot to get a minimum inductance.

$$\alpha_s \leq \frac{2\pi}{Q_r} - \alpha_r \text{ mech. degrees} \quad (2.1.4-2)$$

If unidirectional current is energizing the coil, when the rotor moves from aligned to unaligned position, we get negative torque (generator mode).

$$M_e = \frac{1}{2} i^2 \cdot \frac{dL}{d\gamma} = \frac{1}{2} \frac{L_q - L_d}{\alpha} < 0 \quad (2.1.4-3)$$

In motor mode the induced voltage is positive (Fig.2.1.4-2b), and in generator mode it is negative. Thus with positive current we get positive internal power in motor mode, fed from the grid to the mechanical system, and negative internal power flow in generator mode, being fed by the mechanical system into the grid.

$$\dot{U}_i = \dot{I} \cdot \frac{L_d - L_q}{\alpha} \cdot \Omega_m > 0, \quad \dot{P}_d = \dot{U}_i \dot{I} / 2 > 0 \quad \text{motor} \quad (2.1.4-4)$$

$$\dot{U}_i = \dot{I} \cdot \frac{L_q - L_d}{\alpha} \cdot \Omega_m < 0, \quad \dot{P}_d = \dot{U}_i \dot{I} / 2 < 0 \quad \text{generator} \quad (2.1.4-5)$$

With a theoretically linear change of inductance it does not make any sense to impress current longer than angle $\alpha$, as in the region $\tilde{\alpha} = |\alpha_s - \alpha_r|$ the change $dL/d\gamma = 0$. So no torque is produced, but the current will generate resistive losses in the coils. In Fig.2.1.4-3 the complete pattern of changing inductance of all three phases a, b, c of a 6/4-machine is depicted. Each phase is energized with a unidirectional current pulse during the angle $\alpha$. This angle is
Motor development counted in “electrical” degrees, so that $\alpha / Q_r = \Delta \gamma$ yields the mechanical angle. One rotor slot pitch corresponds with 360° electrical degrees. As the tooth width is chosen equal to the slot width, we get under these ideal conditions $\alpha = 120^\circ$ el., meaning that the duration of the current impulse should be $120^\circ$ el. With three phases, the three H-bridges generate current pulses with $120^\circ$el. duration and pausing for $240^\circ$el. in between. By that a theoretically smooth torque without any ripple is generated. No time overlap between different phase current occurs. From Fig.2.1.4-3 it can be seen, that the frequency of the stator current pulse is

$$f_s = n \cdot Q_r$$  \hspace{1cm} (2.1.4-6)

Conclusions:
As long as there is no current overlap between adjacent phases (like in Fig.2.1.4-3), the torque per phase is also the value of the constant torque of the SR-machine. If the “current angle” is smaller $\vartheta_W < \alpha$, then the torque shows gaps between the impulses of each phase, thus creating a torque ripple. In that case the average torque is reduced by $\vartheta_W / \alpha$.

2.1.5 Calculating torque in saturated SR machines

As torque rises with the square of current, high motor utilization means high current and thus saturation of iron. In order to utilize directly the saturated flux linkage characteristics $\psi(i)$ for calculating torque, the magnetic co-energy $W^*$ is used (Fig.2.1.5-1). As magnetic energy

$$W_{mag} = \int_{V_0}^{V} \int_{0}^{B} (H \cdot dB) \cdot dV = \int_{0}^{\psi} \int_{0}^{i} d\psi$$

we get in the unsaturated (linear) case $\psi = L \cdot i$

$$W_{mag} = \int_{0}^{\psi} \int_{0}^{i} d\psi = L \int_{0}^{i} di = L \frac{i^2}{2}$$  \hspace{1cm} (2.1.5-2)

which corresponds to the triangular area in Fig.2.1.5-1a of linear $\psi(i)$-curve. In a non-linear case $W_{mag}$ corresponds to the area left of the non-linear $\psi(i)$-curve in Fig.2.1.5-1b.

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Magnetic co-energy is defined as

\[ W^* = \psi \cdot i - W_{\text{mag}} \]  \hspace{1cm} (2.1.5-3)

and corresponds to the area to the right of the \( \psi(i) \)-curve in Fig. 2.1.5-1.

Fig. 2.1.4-3: Three phase 6/4-SR machine: Unidirectional current impression is done for \( \alpha = 120^\circ \) el. for each phase, yielding a theoretically smooth torque. A current impression of 180° at this ideal linear change of inductance will only increase resistive losses, but not torque.
When the rotor is turning, the magnetic flux linkage changes from one characteristic to the next (see Fig. 2.1.2-2). Assuming linear characteristics in Fig. 2.1.5-2 and operation with constant current $i_0$, a movement of the rotor towards aligned position means a changing from a lower to an upper $\psi(i)$-curve. We assume a (very small) rotor step $d\alpha$, then the change of magnetic energy and co-energy by moving from characteristic 1 to characteristic 2 yields an increase of magnetic energy and of co-energy, as the total flux linkage is now larger by the value $d\psi$. By comparing the areas in Fig. 2.1.5-2 (left), we note

$$W_{mag,2} = W_{mag,1} + dW_{mag} = W_{mag,1} + i_0 \cdot d\psi - dW^*.$$  

(2.1.5-4)

![Diagram showing magnetic energy and co-energy](image)

**Fig. 2.1.5-1:** Magnetic energy $W_{mag}$ and co-energy $W^*$ for a) linear (unsaturated) and b) non-linear (saturated) flux linkage

From the voltage equation we get the energy balance during the very small step $d\gamma$, corresponding to the time step $dt$:

$$u = R \cdot i_0 + \frac{d\psi}{dt} \Rightarrow dW_e = u \cdot i_0 \cdot dt = R \cdot i_0^2 \cdot dt + i_0 \cdot d\psi = R \cdot i_0^2 \cdot dt + dW_{mag} + dA_m.$$  

(2.1.5-5)

A very small time step leads to an increase of loss and magnetic energy. The delivered torque produces mechanical work:

$$dA_m = M_e \cdot d\gamma = M_e \cdot \Omega_m \cdot dt.$$  

(2.1.5-6)

By comparing (2.1.5-4) and (2.1.5-5), we see
\[ i_0 \cdot d\psi = dW_{\text{mag}} + dW^* = dW_{\text{mag}} + dA_m \quad \Rightarrow \quad dW^* = dA_m, \quad (2.1.5-7) \]

thus getting the following torque equation

\[ M_e(\gamma, i) = \frac{dW^*}{d\gamma}. \quad (2.1.5-8) \]

When changing from \( q \)- to \( d \)-position, the change in co-energy \( \Delta W^* \) is the area between the \( \psi_q(i) \)-curve and the \( \psi_d(i) \)-curve. For maximizing the torque of a SR motor, this area has to be as big as possible, which demands a very high motor current. If the motor is unsaturated, the magnetic energy and co-energy are equal. This means that increase of co-energy gives the same increase to magnetic energy. As with each switch-on and switch-off of one phase the complete magnetic energy has to be put into the system and afterwards taken out, the feeding inverter must be rated for this additional amount of energy flow either by over-sizing voltage or current rating. In saturated machines, with increasing current the co-energy rises much stronger than magnetic energy (Fig.2.1.5-1b), which is very economical for inverter rating. Therefore SR machines should be operated as highly saturated machines, which is quite contrary to many other electric machines.

**Conclusions:**

Torque calculation can be done from the map of \( \psi(i) \)-curves, evaluating the change of co-energy with change of rotor angle \( \gamma \) for a given current \( i \). SR machines shall be operated highly saturated in order to limit inverter rating by limiting switched magnetic energy.

In low current operation (no saturation) co-energy difference between \( d \)- and \( q \)-position equals a triangle area. Triangle surface is proportional to \( i^2 \). Thus torque rises with square of current. At high saturation increase of co-energy difference between \( d \)- and \( q \)-position increases linear with rising current. Therefore torque of saturated SR machines rises linear with current (Fig.2.1.5-3).

**2.1.6 SR machine operation at real conditions**

**a) Real change of inductance between unaligned and aligned position:**

In Fig.2.1.6-1a the change of inductance - calculated with Finite Elements - is shown for the 8/6-motor of Fig.2.1.1-1b, being compared with the linear approximation of Section 2.1.2 and...
with measurement. Thus the angle of real change of inductance is larger than the angle $\alpha$. The derivative $dL/d\gamma$ is not constant, but looks like a hump (Fig.2.1.6-1b). Thus even with constant current the torque $M_e(\gamma) = (i^2/2) \cdot dL/d\gamma$ is not any longer constant, but shows a considerable **torque ripple**. In the same way induced voltage is also now hump-like shaped: $u_i(\gamma) = i \cdot dL/d\gamma \cdot \Omega_m$.

![Fig.2.1.6-1: Numerical calculation of inductance and torque for the 8/6-motor of Fig.2.1.1-1b, compared with measurement. Although current is constant, torque is not, showing a considerable ripple.](image)

This torque ripple may be reduced a little bit, if the current angle is increased: $\vartheta_W > \alpha$. Maximum possible angle is $\vartheta_W = 180^\circ$. Bigger angle would already generate braking torque. In Fig. 2.1.6-2a the torque ripple is shown for current angle $120^\circ$. By increasing angle to $180^\circ$ (Fig. 2.1.6-2b), the torque contributions of adjacent phases overlap, resulting in a smoother total torque and a slight increase in average torque on the expense of higher resistive losses.

![Fig.2.1.6-2: Torque ripple in SR machines: a) Due to non-linear change of inductance torque per tooth and phase shows a hump-like shape, thus reducing average torque and generating torque ripple b) If current angle is increased from $120^\circ$ to $180^\circ$, torque ripple is reduced a little bit and average torque is raised.](image)
Conclusions:
Real SR machines show considerable torque ripple already at low speed, when operated with constant current. Frequency of torque ripple is given by stator frequency $f_s = n \cdot Q$ per phase and number of phases $m$, thus $f_{puls} = n \cdot Q \cdot m$.

Average resistive losses per period $T = 1/f_s$ are calculated with

$$P_{Cu} = m \cdot R \cdot \frac{1}{T} \int_0^T i^2 \cdot dt = m \cdot R \cdot I^2$$

(2.1.6-1)

where r.m.s.-value of unipolar current is depending on current angle, which for $120^\circ$ gives a ratio of time of current flow vs. period of $x_{120} = 120^\circ/360^\circ = 1/3$ and for $180^\circ$ the value $x_{180} = 1/2$:

$$I = \frac{1}{T} \int_0^T i^2 \cdot dt = \frac{1}{T} \int_0^{xT} i^2 \cdot dt = \frac{1}{T} \cdot I^2 \cdot xT = \sqrt{x} \cdot I$$

(2.1.6-2)

Conclusions:
For $180^\circ$ current angle the resistive losses are 50% higher than with $120^\circ$ current angle ($x_{180}/x_{120} = 3/2 = 1.5$).

b) Real shape of unidirectional current:
In order to get block shaped unidirectional current, the H-bridge per phase is chopping the DC link voltage, and tries to keep by hysterisis current control the current amplitude within a certain small hysteresis band. The H-bridge of Fig.2.1.1-a is chopping the DC link voltage $U_d$ e.g. by switching upper transistor T2 on and off. Current flow continues – driven by stored magnetic energy $W_{mag}$ - with T2 “off” by flowing through free-wheeling diode D1. Turning off current is accomplished by switching off T1 and T2. The current continues to flow via diodes D1 and D2 against the direction of DC link voltage and is therefore reduced rather quick. Stored magnetic energy is fed back to DC link. From voltage equation

$$\frac{0}{dL} \frac{0}{di} \frac{0}{d\gamma} = U_d \quad \text{or} \quad -U_d \quad \text{or} \quad 0$$

(2.1.6-3)

we see at low speed $\Omega_m$, that induced voltage is much smaller than $U_d$, thus being neglected. If we also neglect resistance, current rises and decreases linear according to $u = L \cdot di/dt$ (Fig.2.1.6-3, left). Thus the ideal block shaped unipolar current is more or less well reached by the chopping operation. At high speed the time duration $T_w ~ 1/n$ for current pulse is very short according to high current frequency. The time constant of phase winding $T_e = L/R$ is now even larger than $T_w$, so current rise is much longer than the wished time duration of current pulse. No longer chopping mode is possible. It is only possible to switch DC link voltage on and off. Therefore current pulse shape is no longer of block shape, but determined by the difference of constant DC link voltage and hump-like shaped induced voltage $u_i(\gamma)$ (Fig.2.1.6-3, right). With this current shape $i(\gamma)$ the torque ripple increases drastically: $M_r(\gamma) = (i^2(\gamma)/2) \cdot dL/d\gamma$. In Fig.2.1.6-4 unidirectional current for very low (left) and very high (right) speed is shown. Please note, that rise and fall of current at low speed is determined mainly by inductance, which increases with time towards aligned
position. Thus current rise and fall of current ripple in hysteresis band is slowed changes is slowed down with inductance increase.

![Graph 1](image1.png)

**Fig.2.1.6-3:** Real current shape: Left: At low speed hysteresis control of current allows generating rather block shaped unidirectional current. Right: At high speed time is too short to chop DC link voltage, so only “voltage on” and “off” is possible, leading to distorted current pulse, which generates increased torque ripple

![Graph 2](image2.png)

**Fig.2.1.6-4:** Real current shape: Left: At very low speed, Right: At very high speed time (with time scale extremely enlarged, compared to left figure)

### 2.1.7 SR Drive operation – torque-speed characteristic

Maximum possible torque vs. speed is called torque-speed characteristic, which is consisting of mainly two sections: **current limit** and **voltage limit**.
a) Voltage limit:
At high speed the induced voltage (back EMF), which is increasing with increasing speed, limits current flow, as stator voltage cannot surpass maximum value DC link voltage $U_d$. The induced voltage and the resistive and inductive voltage drop equal the DC link voltage (“voltage limit”). Neglecting resistance $R = 0$ and assuming constant back EMF $u_i(\gamma) = \dot{U}_i$ and constant current $i(\gamma) = \dot{I}$, the condition for voltage limit is:

$$u = U_d = R \cdot \dot{I} + L \cdot \frac{d\dot{I}}{dt} + \dot{U}_i \Rightarrow U_d = \dot{U}_i = \dot{I} \cdot \frac{dL}{d\gamma} \cdot \Omega_m$$  \hspace{2cm} (2.1.7-1)

Current flow at voltage limit is

$$\dot{I} = \frac{U_d}{dL_d/d\gamma} \cdot \frac{1}{\Omega_m} \approx \frac{U_d}{(L_d - L_q)/\alpha} \cdot \frac{1}{\Omega_m}$$  \hspace{2cm} (2.1.7-2)

Possible current flow rises with inverse of decreasing speed, until it reaches the inverter current limit $\dot{I}_{max}$ at speed

$$n_g = \frac{1}{2\pi} \cdot \frac{U_d}{\dot{I}_{max} \cdot ((L_d - L_q)/\alpha)}$$  \hspace{2cm} (2.1.7-3)

Thus torque-speed characteristic at voltage limit is therefore

$$M_e \approx \frac{1}{2(L_d - L_q)/\alpha} \cdot \left( \frac{U_d}{\Omega_m} \right)^2$$  \hspace{2cm} (2.1.7-4)

Conclusions:
At the voltage limit the maximum possible torque of SR drives decreases with the square of rising speed.

b) Current limit:

Fig. 2.1.7-1: Torque-speed characteristic of SR machine, a) for ideal block-shaped current, b) considering real current shape, which deviates from block shape with rising speed
Inverter current limit is also thermal limit of inverter, as thermal time constant of semiconductor devices is below 1 s. Motor thermal current limit ("rated current") usually is 50\% of this inverter current limit. So short time overload capability of machine is 100\%, as its thermal time constant is - rising with motor size and depending on cooling system – about several minutes to typically half an hour. Maximum torque $M_{\text{max}}(I_{\text{max}})$ of drive system is available in the base speed region $0 \leq n \leq n_g$ (Fig. 2.1.7-1a). In real SR machines with rising speed the current shape cannot be kept as ideal block. Therefore torque ripple increases with rising speed and average torque decreases. Hence real speed torque characteristics, mapping average torque versus speed, have maximum torque at low speed (Fig. 2.1.7-1b).

2.1.8 Inverter rating

a) Static rating:
In each phase per switching instant the total electric energy must be put into the motor phase, not only the mechanic energy and loss energy, but also total magnetic energy. Each phase is only energized during “current angle” $\theta_W$. Neglecting losses, we consider total energy per switching as

$$W_{\text{tot}} = W_{\text{mag}} + A_m = W_{\text{mag}} + \Delta W^*$$

In Fig. 2.1.5-3 a simplified flux linkage characteristic with $L_q << L_d$, assuming $L_q = 0$, and with constant saturated flux linkage $\psi_d = \psi_{d,\text{sat}}$, allows estimation of ratio

$$\frac{W_{\text{tot}}}{\Delta W^*} = \frac{W_{\text{tot}}}{W_{\text{tot}} - W_{\text{mag}}} = \frac{i_{\text{max}} \cdot \psi_{d,\text{sat}}}{i_{\text{max}} \cdot \psi_{d,\text{sat}} - (i_{\text{sat}} \cdot \psi_{d,\text{sat}}/2)} = \frac{1}{1 - (i_{\text{sat}}/(2i_{\text{max}}))}. \quad (2.1.8-2)$$

Example 2.1.8-1:

$i_{\text{max}}/i_{\text{sat}} = 2$: $W_{\text{tot}}/\Delta W^* = 4/3 = 1.33$

For static energy demand the inverter must be capable of 133\% motor power rating to switch also the total amount of magnetic energy.

With three phase motors, the three H-bridge inverters need a total amount of six IGBTs and six free-wheeling diodes like three phase inverters for rotating field AC machines like induction or synchronous machines.

b) Dynamic rating:
The electric winding time constant

$$T_e = \frac{L}{R} \sim \frac{W_{\text{mag}}}{P_{\text{Cu}}} \quad (2.1.8-3)$$

rules the current rise in simple coils. In SR machines also the back EMF must be considered, which rises with rising speed. Neglecting resistance at high speed, voltage equations yields

$$u = L \cdot \frac{di}{dt} + i \cdot \frac{dL}{dy} \cdot \Omega_m = U_d \quad . \quad (2.1.8-4)$$
The solution of this 1\textsuperscript{st} order differential linear equation is

\[ i = \frac{U_d}{\Omega_m \cdot dL \cdot d\gamma} \left( 1 - e^{-t/T_U} \right), \quad T_U = \frac{L}{\Omega_m \cdot dL \cdot d\gamma}, \quad (2.1.8-5) \]

which shows that for reaching sufficient current within time \( T_U \), a high DC link voltage is necessary. So dynamic condition demands considerable voltage and thus big inverter rating for getting rather block shaped current also at high speed.

\textbf{Conclusions:}

If inverter rating has to be limited because of costs, there is no sufficient surplus DC link voltage to impress block shaped unidirectional current also at high speed. Therefore current deviates from ideal block shape at high speed, causing increased torque ripple and decreased average torque, thus reducing motor power output.

c) Inverter current control:

(i) At low speed:

\textbf{Block current} by hysteresis control with constant current angle \( \varphi_W \), usually 180° or 120° (increased torque ripple, decreased resistive losses). Current rise and fall time \( t_r \) and \( t_f \) very short in relation to current impulse duration \( T_W \).

(ii) Increased speed:

Current impulse duration \( T_W \) decreases with increased speed, \( t_r \) and \( t_f \) are now significant parts of current flow duration, so current impulse gets trapezoidal shape, and average torque decreases.

(iii) At high speed:

The current impulse duration \( T_W \) is too short for chopping with hysteresis control. So only voltage “switch on – switch off” mode is possible. Current angle, determined by switch on and switch-off angle, adjusted to speed, so that average torque decreases with 1/\( n \) (constant power operation).

(iv) Voltage limit:

No adjusting of current angle possible, because of too short time \( T_W \); average torque decreases with 1/\( n^2 \).

Fig.2.1.8-1: Maximum torque & power, depending on speed with inverter control according to sections (i) ... (iv), see text.
2.1.9 Motor technology and performance

a) Advantages:
- simple, low cost motor construction
- no distributed AC winding; simple tooth-wound coils; no overlap in winding overhangs
- smaller number of slots than in comparable AC machines, simple manufacturing
- robust rotor without any winding or magnets, thus well suited for high speed
- low rotor inertia
- Independent feeding of phases: if one phase fails, motor with $m \geq 3$ is still capable of self starting and running at reduced torque without overloading the “healthy” windings
- Compared with inverter fed induction machines, SR machine has increased efficiency due to lower rotor losses (Example 2.1.9-1).

b) Disadvantages:
- High torque ripple with increased speed. At very low speed current profiling by inverter current control is possible to cancel torque ripple, but not at elevated speed due to limited inverter rating.
- Increased number of phases will reduce torque ripple, but number of semiconductor switches rises (expensive!)
- No standard three phase AC inverter applicable; special H-bridge inverter is necessary.
- Danger of exciting torsion resonance vibrations by torque ripple at variable speed operation
- **Magnetically excited acoustic noise:** Pulsating radial magnetic pull with frequency $f_{puls} = n \cdot Q_e \cdot m$ causes radial vibrations of stator yoke and housing, which compresses / decompresses air with same frequency, causing acoustic sound. Acoustic sound is especially big, if frequency $f_{puls}$ coincides with eigen-frequency of stator yoke and housing (resonance). As flux per pole is only flux per teeth, thus rather small, stator yoke is very thin and “rings like a bell” (Fig.2.1.9-1).
- Strong decrease of maximum torque with rising speed due to increased torque ripple.
- Increased inverter rating due to switching on/off of magnetic energy at each phase switching.
- Usually rotor position sensor needed for optimum inverter-motor operation

![Fig.2.1.9-1: Measured sound pressure level of 7.5 kW 12/8 SR machine (Fig.2.1.1-2) in anechoic chamber (1: rated current, 2: no-load current). At no-load torque and current is very small, so air gap flux density $B_0 \sim i$ and exciting radial magnetic pull $f_1 \sim B_0^2 \sim i^2$ is small, leading to low magnetic noise. At rated load noise is rather big, showing sharp maximum when exciting frequency coincides with stator resonance frequencies.](image-url)
c) Electromagnetic motor utilization:
Torque per motor active volume may be compared with standard induction machines. For the same cooling system about 80% of utilization of induction machine is reached.
In spite of small flux per pole (tooth flux = pole flux) this can be achieved by
- high slot fill factor due to tooth-wound coils,
- short winding overhangs due to tooth-wound coils, thus decreasing resistive losses
- reduced iron losses in rotor especially at low speed.

Example 2.1.9-1:
Comparison of inverter fed induction and SR motor (see Fig.2.1.1-2) for the same rated power and speed at identical cooling (totally enclosed, fan mounted on shaft):
Data: 7.5 kW, shaft height 132 mm, base speed 1500/min, top speed 3000/min, Thermal Class F, three-phase four-pole machines

<table>
<thead>
<tr>
<th>Motor data</th>
<th>Switched Reluctance Machine</th>
<th>Induction Machine</th>
<th>SRD/IM Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer/inner stator diameter $d_{sa}/d_{si}$ (mm)</td>
<td>210 / 120.9</td>
<td>200 / 122.6</td>
<td>+5% /-1%</td>
</tr>
<tr>
<td>Outer rotor diameter/ air gap $d_{o}/\delta$ (mm)</td>
<td>120 / 0.45</td>
<td>122 / 0.3</td>
<td>-1%/+50%</td>
</tr>
<tr>
<td>Tooth number: Stator/ Rotor $Q_{s}/Q_{r}$</td>
<td>12 / 8 unskewed</td>
<td>48 / 36 skewed</td>
<td></td>
</tr>
<tr>
<td>Yoke height stator/rotor $h_{s}/h_{r}$ (mm)</td>
<td>14 / 17.5</td>
<td>21.5 / 16</td>
<td></td>
</tr>
<tr>
<td>Tooth width stator/ rotor $b_{s}/b_{r}$ (mm)</td>
<td>16 / 16.7</td>
<td>4.3 / 6</td>
<td></td>
</tr>
<tr>
<td>Stack length $l_{as}$ (mm)</td>
<td>193</td>
<td>135</td>
<td>+42%</td>
</tr>
<tr>
<td>Stack length + end winding overhangs (mm)</td>
<td>193 + 2x18 = 229</td>
<td>135 + 2x47 = 229</td>
<td>0%</td>
</tr>
<tr>
<td>Stator resistance per phase (20 °C) $R_{s}$ (Ohm)</td>
<td>0.85</td>
<td>0.512 Y</td>
<td>+64%</td>
</tr>
<tr>
<td>Armature winding / slot space factor</td>
<td>One coil per stator tooth / 0.44</td>
<td>Single-layer, full-pitched / 0.38</td>
<td>-/+16%</td>
</tr>
<tr>
<td>Number of turns per phase $N_{t}$</td>
<td>244</td>
<td>112</td>
<td>+118%</td>
</tr>
<tr>
<td>Stator frequency $f_{s}$ for $n = 0..3000/min$</td>
<td>0 ... 400 Hz</td>
<td>0 ... 100 Hz</td>
<td>+300%</td>
</tr>
<tr>
<td>Cylindrical rotor volume $d_{ra}^2 \pi l_{Fe}/4$</td>
<td>2.18 dm³</td>
<td>1.58 dm³</td>
<td>+38%</td>
</tr>
<tr>
<td>Rotor moment of inertia (measured) $J_{rot}$ (kgm²)</td>
<td>0.0195 kgm²</td>
<td>0.024 kgm²</td>
<td>-19%</td>
</tr>
</tbody>
</table>

Table 2.1.9-1: Some basic design data for the Switched Reluctance Machine and the Induction Machine.

Comparison of measured loss balance at rated speed:

<table>
<thead>
<tr>
<th></th>
<th>Switched Reluctance Machine</th>
<th>Induction Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input / Output power $P_{in}/P_{out}$</td>
<td>9440 W/ 8480 W</td>
<td>9950 W/ 8480 W</td>
</tr>
<tr>
<td>Phase current $I_{p}$</td>
<td>13.3 A/ 27.5 A</td>
<td>17.45 A/ 30 A</td>
</tr>
<tr>
<td>Stator frequency $f_{s}$</td>
<td>200 Hz</td>
<td>52 Hz ($U_{d,k} = 225.5V$)</td>
</tr>
<tr>
<td>Armature temperature rise</td>
<td>110 K</td>
<td>101 K</td>
</tr>
<tr>
<td>Iron losses / friction &amp; windage losses</td>
<td>200 W/ 165 W</td>
<td>265 W/ 55 W</td>
</tr>
<tr>
<td>Stator copper losses/cage losses</td>
<td>595 W/ 0 W</td>
<td>650 W/ 350 W</td>
</tr>
<tr>
<td>Additional losses $J_{r}$</td>
<td>0 W</td>
<td>150 W</td>
</tr>
<tr>
<td>Stator current density $J_{s}$</td>
<td>5.25 A/mm²</td>
<td>8.23 A/mm²</td>
</tr>
<tr>
<td>Current loading $A = 2mN_{j}I_{p}/(d_{si}\pi)$</td>
<td>513 A/cm</td>
<td>305 A/cm</td>
</tr>
<tr>
<td>Motor efficiency $\eta_{mot}$</td>
<td>89.8 %</td>
<td>85.2 %</td>
</tr>
<tr>
<td>Inverter efficiency $\eta_{inv}$</td>
<td>96.6 %</td>
<td>97.0%</td>
</tr>
<tr>
<td>Drive efficiency $\eta$</td>
<td>86.7 %</td>
<td>82.6 %</td>
</tr>
</tbody>
</table>

Table 2.1.9-2: Measured results of thermal load run for 1500/min, 54 Nm and $U_{d} = 540$ V. Cooling was performed with a shaft-mounted fan of 230mm outer diameter and 7 radial fan blades.

Conclusions:
Short winding overhangs allow increased iron stack length, so for the same outer motor dimensions inverter fed induction machine and SR machine have nearly the same power output at same motor size. At low speed due to lower motor losses SR torque may be
increased (high starting torque). SR motor shows increased efficiency when compared with induction machine due to lack of cage losses.

Fig 2.1.9-2: Comparison of measured motor efficiency of 7.5 kW, four-pole 12/8 SR machine (SRD) and inverter-fed standard induction machine (IM) at 54 Nm, 100 K armature temperature rise for different motor speed. Inverter efficiency of about 97% has to be added in both cases.

2.1.10 Applications of SR drives

Standard variable speed SR drives have been designed and manufactured to compete with inverter-fed variable speed induction machines. More importance was gained by using the SR drive in special applications, where variable speed drives for
- cheap mass production (e.g. drive in electric cars, pump and compressor drives),
- special purpose in rough environment (e.g. mills in coal mines, explosion hazard utilities such as oil fields)
- robust high speed drives (e.g. starter-generator for air craft engine) are needed.

For some of these cases the magnetic noise of motor is not of interest, as the driven load is already noisy (e.g. air craft engine).

Example 2.1.10-1:
Starter-generator for military aircraft jet (US Air Force, manufacturer GE)

- Rated power: 250 kW, rated speed 13 500/min, rated torque 177 Nm, rated current 750 A, DC link voltage 270 V
- Maximum speed 22200/min, overspeed 26000/min, overall system efficiency: 90%
- Three phase four pole 12/8 SR machine,

For reliability two independent three phase systems are arranged with each 125 kW power.

In case of failure motor is “fail-silent” = no current = no force = no induced voltage, so risk of fire due to short circuit is minimized.

In order to decrease motor size and mass, motor is intensively cooled by oil. So motor mass is only 70 kg, yielding “power weight” of \( P/m = 3.6 \) kW/kg.

In low speed region 0 .. 13500/min SR machine is used to start compressor of air craft engine with constant torque 177 Nm.
In high speed region 13500 ... 26000/min SR machine is driven by aircraft engine as generator with constant power output 250 kW (Fig.2.1.10-1).

![Torque-speed characteristic and torque-current curve](image)

**Fig.2.1.10-1**: 250 kW four-pole SR machine as starter-generator for aircraft; a) Torque-speed characteristic, b) Measured and calculated torque-current-curve (above 400 A the SR machine is saturated).

### 2.2 Synchronous reluctance machines

#### 2.2.1 Basic function of synchronous reluctance machine

The stator of the synchronous reluctance machine is slotted, with a distributed AC winding like a PM synchronous machine. So stator winding is usually fed from sinus three-phase voltage system; current in winding is also three-phase sinus system. The rotor is consisting of deep slots between the poles in the \(q\)-axis (“inter-pole gaps”), whereas with the \(d\)-axis the air gap is as small as possible. Fig.2.2.1-1a shows a typical 4 pole rotor with slot width of 36° and pole width 54°, with resulting pole pitch 90°. In the pole region slots containing squirrel cage segments for asynchronous starting at fixed stator voltage (“line start”) are arranged. After asynchronous starting, when the rotor has accelerated up to synchronous speed of stator rotating magnet field, this stator field exerts tangential magnetic pull on the rotor, if the rotor poles are not in \(d\)- or \(q\)-position (Fig.2.2.1-2). In \(d\)- and \(q\)-position total tangential magnetic
Motor development

Reluctance machines

pull on rotor is zero, so no torque is generated. In positions between \(d\)- and \(q\)-position asymmetric flow of flux across rotor and air gap generates resulting tangential pull and therefore synchronous reluctance torque. This torque tries to align rotor in \(d\)-position against load torque, thus generating mechanical power output (motor mode).

Fig 2.2.1-1: Different shapes (at different scale) of rotor cross section of 4-pole, 3-phase synchronous reluctance machines (Steady state power at Thermal Class F, totally enclosed, shaft-mounted fan, 50 Hz, synchronous speed 1500/min). Rotor a) 2.2 kW, shaft height 112 mm, rotor b) 550 W, shaft height 80 mm (Siemens AG, Germany).

If rotor is driven mechanically, and rotor is leading ahead \(d\)-position of stator field, polarity of torque is reversed, thus operating as generator with power flow from shaft to grid. But always magnetizing current from grid is needed to excite air gap flux, so always current will be lagging behind voltage.

Conclusions:

Synchronous reluctance machine is fed by three-phase sinus voltage system, and may be operated as motor or generator. Due to magnetizing current the stator current is always lagging (inductive current).

No position sensor is needed, but load angle \(\vartheta\) between rotor axis and stator field (Fig. 2.2.2-1) increases with increased torque demand up to maximum torque at 45° load angle. This maximum torque is static pull-out torque \(M_{p0}\). By surpassing this angle and torque, the rotor
is pulled out of synchronism and runs asynchronously with the starting cage segments being induced and producing **asynchronous torque**.

Rotor is very robust and cheap, as it contains neither winding and magnets nor position sensor. Rotor speed in synchronism is exactly **synchronous speed** according to Section 1.1.1

\[ n_{\text{syn}} = \frac{f_s}{p} \]  

**(2.2.1-1)**

**Example 2.2.1-1:**

<table>
<thead>
<tr>
<th>Pole count</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed 1/min at 50 Hz</td>
<td>3000</td>
<td>1500</td>
<td>1000</td>
<td>750</td>
</tr>
<tr>
<td>Speed 1/min at 60 Hz</td>
<td>3600</td>
<td>1800</td>
<td>1200</td>
<td>900</td>
</tr>
</tbody>
</table>

### 2.2.2 Voltage and torque equation of synchronous reluctance machine

Comparing synchronous reluctance machine with PM synchronous machine of section 1, we see that back EMF \( U_p \) is zero, as no rotor excitation exists. On the other hand stator main inductance \( L_h \) for \( d \)-axis with small air gap is much bigger than for \( q \)-axis with big air gap of rotor inter-pole gap: \( L_{hd} > L_{hq} \). When each phase of stator winding is fed by stator voltage \( U_s \), the stator current \( I_s \) will be lagging by phase angle \( \varphi \). By decomposing stator current in two components with 90° phase shift, we get \( d \)-component of current \( I_d \), magnetizing the rotor along \( d \)-axis, and \( q \)-component of current \( I_q \), magnetizing current along \( q \)-axis. Air gap field of \( d \)-component experiences small air gap and therefore self-induced voltage \( \omega_s L_{hd} I_d \) of that stator field component is big according to self-inductance \( L_{hd} \), whereas in \( q \)-axis only smaller \( L_{hq} \) will result in smaller self-induced voltage \( \omega_s L_{hq} I_q \).

Thus total voltage per phase is given by these two components of **self-induced voltage of air gap field** and of self-induced voltage due to stator **stray flux** in slots and winding overhangs and **resistive voltage** drop. As voltages and currents are sinusoidal, complex phasor calculation may be used. The real axis of complex co-ordinate system is chosen to coincide with rotor \( d \)-axis, and imaginary axis with rotor \( q \)-axis.

\[ I_s = I_d + j I_q = I_d + I_q \]  

**(2.2.2-1)**

Voltage equation:

\[ U_s = R_s I_s + j \omega_s L_{s\sigma} I_s + j \omega_s L_{hd} I_d + j \omega_s L_{hq} I_q = R_s I_s + j \omega_s L_{s\sigma} I_s + U_h \]  

**(2.2.2-2)**

With defining **synchronous \( d \)- and \( q \)-axis inductance**

\[ L_d = L_{s\sigma} + L_{hd}, \quad L_q = L_{s\sigma} + L_{hq} \]  

**(2.2.2-3)**

and **synchronous \( d \)- and \( q \)-axis reactance** \( X_d = \omega_s L_d \), \( X_q = \omega_s L_q \) and neglecting resistive losses, voltage equation is

\[ U_s = j X_d I_d + j X_q I_q = j X_d I_d - X_q I_q \]  

**(2.2.2-4)**
In Fig.2.2.2-1 for a two-pole synchronous reluctance machine this voltage equation is depicted as complex phasor diagram, showing
a) lagging stator current with phase angle $\phi$,
b) decomposition of current in $d$- and $q$-component,
c) current angle $\beta$ between direction of current phasor and $d$-axis,
d) load angle $\delta$ between stator voltage phasor and $q$-axis
e) motor operation, as $\cos \phi$ is positive \( (|\phi| < \pi/2) \).

![Figure 2.2.2-1: Phasor diagram for synchronous reluctance machine for motor operation. Flux linkage phasors may be taken as direction of air gap flux components.](image)

In addition, the values of air gap flux linkages

$$\Psi_{hd} = L_{hd} \cdot I_d \cdot \sqrt{2}, \quad \Psi_{hq} = L_{hq} \cdot I_q \cdot \sqrt{2}$$

(2.2.2-5)

indicate, that resulting air gap flux

$$\Psi_h = \Psi_{hd} + \Psi_{hq}$$

(2.2.2-6)

has its direction much closer to $d$-axis than current phasor. Note, that if $L_{hd} = L_{hq}$, then direction of resulting flux would be the direction of current phasor, so reluctance rotor directs flux into $d$-axis direction, thus generating reluctance torque.

Electrical real $P_e$ is converted in motor mode into mechanical power. Neglecting losses, voltage equation (2.2.2-4) yields

$$P_e = m \cdot U_s \cdot I_s \cdot \cos \phi = m \cdot (U_{s,Re} \cdot I_s,Re + U_{s,Im} \cdot I_s,Im) = m \cdot (-X_q I_d I_q + X_d I_d I_q) = \Omega_m M_e \Rightarrow$$

$$M_e = \frac{P \cdot m}{\omega_s} (X_d - X_q) \cdot I_d I_q$$

(2.2.2-7)

Conclusions:
A big difference between $d$- and $q$-axis inductance is needed to generate sufficient torque (typically $L_d/L_q \sim 5$). Current component $I_d$ may be taken as magnetizing current, exciting the
“main” flux in d-direction, whereas perpendicular current component \( I_q \) may be taken as torque-delivering current.

If synchronous reluctance machine is fed from inverter with field oriented torque and current control, \( d \)- and \( q \)-current may be controlled independently, using equ. (2.2.2-7). When fed from grid with constant voltage amplitude, current is not available for control. From Fig.2.2.2-1 we take for \( R_s = 0 \), considering positive angle \( \theta \) in mathematical positive direction (counter-clockwise direction), being counted from stator voltage to \( q \)-axis:

\[
U_s \cos \theta = I_d X_d, \quad -U_s \sin \theta = I_q X_q
\]

Thus load angle is negative for motor mode and positive for generator mode. Substituting this in (2.2.2-7), and using \( \sin 2\theta = 2\sin \theta \cos \theta \), torque equation for impressed voltage is derived:

\[
M_e = -\frac{p \cdot m}{\omega_i} \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin(2\theta)
\]  

(2.2.2-8)

Fig.2.2.2-2: Torque vs. load angle of synchronous reluctance machine and schematic view of rotor position vs. stator field for no-load (\( d \)-position: \( \theta = 0 \)), maximum torque (pull out torque) in generator (\( \theta = 45^\circ \)) and motor mode (\( \theta = -45^\circ \)) and unstable operation at \( q \)-position no-load (\( \theta = 90^\circ \))
Conclusions:
At no-load load angle is zero (\( \vartheta = 0^\circ \)). Maximum torque is delivered at (\( \vartheta = \pm 45^\circ \), generator/motor), whereas q-position (\( \vartheta = 90^\circ \), zero torque) is unstable. A slight movement of rotor would snap the rotor into d-position, where flux is maximum. So only for \(-45^\circ \leq \vartheta \leq 45^\circ\) stable operation for impressed stator voltage is possible.

2.2.3 Operation of synchronous reluctance machine at constant voltage and frequency

a) Phasor diagram and reluctance circle diagram (\( R_s = 0 \)):
Line operated synchronous reluctance machine operates at constant voltage and frequency. Neglecting resistance and iron losses, voltage and torque equation (2.2.2-4), (2.2.2-7) have to be used. In Fig.2.2.3-1 phasor diagram for generator and motor mode is shown with voltage phasor \( U_s \) put in real axis. Note that d-axis current (magnetizing current) is always directed in positive d-axis \( I_d > 0 \). It is necessary to excite the magnetic air gap field of d-axis, which in PM machine is done by the rotor magnets. In generator mode q-axis is leading the voltage phasor, thus defining positive load angle \( \vartheta \). In that case q-axis current is directed in negative q-axis \( I_q < 0 \), yielding negative (braking) torque according to (2.2.2-7). In motor mode q-axis is lagging the voltage phasor, thus defining negative load angle \( \vartheta \). In that case q-axis current is directed in positive q-axis \( I_q > 0 \), yielding positive (driving) torque according to (2.2.2-7). The phase shift between voltage and current \( \varphi \) is always lagging, as machine is needing inductive reactive power to be magnetized. The power factor \( \cos \varphi \) is positive in motor mode, thus showing a positive electric power. That means, machine is consuming electric power, converting it into mechanical power as a motor. The power factor \( \cos \varphi \) is negative in generator mode, yielding negative electric power. That means, machine is delivering power as a generator.

![Phasor diagram](image)

Fig.2.2.3-1: Simplified phasor diagram for neglected stator resistance: left: motor, right: generator

<table>
<thead>
<tr>
<th></th>
<th>Motor</th>
<th>Generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load angle ( \vartheta )</td>
<td>(&lt; 0)</td>
<td>(&gt; 0)</td>
</tr>
<tr>
<td>Phase shift ( \varphi )</td>
<td>(0 \ldots 90^\circ)</td>
<td>(90^\circ \ldots 180^\circ)</td>
</tr>
<tr>
<td>d-current</td>
<td>(&gt; 0)</td>
<td>(&gt; 0)</td>
</tr>
<tr>
<td>q-current</td>
<td>(&gt; 0)</td>
<td>(&lt; 0)</td>
</tr>
<tr>
<td>Electric power</td>
<td>(&gt; 0)</td>
<td>(&lt; 0)</td>
</tr>
<tr>
<td>Torque and mechanical power</td>
<td>(&gt; 0)</td>
<td>(&lt; 0)</td>
</tr>
</tbody>
</table>

Table 2.2.3-1: Basic electric quantities of synchronous reluctance machine
We ask: How does current input of synchronous reluctance change with changing load? The answer is: The current phasor locus for changing load (torque or load angle) is a circle, the so-called reluctance circle diagram (Fig.2.2.3-2).

**Proof:**

With voltage phasor put in real axis we get: \( U_s = U_s \)

Then we get for \( q \)- and \( d \)-current: \( I_q = I_q \cdot e^{j\vartheta} \), \( I_d = -jI_d \cdot e^{j\vartheta} \) or

\[
I_q = I_q \cdot (\cos \vartheta + j\sin \vartheta), \quad I_d = I_d \cdot (\sin \vartheta - j\cos \vartheta)
\]

We decompose stator phasor into real and imaginary part: \( I_s = I_d + jI_q = I_{s,\text{Re}} + j \cdot I_{s,\text{Im}} \)

Thus we get: \( I_{s,\text{Re}} = I_q \cos \vartheta + I_d \sin \vartheta \), \( I_{s,\text{Im}} = I_q \sin \vartheta - I_d \cos \vartheta \)

From voltage phasor diagram we see: \( U_s \cos \vartheta = X_d I_d \), \( U_s \sin \vartheta = -X_q I_q \). Using this for current components we finally get (with \( 2\sin x \cos x = \sin 2x \), \( \sin^2 x = (\cos 2x - 1)/2 \), \( \cos^2 x = (\cos 2x + 1)/2 \)):

\[
I_{s,\text{Re}} = -\frac{U_s}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin 2\vartheta, \quad I_{s,\text{Im}} = -\frac{U_s}{2} \left( \frac{1}{X_q} + \frac{1}{X_d} \right) + \frac{U_s}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \cos 2\vartheta
\]

Comparing these two components with the circle in Fig.2.2.3-2 we understand, that the centre point of circle \( M \) is lying on negative imaginary axis at \( -\frac{U_s}{2} \left( \frac{1}{X_q} + \frac{1}{X_d} \right) \) and that circle radius is \( \frac{U_s}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \).

Note that real part of current yields input power, so again torque may be derived from circle diagram by
Motor development

\[
P_e = 3U_s I_{s,Re} = \Omega_m M_e = P_m \quad \Rightarrow \quad M_e = -\frac{p}{\omega_s} \cdot \frac{U_s^2}{2} \cdot \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin(2\vartheta)
\]

which is identical with (2.2.2-8). Thus real part of current is directly proportional to torque.

b) Reluctance circle diagram at \( R_s > 0 \):

Usually synchronous reluctance machines are built only for small power as cheap motors, because for larger machines the big amount of magnetizing current is not very economical. With small machines (typically 100 W ... 1 kW) influence of stator resistance usually may not be neglected. Of course, for detailed knowledge also iron, friction and additional losses have to be considered, but are here omitted for simplicity. Using instead of (2.2.2-4) now

\[
U_s = jX_d I_d + jX_q I_q + R_s (I_d + I_q)
\]

again phasor diagram and reluctance circle diagram can be derived. Without proof we note that the centre of circle \( M \) is shifted upwards. So real part of current not only represents torque, but also copper losses in stator resistance. Therefore motor pull out torque is **reduced**, as electrical input power is partially consumed by copper losses, before the remaining part is converted into mechanical power. In generator mode (braking) pull out torque is increased, thus increasing mechanical torque demand to drive machine. Increased mechanical input power is now necessary also for feeding copper losses. Instead of (2.2.2-8) we get a rather complicated expression for torque, which for \( R_s = 0 \) is of course identical with (2.2.2-8):

\[
M_e = -\frac{p \cdot m}{\omega_s} \cdot \frac{U_s^2}{2(R_s^2 + X_d X_q)} \left( R_s \cdot (X_d - X_q) + \sqrt{(R_s^2 + X_d^2)(R_s^2 + X_q^2)} \cdot \sin(2\vartheta - 2\alpha) \right)
\]

\[
2\alpha = \arctan \left( \frac{R_s (X_d + X_q)}{X_d X_q - R_s^2} \right) > 0
\]

**Example 2.2.3-1:**

Shaft height 112 mm, pole count \( 2p = 4 \), rated voltage and current: \( U_N = 380 \text{ V}, Y, I_N = 9 \text{ A}, \)

50 Hz, rated impedance \( Z_N = U_N/\sqrt{3} I_N = 24.4 \text{ } \Omega \); warm resistance: \( R_s/Z_N = 5\% \), \( X_d/Z_N = 165\% \), \( X_q/Z_N = 33\% \)

Thus we get: \( 2\alpha = 10.3^\circ \), \( 1 - \varepsilon = R_s^2 / (X_d X_q) = 0.995 \)

Due to the losses in stator resistance not only motor pull out torque is decreased, but also the motor pull out load angle is not any longer \(-45^\circ \), but \(-45^\circ + 10.3^\circ / 2 = -39.9^\circ \).

2.2.4 Stator flux linkage, Saturation of iron

In spite of this elaborate equation (2.2.3-2) the truth is, that economically designed motors have saturated iron parts. So assumption of constant reactances \( X_d, X_q \) is in reality not true, and for good calculation of machine saturation has to be considered. So theory of circle diagram is only good for basic understanding of machine performance, but NOT for telling the truth on real motor performance of saturated machines.
a) NO saturation considered:
If no saturation is considered, \(d\)- and \(q\)-inductivities differ mainly due to different air gaps in pole region and gap region (Fig. 2.2.4-1). So, if we consider always constant current \(I_s\) in stator winding, but with changing angle \(\beta\) with respect to \(d\)-axis, the flux linkage components are

\[
\Psi_d / \sqrt{2} = L_d I_d = L_d I_s \cos \beta, \quad \Psi_q / \sqrt{2} = L_q I_q = L_q I_s \sin \beta
\] (2.2.4-1)

Thus the locus of flux linkage \(\Psi_s\) (and thus amplitude of fundamental of air gap flux density \(B_0\)) for varying current phasor \(I_s\) on a circle is an ellipse.

\[
\left( \frac{\Psi_d}{\sqrt{2} \cdot L_d I_s} \right)^2 + \left( \frac{\Psi_q}{\sqrt{2} \cdot L_q I_s} \right)^2 = 1
\] (2.2.4-2)
Fig. 2.2.4-1: Locus of flux linkage for constant, but phase shifted stator current for unsaturated iron is an ellipse

b) *WITH iron saturation considered:*

![Diagram](https://via.placeholder.com/150)

Fig. 2.2.4-2: Comparison of d- and q-axis flux linkage: a) **Unsaturated iron:** Different inclination of flux linkage increase is defined by different (small and large) air gap in d- and q-axis. b) **Iron saturation** causes non-linear increase of d-axis flux linkage. As q-axis flux linkage is much smaller, iron flux density is so low, that no saturation occurs within range of rated current.

Calculating saturated flux linkage for different phase shifts $\beta$ and given current amplitude is done usually numerically, as shown in Fig. 2.2.1-2. Stator winding is loaded in the three phases with current in such a way, that for phase shift $\beta = 0^\circ$ the excited air gap flux lines are directed in d-axis, and for $\beta = 90^\circ$ they are in q-axis (across the pole gap). For other angles $\beta$ the direction of flux lines is in between. For each of these chosen angles the air gap flux density distribution $B_{\beta}(x)$ is calculated for given current. By *Fourier* analysis the fundamental amplitude $B_{\delta 1}$ is derived. With pole pitch, stack length, number of turns per phase of stator winding and winding factor of fundamental harmonic we get air gap flux linkage

$$
\Psi_h = N_s k_{w1} \cdot \frac{2}{\pi} \tau_p l_{Fe} B_{\delta 1}
$$

(2.2.4-3)

Varying the current $I_s$, we get characteristics like those in Fig. 2.2.4-3 (taken for the geometry of the motor of Fig. 2.2.1-1a). So for each point of operation of reluctance machine (Fig. 2.2.2-1) a certain current amplitude and angle $\beta$ is given. Then taking the value of the flux linkage from the map of characteristics Fig. 2.2.4-3 (with interpolation for angles in between the numerically calculated ones) the induced voltage is calculated, thus completing the phasor diagram. Please note, the it is NOT sufficient to take only the flux linkage characteristic for d- and q-axis by decomposing the current according $I_d = I_s \cos \beta$, $I_q = I_s \sin \beta$. If one would take these two current components, calculating with them the corresponding d- and q-flux
linkage from the two numerically derived characteristics, and composing with them the total flux linkage, one would grossly over-estimate the flux linkage. This is shown by the thus derived flux linkage locus curves Fig.2.2.4-4b. The real curves Fig.2.2.4-4a, calculated with the accurate flux linkage curves of Fig.2.2.4-3, are resembling nearly ellipses (which are depicted as dotted lines in Fig.2.2.4-4) and yield a much smaller flux for angles $\beta$ between $d$- and $q$-axis. For small currents no saturation occurs and curves a and b close up to ideal ellipses, as is shown in Fig.2.2.4-4 for 50% rated current.

Fig.2.2.4-3: Numerically calculated flux linkage characteristics at different current angle $\beta$, rotor of Fig.2.2.1-1a

Fig.2.2.4-4: Locus of air gap flux density amplitude $B_\delta$, which is proportional to flux linkage, for different stator current (50%, 90%, 330% of rated current) for different current angle $\beta$. Curves (a) are derived with the flux linkage characteristic of the "real" angle $\beta$, whereas curves (b) are derived only with the characteristics of $d$- and $q$-axis, which yields wrong results except for $d$- and $q$-axis. (dotted line -----: ellipses).

**Example 2.2.4-1:**

Four pole synchronous reluctance machine, rotor of Fig.2.2.1-1a:
Comparison of measured and calculated torque, electrical power, power factor, current, efficiency (Fig.2.2.4-5) for constant stator voltage 380 V, 50 Hz and motor and generator mode. Calculation is done using the flux linkage characteristics of Fig.2.2.4-3.

The corresponding locus of current phasor for constant voltage and frequency is depicted in Fig.2.2.4-6. Due to saturation it is **no longer a circle**. Good coincidence between calculation and measurement is only given up to rated load.
**Conclusions:**

Saturation and two-dimensional flux density distribution has to be taken into account for reliable calculation results, so usually numerical field calculation is needed for calculating reluctance machines. Due magnetic coupling of d- and q-axis by common flux in stator yoke – which adds to total saturation – it is necessary to calculate the total flux linkage for each current phasor. Considering d- and q-flux independently would not take into account this additional saturation effect in stator yoke, thus yielding too big flux for positions between d- and q-axis.

### 2.2.5 Synchronous reluctance machine performance and application

a) **Features and performance:**

Due to low power factor (e.g. below 0.6 in Example 2.2.4-1) synchronous reluctance motors are only built for smaller power range typically below 5 kW at 1500/min. The rotor iron sheets are often taken from induction motor series any are punched out to get the gaps. Thus the small pole air gap of induction machines can be used to get big d-inductance, whereas for small q-inductance the parameter ratios

- Gap width / pole pitch,
- Gap depth / Gap width
have to be optimized to get a big ratio $X_d/X_q$. This is done often empirically for small motors. **Pull out torque** must be at least 1.35-faches $M_N$ to satisfy international standard IEC 60034-1. The rotor slots of the motor are used to form a cage for asynchronous line-starting of the machine. Often the rotor gaps are filled with aluminium for die-cast aluminium cage reluctance rotors to increase asynchronous **starting torque at slip** $s = 1$. Being operated from the grid with impressed stator voltage system, rotor may start speed slow oscillations at load steps like any synchronous machine, which is operated with independent voltage system. Rotor cage in that case acts as a **damper** (amortisseur winding), where currents are induced during these oscillations, causing an additional braking torque, which damps the oscillations rather quickly.

Good **synchronizing** must be achieved by big ratio $X_d/X_q$ to ensure, that after asynchronous starting the rotor is pulled from slip $s > 0$ into synchronism $s = 0$ also, when the motor is already loaded. The **product of rated efficiency and power factor** $\eta \cdot \cos \varphi$ is shown in Fig.2.2.5-1 in comparison with line-operated induction motors and is lower due to the increased demand of magnetizing current, caused by the larger air gap of the pole gaps.

![Graph showing product of measured rated efficiency and power factor for different motors](image)

**Example 2.2.5-1:**
Comparison of 4-pole induction and synchronous reluctance machine, identical stator (shaft height 112 mm, 36 stator slots), totally enclosed, fan cooled (TEFC), 380 V Y, 9 A, 50 Hz

<table>
<thead>
<tr>
<th></th>
<th><em>Induction motor</em></th>
<th><em>Reluctance motor</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output power</td>
<td>4 kW</td>
<td>2.2 kW</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.83</td>
<td>0.46</td>
</tr>
<tr>
<td>Efficiency</td>
<td>84 %</td>
<td>78 %</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>1447 /min</td>
<td>1500 /min</td>
</tr>
</tbody>
</table>

**Table 2.2.5-1:** Comparison of 4-pole induction and synchronous reluctance machine with identical stator

**Conclusions:**
**Steady state torque per volume for a certain temperature rise of stator winding is by 30% to 50% lower for synchronous reluctance machines, when compared with induction machines.**
2) Inverter operation:
Usually synchronous reluctance machines are operated with inverter without any speed control due to synchronous capability of motor. Thus again rotor oscillations may happen, which have to be damped by rotor cage. With changing speed and frequency it may happen, that at certain speed the cage damping is much weaker than at other speed levels. So some rules were found to ensure stable non-oscillating operation for a wide speed range:
- reduction of ratio $\frac{X_d}{X_q}$ (which of course is decreasing motor utilization),
- aim a ratio of rotor cage resistance in $d$- and $q$-axis of about $R_{rd}/R_{rq} = 0.5$,
- reduce rotor cage $d$-axis resistance $R_{rd}$,
- increase stator and rotor $d$-axis stray inductance $L_{sd}, L_{r\sigma d}$
- decrease stator resistance and rotor $q$-axis stray inductance $R_s, L_{r\sigma q}$

Often these oscillations occur for machines with rated frequency 50 Hz at lower stator frequencies below 30 Hz. By increasing the ratio of $\frac{\Phi}{s} / f_s$, which means increase of flux and thus saturation, the $d$-axis reactance is saturated and therefore the ratio $X_d/X_q$ is reduced, as mentioned above. Usually the oscillations vanish.

c) Applications:
Synchronous reluctance machines are special drives, which are used often in textile industry, where e.g. acryl threads for synthetic textiles are manufactured. Many threads are taken from one big lump of molten acryl mass and are cooled and wound up synchronously by synchronous machines. Often more than 100 machines are working in parallel. So often cheap synchronous reluctance machines are used, fed from one main inverter at variable speed.

The synchronous reluctance machine is a "fail-silent" machine. In case of failure it can be switched off without further harming the system, in contrary to PM synchronous machines, where switched off, but still turning machines induce back EMF, causing losses.

Conclusions:
The synchronous reluctance machine is a special cheap, low power machine for special purposes, where synchronous speed is necessary.

2.2.6 Asynchronous starting of reluctance machines

a) Asynchronous starting torque:
Due to the rotor gaps the asynchronous starting torque is not constant for a certain slip. The stator air gap flux is rotating faster with the speed difference

$$\Delta n = n_{syn} - n = n_{syn} - (1 - s) \cdot n_{syn} = s \cdot n_{syn} = s \cdot f_s / p \quad \text{(2.2.6-1)}$$

Therefore the asynchronous torque is changing twice its value, when the stator wave passes on rotor pole pair, as two rotor gaps are passed, where the flux and thus the torque are decreased. So asynchronous torque during start up has an average value and a component, pulsating with double slip frequency $2s \cdot f_s$.

b) Asynchronous reluctance torque:
We assume for simplicity that the inverse air gap is varying along rotor co-ordinate $x_r$ sinusoidal:
yielding air gap in \( d \)-axis \( \delta_d = \frac{\delta_0\delta_1}{\delta_0 + \delta_1} \) and in \( q \)-axis \( \delta_q = \frac{\delta_0\delta_1}{\delta_0 - \delta_1} \). This “air gap distribution” is moving synchronously with rotor speed \( n \), thus with respect to stator co-ordinate \( x_s \) we note

\[
x_s = x_r + v \cdot t = x_r + 2p \tau_p \cdot n \cdot t = x_r + (1 - s) \cdot 2f_s \tau_p \cdot t.
\]

(2.2.6-3)

The stator air gap flux density wave, excited by sinusoidal distributed stator ampere turns with amplitude \( V_s = \frac{m}{np} \cdot N_s k_w \sqrt{2} I_s \),

\[
V_s(x_s, t) = V_s \cdot \cos \left( \frac{x_s \pi}{\tau_p} - \omega_s t \right)
\]

(2.2.6-4)

is modulated by air gap. With abbreviation \( \gamma = x_s \cdot \pi / \tau_p \) we get

\[
B_{\delta}(\gamma, t) = \mu_0 V_s(\gamma, t) / \delta(\gamma, t) = \mu_0 V_s \cos(\gamma - \omega_s t) \cdot \left[ \frac{1}{\delta_0} + \frac{1}{\delta_1} \cdot \cos(2\gamma - 2(1 - s) \omega_s t) \right].
\]

(2.2.6-5)

By using \( \cos \alpha \cos \beta = (\cos(\alpha + \beta) + \cos(\alpha - \beta)) / 2 \), three field components are distinguished by modulation:

- the stator fundamental with average air gap: \( B_{s,1} = (\mu_0 V_s / \delta_0) \cos(\gamma - \omega_s t) \)

- a field of same pole number, but different frequency: \( B_{s,3} = (\mu_0 V_s / 2\delta_1) \cos(\gamma - (1 - 2s) \omega_s t) \)

- a field of 3-times pole number and different frequency: \( (\mu_0 V_s / 2\delta_1) \cos(3\gamma - (3 - 2s) \omega_s t) \).

The three-fold pole pair field induces in the three phase windings voltage, which are IN phase. So, in three phase star connected stator winding systems with insulated star point, no currents can be driven by these voltages. Hence the influence of this wave is neglected. The second field induces in the stator windings a voltage system \( U_3 \) with frequency \( f_3 = (1 - 2s)f_s \), which causes an additional (small) stator current \( I_3 \). The corresponding sinusoidal distributed current ampere turns \( V_3 = \frac{m}{np} \cdot N_s k_w \sqrt{2} I_3 \) excite an additional air gap field \( B_{s3}(\gamma, t) = \mu_0 V_3(\gamma, t) / \delta(\gamma, t) = B_{s,1} \cos(\gamma - \omega_s t) + B_{s,2} \cos(\gamma - (1 - 2s) \omega_s t) + \ldots \),

which again contains – with the same calculation as above - a field with three-fold pole pair (neglected here) and two field components \( B_{s,1}, B_{s,2} \).

The two waves \( B_{s,1}, B_{s,3} \) have the same wave length and frequency, thus the same velocity, so they produce a constant torque \( M_1 = B_{s,1} \cdot B_{s,3} \sim I_s \cdot I_3 \). The same holds true for \( B_{s,2} \), \( B_{s,3} \), again resulting in a constant torque \( M_2 = B_{s,2} \cdot B_{s,3} \sim I_s \cdot I_3 \). Therefore the constant part of asynchronous reluctance torque is formed by the sum of these to components:

\[
M_a = M_1 + M_2 \sim I_s I_3.
\]

(2.2.6-6)
On the other hand, the two waves $B_{s,1}, B_{3,2}$ have the same wave length, but not the same frequency and velocity, so they produce a pulsating torque $M_{1p} \sim B_{s,1} \cdot B_{3,2} \sim I_s \cdot I_3$. Difference of frequencies is $f_s - (1-2s) f_s = 2s \cdot f_s$, so frequency of pulsating torque is again double slip frequency. The same holds true for $B_{s,2}, B_{3,1}$, again resulting in a pulsating torque $M_{2p} \sim B_{s,2} \cdot B_{3,1} \sim I_s \cdot I_3$, and also for $M_{3p} \sim B_{s,1} \cdot B_{s,3} \sim I_s^2$, $M_{4p} \sim B_{s,1} \cdot B_{s,2} \sim I_3^2$, all pulsating with double slip frequency. So the pulsating torque amplitude (Fig.2.2.6-2) is constituted of four parts

$$M_p = M_{1p} + M_{2p} + M_{3p} + M_{4p} \sim I_s I_3, I_s^2, I_3^2.$$

(2.2.6-7)

Please note that at half synchronous speed

$$n = n_{syn}/2 \iff s = 0.5$$

(2.2.6-8)

the frequency $(1-2s) f_s$ is zero, so in that special point no current $I_3$ is induced in stator: $I_3 = 0$. At that special point the asynchronous reluctance torque $M_a$ vanishes and the pulsating torque has a minimum, now depending only on $I_3^2$ (Goerges-phenomenon, Fig.2.2.6-1b).

$$M_a(s = 0.5) = 0, \quad M_p = \text{Min}.$$  

(2.2.6-9)

The current $i_3(t) = \hat{I}_3 \cdot \sin((1-2s) \omega_s t)$ changes also sign at $s = 0.5$, therefore the real power flow of this current is also changing direction, being motor at $n < n_{syn}/2$ and generator at $n > n_{syn}/2$. Therefore the asynchronous reluctance torque is a driving (positive) torque for $n < n_{syn}/2$ and a braking (negative) torque for $n > n_{syn}/2$.

**Example 2.2.6-1:**

Asynchronous operation of a small synchronous reluctance torque without rotor cage in order to measure the asynchronous reluctance torque. The motor was driven by coupled external DC machine asynchronously, while stator was fed by the grid with 50 Hz constant voltage. Stator current $I_s$ ranges between 0.35 A and 0.4 A, whereas additional current $I_3$ is smaller with about 0.18 A and zero at half synchronous speed (Fig.2.2.6-1a). Pulsating torque is much bigger as asynchronous reluctance torque, as it is composed by four, and not only by to components (Fig.2.2.6-1b).

**Conclusions:**

During asynchronous start up the machine not only produces an asynchronous starting torque, but also an additional asynchronous reluctance torque, which is caused by the difference of d- and q-axis inductance. Both torque components consist of a constant and a pulsating value, causing the machine to vibrate. So asynchronous starting is usually much more noisy than starting of three phase induction motors.

### 2.2.7 Special rotor designs for increased ratio $X_d/X_q$

By using flux barriers in the rotor the value of $X_q$ may be reduced further without influencing $X_d$ substantially (Fig.2.2.7-1). By increasing the ratio $X_d/X_q$ the locus of stator current
Fig. 2.2.6-1: Asynchronous operation of small synchronous reluctance machine: (a) Calculated and measured (dots) stator and additional stator current $I_3$, (b) asynchronous reluctance torque $M_a$ and pulsating torque amplitude $M_p$ (Source: Bausch, Jordan et al, ETZ-A).

Fig. 2.2.6-2: Calculated asynchronous starting: Comparison of induction machines (ASM), Synchronous reluctance machine (SRM), Permanent magnet synchronous machines with rotor cage (PSM, see Chapter 3). Above: speed, below: starting torque of SRM. Pulsating torque with decreasing frequency clearly visible (Source: Bunzel, E., elektrie).

becomes more like a circle. The power factor increases, which reduces the amount of magnetizing current. Therefore the copper losses are reduced, as for the same torque a lower current is needed. Thus efficiency is increased. A lot of patents have been issued on this topic of the “best” rotor configuration. With the rotors of Fig. 2.2.6-1 $X_d/X_q$ could be increased from about 5 to 10. Power factor increased up to 0.7 ... 0.8, thus nearly reaching the value of induction machines and efficiency of 0.85 ... 0.9 was possible. With these values also bigger synchronous reluctance machines with rated power at 1500/min between 20 ... 50 kW and more may be economical solutions, especially with rotor a) of Fig. 2.2.6-1, whereas rotor b) and c) are rather expensive in manufacturing. The magnetic barriers of rotor c), which reach
the rotor surface, act like additional rotor slots. As it is explained in Chapter 4, the slot openings cause a local reduction in magnetic field. This distortion acts like an additional field harmonic, causing pulsating radial magnetic forces, which not only excite stator vibrations, but also acoustic noise.

Fig. 2.2.7-1: Different special rotor designs to increase ratio $X_d/X_q$: a) Flux barrier rotors with punched-out barriers (left: Rated torque/speed 58 Nm / 1500/min, right: 265 Nm / 1500/min (Source: M. Kamper), b) segmented rotor with non-magnetic shaft (P. Lawrenson, S. Gupta), c) axially laminated rotor, containing of different stack sections (Source: F. Taegen, 1990, Archiv f. Elektrotechnik)