5. Inverter-fed induction machines

5.1 Basic performance of variable-speed induction machines

Induction machines operate at fixed stator frequency between no-load (synchronous speed) and rated slip, so speed differs only a few percent ("fixed speed drive"). Cage induction machine with continuously varying speed is possible with frequency inverter, which generates a three-phase voltage system with variable frequency, which yields variable synchronous speed and therefore variable speed induction machine. Most inverters are voltage-source inverters with DC voltage link. (Current source inverters with a big choke in the DC current link are used only for bigger power and are nowadays rather rare in use). Grid input with its fixed frequency and voltage is rectified and smoothened by a DC link capacitor. Most of the smaller inverters operate with diode rectifier, getting as DC link voltage

\[ U_d = (3/\pi) \cdot \sqrt{2} U_{LL} \]  

(5.1-1)

**Example 5.1-1:**

400 V grid, \( U_d = (3/\pi) \cdot \sqrt{2} \cdot 400 = 540 \text{ V} \)

From this DC link voltage a new variable frequency voltage system is generated by pulse width modulation (PWM). Output line-to-line voltage is a series of pulses of \( +U_d \) or \( -U_d \), which may be Fourier-analyzed.

\[ u_{LL}(t) = \sum_{k=1}^{\infty} \hat{U}_{LL,k} \cdot \cos(k \cdot \omega_s t) \]  

(5.1-2)

*Fourier* fundamental line-to-line sinus voltage \( \hat{U}_{LL,k=1} \) and its corresponding phase voltage \( \hat{U}_{s,k=1} \) is \( l \sqrt{3} \) (if motor stator winding is star-connected) is the necessary input for variable speed motor operation. Higher time harmonics \( \hat{U}_{LL,k>1} \) cause additional motor current with higher frequency \( f_k = k f_s \), which cause additional losses, pulsating torque and generate audible magnetic noise. Neglecting these parasitic effects, variable speed motor operation is determined by variable \( U_s = \hat{U}_{s,k=1} / \sqrt{2} \) and variable \( f_s \).

a) Influence of stator resistance \( R_s \) neglected:

Stator voltage equation yields

\[ U_s = j\omega_s L_{sl}\sigma L_s + j\omega_s L_h (L_s + L_r') = j\omega_s \Psi_s / \sqrt{2} \]  

(5.1-3)

so constant flux linkage in the machine at variable frequency is only possible for voltage change proportional to frequency:

\[ U_s \sim f_s \]  

(5.1-4)

Torque-speed characteristic for \( R_s = 0 \) is *Kloss* function, which can also be expressed in terms of rotor frequency \( f_r = s \cdot f_s \):
Breakdown torque $M_b$ occurs at **rotor breakdown frequency**

$$\omega_b = \pm \frac{R_r'}{\alpha X_r'}, \quad \omega_s = \pm \frac{R_r'}{\alpha L_r'} \quad (+: \text{motor, -: generator})$$  \hspace{1cm} (5.1-6)

which is independent of stator frequency. Note that also breakdown torque is independent of stator frequency, if $U_s \sim f_s$. With $U_s = \omega_s \Psi_s / \sqrt{2}$ we get

$$R_s = 0: \quad M_b = \pm \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1 - \sigma}{\alpha X_s} = \pm \frac{m_s}{2} \frac{p}{\alpha L_s} \frac{1 - \sigma}{\Psi_s^2} \quad (+: \text{motor, -: generator})$$ \hspace{1cm} (5.1-7)

Therefore variable stator frequency means shifting of torque-speed characteristic parallel to abscissa with unchanged shape of characteristic (Fig. 5.1-1). Motor operation range is always for each of these characteristics between $0 \leq \omega_r \leq \omega_b$, if maximum torque (breakdown torque) with its overload current is possible due to sufficient inverter current limit.

**Fig. 5.1-1:** Inverter operated induction motor, stator resistance neglected, only voltage fundamental of PWM inverter output voltage pattern considered: Parallel shift of torque-speed characteristic with variable stator frequency $f_s$ and $U_s/f_s = \text{const.}$

With rising frequency $f_s$ stator voltage must rise, too. By broadening the voltage impulses and narrowing the voltage gaps (zero voltage) in between, the fundamental amplitude increases. Thus the **modulation degree**

$$m = \left(2 / \sqrt{3}\right) \cdot \hat{U}_{LL,k=1} / U_d$$ \hspace{1cm} (5.1-8)

rises with increased frequency up to unity, where usually **rated frequency** is defined: $f_s = f_N$. If $m > 1$ (over-modulation), maximum inverter output voltage is reached, if the voltage pulses
of PWM are united in one positive and one negative impulse per period with duration of impulses equal to 1/3 of period each ("six step mode").

\[
\hat{U}_{LL,k=1,max} = \frac{4}{\pi} \cdot U_d \cdot \sin\left(\frac{\pi}{3}\right) = 1.1U_d
\]  

(5.1-9)

Further increase of frequency yields automatically a weakening of flux, as the magnetizing current in the machine decreases (flux weakening operation):

\[
\Psi_s = \hat{U}_{s,max} / \omega_s \sim 1 / \omega_s
\]  

(5.1-10)

As torque is proportional to product of stator flux and rotor current, torque is decreasing in field weakening range even at constant current.

\[
M_e \sim \Psi_s I_r \sim 1 / \omega_s
\]  

(5.1-11)

and motor power \( P = 2\pi m M_e \approx (\omega_s / p) \cdot M_e \) is constant ("constant power range").

Breakdown torque is decreasing at constant voltage operation with

\[
M_b = \frac{m_s}{2} \cdot \frac{p}{\omega_s} U_{s,max}^2 \frac{1 - \sigma}{\sigma \omega_s L_s} \sim 1 / \omega_s^2.
\]  

(5.1-12)

Therefore constant rated power range is limited at that frequency, where decreasing breakdown torque reaches torque demand for rated power \( M_e = P_N \cdot (p / \omega_s) \) (Fig. 5.1-2). In order to keep an overload margin, maximum frequency \( f_{s,max} \) is defined, where ratio of breakdown torque versus torque at rated current is 1.6 (60\% overload margin).

\[
U_s = R_s I_s + j \omega_s L_s \sigma I_s + j \omega_s L_h (I_s + I_r') \bigg|_{\omega \to 0} \to R_s I_s
\]  

(5.1-13)

b) Influence of stator resistance \( R_s \) considered:

Especially at low speed and therefore low stator frequency the inductive voltage drop decreases rapidly, whereas resistive voltage drop remains constant.
At zero frequency (DC current feeding) only resistive voltage drop remains. Therefore voltage control must consider resistive voltage drop by adding this to the $U_s/f_s$-characteristic. If this is not done, then breakdown torque (+: motor, -: generator)

$$M_b = \pm \frac{m_s}{2} \frac{p}{\omega_s^2} U_s^2 \left( \pm \frac{R_s}{\omega_s} + \frac{1}{(1-\sigma)\omega_s^2 L_s} \sqrt{(R_s^2 + \omega_s^2 L_s^2) (R_s^2 + \sigma^2 \omega_s^2 L_s^2)} \right)$$

(5.1-14)

decreases rapidly with decreasing frequency (Fig. 5.1-4a). In order to keep breakdown torque constant, $U_s(\omega_s)$ is a rather complicated function, derived from (5.1-14). In commercial inverters this function is usually linear with an offset $\Delta U = R_s I_N$ at zero speed (Fig. 5.1-4b). For that simplification breakdown torque decreases also with decreasing frequency, but stays above rated torque.
c) Drive technology:

**Inverter switches** are usually **insulated gate bipolar transistors (IGBT)** with free wheeling diodes, operating at voltages between 200 V and 6000 V with switching frequencies up to 20 kHz for lower and up to about 500 Hz for higher voltage rating. For higher voltage and switching power gate turn off thyristors (GTO) and thyristors with auxiliary circuit to switch current off are in use at switching frequencies below 500 Hz. Below 200 V metal oxide silicon field effect transistors (MOSFET) are used as switches with switching frequencies up to 50 kHz.

Induction motors may be operated at inverter supply without any speed sensor (**voltage control**). Speed is not constant ant constant stator frequency, but varies according to load and slip. At low frequency the short rotor time constant, which determines change of rotor flux, is rather small in comparison to voltage period \( T = \frac{1}{f_s} \). Therefore rotor flux may be regarded as constant during one period, thus acting like a synchronous machine rotor. Therefore **speed oscillations** may occur, if load is changed, in the same way as it is the case for synchronous machines. If the rotor is equipped with a speed sensor, **speed control** – incorporated in the inverter - is possible, thus avoiding change of speed with slip and possible speed oscillations at low speed. Speed and underlying torque control allow the inverter-fed induction machine to operate as an adjustable speed drive with high dynamic performance, if a vector control algorithm is used for torque control. Depending on the power of the used microprocessor (e.g. 16 bit or 32 bit), elaborate digital control is possible. Using numerical motor models with observer-assisted control, even the speed sensor may be spared, calculating the actual speed from voltage and current measurement (**speed sensorless control**).

---

**Example 5.1-2:**

Speed controlled inverter-fed induction motor with speed sensor and digital vector control (1 ms cycle time for measuring and calculating) :
Motor rated data: 400 V Y, 24 Nm, 1500/min, DC link voltage 600 V, inverter rated current 40 A, load inertia equal to motor inertia (each 0.011 kg.m²)

Two examples of motor control:

a) **Step in speed set-point value \( n_{set} \) for zero to 10/min**: Actual speed \( n_{act} \) in Fig. 5.1-5a is shown as calculated value from microprocessor. After about 4 ms actual speed reaches set-point speed for first time and with some overshoot and damped oscillation finally after 16 ms actual speed is constant 10/min.
b) Load step from no-load to 10% of rated torque and back to no-load at set-point speed 1500/min: Actual speed (taken from speed sensor signal) dips at increased load and reaches set-point value again after 50 ms. At decreased load a speed over-shoot occurs for about 50 ms (Fig. 5.1-5b).

5.2 Drive characteristics of inverter-fed standard induction motors

a) Thermal steady state performance:
Standard motors are usually balanced for synchronous speed at 50 Hz, but are often capable of about running to twice frequency without additional measures, e.g. motor size 160 mm may be run up to 6000/min. Smaller motors may run with even elevated speed, bigger motors with less, as natural bending frequency of rotor decreases with increased motor size. Shaft mounted fan will produce increased air flow and noise at elevated speed, so often for higher speed operation fans with decreased diameter are used. Rough estimate for change of sound pressure level of fans with speed and fan diameter is

\[ \Delta L_p = 50 \cdot \log\left(\frac{n_1}{n_2}\right) + 70 \cdot \log\left(\frac{d_1}{d_2}\right) \]  

(5.2-1)

Example 5.2-1:
Increase of fan sound pressure level at doubled speed: \( \Delta L_p = 50 \cdot \log(2) = 15 \) dB

Cooling air flow generated by fan rises linear with speed:

\[ \dot{V} \sim \frac{n}{n_N} \sim \frac{f_s}{f_N} \]  

(5.2-2)

Iron losses in stator rise proportional to square of flux linkage, eddy current loss component with square and hysteresis loss component linear with frequency, so an average exponent \( x \) is assumed.

\[ P_{Fe} \sim (\Psi / \Psi_N)^2 \cdot \left(\frac{f_s}{f_N}\right)^x \]  

\[ x \approx 1.8 \]  

(5.2-3)

Friction losses due to bearings rise nearly linear with speed, but are usually much smaller than windage losses due to power consumption of shaft-mounted fan. Fan power consumption rises with cube of speed, so friction and windage losses rise with an exponent \( y = 2.5 \ldots 3 \).

\[ P_{fr+w} \sim \left(\frac{n}{n_N}\right)^y \sim \left(\frac{f_s}{f_N}\right)^y \]  

\[ y \approx 2.5 \ldots 3 \]  

(5.2-4)

Stray load losses (additional losses) due to space harmonic effects rise with speed and current:

\[ P_{adv} \sim \left(\frac{n}{n_N}\right)^z \cdot \left(\frac{I_s}{I_N}\right)^2 \]  

\[ z \approx 1.5 \ldots 2 \]  

(5.2-5)

Considering the thermal power limit of the inverter-fed standard induction machine, we have to consider three frequency ranges (Fig. 5.2-1):
- \( f_s > f_N \): Flux weakening range: Flux linkage decreases with \( \Psi \sim 1/f_s \), so iron losses are nearly constant. Motor is operated at rated current, so copper losses are constant. As motor is cooled better at high speed due to increased air flow, constant rated power operation is no
thermal problem even with increased friction and stray load losses. Upper frequency limit is given, when decreasing breakdown power reaches 160% of motor rated power.

- \( f_{th} < f_s < f_N \): Constant torque range: With decreasing speed at constant current iron losses decrease with square of speed, air flow only linear with speed, so down to \( f_{th} \sim 0.5f_N \) operation with rated flux and current (thus rated torque) is no thermal problem. Power decreases linear with speed.

- \( f_s < f_{th} \): Reduced torque operation: Below about 50% rated speed air flow is so small, that copper losses must be reduced by reduction of stator current. Thus torque \( M_e \) is decreased. Moreover breakdown torque usually decreases also with decreasing speed (Fig. 5.1-4), so with a demanded overload margin of \( M_b/M_e = 1.6 \) a decrease in torque is necessary, likewise.

![Fig. 5.2-1: Standard induction machine with shaft-mounted fan, operated at variable frequency. Steady state thermal torque versus speed and overload capability.](image)

b) Increase of motor power with delta connected winding:

Star connected motor has phase voltage by \( 1/\sqrt{3} \) lower than line-to-line voltage: \( U_s = U_{LL} / \sqrt{3} \). Flux linkage is \( \Psi_{sN} = \dot{U}_s / \omega_s \). Operating motor with constant flux is possible up to angular frequency \( \omega_s = \dot{U}_{s,\text{max}} / \Psi_{sN} = \dot{U}_{LL,\text{max}} / (\Psi_{sN} \cdot \sqrt{3}) \). If the SAME motor is delta connected, line-to-line voltage of inverter is identical with motor phase voltage, so maximum angular frequency for constant flux operation is \( \omega_s = \dot{U}_{s,\text{max}} / \Psi_{sN} = \dot{U}_{LL,\text{max}} / \Psi_{sN} \), thus being increased by factor \( \sqrt{3} = 1.73 \). With constant rated phase current \( I_{sN} \) and rated flux linkage operation with rated torque is therefore possible at an 173% increased speed range. Therefore motor power is raised up to 173% with the same motor (Fig. 5.2-2) due to speed increase at constant flux. At delta connection line current is \( I_L = \sqrt{3} \cdot I_{sN} \), thus 173% of phase current, therefore inverter rating has to be increased by 73%.

Conclusions:

By switching winding from star to delta for the same inverter line-to-line voltage, maximum speed for rated flux is increased by 73% (e.g. 87 Hz instead of 50 Hz). Motor of same size is now capable of 73% more power, because only speed is raised, whereas torque remains constant. But inverter power has increased also, so a bigger inverter with 73% additional rating is necessary for delta operation with rated flux up to e.g. 87 Hz.
Fig. 5.2-2: Increase of constant torque range and motor output power by switching the winding from a) star to b) delta connection

**Example 5.2-1:**
4-pole induction motor, power factor 0.85, efficiency 90%

<table>
<thead>
<tr>
<th>Stator winding connection</th>
<th>Star</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter maximum output voltage line-to-line ( U_{LL,\text{max}} )</td>
<td>400 V</td>
<td>400 V</td>
</tr>
<tr>
<td>Motor maximum phase voltage ( U_{s,\text{max}} )</td>
<td>230 V</td>
<td>400 V</td>
</tr>
<tr>
<td>Motor frequency ( f_s ) at ( U_{s,\text{max}} )</td>
<td>50 Hz</td>
<td>87 Hz</td>
</tr>
<tr>
<td>Motor rated phase current ( I_{sN} )</td>
<td>100 A</td>
<td>100 A</td>
</tr>
<tr>
<td>Motor rated line current ( I_{sN} )</td>
<td>100 A</td>
<td>173 A</td>
</tr>
<tr>
<td>Motor torque</td>
<td>336 Nm</td>
<td>336 Nm</td>
</tr>
<tr>
<td>Motor output power</td>
<td>52.3 kW</td>
<td>90.6 kW</td>
</tr>
<tr>
<td>Inverter power rating</td>
<td>69 kVA</td>
<td>119.4 kVA</td>
</tr>
</tbody>
</table>

Inverter voltage-frequency characteristic has to be changed, when motor is switched from star to delta:

- Star: \( U_{LL} / f_s = 400V / 50Hz = 8V / Hz \)
- Delta: \( U_{LL} / f_s = 400V / 87Hz = 4.6V / Hz \)

c) **Drive applications:**
Inverters supplied standard induction motors are used widely for many industrial purposes such as cranes, cater-pillars, mining, mills, pulp & paper machinery, wire production, extruders, woodcraft, drilling (e.g. off-shore platforms), compressors, fans, pumps.

In **fan and pump drives**, where the fluid flow and the generated pressure is adjusted via speed: \( \dot{V} \sim n, \Delta p \sim n^2 \), power \( P = \dot{V} \Delta p \sim n^3 \) increases with speed, so no flux weakening is possible at maximum speed. But with decreasing speed torque demand decreases with \( M \sim n^2 \), so often voltage-frequency characteristic is chosen \( U_{LL} \sim f_s^2 \). Thus the flux and therefore magnetizing current decrease linear with frequency. As torque is \( M \sim \Psi_s I_s \), also load current \( I_s \) may decrease linear with decreasing frequency, leading to lowered motor losses.
5.3 Features of special induction motors for inverter-operation

a) Features of high-performance inverter-fed induction machines:
High performance inverter-fed induction drives must be able to produce full torque also at stand-still for continuous duty, so an external fan, operating independent from motor speed, is necessary for cooling. A winding temperature sensor e.g. within winding overhang such as positive temperature coefficient sensor (PTC) is often used as thermal protection against overheating. Increased motor utilization is often necessary to get increased power out of a given volume such as for drives for tooling machinery or electric cars. If water-jacket cooling is chosen instead of indirect air cooling, stator winding heat transfer coefficient raises up to 300%. In that case losses may increase up to 300% for the same winding temperature. Motor power may be increased by 40%.

Special attention is paid to low inertia and to overload capability (e.g. 180% for 4 min. in a 10 min. cycle, S6-40%) to get a drive with high dynamic performance. Low inertia is gained by low ratio $d_{si}/l_{Fe}$, but care must be taken not to get too low rotor natural bending frequency, it the rotor gets too "thin". High-performance motors are operated usually with a high resolution speed sensor for positioning purposes at low speed (e.g. 2048 increments per revolution), but must be capable also of high speed (e.g. motor size 100 mm up to 10 000/min, motor size 180 mm up to 6000/min). So speed range for operation ranges typically from 0.01 /min up to 12 000/min. If a speed sensor with sinusoidal signals is used, the 2048 sinus increments may be interpolated by sine function with 2048 interpolation points per period, yielding a resolution of 4 000 000 /rev. Accuracy of positioning is of course lower by at least factor 10, as the sinus increments differ from ideal sine wave.

Especially in tooling machines a wide constant power speed range is necessary, as the necessary cutting power in milling or high speed cutting (HSC) process is demanded independently of speed. With low motor utilization and low stray inductance a high ratio of breakdown torque versus rated torque is possible, yielding to constant power range of $n_{max} : n_N = 5:1$ or $6:1$. With star-delta switching of stator winding constant power range ratios 12:1 up to 16:1 are possible ("wide range motors"). By using an additional planetary gear with two stages, 1:1 and e.g. 1:4 a further extension of constant power range is possible. A 24 V-DC motor operates the gear, which needs about 400 ms to switch the gear at motor stand still form gear ratio 1:1 to gear ratio 1:4.
Drive applications are mainly spindle drives in tooling machines, but also in other motion control applications, mainly production machines with high degree of automation, high dynamic performance and increased accuracy of speed and position control.

b) Induction motor design (Grid-operated vs. inverter-operated induction machines)

Grid-operated machines must start from the line at fixed frequency with slip range from 1 to zero, whereas inverter-operated machines start with variable frequency, being operated only between rated slip and zero slip. So the motor design for grid- and inverter-operated machines is quite different.

<table>
<thead>
<tr>
<th>Aim</th>
<th>air gap</th>
<th>$L_{\sigma}$</th>
<th>$R_{r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big breakdown torque</td>
<td>-</td>
<td>small 1)</td>
<td>-</td>
</tr>
<tr>
<td>Small magnetizing current</td>
<td>small 2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>low starting current</td>
<td>-</td>
<td>big</td>
<td>big</td>
</tr>
<tr>
<td>big starting torque</td>
<td>small</td>
<td>big 3)</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3-1: Choice of equivalent circuit motor parameters for line-operated induction machines

Remarks:
1) many slots, no skew, no deep slots, big slot openings
2) low saturation
3) big current displacement, deep rotor slots, special rotor cage (e.g. Siluminium cage for increased resistance). For good starting performance skewing is necessary to minimize harmonic torque components

Table 5.3-1 shows that even for grid operation the different aims to optimize motor performance lead to contradicting demands for equivalent circuit parameters, so always a compromise is necessary.
Motor development

Inverter-fed induction machines

<table>
<thead>
<tr>
<th>Aim</th>
<th>air gap</th>
<th>$L_\sigma$</th>
<th>$R_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big breakdown torque</td>
<td>-</td>
<td>small $^1$</td>
<td>-</td>
</tr>
<tr>
<td>Small magnetizing current</td>
<td>small $^2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>low additional losses</td>
<td>big $^3$</td>
<td>big $^4$</td>
<td>Small $^5$</td>
</tr>
</tbody>
</table>

Table 5.3-2: Choice of equivalent circuit motor parameters for inverter-operated induction machines

1) many slots, no skew, no deep slots, big slot openings
2) low saturation
3) small current displacement, round or oval rotor slots
4) low field harmonics, small slot openings
5) Stray inductance must limit current harmonics. Avoid skewing to avoid inter-bar currents.

Table 5.3-2 shows that also for inverter-fed induction machines the different aims to optimize motor performance lead to contradicting demands for design parameters, again making a compromise is necessary.

Pole-count for high-speed:
For high speed operation low pole count is aimed to keep stator frequency low for low iron losses, current displacement in stator winding and for limiting inverter switching frequency. High-performance induction machines for inverter-operation are usually 4-pole machines, so frequency at high speed is still low enough. Two-pole machines have only 50% of that frequency at same speed, but are used not so often, as they have some disadvantages:
- Due to big flux per pole stator and rotor yoke must be big, thus increasing motor mass.
- Coil span is – depending on chording – nearly half air gap circumference, so winding overhangs are rather big, demanding space and causing increased copper losses.
- In case of **rotor dynamic eccentricity** due to elastic rotor bending the air gap is smaller on one side than on opposite side, causing an unbalanced magnetic pull on the rotor. With two pole machines this eccentricity leads to distortion of two-pole air gap field, causing a radial pulsating force in direction of minimum air gap with frequency $f = 2 \cdot n$, whereas with $2p > 2$ this frequency is usually $f = n$. So excitation of natural rotor bending frequency $f_b$ occurs with two-pole machines already at a speed $n = f_b / 2$, which may limit speed range.

c) Design criteria for motors with wide field weakening range:
(i) Oversizing of motor:
Due to oversizing the demanded rated torque $M_N$ of the motor is smaller than thermal possible rated torque $M_{N,th}$ of motor. Therefore increase of ratio breakdown torque / rated torque $M_b/M_N$ is given, yielding an increased constant power (= field weakening) range, but at a cost of a bigger motor.

Example 5.3-1:
Motor with rated torque $M_{N,th} = 150 \text{Nm}$ (thermally possible), ratio $M_b/M_{N,th} = 2.5$. Field weakening range:

$$\frac{M_e}{M_{N,th}} = \frac{M_{bN}}{M_{N,th}} \frac{M_{N,th}}{(n/n_N)^2} \Rightarrow \frac{1}{n/n_N} = \frac{2.5}{(n/n_N)^2} \Rightarrow \frac{n_{\text{max}}}{n_N} = 2.5$$

Motor oversized by 30%: $M_N = 150 \text{Nm} \cdot 1.3 = 15 \text{Nm}$.

$$\frac{M_e}{M_{N,th}} = \frac{M_{bN}}{M_{N,th}} \frac{M_{N,th}}{(n/n_N)^2} \Rightarrow \frac{1/1.3}{n/n_N} = \frac{2.5}{(n/n_N)^2} \Rightarrow \frac{n_{\text{max}}}{n_N} = 2.5 \cdot 1.3 = 3.25$$

Field weakening range is increased by 30%.

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(ii) Increase of inverter current rating:
Motor is rewound with less windings per phase: \( N_{sx} = N_s / x \), so rated motor voltage is reduced to \( U_{Nx} = U_N / x \). For the same power and torque current has to be increased by \( I_{Nx} = I_N \cdot x \). This is thermally possible, as current density \( J \) in motor remains the same, as constant slot cross section \( A_Q \) offers increased conductor cross section \( q_{Cu} \) due to decreased number of conductors:

\[
q_{Cu} = \frac{q_{Cu}}{x N_s} \Rightarrow q_{Cu} = \frac{q_{Cu}}{x N_{sx}} \Rightarrow q_{Cu} = x \cdot q_{Cu},
\]

so rated motor voltage is reduced to \( U = \frac{U_{N_N}}{x} \). For the same power and torque current has to be increased by \( x \):

\[
I = \frac{I_N}{x} \Rightarrow I = \frac{I_N}{x} \cdot q_{Cu},
\]

Thermal possible, as current density \( J \) in motor remains the same, as constant slot cross section \( A_Q \) offers increased conductor cross section \( q_{Cu} \) due to decreased number of conductors:

\[
q_{Cu} = \frac{q_{Cu}}{x N_s} \Rightarrow q_{Cu} = \frac{q_{Cu}}{x N_{sx}} \Rightarrow q_{Cu} = x \cdot q_{Cu},
\]

and therefore we get

\[
\frac{U}{I} = \frac{U_{N_N}}{x \cdot I_N \cdot q_{Cu}} = \frac{U_{N_N}}{x \cdot I_N \cdot q_{Cu}} = \frac{U_{N_N}}{x \cdot I_N \cdot q_{Cu}}.
\]

Inverter rated voltage remains \( U_{N_N} \), but due to increased current \( I_N \) inverter power increases by factor \( x \). Due to \( 2 \cdot \frac{U_{N_N}}{x} \) breakdown torque increases by \( 2 \cdot \frac{U_{N_N}}{x} \):

\[
M_b = \frac{m_s}{2} \frac{p}{\omega_s^2} \frac{U_{N_N}^2}{N_s^2} \left( 1 - \frac{\sigma}{\omega_s^2} \right) \Rightarrow M_b = \frac{m_s}{2} \frac{p}{\omega_s^2} \left( \sqrt{3} U_{N_N} \right)^2 \left( 1 - \frac{\sigma}{\omega_s^2} \right) = 3M_{bY},
\]

Field weakening range is extended by factor, but needs now a switch with six connections for the six winding terminals U-X, V-Y, W-Z.

Example 5.3-2:
Motor with rated voltage 400 V is rewound to 300 V: \( x = 4/3 = 1.33 \). Inverter is oversized by 33\%, breakdown torque increases by 1.33\^2 = 77\%. Field weakening range is increased by 77\%.

(iii) Star-delta switching of stator winding:
By switching from Y to D, the maximum phase voltage increases by \( \sqrt{3} \), thus raising the breakdown torque by factor 3:

\[
M_{bY} = \frac{m_s}{2} \frac{p}{\omega_s^2} \frac{U_{Y}^2}{N_s^2} \left( 1 - \frac{\sigma}{\omega_s^2} \right), \quad M_{bD} = \frac{m_s}{2} \frac{p}{\omega_s^2} \left( \sqrt{3} U_{Y} \right)^2 \left( 1 - \frac{\sigma}{\omega_s^2} \right) = 3M_{bY}.
\]

Field weakening range is extended by factor, but needs now a switch with six connections for the six winding terminals U-X, V-Y, W-Z.

Example 5.3-3:
a) 4-pole Y-connected motor with \( N_s = 111 \) turns per phase operates with rated line-to-line voltage 400 V, rated phase current 17 A at rated speed 1500/min and power 9 kW (power factor 0.85, efficiency 0.9). Constant power operation is possible up to 4500/min, where breakdown torque is reached (Fig. 5.3-3). By switching in D-connection, full voltage 400 V is reached at \( \sqrt{3} \cdot 1500 = 2600/min \). Inverter current is kept constant 17 A, so power is kept constant. Motor phase current decreases by \( 17 / \sqrt{3} = 10 \) A, leaving the motor now oversized. But due to increased breakdown torque by factor 3 constant power operation with 9 kW is possible up to \( 3 \cdot 4500 = 13500/min \).

b) For comparison: If the same motor size 4-pole Y-connected motor without Y-D-switching should be used for the same constant power range 9 kW, 13500/min, the motor must be
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equipped with \( N_s = 111/\sqrt{3} = 66 \) turns per phase, yielding an increase in inverter current rating of \( 17 \cdot \sqrt{3} = 30 \) A, which usually is more expensive than the Y-D-switch for 17 A.

Fig. 5.3-3: Field weakening with constant power 9 kW: (a) Y-connected motor with decreased number of turns per phase, (b) Y-D-switching of winding, c) Increase of inverter current rating for b)

(iv) Series-parallel switching of stator winding:

Fig. 5.3-4: Possible motor current, voltage and power rating for the same motor size and torque, but a) series and b) parallel connection of each phase: With parallel winding number of turns is only 50\%, so for the same rated voltage the rated frequency and current is 200\%, thus increasing motor power by factor 2

If e.g. for a four pole motor the number of turns per winding \( N_i \) is split into two halves (one for first and one for second pole pair, each \( N_i/2 \)), one can switch those two halves either in series or in parallel. In parallel the total number of turns per phase is with \( N_i/2 \) only 1/2 of the
Motor development

series value $N_s$, thus breakdown torque increases by factor \((N_s/(N_s/2))^2 = 4\), allowing 4-times larger field weakening range in parallel than in series (Fig. 5.3-5). In parallel switching mode for the SAME inverter current rating the current per parallel winding branch is only 50% of rated current, so motor is oversized in parallel winding field weakening operation by factor 2 (Fig. 5.3-4). As each phase is split into two halves, a series-parallel switch for 12 terminals is needed: U1-X1, U2-X2, V1-Y1, V2-Y2, W1-Z1, W2-Z2.

Fig. 5.3-5: Series-parallel switching for increasing constant power range by factor 4

a) Breakdown power $P_b = 2\pi M_b \sim 1/n$, current, voltage and power for series connected winding
b) Operation with parallel connected winding allows increase of field weakening by factor 4 from 1:2.5 to 1:10

Fig. 5.3-6: Catalogue examples for maximum motor power versus speed at voltage source inverter supply for motor size 160 mm: S1: thermal steady state operation, S3-60%: intermittent operation: 6 minutes overload, 4 minutes stand still, S6-60%: intermittent operation: 6 minutes overload, 4 minutes no load
a) High performance 4-pole induction motor with star-delta switch to enlarge constant power range, rated speed at Y: 500/min, rated frequency 16.7 Hz
b) Standard induction 4-pole motor, D connected winding, rated speed 1460 /min, rated frequency 50 Hz
5.4 Influence of inverter harmonics on motor performance

a) Inverter voltage time harmonics:
Inverter line-to-line output voltage is a series of rectangular shaped voltage impulses of varying duration due to PWM (Fig. 5.4-2). Fourier analysis yields voltage harmonics, depending on modulation degree \( m \). Usually the switching frequency of inverter transistors is fixed \( f_T = \text{const.} \), independent of fundamental frequency \( f_k = f_s \) (asynchronous switching). For bigger power ratings e.g. with GTO-inverters the ratio \( f_T / f_s \) is fixed and an integer odd number, but may be changed for different values, e.g. 9, 15, ... (synchronous switching). In case of asynchronous switching usually switching frequency is high enough \( f_T / f_s \gg 1 \) (e.g. 3 ... 10 kHz), that one can always take \( f_T / f_s \approx \text{integer odd number} \). In Fig. 5.4-2 example for synchronous switching \( f_T / f_s = 12000 \, \text{Hz}/800 \, \text{Hz} = 15 \) is given for different modulation degrees.

Often in inverters upper limit is modulation degree \( m = 1 \), leaving over-modulation as additional voltage margin for current control. At over-modulation PWM converts into six-step voltage mode (compare Fig. 5.4-2a with Fig. 5.4-2c). Phase voltage for star connected stator winding is derived from line-to-line voltage according to Fig. 5.4-1a:

\[
\begin{align*}
  u_{S1}(t) - u_{S2}(t) &= u_{L1-L2}(t) \\
  u_{S2}(t) - u_{S3}(t) &= u_{L2-L3}(t) \\
  u_{S1}(t) + u_{S2}(t) + u_{S3}(t) &= 0
\end{align*}
\]

Thus phase voltage of phase 1 ("U") is

\[
u_{S1}(t) = \frac{2u_{L1-L2}(t) + u_{L2-L3}(t)}{3},
\]

yielding in case of over-modulation a step-like time-function (Fig. 5.4-1b) with amplitude \( 2U_d/3 \).

![Fig. 5.4-1: Star connection of stator winding: a) Phase voltage \( u_s \) and line-to-line voltage \( u_L \), b) Time-function of line-to-line voltage \( u_L \) and phase voltage \( u_s \) in case of six step operation](image)
In case of delta connected winding line-to-line voltage is also phase voltage. Fig. 5.4-1b shows clearly, that in case of star connection the voltage time-function fits better to ideal sine time-function than in case of delta connection.

**Conclusions:**
*Inverter fed machines are usually operated in star connection, as phase voltage time function fits better to ideal sine wave.*

**Frequencies** of voltage harmonics are \( f_{sk} = k \cdot f_s \), and inverter generates three phase voltage system for each time harmonic \( k \).

\[
\begin{align*}
u_{uk}(t) &= \hat{U}_{Sk} \cos(k \cdot \omega_s t) \\
u_{vk}(t) &= \hat{U}_{Sk} \cos(k \cdot (\omega_s t - 2\pi/3)) \\
u_{wk}(t) &= \hat{U}_{Sk} \cos(k \cdot (\omega_s t - 4\pi/3))
\end{align*}
\] (5.4-5)

**Example 5.4-1:**
- \( k = 1 \): phase sequence U, V, W: *clockwise* rotating air gap field 
- \( u_{U1}(t) = \hat{U}_{S1} \cos(\omega_s t) \quad u_{V1}(t) = \hat{U}_{S1} \cos(\omega_s t - 2\pi/3) \quad u_{W1}(t) = \hat{U}_{S1} \cos(\omega_s t - 4\pi/3) \)
- \( k = 5 \): phase sequence U, V, W: clockwise rotating air gap field

\[
\begin{align*}
u_{U5}(t) &= \hat{U}_{S5} \cos(5\omega_s t) \\
u_{V5}(t) &= \hat{U}_{S5} \cos(5\omega_s t - 10\pi/3) \\
u_{W5}(t) &= \hat{U}_{S5} \cos(5\omega_s t - 20\pi/3)
\end{align*}
\]

This can also be written as

\[
\begin{align*}
u_{U5}(t) &= \hat{U}_{S5} \cos(5\omega_s t) \\
u_{V5}(t) &= \hat{U}_{S5} \cos(5\omega_s t - 4\pi/3) \\
u_{W5}(t) &= \hat{U}_{S5} \cos(5\omega_s t - 2\pi/3)
\end{align*}
\]

which means that phase V and W for \( 5^{th} \) harmonic have changed phase shift, yielding:

- \( k = 5 \): phase sequence U, V, W: *counter-clockwise* rotating air gap field.

**Time-harmonic** three-phase voltage system, applied to stator winding as stator voltage \( U_{sk} \) with frequency \( f_{sk} \) causes **time-harmonic** three-phase current system \( I_{sk} \), which excites air gap flux density \( B_{sk} \), which may be expressed as *Fourier* sum of **space-harmonics**

\[
B_{sk}(x_s, t) = \sum_{\nu=1,-5,7,...}^{\infty} B_{sk,v} \cdot \cos \left( \frac{v \cdot \pi x_s}{\tau_p} - k \cdot \omega_s t \right) \quad \text{(5.4-6)}
\]

with wave speed of fundamental space harmonic of \( k^{th} \) current system

\[
v_{syn,k,v=1} = 2 \cdot k f_s \cdot \tau_p \quad . \quad \text{(5.4-7)}
\]

So inverse rotation of air gap field of e.g. \( 5^{th} \) time-harmonic current system is described mathematically correct, if we define the ordinal number \( k \) as negative number \( k = -5 \). Result of *Fourier* analysis of Fig. 5.4-2 is: Line-to-line voltage spectrum contains in case of synchronous switching as **ordinal numbers** \( k \) all odd numbers, which are not dividable by 3 (due to the 3-phase system):

\[
k = 1 + 6g \quad g = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \quad \text{(5.4-8)}
\]

Negative sign means that sequence of phase voltages change their position (U, W, V instead of U, V, W), yielding current system in motor, which excites **inverse rotating** air gap magnetic fields.

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Conclusions:
Do not mix up the formulas $k = 1 + 6g$ for ordinal number of three-phase time-harmonic voltage and current harmonics, and $v = 1 + 6g$ for ordinal number of space-harmonics of air-gap field, excited by three-phase winding system. First is defined by inverter, independent of motor design, second is defined by winding arrangement in motor, independent of inverter design. In both cases negative sign of ordinal number means, that the air gap field is inverse rotating.

Fig. 5.4-2: Line-to-line pulse width modulated voltage in per unit of DC link voltage $U_d$, and its Fourier spectrum for synchronous switching mode $f_s/f_s = 12000\,\text{Hz}/800\,\text{Hz} = 15$ for different modulation degree $m$

a) PWM with $m = 0.5$, b) PWM with $m = 1$, c) over-modulation $m = 5$, which yields six step voltage supply

Fig. 5.4-2 shows that for low modulation degree dominating amplitudes of voltage time-harmonics occur at frequencies $2f_T \pm f_s \approx 2f_T$ (Fig. 5.4-2a), and therefore often $f_p = 2f_T$ is defined as pulse frequency, as number of positive and negative voltage impulses in line-to-
line voltage is given by \( f_p / f_s \) (e.g. Fig. 5.4-2: \( f_T / f_s = 15 \Rightarrow f_p / f_s = 30 \) : 15 positive and 15 negative voltage impulses). At modulation degree near unity dominating amplitudes of voltage time-harmonics occur at switching frequency \( f_T \pm 2f_s \approx f_T \) (Fig. 5.4-2b), whereas with over-modulation (six step mode) voltage harmonics decrease with inverse of ordinal number (Fig. 5.4-2c).

**Conclusions:**

At medium modulation degree twice switching frequency voltage harmonics ("pulse-frequent" voltage ripple) is dominating as harmonic voltage content (Fig. 5.4-3).

**Example 5.4-2:**

*Fourier* voltage harmonics in six-step mode:

a) Line-to-line voltage:

\[
\begin{align*}
U_{LL}(t) &= \sum_{k=1}^{\infty} \hat{U}_{LL,k} \cdot \cos(k \cdot \omega_s t) \\
\hat{U}_{LL,k} &= \frac{4U_d}{\pi} \cdot \frac{\sin(|k| \cdot \pi / 3)}{|k|} = \frac{2U_d}{k \cdot \pi} \cdot \sqrt{3} \quad k = 1, -5, 7, -11, 13, -17, 19, ...
\end{align*}
\]

(5.4-9)

b) For deriving phase voltage, one has to evaluate eq. (5.4-4) for each harmonic. If we take time co-ordinate so that like in (5.4-9) fundamental voltage amplitude occurs at \( t = 0 \), we get:

\[
\begin{align*}
U_S(t) &= \sum_{k=1}^{\infty} \hat{U}_{S,k} \cdot \cos(k \cdot \omega_s t) \\
\hat{U}_{S,k} &= \frac{2U_d}{\pi} \cdot \frac{\sin(|k| \cdot \pi / 2)}{|k|} = \frac{2U_d}{k \cdot \pi} (-1)^{g} \quad k = 1 + 6g = 1, -5, 7, -11, 13, -17, 19, ...
\end{align*}
\]

(5.4-10)
**Conclusions:**

Phase voltage amplitudes are \( \hat{U}_{S,k} = \hat{U}_{LL,k} \sqrt{3} \) in star-connected stator winding and \( \hat{U}_{S,k} = \hat{U}_{LL,k} \) in delta connected stator winding.

**b) Current time harmonics and additional motor losses due to inverter supply:**

Applying harmonic phase voltage as stator voltage \( U_{sk} \) to stator winding yields a stator harmonic current \( I_{sk} \), which excites a step-like air gap field, that can be written as Fourier sum of field space harmonics (5.4-6). As \( I_{sk} \) for \( |k| > 1 \) is usually much smaller than \( I_{s,k=1} = I_{s,1} \), for higher time-harmonics only the fundamental of excited air gap field is considered \( B_{s\delta k,v=1} \), whereas higher space harmonics \( B_{s\delta k,|v|>1} \) are neglected. Slip of the fundamental field wave of \( k^{th} \) current harmonic is according to (5.4-7)

\[
s_k = \frac{n_{syn,k} - n}{n_{syn,k}} = \frac{n_{syn} \cdot k - n}{n_{syn} \cdot k} = 1 - \frac{1}{k} \cdot (1 - s)
\]

(5.4-11)

inducing rotor cage with rotor harmonic frequency

\[
f_{rk} = s_k \cdot f_{sk} = s_k \cdot |k| \cdot f_s
\]

(5.4-12)

As only fundamental field wave is considered, we can use fundamental wave equivalent circuit of induction machine for deriving harmonic current \( I_{sk} \) (Fig. 5.4-4).

**Fig. 5.4-4:** Equivalent circuit for calculating time harmonic current \( I_{sk} \) with fundamental air gap field wave

As \( |k| \geq 5 \), slip is \( s_k \approx 1 \), that means: Due to high frequency space fundamental of \( k^{th} \) harmonic current moves so fast with respect to rotor, that rotor seems to be at stand-still.

Therefore absolute value of \( j|k| \omega_s L_h \) is much bigger than absolute value of \( R'_r / s_k + j|k| \omega_s L'_r \) \( \approx R'_r + j|k| \omega_s L'_r \), so we can simplify: \( I_{s,k} \approx -I'_{r,k} \), getting

\[
s_k \approx 1 \implies I_{s,k} \approx \frac{U_{s,k}}{\sqrt{(R_s + R'_r)^2 + (k \omega_s L'_r)^2}} \approx \frac{U_{s,k}}{|k| \omega_s (L_{s\sigma} + L'_{r\sigma})}
\]

(5.4-13)

**Conclusions:**

Harmonic currents \( I_{s,k} \) are nearly independent from load (= slip \( s \)) and are limited mainly by stray inductance. They occur already at no-load \( s = 0 \) in full extent, causing additional losses

\[
P_{ad,inv} \approx \sum_{|k|>1} \infty \frac{(R_s + R'_r) I_{s,k}^2}{k}
\]

(5.4-14)
Due to high frequency considerable current displacement mainly in rotor conductors will occur, increasing AC bar resistance and additional losses $P_{ad,inv}$ due to inverter supply. Therefore rotor bars are designed especially for inverter supply. Round bars would yield lowest current displacement, but bars are as broad as high, thus leading to narrow rotor teeth with high iron saturation. Therefore **oval shaped slots** with constant width teeth in between are preferred (Fig. 5.4-5). This oval shape is usually only possible if cage is made from die-cast aluminium.

In case of **semi-closed** rotor slot opening (Fig. 5.4-5, slot A) stray inductance is lower than in case of **closed slots** (Fig. 5.4-5 B, C), as the iron bridge in case B, C has a lower magnetic resistance and therefore guides a bigger rotor stray flux. So slots B, C would limit current harmonics stronger. Further, air gap wave will induce in aluminium surface of slot opening additional eddy current, which in case of closed slots with iron stack made of insulated iron sheets is not possible.

Comparing slot B and C, one has to consider that in order to ensure small air gap, the rotor surface is often tooled. Tooling process might bridge the insulation between adjacent iron sheets, leading to a thin layer conducting iron rotor surface, where the air gap field again might induce eddy currents, causing additional rotor losses. In case of slot C, the small additional slot above the iron bridge interrupts the rotor surface and the eddy currents. Moreover it adds an additional by-pass for rotor stray flux, thus increasing rotor stray inductance and limiting stronger stator and rotor current harmonics.

**Fig. 5.4-5:** Typical oval rotor slots for aluminium die cast cages, especially for inverter supplied induction machines: A: semi-closed slot, B: closed slot, C: closed slot with additional gap

**Example 5.4-3:**
Comparison of effect of different rotor slots A and C for three-phase unskewed motor
Data: $2p = 4, 15$ kW, 380 V D, 30 A, air gap 0.45 mm, iron stack length 195 mm, rotor outer diameter 145 mm, slots in stator / rotor: 36 / 28.
Calculation of additional losses in bars numerically with finite elements for inverter operation at $f_s = 50$ Hz and 15 kW, synchronous inverter switching, $f_T = 750$ Hz ($f_T/f_s = 15$), as compared to sinusoidal operation at 50 Hz from grid:

$$P_{ad,r,inv} = Q_r \sum_{|k|>1} R_r \cdot I_{r,k}^2$$

Slot shape A: 190 W, slot shape C: 60 W.

**Conclusions:**
For inverter operation often oval shaped die cast bars are used, where bars with big stray induction reduce additional cage losses, but increased stray inductance also reduces breakdown torque. So often as compromise slot shape B is used.
Increased losses e.g. with slot shape A yield increased temperature in motor, typically in stator winding up to 5 K, which sometimes might lead to slight motor de-rating to stay within thermal limit of applied thermal motor class.

A further method to decrease current harmonics is to raise inverter switching frequency $f_T$. From Chapter 4.2 we know that due to current displacement AC bar resistance increases by ratio $k_{E_{bar}} \equiv \frac{h_{bar}}{d_{E,k}}$ with penetration depth (conductivity $\kappa$, permeability $\mu$ of rotor bar conductor)

$$d_{E,k} = 1/\sqrt{\frac{\mu \cdot \kappa \cdot \pi \cdot f_{rk}}{f_s}} = 1/\sqrt{\frac{\mu \cdot \kappa \cdot \pi \cdot |k| \cdot f_s}} \quad (5.4-15)$$

Losses in bar conductors are therefore for $k^{th}$ harmonic

$$P_{ad,r,inv} = Q_r \cdot R_{r,s} \cdot I_{r,k}^2 = Q_r \cdot k_{R_{rk}} \cdot R_{r,s} \cdot I_{r,k}^2 \equiv Q_r \cdot k_{R_{rk}} \cdot R_{r,s} \cdot \frac{U_{s,k}^2}{(|k| \omega_s (L_{s\sigma} + L_{r\sigma}'))^2} \quad \text{or}$$

$$P_{ad,r,inv} \sim |k| \cdot U_{s,k}^2 / |k|^2 \quad (5.4-16)$$

Increasing switching frequency e.g. at medium modulation degree means increasing of ordinal number $|k| = 2 f_T / f_s$, where dominant voltage harmonic $U_{s,k}$ occurs. With increase of $k$ voltage harmonic is reduced, but current harmonic is reduced much stronger by $U_{s,k}/k$, so losses in rotor bars decrease according to (5.4-16). On the other hand switching losses in power transistors of inverter rise linear with switching frequency, so optimum for minimum system losses (= overall drive efficiency) has to be found.

**Example 5.4-4:**
Standard 2-pole motor, 3 kW, 380 V Y is fed by voltage source inverter with asynchronous switching with rating 8.3 kVA, 12 A rated current, 400 V. Motor is operated at $f_s = 50$ Hz with slip $s = 4.5\%$ at 10 Nm (rated power 3 kW), loading the inverter with 54% of rated inverter current. Motor and inverter efficiency was measured by directly measuring input and output power both of motor and inverter for different inverter switching frequency.

<table>
<thead>
<tr>
<th>grid operation</th>
<th>inverter operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s$ / Hz</td>
<td>50</td>
</tr>
<tr>
<td>$f_T$ / Hz</td>
<td>-</td>
</tr>
<tr>
<td>$f_p = 2 f_T$ / kHz</td>
<td>4.8</td>
</tr>
<tr>
<td>Efficiency motor</td>
<td>81.9 %</td>
</tr>
<tr>
<td>Efficiency inverter</td>
<td>-</td>
</tr>
<tr>
<td>Overall efficiency</td>
<td>81.9 %</td>
</tr>
</tbody>
</table>

**Table 5.4-1:** Measured motor, inverter and overall efficiency of 3 kW motor at different inverter switching frequency

As inverter is only partially loaded, its efficiency is lower than at rated point, which is 97%. This is due to constant load independent losses such as current supply for controller electronics and fan blower.

**Conclusions:**
At $f_p = 9.6$ kHz overall motor efficiency is higher (motor heating is lower), and overall efficiency is the same as at 4.8 kHz. At 19.2 kHz current ripple reduction in motor is already negligible, but inverter losses increase considerably.

In Fig. 5.4-6 and Fig. 5.4-7 the corresponding measured time functions of line-to-line voltage and of motor phase current along with the Fourier spectra are shown for switching frequency 2.4 kHz and 4.8 kHz. Inverter operates at DC link voltage $U_d = 525$ V. Note that oscilloscope...
resolution was not sufficient to display PWM pattern of line-to-line voltage for a whole period appropriate. Only in the zoomed time function the voltage pulse is visible correctly.

<table>
<thead>
<tr>
<th>$f_\sigma$</th>
<th>$U_{LL,k}$</th>
<th>$I_{sk}$</th>
<th>$I_{sk} / I_{sk=1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Hz</td>
<td>372 V</td>
<td>6.2 A</td>
<td>100 %</td>
</tr>
<tr>
<td>4750 Hz</td>
<td>89.5 V</td>
<td>0.18 A</td>
<td>2.9 %</td>
</tr>
<tr>
<td>4850 Hz</td>
<td>91.3 V</td>
<td>0.18 A</td>
<td>2.9 %</td>
</tr>
<tr>
<td>9550 Hz</td>
<td>38.6 V</td>
<td>0.035 A</td>
<td>0.6 %</td>
</tr>
<tr>
<td>9650 Hz</td>
<td>24.8 V</td>
<td>0.027 A</td>
<td>0.4 %</td>
</tr>
</tbody>
</table>

Table 5.4-1: Measured Fourier spectrum of line-to-line voltage and phase motor current at 2.4 kHz inverter switching frequency (see Example 5.4-4)

Note that overall r.m.s. of voltage e.g. at 2.4 kHz switching frequency

$$U_{LL,rms} = \sqrt{\sum_{k=1}^{\infty} U_{LL,k}^2}$$

(5.4-17)

is with $U_{LL,rms} = 403 \text{ V}$ much bigger than r.m.s.-value of fundamental (372 V) due to voltage harmonics. Current harmonics are small, so total current r.m.s. $I_{s,rms} = 6.25 \text{ A}$ is nearly the same as r.m.s. of fundamental 6.2 A. Current is close to sine wave, so current measured peak 9.65 A is nearly the same as $\sqrt{2} \cdot I_{s,k=1} = \sqrt{2} \cdot 6.2 = 9.2 \text{ A}$. From Table 5.4-1 we derive with (5.4-13) total motor stray inductance

$$L_{ss} + L'_{\sigma} = L_{\sigma} = \frac{U_{LL,k}}{\omega_{sk} \cdot I_{sk}} = \frac{89.5 / \sqrt{3}}{2\pi \cdot 4750 \cdot 0.18} = 9.61 \text{ mH}$$

and current amplitude at e.g. 9550 Hz with

$$I_{sk} = \frac{U_{LL,k}}{\omega_{sk} \cdot (L_{ss} + L'_{\sigma})} = \frac{38.6 / \sqrt{3}}{2\pi \cdot 9550 \cdot 0.00961} = 38.6 \text{ mA}$$

which fits well to the measured value 35 mA, proving the value of presented theory.Measured current ripple is the sum of all current Fourier harmonics and is of triangular shape, as it is explained in Chapter 1. In case that voltage impulse width is the same as voltage gap in between, peak-to-peak of current ripple is maximum. Considering in Y-connection the stray inductance of two phases for line-to-line voltage value $U_d$ and frequency of voltage impulses as double switching frequency, we get as maximum peak-to-peak current ripple 2.8 A, which fits well to the measurement Fig. 5.4-6.

$$u = L \cdot \frac{di}{dt} \Rightarrow U_d = 2L_{\sigma} \cdot \frac{\Delta i}{\Delta t}, \quad \Delta t = T_p / 2 = 1/2f_p$$

$$\frac{\Delta i}{4L_{\sigma}f_p} = \frac{525}{4 \cdot 0.00961 \cdot 4800} = 2.8 \text{ A}$$

The 5th and 7th inverter voltage harmonic are rather small at this modulation degree, e.g. 5th voltage line-to-line r.m.s. measured value is 2.9 V at 5 \cdot 50 = 250 Hz, yielding with (5.4-13) harmonic current $I_{s,5} = 2.9/(\sqrt{3} \cdot 2\pi \cdot 250 \cdot 0.00961) = 0.11 \text{ A}$; measured value is 0.09 A.

Note that this motor has been already investigated at sinusoidal voltage supply (Example 4.4.1-5), showing due to rotor slot space harmonics additional stator current harmonics at about 950 Hz and 1150 Hz with r.m.s. current harmonics 0.08 A and 0.05 A, which are also visible in Fig. 5.4-6 and Fig. 5.4-7.

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Fig. 5.4-6: Above: Measured line-to-line voltage PWM inverter output at pulse frequency 4.8 kHz (switching frequency 2.4 kHz) of example 5.4.4 and corresponding Fourier spectrum, showing considerable voltage harmonics at $2f_t \pm f_s$ and $4f_t \pm f_s$. Below: Motor phase current time-function, showing current ripple with frequency $f_r \approx 2f_t = 4.8$ kHz, which is determined in corresponding Fourier spectrum as $2f_t \pm f_s$. Note that current harmonics decrease much stronger with increasing ordinal number, so that at $4f_t \pm f_s$ nearly no current amplitude is visible any longer.
Fig. 5.4-7: As Fig. 5.4-6, but at pulse frequency 9.6 kHz (switching frequency 4.8 kHz). Note that compared with Fig. 5.4-6 due to doubling of switching frequency current ripple is reduced by 50%
c) Inverter-induced torque ripple:
Fundamental air gap field amplitudes $B_{s\delta k, v=1}$ of stator harmonic currents $I_{sl}$ produce with rotor harmonic currents $I_{lk}$ tangential Lorentz-forces, and therefore additional torque components. Space harmonics $|\nu| > 1$ are neglected, as their amplitude is much smaller.

- Stator field $B_{s\delta k, v=1}$ generates with rotor harmonic current $I_{sl}$ of SAME ordinal number $k$ a constant torque $M_{e, kkk}$, which is proportional to product $M_{e, kkk} \sim B_{s\delta k, v=1} \cdot I_{sl}$. As $B_{s\delta k, v=1} \sim I_{sl}$, this torque is $M_{e, kkk} \sim I_{sl} \cdot I_{sl}$. As both amplitudes are small for $|k| > 1$, their product is very small, so these constant torque contributions $M_{e, kkk}$ for $|k| > 1$ are negligible. The torque $M_{e, kkk}$ for $k = 1$ is the well known fundamental torque of Kloss function.

- Stator field $B_{s\delta k, v=1}$, excited by fundamental current $I_{sl} = 1$, generates with rotor harmonic current $I_{sl}$ of DIFFERENT ordinal number $k$ a pulsating torque $M_{e, 1k}$, which is proportional to product $M_{e, 1k} \sim B_{s\delta k, v=1} \cdot I_{sl}$ or due to $B_{s\delta k, v=1} \sim I_{sl}$ it is $M_{e, 1k} \sim I_{sl} \cdot I_{sl}$. Due to rather big fundamental current this torque ripple is not negligible.

Rotor bar current with amplitude $\dot{I}_{rk}$ and frequency $f_{rk} = s_k \cdot f_{sk}$ is distributed in the rotor bars along rotor circumference. Due to phase shift between adjacent bar currents on can describe current distribution along rotor circumference as ideal sinusoidal, so that per rotor circumference element $d\gamma$ we have the rotor current element

$$d\dot{I}_{rk}(x_r, t) = \frac{Q_r}{2p \tau_p} \cdot \dot{I}_{rk} \cdot dx_r \cdot \sin(x_r \frac{\pi}{\tau_p} - s_k \cdot \omega_s t)$$  \hspace{1cm} (5.4-18)

With respect to stator co-ordinate system with $x_r = x_s - (1 - s) \cdot 2f_s \tau_p \cdot t$ and considering harmonic slip $s_k \cdot f_{sk} = (1 - \frac{1}{k}) \cdot (1 - s) \cdot k \cdot f_s$ we get

$$d\dot{I}_{rk}(x_s, t) = \frac{Q_r}{2p \tau_p} \cdot \dot{I}_{rk} \cdot dx_s \cdot \sin(x_s \frac{\pi}{\tau_p} - k \cdot \omega_s t)$$  \hspace{1cm} (5.4-18)

Tangential Lorentz-force $dF_k = d\dot{I}_{rk}(x_s, t) B_{s\delta k}(x_s, t)l_{Fe}$ yields torque $dM_{e, 1k} = dF_k \cdot (d_s / 2)$ and with $d_s / 2 \approx \tau_p / \pi$ torque is the summation of $dM_{e, 1k}$ along rotor circumference

$$M_{e, 1k} = \int dM_{e, 1k} = \frac{p \tau_p l_{Fe}}{\pi} \int_0^{2p \tau_p} \frac{Q_r}{2p \tau_p} \cdot \dot{I}_{rk} \cdot \sin(x_s \frac{\pi}{\tau_p} - k \cdot \omega_s t) \cdot B_{s\delta k} \cdot \cos(x_s \frac{\pi}{\tau_p} - \omega_s t) \cdot dx_s$$  \hspace{1cm} (5.4-19)

$$M_{e, 1k} = \frac{p \tau_p Q_r \dot{I}_{rk} B_{s\delta k} l_{Fe}}{2\pi} \cdot \sin((1 - k) \omega_s t) = \text{sgn}(g) \frac{p \tau_p Q_r \dot{I}_{rk} B_{s\delta k} l_{Fe}}{2\pi} \cdot \sin(6|g| \omega_s t)$$  \hspace{1cm} (5.4-20)

Conclusions:
As $f_{lk} = |1-k|f_s = 6|g|f_s$, the pulsating torque occurs with multiples of six time stator fundamental frequency. Each two harmonics, e.g. $k = -5$ and $k = 7$, contribute with their torque amplitudes e.g. $M_{e, 1-5}$, $M_{e, 1, 7}$ to one resulting pulsating torque with e.g. $6f_s$ as the sum of both torque amplitudes: $M_{e, 6f_s} = M_{e, 1-5} + M_{e, 1, 7}$.
**Proof (here for six step operation):**

Time-harmonic ordinal number: \( k = 1 + 6g > 1, \quad g = \pm 1, \pm 2, \ldots \)

Phase voltage amplitude: 
\[
\hat{U}_{s,k} = \frac{2U_d}{k \cdot \pi} \cdot (-1)^g
\]

Rotor harmonic currents: 
\[
\hat{i}_{rk} = \hat{u}_{rk} = -\frac{m_s N_s k_{ws}}{Q_r / 2} \cdot \frac{\hat{U}_{s,k}}{\|k\omega_s L_{\sigma}}
\]

Torque amplitude: 
\[
\hat{M}_{e,1k} = -\text{sgn}(g) \cdot \frac{p_T r_p Q_s B_{\delta,s} l_{Fe}}{2\pi} \cdot \frac{M_{e,6g[|f_s|} \cdot 1 \cdot 2U_d}{\|k\omega_s L_{\sigma}} \cdot (-1)^g
\]

The sign of \( \frac{\text{sgn}(g)}{k} \cdot (-1)^g \) is for a harmonic pair e.g. \( k = -5, 7 \) always the same, so both torque amplitudes add as the sum to resulting torque amplitude.

**Example 5.4-5:**

Standard 8-pole induction motor, 440 V Y, 60 Hz, 2.6 kW, 5.6 A, 28.4 Nm, 860/min fed by inverter supply at \( f_s = 50 \) Hz from 50 Hz-grid. 380 V, \( U_d = 525 \) V, PWM, switching frequency 2.4 kHz

Motor design data: \( m_s = 3, \frac{Q_s}{Q_l} = 48/44, l_{Fe} = 80 \) mm, \( L_{\sigma} = 19.6 \) mH, two-layer winding, \( q = 2 \), coil chording 5/6, \( k_{ws} = 0.933, N_s = 344 \), stator bore diameter 155 mm

Stator field fundamental: \( B_{\delta,s} = 1 \) T, \( \hat{i}_{rk} = -\frac{m_s N_s k_{ws}}{Q_r / 2} \cdot \frac{\hat{U}_{s,k}}{\|k\omega_s L_{\sigma}}
\]

| \( |k| \) | \( f_k \) | \( \hat{U}_{s,k} \) | \( \hat{i}_{rk} \) | \( \hat{M}_{e,1k} \) | \( \hat{M}_{e,6g[|f_s|} \) | \( f_{ik} \) |
|-----|-----|-----|-----|-----|-----|-----|
| -   | Hz  | V   | A   | Nm | Nm | Hz |
| 5   | 250 | 2.5 | 3.55 | 0.55 | 0.94 | 300 |
| 7   | 350 | 2.5 | 2.54 | 0.39 | 0.28 | 9600 |
| 95  | 4750 | 73.1 | 5.5 | 0.85 | 1.68 | 4800 |
| 97  | 4850 | 74.2 | 5.4 | 0.83 |   |   |
| 191 | 9550 | 31.4 | 1.1 | 0.17 |   |   |
| 193 | 9650 | 20.2 | 0.7 | 0.11 |   |   |

Table 5.4-2: Calculated low frequency torque ripple 300 Hz at PWM is 0.94/28.4 = 3.3 % in air gap

\[
f_k < f_0 : \quad \hat{M}_s \equiv \frac{\hat{M}_{e,6g[|f_s|} \cdot J_L}{J_L + J_M} \quad f_k > f_0 : \quad \hat{M}_s \equiv \frac{\hat{M}_{e,6g[|f_s|} \cdot c}{J_M \cdot (2\pi f_k)^2}
\]  

(5.4-21)

Shaft torque ripple is below torsion resonance determined by ratio of load versus motor inertia (Chapter 1.1), and above resonance by the inverse square of pulsation frequency. For \( f_0 = 200 \) Hz, \( f_s = 10 \) Hz we get: \( 6f_s = 60 \) Hz < 200 Hz, \( 2f_T = 4.8 \) kHz > 200 Hz. With \( J_L = J_M \) we get: 
\[
\hat{M}_s / M_N \equiv 0.5 \cdot \hat{M}_{e,6g[|f_s|} / M_N = 1.7% \text{, whereas at 4800 Hz shaft torque ripple is 0.0015 Nm.}
\]

**Conclusions:**

Although voltage amplitude of 5\textsuperscript{th} and 7\textsuperscript{th} voltage harmonic is low, compared to pulse frequency harmonic voltage amplitude, the 6\textsuperscript{th} harmonic torque ripple is dominating, as switching frequency torque ripple is attenuated by inertia and elastic coupling. Natural torsion frequency \( f_0 \) of drive train is 20 ... 200 Hz, so torsion resonance might be excited. Note that the frequency of the 5\textsuperscript{th} and 7\textsuperscript{th} voltage harmonic vary with speed, so during speed variation torsion resonance might be hit.
Example 5.4-6:
Measured torque ripple with accelerometer:
Standard induction motor: 2 pole, 750 W, 2850/min, 2.51 Nm, 400 V D, 50 Hz, 1.6 A
Voltage-source IGBT Inverter: 1500 VA, 2.2 A, 400 V
Motor operated at \( f_s = 1.2 \) Hz, \( n = 20/\text{min},\ s = 0.72,\ M = 1.24 \) Nm = 50% rated torque
a) PWM, asynchronous switching: \( k = 1 + 6g = 1, -5, 7, -11, 13, \ldots\ \ g = 0, \pm 1, \pm 2, \ldots\)
b) PWM only for two thirds of one period, one third no switching (zero voltage): This switching mode leads to lower inverter switching losses, but voltage output pattern is not symmetrical to abscissa, so voltage harmonics with even ordinal numbers occur: \( k = 1 + 3g = 1, -2, 4, -5, 7, -8, 10, -11, 13, \ldots\ \ g = 0, \pm 1, \pm 2, \ldots\)

Calculation of lowest torque ripple frequency (Table 5.4-3) and measurement of torque ripple \( \hat{\omega}_M \) according to Chapter 1. Load inertia is much bigger than motor inertia, so shaft torque ripple is equal to air gap torque ripple.

\[
\hat{\omega}_M = \frac{\hat{M}_{cogging}}{M_{\text{average}}} = \frac{(M_{\text{max}} - M_{\text{min}})}{2} / \frac{(M_{\text{max}} + M_{\text{min}})}{2}
\]  

(5.4-22)

<table>
<thead>
<tr>
<th></th>
<th>a) symmetrical PWM</th>
<th>b) asymmetrical PWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest torque ripple frequency</td>
<td>( f_{1k} = 6</td>
<td>g</td>
</tr>
<tr>
<td>Measured torque ripple ( \hat{\omega}_M )</td>
<td>14.4%</td>
<td>35.0%</td>
</tr>
<tr>
<td>Torque ripple amplitude</td>
<td>0.18 Nm</td>
<td>0.43 Nm</td>
</tr>
</tbody>
</table>

Table 5.4-3: Calculated low frequency torque ripple at PWM for different PWM feeding

Conclusions:
With asymmetrical PWM increase of voltage harmonics occurs, leading to increased current harmonics and to double torque ripple amplitude at low frequency, which might excite torsion resonance.

Fig. 5.4-8: Measured low frequency torque ripple at low speed with a) symmetrical PWM, b) asymmetrical PWM. High frequency torque ripple e.g. due to inverter switching is not seen, as accelerometer is low pass filter. In case b) superposition of torque ripple with 3.6 Hz and 7.2 Hz is visible.

d) Acoustic noise
In addition to the magnetically excited acoustic noise at sinus voltage supply, caused by the slot harmonics, the additional air gap waves of the current harmonics \( I_{sk} \) will add to acoustic noise with new tonal frequency, which is typically inverter pulse frequency \( f_p = 2f_T \). Like for
torque ripple, also here only the fundamental wave amplitudes $B_{\delta k,v=1} \sim I_{sk}$ are considered, as the space harmonics amplitudes of higher order $|k| > 1$ are small. So dominant field wave $B_{\delta k,v=1} \sim I_{sk} = I_s$ of fundamental current is modulated by $B_{\delta k,v=1} \sim I_{sk}$:

- Stator fundamental field wave, excited by magnetizing current $I_m = I_s + I_{r,sk}$ with stator frequency $f_s$:

$$B_{\delta k=1,v=1}(x_s,t) = B_{\delta k} \cdot \cos \left( \frac{\pi x_s}{p} - 2\pi f_s t \right) = B_{\delta k} \cdot \cos \alpha . \quad (5.4-23)$$

- Stator harmonic field wave, excited by magnetizing current $I_{mk} = I_{sk} + I_{r,sk} I_s$ with stator harmonic frequency $f_{sk}$:

$$B_{\delta k,v=1}(x_s,t) = B_{\delta k} \cdot \cos \left( \frac{\pi x_s}{p} - 2\pi k \cdot f_s t \right) = B_{\delta k} \cdot \cos \beta . \quad (5.4-24)$$

The magnetic pull due the superposition of these waves is

$$f_n(x_s,t) = \frac{B_n^2(x_s,t)}{2\mu_0} - (\sum_{k=1}^\infty B_{\delta k})^2 \Rightarrow f_{n,1,k} \sim \sum_{k=1}^\infty B_{\delta k}^2 + 2B_{\delta k}B_{\delta k'} , \quad k \neq k'. \quad (5.4-25)$$

Mainly the mixed products $B_{\delta k}B_{\delta k'} = B_{\delta k}B_{\delta k=1}$ of fields of the $k^{th}$ harmonic and of the fundamental current are of interest, because they are big enough. The squares $B_{\delta k}^2$ for $|k| > 1$ and the mixed products $B_{\delta k}B_{\delta k'}$ for $k \neq k' \neq 1$ are very small due to the small field amplitudes. With use of the trigonometric formula $2\cos \alpha \cdot \cos \beta = \left[ \cos(\alpha + \beta) + \cos(\alpha - \beta) \right]$ the radial force density waves, exert an oscillating pull on stator and rotor iron stack, are

$$f_{n,1,k}(x_s,t) = \frac{B_{\delta k}B_{\delta k'}}{2\mu_0} \cdot \cos \left( 2r \cdot \frac{\pi x_s}{2p \tau} - 2\pi f_{Ton,k} t \right) . \quad (5.4-26)$$

The number of positive and negative half-waves of force density along machine circumference equals the number of nodes $2r$ in between:

$$2r = 2p \cdot |1 \pm 1| = 4p ("+") \quad \text{and} \quad 0 ("-") \quad (5.4-27)$$

The tonal frequency $f_{Ton,k}$ of force wave variation is

$$f_{Ton,k} = f_s \cdot |1 + k| \quad \text{for} \quad 2r = 4p \quad (5.4-28)$$  
$$f_{Ton,k} = f_s \cdot |1 - k| \quad \text{for} \quad 2r = 0 \quad (5.4-29)$$

As already explained, the deformation of stator yoke as elastic ring is small for high node force wave, and the acoustic sound wave of this multi-pole excitation is not far reaching. On the other hand, the vibration mode $2r = 0$ yields to "in-phase" vibration of the whole motor surface as a very good loud-speaker. As already shown in this chapter, the dominating current harmonic amplitudes occur at medium modulation degree with frequencies $f_{sk} = 2f_T \pm f_s$.

Thus tonal frequency is $f_{Ton,k} = f_s \cdot |1 - k| = f_{sk} - f_s = 2f_T$ or $f_{Ton,k} = 2f_T - 2f_s$. 

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Conclusions:
Inverter caused magnetic noise is usually excited with twice switching frequency ("pulse frequency").

Force amplitude is depending on motor main flux and stator current harmonics. Therefore inverter-induced magnetic noise occurs already at no-load, as both main flux $\Phi$ and current harmonics $I_{sk}$ are at their full value already at no-load.

$$\hat{F}_{n,1k} = \frac{1}{2\mu_0} B_{kh} \cdot \frac{\mu_0 \cdot \sqrt{2}}{\pi} \cdot \frac{m_s N_s k_{ws} I_{sk}}{p} - I_m \cdot I_{sk} \sim \Phi \cdot I_{sk}$$  \hspace{1cm} (5.4-30)

By increasing inverter switching frequency acoustic noise may be reduced, because
- harmonic current is reduced,
- at high frequencies beyond 6 kHz human ear is decreasingly sensitive,
- vibration amplitude of mechanical structure decreases with increased frequency.

Magnetically excited acoustic noise depends on speed range: In the constant torque range $0 \leq n \leq n_N$ flux is constant and big, so noise is big. In the constant power range flux is weakened by $\Phi = \Phi_N \cdot (n_N / n)$ and therefore magnetic noise decreases. In case of standard induction motors with shaft mounted fan the mechanical noise of the fan increases with speed, containing mainly the "fan blade frequency" $f_z = z \cdot n$ with $z$ as number of fan blades.

Example 5.4-7:
Measurement of sound pressure level $L_{PA}$ in 1 m distance of motor in acoustic chamber, where walls contain sound-absorbing glass fibre cones, so no reflections of sound waves occur. Sound pressure level $L_p$ (with human audibility level $p_0 = 2 \cdot 10^{-5}$ Pa) is measured as average value of 8 microphones

$$\overline{L}_{PA} = 20 \cdot \lg \left( \frac{p}{p_0} \right) \cdot a$$  \hspace{1cm} (5.4-31)

and is weighted with factor $a$ to simulate frequency-dependent sensitivity of human ear.

4 pole cage standard induction motor, size 71 mm (shaft height), with shaft-mounted fan and data: 250 W, 400 V D, 50 Hz, fed from IGBT-inverter with 400 VA, capable of different switching frequencies 1 kHz, 2 kHz, 4 kHz, 8 kHz, leading to audible noise with pulse frequency 2 kHz, 4 kHz, 8 kHz, 16 kHz. Fig. 5.4-9 shows, that increase of switching frequency above 2 kHz decreases $\hat{L}_{PA}$ considerably up to 25 dB(A) at low speed. With switching frequency 8 kHz the current ripple (at 16 kHz) is so small, that current is nearly sinusoidal. So no difference in noise to ideal sinusoidal voltage operation (generated from synchronous test-field generator with variable frequency) occurs. Above 1500/min flux is weakened and sound pressure level is reduced at 1 and 2 kHz switching frequency. At high speed the noise of the fan is dominating. Noise of fan increases theoretically with

$$\Delta L_p = 50 \cdot \lg \left( \frac{n_2}{n_1} \right)$$  \hspace{1cm} (5.4-32)

thus between 1500/min and 6000/min by 30 dB, which is well in coincidence with measurement at 8 kHz switching frequency.

For big motors it is not easy to raise inverter switching frequency to reduce audible noise, as due to increased switching losses the power transistors must be oversized, which is expensive at big power. As audible tone with pulse frequency (e.g. 2 kHz) may be experienced really
nasty, some manufacturers of inverters offer a control strategy with wobbling of switching frequency. Switching frequency varies between e.g. 1 kHz and 2 kHz periodically with e.g. a sine function. So the tonal frequency varies also. One experiences a frequency mix instead of a tonal fixed frequency. This sound is to many people more convenient, but sometimes resembles to that kind of noise, if motor bearings were damaged.

![Graph](image)

**Fig. 5.4-9:** Measured A-weighted sound pressure level at 1 m distance of 250 W, PWM inverter-fed induction motor, varying with speed and switching frequency \( f_s \), corresponding with pulse frequency \( f_p = 2f_s \).

e) **Influence of air gap field space harmonics at inverter-operation:**

Asynchronous and synchronous harmonic torque due to slotting occur at big slip (typically between \( s = 0.8 \ldots 1.2 \)), so inverter-fed motors – being operated usually between full-load slip and zero slip – should not be influenced by these parasitic torque components. Nevertheless, at low speed, full-load slip increases up to unity, so that space harmonic torque will be in the range of motor operation. Yet, motor current is in the range of rated current and not –as with line-started induction motors – in the range of 500% rated current. So these parasitic torque components remain small and negligible (see Example 5.4-8).

**Example 5.4-8:**
Calculation of space harmonic torque with numerical finite element calculation method for unskewed 4-pole cage induction motor, 15 kW, 380 V, D, 50 Hz, 30 A, 1430/min, 100 Nm, \( Q/Q_r = 36 / 28 \).

a) Harmonic torque at \( f_s = 50 \) Hz line-operation (This was calculated in Example 4.4.2-1):

(i) Asynchronous harmonic torque due to \( \nu = -11 \) stator field harmonic with harmonic synchronous speed \( n = (1 - s) \cdot n_{syn} = (1 - 1.09) \cdot 1500 = -136 \) / min.

(ii) Synchronous harmonic torque at slip 0.86 due to harmonics \( \nu = -\mu = 13 \) occurs at 215/min with amplitude 150 Nm.

b) Harmonic torque at \( f_s = 1.5 \) Hz inverter-operation (Fig. 5.4-10):

(ii) Synchronous harmonic torque at slip 0.86 due to harmonics \( \nu = -\mu = 13 \) occurs at 6.3/min with amplitude 5 Nm.

<table>
<thead>
<tr>
<th></th>
<th>a) 50 Hz line operation</th>
<th>b) 1.5 Hz inverter operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed at rated torque 98 Nm</td>
<td>1500/min</td>
<td>45/min</td>
</tr>
<tr>
<td>slip at rated torque 98 Nm</td>
<td>1460/min</td>
<td>5/min</td>
</tr>
<tr>
<td>rotor frequency at 98 Nm</td>
<td>2.6 %</td>
<td>88.9 %</td>
</tr>
<tr>
<td>current at slip 0.86 (synchronous harmonic torque)</td>
<td>1.3 Hz</td>
<td>1.3 Hz</td>
</tr>
<tr>
<td>speed at synchronous torque</td>
<td>550 % rated current</td>
<td>100% rated current</td>
</tr>
<tr>
<td>Synchronous torqu</td>
<td>215/min</td>
<td>6.3 /min</td>
</tr>
<tr>
<td>Synchronous torque amplitude</td>
<td>150 Nm</td>
<td>5 Nm</td>
</tr>
</tbody>
</table>

**Table 5.4-4:** Harmonic synchronous torque at inverter-operation is very small, as motor operates only up to due rated motor current, avoiding the big starting currents of line-operated machines.
According to Chapter 4.4 the synchronous torque amplitude $M_{evv} \sim I_s I_r \sim I_s^2$ depends on square of stator current. At inverter operation current stays in the range of rated current, avoiding big starting currents. In our example, we get therefore only 5 Nm instead of 150 Nm as synchronous torque amplitude.

$$M_{evv,b) \left( I_s, b \right)} = \left( \frac{I_s}{I_s,a} \right)^2 \cdot M_{evv,a) \left( 1, 5.5 \right)} = \left( \frac{1}{5.5} \right)^2 \cdot 150 = 5 \text{ Nm}$$

As asynchronous harmonic torque depends in the same manner on $I_s^2$, their values are reduced likewise.

![Graph of calculated torque-speed characteristic for 15 kW four pole cage induction motor](image1)

**Fig. 5.4-10:** Calculated torque-speed characteristic for 15 kW four pole cage induction motor
a) at 50 Hz, 400 V, b) at 1.5 Hz, voltage $12 \text{ V} + I_s R_s = 23 \text{ V}$ to keep main flux constant in machine. Synchronous and asynchronous harmonic torque are strongly reduced at inverter operation due to reduction of current from starting current values (550% rated current) down to rated current range.