8. Stepper motors

Electromechanical actuators, which transform electrical impulses into mechanical steps, are called **stepper motors**. A complete **stepper drive** consists of

- an electronic control unit,
- a power amplifier, including a grid-side front end, and
- a stepper motor.

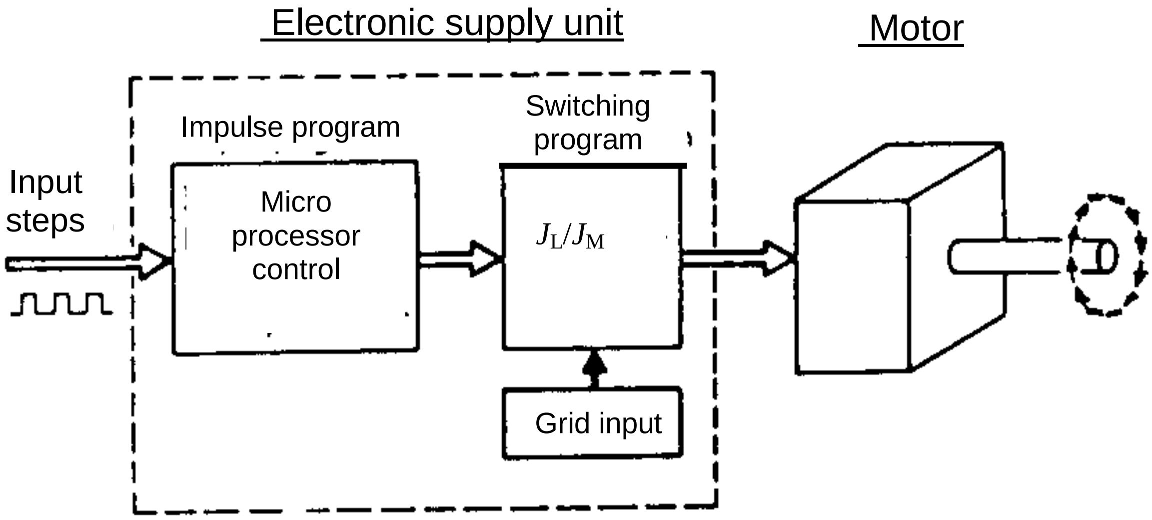


Fig. 8-1: Basic scheme of a stepper drive (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)

In order to keep the cost for a stepper drive as low as possible, the stepper motor is operated in a **feed forward control**, without any use of a position feed-back. Hence the condition, that the number of electrical impulses is always identical with the number of mechanical steps, must be guaranteed under all circumstances. Stepper motors usually operate according to the **synchronous motor** principle in three different variants (Table 8-1):

- Variable reluctance stepper motors (**VR-motor**)
- Permanent magnet stepper motors (**PM-motor**)
- **Hybrid** stepper motors, which operate on a combination of the PM and VR stepper principle

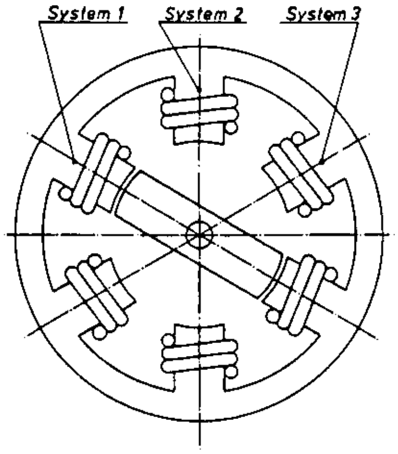
Therefore stepper motors, like grid-operated synchronous motors, have a **static pull-out torque**, which is the maximum available electromagnetic torque at steady state operation. If the load torque at the shaft is bigger than the pull-out torque, the rotor is pulled out of synchronism and falls behind the stator magnetic field; the motor is **losing steps**. During acceleration and deceleration the rotor tends **to oscillate** like grid-operated synchronous machines due to the elastic attributes of the magnetic field forces and the rotor and load inertia. Hence stepper motors are usually manufactured only up to a rated power of about 1 kW. For bigger power the position-controlled synchronous servo drives („brushless DC drives“), which are not endangered by pull-out torque and intrinsic oscillation, are the more economical solution.

Type of stepper motor	**PM-motor**	**VR-motor**	**Hybrid motor**
Electromagnetic utilization	high	low	medium
Damping of oscillations	good	bad	good
Detent torque (= at open circuit) ?	yes	no	yes
Minimum necessary phase count	2	3	2
Cost	low	high	high

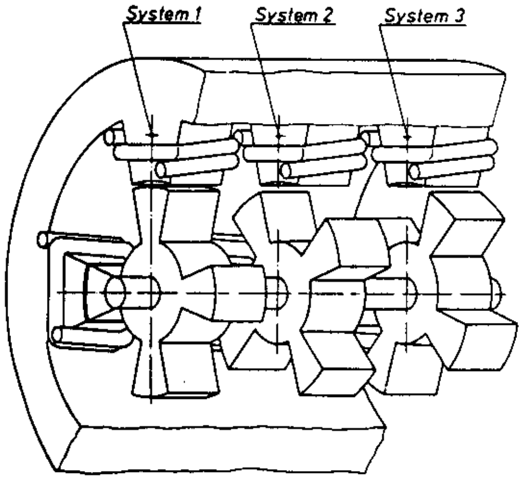
Table 8-1: Three variants of stepper motor principles

8.1 Basic principle of operation of steppers

The multi-phase stator winding may be arranged in the stepper motors either
- in one plane (Fig. 8.1-1a) or
- in different planes along the rotating axis (Fig. 8.1-1b).



a)



b)

Fig. 8.1-1: Design examples of the multi-phase stator winding of a VR-motor with m phases and Q_r rotor teeth
a) stator winding phases in one plane: $m = 3$, $Q_r = 2$
b) stator winding phases in three planes along the axis: $m = 3$, $Q_r = 4$

We assume that each stator phase is fed independently by an impulse voltage, so each phase is energized separately. If the feeding leads to block currents (impulses) per phase, the stator magnetic field is directed into the axis of phase U, if phase U is energized only. A PM rotor of the same pole count as the stator winding or a reluctance rotor of $Q_r/p = 2$ rotor teeth per pole pair (like a conventional synchronous reluctance machine) will be aligned into the stator field direction due to the magnetic tangential drag. By feeding afterwards phase W with a negative block current (**bipolar feeding**) the stator field performs a step of 60°el., which the above described rotors will follow due to the magnetic tangential drag. In case of **uni-polar feeding** (= always positive phase current) the next phase to be energized may e.g. phase V, which is shifted spatially by 1/3 of $2\tau_p$ to phase U. Hence the stator field will make a step of 120°el.. A PM will have to move also by 120°, whereas the VR rotor moves only by 60°, because it is magnetized in the opposite direction and aligns like that into the direction of phase V. Energizing next phase W with positive phase current moves the PM rotor by another 120°, and the VR rotor by another 60°, being now magnetized again into the same direction as before in case of phase U. According to Fig. 8.1-1a the number of mechanical steps z_p during one stator period $T_s = 1/f_s$ for a VR-rotor is given with m stator phases and Q_r/p rotor teeth per pole pair as

$$z_p = (Q_r / p) \cdot m \qquad (8.1\text{-}1)$$

At each step the rotor is shifted by the **rated stepping angle** (in electrical degrees, as a shift by 2π is a movement per one pole pair)

$$\alpha_e = 2\pi / z_p \quad . \qquad (8.1\text{-}2)$$

The number of steps per rotor revolution is p-times bigger:

$$z = p \cdot z_p = Q_r \cdot m \qquad (8.1\text{-}3)$$

Hence the possibility to increase z acc. to (8.1-3) is increasing
- the number of poles $2p$ and/or
- the number of teeth per pole Q_r/p.

The **rated stepping angle** in mechanical degrees is the fraction $1/p$ of the value in electrical degrees:

$$\alpha = \alpha_e / p = 2\pi/(z_p \cdot p) = 2\pi / z \quad . \qquad (8.1\text{-}4)$$

In case of $2p = 2$ there are the following identities: $\alpha = \alpha_e, \quad z = z_p$.
By increasing the number of rotor teeth per pole by the factor 2, getting $Q_r/p = 4$, the VR-rotor moves only by 30° (electrically), if the stator three-phase winding is energized with uni-polar block current first in phase U and then in phase V. If first teeth 1 and 3 have been magnetized by the magnetic field of phase U, at the next step the teeth 2 and 4 are magnetized by the magnetic field of phase V. Comparing the cases $Q_r/p = 2$ and $Q_r/p = 4$, the rotor movement is opposite for the two cases, if the stator phases are energized in the same way, e.g. U-V-W-U-V-W-…

Example 8.1-1: $2p = 2$ stepper motor
a) VR-motor: Fig. 8.1-1a: $Q_r = 2$, $m = 3$, $z = 6$, $\alpha = 360°/6 = 60°$
b) VR-motor: Fig. 8.1-1b: $Q_r = 4$, $m = 3$, $z = 12$, $\alpha = 360°/12 = 30°$

In case of PM stepper motors the rotor teeth number Q_r is replaced by the rotor pole pair number p. Instead of Q_r/p we put 1 and get

$$z = p \cdot z_p = p \cdot m \quad . \qquad (8.1\text{-}5)$$

Facit:
Whereas in conventional synchronous machines the m-phase, 2p-pole stator winding is fed with m sinusoidal phase voltages of the frequency f_s, which are phase shift by $2\pi/m$, resulting in a rotational stator magnetic field with rotational speed $n_{syn} = f_s / p$, the movement of the stator field in stepper motors is not continuous, but step-wise. The stator winding is fed by impulse e.g. block voltages, resulting in case of neglected stator inductances, considering only the stator winding resistance, in block currents and block flux linkages.

8.1.1 Full-step and half-step operation

Full step operation:
If each phase a, b, c, d, … is energized separately by its phase voltage, the stator field moves from phase axis to phase axis, hence by the angle $2\pi/m$ (in electrical degrees). The mechanical step of the rotor depends on the type of rotor, as explained before.
In order to increase z by a factor of 2 we introduce the half-step operation.

Half step operation:
If after a single phase excitation two adjacent phases are energized at the same time (e.g. a, a-b, b, b-c, c, c-d, ….) both with the same current polarity e. g. due to uni-polar feeding, the resulting field is positioned along the axis between the two phases. Hence the angle of stator

field movement is halved, resulting in an electrical angle π/m. Independent of the type of rotor the number of rotor steps therefore is doubled in comparison to full step operation and the mechanical stepping angle is halved. In case of VR-rotors we get for the number of steps per rotor revolution:

VR: $z = Q_r \cdot m / k_B$, PM: $z = p \cdot m / k_B$ (8.1.1-1)

(k_B = 1: Full step operation, k_B = 0.5: Half step operation)

In this mode of operation alternatively one or two phases are magnetized. As the phase current amplitude i is always the same, the magnetic field amplitude may change its amplitude. For easy understanding we punt in the cross section of the machine a complex coordinate system and describe the magnetic field per phase as a vector of flux linkage (a complex phasor), that is aligned in direction of the phase axis. In case of m = 3 and uni-polar feeding the field amplitude should not change, being first e.g. $\underline{\psi}_U$ and then $\underline{\psi}_U + \underline{\psi}_V = \underline{\psi}_U + \underline{\psi}_U \cdot e^{j \cdot 2\pi/3} = \underline{\psi}_U \cdot e^{j \cdot \pi/3}$. But already in case of m = 4 the field is bigger in case of two energized adjacent phases, leading first e.g. to a flux linkage $\underline{\psi}_a$ and then to $\underline{\psi}_a + \underline{\psi}_b = \underline{\psi}_a + \underline{\psi}_a \cdot e^{j \cdot 2\pi/4} = \sqrt{2} \cdot \underline{\psi}_U \cdot e^{j \cdot \pi/4}$. Due to the changing magnetic field also the electromagnetic torque $m \sim i \cdot \psi$ is changed, being smaller with only one phase energized. The stepping torque changes step by step between a small and a big torque, hence between a hard and a soft step.

- **Hard** step: Two adjacent phases are energized, big torque
- **Soft** step: One phase energized, small torque.

8.1.2 Variable reluctance with additional stator teeth (*Vernier* machine)

In Fig. 8.1.2-1 a VR-motor is shown, where the pole surface of the stator and rotor is toothed with stator and rotor teeth of different tooth count or slot pitch, respectively. The condition $Q_r \neq Q_s$ is essential to avoid a big magnetic cogging torque, which would fix the rotor position with stator teeth and rotor teeth aligned. Due to the additional slot openings of the stator the resulting magnetic air gap field is modulated. The air gap δ is varying due to the stator slotting roughly with the stator tooth count Q_s.

$$\delta(x_s) = \delta \cdot (1 + \lambda \cdot \cos((Q_s / p) \cdot (x_s \pi / \tau_p))) \quad (8.1.2\text{-}1)$$

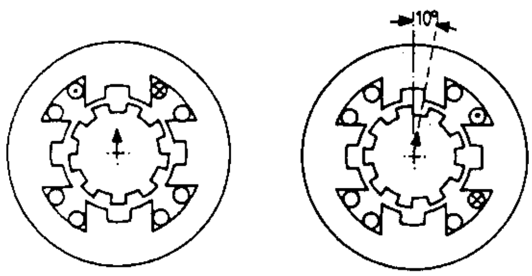
The parameter λ increases with increasing stator slot opening and decreasing air gap. The stator m.m.f. per phase may be expanded into a FOURIER series. Its fundamental is a m.m.f. wave of pole count 2p, which – together with the air gap – gives the resulting magnetic air gap field.

$$V_{phase,\nu=1}(x_s) = \hat{V} \cdot \cos(x_s \pi / \tau_p) \quad (8.1.2\text{-}2)$$

$$B(x_s) = \mu_0 \hat{V} \cdot \cos(x_s \pi / \tau_p) / \delta(x_s) = B \cdot \cos(x_s \pi / \tau_p) + (\lambda/2) \cdot B \cdot \cos((1 \pm Q_s / p) \cdot x_s \pi / \tau_p)$$

By the modulation of the stator field due to the stator pole slotting an additional magnetic field wave pair $(\lambda/2) \cdot B \cdot \left(\cos((1 + Q_s / p) \cdot x_s \pi / \tau_p) + \cos((1 - Q_s / p) \cdot x_s \pi / \tau_p)\right)$ appears with the average pole count $2p' = 2p \cdot \left((Q_s / p - 1) + (Q_s / p + 1)\right)/2 = 2Q_s$. The stator slotting gives by modulation of the stator field an additional harmonic field wave of the pole count $2p' = 2Q_s$. The rotor teeth are interacting with this space harmonic field wave and experience a much higher pole count $2p' = 2Q_s >> 2p$, which now defines the number of steps per revolution. Hence the rotor teeth per pole Q_r / p' are of interest to define the number of mechanical steps z_p during one stator period $T = 1/f_s$ for a VR-rotor. Therefore the number of rotor teeth must be refined with respect to the number of stator teeth. With m stator phases and Q_r/p' rotor teeth per "new" pole pair the number of steps per electrical period is $z_p = (Q_r / p') \cdot m$. The number of steps per rotor revolution is p'-times bigger, resulting again in $z = p' \cdot z_p = Q_r \cdot m$ (*Vernier* principle), but now with a much bigger number of rotor teeth with small slot openings and small slot depth.

Facit:
The stator slotting gives by modulation of the stator field an additional harmonic field wave of the pole count $2p' = 2Q_s$. The rotor teeth are interacting with this space harmonic field wave and experience a much higher pole count $2p' = 2Q_s >> 2p$, which now defines the number of steps per revolution.



a) Phase 1 energized b) Phase 2 energized

Fig. 8.1.2-1: 2-pole VR-motor with stator and rotor slotting: $Q_s = 8$, $Q_r = 9$, $m = 4$

Example 8.1.2-1:
Vernier VR-motor acc. to Fig. 8.1.2-1: $2p = 2$, $Q_s = 8$, $Q_r = 9$, $m = 4$:

$2p' = 2Q_s = 16$, $Q_r / p' = 9/8$, $z_p = (Q_r / p') \cdot m = (9/8) \cdot 4 = 9/2$

$z = p' \cdot z_p = 8 \cdot (9/2) = Q_r \cdot m = 9 \cdot 4 = 36$, $\alpha = 360°/36 = 10°$

8.1.3 Comparison of basic stepper motor functionality

VR stepper motors:
According to Chapter 2 a switched reluctance motor, which is the special case of a VR-motor with position encoder feed back, cannot self-start, if it is only manufactured with 2 phases and symmetrical rotor. In order to get a defined direction of self-starting, at minimum three phases are necessary. In open circuit (= zero stator current) the torque is zero. Hence at switched off motor no torque is supplied to keep the motor in the wanted position. On the other hand a very fine slotting can be manufactured with problems, so with the *Vernier* principle a rather high number of steps per revolution is possible.

PM stepper motors:
PM-motors have due to the rotor permanent magnet field also in 2-phase stators a defined starting torque and starting direction, so the minimum phase number for stepper operation is 2. At open circuit the cogging torque of the rotor permanent magnet field with the stator

slotting provides sufficient torque to keep the rotor at the desired position before switch-off. On the other hand it is very difficult to magnetize a cylindrical rotor permanent magnet material with a high pole count, as the magnetization coils for a N-S-N-S-polarization with a very small pole width are difficult manufacture. Hence the number of steps per revolution is considerably smaller than in VR-motors. So rotors for a high number of steps per revolution must be VR-rotors with a very accurate manufacturing in order to keep the relative error of the individual stepping angle below 3% ... 5%.

Hybrid stepper motors:
In hybrid motors a rotor PM excitation is combined with toothed rotor ring structures to generate a variable reluctance in the air gap. Hence the advantages of PM stepper motors with minimum phase count 2 and open circuit torque are combined with the advantages of VR stepper motors with a high number of steps per revolution (Table 8.1-3-1).

type of motor	VR/H	VR/H	VR/H	VR/H/PM	PM	PM	PM	PM	PM
rotor teeth Q_r	100	90	50	24					
rotor poles $2p$				24	20	16	12	6	4
z (at m = 2)	200	180	100	48	40	32	24	12	8
α / ° (mech.)	1.8	2	3.6	7.5	9	11.25	15	30	45

Table 8.1.3-1: Typical numbers of rotor teeth for different kinds of stepper motors
(VR: variable reluctance stepper motor, H: hybrid stepper motor, PM: permanent magnet stepper motor)

8.1.4 Micro stepping

If the stator winding of the 3-phase, 2-pole VR-motor with 2 rotor teeth of Fig. 8.1-1a is fed with uni-polar phase currents according to Fig. 8.1.4-1, the rotor will need for one revolution two electrical periods $2T_s$ and will perform six steps per revolution. During one electrical period the rotor performs 3 mechanical steps.

Electrical stator frequency $f_s = 1/T_s$, stepping frequency $f = 1/T = 3f_s$

By applying a **bipolar current feeding** with e.g. k_z = 20 current steps per period T_s (Fig. 8.1.4-2), where the stepped current curve approximates a sine wave, the rotor will perform instead of 3 steps per period T_s now 20 “micro” steps. This can be understood easily, if the magnetic field per phase is regarded as a vector, which is aligned into positive phase axis, when fed with positive phase current. Due to *Ampere´s* law the length of the vector is directly proportional to the phase current, e.g. $\psi_U \sim i_U$. Hence the vector of the resulting magnetic field is a superposition of the phase magnetic field vectors. In a three-phase system (m = 3) the direction of the three phase axes is shifted in space by 120° (electrically). Hence the phase magnetic field vector of V is shifted with respect to U by $2\pi/3$ or $\underline{a} = e^{j\cdot 2\pi/3}$, and the phase vector of W by 240° or $\underline{a}^2 = e^{j\cdot 4\pi/3}$. The resulting magnetic field vector, expressed as the resulting magnetic flux linkage, is given as superposition of the three phase flux linkage vectors.

$$\underline{\psi} = \underline{\psi}_U + \underline{\psi}_V + \underline{\psi}_W \sim i_U + \underline{a}\cdot i_V + \underline{a}^2 \cdot i_W \qquad (8.1.4\text{-}1)$$

Hence the position of the resulting field my be adjusted by the current values i_U, i_V, i_W nearly continuously. If these values approximate a sine wave function with e.g. 20 steps per period $2\pi/20$, the resulting field will take 20 positions per period with almost constant amplitude ψ.

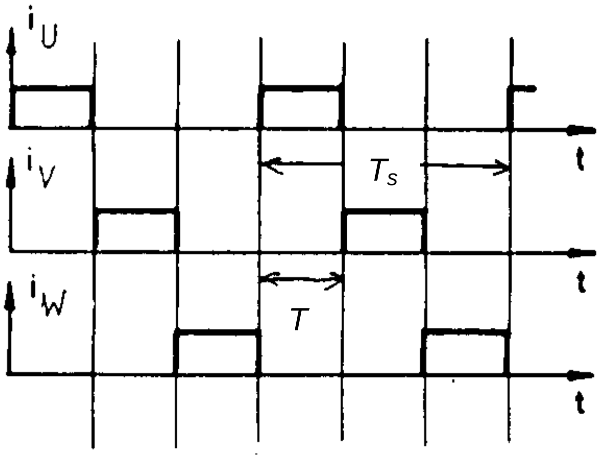


Fig. 8.1.4-1:
Uni-polar current feeding of a three-phase stator winding

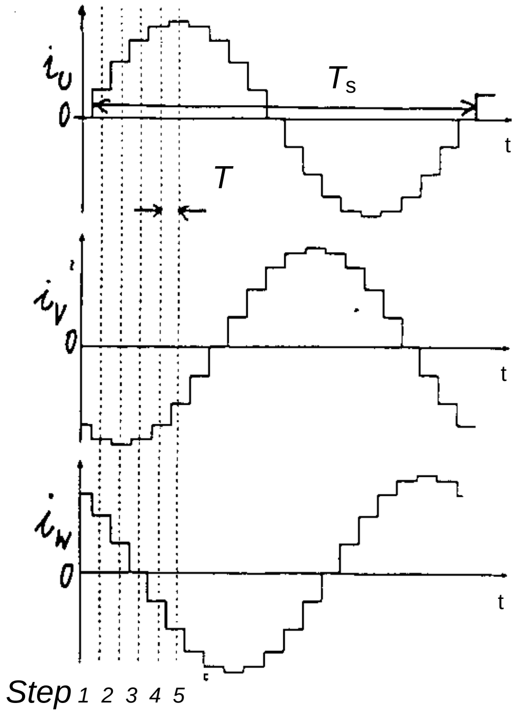


Fig. 8.1.4-2:
Bipolar current feeding of a three-phase stator winding with **micro-stepping** (here: $T_s/T = 20$)

Proof:

$\Omega_s = 2\pi / T_s$, $k_z = 20$, $t_i = 0, T_s/20, 2T_s/20, 3T_s/20, \ldots$:

$i_U(t_i) = \hat{I}\cdot\cos(\Omega\cdot t_i), i_V(t_i) = \hat{I}\cdot\cos(\Omega\cdot t_i - 2\pi/3), i_W(t_i) = \hat{I}\cdot\cos(\Omega\cdot t_i - 4\pi/3)$

With $i_U(t_i) = \hat{I}\cdot\dfrac{e^{j\Omega_s\cdot t_i} + e^{-j\Omega_s\cdot t_i}}{2}$, $i_V(t_i) = \hat{I}\cdot\dfrac{e^{j(\Omega_s\cdot t_i - 2\pi/3)} + e^{-j(\Omega_s\cdot t_i - 2\pi/3)}}{2}$

and $i_W(t_i) = \hat{I}\cdot\dfrac{e^{j(\Omega_s\cdot t_i - 4\pi/3)} + e^{-j(\Omega_s\cdot t_i - 4\pi/3)}}{2}$ one gets

$$\underline{\psi}(t_i) \sim \hat{I}\cdot\left[\frac{e^{j\Omega_s\cdot t_i} + e^{-j\Omega_s\cdot t_i}}{2} + e^{j\frac{2\pi}{3}}\cdot\frac{e^{j(\Omega_s\cdot t_i - 2\pi/3)} + e^{-j(\Omega_s\cdot t_i - 2\pi/3)}}{2} + \right.$$

$$\left. + e^{j\frac{4\pi}{3}}\cdot\frac{e^{j(\Omega_s\cdot t_i - 4\pi/3)} + e^{-j(\Omega_s\cdot t_i - 4\pi/3)}}{2}\right] = \hat{I}\cdot\left[\frac{3e^{j\Omega_s t_i}}{2} + \frac{3e^{-j\Omega_s t_i}}{2}\cdot\left(1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}}\right)\right] =$$

$$= \underline{\underline{(3\hat{I}/2)\cdot e^{j\Omega_s\cdot t_i}}}$$

The length of the resulting field vector remains constant. The angle of the field vector position is changing in steps $\Omega_s t_i = 2\pi\cdot t_i / T_s = 0, 2\pi/20, 2\cdot 2\pi/20, \ldots$ The stepping angle is therefore refined by k_z.

VR: $\alpha = 2\pi/(Q_r\cdot k_z)$ PM: $z = 2\pi/(p\cdot k_z)$ (8.1.4-2)

The driving electronic equipment is more complicated fort the micro stepping operation than for the conventional half step operation.

Example 8.1.4-1:
VR stepper motor: $m = 3$, $Q_r = 50$
a) Full step operation, uni-polar current feeding: $\alpha = 2\pi/(Q_r m) = 360°/(50x3) = 2.4°$
b) Micro step operation $k_z = 20$: $\alpha = 2\pi/(Q_r k_z) = 360°/(50x20) = 0.36°$

Facit:
By increasing phase count, pole count and half-step or micro-step operation the number of steps may be increased for all kinds of stepper motors. In addition in VR- and hybrid stepper motors the number of steps per revolution may be increased by increasing number of rotor teeth.

8.2 Stepper motor design

8.2.1 Hetero-polar motors

The polarity of the air gap flux changes its sign along the motor air gap circumference (Fig. 8.1-1) as north and south poles. Most of the VR- and PM-stepper motors are built as hetero-polar motors. A low cost design for the stator winding is the **claw pole stator winding**. Each phase of the stator winding is manufactured as a ring coil, which is surrounded by an iron yoke made of claws, which alternatively enclose the ring coil from the left and the right side. The ring coil field with closed field lines preferably passes in this iron yoke, but is forced at the end of the claws to cross the air via the air gap. Hence the field lines leave at e.g. positive current polarity the left claw tips and enter the right claw tips. Thus the left claws may be regarded as north poles and the right ones as south poles. Thus an alternating (“hetero-polar”) air gap field distribution is generated. The number of claws per ring coil therefore is even and equals the number of poles. The different phases are arranged in different planes along the motor axis.

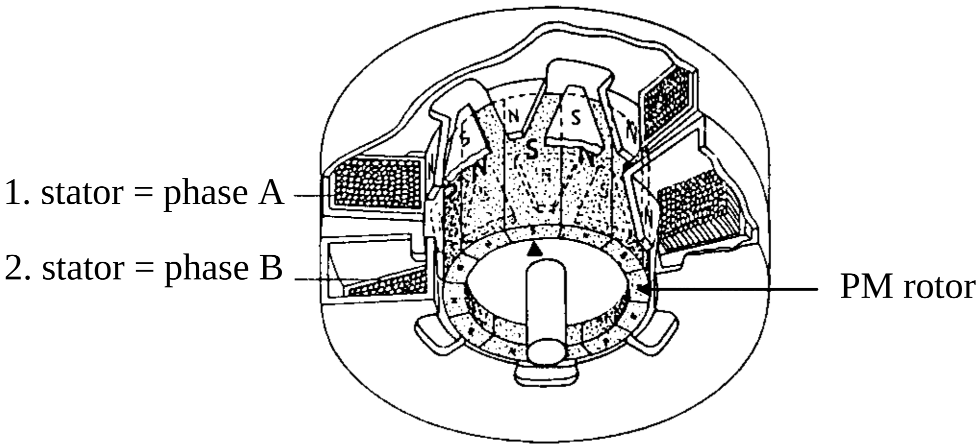


Fig. 8.2.1-1: PM-claw pole stepper motor, $m = 2$, $2p = 12$ (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)

In Fig. 8.2.1-1 a two-phase motor with 2 ring coils is depicted. Each ring coil is surrounded by 6 right and 6 left claws, fitting to a 12-pole PM cylindrical rotor. The necessary 90° el. shift between the two phases is realized by shifting the second phase claws by half claw pitch (= half pole pitch), whereas the rotor is magnetized without any axial skew. Of course it would be also possible to arrange the claws of the two phases aligned and shift the rotor magnetization of the second half (beneath the second phase) by half pole pitch. But this kind of magnetization is difficult to realize and is therefore not in use.

Note, that this kind of motor very closely resembles to a two-phase transversal flux motor, where the stator U-yokes are replaced by the stator claws!

Example 8.2.1-1:
PM-claw pole stepper motor (Fig. 8.2.1-1):
Number of steps per revolution $z = 2p.m$ = 12x2 = 24, stepping angle α = 360°/24 = 15° (mech. degrees)

In Fig. 8.2.1-2 the bipolar current feeding for the two phases A and B is shown for the motor of Fig. 8.2.1-1 for full step operation step by step. The current polarity in A is reversed prior to the current in B. The direction of rotation is reversed by changing the sequence of current reversal; the current polarity in B is then reversed before the current in A changes its polarity. It is clearly visible, that two phases are sufficient in PM motors to define the direction of motion at starting.

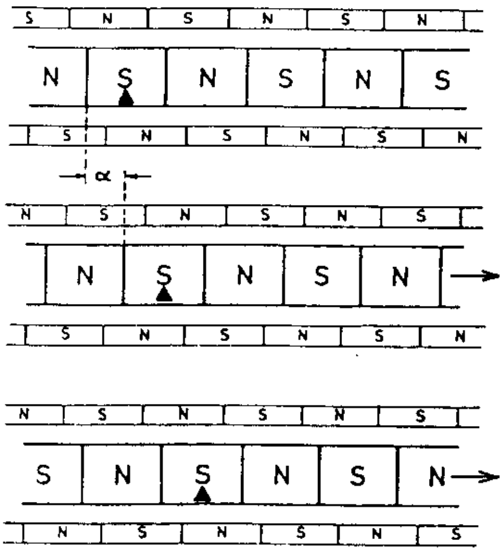
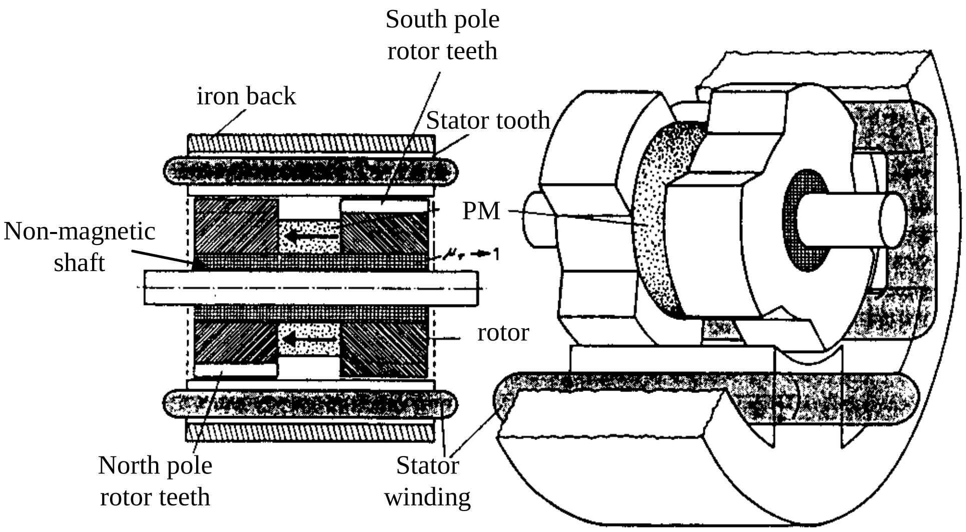


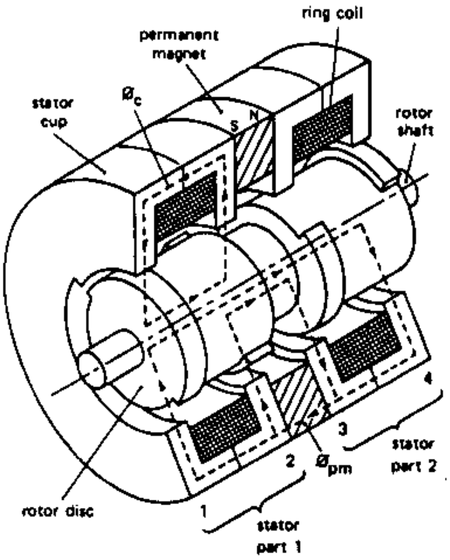
Fig. 8.2.1-2: Current feeding sequence of a two-phase claw pole PM stepper motor

8.2.2 Homo-polar motors

Rotor permanent magnets:
The flux polarity does not change along the motor air gap circumference, but it is changing in axial direction. Hence the air gap flux density along the rotor circumference is called homo-polar. Mainly **hybrid stepper motors** are designed as homo-polar machines. In Fig. 8.2.2-1a the two-phase stator winding (m = 2) is unskewed. The permanent magnet rotor has at its outer side two additional toothed iron rings, which are shifted by half tooth pitch in circumference direction. The rotor shaft is made of non-magnetic iron to keep the permanent magnet inner stray flux as small as possible. The homo-polar permanent magnet flux leaves the first rotor half with positive polarity and enters the second half, hence defining there negative polarity. So all teeth of the first ring are north poles, and the teeth of the second ring are south poles. If the rotor magnet is omitted, only the iron tooth rings remain and we get a VR-motor in homo-polar design. For the same stator *Ampere*-turns this VR-motor produces a smaller torque than the hybrid stepper motor due to the missing PM flux.



a)



b)

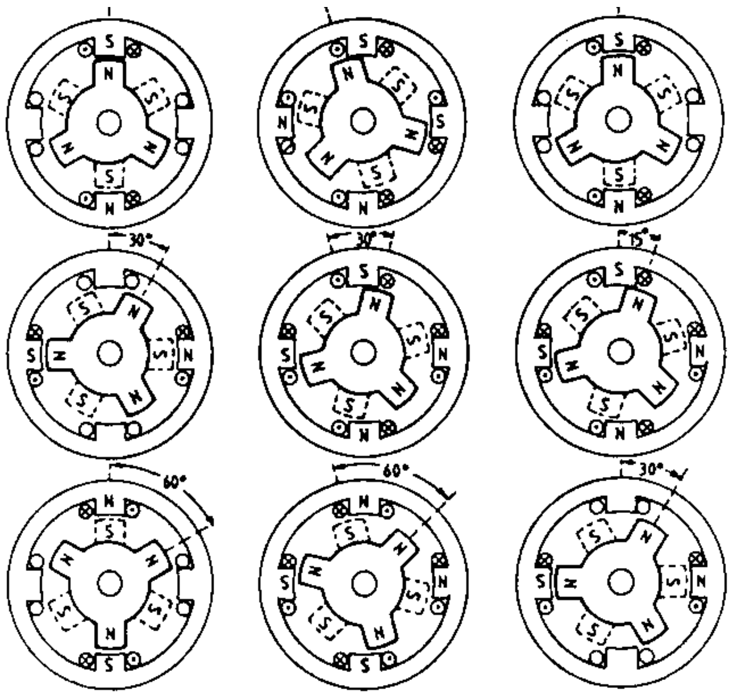
Fig. 8.2.2-1: 2-phase hybrid stepper motors with homo-polar flux arrangement and bipolar current operation:
a) Permanent magnet in the rotor, 3 rotor teeth per half machine, 4 stator teeth (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987), b) Permanent magnet in the stator, claw pole stator, 1 tooth in stator and rotor per radial plane: Φ_c: coil flux, Φ_{PM}: PM flux (*Philips Comp.*)

The bipolar current operation for the stator winding of the motor of Fig. 8.2.2-1a is shown in Fig. 8.2.2-2 for

(i) full step operation with one excited stator phase,
(ii) full step operation with two excited stator phases (increased torque),
(iii) half step operation, exciting alternatively $m = 2$ and $m - 1 = 1$ phases.

Stator permanent magnets:
In the same way the permanent magnet may be also arranged **in the stator** as e.g. a ring magnet with axial direction of magnetization to guide the flux Φ_{PM} with positive polarity in the first axial half of the machine (here: first phase) and with negative polarity in the second half (= second phase) (Fig. 8.2.2-1b, stator claw pole arrangement with one claw per half side (cup)). If the first phase is magnetized with positive current, resulting in a positive coil flux Φ_c, Φ_{PM} and Φ_c add on the left stator cup of phase 1 and substract on the right cup. Hence the rotor moves in aligned tooth position on the left cup (plane 1). For the next step phase 1 is switched off and phase 2 is magnetized (full step operation) e.g. with positive current,

aligning stator and rotor tooth in plane 4, where Φ_{PM} and Φ_c add. A step of 90° is performed in counter-clockwise operation. Then phase 2 is turned off and phase 1 is energized with negative coil current. Due to negative Φ_c the fluxes Φ_{PM} and Φ_c add on the right stator cup of phase 1 (plane 2), where the teeth align. In total 2x90° =180° rotational angle have been performed. Then phase 2 is energized with negative current and so on.



(i) (ii) (iii)

Fig. 8.2.2-2: Current pattern in the stator winding of the hybrid stepper motor of Fig. 8.2.2-1: full step operation with (i) one excited stator phase, (ii) with two excited stator phases, (iii) half step operation (*Stölting et al., El. Kleinmaschinen, Teubner*, 1987)

Example 8.2.2-1:
Hybrid stepper motor of Fig. 8.2.2-1a: m = 2. Sum of the teeth of both tooth rings: Q_r = 6
k_B = 1: Full step operation (Fig. 8.2.2-2 (i) and (ii): $z = Q_r m / k_B = 12$, α = 360°/12 = 30°
k_B = 0.5: Half step operation: $z = Q_r m / k_B = 24$, α = 360°/24 = 15°

Example 8.2.2-2:
Hybrid stepper motor of Fig. 8.2.2-1b: m = 2. Sum of the teeth of the two tooth rings per phase: Q_r = 2
k_B = 1: Full step operation: $z = Q_r m / k_B = 4$, α = 360°/4 = 90°

8.2.3 Single phase stepper motors

Single phase stepper motors are used e.g. as drives for watches and are very simple and low cost motors. They are PM stepper motors with a PM rotor and asymmetric stator poles (teeth) to define a direction of motion.

a) In Fig. 8.2.3-1 the single phase (m = 1) stator winding carries no current at the time instant *a*. The rotor is positioned according the cogging torque, which is generated by the PM rotor flux and the stator teeth.
b) At time instant *b* the stator coil is energized with positive current (voltage U_c), exciting a flux Φ_c. Hence the left pole shoe (tooth) is magnetized as north-pole and the right one as south-pole. Due to that magnet field the rotor is turned anti-clockwise.

c) At time instant *c* the current is again zero and the rotor is fixed due to the cogging torque.
d) At time instant d the stator winding is energized with reversed current (voltage $-U_c$), exciting a flux $-\Phi_c$. The left pole shoe (tooth) is magnetized as south-pole and the right one as north-pole. Hence the rotor continues its anti-clockwise rotation.

Note: If the stator winding is energized at time instant b with negative current (left pole shoe is south-pole), the rotor would not have moved at all. Only positive current allows rotation, which shows that it is possible to get **a defined sense of rotation** also with single phase

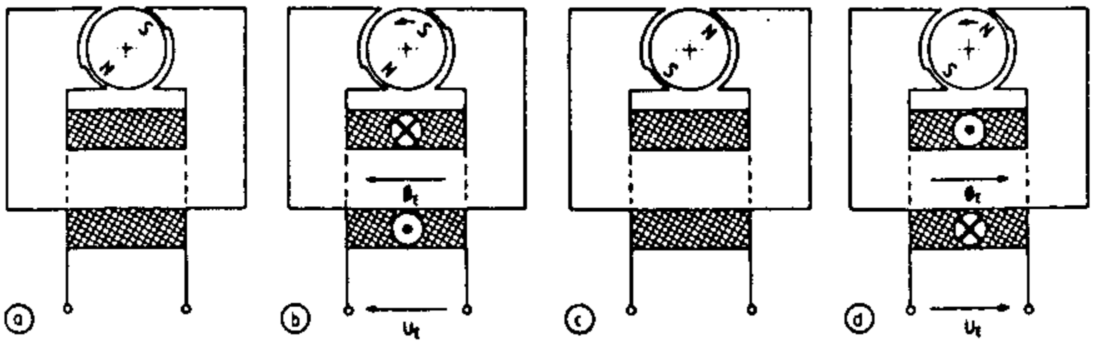


Fig. 8.2.3-1: Single phase PM stepper motor with asymmetric poles by narrower air gap at one pole side (*Stölting et al., El. Kleinmaschinen, Teubner*, 1987)

Facit:
VR- and PM-stepper motors are mainly manufactured as hetero-polar motors, whereas hybrid steppers are mainly homo-polar machines. Single phase steppers need an asymmetric magnetic circuit and are manufactured as PM machines.

8.3 Driving circuits of stepper motors

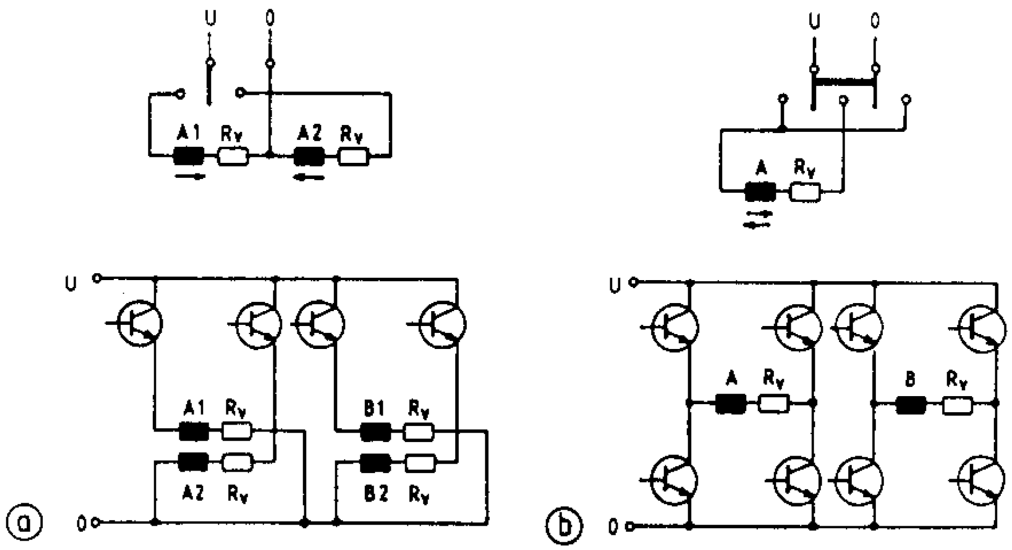


Fig. 8.3-1: Driving circuits for stepper motors with a) uni-polar, b) bipolar current. Free-wheeling diodes are omitted here for clarity. The resistance R_v are used to reduce the electric time constant (*Stölting et al., El. Kleinmaschinen, Teubner*, 1987)

a) The two branches A1, A2 of phase A and B1, B2 of phase B are energized subsequently always with positive current, so always one branch per phase is without current, which gives only 50% motor winding utilization.

b) By using four switches instead of two per phase the phases A and B may be energized with bi-polar current, so always the complete phase winding is energized, yielding a high motor utilization at the expense of double amount of power switches.

The electronic driving circuits for stepper motors work either with **uni-polar** current impulses (current impulses of one polarity) or with **bipolar** current (AC current) (Fig. 8.3-1).

Kind of winding current	**uni-polar**	**Bipolar**
Number of winding branches	two per phase	one per phase
Number of power switches	two per phase	four per phase
Polarity of current	positive	positive and negative
Amount of circuit elements	low	high
Motor utilization	low	high

Table 8.3-1: Uni-polar and bipolar current feeding for stepper motor windings

The ideal current wave form is the rectangular impulse, but the current rise at voltage turn-on is limited by the **electrical time constant** $T_e = L_s/R_s$, which is defined by the winding resistance R_s and inductance L_s per phase (Fig. 8.3-2). Hence the electrical switching frequency f_s cannot be shorter than $1/T_e$: $f_{s,\max} < 1/T_e$.

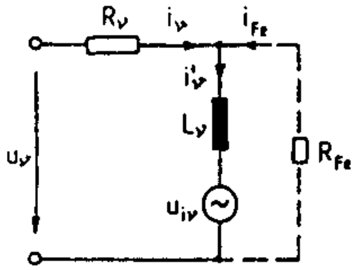


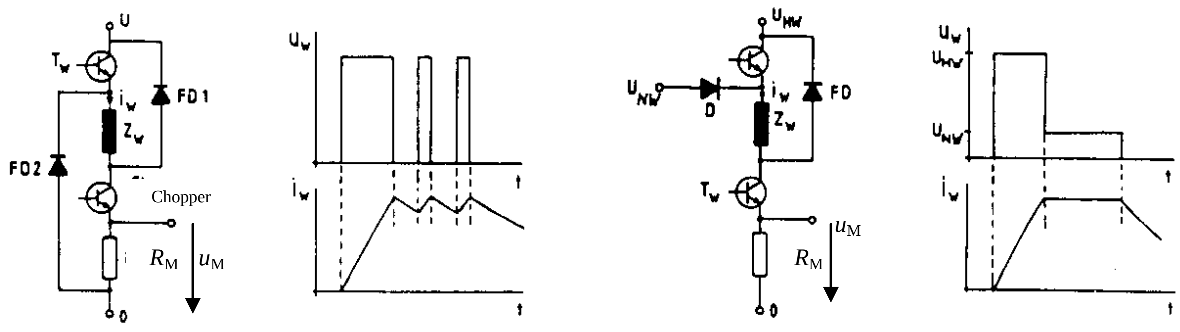
Fig. 8.3-2: Electrical equivalent circuit of the ν^{th} stator phase (R_v, L_v: phase resistance and inductance, u_{iv}: back EMF per phase, R_{Fe}: equivalent resistance for considering the iron losses per phase) (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)

Measures to raise the switching frequency:

Constant voltage operation – serial resistance:
By adding an additional serial resistance per phase R_v the electrical time constant is reduced $T_e = L_s/(R_s + R_v)$, which allows an increase of $f_{s,\max} < 1/T_e$. This may be used for operation with constant supply voltage U (which has to be increased for constant current: $U = I \cdot (R_s + R_v)$), but the losses are increased by R_v/R_s (Fig. 8.3-1).

Constant current operation – chopping mode:
By raising the supply voltage U the inclination of current slope is increased: $di/dt = U/L_s$. Hence the rise time t_r to reach the rated value I is decreased: $t_r = I/(di/dt) = L_s \cdot I/U$. As $I << U/R_s$, the supply voltage must be chopped to adjust the average voltage $U_{av} = R_s \cdot I < U$ to keep the current at its rated value (Fig. 8.3-3a). The switching losses have to be covered in the power converter, and slightly higher losses in the stator winding due to the switching-frequent current ripple.

Bi-level voltage operation:
The winding is switched on with a high voltage U_{HW} to reduce the current rise time according to $t_r = L_s \cdot I/U_{HW}$. After reaching the rated current, the voltage is lowered to the value $U_{NW} = R_s \cdot I < U_{HW}$ (Fig. 8.3-3b).



a) b)

Fig. 8.3-3: Driving circuits for stepper motors (winding impedance Z_w) (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)

a) Chopping mode: The phase is turned on and off with the upper transistor T_w to the DC link voltage U. The lower transistor is chopping. The current with its ripple is measured at the measuring resistance R_M, (shunt), delivering the voltage u_M. The free-wheeling diode FD1 takes over the current during chopping and FD2 after final turn-off of the phase.

b) Bi-level-mode for stepper motor current feeding: The upper transistor switches the phase to the high voltage U_{HW}. During that time the diode D blocks the current not to enter the low voltage circuit U_{NW}. After the upper transistor is switched off, the winding is operated with $U_{NW} < U_{HW}$. The phase is turned off with the transistor T_w, and the free-wheeling diode FD takes over the residual winding current to decay via U_{HW}.

Due to the inertia of the rotor the motor tends to overshoot the last step, where it should come to a stand still. Hence this over-shooting must be damped effectively.

Measures to damp the over-shoot at the last step:

Mechanical damper unit:
Mechanical damper units have been used in the earlier days of stepper motors, but are nowadays practically out of use.

Short circuit damper:
If those winding phases, which are not energized at the last steps, are short circuited by the power electronics supply, the voltage, which is induced there by the moving rotor, will cause a short circuit current. The short circuit currents produce with rotor a braking torque, that damps the overshoot. As the stepper motors are usually of small power rating, their winding resistance per phase R_s is rather big. Hence the short circuit current is not very big, compared to the rated current, so the braking and damping usually is weak.

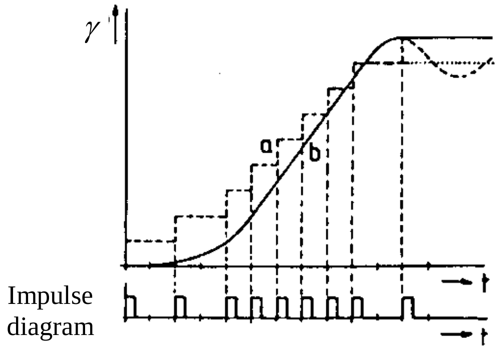


Fig. 8.3-4: Delayed last step: According to the current impulses (below) the wanted rotor position (dashed line *a*), expressed as rotational angle γ, is followed by the real motion (solid line b). At the maximum overshoot of step *N*-1 the last current impulse is commanded, hence avoiding the overshoot at step *N* (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987).

Electronic damping:
a) Delayed last step:
If the current impulse for the last step is given delayed during the maximum overshoot of the step before, the rotor will have lost already a part of its kinetic energy and will come to rest without further overshoot (Fig. 8.3-4).
b) Back phase damping:
The final step N+1 is a step into opposite direction of motion to brake the rotor at the step N.

Facit:
Uni-polar and bipolar impulse current feeding is used to operate the stepper motors stepwise, but always in open-loop. The drive unit must avoid overshoot at the last step and should be able to realize high switching frequencies.

8.4 Torque characteristics of stepper motors

8.4.1 Static torque characteristic

Like in conventional feed-forward operated synchronous machines, operated by voltage supply, there exist two kinds of static torque components:

a) **synchronous torque M_{syn}** (interaction of rotor magnetic field with stator current in the stator winding),
b) **reluctance torque M_{rel}** (interaction of stator magnetic field of stator currents, which is modulated by rotor reluctance, with stator currents in the stator winding).

Cogging torque:
In Fig. 8.4.1-1 for a **PM-stepper motor** the synchronous torque due to the interaction of the rotor PM field with the stator currents is depicted. The reluctance torque in this case is given by the interaction of the rotor PM field with its modulated field component due to the stator slotting. This cogging torque occurs already at no load (= stator open circuit) and is an unwanted effect. The resulting torque of the PM stepper motor is therefore a superposition of both torque components. The stator field is not modulated by the rotor, as the rare earth rotor PM material has a permeability of nearly unity, so there is no rotor reluctance active for the stator field.

Holding torque M_H:
The holding torque is the maximum torque, with which an excited stepper motor can be loaded without performing a continuous motion. In Fig. 8.4.1-1 this torque corresponds with the static pull out torque M_{p0} of an electrically excited synchronous machine, which occurs at a load angle ϑ of ±90° (el.). The disturbing influence of the cogging torque increases M_H slightly and shifts this value to $|\vartheta| < 90°$. If the stator slot openings are small, this influence may be neglected.

Detent torque M_D:
The detent torque is the maximum torque, with which an un-excited stepper motor can be loaded without performing a continuous motion. Like in Fig. 8.4.1-1, this torque corresponds in PM and hybrid stepper motors with the cogging torque.

The stable operation range for PM stepper motors without VR or cogging effect is given as $-90° \le \vartheta \le 90°$ (Fig. 8.4.1-2a).
In **VR motors** only the reluctance torque M_{rel} exists, when the stator winding is excited. This torque is essentially determined by rotor slot width and depth. No detent torque exists.

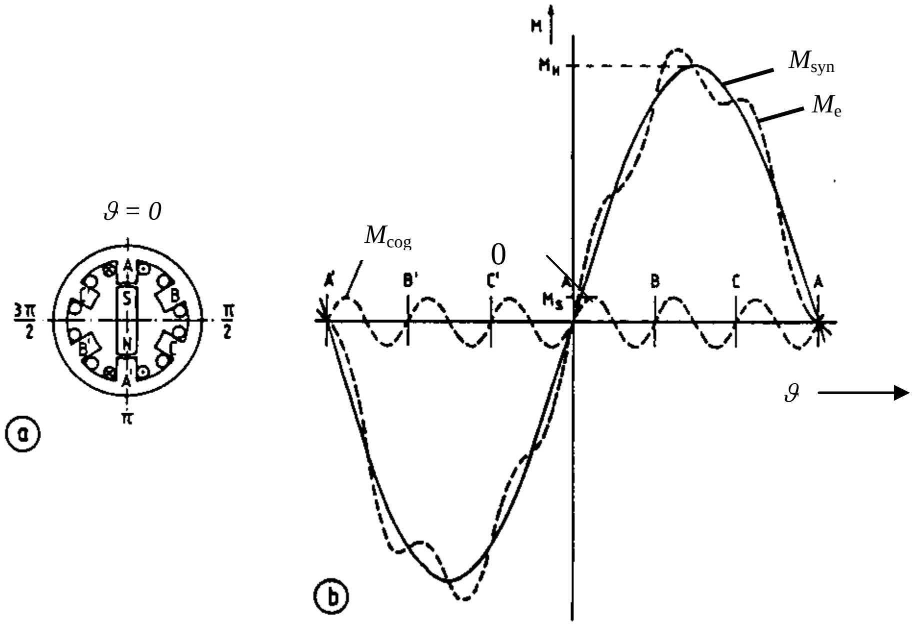
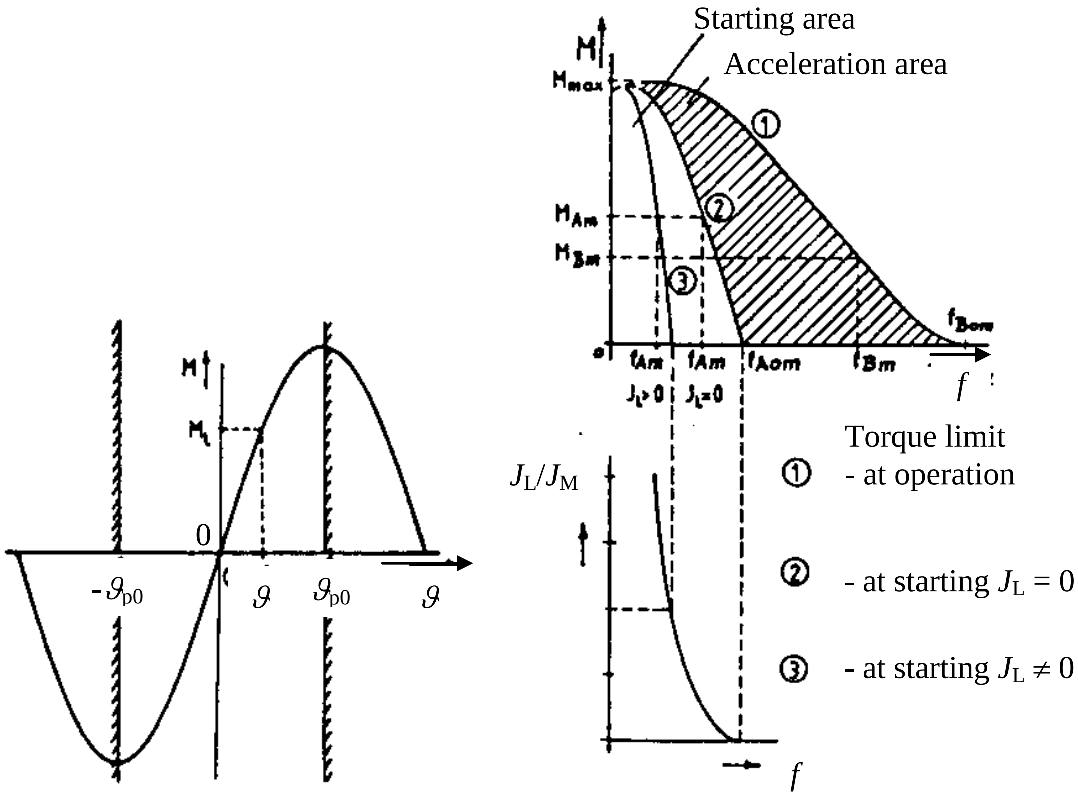


Fig. 8.4.1-1: Three-phase, 2-pole PM-stepper motor (m = 3, $2p$ = 2): a) cross section: Rotor S-pole at position A, load angle is zero, b) static torque for increasing/decreasing load angle. For the cogging torque six periods with the positions A, B, C, A´, B´, C´ exist. (*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)



a) b)

Fig. 8.4.1-2: a) Static load angle ϑ of a stepper motor, that is loaded at the shaft with a torque M_s, b) dynamic torque characteristic of a stepper motor in dependence of the stepping frequency f : 1: Torque limit at operation with constant f, 2: Torque limit at no-load starting (no load coupled: J_L = 0), 3: as 2, but with coupled load inertia J_L > 0 (J_L, J_M: Load and motor rotor inertia)
below: Admissible load inertia for starting with zero load torque
(*Stölting et al., El. Kleinmaschinen, Teubner,* 1987)

8.4.2 Dynamic torque characteristics

Torque limit at operation:
The holding torque can only be produced, if the stator current rises to the rated value. As this needs some time due to the electric time constant of the winding, the holding torque can be only produced at **stand still** (n = 0). When the stator winding of the rotating motor (rotational speed n) is supplied with current impulses with the stepping frequency

$$f = z \cdot n \quad , \qquad (8.4.2\text{-}1)$$

the average current per phase is smaller than the maximum (rated) value at constant voltage operation due to the current rise time. Hence the average motor torque is also smaller and so the admissible load torque decreases, too (curve 1 in Fig. 8.4.1-2b). It decreases with increasing stepping frequency, as due to the rising speed the duration of the current impulses decrease ~ $1/n$. The current rise time t_r becomes a more dominating part of the duration of the current impulse, and the average current per impulse decreases with $1/n$ or with $1/f$.

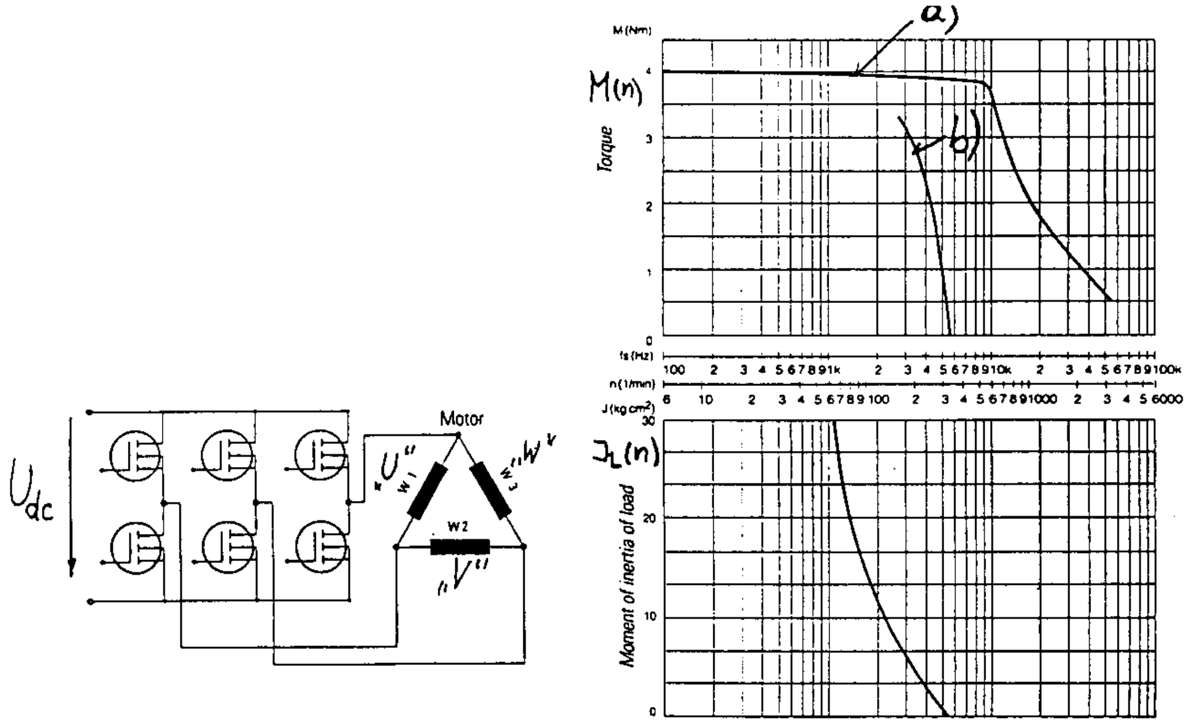
Torque limit at starting:
Starting the motor means to synchronize the rotor movement with the stepwise movement of the stator field. In Chapter 3 for asynchronously starting PM-machines it was stated, that the slip of the rotor with respect to the moving stator field must get smaller than a critical slip to allow synchronization of the rotor movement with the moving stator field. That critical slip decreased with increasing inertia and stator frequency.
Here the starting torque M_e is either the synchronous or the reluctance torque or both. According to *Newton´s* equation $(J_L + J_M) \cdot d(2\pi n)/dt = M_e - M_s$ (J_L, J_M: load and motor rotor inertia) the motor can start at a certain load inertia J_L with only a certain upper limit of the stepping frequency. This stepping frequency, supplied by the driving electronics, must be sufficiently low to allow safe motor synchronization. Hence for a given load inertia the maximum possible loading torque M_s at a given stepping frequency f for safe synchronization is given as a torque limit at starting (Fig. 8.4.1-2b, curve 3). The lower the load inertia, the higher the load torque at starting may be.
The maximum admissible load torque occurs at J_L = 0 (Fig. 8.4.1-2b, curve 2). From curves 2 and 3 for zero load torque the admissible load inertia in relation to the motor inertia for starting is determined in dependence of the starting frequency (Fig. 8.4.1-2b: $J_L/J_M(f)$).

Acceleration operation:
Once the motor is synchronized, it may be operated for a certain load torque also at higher stepping frequencies without losing steps (shaded area in Fig. 8.4.1-2b), hence with a faster rotor movement.

Example 8.4.2-1:
Dynamic torque curve of Fig. 8.4.1-2b): With the load torque M_{Am} and the load inertia J_L a starting frequency of $f_{Am}(J_L>0)$ is possible. This frequency may be increased to $f_{Am}(J_L=0)$, if no additional load inertia is acting. The maximum starting frequency is at zero load torque: f_{A0m}. With the load torque M_{Bm} during operation a maximum frequency f_{Bm} may be reached. The maximum operational frequency is at zero load torque: f_{B0m}.

Example 8.4.2-2:
Hybrid stepper motor (Fig. 8.4.2-1): m = 3, delta connected, dc link voltage 325 V, Q_r = 50, micro-step-operation, 1000 steps per revolution, motor inertia J_M = 0.00022 kgm^2. The holding torque is M_H = 4.0 Nm at I_{rms} = 2.0A per phase.

- Stepping angle: $\alpha = 360° / 1000 = 0.36°$
- At J_L = 0 (= no load torque) the maximum admissible stepping frequency at starting is acc. to Fig. 8.4.2-1 (ii) b) f_{max} = 5.3 kHz. The corresponding speed is $n = f / z = (5300/1000) \cdot 60 = 318/\text{min}$.



(i) (ii)

Fig. 8.4.2-1: Hybrid stepper motor: (i) Three-phase bipolar power electronic circuit and delta-connected stator winding, (ii) dynamic torque characteristic: a) Torque limit at operation with constant *f*, b: Torque limit at no-load starting (no load coupled: J_L = 0), below: admissible load inertia for starting with zero load torque (*Berger-Lahr,* 1994)