3. PM synchronous machines with rotor cage

Source: Siemens AG, Germany
Two-pole PM synchronous machines with rotor cage

Two-pole PM synchronous machine with cage rotor and rare earth permanent magnets. Note non-magnetic (welded with non-magnetic steel) gap between N- and S-pole to reduce PM stray flux.

Source: Siemens AG, Germany
PM rotor flux concentration

Comparison of rotor PM arrangement for (left) surface mounted and (right) buried magnets; flux concentration factor $k_M$ depends on pole count $2p$

\[
k_M = \frac{2b_M}{\tau_p} = \frac{d_{ra} - d_{ri}}{d_{ra} \pi} = \frac{2p}{\pi} \left(1 - \frac{d_{ri}}{d_{ra}}\right)
\]

<table>
<thead>
<tr>
<th>Pole count $2p$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_M$</td>
<td>0.45 &lt; 1</td>
<td>0.9 &lt; 1</td>
<td>1.34 &gt; 1</td>
<td>1.78 &gt; 1</td>
</tr>
</tbody>
</table>
Flux concentration with special arrangement of rotor magnets is also possible for 4 pole machines: a) Axial cross section: 1: PM main flux, 2: PM stray flux, 3: permanent magnets (PM), 4: non-magnetic flux barrier to reduce PM stray flux, 5: rotor cage, b) side view.

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<tr>
<td>$k_M$</td>
<td>1.27 &gt; 1</td>
<td>1.9 &gt; 1</td>
</tr>
</tbody>
</table>

Source: Siemens AG, Germany
4- and 6-pole PM synchronous machines with rotor cage

4- and 6-pole machine with buried ferrite magnets: $B_R = 0.4$ T, $H_C = 300$ kA/m at 20°C, magnet height 15 mm, air gap 1 mm. Air gap flux density at no-load is calculated, assuming ideal iron. The closed loops of flux lines pass one magnet (height $h_M$) and two times the air gap $\delta$.

The closed loops of flux lines pass one magnet (height $h_M$) and two times the air gap $\delta$.

$$B_\delta = \frac{B_R}{1 + \frac{\mu_M \cdot 2\delta}{k_M \cdot \mu_0 \cdot h_M}}$$

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<tbody>
<tr>
<td>$k_M$</td>
<td>1.27</td>
<td>1.9</td>
</tr>
<tr>
<td>$B_\delta / T$</td>
<td>0.43</td>
<td>0.6</td>
</tr>
</tbody>
</table>
**d- and q-axes of PM synchronous machine rotor**

Simplified 2-pole arrangement with buried rotor magnets:

The stator \(d\)-flux has to cross the rotor magnets, thus yielding a lower \(d\)-inductance (left). The stator \(q\)-flux can avoid the rotor magnets, which results in a higher \(q\)-inductance (right).
Phasor diagram of PM machine with buried magnets \((L_d \neq L_q)\) with neglected stator resistance for generator operation. At positive \(U_{sd}\) and \(U_{sq}\) the current components \(I_{sd}\) and \(I_{sq}\) are negative. The load angle \(\vartheta\) is positive.
OPERATION at "rigid" grid: \( U_s = \text{constant} \)

We choose: \( d \)-axis = Re-axis, \( q \)-axis = Im-axis of complex plane:

\[
U_s = U_{sd} + jU_{sq} \quad I_s = I_{sd} + jI_{sq} \quad U_p = jU_p
\]

\( R_s = 0: \)

\[
U_s = jX_d I_{sd} + jX_q I_{sq} + U_p \quad \Rightarrow \quad U_s = jX_d I_{sd} - X_q I_{sq} + jU_p
\]

Active power \( P_e \):

\[
P_e = m_s U_s I_s \cos \varphi = m_s \cdot \text{Re}\left\{U_s I_s^*\right\} = m_s \left( U_{sd} I_{sd} + U_{sq} I_{sq} \right)
\]

\[
P_e = m_s (-X_q I_{sq} I_{sd} + X_d I_{sd} I_{sq} + U_p I_{sq})
\]

Electro-magnetic torque:

\[
M_e = \frac{P_m}{\Omega_{\text{syn}}} = \frac{P_e}{\Omega_{\text{syn}}} = \frac{m_s}{\Omega_{\text{syn}}} \cdot \left( U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right)
\]

- Two torque components:
  a) prop. \( U_p \) as with round rotor machines
  b) "Reluctance”torque due to \( X_d \neq X_q \). NO rotor excitation is necessary.
Torque as function of load angle $\vartheta$

$\bar{U}_s = jX_d I_{sd} - X_q I_{sq} + jU_p \Rightarrow \begin{cases} U_{sd} = -X_q I_{sq} \\
jU_{sq} = jX_d I_{sd} + jU_p \Rightarrow I_{sd} = \frac{U_{sq} - U_p}{X_d} \end{cases}$

$U_s = U_{sd} + jU_{sq} \begin{cases} U_{sd} = U_s \sin \vartheta \\
U_{sq} = U_s \cos \vartheta \end{cases}$

$M_e = \frac{m_s}{\Omega_{\text{syn}}} \cdot \left( U_p \cdot I_{sq} + (X_d - X_q) \cdot I_{sd} \cdot I_{sq} \right) = \frac{m_s}{\Omega_{\text{syn}}} \left( -\frac{U_p U_s \sin \vartheta}{X_q} - \frac{X_d - X_q}{X_d X_q} \cdot U_s \sin \vartheta \cdot (U_s \cos \vartheta - U_p) \right)$

$M_e = -\frac{p \cdot m_s}{\omega_s} \left( \frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$
The torque $M_e$ of synchronous PM machine, operated from grid with fixed voltage and frequency, is determined by permanent magnet torque $M_{\text{syn}}$, which depends linear on stator voltage, and by reluctance torque $M_{\text{rel}}$, which depends on square of stator voltage ($L_d < L_q$).

$$M_e = -\frac{p \cdot m}{\omega_s} \left( \frac{U_s U_p}{X_d} \sin \vartheta + \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\vartheta \right)$$

Demand:

$$\left. \frac{dM_e}{d\vartheta} \right|_{\vartheta=0} \geq 0: \quad \frac{U_p}{X_d} + U_s \cdot \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \geq 0$$

At $X_q = 2X_d$ a minimum back EMF of $U_p \geq U_s / 2$ is necessary!
Asynchronous operation due to pull-out

- When the machine is loaded higher than the maximum electromagnetic torque $M_{p0}$ (pull-out torque), the rotor is pulled out of synchronism by the load torque.

- In the asynchronously running rotor the rotor currents are induced in rotor cage, generating an asynchronous torque.

- In motor operation the stator field, running at synchronous speed, is now faster than the turning rotor.

- So the stator field will oppose the rotor permanent magnet flux at certain instants ("phase opposition"), causing danger of irreversible demagnetization of rotor magnets.

- At phase opposition a large current is consumed:

$$I_s = \frac{U_s + U_p}{X_d} > I_N$$

- The rotor cage self-field opposes the stator field and reduces the inner rotor field, hence shielding inner PM against demagnetization.
Asynchronous starting

- Three-phase AC stator current system $I_s$ with frequency $f_s$: causes field wave

$$B_s(x,t) = \hat{B}_s \cos\left(\frac{x_s \pi}{\tau_p} - \omega_s t\right)$$

- Induces rotor cage: AC rotor current system $I'_r$ with frequency $f_r = s f_s$: yields asynchronous starting torque $M_{\text{asyn}}$

- Rotor permanent magnet field rotates with rotor speed $n = (1-s)n_{\text{syn}}$:

$$B_p(x,t) = \hat{B}_p \cos\left(\frac{x_s \pi}{\tau_p} - (1-s) \cdot \omega_s t\right)$$

- Induces stator winding: Causes three-phase AC stator current system $I_p$ with frequency $f = (1-s)f_s$: yields asynchronous braking torque $M_p$

- Interaction between $B_s$ and $B_p$ causes pulsating torque $M_p$ with $\omega_p$

$$M_p(t) = \hat{M}_p(s) \cdot \sin(s \cdot \omega_s t)$$

$$\omega_p = \omega_s - (1-s) \cdot \omega_s = s \cdot \omega_s$$
Asynchronous braking torque $M_p$

- Voltage equation for induced additional current system $I_p$ in the stator winding:

$$0 = R_s I_p + j \omega \Psi_s / \sqrt{2} = R_s I_p + j \omega L_d I_p + j \omega \hat{\Psi}_p / \sqrt{2}$$

Assumption: $L_d = L_q$

- Additional current system $I_p$ in the stator winding with $\omega = (1 - s) \cdot \omega_s$

$$I_p = \frac{-j \omega \hat{\Psi}_p / \sqrt{2}}{R_s + j \omega L_d}$$

- Losses in the stator winding due to current system $I_p$:

$$M_p = -\frac{P_{Cu,p}}{\Omega_m} = -\frac{p \cdot m \cdot R_s \cdot ((1 - s) \cdot \omega_s)^2 \cdot \hat{\Psi}_p^2 / 2}{R_s^2 + (\omega_s (1 - s) \cdot L_d)^2} = -\frac{(1 - s) \cdot \omega_s \cdot p \cdot m \cdot R_s \cdot \hat{\Psi}_p^2 / 2}{R_s^2 + (\omega_s (1 - s) \cdot L_d)^2}$$

$$M_p = -\frac{\omega \cdot p \cdot m \cdot R_s \cdot \hat{\Psi}_p^2 / 2}{R_s^2 + (\omega L_d)^2}$$

- Maximum braking torque ($m = 3$):

$$dM_p / d\omega = 0 : n^* = \omega^* / (2\pi p) = R_s / (2\pi p \cdot L_d)$$

$$M_{p,\text{max}} = M_p(\omega^*) = \frac{3 p \cdot \hat{\Psi}_p^2}{2 \cdot 2 L_d} = \frac{3 p \cdot U_{pN}^2}{2 \cdot \omega_N^2 L_d}$$
Torque at asynchronous starting

Average asynchronous starting torque $M_{\text{asyn}}$ and permanent magnet braking torque $M_p$, yielding saddle shaped resulting torque $M_{\text{res}}$

$M_{p,\text{max}} = M_p(\omega^*) = \frac{3p}{2} \cdot \frac{\Psi_p^2}{2L_d} = \frac{3p}{2} \cdot \frac{U_{pN}^2}{\omega_N^2 L_d}$
Pulsating torque amplitude

- Pulsating torque amplitude due to stator field \( B_s (x,t) = \hat{B}_s \cos \left( \frac{x_s \pi}{\tau_p} - \omega_s t \right) \)

and additional field \( B_p (x,t) = \hat{B}_p \cos \left( \frac{x_s \pi}{\tau_p} - (1 - s) \cdot \omega_s t \right) \)

- Additional field amplitude \( B_p \) is proportional to induced current \( I_p \).

\[
\hat{M}_P(s) = F \cdot z \cdot \frac{d_{si}}{2} = \frac{\hat{B}_s}{\sqrt{2}} \cdot \frac{\hat{I}_p(s)}{\sqrt{2}} \cdot l_{Fe} \cdot z \cdot \frac{d_{si}}{2} = \frac{\hat{B}_s}{\sqrt{2}} \cdot I_p(s) \cdot l_{Fe} \cdot 2mN_s k_{ws} \cdot \frac{p \tau_p}{\pi}
\]

\[
I_p(s) = - \frac{j \omega(s) \hat{\Phi}_p / \sqrt{2}}{R_s + j \omega(s) L_d} \approx - \frac{\omega(s) \hat{\Phi}_p / \sqrt{2}}{\omega(s) L_d} = \frac{U_p}{\omega_s L_d} = \frac{U_p}{X_d}
\]

\[
\hat{M}_P(s) = m \cdot p \cdot N_s k_{ws} \cdot \frac{(2/\pi) \tau_p l_{Fe} \hat{B}_s}{\sqrt{2}} \cdot \frac{U_p}{X_d} = m \cdot p \cdot \frac{\omega_s}{\omega_s} N_s k_{ws} \cdot \frac{\Phi_h}{\sqrt{2}} \cdot \frac{U_p}{X_d}
\]

\[
\hat{M}_P(s) = \frac{m \cdot p}{\omega_s} \cdot U_h \cdot \frac{U_p}{X_d} \approx \frac{m \cdot p}{\omega_s} \cdot U_s \cdot \frac{U_p}{X_d}
\]

\[
U_h = \omega_s N_s k_{ws} \cdot \Phi_h / \sqrt{2} \approx U_s
\]
Calculated asynchronous starting

Comparison of
- Induction machines (ASM),
- Synchronous reluctance machine (SRM)
- Permanent magnet synchronous machines with rotor cage (PSM)

ASM has highest starting torque and no pulsating torque (apart from the pulsation due to the switching transient at the start).

SRM has a lower average starting torque due to the GOERGES-saddle and a pulsating torque with \( \left| f_s - f_3 \right| = 2sf_s \).

PSM has the lowest average starting torque due to the braking torque \( M_p \) and a pulsating torque with \( sf_s \). In case of buried magnets the SRM-torque effect (GOERGES) adds!

Source: Bunzel, E.; Elektrie, 1987
Synchronization after asynchronous start-up

- At synchronous speed the slip is zero: The asynchronous torque of the cage is zero.
- The speed of PM rotor is synchronous speed, so the frequency of the stator current system $I_p$ is: $f_3 = (1 - s) f_s = f_s$
- Hence current $I_s$ and $I_p$ unite as the total stator current $I_s$ at synchronous speed.
- The pulsating torque becomes the constant synchronous torque!

- At $s << 1$ the frequency $s \cdot f_s$ corresponds with a very slowly increasing load angle $\vartheta(t)$: $M_P(t) = \hat{M}_P(s) \cdot \sin(s \cdot \omega_s t + \vartheta) \Rightarrow \hat{M}_e \cdot \sin(\vartheta)$
- With the assumption $R_s = 0$, $L_d = L_q$ we get:

$$M_e = -\frac{p \cdot m \cdot U_s U_p}{\omega_s X_d} \cdot \sin(\vartheta)$$
Torque components shortly before synchronization

- At \( s << 1 \) we get a very slowly changing load angle \( \mathcal{A}(t) \):
- Here: \( R_s \neq 0, L_d \neq L_q \)

Torque at low slip, rotor passing from \(-180^\circ\) to \(180^\circ\), when slipping by one pole pair
Synchronization - Critical slip (1)

- Slipping rotor with small slip, passing the load angle from \(-180^\circ\) to \(180^\circ\):

\[
\Delta \gamma = \gamma_s - \gamma_r = \theta
\]

\[
\Delta \dot{\gamma} = \dot{\gamma}_s - \dot{\gamma}_r = \Omega_{\text{syn}} - \Omega_m(t) = s(t) \cdot (\omega_s / p)
\]

\[
\Delta \ddot{\gamma} = \ddot{\gamma}_s - \ddot{\gamma}_r = \dot{s}(t) \cdot (\omega_s / p) \quad \ddot{\gamma}_s = \dot{\Omega}_{\text{syn}} = 0
\]

\[
(J_M + J_L) \frac{d^2 \gamma_r}{dt^2} = M_e - M_s \quad \Rightarrow \quad -J \cdot \frac{\omega_s}{p} \cdot \frac{ds}{dt} = M_e(\theta(t)) - M_s
\]

Assumptions:
(1) No load torque: \(M_s = 0\),
(2) No reluctance effect: \(L_d = L_q\): \(M_{\text{rel}} = 0\), \(M_{\text{asyn}, \sim} = 0\).
(3) At small slip \(s \ll 1\): Asynchronous torque is nearly zero \(M_{\text{asyn, av}} \approx 2s \cdot M_{b} / s_b \approx 0\).

Hence the torque during synchronization is only:

\[
M_e = M_{\text{syn}} + M_{\text{rel}} + M_{\text{asyn}} \approx M_{\text{syn}} = -M_{p0} \cdot \sin \theta
\]
Synchronization - Critical slip (2)

- At \( t = 0 \): Rotor position relative to stator field is: \( \vartheta_1 = -\pi \); rotor slip \( s_1 \ll 1 \).

- Rotor passes from \( \vartheta_1 = -\pi \) to \( \vartheta_2 = 0 \) during the time \( t_s = 1/(2 \cdot s_{av} \cdot f_s) \), being accelerated by \( M_{\text{syn}} \), to the smaller slip \( s_2 < s_1 \).

- Average slip during that time is \( s_{av} = (s_1 + s_2)/2 \).

- The final chance to be pulled into synchronism is at \( t = t_s \), because afterwards \( M_{\text{syn}} \) becomes negative. So if \( s_2 = 0 \), the rotor synchronizes.

- The corresponding slip \( s_1 \) is then the maximum admissible slip \( s_{cr} \) for synchronization.

\[
\int_{0}^{t_s} - J \cdot \frac{\omega_s}{p} \cdot \frac{ds}{dt} \cdot dt = \int_{s_1}^{s_2} - J \cdot \frac{\omega_s}{p} \cdot ds = J \cdot \frac{\omega_s}{p} \cdot (s_1 - s_2) = J \cdot \frac{\omega_s}{p} \cdot s_1 = \int_{0}^{t_s} M_{\text{syn}}(\vartheta(t)) \cdot dt =
\]

\[
= \frac{t_s}{\pi} \int_{-\pi}^{0} - M_{p0} \cdot \sin(\vartheta) \cdot d\vartheta = \frac{t_s}{\pi} \cdot 2M_{p0} = \frac{1}{2\pi \cdot s_{av} f_s} \cdot 2M_{p0} = \frac{1}{2\pi \cdot (s_1/2) \cdot f_s} \cdot 2M_{p0}
\]

- Critical slip \( s_{cr} \) for synchronization:

\[
s_1 = s_{cr} = \frac{1}{\Omega_{\text{syn}}} \cdot \sqrt{\frac{4M_{p0}}{J \cdot p}} = \frac{1}{\omega_s / p} \cdot \sqrt{\frac{2P_{p0}}{\pi \cdot (J_M + J_L) \cdot f_s}}
\]

\[
s_{cr} \approx \frac{0.5}{\omega_s / p} \cdot \sqrt{\frac{P_{p0}}{(J_M + J_L) \cdot f_s}}
\]

\[\sqrt{2/\pi} = 0.8 > 0.5\]
Pull-in and pull-out of PM synchronous machines with rotor cage

Condition for synchronization is therefore: Slip is less than CRITICAL SLIP

\[ s_{cr} \approx \frac{0.5}{\omega_s / p} \sqrt{\frac{P_{p0}}{(J_M + J_L) \cdot f_s}} \]

**Example:**

Barium ferrite six-pole synchronous permanent magnet motor, shaft height 160mm, totally enclosed, fan-cooled:

50 ... 150 Hz, 1000 ... 3000/min, 125 ... 380 V, Y, 42 A, rated torque 49.7 Nm, overload capability 150%. total momentum of inertia (motor and load): 1.5 kgm²

\[
s_{cr} = \frac{0.5}{2\pi \cdot 50/3} \cdot \sqrt{\frac{7806}{50 \cdot 1.5}} = 0.049 = 4.9\% 
\]
Measured time function of starting torque of PM synchronous machine with squirrel cage rotor at fixed stator voltage and frequency:

a) Synchronisation of motor visible,

b) Increased load inertia by factor 2.4: No synchronisation is possible!

The pulsating torque with $s f_s$ is clearly visible!
Asynchronous start-up of electrically excited (big) synchronous motors

- Laminated rotor poles need starting copper cage (= heat sensitive)
- Massive rotor poles do not need rotor cage: massive pole surface (conductive iron) is carrying eddy currents

**Advantages:**

a) no heat expansion problem of cage!

b) 10 ... 20 times bigger iron rotor resistance shifts break down slip to nearly unity = much bigger starting torque.

**Goerges** phenomenon due to rotor saliency and short-circuited field winding at slip 1/2!
Massive pole synchronous motors

- 4 pole motor
- Screwed poles
- During start-up the field winding is short-circuited:
  a) to avoid induced over-voltage, which is deadly for the power electronics of the excitation rectifier
  b) to avoid a braking torque $M_p$
  c) to add a small asynchronous torque of the field winding as a second “cage”

Source: Andritz Hydro, Austria
Outer-rotor PM synchronous machines with rotor cage

Cross section of outer rotor 4-pole PM synchronous motor with ferrite surface mounted magnets and squirrel cage: 1: inner stator AC three-phase winding, 2: outer rotor iron back, 3: squirrel cage, 4: ferrite magnets

Application: Textile fibre fabrication

Source: Siemens AG, Germany
PM synchronous motor group drives for synthetic thread fabrication

Source: Siemens AG, Germany