5. Inverter-fed induction machines

Source: ELIN EBG Motoren GmbH, Austria
5. Inverter-fed induction machines

5.1 Basic performance of variable-speed induction machines

5.2 Drive characteristics of inverter-fed standard induction motors

5.3 Features of special induction motors for inverter operation

5.4 Influence of inverter harmonics on motor performance
5. Inverter-fed induction machines

5.1 Basic performance of variable-speed induction machines

Source: ELIN EBG Motoren GmbH, Austria
Stator winding air gap flux density distribution

**Voltage-source inverters** with DC voltage link:

400 V grid, \( U_d = \left( \frac{3}{\pi} \right) \cdot \sqrt{2} \cdot 400 = 540 \) V

With neglected stator resistance (\textit{Kloss} function) break down torque is independent of frequency, if voltage fundamental varies linear with frequency:

\[
U_s \sim f_s \quad f_r = s \cdot f_s
\]

\( R_s = 0 \):

\[
\frac{M_e}{M_b} = \frac{2}{s_b + \frac{s}{s_b}} = \frac{2}{\frac{\omega_b}{\omega_r} + \frac{\omega_r}{\omega_b}}
\]

\[
\frac{U_s}{\omega_s} = \frac{\Psi_s}{\sqrt{2}} \quad s_b = \frac{s_b f_s}{s} \quad \omega_b = \frac{f_b}{f_r}
\]
Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\varphi_{L1}(t)$ at terminal L1 varies with that PWM signal

b) Difference of two terminal potentials delivers line-to-line voltage $u_{L1-L2}(t)$

\[ f_s = \frac{1}{T_s} \]

\[ f_p = \frac{2}{T_{\text{switch}}} \]

Source:
Kleinrath, H.; Springer, 1980
Constant power range

Voltage stays constant: Power \( \sim U \cdot I = \text{const} \); torque decreases inverse to frequency: 
\[
M_{eN} = P_N \cdot \frac{p}{(\omega_s - \omega_{r,N})} \rightarrow M_e \approx M_{mot} = P_N \cdot \frac{p}{\omega_s} \Leftrightarrow M_e \sim I_s \Psi_s \sim I_s U_s / \omega_s
\]

Breakdown torque decreases inverse to square of frequency:
\[
M_b = \frac{m_s}{2} \frac{p}{\omega_s} U_{s,\text{max}}^2 \frac{1 - \sigma}{\sigma \omega_s L_s} \approx 1 / \omega_s^2
\]

\[
\Psi_s = k_{ws} N_s \Phi
\]

Constant rated power range is limited, where decreasing breakdown torque reaches torque demand for rated power: e.g. \( 2.7f_N \).

Usually maximum frequency \( f_{s,\text{max}} \) is defined, where \( M_b / M_N = 1.6 \) (60% overload margin).
Variable speed torque-speed curves with voltage limit

- Constant torque
- Constant power
- Voltage limit

\[ U_s \text{ at } R_s > 0 \]

\[ U_{s1} = I_{sN} R_s \]

Offset

\[ \sim \frac{1}{f_s^2} \]

\[ \sim \frac{1}{f_s} \]
Variable speed induction motor at \( R_s > 0 \)

a) Linear rise of voltage with frequency \( U_s \sim f_s \):
   Breakdown torque decreases with decreasing speed

b) Additional voltage offset \( U_{s1} = R_s I_s \) at zero speed to compensate resistive voltage drop: Breakdown torque stays above rated torque

c) \( U_s(f_s) \)

\[
0 \leq f_s \leq f_N:
\begin{align*}
a) & \quad U_s \sim f_s \\
b) & \quad U_s = (U_N - U_{s1}) \cdot \frac{f_s}{f_N} + U_{s1} \\
c) & \quad U_s(f_s)
\end{align*}
\]
5. Inverter-fed induction machines

5.2 Drive characteristics of inverter-fed standard induction motors

Source: Siemens AG
Loss variation in inverter-fed standard induction motors

- Cooling air flow of shaft-mounted fan rises linear with speed: \( \dot{V} \sim n / n_N \sim f_s / f_N \)
- Iron losses in stator: rise with square of flux, with frequency exponent \( x \approx 1.8 \):
  \[
P_{Fe} \sim (\Psi / \Psi_N)^2 \cdot (f_s / f_N)^x \quad \text{Hysteresis} \sim f, \text{eddy currents} \sim f^2
\]
- Friction losses (bearings \( n \ldots n^2 \)), windage losses (fan \( n^3 \)): exponent \( y = 2.5 \ldots 3 \):
  \[
P_{fr+w} \sim (n / n_N)^y \sim (f_s / f_N)^y
\]
- Stray load losses (caused by space harmonic effects): \( z \approx 1.5...2 \)
  \[
P_{ad1} \sim (n / n_N)^z \cdot (I_s / I_N)^2
  \begin{align*}
  &\quad \text{- tooth flux pulsations} \\
  &\quad \text{- harmonic rotor currents} \; I_{r_v} \\
  &\quad \text{- harmonic inter-bar currents} \; I_{q_v}
  \end{align*}
\]
Variable air flow in inverter-fed standard induction motors with shaft-mounted fan

- $f_s > f_N$: **Flux weakening**: $\Psi \sim 1/f_s$:
  Iron & copper losses constant;
  frequency limit: breakdown/rated power reaches 160%.

- $f_{th} < f_s < f_N$: **Constant torque range**:
  Iron losses and air flow decrease: at $f_{th}$, e.g. $f_{th} \sim 0.5f_N$ thermal limit.

- $f_s < f_{th}$: **Reduced torque operation**:
  Reduction of current to decrease copper losses due to low air flow
Variable speed induction motor with shaft-mounted fan cooling

- Cooling air flow of shaft-mounted fan rises linear with speed: \( \dot{V} \sim \frac{n}{n_N} \sim \frac{f_s}{f_N} \)

Source: Siemens AG
Y-D (star-delta) to increase power by speed

- **Star**: Phase voltage \( U_Y = \frac{U_{inv}}{\sqrt{3}} \), phase current \( I_Y = \text{inverter current} \ L_{inv,Y} \).
- **Delta**: Phase voltage \( U_\Delta = \text{Line-to-line voltage} \ U_{inv} \), Phase current \( I_\Delta = \frac{L_{inv,\Delta}}{\sqrt{3}} \).

\[
U_Y \sqrt{3} = U_\Delta \quad f_{s,\Delta} = \sqrt{3} f_{s,Y} \quad \text{for same flux} \implies \text{Inverter current:} \quad I_Y \sqrt{3} = I_\Delta
\]

Increase of phase voltage in “delta” by 73% allows increase of speed by 73% from 50 Hz to 87 Hz for constant torque. So power increases by 73%, but also current!
Motor power increase with delta connection

*Example:* 4-pole induction motor, power factor 0.85, efficiency 90%

<table>
<thead>
<tr>
<th>Stator Winding Connection</th>
<th>Star</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter maximum output voltage line-to-line $U_{LL,\text{max}}$</td>
<td>400 V</td>
<td>400 V</td>
</tr>
<tr>
<td>Motor maximum phase voltage $U_{s,\text{max}}$</td>
<td>230 V</td>
<td>400 V</td>
</tr>
<tr>
<td>Motor frequency $f_s = f_N$ at $U_{s,\text{max}}$</td>
<td>50 Hz</td>
<td>87 Hz</td>
</tr>
<tr>
<td>Motor rated phase current $I_{sN}$</td>
<td>100 A</td>
<td>100 A</td>
</tr>
<tr>
<td>Motor rated line current $I_{sN}$</td>
<td>100 A</td>
<td>173 A</td>
</tr>
<tr>
<td>Motor rated torque $M_N$</td>
<td>336 Nm</td>
<td>336 Nm</td>
</tr>
<tr>
<td>Motor output power $P = 2\pi f_N M_N / p$</td>
<td>52.3 kW</td>
<td>90.6 kW</td>
</tr>
<tr>
<td>Inverter power rating $S = \sqrt{3} U_{LL,\text{max}} I_{sN}$</td>
<td>69 kVA</td>
<td>119.4 kVA</td>
</tr>
</tbody>
</table>
Increase of constant torque range & output power

By switching the winding from a) **star** to b) **delta** connection!

Diagram showing the relationship between 

- $\frac{P}{P_N}$
- $\frac{M_{\text{mot}}}{M_N}$
- $\frac{U_s}{U_N}$

with respective operating points:

- a) $400 \, \text{V Y} \, 50\, \text{Hz}$
- b) $230 \, \text{V Δ} \, 50\, \text{Hz}$
- $400 \, \text{V Δ} \, 87\, \text{Hz}$

Key parameters:

- $f_{th}$
- $f_{NY}$
- $f_{N\Delta}$
- $f_{\text{max}}$
**Application: Variable speed drive for pumping**

**Example:**
The flow rate $Q$ shall be changed!

- **Flow rate variation**
  - **a) by throttling:** Pump operates at fixed speed
  - **b) by speed variation:** Lower pump speed = lower flow rate. No throttling losses = better overall efficiency. **Up to 60% lower total losses!**

Source: KSB, Frankenthal
Variable speed pumping reduces losses

Electric energy consumption (%)

100%

Throttling

Speed variation of pump

Energy savings 60%

Flow rate (%)

Source: Siemens AG, Pictures of the future
Variable speed elevators save energy

**Elevator:**

1 Ton pay-load, 17 m delivery head, 5 stops

a) **Old drive:**
   - Fixed speed drive at the grid: 8.8 kW-induction motor,
   - Speed variation by pole-changing („slow-fast“)
   - Standard gear
   - Mechanical braking during stops (Braking energy as heat)

b) **New drive:**
   - 7.5 kW induction motor with raised efficiency
   - Speed variation via inverter-feeding
   - Low loss gear with synthetic oil
   - Energy recovery during braking via the inverter (with Active Front End)

**Total energy saving per ride: 81 % at full load (best case)**

**Pay-back time for the more expensive “New drive” 5.5 years at a rate of 400 daily rides!**

*Source: ZVEI, Frankfurt, Germany*
5. Inverter-fed induction machines

5.3 Features of special induction motors for inverter operation

Source: ELIN EBG Motoren GmbH, Austria
Induction motor design: Grid vs. inverter-operation

- Equivalent circuit motor parameters for line- vs. inverter operated induction machines

<table>
<thead>
<tr>
<th>Aim</th>
<th>Grid / Inverter</th>
<th>air gap</th>
<th>$L_\sigma$</th>
<th>$R_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big breakdown torque</td>
<td>-</td>
<td>small</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Small magnetizing current</td>
<td>small</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>low starting current</td>
<td>-</td>
<td>big</td>
<td>big</td>
<td></td>
</tr>
<tr>
<td>big starting torque</td>
<td>-</td>
<td>small</td>
<td>big</td>
<td></td>
</tr>
<tr>
<td>Low additional losses</td>
<td>big</td>
<td>big</td>
<td>small</td>
<td></td>
</tr>
</tbody>
</table>

- Low leakage: many slots, no skew, no deep slots, big slot openings
- Small magnetizing current: small air-gap, low iron saturation

**Line-operation:**
- Big starting torque: big current displacement, deep rotor slots, special rotor cage
- Good starting performance: skewing is necessary to minimize harmonic torque.

**Inverter operation:**
- Small current displacement: round or oval rotor slots
- Low additional losses: Avoid skewing to avoid inter-bar currents.
- Sufficient leakage: small slot openings, stray inductance must limit current harmonics.

$$M_b = \frac{m_s}{2} \frac{p}{\omega_s} U_{s,max}^2 \frac{1 - \sigma}{\sigma \omega_s L_s} \sim 1/\omega_s^2$$
Tooling machine: Variable speed main spindle drive

- Belt as mechanical gear for big torque at the spindle at low speed

Source: Siemens AG
Increase of constant power range by switching a mechanical gear

**Example:**

\(i = 4:\)

\(n_N = 500/\text{min}\)

\(n_{\text{max}} = 7500/\text{min}\)

\(n_N' = 125/\text{min}\)

Extension of constant power range from

500 … 7500 = 1:15

to 125 … 7500/\text{min} = 1:60

- Gear transfer ratio \(i > 1\): Increase of constant power range at low speed e.g. \(i = 4\)

Source: Siemens AG
Planetary gear

\[
\begin{align*}
1: \quad \omega \cdot r &= \omega_1 \cdot r_1 + \omega_2 \cdot r_2 \\
2: \quad \omega_3 \cdot r_3 &= \omega_1 \cdot r_1 - \omega_2 \cdot r_2
\end{align*}
\]

\[
\omega_1 = \frac{\omega \cdot r + \omega_3 \cdot r_3}{2 \cdot r_1}
\]

\[
i = \frac{\omega}{\omega_1} = \frac{r + r_3 \cdot (\omega_3 / \omega)}{2 \cdot r_1}
\]

Case I: \( \omega_3 = 0 \): \( i = \frac{r}{2r_1} \)

Case II: \( \omega_3 = \omega \): \( i = \frac{r}{2r_1} + \frac{r_3}{2r_1} \)

Special case:
\( r_2 = r \rightarrow r_1 = 2r, \quad r_3 = 3r \)

Case I: \( \omega_3 = 0 \): \( i = \frac{1}{4} \)

Case II: \( \omega_3 = \omega \): \( i = \frac{1}{4} + \frac{3}{4} = 1 \)

Source: NKE, Germany
Switching planetary gear with $i = 1$ and $i = 4$

1: gear housing  2: motor
3: sun wheel
4: planetary wheel
5: hollow wheel
6: planetary wheel support
7: switching unit
8: shifting sleeve
   position I: gear ratio 1: $i$
   position II: gear ratio 1: 1
9: motor shaft  10: gear shaft
11: housing  12: belt pulley

The motor has a holding brake at switched-off power.

Source: Siemens AG
Induction motors for wide field weakening range

- Over-sizing of motor: $M_b/M_N$ increased

- Increase of inverter current rating $I_s$: $N_s$ reduced,
  $I_sN_s = \text{const. for constant rated torque}$
  $= \text{increased conductor cross section } A_c \text{ at constant current}$
  $\text{density } J_s = I_s/A_c \text{ (slot fill factor } k_Q = N_cA_c/A_Q = \text{const.}, \ I_s)$
  $\text{inductance } L_s \sim (N_s)^2 \text{ decreased, } M_b \sim 1/L_s \text{ increased}$

- Star-delta switching of stator winding

- Series-parallel switching of stator winding
Star-delta switching of stator winding for same motor volume to increase constant power range by 1:3

Source: Huth, G.; ETG-Fachtagung, 1989

Ns reduced: \( U_{N,\text{mot}} < U_{N,\text{inv}} \)

a) 4-pole Y-connected motor with 66 turns
b) As a), but 170% higher turns number 111.

At DELTA connection 300% increased breakdown torque. So constant power operation with 9 kW up to 13500/min. Increased Δ-power NOT used!
Series-parallel switching of stator winding

High performance 4-pole induction motor with star-delta switch to enlarge constant power range, rated speed at Y: 500/min, rated frequency 16.7 Hz

Standard induction 4-pole motor, D connected winding, rated speed 1460/min, rated frequency 50 Hz

Source: Siemens AG

Factor “3”

No Y-Δ-switching used!
Series-parallel switching of stator winding (1)

Series:
- \( N_s \)
- \( \frac{N_s}{2} \)

Parallel:
- \( \frac{N_s}{2} \)

Parallel: \( P_{b)} = 2P_{a)} \) series

a) series \{ connection for same torque \( M_N \)
b) parallel
Series-parallel switching of stator winding (2)

- Increasing constant power range by factor 4
- Increase of field weakening by factor 4 from 1:2.5 to 1:10

Parallel: $P_b = 2P_a$) series
NOT used!
5. Inverter-fed induction machines

5.4 Influence of inverter harmonics on motor performance

Source: ELIN EBG Motoren GmbH, Austria
Inverter with voltage six step operation

- Bridge rectifier with thyristors on grid side
  GR (firing angle $\alpha$) generates variable DC voltage $U_d$ in DC link ZK; voltage smoothed by capacitor.
- **Inverter WR** generates by six-step switching from $U_d$ a block shaped line-to-line output voltage between terminals L1, L2, L3.

- **DC link voltage** $U_d$ is changed by $\alpha$ proportional with output frequency $f_{\text{mot}} = 1/T$
- **Grid side energy feed-back only possible with 2nd anti-parallel thyristor bridge:** At $\alpha > 90^\circ$ positive $U_d$ and negative $I_d$ give negative dc link power = power to the grid (gener. braking).

Source: Kleinrath, H.; Springer, 1980
Voltage harmonics at six-step operation (see Chapter 1)

- **Inverter output phase voltage:**
  - \( u_{S1} - u_{S2} = u_{L1-L2} \)
  - \( u_{S2} - u_{S3} = u_{L2-L3} \)
  - \( u_{S1} + u_{S2} + u_{S3} = 0 \)
  
  we get:
  
  \[
  u_{S1} = \frac{2u_{L1-L2} + u_{L2-L3}}{3}
  \]

- Block shaped line-to-line voltage, expanded as FOURIER-series:
  
  \[
  u_L(t) = \sum_{k=1, -5, 7, ..}^{\infty} \hat{U}_{L,k} \cdot \cos(k \cdot \omega_s t)
  \]

  \[
  k = 1 + 6g, \quad g = 0, \pm1, \pm2, ...
  \]

  \[
  \Rightarrow \quad k = 1, -5, 7, -11, 13, ...
  \]

  \[
  \hat{U}_{L,k} = \frac{2}{\pi} \sqrt{3} \frac{U_d}{k}
  \]

**Electrical machine is fed with a mix of harmonic voltages of different amplitude, frequency and phase angle. Only fundamental (ordinal number \( k = 1 \)) is desired. Voltage harmonics (\(|k| > 1\)) cause harmonic currents in electric machine with additional losses, torque pulsation, vibrations and acoustic noise.**
Harmonic voltage systems (see Chapter 1)

- **General rule:** Positive and negative systems occur alternatively: Ordinal number $k$ has a positive or negative sign: $k = +1, -5, +7, -11, +13, \ldots$

  \[
  u_{U_k}(t) = \hat{U}_k \cdot \cos(k \omega t) \\
  u_{V_k}(t) = \hat{U}_k \cdot \cos(k \omega t - 2\pi / 3) \\
  u_{W_k}(t) = \hat{U}_k \cdot \cos(k \omega t - 4\pi / 3)
  \]

  \[
  k = 1 + 6g \\
  g = 0, \pm 1, \pm 2, \ldots
  \]
Pulse width modulation (PWM)

- **At grid side:** Diode rectifier GR
  (= firing angle $\alpha = 0$): generates constant DC link voltage $U_d$, which is smoothed by capacitor: $U_d \sim U_{\text{grid}} = \text{const}$.

- Motor side inverter WR generates from $U_d$ by pulse width modulation a line-to-line voltage between L1, L2, L3. Width of pulses is defined by comparison of saw tooth signal $u_{SZ}$ (switching frequency $f_{\text{sch}}$) with AC reference signal $u_{\text{ref}}$, which pulsates with desired stator frequency $f_s$. With comparator a PWM-signal is generated to control power switches. Reference signal is most often sine wave.

- Amplitude $A_1$ of $u_{\text{ref}}$ defines amplitude of fundamental of PWM voltage at motor terminal. So it is varied proportional to $f_{\text{mot}}$.

- Grid side: $\cos \varphi = 1$ No power flow back into grid possible. (For that a grid-side inverter and a grid-side inductance is necessary !). Therefore generator braking power has to be dissipated in "brake"-resistors, which are connected in parallel with capacitor in DC link.
Generation of PWM voltage

a) Comparison of saw tooth and reference signal lead to PWM control signal for power switches: Potential $\phi_{L1}(t)$ at terminal L1 varies with that PWM signal

b) Difference of two terminal potentials delivers line-to-line voltage $u_{L1-L2}(t)$

Example:
Synchronous switching mode

$T_s / T_{switch} = \text{integer}$

- $f_s = f_{mot} = 1/T_s$
- $f_T = 1/T_{switch}$

$m = A_{1} = \hat{U}_{\text{phase}} / (U_d / 2)$ modulation degree

$f_p = 2/T_{switch}$

Source: Kleinrath, H.; Springer, 1980
Voltage harmonics: Six-step and PWM

- **Six-step modulation**: FOURIER spectrum of line-to-line inverter output voltage:

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>-5</th>
<th>7</th>
<th>-11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{U}<em>{Lk} / \hat{U}</em>{L1}$</td>
<td>1</td>
<td>-0.2</td>
<td>0.14</td>
<td>-0.1</td>
<td>0.08</td>
</tr>
</tbody>
</table>

- **PWM**: FOURIER spectrum of terminal electric potential $\varphi_{L1}(t)$ and of line-to-line voltage $u_{L1-L2}(t)$ (at modulation degree $m = A_1 = 0.5$ and switching frequency ratio $f_T / f_s = 9$)

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
<th>17</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\varphi}_k / (\hat{U}_d / 2)$</td>
<td>0.5</td>
<td>$&lt;10^{-5}$</td>
<td>0.001</td>
<td>0.09</td>
<td>1.08</td>
<td>0.09</td>
<td>0.002</td>
<td>0.04</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>$\hat{U}<em>{L,k} / \hat{U}</em>{L,k=1}$</td>
<td>1</td>
<td>0</td>
<td>0.002</td>
<td>0.18</td>
<td>0</td>
<td>0.18</td>
<td>0.004</td>
<td>0</td>
<td>0.72</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Spectrum of terminal potential $\varphi_L$ shows big amplitude of fundamental, of switching harmonic ($k = 9$) and at **about twice switching frequency** $f_p = 2 f_T$ ($k = 17$ and 19).

$$k = \left| \frac{f_p}{f_s} \pm 1 \right| \Rightarrow k = \left| 18 \pm 1 \right| = 17, 19$$

Voltage harmonics with ordinal numbers, divisible by 3, do **not** occur in line-to-line voltage, because $f_T / f_s$ is divisible by 3!

At high switching frequency $f_T$ the amplitudes of all low voltage harmonics $f_k < f_T$ are small.
FOURIER-Spectrum of voltage harmonics: Six-step operation

\[ \frac{U_{LL,k}}{U_d} \sim \frac{1}{k} \]

FOURIER-Spectrum of voltage harmonics: PWM at 50% modulation degree

\[ k = 1 : m = \frac{U_{\text{phase,1}}}{U_d / 2} = 0.5 : \frac{U_{LL,1}}{U_d} = \sqrt{3} \cdot m / 2 = 0.43 \]

Fundamental frequency \( f_s = 800 \text{ Hz} \)

Switching frequency \( f_T = 12000 \text{ Hz} = 15f_s \)

Source:
FOURIER-Spectrum of voltage harmonics: PWM at 100% modulation degree

\[ k = 1 : m = \frac{\hat{U}_{\text{phase,1}}}{(U_d / 2)} = 1 : \frac{\hat{U}_{\text{LL,1}}}{U_d} = \sqrt{3} \cdot m / 2 = 0.866 \]

\[ u_{\text{LL}} / U_d \]

Fundamental frequency \( f_s = 800 \) Hz

Switching frequency \( f_T = 12000 \) Hz = 15\( f_s \)

Source:
Example: Voltage harmonics at “synchronous” PWM

Reference signal: sinusoidal

- 1: amplitude $\hat{U}_{k=1}$
- 2,3: amplitude $\hat{U}_k$ : $f_T-2f_1$ and $f_T+2f_1$
- 4,5: amplitude $\hat{U}_k$ : $2f_T-f_1$ and $2f_T+f_1$

$(f_T=12\,\text{kHz}, \ f_{s1} =0,8\,\text{kHz})$


Modulation ratio

$$m = \frac{\hat{U}_{LL,1}}{(\sqrt{3}/2) \cdot U_d}$$
**Example: Current harmonics at PWM**

Reference signal: trapezoidal

Reference signal: rectangular

Current ripple amplitude decreases inverse with increasing switching frequency

Switching ratio:

\[ \frac{f_T}{f_s} = 6 \]

Switching ratio:

\[ \frac{f_T}{f_s} = 9 \]
Voltage harmonics cause current harmonics

- The **voltage harmonics per phase** $U_{s,k}$ (frequency $k$-times fundamental frequency $kf_s$) cause current harmonics per phase $I_{s,k}$ in stator winding.
- These 3-phase harmonic current systems excite in the air gap a “high-speed” magnetic field wave (with pole count $2p$ due to the winding):
  
  \[ n_{\text{syn},k} = \frac{k \cdot f_s}{p} \]

- **Rotor slip with $k$th high-speed field** $s_k$:

\[
s_k = \frac{n_{\text{syn},k} - n}{n_{\text{syn},k}} = \frac{kn_{\text{syn}} - n}{kn_{\text{syn}}} = 1 - \frac{1}{k} \cdot \frac{n}{n_{\text{syn}}} = 1 - \frac{1}{k} \cdot (1 - s) \approx 1
\]

As harmonic slips $s_k$ are nearly unity, independent of base slip $s$, harmonic current amplitudes $I_{s,k}$ are nearly independent from load. Current harmonics are already present at no-load ($s = 0$) to full extent.

High speed fields $2p$ induce the rotor cage, causing rotor current harmonics with high frequency:

\[
f_{rk} = s_k f_{s,k} \approx f_{s,k}
\]

causing big eddy currents in rotor bars and **big additional rotor losses**!

\[
s_k \approx 1 \quad \Rightarrow \quad I_{s,k} \approx \frac{U_{s,k}}{\sqrt{(R_s + R'_r)^2 + (k \omega_s)^2 \cdot (L_{s\sigma} + L'_{r\sigma})^2}} \approx \frac{U_{s,k}}{|k| \omega_s (L_{s\sigma} + L'_{r\sigma})}
\]
Example: Current harmonics at six-step modulation

- Amplitudes of current harmonics at six-step operation:

\[ I_{s,k} \approx \frac{U_{s,k}}{k|\omega_s (L_{s\sigma} + L'_{r\sigma})^2} \sim \frac{1}{k^2} \]

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>-5</th>
<th>7</th>
<th>-11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{U}<em>{Lk} / \hat{U}</em>{L1} )</td>
<td>1</td>
<td>0.2</td>
<td>0.14</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>( I_{s,k} / I_{s,k=1} )</td>
<td>1</td>
<td>0.04</td>
<td>0.02</td>
<td>0.008</td>
<td>0.006</td>
</tr>
</tbody>
</table>

\( s = 1, s_k \approx 1 \):

- Amplitudes of current harmonics decrease with inverse of square of ordinal number k, because the leakage inductances smooth the shape of current (= they reduce the current harmonics!)

Source: Kleinrath, H.; Springer, 1980

Torque ripple

\[ \frac{2}{3} U_d \]

\[ i_U \]

\[ u_{SU} \]

\[ T/2 \]

\[ t \]

\[ M_e (t) \]

\[ T \]

\[ t \]

FOURIER sum of 25 current harmonics

Exact solution of dynamic machine equation with space vector theory
Additional machine losses at inverter-supply

- Additional cage losses:
  
  Current displacement: AC deep bar resistance increases by ratio \( k_{Rk} \approx h_{bar} / d_{E,k} \) with penetration depth (conductivity \( \kappa \), permeability \( \mu \) of rotor bar conductor)

  \[
  d_{E,k} = 1 / \sqrt{\mu \cdot \kappa \cdot \pi \cdot f_{rk}} \approx 1 / \sqrt{\mu \cdot \kappa \cdot \pi \cdot |k| \cdot f_s}
  \]

  Losses in bar conductors for \( k^{th} \) harmonic of voltage/current:

  \[
  P_{ad,r,inv,k} = Q_r \cdot R_r \cdot I_{r,k}^2 = Q_r \cdot k_{Rk} \cdot R_r \cdot I_{r,k}^2 \approx Q_r \cdot k_{Rk} \cdot R_r \cdot \frac{U_{s,k}^2}{(|k| \omega_s (L_s \sigma + L_r \sigma))^2}
  \]

  \[
  P_{ad,r,inv,k} \approx \sqrt{|k|} \cdot U_{s,k}^2 / |k|^2
  \]


- Additional “iron losses”: Rotor surface eddy current losses; losses in the end shields, ...

- Additional inter-bar current losses: rather small, as only field fundamental waves \( \nu = 1 \) are of considerable amplitude for \( k \neq 1 \). But field waves \( \nu = 1 \) do not produce much inter-bar currents in skewed cages.
Influence of rotor slot shape on additional cage losses at low switching frequencies $f_T = \text{high power motors}$

**Example:** $f_sN = 50 \text{ Hz}$, Model motor 15 kW for investigating losses in high-power motors, synchronous switching, $f_T = 750 \text{ Hz} \ (f_T/f_s = 15)$:

Additional cage losses: numerically determined:
- Slot shape A: **190 W**, slot shape C: **60 W**.

- Slit gives independence of leakage inductance of bridge saturation
- Closed bridge increases leakage inductance = reduction of harmonic currents
- Surface eddy currents
- Current displacement

*Source:* Arkkio, A.; ICEM, 1992
Increase of switching frequency to decrease add. losses

**Example:**
- 2-pole motor, 3 kW, 380 V Y; voltage source inverter 8.3 kVA, asynchronous switching \((f_T = \text{const.})\) with rating 8.3 kVA, 400 V.
- Motor is operated at \(f_s = 50\) Hz, slip \(s = 4.5\%\) at 10 Nm, 3 kW.
- Motor and inverter efficiency were measured by directly measuring input and output power both of motor and inverter for different inverter switching frequencies.

<table>
<thead>
<tr>
<th>(f_s / \text{Hz})</th>
<th>(f_T / \text{Hz})</th>
<th>(f_p = 2f_T / \text{kHz})</th>
<th>Efficiency motor</th>
<th>Efficiency inverter</th>
<th>Overall efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-</td>
<td>4.8</td>
<td>81.9 %</td>
<td>96.9 %</td>
<td>81.9 %</td>
</tr>
<tr>
<td></td>
<td>2 400</td>
<td>9.6</td>
<td>81.3 %</td>
<td>96.9 %</td>
<td>78.8 %</td>
</tr>
<tr>
<td></td>
<td>4 800</td>
<td>19.2</td>
<td>81.4 %</td>
<td>96.8 %</td>
<td>78.8 %</td>
</tr>
</tbody>
</table>

**Facit:**
At \(f_p = 9.6\) kHz the motor efficiency is higher (motor heating is lower), and the overall efficiency is the same as at 4.8 kHz. At 19.2 kHz the current ripple reduction in the motor is already negligible, but inverter switching losses increase considerably.
Measured PWM voltage

Measured *Fourier* spectrum of line-to-line voltage, $f_T = 2.4 \text{ kHz}$, $f_p = 2f_T = 4.8 \text{ kHz}$, 2-pole induction motor, 3 kW, load operation, $U_{LL,\text{rms}} = 403 \text{ V}$

Source: Siemens AG

4850 Hz
4750 Hz
Measured motor current at PWM feeding

Measured Fourier spectrum of phase motor current, \( f_T = 2.4 \, \text{kHz} \), \( f_p = 2f_T = 4.8 \, \text{kHz} \), 2-pole induction motor, 3 kW, load operation, \textbf{r.m.s. current 6.25 A}

Source: Siemens AG
Determination of harmonic stator/rotor currents

Example:
Measured *Fourier* spectrum of line-to-line voltage & phase motor current, $f_T = 2.4$ kHz

<table>
<thead>
<tr>
<th>$f_s$</th>
<th>$U_{LL,k}$</th>
<th>$I_{s,k}$</th>
<th>$I_{s,k}/I_{s,k=1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Hz</td>
<td>372 V</td>
<td>6.2 A</td>
<td>100 %</td>
</tr>
<tr>
<td>4750 Hz</td>
<td>89.5 V</td>
<td>0.18 A</td>
<td>2.9 %</td>
</tr>
<tr>
<td>4850 Hz</td>
<td>91.3 V</td>
<td>0.18 A</td>
<td>2.9 %</td>
</tr>
<tr>
<td>9550 Hz</td>
<td>38.6 V</td>
<td>0.035 A</td>
<td>0.6 %</td>
</tr>
<tr>
<td>9650 Hz</td>
<td>24.8 V</td>
<td>0.027 A</td>
<td>0.4 %</td>
</tr>
</tbody>
</table>

-Time harmonic current:
e.g. at 9550 Hz: $I_{sk} = \frac{U_{LL,k}}{\omega_{sk} \cdot (L_{s\sigma} + L'_{r\sigma})} = \frac{38.6}{2\pi \cdot 9550 \cdot 0.00961} = 38.6 \text{ mA (measured: 35 mA)}$

-Peak-to-peak current ripple: Two phases in series: $2L_{\sigma}$ active.

$$u = L \cdot \frac{di}{dt} \Rightarrow U_d = 2L_{\sigma} \cdot \Delta i / \Delta t, \quad \Delta t \approx T_p / 2 = 1/(2f_p)$$

$$\Delta i = \frac{U_d}{4L_{\sigma}f_p} = \frac{525}{4 \cdot 0.00961 \cdot 4800} = 2.8 A$$

$$2.8 A / (2 \sqrt{2}) \approx 1 A \approx \sum_k |I_{k}|$$
Measured PWM voltage at increased switching frequency

Measured Fourier spectrum of line-to-line voltage, \( f_T = 4.8 \text{ kHz} \), \( f_p = 2f_T = 9.6 \text{ kHz} \),
2-pole induction motor, 3 kW, load operation, \( U_{LL,\text{rms}} = 401 \text{ V} \)

Source: Siemens AG
Measured motor current at increased switching frequency

Measured *Fourier* spectrum of phase motor current, $f_T = 4.8$ kHz, $f_p = 2f_T = 9.6$ kHz, 2-pole induction motor, 3 kW, load operation, r.m.s. current 6.27 A

Source: Siemens AG

$f_p \rightarrow 2f_p: \Delta i \rightarrow \Delta i/2$

9650 Hz
Measured motor current at a) PWM and b) grid operation

Measured *Fourier* spectrum of phase motor current, $f_T = 4.8$ kHz, $f_p = 2f_T = 9.6$ kHz, 2-pole induction motor, 3 kW, load operation, **r.m.s. current 6.27 A**

- Nearly no difference between current wave form at PWM with high switching frequency and sine wave voltage operation.
- Harmonic content in the stator current is caused mainly by rotor slot harmonics of the air-gap field, excited by the fundamental rotor current.

Source: Siemens AG
No-load iron and additional losses at sinus and inverter supply

270 kW, 400 V, 2-poles, high-speed induction machine, max. 16000/min

Comparison of no-load iron and additional losses
a) Sinus supply
b) IGBT-PWM-Inverter-supply, 4 kHz, star connection Y,
c) delta connection D.

Dependence of stator fundamental frequency $f_s$.
During measurements air gap flux density had been maintained constant.

Measured!

$\Delta$: Lower modulation degree = higher harmonic voltage amplitudes = higher add. losses!
Inverter-induced torque ripple

- Fundamental air gap field amplitudes $B_{\delta sk,v=1}$ of stator harmonic currents $I_{sk}$ produce with rotor harmonic currents $I_{rk}$ tangential Lorentz-forces, and therefore additional torque components.

- Stator field $B_{\delta sk=1,v=1}$, excited by fundamental current $I_{s,k=1}$, generates with rotor harmonic current $I_{rk}$ of DIFFERENT ordinal number $k$ a pulsating torque $M_{e,1k}$, which is proportional to product $M_{e,1k} \sim B_{\delta sk=1,v=1} \cdot I_{rk}$ or due to $B_{\delta sk=1,v=1} \sim I_{sk=1}$ it is $M_{e,1k} \sim I_{s1} \cdot I_{rk}$ . Pulsating frequency: $f_{1k} = |f_s - f_{sk}|$

- As $f_{1k} = |1 - k|f_s = 6|g|f_s$ , the pulsating torque occurs with multiples of six time stator fundamental frequency.
- Each two harmonics contribute with their torque amplitudes to one resulting pulsating torque with the sum of both torque amplitudes:

Example:

$k = -5$ and $k = 7$: $\hat{M}_{e,1,-5}$, $\hat{M}_{e,1,7}$, frequency $6f_s$

resulting amplitude $\hat{M}_{e,6f_s} = \hat{M}_{e,1,-5} + \hat{M}_{e,1,7}$.
Torque ripple amplitude at six-step operation

Fundamental field waves of stator current time harmonic $k = -5$ & $k = 7$ produce a pulsating torque with the fundamental field wave of the fundamental stator current $k = 1$ with $f_{1k=-5} = 6f_s$ & $f_{1k=7} = 6f_s$

Ripple frequency = 6-times fundamental frequency and multiples

Source: Kleinrath, H.; Springer, 1980
**Air-gap torque ripple amplitude**

\[
M_{e,1k} = \frac{p \tau_p \cdot Q_r \cdot \hat{I}_{rk} \cdot B_{\delta s} \cdot l_{Fe}}{2\pi} \cdot \sin\left((1-k)\omega_s t\right)
\]

\[B_{\delta s} = B_{\delta s,k=1,\nu=1}\]

**Example:**

8-pole induction motor, 440 V Y, 60 Hz, 2.6 kW, 5.6 A, 28.4 Nm, 860/min, inverter supply at \(f_s = 50 \text{ Hz}, U_d = 525 \text{ V}, \) PWM, switching frequency \(f_T = 2.4 \text{ kHz}\)

Motor data: \(m_s = 3, Q_s/Q_r = 48/44, l_{Fe} = 80 \text{ mm}, L_\sigma = 19.6 \text{ mH}, k_{ws} = 0.933, N_s = 344,\)

stator field fundamental: \(B_{\delta s} = 1 \text{T}\)

\[
\hat{I}_{rk} = -\frac{m_s N_s k_{ws}}{Q_r / 2} \cdot \frac{\hat{U}_{s,k}}{|k|\omega_s L_\sigma}
\]

\(2f_T/f_s \pm 1 = 95, 97: k = -95, 97\)

Air gap torque ripple: \(1.68/28.4 = 5.9 \%\)

Shaft torque ripple: 0.0015 Nm (\(= 0.005 \%\)) due to rotor inertia and elastic coupling

| \(|k|\) | \(f_k\) | \(\hat{U}_{s,k}\) | \(\hat{I}_{rk}\) | \(\hat{M}_{e,1k}\) | \(\hat{M}_{e,6|g|f_s}\) | \(f_{1k}\) |
|---|---|---|---|---|---|---|
| -  | Hz  | V  | A  | Nm | Nm | Hz  |
| 95 | 4750| 73.1|5.5 | 0.85|
| 97 | 4850| 74.2|5.4 | 0.83|

1.68  4800
Shaft-torque much smoother than air-gap torque

Rotor of motor coupled to rotating load via an elastic coupling

- coupling stiffness $c$
- inertia of motor and load $J_M, J_L$

Resonance frequency $f_0$:
big … small motors: 20 Hz … 200 Hz

Pulsating shaft torque: $m_s(t) = c \cdot \gamma(t) = \frac{\hat{M}}{J_M} \cdot \frac{c}{\omega_0^2 - \omega^2} \cdot \sin(\omega t)$

Low exciting frequency $f < f_0$: $m_s(t) \approx \frac{\hat{M}}{J_M} \cdot \frac{J_L}{J_M + J_L} \cdot \sin(\omega t)$

High exciting frequency $f > f_0$: $m_s(t) \approx -\frac{\hat{M}}{J_M} \cdot \frac{c}{\omega_0^2} \cdot \sin(\omega t)$

Facit: Switching frequency torque ripple smoothed strongly by rotor inertia and elastic coupling above $f_0$!
Air-gap torque ripple amplitude

Shaft torque ripple is below torsion resonance $f_0$ determined by ratio of load versus motor inertia, above resonance by the inverse square of pulsation frequency

$$f_{1k} < f_0 : \hat{M}_s \approx \frac{\hat{M}_{e,6g} f_s \cdot J_L}{J_L + J_M}$$

$$f_{1k} > f_0 : \hat{M}_s \approx \frac{\hat{M}_{e,6g} f_s \cdot c}{J_M \cdot (2\pi f_{1k})^2}$$

**Example:**

$f_0 = 200 \text{ Hz}, f_s = 10 \text{ Hz}, J_L = J_M$

a) $f_{1k} = 6f_s = 60 \text{ Hz} < 200 \text{ Hz}$

b) $f_{1k} = 2f_T = 4.8 \text{ kHz} > 200 \text{ Hz}$.

**Torque ripple:**

- in air gap: 3.3% 5.9%
- at the shaft: 1.7% 0.005%

*Although $6f_s$-torque ripple in the air gap is lower, the shaft torque ripple with $6f_s$ is dominating over high harmonic torque components.*

*Due to speed and frequency variation the torsion resonance might be hit.*
**Measured shaft torque ripple: Low $n$, high $J_L$**

**Example:** Low speed 20/min

Induction motor: 2 pole, 750 W, 2850/min, 2.51 Nm, 400 V D, 50 Hz, 1.6 A

Voltage-source IGBT Inverter: 1500 VA, 2.2 A, 400 V

Motor operated at $f_s = 1.2$ Hz, $n = 20$/min, $s = 0.72$, $M = 1.24$ Nm = 50% rated torque

a) PWM, asynchronous switching: $k = 1 + 6g = 1, -5, 7, -11, 13, ...$  $g = 0, \pm 1, \pm 2, ...$

b) PWM voltage output pattern is not symmetrical to abscissa:
Voltage harmonics with even ordinal numbers occur:
$k = 1 + 3g = 1, -2, 4, -5, 7, -8, 10, -11, 13, ...$  $g = 0, \pm 1, \pm 2, ...$

<table>
<thead>
<tr>
<th></th>
<th>a) symmetrical PWM</th>
<th>b) asymmetrical PWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest torque ripple frequency</td>
<td>$f_{1k} = 6</td>
<td>g</td>
</tr>
<tr>
<td></td>
<td>$= 6 \cdot 1.2 = 7.2$ Hz</td>
<td>$= 3 \cdot 1.2 = 3.6$ Hz</td>
</tr>
<tr>
<td>Measured torque ripple $\hat{\omega}_M$</td>
<td>14.4%</td>
<td>35.0%</td>
</tr>
<tr>
<td>Torque ripple amplitude</td>
<td>0.18 Nm</td>
<td>0.43 Nm</td>
</tr>
</tbody>
</table>
Measured torque ripple

<table>
<thead>
<tr>
<th></th>
<th>a) symmetrical PWM</th>
<th>b) asymmetrical PWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest torque ripple frequency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{1k} = 6g</td>
<td>f_s = 6f_s =$</td>
<td>$f_{1k} = 3g</td>
</tr>
<tr>
<td>$= 6 \cdot 1.2 = 7.2\text{Hz}$</td>
<td>$= 3 \cdot 1.2 = 3.6\text{Hz}$</td>
<td></td>
</tr>
<tr>
<td>Torque ripple amplitude</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.18 \text{Nm}$</td>
<td>$0.43 \text{Nm}$</td>
<td></td>
</tr>
</tbody>
</table>
Inverter-induced acoustic noise

- In addition to the magnetically excited acoustic noise at sinus voltage supply, additional air gap waves $\nu = 1$ of the current harmonics $I_{sk}$ will add to acoustic noise with new tonal frequencies, mainly with inverter pulse frequency $f_p = 2f_T$.

**Stator fundamental field waves ($\nu = 1$):**

a) excited by magnetizing current $I_m = I_s + I'_r$ with stator frequency $f_s$:

$$B_{\delta k=1,\nu=1}(x_s, t) = B_{\delta s} \cdot \cos \left( \frac{\pi x_s}{\tau_p} - 2\pi f_s t \right) = B_{\delta s} \cdot \cos \left( \frac{2p\pi x_s}{2p\tau_p} - 2\pi f_s t \right) = B_{\delta s} \cdot \cos \alpha$$

b) excited by magnetizing current $I_{mk} = I_{sk} + I'_{rk}$ with stator harmonic frequency $f_{sk}$:

$$B_{\delta k,\nu=1}(x_s, t) = B_{\delta sk} \cdot \cos \left( \frac{2p\pi x_s}{2p\tau_p} - 2\pi \cdot k \cdot f_s t \right) = B_{\delta sk} \cdot \cos \beta$$

Magnetic pull: $f_n(x_s, t) = \frac{B^2(x_s, t)}{2\mu_0} \sim \left( \sum_{k=1}^{\infty} B_{\delta sk} \right)^2 \Rightarrow f_{n,1k} \sim \sum_{k,k'=1}^{\infty} B_{\delta sk}^2 + 2B_{\delta sk}B_{\delta sk'} \quad k \neq k'$

**Radial force density waves**, causing oscillating pull on iron stack: $f_{n,1k} \sim \cos(\alpha \pm \beta)$

$$f_{n,1k}(x_s, t) = \frac{B_{\delta s}B_{\delta sk}}{2\mu_0} \cdot \cos(2r' \cdot \frac{\pi x_s}{2p\tau_p} - 2\pi f_{Ton,k} t)$$
Vibration mode of inverter-induced acoustic noise

**Example:**

\[ 2p = 4, \quad 2r = 0 \text{ or } 8. \]

**Tonal frequency** \( f_{Ton,k} \):

\[
\begin{align*}
  f_{Ton,k} &= f_s \cdot |1 + k| \quad &\text{for} & \quad 2r = 4p \\
  f_{Ton,k} &= f_s \cdot |1 - k| \quad &\text{for} & \quad 2r = 0
\end{align*}
\]

Dominating current harmonic amplitudes at frequencies \( f_{sk} = 2f_T \pm f_s \).

Dominating acoustic noise at

\[
\begin{align*}
  f_{Ton,k} &= f_s \cdot |1 - k| = f_{sk} - f_s = 2f_T \quad &\text{or} & \quad f_{Ton,k} = 2f_T - 2f_s.
\end{align*}
\]

**Inverter caused magnetic noise is usually excited with twice switching frequency ("pulse frequency"). Modal vibration is \( r = 0 \), so the sound is far reaching and well audible.**
Influence of switching frequency on acoustic noise at inverter-operation

Measured A-weighted sound pressure level at 1 m distance:

250 W, PWM inverter-fed induction motor,

Varying switching frequency $f_T$ and pulse frequency $f_p = 2f_T$.

Source: Siemens AG

- 2 kHz → 4 kHz: A-weighting increases $L_p$
- 4 kHz → 8 kHz → 16 kHz: current ripple decreases and so does noise
Influence of space harmonics at inverter-operation

- Asynchronous and synchronous harmonic torque occur at big slip: $s = 0.8 \ldots 1.2$
- At low speed, full-load slip increases up to unity, so space harmonic torques are in the range of motor operation.
- **Motor current:** $\sim$ rated current (not – as during line-start – $\sim 500\%$ of $I_N$). So parasitic torque $\sim I^2$ components are negligible.

**Example:** Unskewed 4-pole cage induction motor, 15 kW, 380 V, D, 50 Hz, 30 A, $Q_s/Q_r = 36 / 28$. Synchronous harmonic torque at slip 0.86: harmonics $\nu = -\mu = 13$, speed 215/min.

<table>
<thead>
<tr>
<th></th>
<th>a) 50 Hz line operation</th>
<th>b) 1.5 Hz inverter operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous speed</td>
<td>1500/min</td>
<td>45/min</td>
</tr>
<tr>
<td>speed at rated torque 98 Nm</td>
<td>1460/min</td>
<td>5/min</td>
</tr>
<tr>
<td>slip at rated torque 98 Nm</td>
<td>2.6 %</td>
<td>88.9 %</td>
</tr>
<tr>
<td>rotor frequency at 98 Nm</td>
<td>1.3 Hz</td>
<td>1.3 Hz</td>
</tr>
<tr>
<td>current at slip 0.86 (synchronous harmonic torque)</td>
<td>550 % rated current</td>
<td>100% rated current</td>
</tr>
<tr>
<td>speed at synchronous torque</td>
<td>(1-0.86)1500=215/min</td>
<td>(1-0.86)45=6.3 /min</td>
</tr>
<tr>
<td>Synchronous torque amplitude</td>
<td>150 Nm</td>
<td>$150(1/5.5)^2 = 5$ Nm</td>
</tr>
</tbody>
</table>
SMALL harmonic torque at inverter operation with rated current

50 Hz
550 % rated current
Big synchronous harmonic torque

1.5 Hz
100 % rated current
Small synchronous harmonic torque

Source: Arkkio, A.; ICEM, 1992
Unskewed induction motors for inverter operation

- Inverter-fed motors start via frequency control with rated current, so the harmonic torques are small. Therefore no skewing is necessary!

- **Advantage** of unskewed induction motors:
  - Minimization of inter-bar currents = reduction of additional losses!
  - Maximum magnetic coupling between stator and rotor winding = Increase of main inductance, no skew leakage reactance = total leakage flux is reduced!

- As in unskewed induction motors there are nearly no inter-bar currents, the rotor slot number may be bigger than the stator slot number: $Q_r > Q_s$!
  
  - **Advantage:** At $Q_r > Q_s$ the ratio $Q_r/p$ increases. The rotor air-gap field gets more sinusoidally distributed = the rotor harmonic leakage flux is reduced = total leakage flux is reduced!

- Reduction of total leakage flux = reduction of **BLONDEL**’s coefficient $\sigma$ = increase of breakdown torque $M_b$ according to **KLOSS**’s function!

\[
M_b \approx \frac{m_s}{2} \frac{p}{\omega_s} U_s^2 \frac{1 - \sigma}{\sigma X_s}
\]

**Example:** Six-pole unskewed cage induction machine: $Q_s = 36$, $Q_r = 42 > 36$