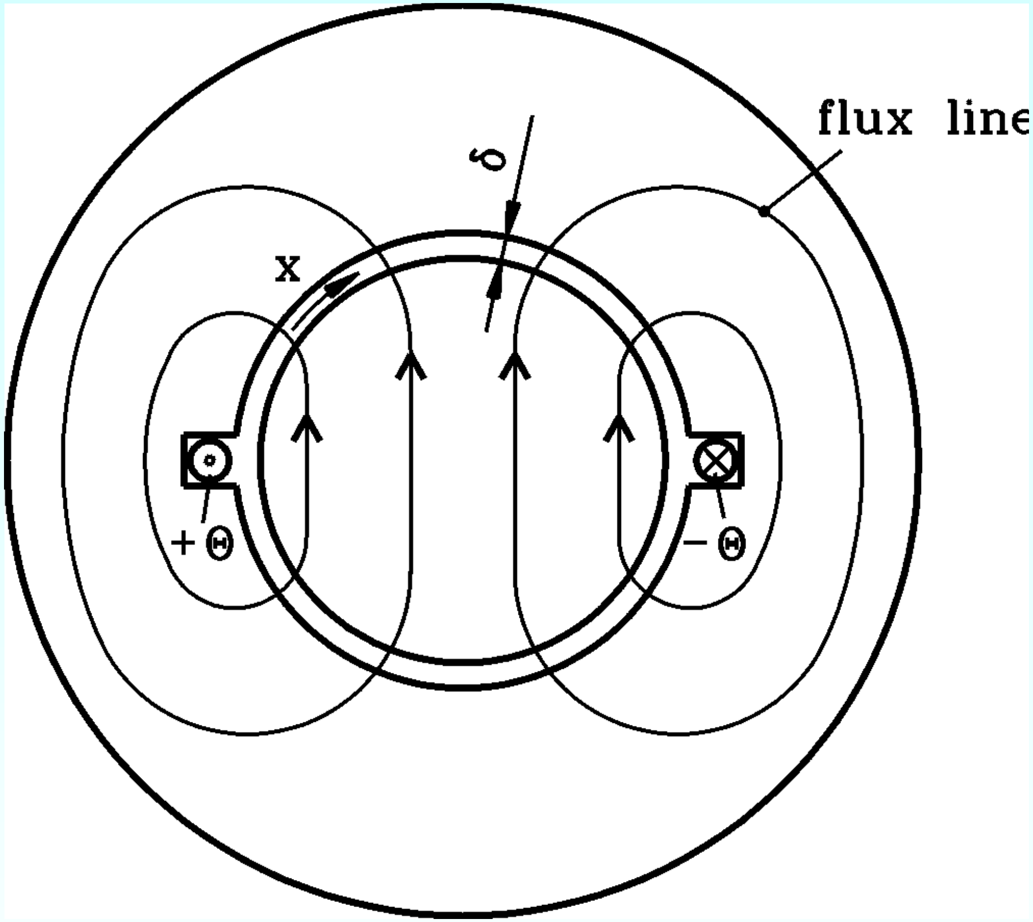
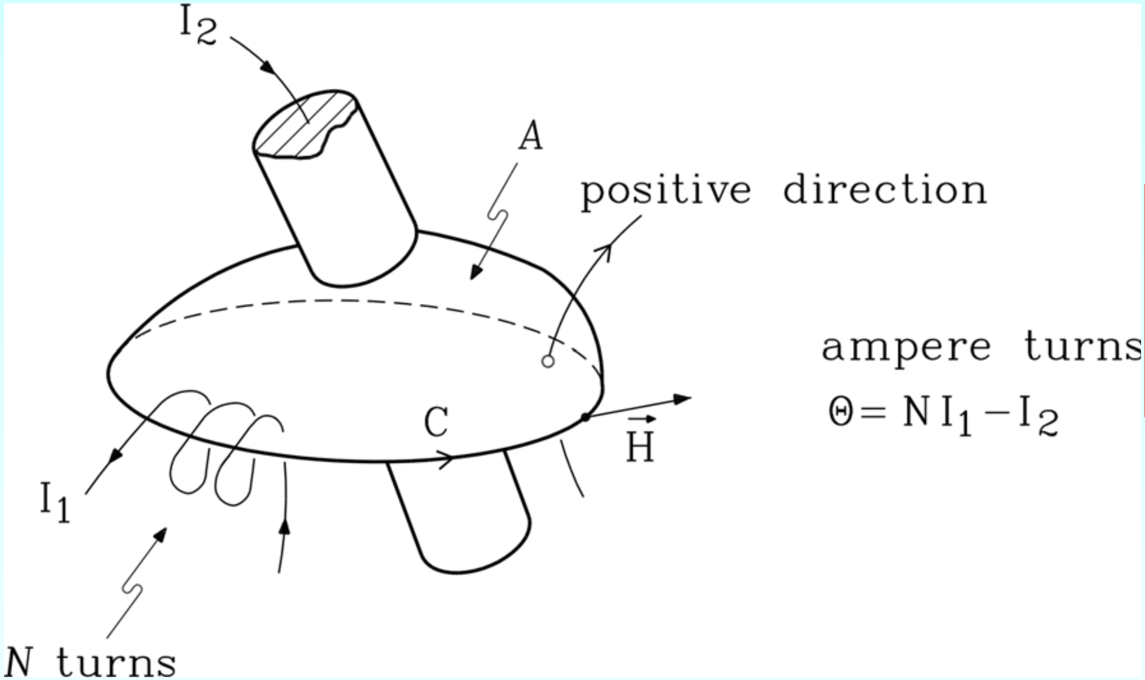
2. Electromagnetic fundamentals



AMPERE´ s law: Excitation of magnetic field by electric current

Example:

Two different currents I_1, I_2 with two different numbers of turns 1 and N and two different flow directions:



$$\oint_C \vec{H} \cdot d\vec{s} = \Theta$$

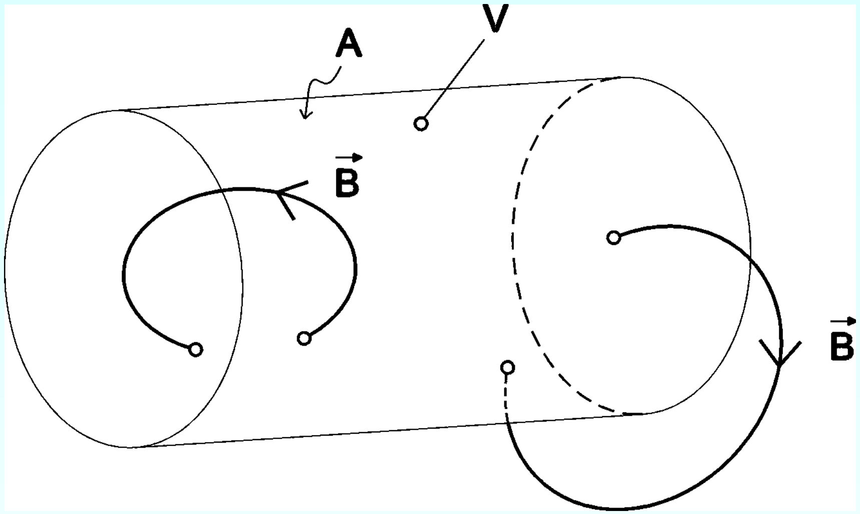
Ampere turns Θ :
$\Theta = N\,I_1 - I_2$

- **The integration of magnetic field strength *H* along closed loop (curve *C*), which spans the area *A*, is equal to the resulting current flow (Ampere turns Θ) penetrating through the area *A*.**
- **Positive field direction is connected to positive current flow direction by RIGHT HAND RULE.**

Law of magnetic flux on closed surfaces

- The total magnetic flux Φ on closed surface *A* of volume *V* is **always ZERO** !

$$\oint_A \vec{B} \cdot d\vec{A} = \Phi = 0$$

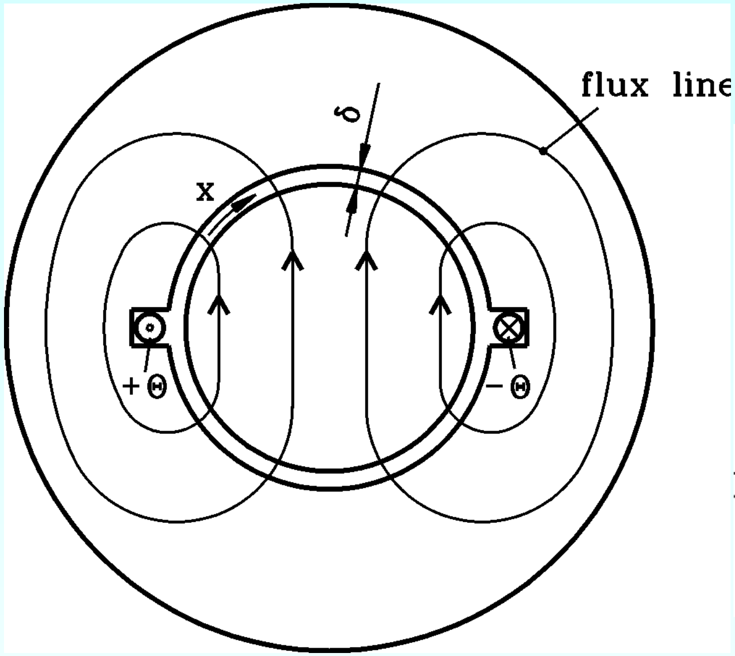


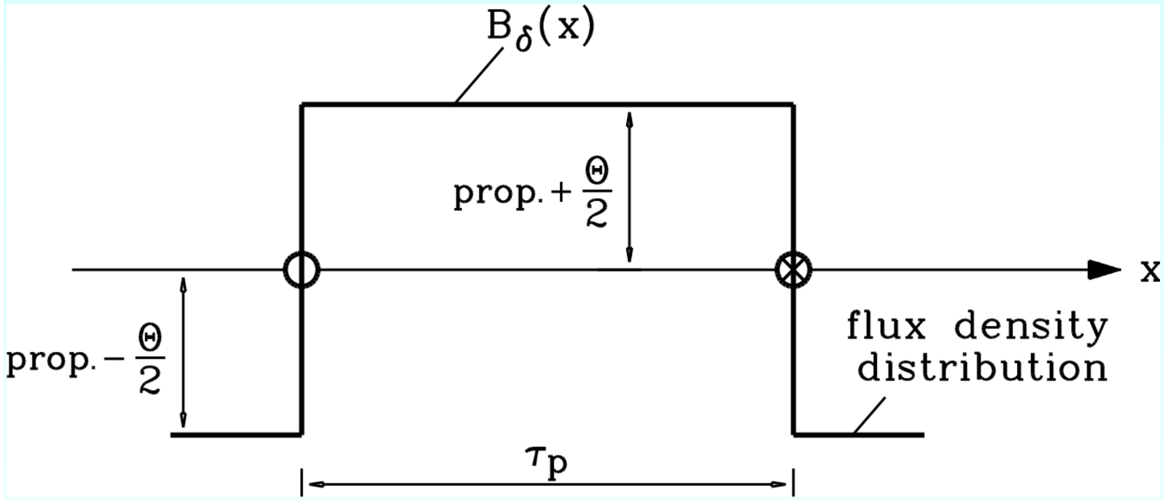
***B*:Magnetic flux density**

- **Normal component of *B*-vector** on both sides of surface *A* is identical: $B_{n,1} = B_{n,2}$
- Magnetic field has always **north- AND south poles**: **NO magnetic monopoles !**
- Minimum pole number is 2: One north and one south pole (*Example:* Earth magnetic field !
- Number of magnetic poles 2*p* (**pole pair number** *p* = 1, 2, 3, ... means 2, 4, 6, ... poles).

Magnetic field of current excited coil in air gap

- *AMPERE´* s law: $\oint_C \vec{H}\cdot d\vec{s} = 2H_{Fe}\Delta_{Fe} + 2H_\delta\delta = 2H_\delta\delta = \Theta$





- $B_\delta = B_{Fe} \Rightarrow H_{Fe} = B_{Fe}/\mu_{Fe} = 0$ ($\mu_{Fe} = \infty$) und $H_\delta = B_\delta/\mu_0$ ($\mu_0 = 4\pi\cdot 10^{-7}$ Vs/Am)
- **Field vectors** $\vec{H}, \vec{B}$ in air gap: only dominating radial components considered !

- Number of turns of coil N_c, coil current I_c : $B_\delta = \mu_0 H_\delta = \mu_0 \dfrac{\Theta}{2\delta} = \mu_0 \dfrac{N_c I_c}{2\delta}$

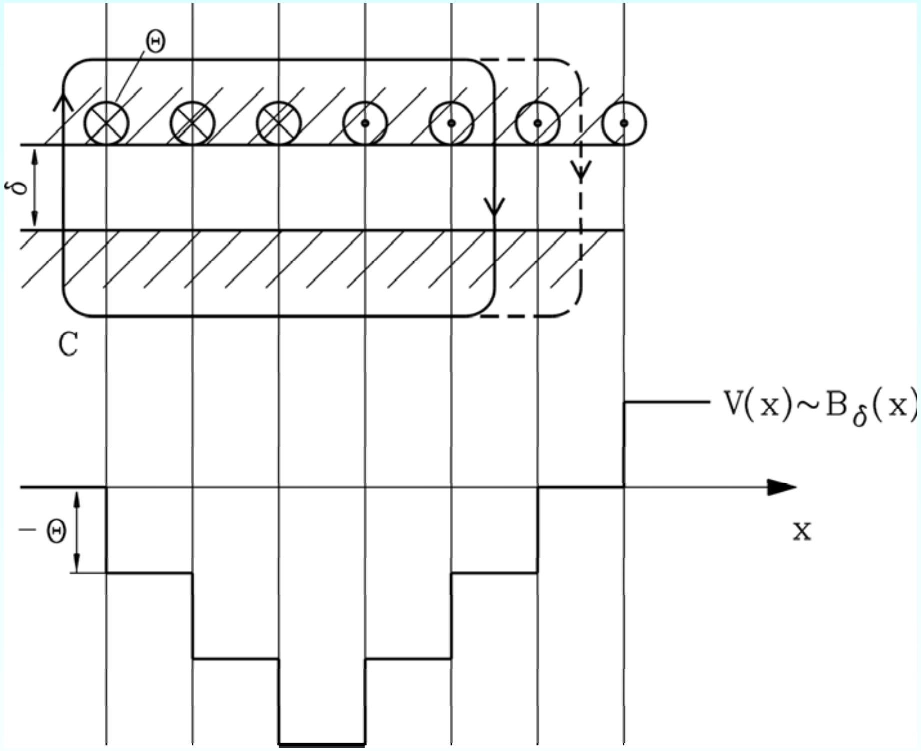
Magnetomotive "force" $V(x)$ and current layer $A(x)$

- As $H_{Fe} = 0$ ($\mu_{Fe} \to \infty$): field lines of H_δ start and end at iron surfaces:

"magnetomotive force *V*" in air gap: $\boxed{V_\delta = H_\delta \cdot \delta} \Rightarrow \boxed{B_\delta(x) = \mu_0 \frac{V_\delta(x)}{\delta}}$

- **" current layer"** *A(x)*: $A = \lim_{b \to 0} \frac{\Theta}{b}$ in slot region, $A = 0$ in tooth region

b: slot width: here *b* = 0 for simplification !



- Calculation of B_δ with use of current layer $A(x)$:

$$B_\delta(x) = \mu_0 H_\delta(x) = \frac{\mu_0}{\delta}\int_0^x A(x)dx = \frac{\mu_0}{\delta}\left(V(x) - V_0\right)$$

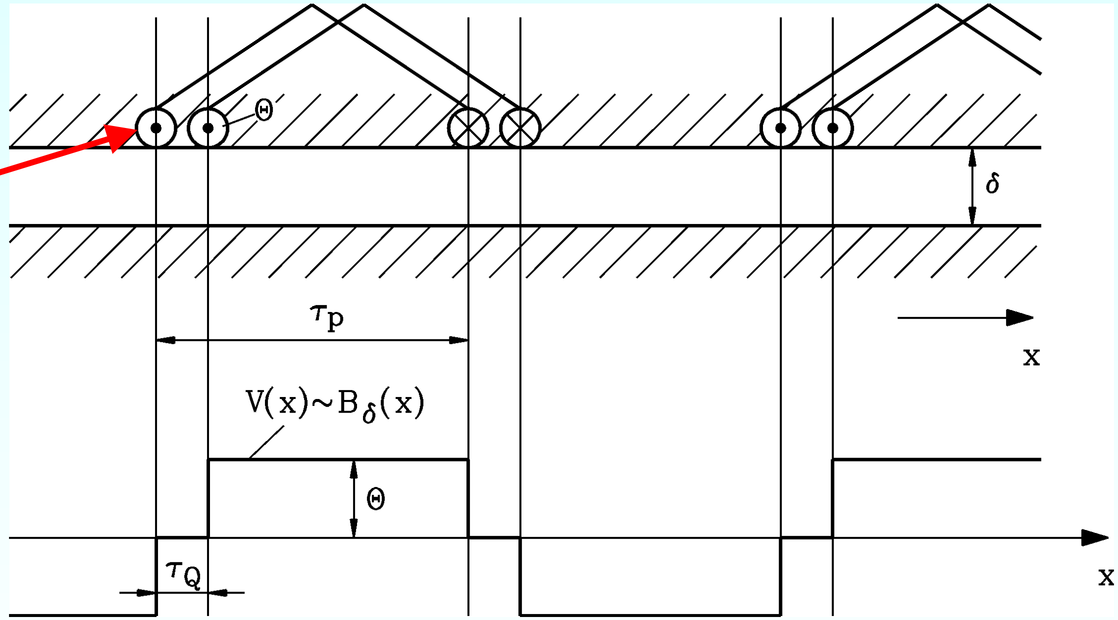
- **Total magnetic flux** at closed surface A_H surrounding rotor in air gap is zero ⇒ This determines integration constant V_0.

$$\oint_{A_H} \vec{B} \cdot d\vec{A} = l_{Fe} \int_{x=0}^{2p\tau_p} B_\delta(x)dx = 0$$

Magnetic air gap field of group of coils

- **Coil group:** The windings per pole are given by more than one coil. Coils are connected in series (*q* coils per group).
- Coil groups distanced by one **pole pitch** τ_p distributed along machine circumference.
- " **Concentrated" Ampere-turns** per coil is Θ.
- **Magnetic air gap field** of coil group is **symmetrical to abscissa** = field curve $B_\delta(x)$ above and below abscissa *x* is identical.
- Flux per pole and per axial length = Area beneath field curve: positive & negative areas are equal: north pole flux = south pole flux.

Example: " Number of coils per pole and phase" $q = 2$.

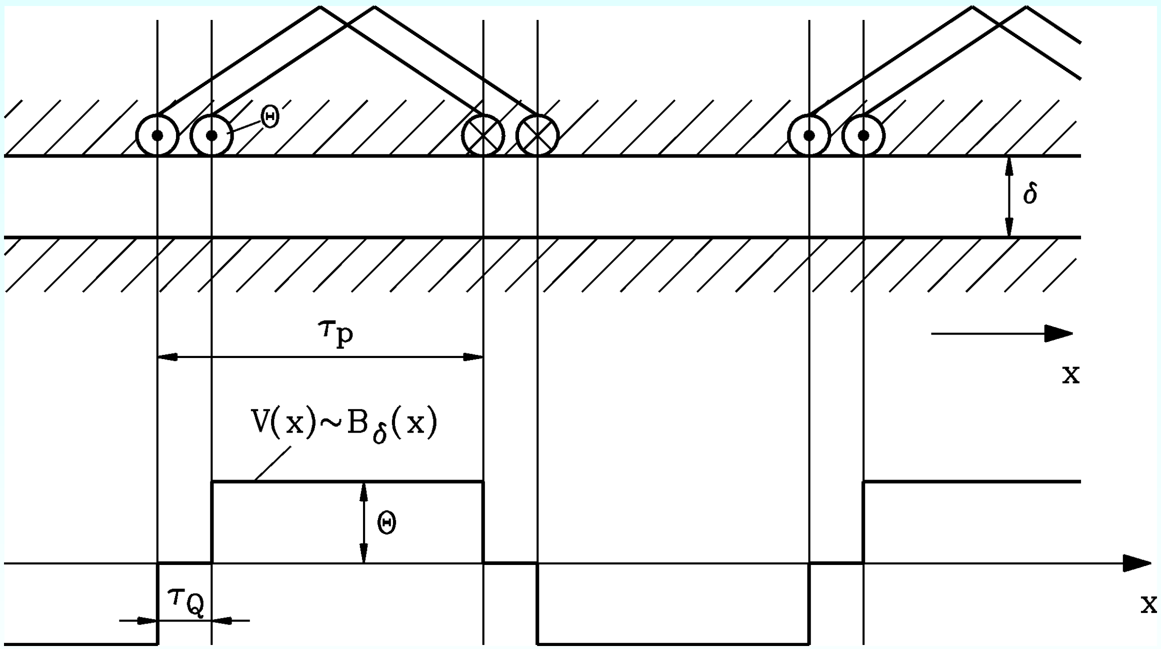


Magnetic alternating field (AC field)

- Feeding the coil groups with sinusoidal alternating current i_c:
  Amplitude $\hat{I}_c$, frequency f, angular frequency $\omega = 2\pi f$, $T = 1/f$: period of oscillation

$$i_c(t) = \hat{I}_c \cos \omega t \qquad \Rightarrow \qquad B_\delta(x,t) = B_\delta(x) \cos \omega t$$

- Air gap field oscillates also sinusoidal with time, BUT maintains **its spatial distribution (its shape =** its distribution along x) ! The amplitude of (radial) field component at locus x changes with time between positive and negative maximum value.



TESLA's idea for rotating (moving) magnetic air gap field

- THREE windings ("phases") U, V, W with positive and negative current flow direction = **6 zones** with notation +U, -W, +V, -U, +W, -V form a WINDING BELT.
- Zones with positive current flow direction chosen so, that phase V is shifted with respect to phase U by $2\tau_p/3$, and phase W by $4\tau_p/3$.
- Winding belt phases U, V, W fed with 3 sinus currents: Each AC current time-shifted with $T/3$ phase shift: $i_U(t)$, $i_V(t)$, $i_W(t)$ (= symmetrical 3-phase AC CURRENT SYSTEM).

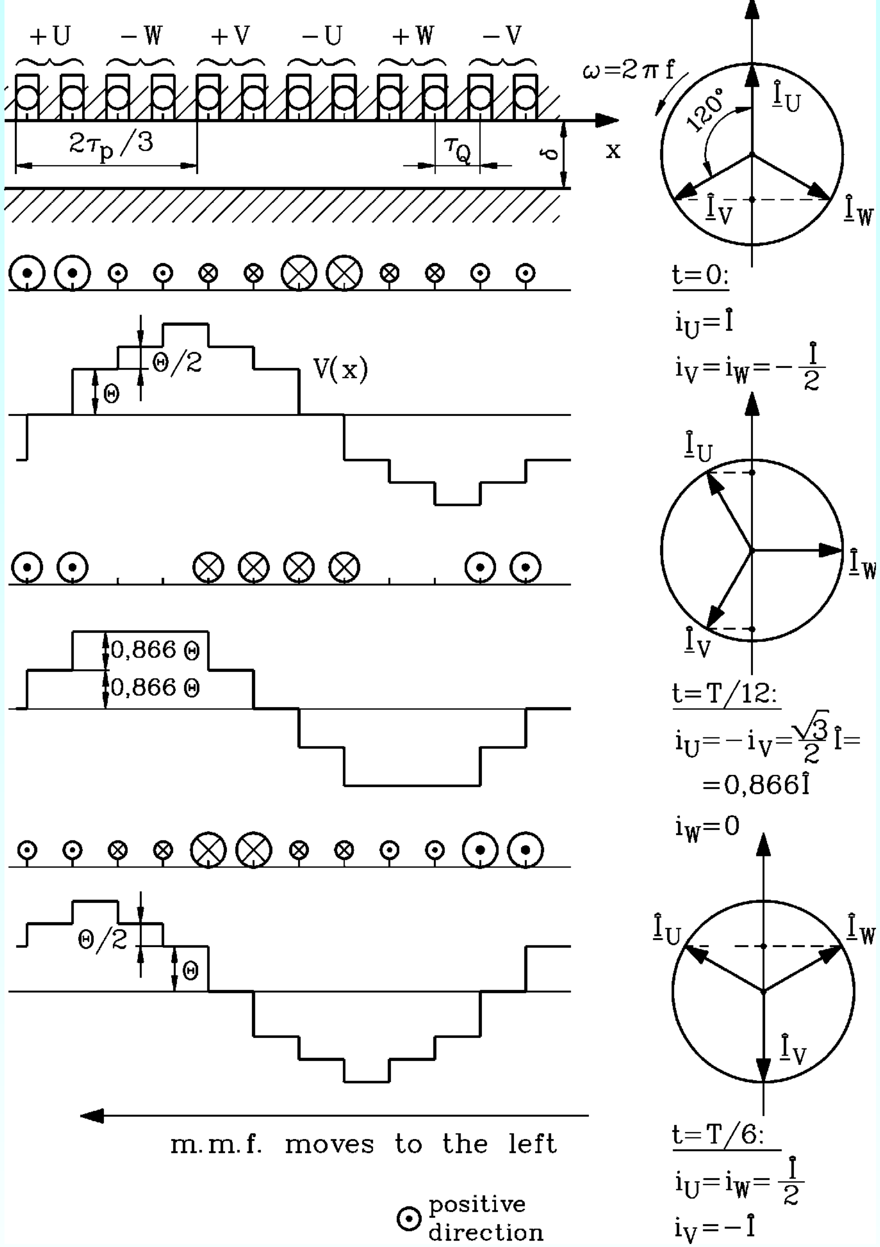
$$i_U(t) = \hat{I}\cos(\omega t + \varphi)$$

$$i_V(t) = \hat{I}\cos(\omega t + \frac{\omega \cdot T}{3} + \varphi)$$

$$i_W(t) = \hat{I}\cos(\omega t + \frac{\omega \cdot 2T}{3} + \varphi)$$

- We use **complex phasor calculus for sinusoidal AC currents & voltages:**

$$i(t) = \mathrm{Re}\left\{\underline{I} \cdot \sqrt{2} \cdot e^{j\omega t}\right\} = \mathrm{Re}\left\{I \cdot e^{j\varphi} \cdot \sqrt{2} \cdot e^{j\omega t}\right\} = \hat{I}\cos(\omega t + \varphi) \quad \Rightarrow \quad \underline{I} = I \cdot e^{j\varphi}$$



Magnetic moving field

- Field curve moves with increasing time *t* to the left !
- After time *T* the field curve has passed the distance $2\tau_p$
- Velocity of linear movement is called

$$v_{syn} = \frac{2\tau_p}{T} = 2f\tau_p$$

synchronous velocity !

Synchronous rotational speed n_{syn} in case of rotating field arrangement:

$$\omega_{syn} = 2\pi n_{syn} = \frac{v_{syn}}{d_{si}/2} = \frac{v_{syn}}{p\tau_p/\pi} = \frac{2\pi f}{p}$$

$$n_{syn} = \frac{f}{p}$$

Linear machines

- **Linear** movement, e.g. drive system for magnetically levitated Hi-speed train (MagLev)
- Cruising speed of MAGLEV train *TRANSRAPID :*
  Data: τ_p = 258 mm, f = 270 Hz (Maximum frequency of feeding inverter)
  $v_{syn} = 2 f \tau_p = 2 \cdot 270 \cdot 0.258 = \underline{139.3}$ m/s = $\underline{501.6}$ km/h

Rotating field machines

- **Rotating part of machine** (= Rotor):

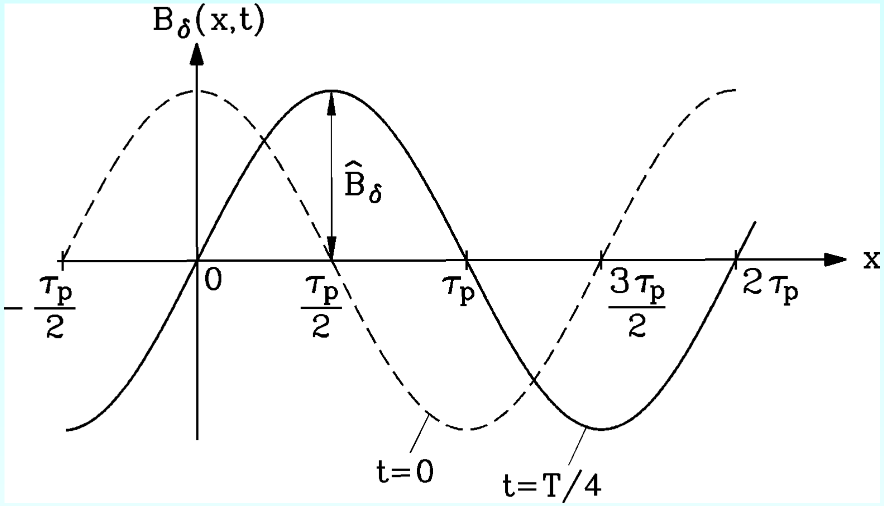
  Two-pole machine ($2p$ = 2): Magnetic field rotates with n_{syn} = 50 Hz = $\underline{3000}$/min
  Sixty-pole hydro generator ($2p$ = 60): Magnetic field rotates with n_{syn} = $\underline{120}$/min

	$2p$	-	2	4	6	8	10	12	14
f = 50 Hz	n_{syn}	1/min	3000	1500	1000	750	600	500	428.6
f = 60Hz	n_{syn}	1/min	3600	1800	1200	900	720	600	514.2

- Changing direction of rotation of magnetic field by changing connection of two terminals !

Rotating waves - Travelling waves

- **Rotating wave:** *x* is stator circumference co-ordinate (*rotating machine*)
- **Travelling wave:** *x* is stator linear co-ordinate (*linear machine*)



$$B_{\delta 1}(x,t) = \hat{B}_{\delta 1} \cos\left(\frac{x \cdot \pi}{\tau_p} - 2\pi f \cdot t\right)$$

Wave velocity: Observer, who is moving with the wave, sees constant argument of cos() = *const.*

$$v_{syn} = \frac{dx}{dt} = \frac{d}{dt}(const. + 2\pi f t)\frac{\tau_p}{\pi} = \underline{\underline{2 f \tau_p}}$$

- **Wave in opposite direction:** $B_{\delta 1}(x,t) = \hat{B}_{\delta 1} \cos(\frac{x \cdot \pi}{\tau_p} + 2\pi f \cdot t) \quad \Rightarrow \quad v_{syn} = -2 f \tau_p$

- *Example:*

  At frequency f = 50 Hz: v_{syn} in *m/s* is SAME number as pole pitch in *cm*: $v_{syn}^{[m/s]} = \tau_p^{[cm]}$

  e.g. 2-pole turbine generator ($2p$ = 2) in thermal power plant: n_{syn} = 3000/min:

- stator bore diameter d_{si} = 1.2 m, pole pitch τ_p = 1.2π/2 = 1.88 m = 188 cm
- v_{syn} = 188 m/s = 676 km/h = rotor surface velocity, as rotor is spinning synchronously with rotating stator magnetic field wave (synchronous machine !)

***FOURIER*-Analysis: Determining fundamental & harmonic waves**

- ***FOURIER*-series:** A periodical function $V(\gamma)$ with period 2π may be described by an infinite sum of sine & co-sine functions with decreasing wave length.

$$V(\gamma) = V_0 + \sum_{\nu=1,2,3,...}^{\infty} \left[\hat{V}_{\nu,a} \cdot \cos(\nu \cdot \gamma) + \hat{V}_{\nu,b} \cdot \sin(\nu \cdot \gamma) \right]$$

- **Ordinal numbers**: $\nu = 1, 2, 3, ...$
- **Amplitudes:** $\hat{V}_{\nu,a} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \cos(\nu \cdot \gamma) \cdot d\gamma \; , \quad \hat{V}_{\nu,b} = \frac{1}{\pi} \int_0^{2\pi} V(\gamma) \cdot \sin(\nu \cdot \gamma) \cdot d\gamma$
- **Average value:** $V_0 = \frac{1}{2\pi} \int_0^{2\pi} V(\gamma) \cdot d\gamma$
- **Magnetomotive force (MMF) of air gap field:**
  - **a) NO UNIPOLAR flux: $V_0 = 0$**
  - **b) MMF *V <u>symmetrical to abscissa</u>*: NO even ordinal numbers**
  - **c) By choosing origin so, that MMF *V* is <u>even</u> function $V(\gamma) = V(-\gamma)$:**
    NO sine-wave functions occur in *FOURIER* sum.

Fundamental and harmonic air gap field waves

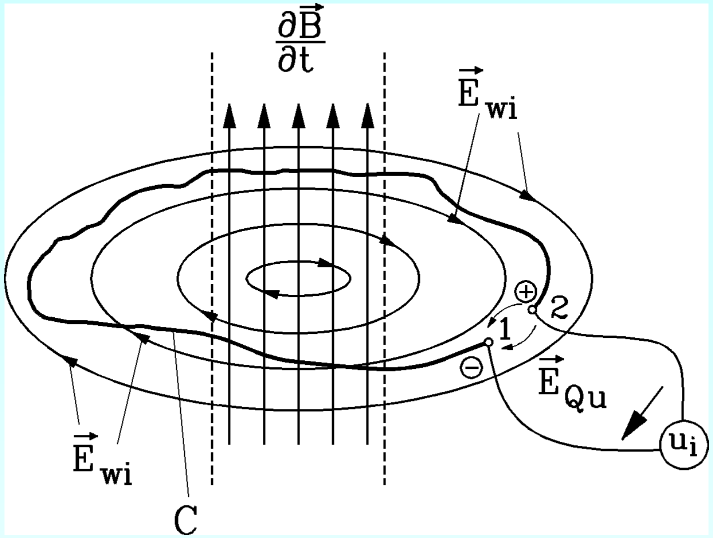
$$V(x,t)=\sum_{\nu=1,-5,7,...}^{\infty} V_\nu(x,t)=\sum_{\nu=1,-5,7,...}^{\infty}\frac{\sqrt{2}}{\pi}\frac{m}{p}N\frac{k_{w,\nu}}{\nu}I\cdot\cos\left(\frac{\nu\pi x}{\tau_p}-\omega t\right)$$

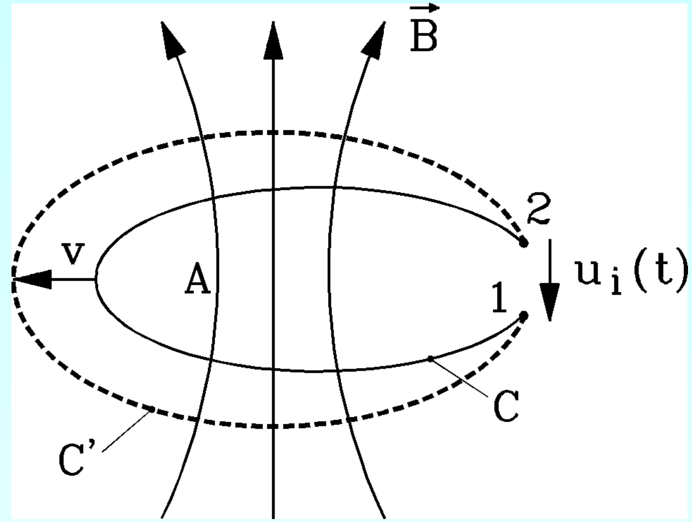
ν = 1, -5, 7, -11, 13, -17,...   Phase number *m*: is usually 3

- **Positive and negative ordinal numbers:** $\nu = 1+2mg \qquad g=0,\pm 1,\pm 2,\pm 3,...$
- Velocity of harmonic waves decreases with 1/ ν: $\qquad v_{syn,\nu}=2f\tau_p/\nu$
- Wave amplitudes (% of fundamental): $\hat{B}_{\delta\nu}/\hat{B}_{\delta 1}$ (%)   **Underlined: "slot harmonics" !**

ν	q = 1, W/τ_p = 2/3, Q/p = 6	q = 2, W/τ_p = 5/6, Q/p = 12	q = 3, W/τ_p = 7/9, Q/p = 18
1	**100**	**100**	**100**
-5	-20	1.4	-0.8
7	14.3	-1.0	-2.2
-11	-9.1	**-9.1**	-1.4
13	7.7	**7.7**	-0.3
-17	-5.6	-0.4	**5.9**
19	5.3	0.38	**-5.3**

FARADAY´s law of induction

a) 

b) 

Each change of flux *Φ*, which is linked to conductor loop *C*, causes an induced voltage u_i in that loop; the induced voltage is the negative rate of change of the linked flux.

$$u_i = -d\Phi / dt$$ Flu□: $$\Phi = \int_A \vec{B} \cdot d\vec{A}$$

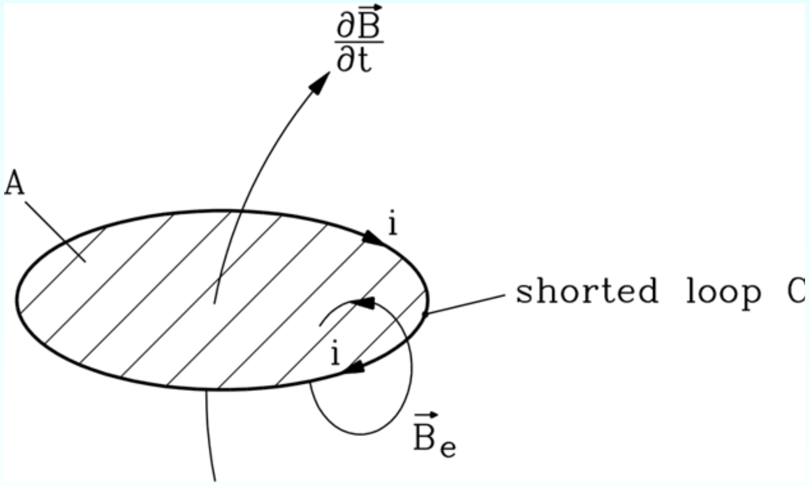
- If coil is used instead of loop with *N* series connected turns, so u_i *is N*-times bigger:
  $$u_i = -N \cdot d\Phi / dt$$
- **"Flux linkage"** $\Psi = N \cdot \Phi \quad \Rightarrow \quad \boxed{u_i = -d\Psi / dt}$
- **Changing of Ψ**: a) *B* is changing, b) Area *A* is changing with velocity *v*

Law of induction: also called: "*LENZ´s* rule"

Lenz´s rule: A change of flux linkage induces voltage u_i, which drives a current i in the loop, which excites a magnetic field B_e, whose direction is opposite to the original change of flux linkage.

- ***Example: Induction in short circuited loop at rest***:

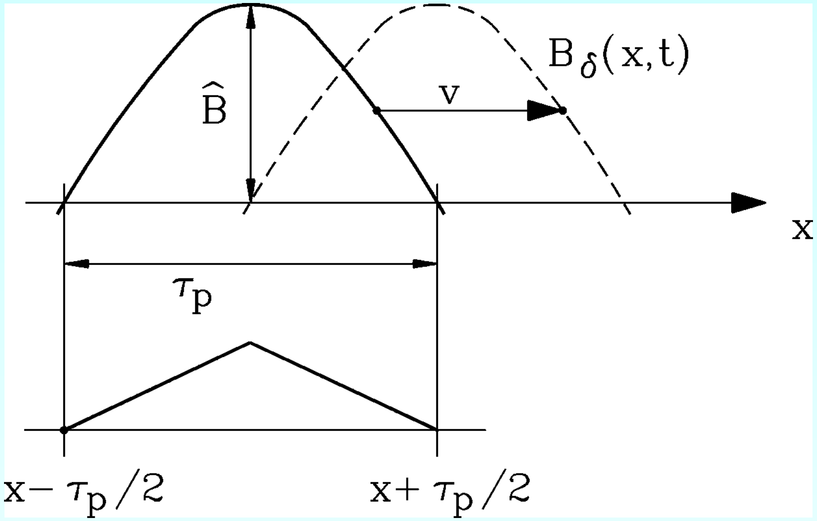
- The change of external field B causes an increase of flux density with orientation from bottom to top. This causes increase of flux in loop area A and **induces electrical field E_{Wi}**.
- E_{Wi} is left hand oriented to $\partial \vec{B} / \partial t$ and drives in loop C a **current i**.
- Current i excites **(*Ampere´s* law !) a right hand oriented** magnetic field B_e .
- Orientation of B_e is opposite to change of original flux density $\partial \vec{B} / \partial t$.

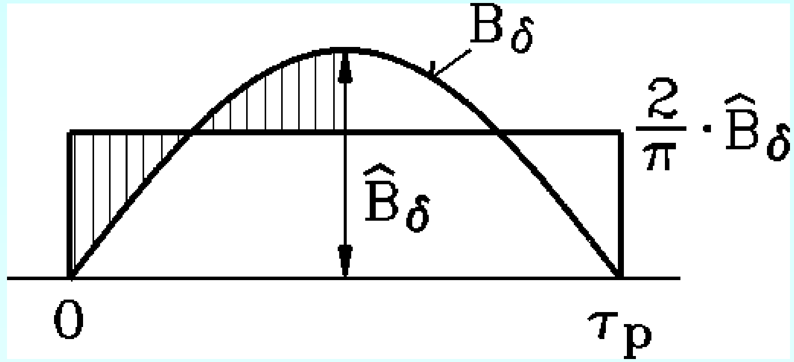


Result:

The „reaction field" B_e acts AGAINST the original flux density change !

Induction of voltage in stator coil





- Sinusoidal moving wave $B_{\delta 1}(x,t) = \hat{B}_{\delta 1}\cos(x\pi/\tau_p - \omega t)$ causes changing coil flux $\Phi(t)$

$$\Phi(t) = l\int_{-\tau_p/2}^{\tau_p/2} B_{\delta 1}(x,t)dx = \frac{2}{\pi}\tau_p l\hat{B}_{\delta 1}\cdot\cos\omega t \;\Rightarrow\; \text{flux linkage}\; \Psi(t) = N_c\Phi(t)$$

- **Induced AC voltage in coil is sinusoidal**: $u_{i,c}(t) = -d\Psi_c(t)/dt = \hat{U}_{i,c}\sin\omega t$

Voltage amplitude: $\hat{U}_{i,c} = \omega N_c\Phi_c = 2\pi f N_c\frac{2}{\pi}\tau_p l\hat{B}_{\delta 1}$ (full-pitched coil)

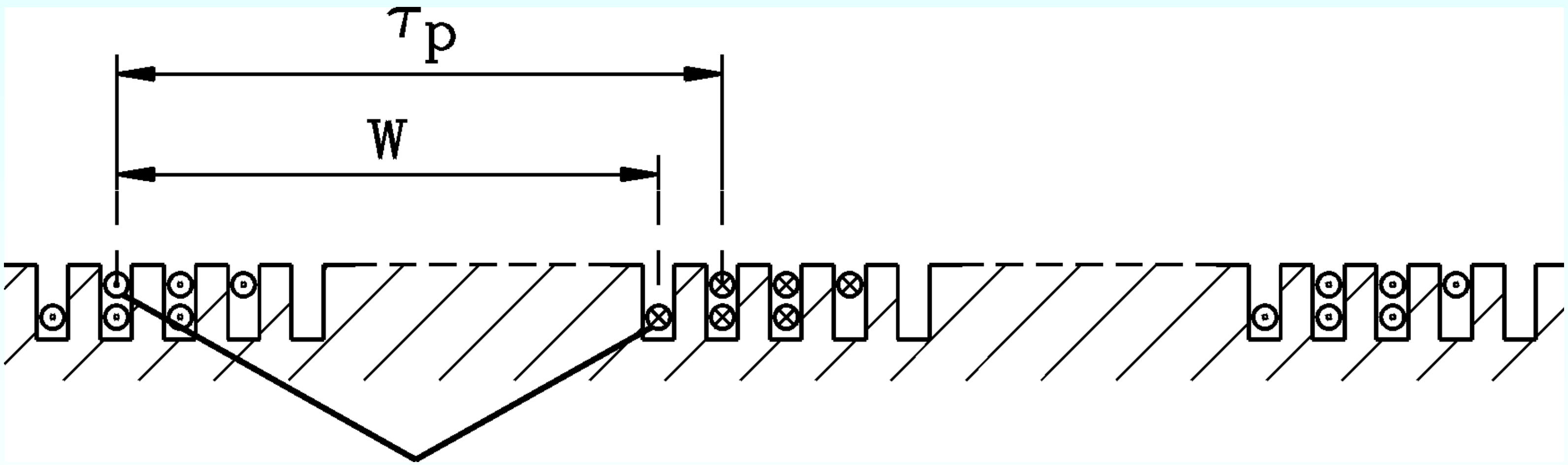
Induction of voltage in pitched coil

- Pitched coil: Coil span is only *W* instead of τ_p :

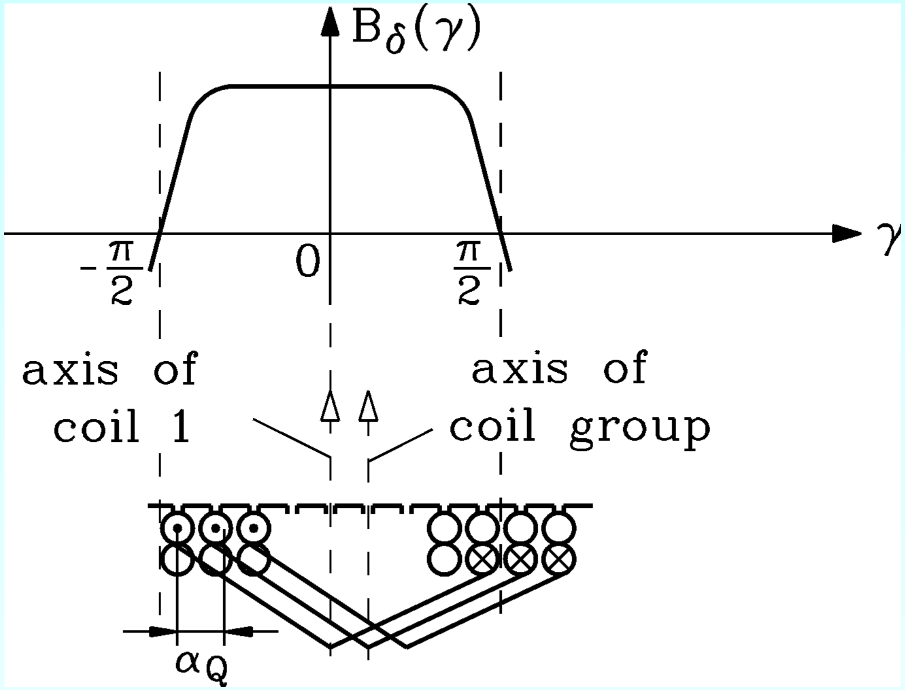
$$\Phi_{c\mu}(t) = l \int_{-W/2}^{W/2} \hat{B}_{\delta\mu} \cos\left(\frac{\mu\pi x}{\tau_p} - \mu\omega t\right) dx = \frac{2}{\pi}\tau_p l \hat{B}_{\delta\mu} \cdot \sin\left(\mu \frac{\pi}{2}\frac{W}{\tau_p}\right) \cdot \cos \omega t$$

Linked coil flux is smaller by **pitch coefficient** $k_{p,\mu}$, compared to full-pitched coil.

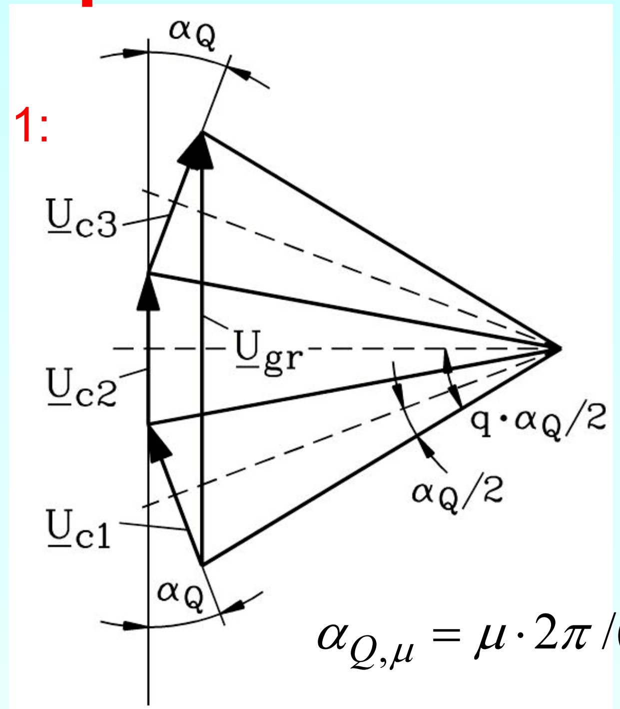
$$k_{p,\mu} = \sin\left(\mu \frac{\pi}{2} \cdot \frac{W}{\tau_p}\right)$$



Induction of voltage in group of coils



$\mu = 1$:



$$\alpha_{Q,\mu} = \mu \cdot 2\pi / (2mq)$$

•The induced sinusoidal AC voltage per coil group is the sum of complex phasors of the q coils. The coil voltage phasors are phase shifted by angle $\alpha_{Q,\mu}$ between adjacent coils:

• **Distribution coefficient:**

$$k_{d,\mu} = \frac{\hat{U}_{i,gr,\mu}}{q\hat{U}_{i,c,\mu}} = \frac{2\sin\left(q\frac{\alpha_{Q,\mu}}{2}\right)}{q \cdot 2\sin\left(\frac{\alpha_{Q,\mu}}{2}\right)} = \frac{\sin\left(\mu\frac{\pi}{2m}\right)}{q \cdot \sin\left(\mu\frac{\pi}{2mq}\right)}$$

Induced voltage per phase

- Machine with $2p$ poles, **two-layer winding: One phase consists of** $2p$ coil groups with q pitched coils per group.
- Induced voltage per phase (r.m.s. value):

Fundamental:

$$U_{i1} = \sqrt{2}\pi f \cdot N \cdot k_{w1} \cdot \frac{2}{\pi} \tau_p l \hat{B}_{\delta 1} \qquad N = 2pqN_c / a \qquad k_{w1} = k_{d1} \cdot k_{p1}$$

μ-th harmonic:

$$U_{i,\mu} = \sqrt{2}\pi \mu f \cdot N \cdot k_{w,\mu} \cdot \frac{2}{\pi} \frac{\tau_p}{\mu} l \hat{B}_{\delta\mu}$$

Example: 12-pole synchronous generator: $n = 500/\text{min}$, $2p = 12$, $f = 50$ Hz

- Stator winding: $N_c = 2$, $q = 2$, $W = 5/6\,\tau_p$, $a = 2$, $\tau_p = 0.5$ m, $l = 1$ m
- Number of turns per phase: $N = 2pqN_c / a = 12 \cdot 2 \cdot 2 / 2 = \underline{\underline{24}}$

μ	$\hat{B}_{\delta\mu}$	$\hat{B}_{\delta\mu} / \hat{B}_{\delta 1}$	f_μ	$\Phi_{c\mu}$	$U_{i,\mu}$	$U_{i,\mu} / U_{i,1}$
-	T	%	Hz	mWb	V	%
1	0.9	**100**	50	276.7	2850.1	**100**
3	0.15	**16.7**	150	-11.3	-254.6	**8.9**
5	0.05	**5.6**	250	0.8	11.4	**0.4**
7	0.05	**5.6**	350	-0.6	-11.4	**0.4**

Facit: *By pitching and by coil group arrangement voltage harmonics are reduced drastically.*

Three phase winding: Self induction leads to magnetizing inductance

- Stator air gap field waves, excited by stator current *I*, induce in stator winding by **self induction the voltage *u_i* !**

$$B_{\delta\nu}(x,t) = \hat{B}_{\delta\nu} \cdot \cos\left(\frac{\nu\pi x}{\tau_p} - \omega t\right) \quad \hat{B}_{\delta\nu} = \frac{\mu_0}{\delta}\frac{\sqrt{2}}{\pi}\frac{m}{p}N\frac{k_{w,\nu}}{\nu}I \quad \nu = 1, -5, 7, -11, 13, \cdots$$

- Stator air gap field waves $B_{\delta\nu}(x,t)$: Speed n_ν is n_{syn}/ν. Hence stator field fundamental and field harmonics induce in stator coils **ALL with the same frequency *f* .**

$$f_\nu = \nu \cdot p \cdot (n_{syn}/\nu) = p \cdot n_{syn} = f$$

- r.m.s. of self-induced voltage per phase for each ν-th field harmonic:

$$U_{i,\nu} = \sqrt{2}\pi f \cdot N \cdot k_{w,\nu} \cdot \frac{2}{\pi}\frac{\tau_p}{\nu} l \hat{B}_{\delta\nu}$$

- **Magnetizing inductance per phase**: $L_{h\nu}$ for ν-th air gap field harmonic wave.

$$U_{i,\nu} = \omega L_{h\nu} I \quad \Rightarrow \quad \boxed{L_{h\nu} = \mu_0 N^2 \frac{k_{w,\nu}^2}{\nu^2}\frac{2m}{\pi^2}\frac{l\tau_p}{p\cdot\delta}}$$

Stray inductance of stator winding per phase

$$L_\sigma = L_{\sigma,Q} + L_{\sigma,b} + L_{\sigma,o}$$

- Air gap field: Fundamental wave = **Magnetizing field (subscript h):** $L_h = L_{h,\nu=1}$
  Magnetizing inductance L_h
- Magnetic field in **slots** (slot stray field) and around the **winding overhang** is NOT linked with rotor winding. It does NOT produce any forces with rotor current. Hence it does NOT contribute to electromechanical energy conversion, and is thus called **stray field (subscript σ).**

- Stray flux induces in stator winding additional voltage by self induction. Hence we define:
  Slot stray inductance $L_{\sigma Q}$, overhang stray inductance $L_{\sigma b}$: $U_{i\sigma,Q+b} = \omega(L_{\sigma Q} + L_{\sigma b})I$

- Air gap field harmonic waves induce stator winding with voltage $U_{i,\nu}$ with the same frequency *f*. So they are summarized as total harmonic voltage : $\sum_{\nu=1,-5,7,...}^{\infty} U_{i,\nu}$

$$L_{h,total} = \frac{\sum_{\nu=1,-5,7,...}^{\infty} U_{i,\nu}}{\omega I} = \sum_{\nu=1,-5,7,...}^{\infty} L_{h\nu} = (1+\sigma_o)L_{h,\nu=1} \Rightarrow \boxed{\sigma_o = \sum_{\nu=1,-5,7,..}^{\infty} \left(\frac{k_{w,\nu}}{\nu \cdot k_{w,1}}\right)^2 - 1}$$

σ_o : **harmonic stray coefficient** (is small: ca. 0.03 ... 0.09).

- Harmonic field waves are linked to rotor, but " disturbe" basic machine function; hence they are summarized in **harmonic stray inductance $L_{\sigma o}$:** $U_{i\sigma,o} = \omega L_{\sigma,o} I, \quad L_{\sigma,o} = \sigma_o L_h$

Active and reactive power in load reference frame

		Active power $P = mUI\cos\varphi$	Reactive power $Q = mUI\sin\varphi$
1.	$-180° \le \varphi < -90°$	$P < 0$, Generator	$Q < 0$, capacitive load
2.	$-90° \le \varphi < 0°$	$P > 0$, Motor	$Q < 0$, capacitive load
3.	$0 \le \varphi < 90°$	$P > 0$, Motor	$Q > 0$, inductive load
4.	$90° \le \varphi < 180°$	$P < 0$, Generator	$Q > 0$, inductive load